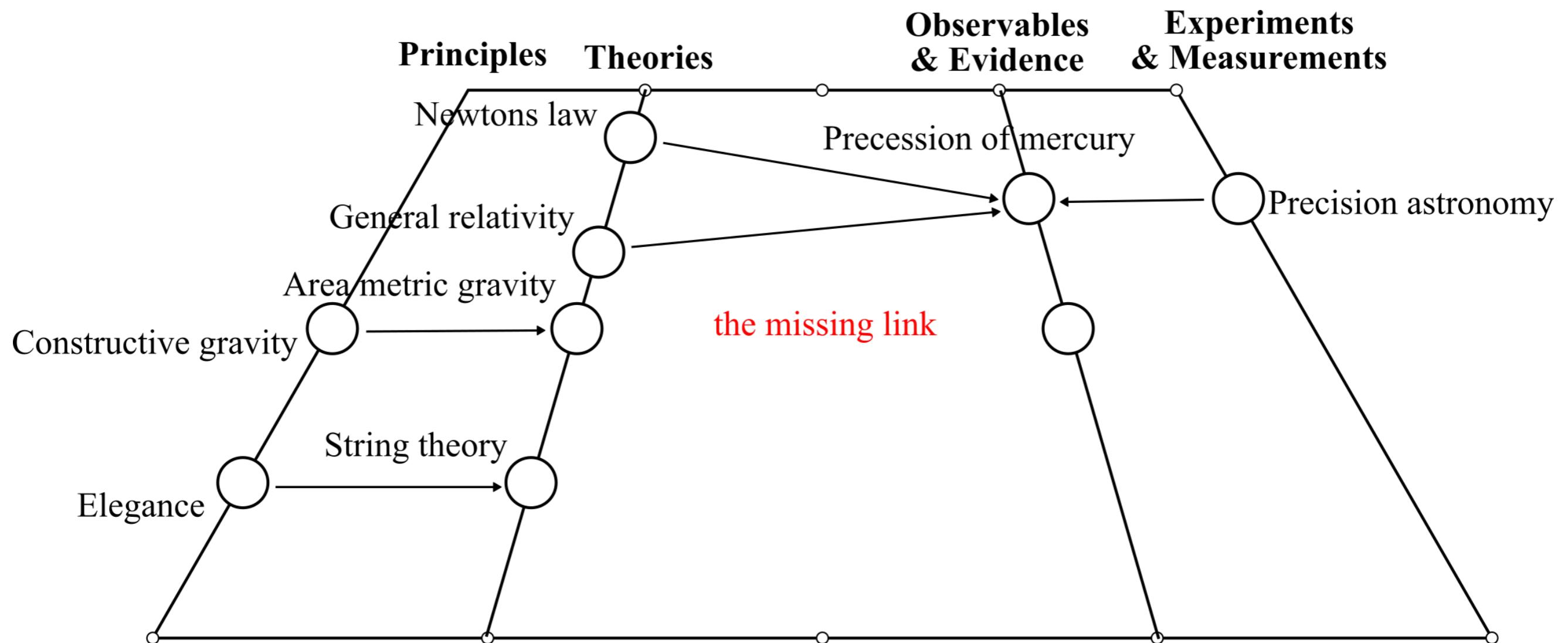


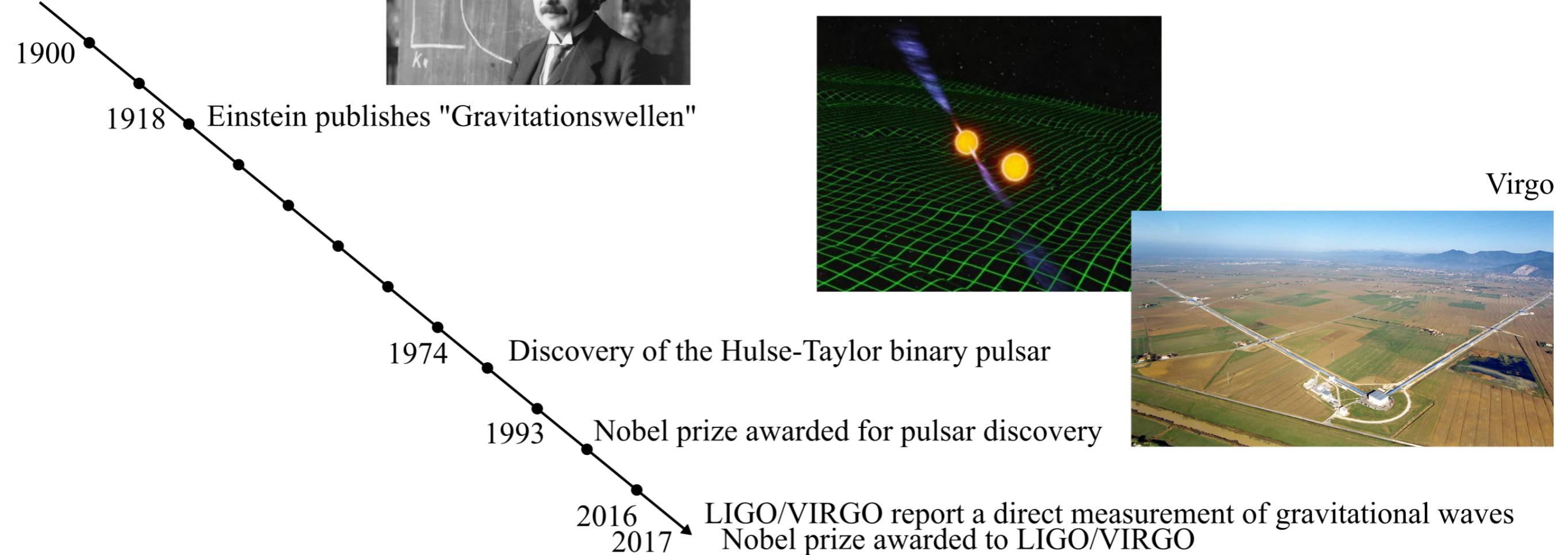
The generation of gravitational waves
by orbiting charges

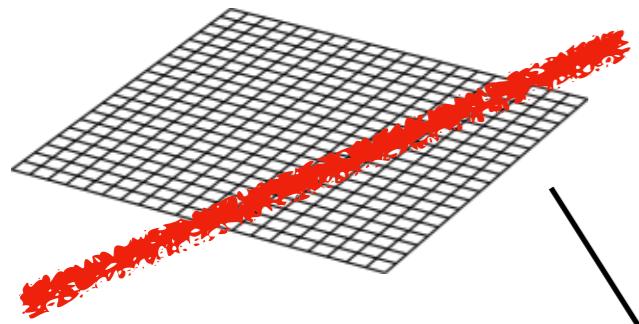
Master's Thesis in Physics
Presented by
Moritz Möller
25 June 2018
Friedrich-Alexander-Universität Erlangen-Nürnberg



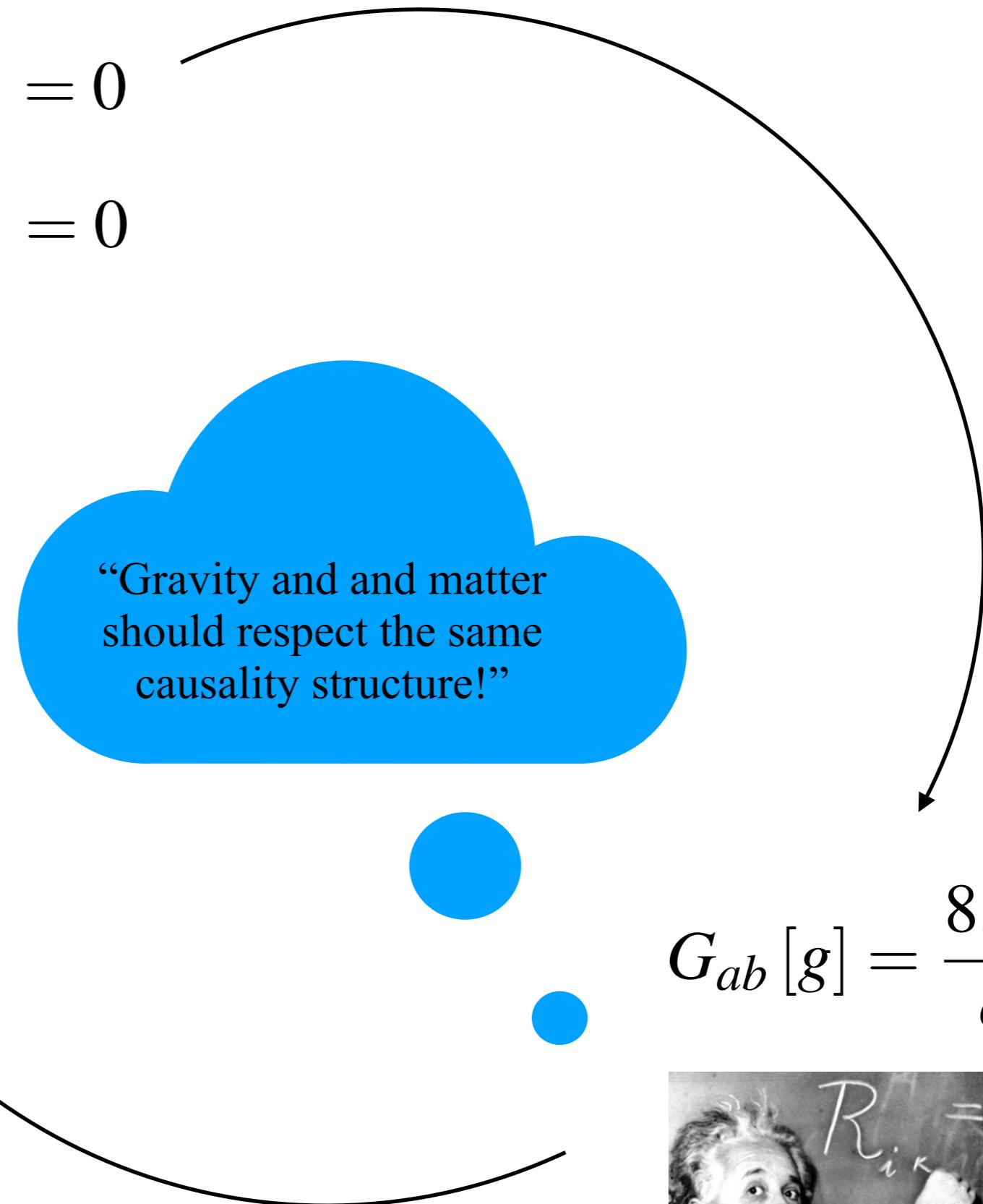
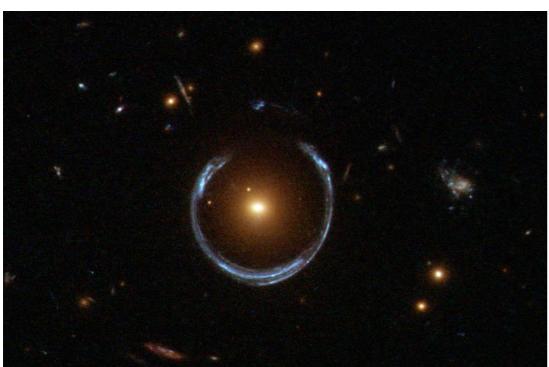
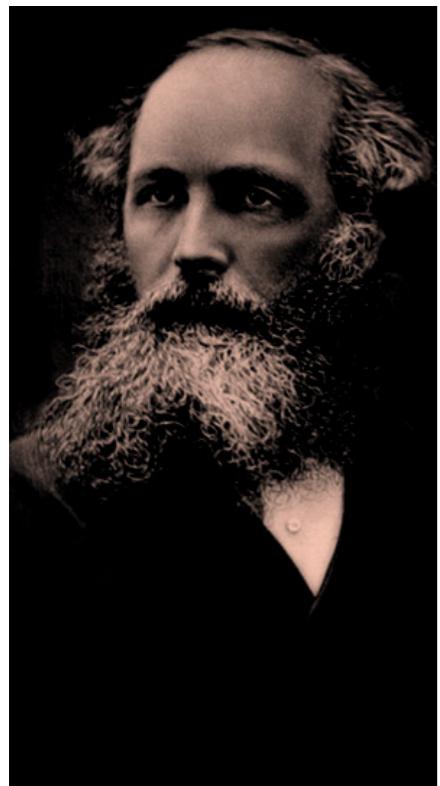
Supervisor: Dr. Frederic P. Schuller
Erstgutachter: Prof. Dr. Gerd Leuchs



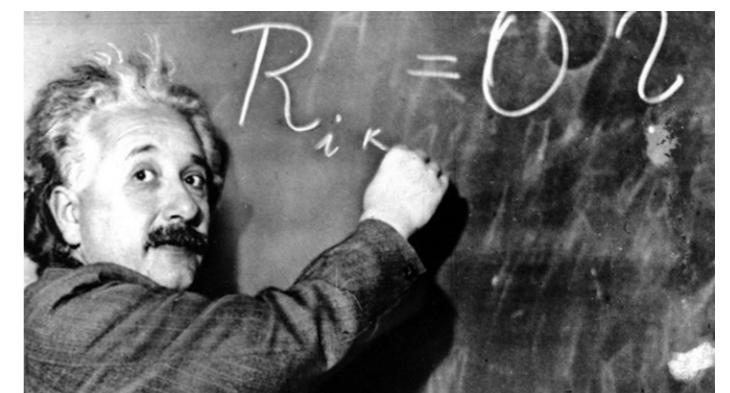


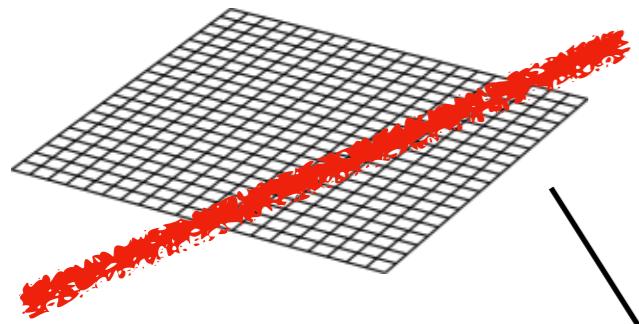


$$\partial_a \left(\sqrt{-g} F_{mn} g^{am} g^{bn} \right) = 0$$



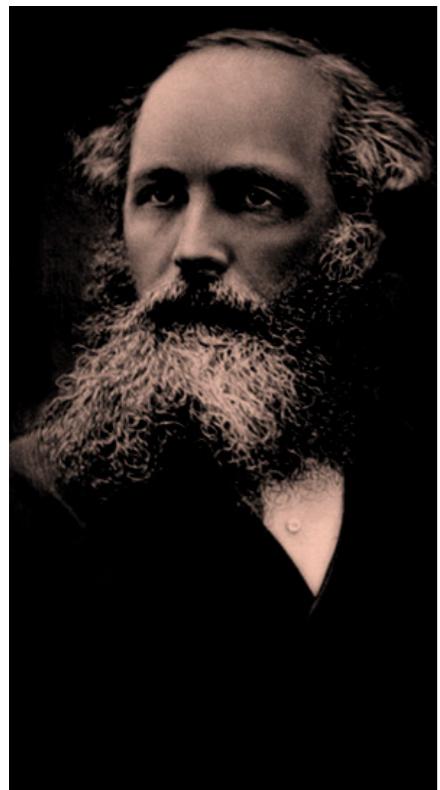
$$G_{ab}[g] = \frac{8\pi G}{c^4} T_{ab}$$



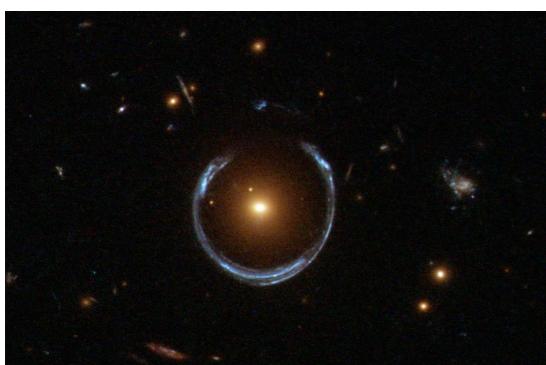


$$\partial_a F_{bc} = 0$$
$$\partial_a \left(\sqrt{-g} F_{mn} g^{am} g^{bn} \right) = 0$$

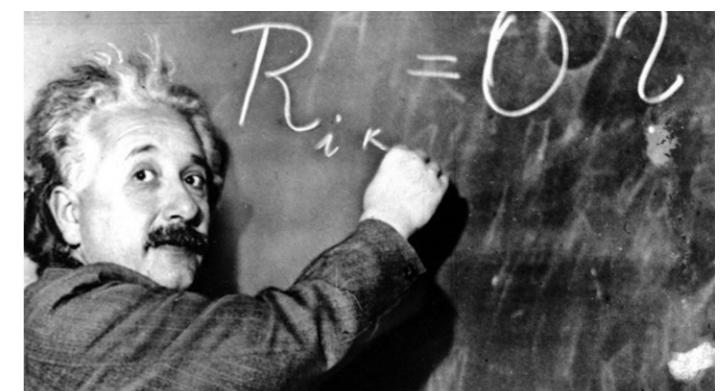
Schuller et al. (2018)



Constructive gravity:
What is the most general
gravity that respects
matter's causal structure?



$$G_{ab}[g] = \frac{8\pi G}{c^4} T_{ab}$$

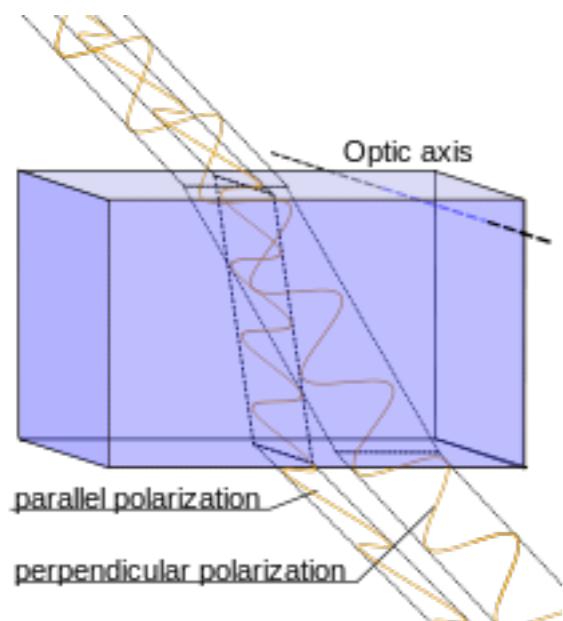


$$\partial_{[a} F_{bc]} = 0$$

$$\partial_a \left(\omega(G) F_{mn} G^{mnab} \right) = 0$$



Constructive gravity:
What is the most general
gravity that respects
matter's causal structure?

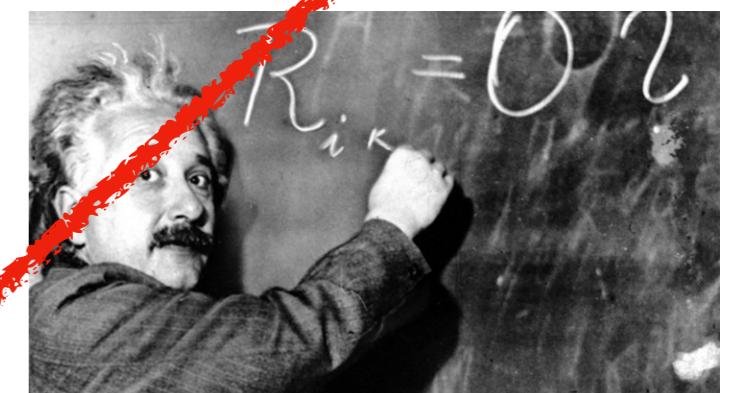


Hehl & Obukhov (2012)

Schuller et al. (2018)



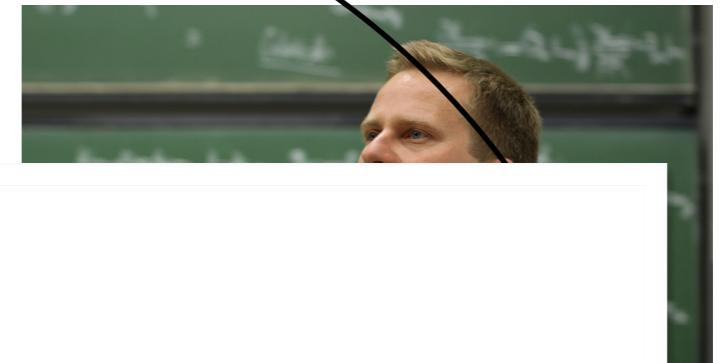
$$G_{ab}[g] = \frac{8\pi G}{c^4} T_{ab}$$



$$\partial_a F_{bc} = 0$$

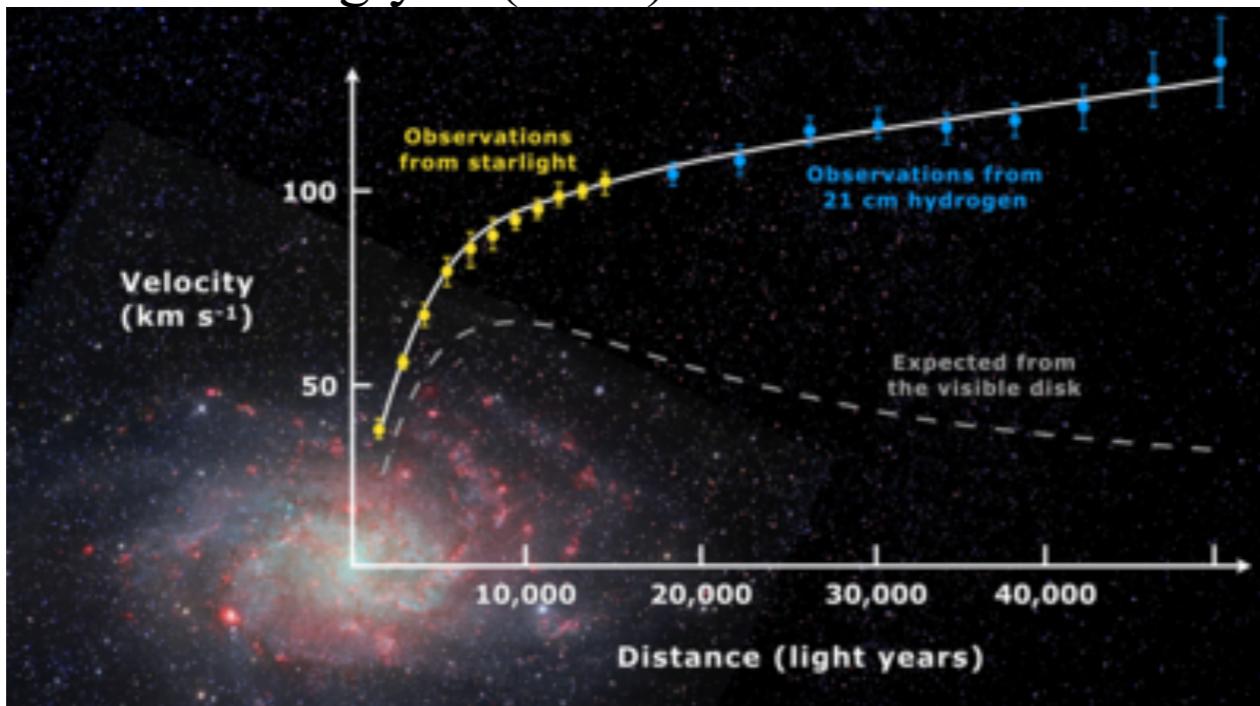
Schuller et al. (2018)

$$\partial_a (\sigma(G) E_{mnab} G^{mnab}) = 0$$

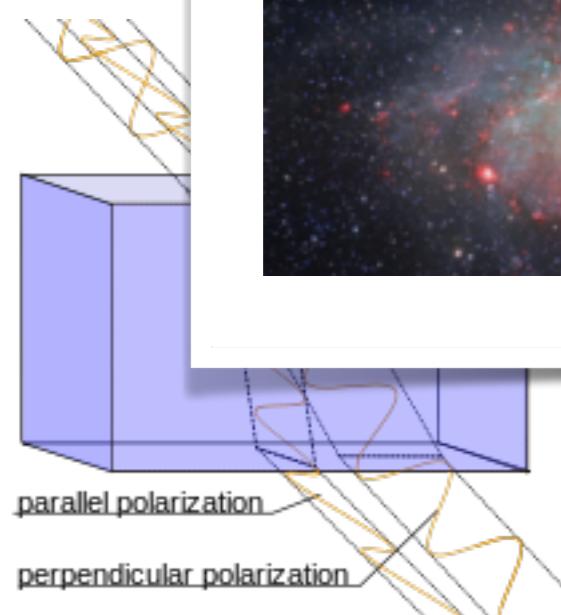
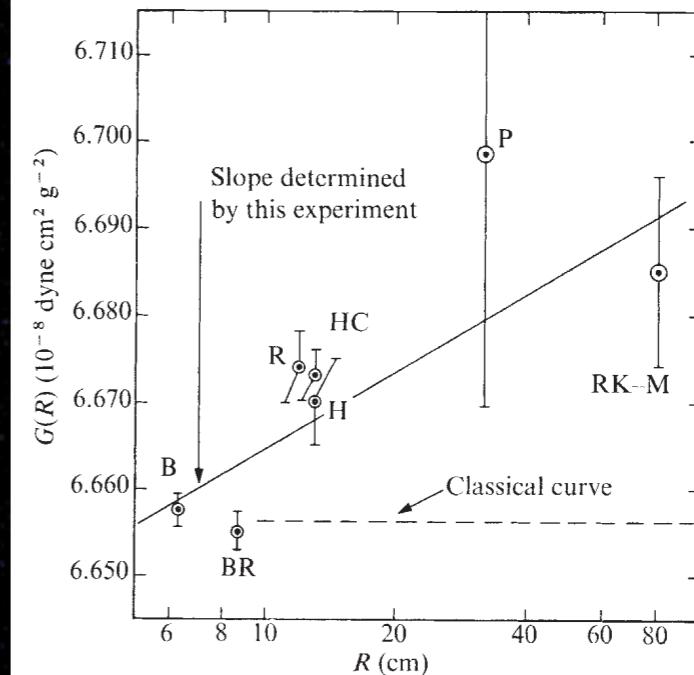


But why?

Kuhn & Kruglyak (1987)



Long (1976)



Constraint equations:

$$-\frac{\delta H_M}{\delta N} - 2\gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] - (18(s_1 + s_2) + 12s_8) \ddot{U} - (12(s_1 + s_2) - 4s_8) \Delta \tilde{U} = 2(s_1 + s_2) \Delta^2 V \quad (193)$$

$$-\frac{\delta H_M}{\delta N} - 2\gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] - (18(s_1 + s_2) + 12s_8) \ddot{U} - (12(s_1 + s_2) - 4s_8) \Delta \tilde{U} = 24(s_1 + s_2) \Delta A \quad (194)$$

$$-\left[\frac{\delta H_M}{\delta N^\alpha} \right]^V = 2(v_1 + v_2) \Delta \dot{V}_\alpha \quad (195)$$

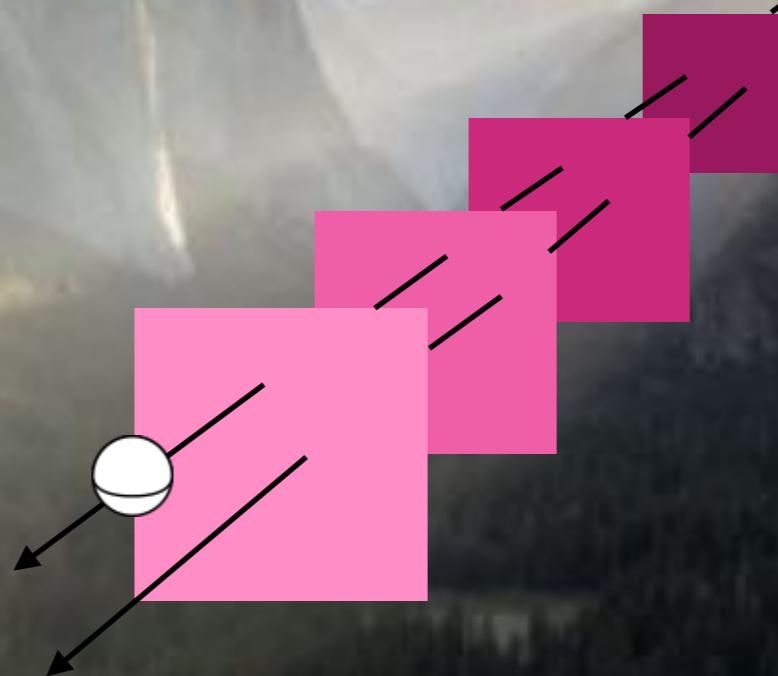
Hehl & Obukhov (2012)

Stritzelberger et al. (2017)

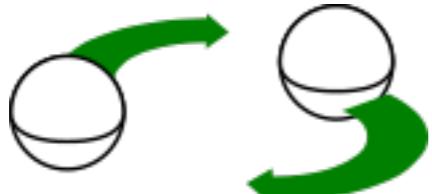
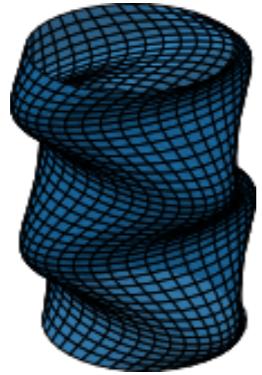


$$S = m \int d\tau \left[P \left(L^{-1} \left(\frac{d\gamma(\tau)}{d\tau} \right) \right)^{-\frac{1}{\deg P}} \right]$$

$$\partial_a (\omega F_{mn} G^{mnab}) = 0$$



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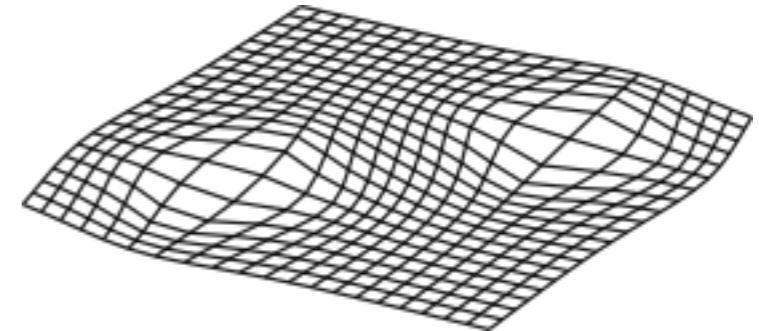
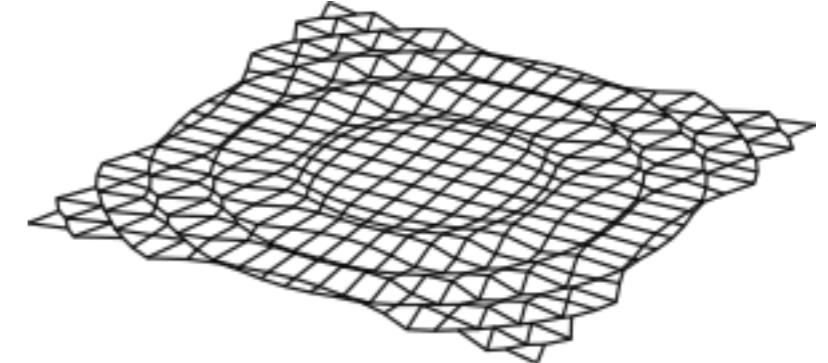
M_3

M_2

G_2

M_1

G_1



G_0

Wave equations:

$$-m_4 \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right]^{\text{TF}} + m_2 \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right]^{\text{TF}} = (m_3 m_4 - m_1 m_2) \square \bar{I}_{\alpha\beta} + (m_4^2 - m_2^2) \bar{I}_{\alpha\beta} \quad (189)$$

$$-m_4 \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right]^{\text{TF}} + m_2 \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right]^{\text{TF}} = (m_3 m_4 - m_1 m_2) \square \bar{U}_{\alpha\beta} + (m_4^2 - m_2^2) \bar{U}_{\alpha\beta} \quad (190)$$

$$- \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right]^{\text{TT}} = (t_1 - t_3) \square V_{\alpha\beta} \quad (191)$$

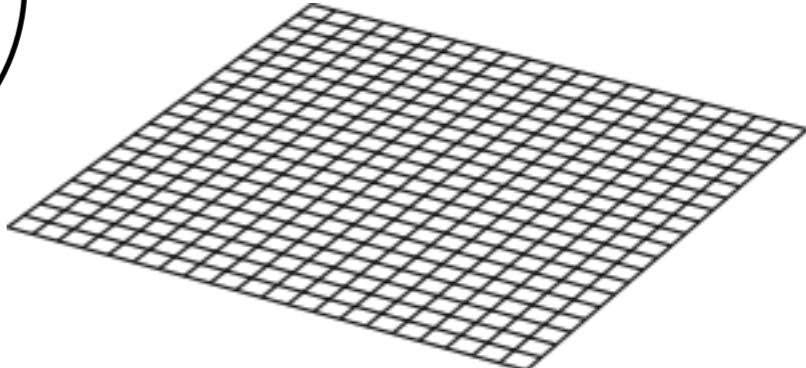
$$(3(s_1 + s_2) + 2s_8) \gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] - 3(s_1 + s_2) \gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] + (3(s_1 + s_2) - s_8) \frac{\delta H_M}{\delta N} \\ = -(108(s_1 + s_2)^2 + 72(s_1 + s_2)s_8 - 18(s_1 + s_2)s_{28} + 12s_8^2) \square \bar{U} + 18(s_1 + s_2)s_{32}\bar{U} \quad (192)$$

Constraint equations:

$$\frac{\delta H_M}{\delta N} + (6(s_1 + s_2) + 4s_8) \Delta \bar{U} = 2(s_1 + s_2) \Delta^2 V \quad (193)$$

$$-\frac{\delta H_M}{\delta N} - 2\gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] - (18(s_1 + s_2) + 12s_8) \ddot{\bar{U}} - (12(s_1 + s_2) - 4s_8) \Delta \bar{U} = 24(s_1 + s_2) \Delta A \quad (194)$$

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Wave equations:

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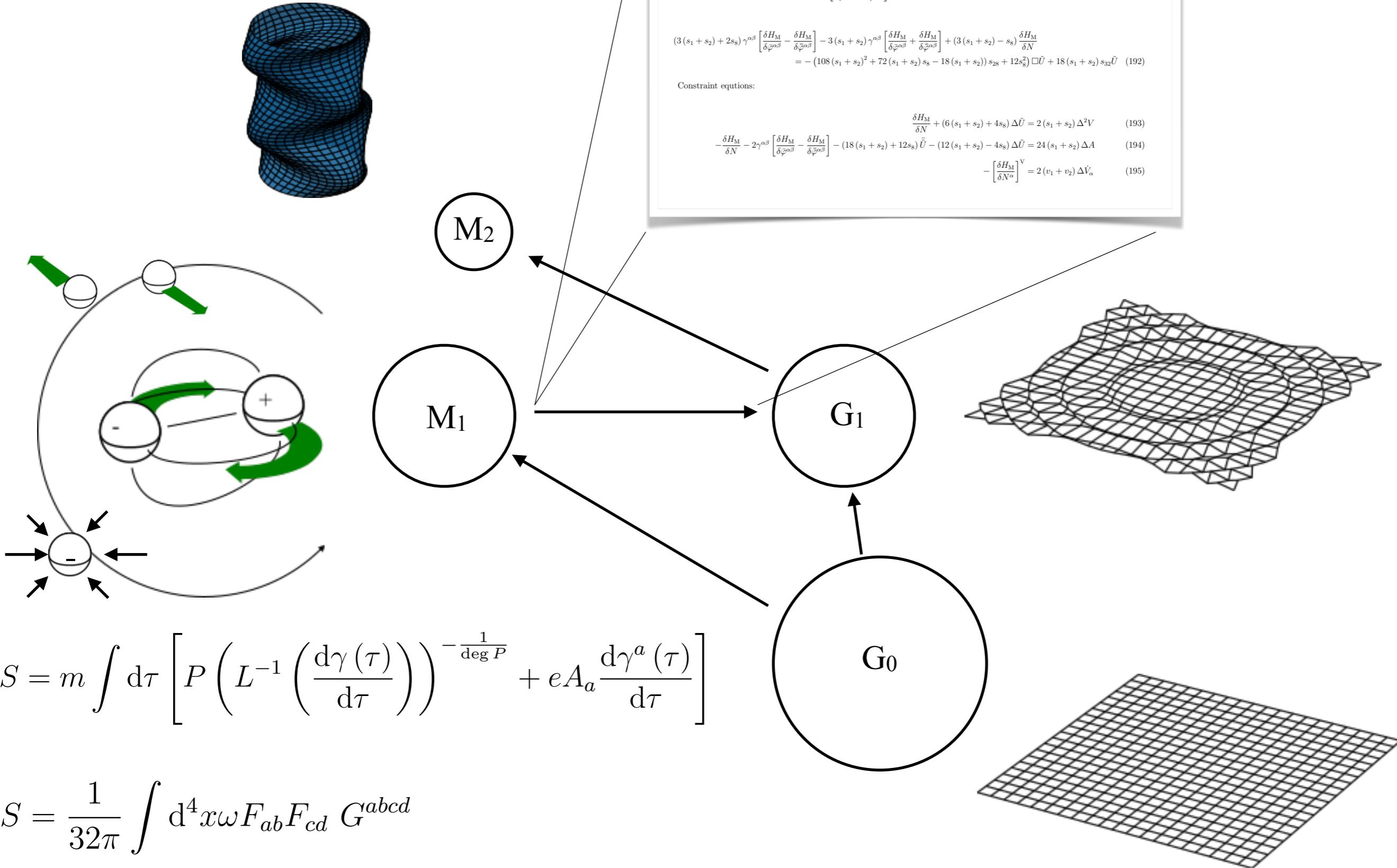
$$(3(s_1 + s_2) + 2s_8) \gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] - 3(s_1 + s_2) \gamma^{\alpha\beta} \left[\frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H_M}{\delta \bar{\varphi}^{\alpha\beta}} \right] + (3(s_1 + s_2) - s_8) \frac{\delta H_M}{\delta N} \\ = -(108(s_1 + s_2)^2 + 72(s_1 + s_2)s_8 - 18(s_1 + s_2)s_{28} + 12s_8^2) \square \bar{U} + 18(s_1 + s_2)s_{32}\bar{U} \quad (192)$$

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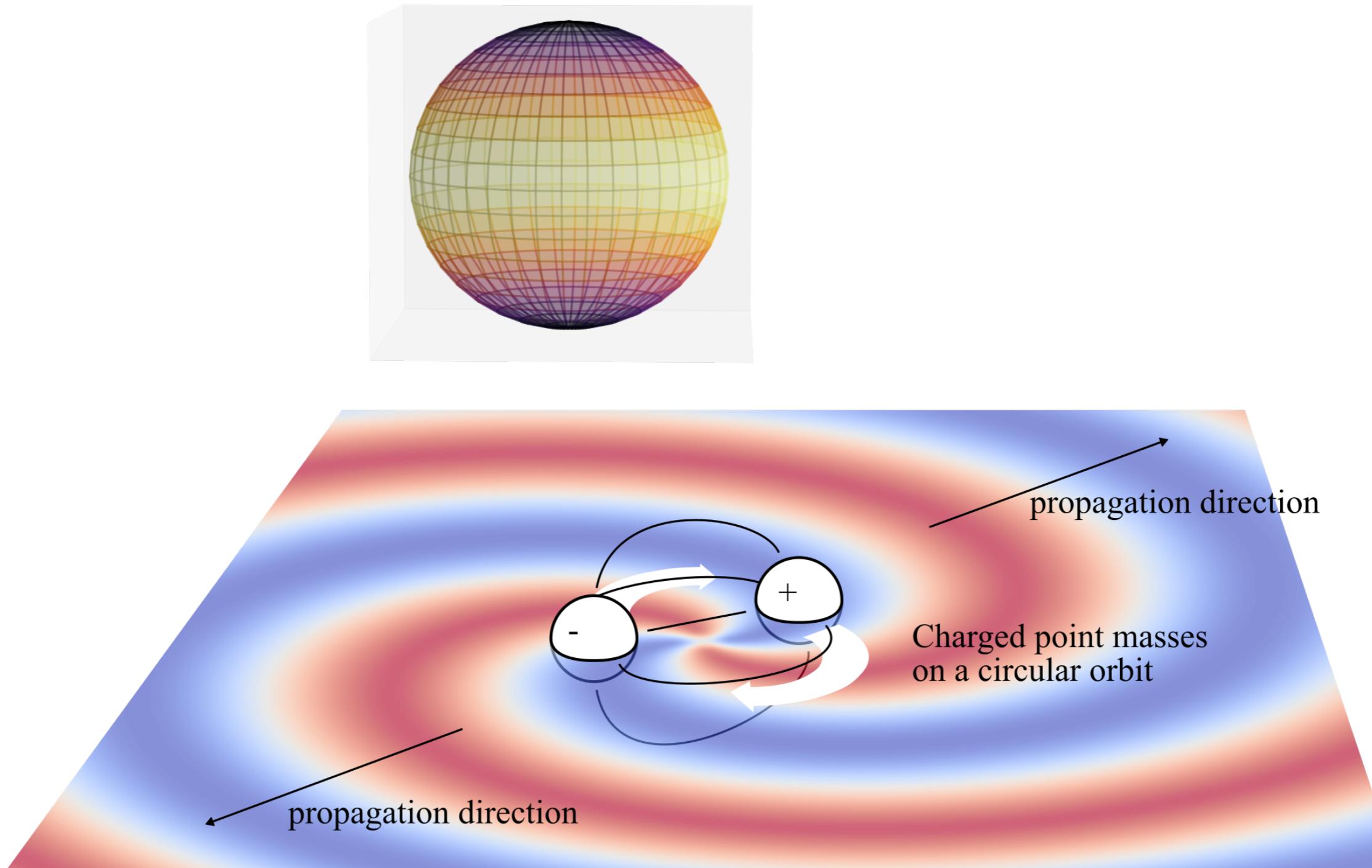
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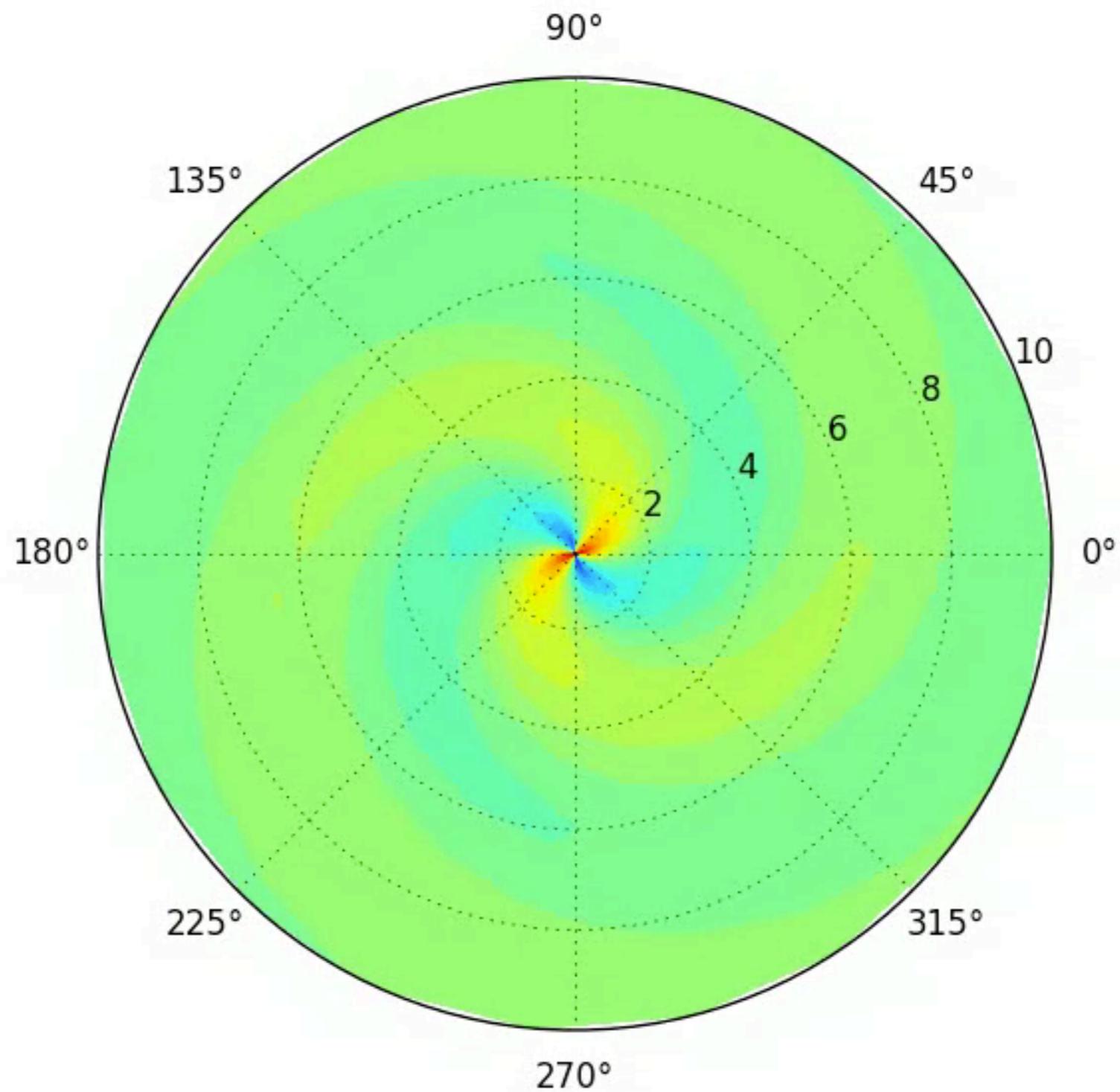
$$- \left[\frac{\delta H_M}{\delta N^\alpha} \right]^V = 2(v_1 + v_2) \Delta \dot{V}_\alpha \quad (195)$$



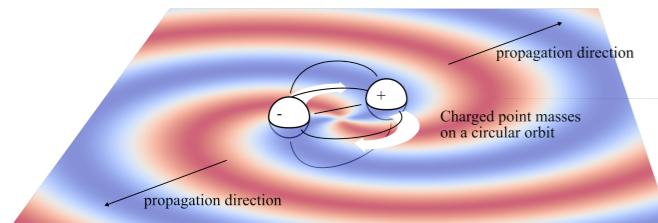
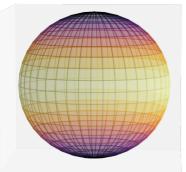
The radiation characteristic



$$\tilde{U}(x, t) = \frac{\tau \mu d^2 \omega_{\text{bin}}^2}{4\pi r} \left(1 - \frac{M_{\tilde{U}}^2}{(2\omega_{\text{bin}})^2}\right) \hat{x}^\alpha \hat{x}^\beta \begin{pmatrix} \cos 2\omega_{\text{bin}} \tilde{t} & \sin 2\omega_{\text{bin}} \tilde{t} \\ \sin 2\omega_{\text{bin}} \tilde{t} & -\cos 2\omega_{\text{bin}} \tilde{t} \end{pmatrix}_{\alpha\beta} + \text{const.} + \mathcal{O}(\omega^3, 1/r^2)$$

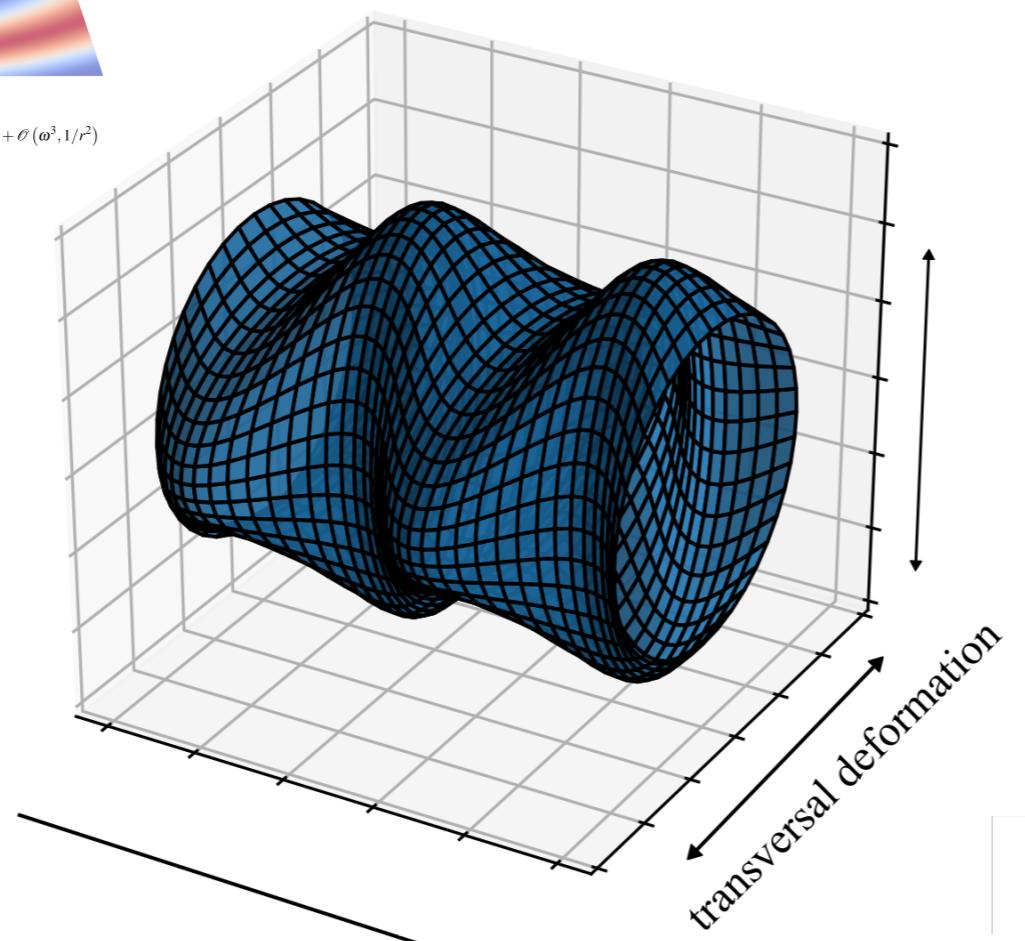


The radiation characteristic



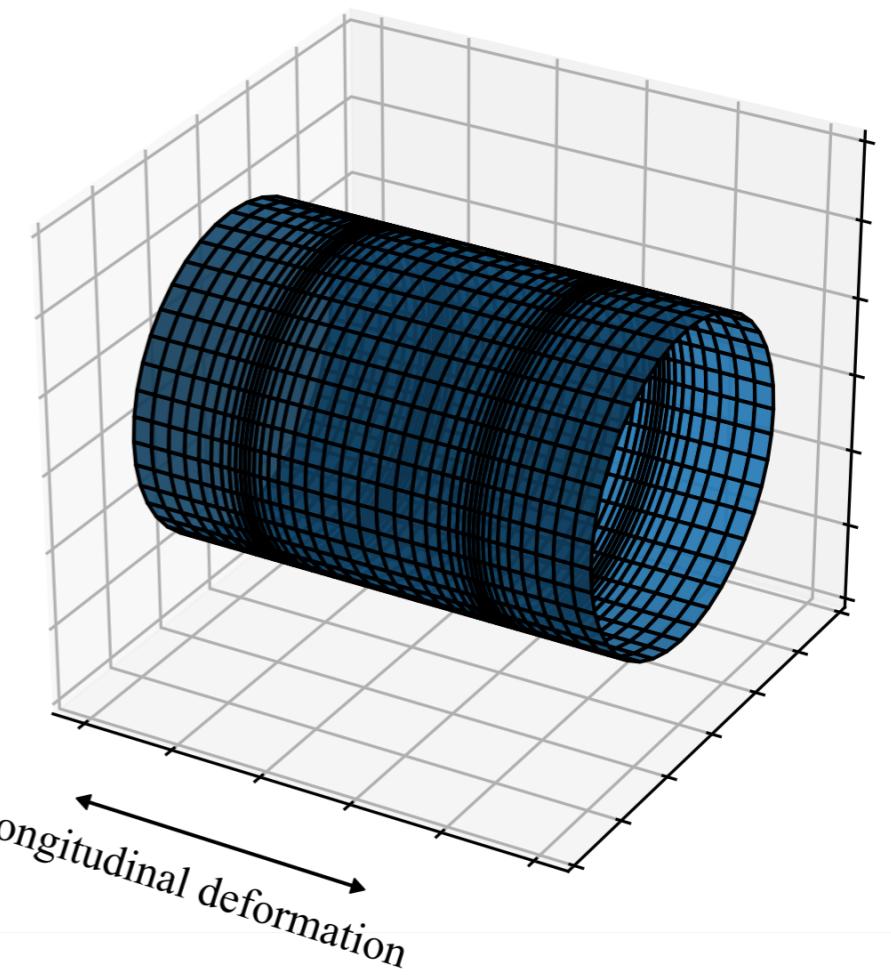
Transverse traceless tensor wave

$$\tilde{U}(x, t) = \frac{\pi \mu d^2 \omega_{\text{bin}}^2}{4\pi r} \left(1 - \frac{M_{\tilde{U}}^2}{(2\omega_{\text{bin}})^2} \right) \hat{x}^\alpha \hat{x}^\beta \begin{pmatrix} \cos 2\omega_{\text{bin}} \tilde{t} & \sin 2\omega_{\text{bin}} \tilde{t} \\ \sin 2\omega_{\text{bin}} \tilde{t} & -\cos 2\omega_{\text{bin}} \tilde{t} \end{pmatrix}_{\alpha\beta} + \text{const.} + \mathcal{O}(r^3, 1/r^2)$$



direction of propagation

Scalar wave



$$d = d_0 - \frac{2A_0}{\omega} \sqrt{1 - M_{\tilde{U}}^2/\omega^2} \sin\left(\frac{d_0}{\lambda}\pi\right) \sin\left(\omega t + \frac{d_0}{\lambda}\pi\right) + \mathcal{O}(1/r^2).$$

