Matrikelnr.:

Name:

Discrete Optimization

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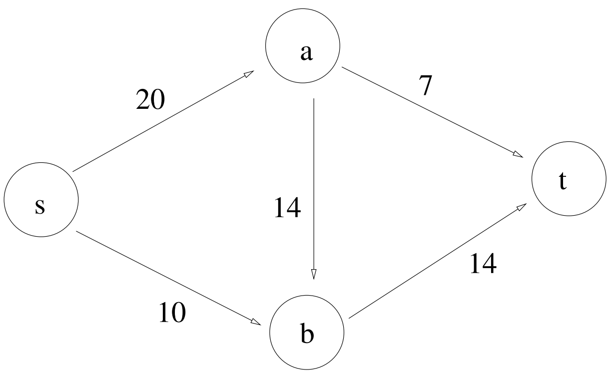
Institut für Formale
Methoden der
Informatik
Abt. Algorithmik
Universität Stuttgart

Exercise Sheet 1

Problem 1

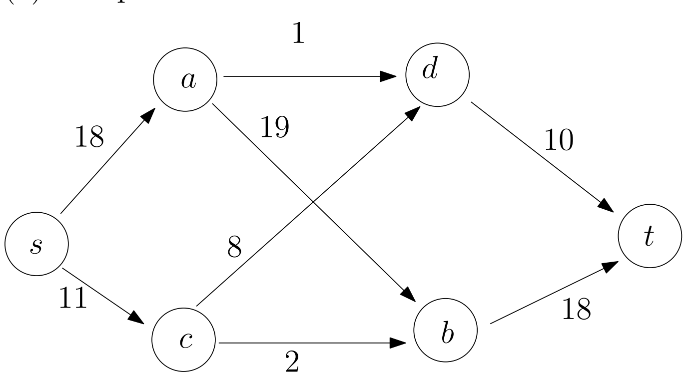
Compute the maximum s-t flow in the following network using the Ford-Fulkerson algorithm:



Assume that in each round the s-t path with maximum bottleneck value is used for augmentation; write explicitly the current flow f as well as the residual network after each augmentation.

Problem 2

(a) Compute the maximum s-t flow in the following network using the Ford-Fulkerson algorithm:



Assume that in each round the s-t path with maximum bottleneck value is used for augmentation; write explicitly the current flow f as well as the residual network after each augmentation.

(b) Provide the certificate of optimality that is implicitly given by the execution of FF.

(c) Is there a always a sequence of augmenting paths that does not use any virtual back edges and still yields a maximum flow? Prove or disprove (the latter you can do via an example).

Problem 3

Computing network flows has various applications which seem completely unrelated to actual 'flow' problems. The following are two examples:

(a) Consider a set S of n students and a set C of n college places (places to study). Goal is to assign students to college places bijectively but taking into account study preferences (e.g. not every University offers the subjects a particular student is interested in). The study preferences are given as a set P of pairs (s_i, c_j), where the latter denotes that student $s_i \in S$ can study at college $c_j \in C$. Clearly, if all possible pairs are in P, there is a feasible assignment from students to colleges. But what if this is not the case?

Devise an algorithm (based on a maxFlow computation) which decides whether a bijective assignment of students to colleges exists which satisfies the study preferences. Also explain how to extract the respective assignment.

(b) Use a flow formulation to determine the maximum number of *node-disjoint* paths between two nodes s and t in a directed network.

Problem 4 In class we considered the case that every edge had a finite integral capacity. In particular, the proof of termination of FF was making use of that fact. Describe how one could handle networks where some of the edges might have capacity ∞.

Problem 5

In class there was the question of ambiguity of the maximum flow and the certificate of optimality for the final flow computed by FF. The following questions address this issue:

(a) Prove or disprove (the latter can be done via an example):

> The maximum flow of a network is always unique (note that we are talking about the flow as an assignment of flow values to the edges, not about the total *flow value*).

(b) Prove or disprove (the latter can be done via an example):

> There is exactly one directed cut whose summed capacities match the maximum flow value.

(c) Prove or disprove (the latter can be done via an example):

> Let A be the set of nodes reachable from s in the last residual network before termination of FF. Then there is no proper subset $A' \subsetneq A$ with $s \in A'$ which induces a directed cut with summed capacity matching the maximum flow value.

(d) Prove or disprove (the latter can be done via an example):

> The certificate of optimality produced by FF is unique — irrespectively of the sequence of augmenting paths chosen during the execution of FF.