Assignment BIO 364 The Physics of Life

Universität Zürich
Institut für Evolutionsbiologie und Umweltwissenschaften
FS 2021

Name: Moritz Kaufmann
E-Mail: morkau@uzh.ch
Matr.Nr.: 14-667-273

Zürich, 08.06.2021

Inhalt

1. Introduction

In the year 1960 E. Lorenz programmed his simple Royal McBee computer to simulate the weather. With his primitive computer Lorenz had reduced weather to only twelve deterministic equations. Nevertheless, the printouts of his created system seemed to behave in a recognizable way. To visualize the patterns Lorenz not only printed out the different numbers but rather created some simple plots.

One day in 1961 he wanted to rerun a previous simulation over a longer time-period. Instead of running the whole simulation again he started from the middle section by just typing in, the numbers of the previous simulation. When he looked at the printout of the rerun simulation, he saw something astonishing, something that planted the seed for a new science. The new run should have exactly mimicked the old one but instead the two simulations started to diverge rapidly from each other (Figure 1). First, he thought the machine has failed again but then he realized the true reason. The problem was that he used the rounded numbers from the first simulation for the second simulation. Instead of 0.506127 he used 0.506. This tiny difference lead to a completely different behavior of the systems in a short amount of time. Lorenz later explained this phenomenon, which makes long time weather predictions impossible, as the butterfly effect [1].

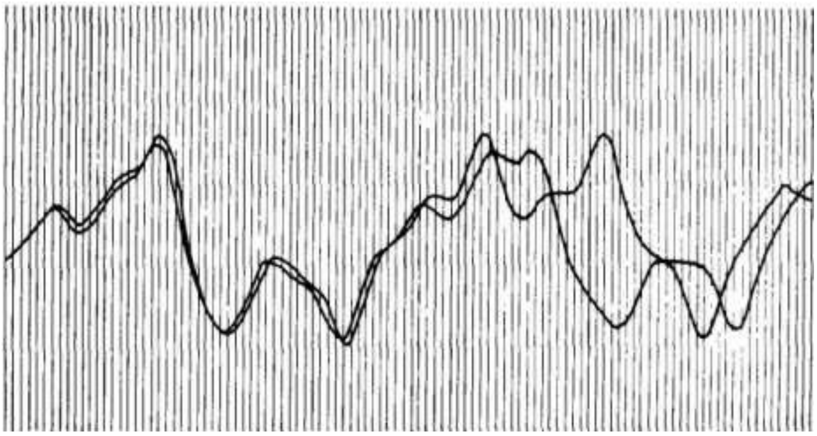

Figure 1: Divergence of weather patterns: The graph shows the two simulations run by Lorenz in 1961. Both graphs seem to start at the same starting point but diverge over time until they behave completely different (Lorenz printout 1961).

2. Deterministic Chaos

The word chaos is derived from the Greek word χάος. It originally referred to the infinite empty space which existed before all things. Nowadays it denotes a state of disorder and irregularity. Physical systems are called deterministic if an equation exists to calculate their future behavior from given initial conditions. In the following, deterministic chaos is referred to as the irregular or chaotic motion generated by nonlinear systems whose dynamical laws uniquely determine the temporal evolution of a state of the system from knowledge of its prior history. In recent years it has become clear that this behavior is abundant in nature and has consequences for many branches of science [2].

There is no universally accepted mathematical definition of chaos but commonly used definitions were originally formulated by R. Devaney [3]. The definitions are as follows:

1. A chaotic system must be sensitive to initial conditions
2. A chaotic system must be topologically transitive
3. A chaotic system must have dense periodic orbits

Sensitivity to initial conditions means that any point in a chaotic system is arbitrarily closely approximated by other points that have significantly different future paths or trajectories. Thus, an arbitrarily small change or perturbation in the current trajectory can lead to significantly different future behavior [4]. Small differences in initial conditions, e.g. due to measurement errors or rounding errors in numerical calculations, can lead to strongly diverging results for such dynamical systems, which generally makes a long-term prediction of their behavior impossible. This can happen even though the future behavior of this system is completely determined by the initial condition and no random elements are involved [5].

How long the behavior of a chaotic system can be effectively predicted depends on three things: how much uncertainty in the prediction can be tolerated, how accurately the current state can be measured, and a time scale that depends on the dynamics of the system called Lyapunov time [6]. In chaotic systems, the uncertainty of a prediction increases exponentially with elapsed time. Therefore, mathematically, the proportional uncertainty in the prediction is more than squared by doubling the prediction time. Consequently, in practice, there is no meaningful prediction over an interval of more than two or three times the Lyapunov time. If no meaningful predictions can be made, the system appears random [7].

3. Lyapunov exponent

It is necessary that noise and vibration signals first are tested for the presence of chaos. Chaos means exponential sensitivity to initial conditions and therefore by definition occurs when there is a positive Lyapunov exponent [8]. Lyapunov exponents are a measure of the predictability and sensitivity of a system to changes in its initial conditions [9]. The Lyapunov exponents can be thought of as the average logarithmic separation or convergence rate of two closely spaced points of two time series X_t and Y_t separated by an initial distance:

$$\Delta R_0 = \|X_0 - Y_0\|_2$$

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} ln \left| \frac{\Delta R_i}{\Delta R_0} \right|$$

In a chaotic system the Lyapunov exponent λ must be positive [10].

4. Logistic map

The logistic map is often cited as a classical example of how complex, chaotic behavior can arise from very simple nonlinear dynamical equations. The map was popularized in a paper by biologist R. May in 1976 [11]. The equation of the logistic map is:

$$x_{n+1} = r x_n (1 - x_n)$$

χ_n represents the ratio of existing population to the maximum possible population and is a number between one and zero. The values for the growth parameter r, can range from zero to 4. The relative simplicity of the logistic map makes it a widely used entry point for the concept of chaos.

5. Material and methods

The dashboard was programmed using R 4.0.3 [12] and R-Studio 1.3.959 [13]. To build the interactive dashboard the R-library shiny 1.6.0 [14] was used. Further packages needed for the application were ggplot2 [15], data.table [16], dplyr [17], tidyr [18] and Rfast [19].To calculate the logistic map and to visualize the cobweb plot, the functions of N. Radziwill were adapted [20]. The used script and the supplemental material were uploaded to https://github.com/MoritzKaufmann/BIO364_assignment.

6. Results

The goal of this assignment was to create a dashboard that visualizes the relationship between bifurcation, the Lyapunov exponent and the cobweb plot of the logistic map. The first panel of the dashboard the bifurcation diagram is integrated as static picture. An interactive plot has not been created because it uses to much computation time to be generated. In the next panel the Lyapunov exponent for the logistic map is shown. In the Lyapunov exponent plot, a region can be selected and by double clicking on the selected region it will be enlarged. In this plot one can explore interesting regions of different grow rates. When double clicking on the plot with no selection made, the plot will enlarge to the original size. The next plot shows the cobweb plot of the underlying logistic map. The growth rate can be adjusted via the slider and the plot will be updated automatically. The last plot finally shows the logistic map and its behavior under different growth rates. Again, the growth rates can be adjusted via the provided slider.

To run the dashboard, the provided script run_dashboard.R can be executed in R-Studio. It will automatically install all needed packages if they have not been installed previously. After the possible installation of the packages the script will launch the dashboard from the Github repository https://github.com/MoritzKaufmann/BIO364_assignment. To get the best layout the dashboard should be viewed in full screen mode. The source code and the used graph for the bifurcation diagram can also be found in the corresponding repository.

7. References

1. James G (1988) Chaos the amazing since of the unpredictable. Vintage
2. Schuster HG, Just W (2006) Deterministic Chaos. Wiley-VCH, Hoboken
3. Devaney R (1989) An Introduction to Chaotic Dynamical Systems, 2nd ed. Studies in Nonlinearity. Chapman and Hall/CRC, Boulder
4. Fractal Foundation What is chaos theory? https://fractalfoundation.org/resources/what-is-chaos-theory/. Accessed 03.06.21
5. Kellert SH (1994) In the wake of chaos: Unpredictable order in dynamical systems, Paperback ed. Science and its conceptual foundations series. Univ. of Chicago Press, Chicago
6. Sussman GJ, Wisdom J (1992) Chaotic evolution of the solar system. Science 257:56–62. https://doi.org/10.1126/science.257.5066.56
7. Goldenfeld N (2004) Sync: The Emerging Science of Spontaneous Order Sync: The Emerging Science of Spontaneous Order Steven Strogatz Hyperion, New York, 2003. $24.95, (338 pp.). ISBN 0-7868-6844-9. Physics Today 57:59–60. https://doi.org/10.1063/1.1784276
8. Gan WS (1996) Techniques in the Application of Chaos Theory in Signal and Image Processing. In: Multidimensional Systems Signal Processing Algorithms and Application Techniques, vol 77. Elsevier, pp 339–387
9. Sprott JC (2003) Chaos and time-series analysis
10. van der Stappen ML (1998) Chaotic hydrodynamics of fluidized beds
11. May RM (1976) Simple mathematical models with very complicated dynamics. Nature 261:459–467. https://doi.org/10.1038/261459a0
12. R Core Team (2020) R: A Language and Environment for Statistical Computing. https://www.R-project.org/
13. RStudio Team (2020) RStudio: Integrated Development Environment for R. http://www.rstudio.com/
14. Chang W, Cheng J, Allaire JJ et al. (2021) shiny: Web Application Framework for R. https://CRAN.R-project.org/package=shiny
15. Wickham H (2016) ggplot2: Elegant Graphics for Data Analysis. Springer-Verlag New York
16. Dowle M, Srinivasan A (2021) data.table: Extension of ‘data.frame‘. https://CRAN.R-project.org/package=data.table
17. Wickham H, François R, Henry L et al. (2021) dplyr: A Grammar of Data Manipulation. https://CRAN.R-project.org/package=dplyr
18. Hadley Wickham (2021) tidyr: Tidy Messy Data. https://CRAN.R-project.org/package=tidyr
19. Manos Papadakis, Michail Tsagris, Marios Dimitriadis et al. (2021) Rfast: A Collection of Efficient and Extremely Fast R Functions. https://CRAN.R-project.org/package=Rfast
20. Radziwill N (2019) Logistic Growth, S-Curves, Bifurcations, & Lyapunov Exponents in R. https://qualityandinnovation.com/wp-content/uploads/2019/09/logistic-growth.html. Accessed 08 Jun 2021