8367

MC-Test: Discrete Optimization WS 2013/14 (fake)

24.12.2013

Matr.no.

Name

Hinweise:

- **Immediately** complete the first page with your name and matriculation number
- Keep the exam sheets closed until told by an instructor to begin with the exam
- Check for completeness of the exam sheets (XX pages, XX questions)
- Make sure that in the top left corner all sheets bear the same number. If not, please ask a supervisor.
- Carefully read each question before answering
- Any attempt of cheating leads to immediate failing of the exam
- All questions assume notation and methodology as taught in the lecture.
- This is **not** an openbook exam! You are only allowed to use pen, pencil, ruler, and eraser. No calculator of any kind is allowed.
- **No mobile phones! Making use of a mobile phone is interpreted as cheating!**
- Tick exactly *one* answer.
- No tick counts as a wrong answer.

Example

The following questions are about (Integer) Linear Programming as covered in class.

Question 1

Which of the following linear programs is in standard form according to our definition.

- a ☐ max $c^T x$ s.t. $Ax \leq b$
- b ☐ min $c^T x$ s.t. $Ax \geq b$
- c ☐ max $c^T x$ s.t. $Ax \leq b, x \geq 0$
- d ☐ min $c^T x$ s.t. $Ax = b$

Question 2

Which of the following statements is wrong?

- a ☐ For most pivoting rules, problem instances are known which make the (primal) Simplex algorithm take an exponential time.
- b ☐ It is not known whether the (dual) Simplex algorithm always terminates in polynomial time.
- c ☐ For many pivoting rules, degeneracies might lead to cycling of the (primal) Simplex algorithm.
- d ☐ The (primal) Simplex algorithm can be proven to terminate in polynomial time.

Question 3

In the context of primal and dual linear programs, which of the following statements is true?

- a ☐ The primal simplex algorithm starts from a corner of the primal feasible region and jumps to a 'better' neighboring corner until no neighboring corner is 'better'.
- b ☐ The primal simplex algorithm starts with an arbitrary (possibly non-feasible) corner and walks towards the optimum vertex.
- c ☐ The dual simplex walks along the boundary of the primal feasible region.
- d ☐ The dual simplex algorithm starts from a corner of the primal feasible region and jumps to a 'better' neighboring corner until no neighboring corner is 'better'.

Question 4

Consider non-degenerate primal linear programs in $\mathbb{R}^2$. Which of the following statements is true?

- a ☐ The primal simplex algorithm performs $O(1)$ steps before reaching the optimum solution.
- b ☐ There are problem instances which require the primal simplex algorithm to visit $\Omega(2^n)$ vertices
- c ☐ Starting from a feasible corner, the primal simplex algorithm performs at most $O(n)$ steps before reaching the optimum vertex.
- d ☐ There are problem instances which require the primal simplex algorithm to visit $\Omega(n^2)$ vertices

Question 5

What is the dual of the primal linear program max $c^T x$ s.t. $Ax \leq b$

- a ☐ min $b^T y$ s.t. $A^T y \geq c$
- b ☐ min $b^T y$ s.t. $A^T y = c$
- c ☐ min $b^T y$ s.t. $A^T y = c,\ y \geq 0$
- d ☐ min $b^T y$ s.t. $A^T y \leq c$

Question 6
Which of the following statements is wrong?

- *a* ☐ The dual simplex algorithm starts with a vertex which is optimal for the constraints defining that vertex.
- *b* ☐ The number of steps the dual simplex performs is always equal to the number of steps the primal simplex algorithm performs on the same problem instance.
- *c* ☐ The objective function value of the dual linear program is equal to the optimum value of the primal linear program.
- *d* ☐ Once the dual simplex reaches a corner which is feasible for all constraints, the optimum has been reached.

Question 7
In the context of primal and dual linear programs, which of the following statements is true?

- *a* ☐ If the primal linear program has arbitrarily large objective function value, the dual linear program cannot be feasible.
- *b* ☐ If the primal linear program is feasible, the dual linear program must be feasible.
- *c* ☐ If the primal linear program is infeasible, also the dual linear program is infeasible.
- *d* ☐ If the primal linear program has arbitrarily large objective function value, the dual linear program must be feasible.

Question 8
Consider a linear program max $c^T x$, $Ax \leq b$ and its dual. Let v_P^{frac} be the objective function value of the optimum fractional solution to the primal LP, v_P^{int} the objective function value of the optimum *integral* solution to the primal LP; accordingly define v_D^{frac} and v_D^{int}. Which of the following relation holds?

- *a* ☐ $v_P^{int} \leq v_P^{frac} \lneq v_D^{frac} \leq v_D^{int}$
- *b* ☐ $v_P^{int} = v_P^{frac} = v_D^{frac} = v_D^{int}$
- *c* ☐ $v_P^{int} \lneq v_P^{frac} = v_D^{frac} \lneq v_D^{int}$
- *d* ☐ $v_P^{int} \leq v_P^{frac} = v_D^{frac} \leq v_D^{int}$

Question 9
We consider linear programming instances in dimension n with m constraints, $m \gg n$. Which of the following statements is true?

- *a* ☐ There are LP instances where the polytope characterizing the feasible region has about $\binom{m}{n}$ vertices.
- *b* ☐ The polytope characterizing the feasible region has at most $n^{1000} m^{9999}$ vertices.
- *c* ☐ There are LP instances where the polytope characterizing the feasible region has about $\binom{n}{m}$ vertices.
- *d* ☐ The polytope characterizing the feasible region has to be empty as $m \gg n$.

Example
The following questions are about network flow problems as covered in class.

Question 10

Assume we are given a *maxFlow* problem instance. Which of the following statements is true?

a ☐ For any feasible flow f of value $v(f)$ there exists a cut of capacity $v(f)$.

b ☐ There are maxFlow problem instances where the capacity of the minimum cut is less than the optimum flow value.

c ☐ A feasible flow f of value $v(f)$ is maximum iff there exists a cut of capacity $v(f)$.

d ☐ There are maxFlow problem instances where the capacity of the minimum cut is more than the optimum flow value.

Question 11

How does the successive shortest path minCostFlow algorithm proceed?

a ☐ In each round, it searches in the residual network for the path which equalizes most surplus with demand.

b ☐ In each round, it searches in the residual network for the cheapest path connecting a surplus node to a demand node, and then sending as much flow as possible across that path.

c ☐ Flow is sent around a cycle of minimum cost.

d ☐ In each round, the path of maximum bottleneck value is used for augmentation.

Question 12

For the cycle cancelling minCostFlow algorithm, what is the idea of proving that for any non-optimal flow, a negative-cost cycle exists in the residual network?

a ☐ We consider the nodes reachable from the surplus nodes and their induced cut.

b ☐ We consider the flow difference $f - f^*$ where f^* is an optimal minCost flow and f the current flow.

c ☐ We consider the nodes reachable from the demand nodes and their induced cut.

d ☐ We consider the flow difference $f^* - f$ where f^* is an optimal minCost flow and f the current flow.

Question 13

Which of the following arguments suffices to prove optimality of the flow found by Ford-Fulkerson after termination?

a ☐ Since there is no more augmenting path in the residual network of the last flow, it has to be optimal.

b ☐ Optimality follows from the fact that $NP \neq P$.

c ☐ When the algorithm terminates, the cut induced by the set of nodes reachable from the source/sink s has capacity equal to the flow value after termination. Since the former is an upper bound on the flow value, optimality follows.

d ☐ Any algorithm which terminates produces an optimal result.

Question 14
Assume we are given a *maxFlow* problem instance. Which of the following statements is wrong?

- *a* ☐ Capacity Scaling guarantees that the Ford-Fulkerson Algorithm terminates in polynomial time.
- *b* ☐ In the Ford-Fulkerson-Algorithm, any rule for choosing an augmenting path amongst the possible augmenting paths guarantees termination of FF.
- *c* ☐ Capacity Scaling guarantees that the Ford-Fulkerson Algorithm terminates in polynomial time, independent of the size of the numbers (capacities) appearing the the problem instance.
- *d* ☐ The Shortest Augmenting Path (Edmonds-Karp-Algorithm) strategy guarantees that the Ford-Fulkerson Algorithm terminates in polynomial time.

Question 15
Which of the following arguments suffices to prove termination of Ford-Fulkerson for a network where all edges have finite capacity?

- *a* ☐ In each iteration the flow increases by at least one. Since the value of the flow is bounded by the sum of the capacities of the edges leaving the source/sink s, FF has to terminate.
- *b* ☐ Termination follow from the fact that $NP \neq P$.
- *c* ☐ In each iteration the flow increases by at least one, hence FF terminates.
- *d* ☐ Any algorithm terminates in finite time.

Question 16
Assume we are given a *maxFlow* problem instance. Which of the following statements is wrong?

- *a* ☐ Capacity Scaling is a refinement of the Ford-Fulkerson algorithm.
- *b* ☐ There are maxFlow instances where no non-zero flow exists.
- *c* ☐ The Ford-Fulkerson algorithm can compute finite time the maximum flow for this problem instance.
- *d* ☐ There is always a non-zero flow in the network.

Question 17
Assume we are given a *minCostFlow* problem instance. Which of the following statements is wrong?

- *a* ☐ There are minCostFlow problem instances where feasible non-zero flows exist but no feasible flow which satisfies all demands and supplies.
- *b* ☐ There might be several optimal flows for a minCostFlow problem instance.
- *c* ☐ There is alwas a feasible flow satisfying all demands and supplies.
- *d* ☐ Some minCostFlow instances might have no feasible non-zero flow at all.

Question 18
Assume we are given a *minCostFlow* problem instance. Which of the following statements is true?

- *a* ☐ All feasible flows are optimal.
- *b* ☐ A flow satisfying all demands and surpluses where its residual network is free of negative-cost cycles is optimal.
- *c* ☐ Feasible flows have negative cost.
- *d* ☐ Any feasible flow satisfying all demands and surpluses is optimal.