

Problem 1:

a)

$$\begin{array}{c|c} s & \\ \hline & i \end{array}$$

$$\frac{1}{9} \times$$

1	1	1	0	0
2	0	0	1	1
0	1	1	0	2
0	0	0	1	3
0	1	2	3	

i

s

Mode / largest probability:  $(1, 0) \rightarrow p_{1,0} = \frac{2}{9}$

$$\begin{aligned} \text{Contrast} &= (0-0)^2 \frac{1}{9} + (0-1)^2 \frac{1}{9} + (0-2)^2 \frac{1}{9} + (1-0)^2 \frac{2}{9} + \\ &\quad (1-3)^2 \frac{1}{9} + (2-1)^2 \frac{1}{9} + (2-2)^2 \frac{1}{9} + (3-3)^2 \frac{1}{9} \\ &= \frac{10}{9} \end{aligned}$$

b) Yes, this would yield the same highest probability and the same contrast, as the combination of neighbouring pixels that are compared stay the same.

Problem 2:

$$a_{11} \cdot u + a_{12} \cdot v = b_1$$

$$a_{12} \cdot u + a_{22} \cdot v = b_2$$

$$\Rightarrow u = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}} = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12}^2} = \frac{\int f_x f_y dx dy \int f_y f_z dx dy - \int f_x f_z dx dy \int f_y^2 dx dy}{\left[ \int f_x f_y dx dy \right]^2}$$

$$\Rightarrow V = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{a_{11}b_2 - b_1a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$= \frac{\int f_1 f_2 dx dy \int f_1^2 dx dy - \int f_1 f_2 dx dy \int f_2^2 dx dy}{\int f_1^2 dx dy \int f_2^2 dx dy - \left[ \int f_1 f_2 dx dy \right]^2}$$