

# A predictive coding model of transitive inference

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## Introduction

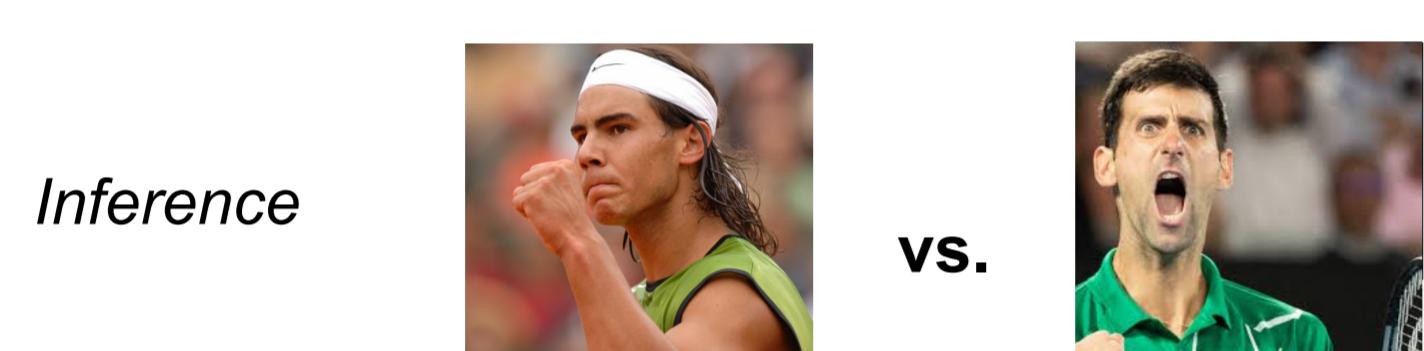
**GOAL** To develop a model that tells us how transitive inference works. Which strategy is used? How is that strategy implemented?

### What is transitive inference?

Try it yourself! Consider these facts:



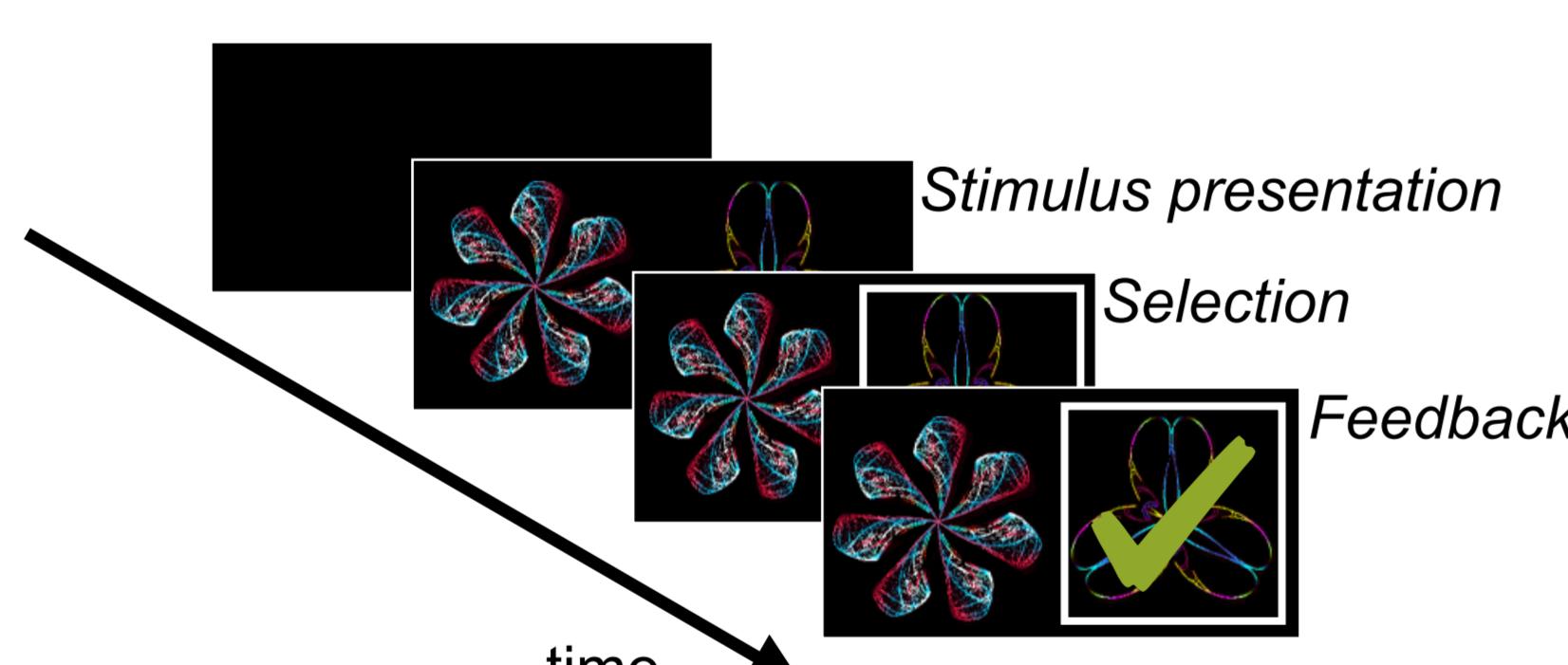
Now make a prediction:



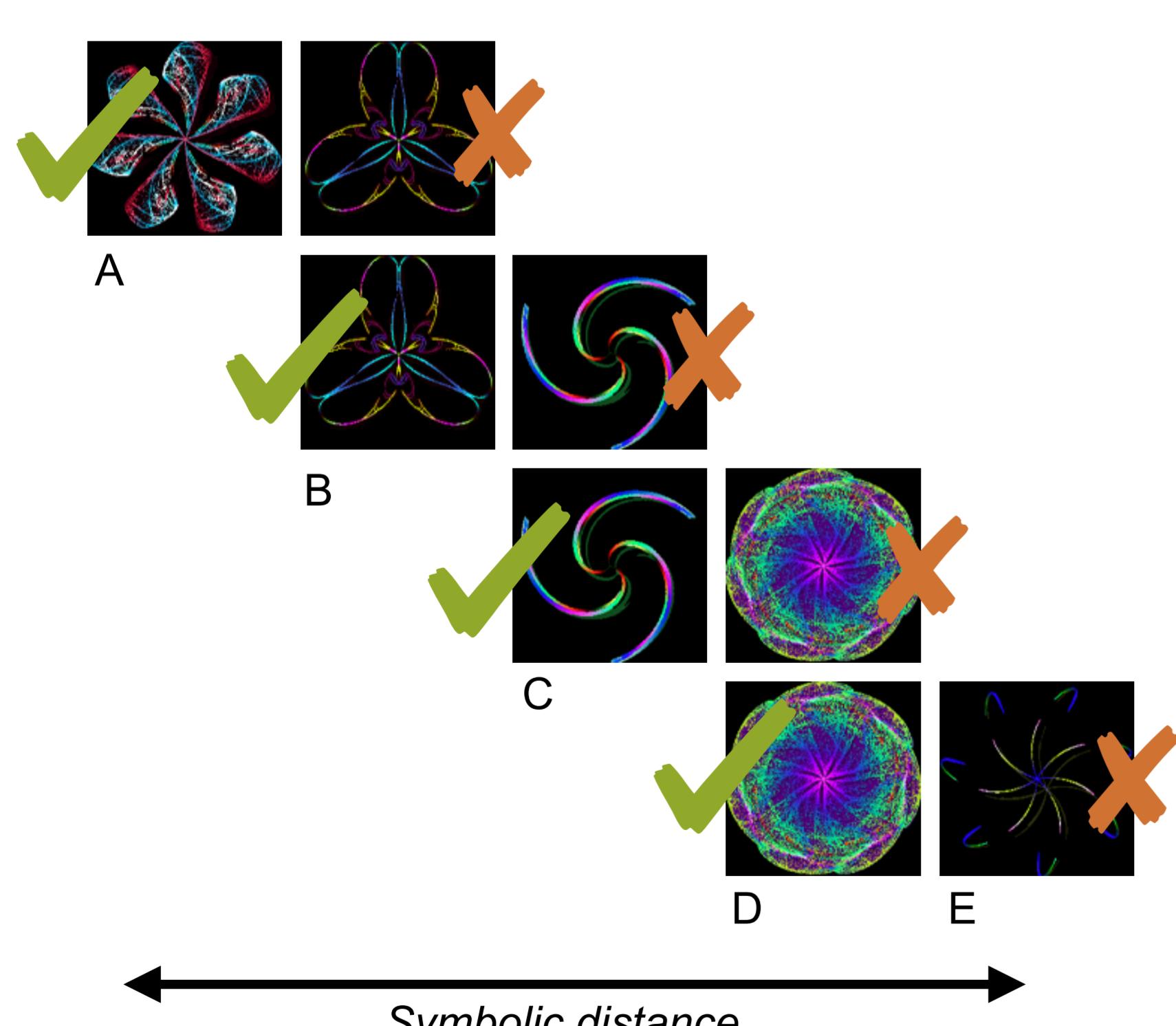
## N-Series Task

A task to measure the ability to do TI in animals and infants

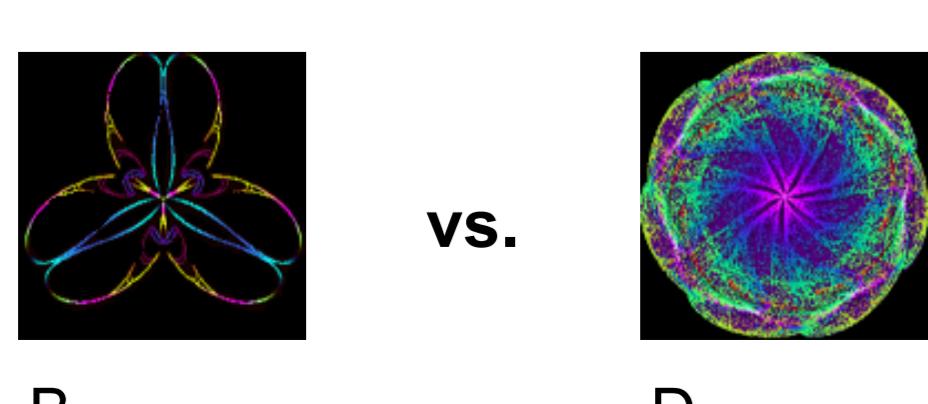
Trial structure



Reinforcement structure during training:



The test:



## A Bayesian Model

### A linear model of reasoning

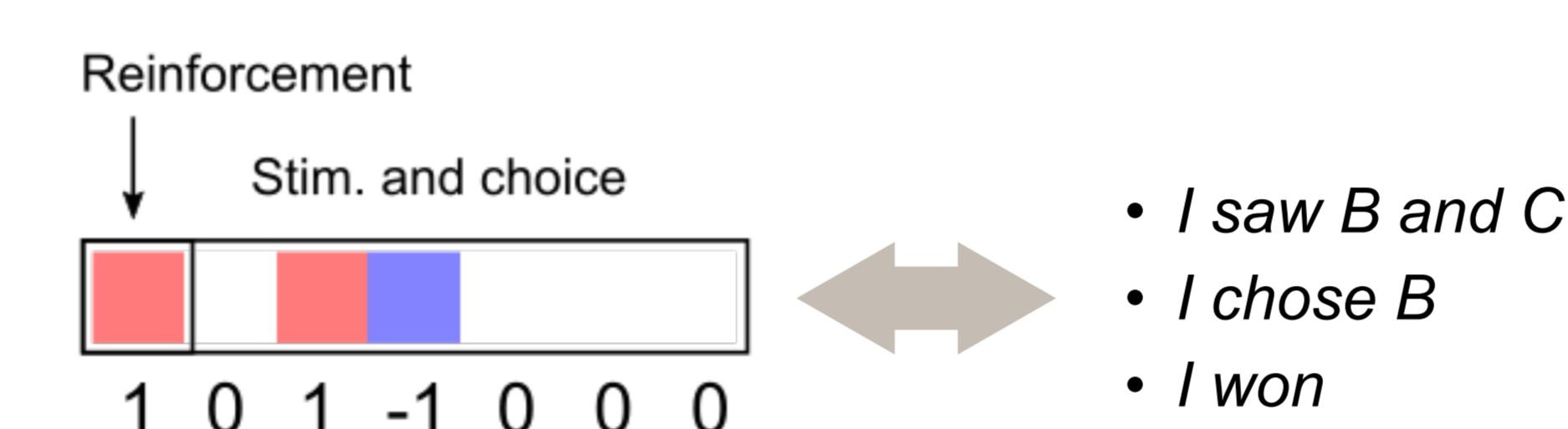
$$\text{FACT} = \text{PREMISE1} + \text{PREMISE2}$$

Vector space of facts

$$\vec{y} = x_1 \vec{w}_1 + \dots + x_n \vec{w}_n = W \vec{x}$$

### Encoding the fact B is better than C

Need a way to encode the experiences such that they can be added.



### Learning premises via Bayesian inference

Likelihood function

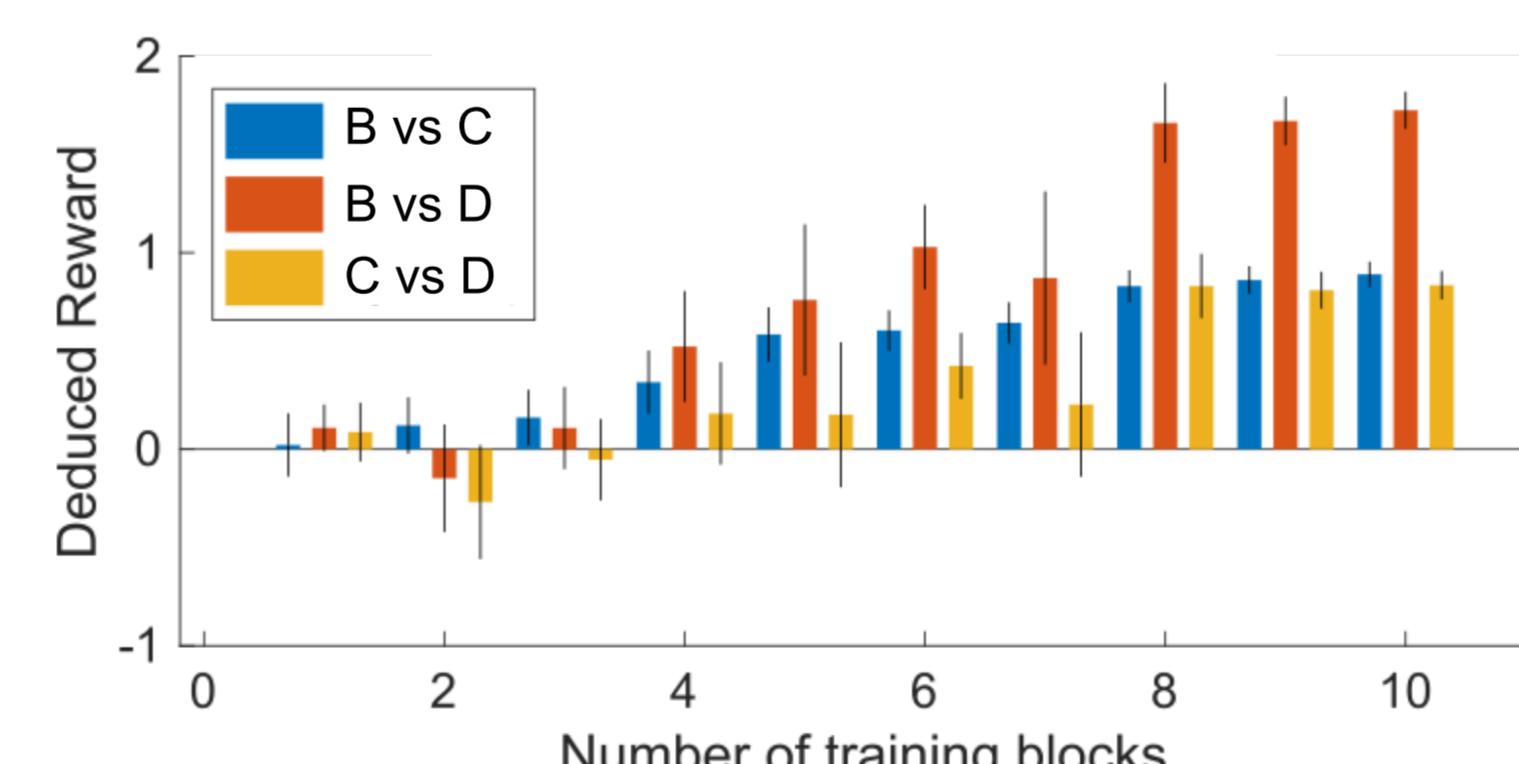
$$\vec{y} \sim \mathcal{N}(W \vec{x}, 1)$$

defines generative density and free energy:

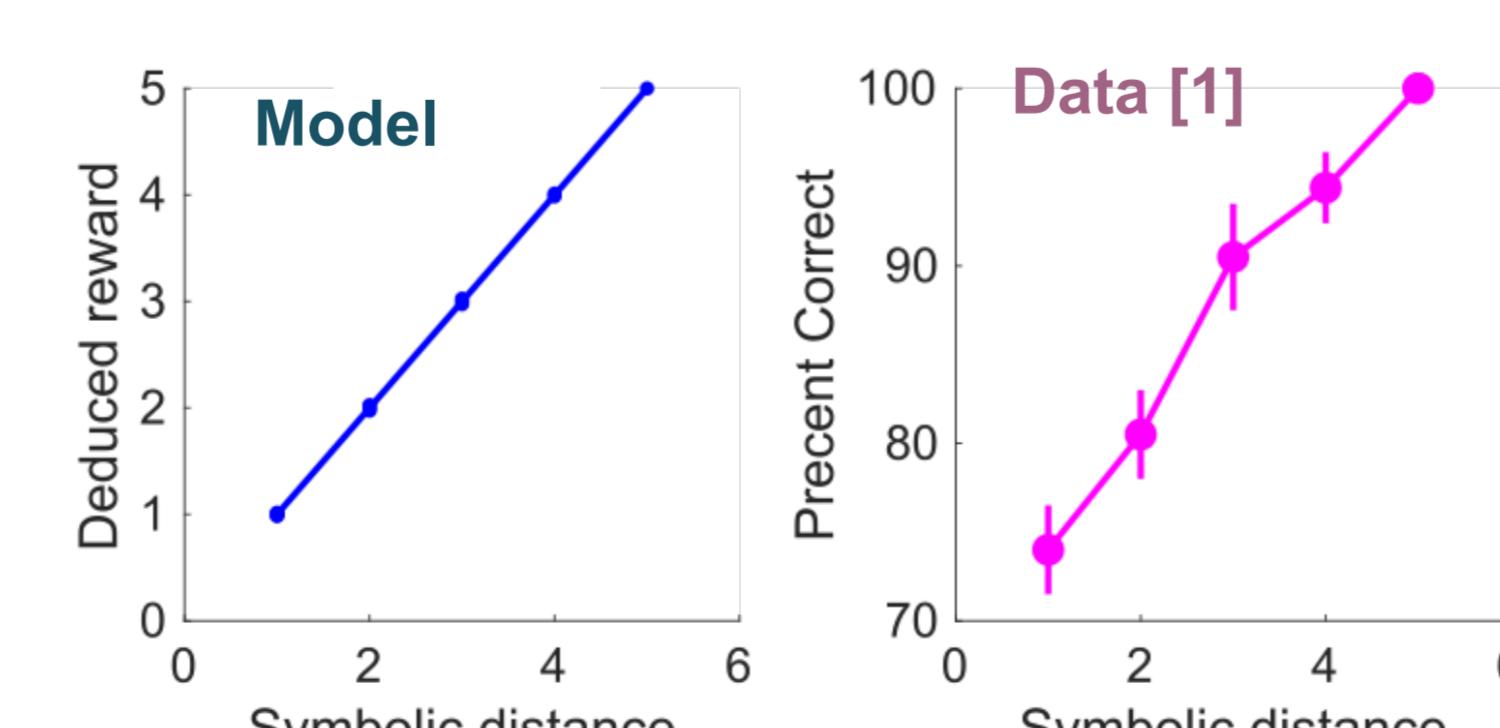
$$P(\vec{y}, W, \vec{x}) \quad F \propto \frac{1}{2} (\vec{y} - W \vec{x})^2$$

## Network Performance

A linear bayesian model correctly learns to infer new facts while learning only the premises:

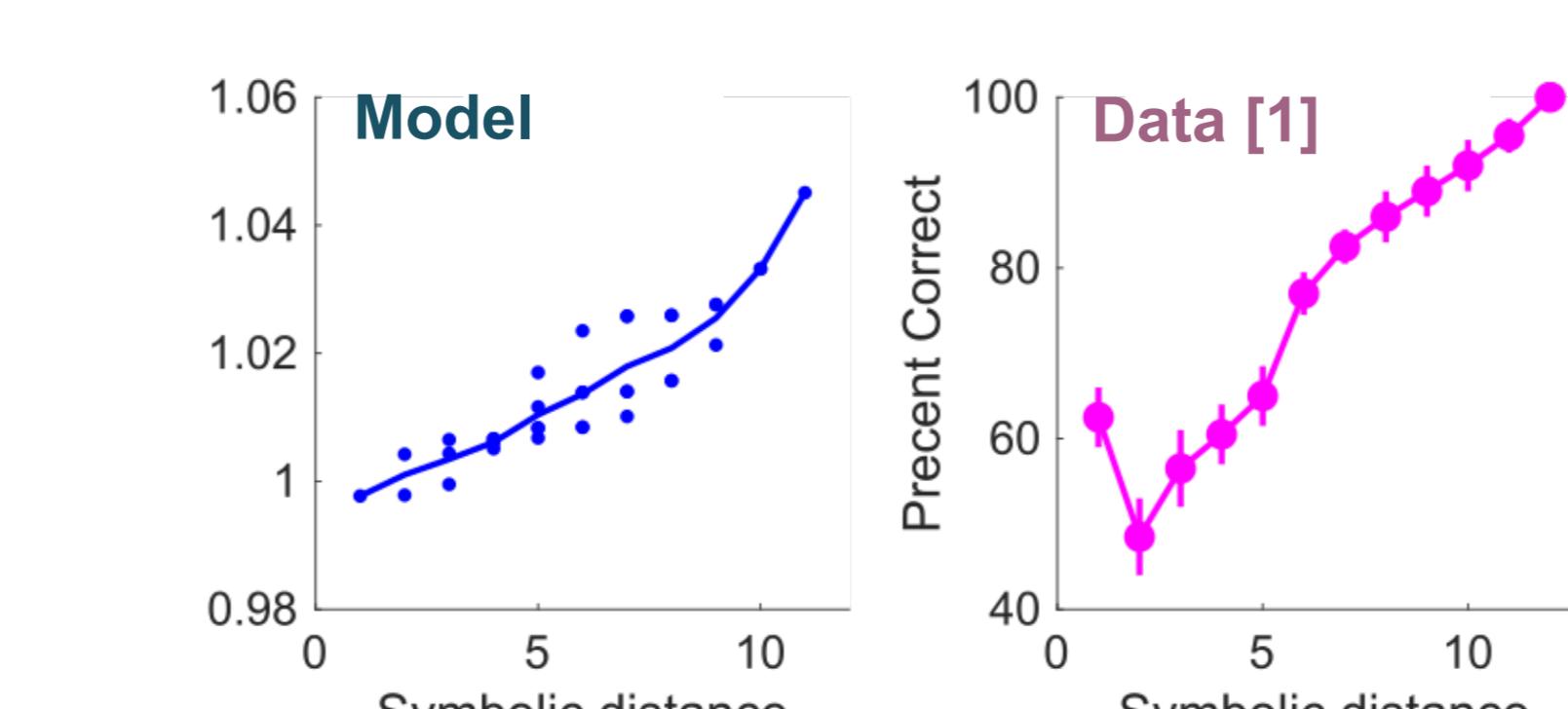


### Symbolic distance effect



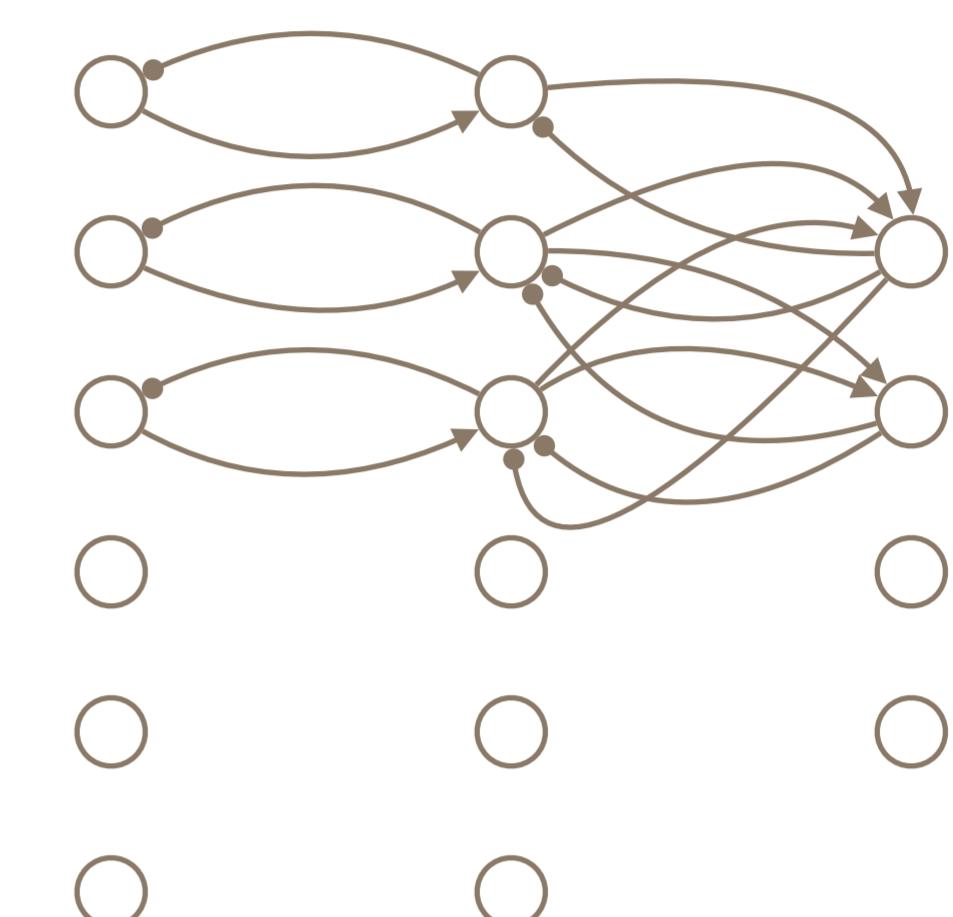
### List linking (other models cannot solve this!)

1. A > B > C > D > E
2. V > W > X > Y > Z
- E > V



## A Network Model

Data layer   Error nodes   Hidden layer

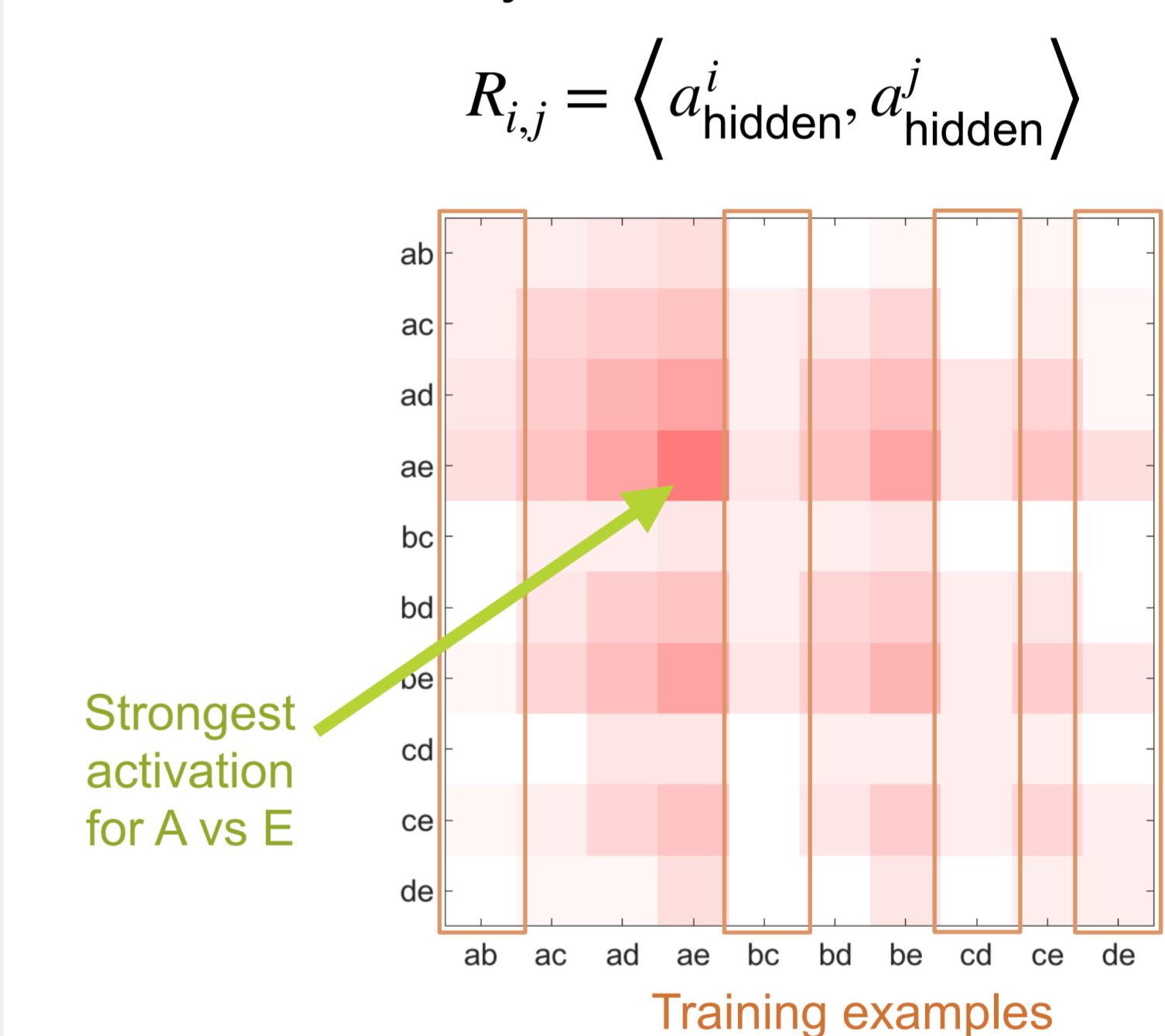


$$\dot{y}_i = -e_i \quad e_i = y_i - W_i^j x_j \quad \dot{x}_i = (W^T)^j_i e_j$$

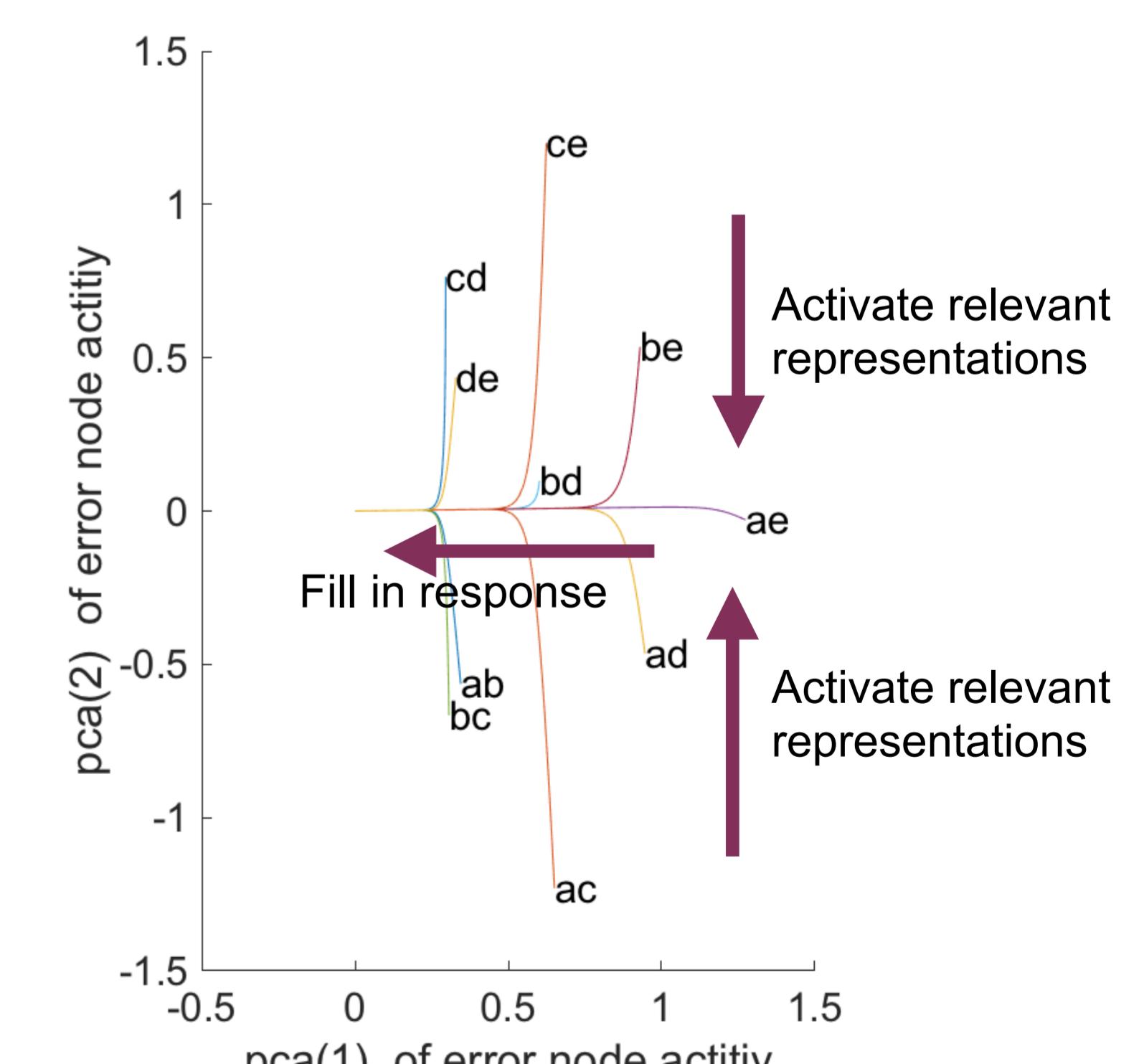
## Neural Results

### Representational similarity

Hidden unit representations mirror the underlying ordinal similarity structure:



### Dynamics in the network: 2 stages of inference



## Discussion

- By choosing a suitable vector representation of propositions like "A>B", transitive inference can be performed.
- This kind of representation reproduces key behavioural effects.
- A neural implementation of this provides testable predictions and gives insight into the network computations.