Exercise 4.2

To interpolate the value at Point P inside the Prism, I first interpolate two points p_1, p_2 in the triangles ABC and DEF using the Barycentric Interpolation. With the two new Points I can then do a Linear Interpolation to find the value at Point P.

p_1:

$$p_1 = (1,0,1)$$

$$f(p_1) = \alpha_1 \, f(A) + \alpha_2 \, f(B) + \alpha_3 \, f(C)$$

$$\alpha_1 = \frac{VOL(p_1, B, C)}{VOL(A, B, C)} = \frac{\det \begin{pmatrix} 1 & 3 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 4 \end{pmatrix}}{\det \begin{pmatrix} 0 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix}} = \frac{5}{12}$$

$$\alpha_2 = \frac{VOL(A, p_1, C)}{VOL(A, B, C)} = \frac{\det \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 4 \end{pmatrix}}{-12} = \frac{1}{3}$$

$$\alpha_3 = \frac{VOL(A, B, p_1)}{VOL(A, B, C)} = \frac{\det \begin{pmatrix} 0 & 3 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}{-12} = \frac{1}{4}$$

$$f(p_1) = \frac{5}{12} * 3 - \frac{1}{3} * 6,75 + \frac{1}{4} * 0 = -1$$

p_2:

$$p_2 = (1,7,1)$$

$$f(p_2) = \alpha_1 \, f(D) + \alpha_2 \, f(E) + \alpha_3 \, f(F)$$

$$f(p_2) = \frac{5}{12} * 18 + \frac{1}{3} * 12 + \frac{1}{4} * 6 = 13$$

P:

$$P = (1,2,1)$$

$$f(P) = (1 - u) * f(p_1) + u * f(p_2), \qquad where \; u = \frac{P_y - p_{1y}}{p_{2y} - p_{1y}} = \frac{2}{7}$$

$$f(P) = \frac{5}{7} * (-1) + \frac{2}{7} * 13 = 3$$