Sourced eq. of motion

for area metric gravity

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linearized and SVT-decomposed

Grav. waves from orbiting charges

In area-metric gravity (linearized theory), define:

$$\boxed{V_A = F_A - E_A}$$

$$\boxed{U_A = F_A + E_A}$$

A stands for all forms of indices.

For non-sourced fields, choose the trivial solution. For sourced fields, evaluate source to order ϵ^2 (e.g. Newtonian source.)

Structure of equations (stability-conditions implemented, $N^\alpha = F = 0$ gauge):

Tensor eq.:

1 massless KG eq.
2 massive KG eq.

$\times$ 2 dof

Vector eq.:

2 massive KG eq.
1 constraint eq.

$\times$ 2 dof

Scalar eq.:

3 massive KG eq.
2 constraint. eq.

$\times$ 1 dof

propagating massless dof: 2

propagating massiv dof: 11

non propagating dof: 4

gauged dof: 4

together $\boxed{\boxed{21 \text{ dof}}}$

Now, to solve all these eq., must only know how to solve:

$$\boxed{\begin{array}{ll} \Box f = g & \partial_{(\alpha} f_{\beta)} = g_{\alpha\beta} \\ (\Box - m^2) f = g & \Delta_{\alpha\beta} f = g_{\alpha\beta} \\ \Delta f = g & \ddot{f} = g \end{array}}$$

$\mathcal{S}$

Scalar equations of motion $\left(\text{Stability condition: } \frac{1}{2}(s_1-3s_2)s_7 = 2s_6 s_3\ ,\quad \text{gauge } F=B=0\right)$

$$\left(3(s_1+s_2)+2s_8\right)\gamma^{\alpha\beta}\left[\frac{\delta H^{(m)}}{\delta\bar{\varphi}^{\alpha\beta}} - \frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{TR} - 3(s_1+s_2)\gamma^{\alpha\beta}\left[\frac{\delta H^{(m)}}{\delta\bar{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{TR} + \left(3(s_1+s_2)-s_8\right)\frac{\delta H}{\delta N}$$

$$= -\left(108(s_1+s_2)^2 + 72(s_1+s_2)s_8 - 18(s_1+s_2)s_{28} + 12 s_8^2\right)\Box\tilde{u} + 18(s_1+s_2)s_{32}\tilde{u}$$

$$-2s_6\left[\frac{\delta H^{(m)}}{\delta\bar{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{STF} + s_7\left[\frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{STF} = \Delta_{\alpha\beta}\left\{\left(s_3 s_7 - (s_1-3s_2)s_6\right)\Box(2C) + \left(s_7^2 - 4s_6^2\right)2C\right\}$$

$$-2s_6\left[\frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{STF} + s_7\left[\frac{\delta H^{(m)}}{\delta\bar{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{STF} = \Delta_{\alpha\beta}\left\{\left(s_3 s_7 - (s_1-3s_2)s_6\right)\Box E + \left(s_7^2 - 4s_6^2\right)E\right\}$$

$$-\frac{\delta H}{\delta N} - \left(6(s_1+s_2)+4s_8\right)\Delta\tilde{u} + 2(s_1+s_2)\Delta^2 E = 6(s_1+s_2)\Delta\tilde{V}$$

$$-\gamma^{\alpha\beta}\left[\frac{\delta H^{(m)}}{\delta\bar{\varphi}^{\alpha\beta}} - \frac{\delta H^{(m)}}{\delta\bar{\bar{\varphi}}^{\alpha\beta}}\right]^{STR} + (s_1+s_2)\Delta^2 E - 9(s_1+s_2)\ddot{\tilde{V}} + 3(s_1+s_2)\Delta\tilde{V} - \left(9(s_1+s_2)+6s_8\right)\ddot{\tilde{u}}$$

$$-\left(3(s_1+s_2)-4s_8\right)\Delta\tilde{u} = 12(s_1+s_2)\Delta a$$

V

Vector equations (Stability condition $-2V_2 V_9 = 2V_8 V_3$ implemented, gauge $B_\alpha = 0$)

$$-\left[\frac{\delta H^{(m)}}{\delta \bar{\varphi}^{\alpha\beta}} - \frac{\delta H^{(m)}}{\delta \bar{\bar{\varphi}}^{\alpha\beta}}\right]^{(V)} = \partial_{(\alpha}\, 2(V_1+V_2)\, \ddot{V}_{\beta)}$$

$$-V_9\left[\frac{\delta H^{(m)}}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta \bar{\bar{\varphi}}^{\alpha\beta}}\right]^{(V)} + 2V_8\left[\frac{\delta H^{(m)}}{\delta \bar{\bar{\bar{\varphi}}}^{\alpha\beta}}\right]^{(V)} = \partial_{(\alpha}\left\{(V_3V_9+2V_2V_8)\,\Box\,(2C_{\beta)}) + (V_9^2-4V_8^2)(2C_{\beta)})\right\}$$

$$-V_9\left[\frac{\delta H^{(m)}}{\delta \bar{\bar{\bar{\varphi}}}^{\alpha\beta}}\right]^{(V)} + 2V_8\left[\frac{\delta H^{(m)}}{\delta \bar{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta \bar{\bar{\varphi}}^{\alpha\beta}}\right]^{(V)} = \partial_{(\alpha}\left\{(V_3V_9+2V_2V_8)\,\Box\,\mathcal{U}_{\beta)} + (V_9^2-4V_8^2)\,\mathcal{U}_{\beta)}\right\}$$

$$-\left[\frac{\delta H^{(m)}}{\delta N^{\alpha}}\right]^{(V)} = \Delta\, 2(V_1+V_2)\,\ddot{V}_{\alpha}$$

~~TT~~

Tensor equations (Stability condition $(t_1+t_3)\,t_{13} = 2t_5\,t_{11}$ is implemented)

$$-\left[\frac{\delta H^{(m)}}{\delta \overline{\varphi}^{\alpha\beta}} - \frac{\delta H^{(m)}}{\delta \overline{\overline{\overline{\varphi}}}^{\alpha\beta}}\right]^{TT} = (t_1 - t_3)\,\Box V_{\alpha\beta}$$

$$-t_{13}\left[\frac{\delta H^{(m)}}{\delta \overline{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta \overline{\overline{\overline{\varphi}}}^{\alpha\beta}}\right]^{TT} + 2t_{11}\left[\frac{\delta H^{(m)}}{\delta \overline{\overline{\overline{\varphi}}}^{\alpha\beta}}\right]^{TT} = \Big(t_5 t_{13} - 2t_{11}(t_1+t_3)\Big)\,\Box\, 2C_{\alpha\beta} + \left(t_{13}^2 - 4t_{11}^2\right) 2C_{\alpha\beta}$$

$$-2t_{11}\left[\frac{\delta H^{(m)}}{\delta \overline{\overline{\overline{\varphi}}}^{\alpha\beta}}\right]^{TT} + t_{13}\left[\frac{\delta H^{(m)}}{\delta \overline{\varphi}^{\alpha\beta}} + \frac{\delta H^{(m)}}{\delta \overline{\overline{\overline{\varphi}}}^{\alpha\beta}}\right]^{TT} = \Big(t_5 t_{13} - 2t_{11}(t_1+t_3)\Big)\,\Box\, u_{\alpha\beta} + \left(t_{13}^2 - 4t_{11}^2\right) u_{\alpha\beta}$$