1 Probability Theory

A probability is a measure of how frequent or likely an event will take place.

Probability Space The probability space is a triplet space containing a sample/outcome space Ω (containing all possible atomic events), a collection of events S (containing a subset of Ω to which we want to assign probabilities) and the mapping P between Ω and S.

Axioms of Probability The mapping P must fulfill the axioms of probability:

1. $P(a) \leq 0$
2. $P(\Omega) = 1$
3. $a, b \in S$ and $a \cap b = \{\} \Rightarrow P(a \cup b) = P(a) + P(b)$

Random Variable A random variable is a function that maps points from the sample space Ω to some range (e.g. Real numbers or booleans). They are characterized by their distribution function. E.g. for a dice roll:

$$X(\omega) = \begin{cases} 0, & \text{if } \omega = heads \\ 1, & \text{if } \omega = tails. \end{cases}$$

Proposition A Proposition is a conclusion of a statistical inference that can be true or false (e.g. a classification of a datapoint). More formally: A disjunction of events where the logic model holds. An event can be written as a **propositional logic model**:
$A = true, B = false \Rightarrow a \wedge \neg b$. Propositions can be continuous, discrete or boolean.

1.1 Probability distributions

Probability distributions assign probabilities to to all possible points in Ω (e.g. $P(Weather) = \langle 0.3, 0.4, 0.2, 0.1 \rangle$, representing Rain, sunshine, clouds and snow). Joint probability distributions give you a probability for each atomic event of the random variables (e.g. $P(weather, accident)$ gives you a 2×4 matrix.)

Cumulative Distribution Function