Machine Learning - Exercise 7

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Assignment 3

We have to show that $P(X,Y|Z) = P(Y|X,Z)P(X|Z)$. We start of on the left side, and transform it into the right side by using appropriate probability transformations.

$$
\begin{aligned}
P(X,Y|Z) &= \frac{P(X,Y,Z)}{P(Z)} \\
&= \frac{P(Y|X,Z)P(X,Z)}{P(Z)}
\end{aligned}
$$

because $P(X,Y,Z) = P(Y|X,Z)P(X,Z)$. And since $\frac{P(X,Z)}{P(Z)} = P(X|Z)$ we can transform the equation into

$$
\frac{P(Y|X,Z)P(X,Z)}{P(Z)} = P(Y|X,Z)P(X|Z)
$$

which is the right side.

Also for the conditionalized version of the Bayes rule, we start at the left side, and transform it into the right side of $P(X|Y,Z) = \frac{P(Y|X,Z)P(X|Z)}{P(Y|Z)}$.

$$
\begin{aligned}
P(X|Y,Z) &= \frac{P(X,Y,Z)}{P(Y,Z)} \\
&= \frac{P(Y|X,Y)P(X,Z)}{P(Y,Z)} \\
&= P(Y|X,Z)\frac{P(X,Z)}{P(Y,Z)} \\
&= P(Y|X,Z)\frac{P(X|Z)P(Z)}{P(Z)P(Y|Z)} \\
&= \frac{P(Y|X,Z)P(X|Z)}{P(Y|Z)}
\end{aligned}
$$