Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Homework Assignment H1

Problem 1 (Edge Detection) 12 Points

Let the following 5×5 2-D signal be given:

$$\begin{array}{|c|ccc|c|}
\hline
0 & 0 & 0 & 0 & 0 \\
\cline{2-4}
0 & 0 & 0 & 0 & 0 \\
5 & 5 & 0 & 0 & 0 \\
5 & 5 & 0 & 0 & 0 \\
\cline{2-4}
5 & 5 & 0 & 0 & 0 \\
\hline
\end{array}$$

For the inner pixels (3×3 - square), compute the structure tensor (with $\rho = 0$, i.e. without convolution). To this end, you should compute

a) the spatial derivatives in x-direction:

$$[f_x]_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2}$$
$$[f_y]_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2}$$

b) the structure tensor:

$$J_0 = \begin{pmatrix} [f_x]^2_{i,j} & [f_x]_{i,j} \cdot [f_y]_{i,j} \\ [f_x]_{i,j} \cdot [f_y]_{i,j} & [f_y]^2_{i,j} \end{pmatrix}$$

We assumed grid sizes $h_x = h_y = 1$ here.

c) Decide for each of the nine pixels if it belongs to a flat area, an edge or a corner. Explain your decisions using the eigenvalues of the structure tensor.

d) For the central pixel, perform convolution with the binomial kernel

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

and decide again for the central pixel. Explain your findings.

Let the following 2D-RGB-signal be given:

$$R = \begin{array}{|ccc|} \hline 5 & 5 & 0 \\ 5 & 5 & 0 \\ 5 & 5 & 0 \\ \hline \end{array}\,, \qquad G = \begin{array}{|ccc|} \hline 0 & 0 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & 5 \\ \hline \end{array}\,, \qquad B = \begin{array}{|ccc|} \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline \end{array}\,.$$

In order to detect the edge in the central pixel, compute

e) the norm of the sum of gradients $|\nabla f_R + \nabla f_G + \nabla f_B|$,

f) the joint colour gradient $\left|\left(|\nabla f_R|,\ |\nabla f_G|,\ |\nabla f_B|\right)^\top\right|$,

g) the joint colour structure tensor $\nabla f_R \nabla f_R^\top + \nabla f_G \nabla f_G^\top + \nabla f_B \nabla f_B^\top$.

Which of these expressions are useful for edge detection? Explain your answer.

Problem 2 (Hough Transform for Circles) **12 Points**

Please download the required file `cv13_ex02.tgz` from the lecture webpage. To unpack the data, use `tar xvfz cv13_ex02.tgz` .

(a) In the file `gradient_map.c`, supplement the routine `gradient_magnitude` with the missing code such that it computes the gradient magnitude via a finite difference approximation. Compile the program with

```
gcc -O3 -o gradient_map gradient_map.c -lm.
```

(b) Try to find appropriate parameters for the standard deviation σ of the Gaussian pre-smoothing kernel and the threshold of the gradient magnitude T_{Edge}. The program will write out the computed edge map.

(c) In the file `hough_transform.c`, supplement the routine `vote_hough` with the missing code such that it implements the voting step of the Hough transform for circles. To this end, you should use the provided routine

```
vote_circle (image, c_list, r_max, r_min, nx, ny, x, y, r) ,
```

that draws a circle with center (x, y) and radius r in the 2D-array "image". Compile the program with

```
gcc -O3 -o hough_transform hough_transform.c -lm.
```

(d) The program will read in the edge map from (b). Adjust the remaining parameters: $r_{\min}, r_{\max}$ for the minimum and maximum radius, respectively, and the thresholding parameter T_{Hough} in the hough space (given as percentage of points on the circle, $0 < T_{\text{Hough}} < 1$) such that all coins are detected.

Submission

Please remember that up to three people can work and submit their solutions together. The theoretical problems have to be submitted in handwritten form before the tutorial. For the programming problem you have to submit the files as follows: After solving the problems, rename the main directory `cv13_ex02` to `cv13_ex02_<your_name>` and use the command

```
tar cvfz cv13_ex02_<your_name>.tgz cv13_ex02_<your_name>
```

to pack the data. The directory that you pack should contain the following files:

- the source files with supplemented code for part (a) and (c)
- the (final) output images for part (b) and (d)
- a text file `readme.txt` that contains
  - information on all people working together for this assignment
  - answers to the questions (e.g. selected parameters)
  - additional comments, if necessary (up to you)

Please make sure that only the final version of the code files and the images are included. Submit the archive via e-mail to your tutor via the address `volz@vis.uni-stuttgart.de` .

Deadline for Submission is Tuesday, November 7th, 2:00 pm (before the tutorial)

Guidelines for the Tutorials

- **Theoretical Homework Assignments** have to be submitted **at the beginning** of the next tutorial (this time November 7th, 2 p.m.). They will be corrected and given back to you in the tutorial after the next.
- **Programming Assignments** have to be submitted **before** the next tutorial. Make sure all relevant results are contained! Submit your solutions via email to `volz@vis.uni-stuttgart.de`. Corrections come with the theoretical assignments.
- **Classroom Assignments** are intended to be solved **within** the next tutorial. They don't have to be submitted, but your tutor will help you to complete them correctly.
- In order to gain **admission to the exam**, you have to achieve 50% of the total points from theoretical homework and programming assignments by the end of the semester.
- **Regular attendance** of the tutorials is not mandatory, but highly recommended as preparation for the exam.

Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Classroom Assignment C2

Problem 1 (Derivative Approximation)

A sufficiently often continuously differentiable function $f(x)$ is sampled with pixel distance h, resulting in a discrete signal (f_i). The goal is to approximate the second derivative $f''(x)$ in pixel i using the four values $f_{i-3}, f_{i-2}, f_{i-1}, f_i$.

1. Deduce the corresponding system of equations which determines the coefficients of the approximation.
2. Determine the order of consistency of the approximation

$$f_i'' = \frac{-f_{i-3} + 4f_{i-2} - 5f_{i-1} + 2f_i}{h^2}$$

   for the second derivative $f''(x)$ in pixel i.