Advanced Computer Graphics Practical Session

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Summer Semester 2023



- Random Sampling
- Projects

- Random Sampling
- Projects

Sampling Triangular Patches

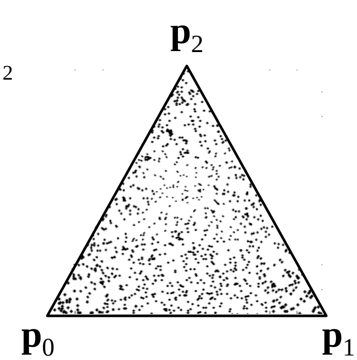
- Triangle in 3D given by vertices $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$
- Arbitrary vertex $\mathbf{q}$ given by barycentric coordinates
- Correct sampling approach

$$\mathbf{q} = \lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 + (1 - \lambda_0 - \lambda_1)\mathbf{p}_2$$

$$\lambda_0 = 1 - \sqrt{\xi_0}$$

$$\lambda_1 = \xi_1 \sqrt{\xi_0}$$

$$\lambda_2 = 1 - \lambda_0 - \lambda_1 = \sqrt{\xi_0}(1 - \xi_1)$$



Generating Random Numbers

- Computer-based approaches **pseudo-random** (i.e. following algorithm)
- Example: `drand48()`

$$x_{n+1} = (a \cdot x_n + c) \bmod m$$

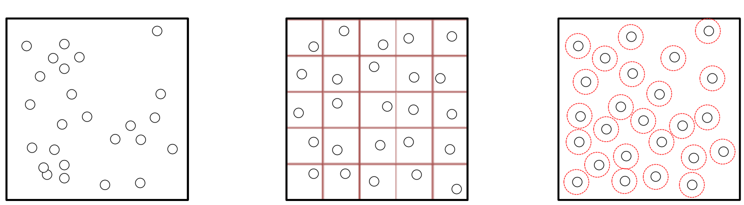
- 48-bit parameters given as

$$m = 2^{48} \qquad a = (5\mathrm{DEECE66D})_{16} \qquad c = (\mathrm{B})_{16}$$

- Linear congruential generator
- Generates non-negative, double-precision, floating-point values, uniformly distributed over $[0.0, 1.0]$

Sampling Strategy

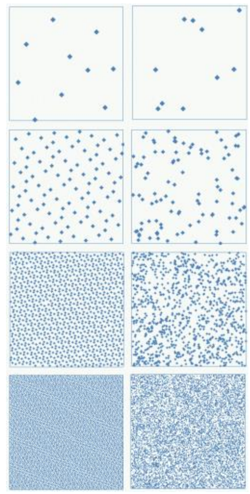
- Fully randomized sampling possibly suboptimal due to clumping of samples
- Stratified or Poisson disk sampling reduces clumping and non-uniformity



Random samples Stratified Poisson disk

Quasi-Random Sampling

- Quasi-random
- Random



Quasi-Random Sampling

- Deterministic sequence of numbers that only appear random, and regularly cover domain
- Values are said to be of low discrepancy
- Example: **Halton sequence**
- Representation of positive integer n with base b

$$n = \sum_{i=1}^{\infty} d_i b^{i-1} \qquad\qquad 0 \le d_i < b$$

- Associated **radical inverse function**

$$\Phi_b(n) = 0.d_1 d_2 \dots d_m \qquad\qquad d_{m+i} = 0 \quad i > 0$$

Halton Sequence

- Example for base 2

n (Base 10)	Base 2 number	$\Phi_2(n)$
1	$(1)_2$	$(0.1)_2 = 1/2$
2	$(10)_2$	$(0.01)_2 = 1/4$
3	$(11)_2$	$(0.11)_2 = 3/4$
4	$(100)_2$	$(0.001)_2 = 1/8$
5	$(101)_2$	$(0.101)_2 = 5/8$
…	…	…

Halton Sequence

- Example for base 2

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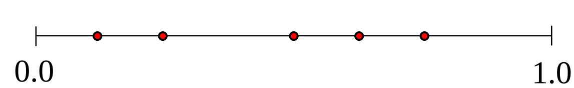
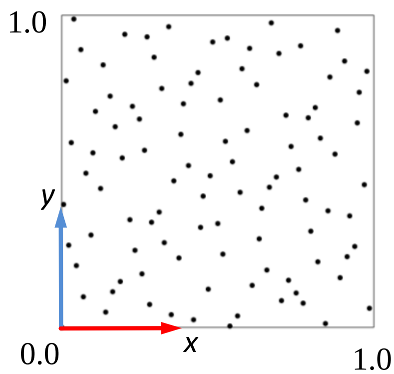


Halton Sequence in 2D

- In higher dimensional cases, choose different prime number as base for each dimension, e.g. Halton-2-3



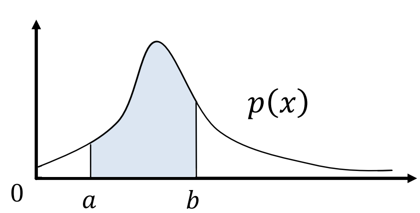
$$(x_i, y_i) = (\Phi_2(i), \Phi_3(i))$$

(100 samples)

Sampling Arbitrary Probability Density Function

- PDF: Probability Density Function
- Probability to belong in an interval $[a, b]$ for a variable X with density function $p(x)$:

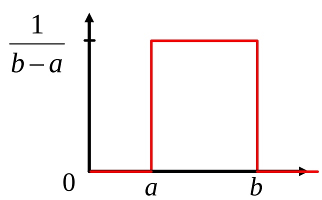
$$P(a \leq X \leq b) = \int_a^b p(x)\, dx$$



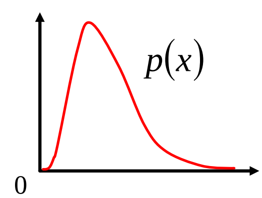
$$\int_{-\infty}^{\infty} p(x) dx = 1$$

Sampling Arbitrary Probability Density Function

- Evaluating integrals via Monte-Carlo method requires random samples according to arbitrary PDF
- Option: **Inversion method**



Uniform distribution



Arbitrary PDF

Inversion Method – Continuous PDF

1) Compute cumulative distribution function for PDF

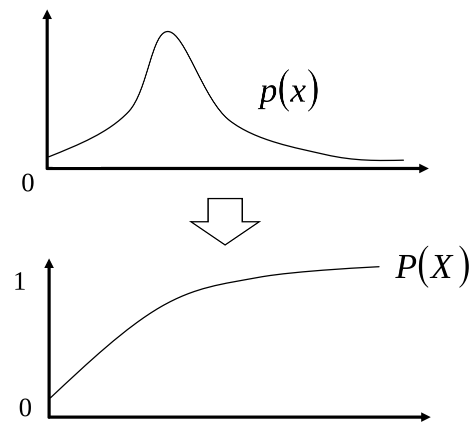
$$P(X) = \int_{-\infty}^{X} p(x)dx$$

2) Compute inverse of CDF (not always feasible)
3) Draw uniformly distributed random number ξ
4) Obtain new random variable, adhering to probability density function $p(x)$

$$X = P^{-1}(\xi)$$

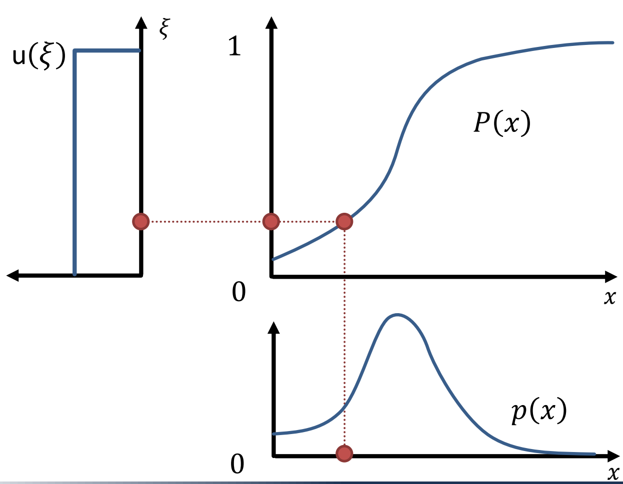
Inversion Method

- Cumulative Distribution Function (CDF)



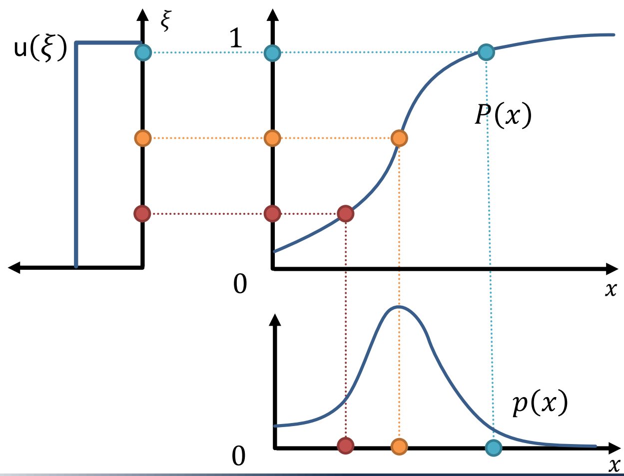
Inversion Method

- Graphically



Inversion Method

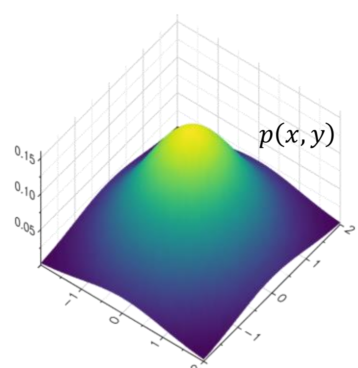
- Mapping graphically



2D Joint Distribution

- Multivariate Probability Density Function
- E.g. 2D random variable

$$P(X,Y) = \int_{-\infty}^{Y} \int_{-\infty}^{X} p(x,y)\, dx\, dy$$



Conditional probability

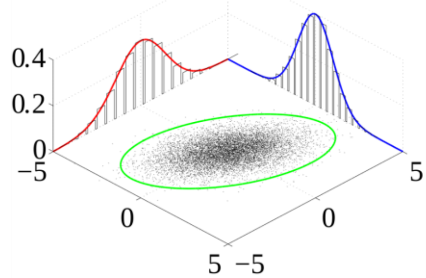
- Marginal densities:

$$p(x) = \int p(x,y)\,dy$$
$$p(y) = \int p(x,y)\,dx$$

- Conditional probability:

$$p(y|x) = \frac{p(x,y)}{p(x)}$$

https://upload.wikimedia.org/wikipedia/commons/thumb/9/95/Multivariate_normal_sample.svg/1019px-Multivariate_normal_sample.svg.png



Conditional Probability

- Probability of outcome given another event occurred

$$P(X|Y) = \frac{P(X,Y)}{P(Y)}$$

- Example: rolling two fair dice
- **Joint probability** of sum being 6 and 2nd die showing 2

$$P(X,Y) = 1/36$$

- **Conditional probability** that sum of two-dice throw equals 6, given that first roll shows 2

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} = \frac{1/36}{1/6} = 1/6$$

2D Joint Distribution Random Sampling

1) Compute **marginal probability density function**

$$p(x) = \int p(x, y) dy$$

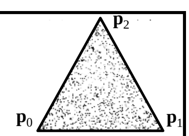
2) Obtain conditional probability density function

$$p(y|x) = \frac{p(x, y)}{p(x)}$$

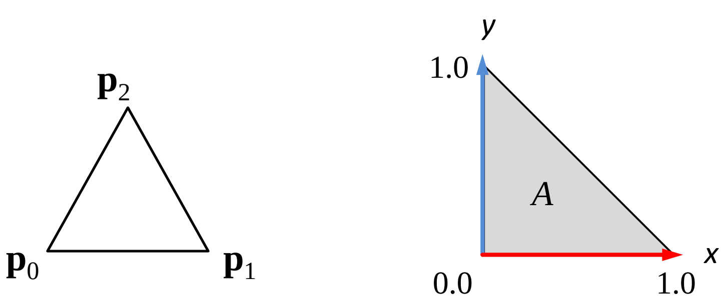
[see last slides]

3) Sample marginal probability function using $p(x)$
4) Sample conditional probability function using $p(y|x)$

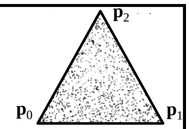
Uniformly Sampling Triangle Area



- Without loss of generality, assume isosceles triangle with area A = ½
- Extends to general case via barycentric coordinates

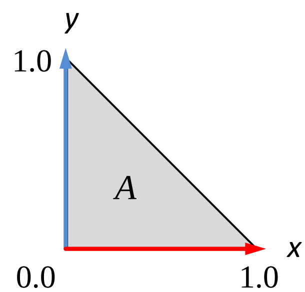


Uniformly Sampling Triangle Area

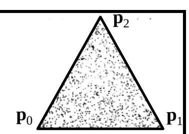


- Determine joint probability density function $p(x,y)$ (constant for uniform probability over area)
- According to definition of PDF

$$\int_A p(x,y)\,dA = 1$$



Uniformly Sampling Triangle Area



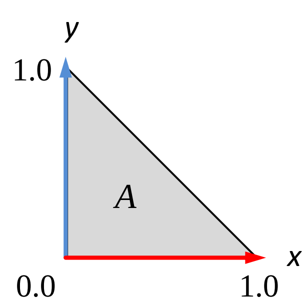
- Determine joint probability density function $p(x,y)$ (constant for uniform probability over area)
- According to definition of PDF

$$\int_A p(x,y)\,dA = 1$$

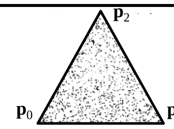
$$p(x,y)\int_A dA = 1$$

$$p(x,y)\frac{1}{2} = 1$$

$$\boxed{p(x,y) = 2}$$



Uniformly Sampling Triangle Area



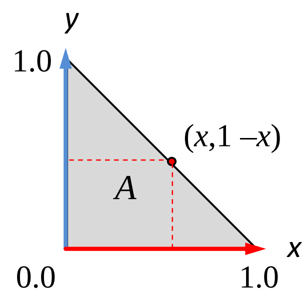
- Compute marginal probability density function

$$p(x) = \int_0^{1-x} p(x, y)\,dy$$

$$p(x) = \int_0^{1-x} 2 \cdot dy$$

$$p(x) = 2y\Big|_0^{1-x}$$

$$\boxed{p(x) = 2 - 2x}$$



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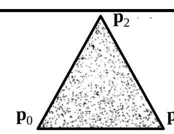


- Determine cumulative distribution functions for applying inversion method

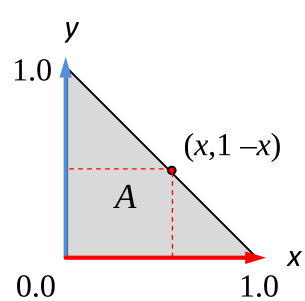
$$P(x) = \int_{-\infty}^{x} p(\hat{x})\,d\hat{x}$$

$$P(x) = \int_0^{x} p(\hat{x})\,d\hat{x}$$

$$P(x) = \int_0^{x} 2 - 2\hat{x} \cdot d\hat{x}$$

$$P(x) = \left[2\hat{x} - \hat{x}^2\right]_0^x$$

$$\boxed{P(x) = 2x - x^2}$$



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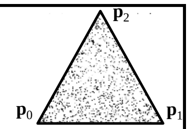
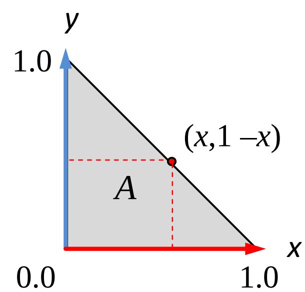


- Compute conditional probability density function

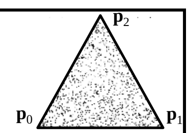
$$p(y|x) = \frac{p(x,y)}{p(x)}$$

$$p(y|x) = \frac{2}{2-2x}$$

$$\boxed{p(y|x) = \frac{1}{1-x}}$$



Uniformly Sampling Triangle Area

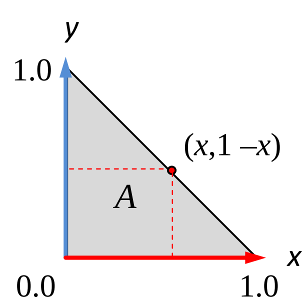


- Determine cumulative distribution functions for applying inversion method

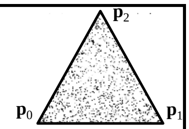
$$P(y) = \int_0^y p(\hat{y}|x)\,d\hat{y}$$

$$P(y) = \int_0^y \frac{1}{1-x}\,d\hat{y}$$

$$\boxed{P(y) = \frac{y}{1-x}}$$



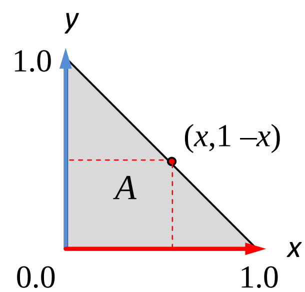
Uniformly Sampling Triangle Area



- Determine cumulative distribution functions for applying inversion method

$$P(x) = 2x - x^2$$

$$P(y) = \frac{y}{1-x}$$



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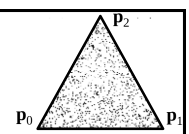
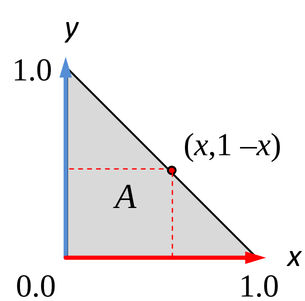
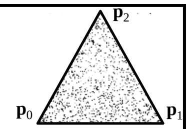


- Invert CDF for sampling with canonical uniformly distributed variable

$$\hat{\xi}_0 = 2x - x^2$$



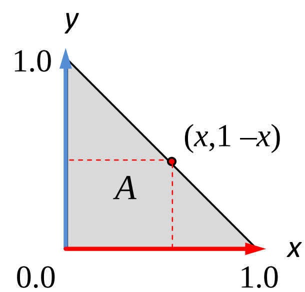
Uniformly Sampling Triangle Area



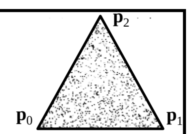
- Invert CDF for sampling with canonical uniformly distributed variable

$$\hat{\xi}_0 = 2x - x^2$$

$$1 - \xi_0 = 2x - x^2$$



Uniformly Sampling Triangle Area



- Invert CDF for sampling with canonical uniformly distributed variable

$$\hat{\xi}_0 = 2x - x^2$$

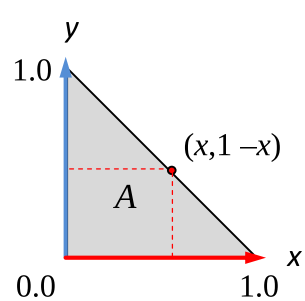
$$1 - \xi_0 = 2x - x^2$$

$$\xi_0 = 1 - 2x + x^2$$

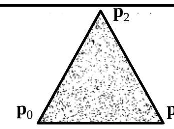
$$\xi_0 = (1 - x)^2$$

$$\sqrt{\xi_0} = 1 - x$$

$$x = 1 - \sqrt{\xi_0}$$



Uniformly Sampling Triangle Area

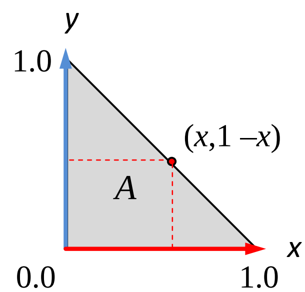


- Invert CDF for sampling with canonical uniformly distributed variable

$$\xi_1 = \frac{y}{1-x}$$

$$\xi_1 = \frac{y}{\sqrt{\xi_0}}$$

$$y = \xi_1 \sqrt{\xi_0}$$



Sampling Triangular Patches

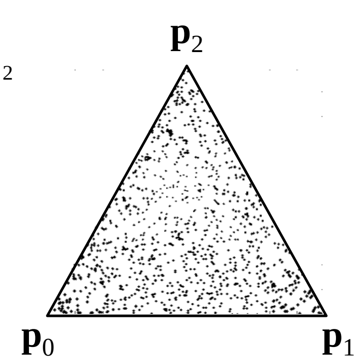
- Triangle in 3D given by vertices $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$
- Arbitrary vertex $\mathbf{q}$ given by barycentric coordinates
- Correct sampling approach

$$\mathbf{q} = \lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 + (1 - \lambda_0 - \lambda_1)\mathbf{p}_2$$

$$\lambda_0 = 1 - \sqrt{\xi_0}$$

$$\lambda_1 = \xi_1 \sqrt{\xi_0}$$

$$\lambda_2 = 1 - \lambda_0 - \lambda_1 = \sqrt{\xi_0}(1 - \xi_1)$$



Uniformly Sampling Disk Area

- Unit disk with max radius $r = 1$

$$A_{disk} = r^2\pi = \pi$$

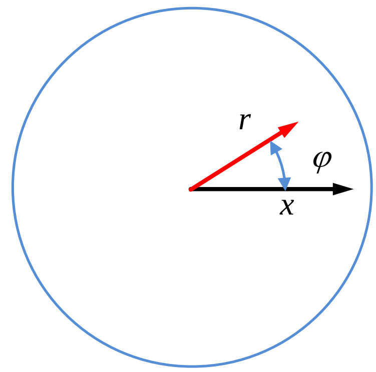
$$\int_A p(x, y)dA = 1$$

$$\rightarrow p(x, y) = 1/\pi$$

- Polar coordinates

$$p(x, y) = p(r, \varphi)/r$$

$$p(r, \varphi) = r\, p(x, y) = \frac{r}{\pi}$$



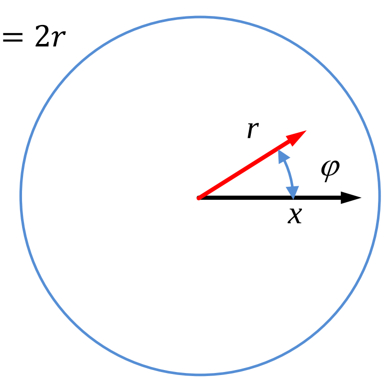
Uniformly Sampling Disk Area

- Marginal density

$$p(r) = \int_0^{2\pi} p(r, \varphi)d\varphi = \int_0^{2\pi} \frac{r}{\pi} d\varphi = 2r$$

- Conditional density

$$p(\varphi|r) = \frac{p(r, \varphi)}{p(r)} = \frac{r/\pi}{2r} = \frac{1}{2\pi}$$



Uniformly Sampling Disk Area

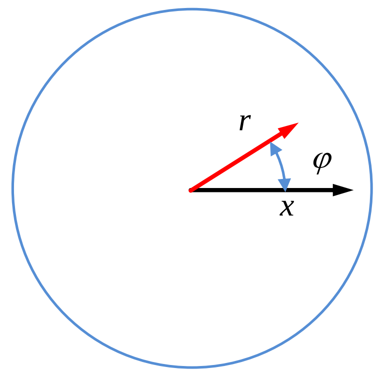
- Cumulative distribution

$$P(r) = \int p(r)dr = 2\frac{r^2}{2} = r^2$$

- Invert

$$\hat{\xi}_0 = r^2$$

$$\rightarrow r = \sqrt{\hat{\xi}_0}$$



Uniformly Sampling Disk Area

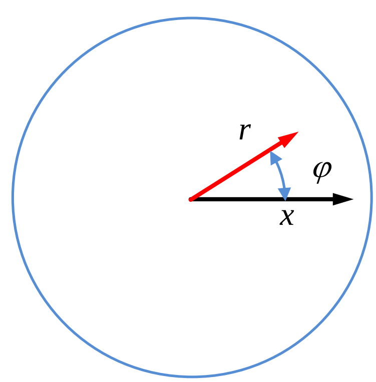
- Cumulative distribution

$$P(\varphi|r) = \int p(r,\varphi)d\varphi = \frac{\varphi}{2\pi}$$

- Invert

$$\hat{\xi}_1 = \frac{\varphi}{2\pi}$$

$$\varphi = \hat{\xi}_1\, 2\pi$$

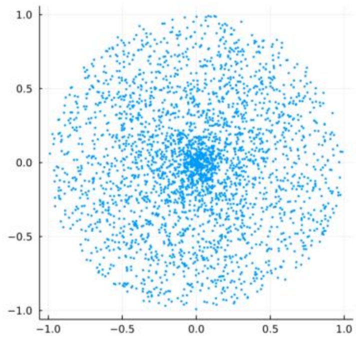


Uniformly Sampling Disk Area

- Polar to cartesian

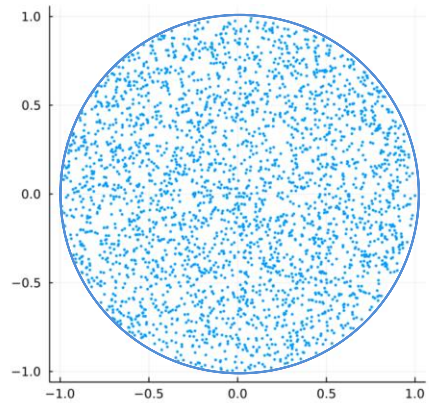
$$x = r\cos(\varphi) = \xi_0 \cdot \cos(\,\xi_1\, 2\pi\,)$$

$$y = r\sin(\varphi) = \xi_0 \cdot \sin(\,\xi_1\, 2\pi\,)$$



$$x = r\cos(\varphi) = \sqrt{\xi_0} \cdot \cos(\,\xi_1\, 2\pi\,)$$

$$y = r\sin(\varphi) = \sqrt{\xi_0} \cdot \sin(\,\xi_1\, 2\pi\,)$$



- Random Sampling
- Projects

Projects

- Select an extension to the Assignment 2 (small-pt)
  1. Add 2D texture mapping support and implement a procedural texture (2D and 3D)
  2. Speed up triangle intersections
  3. Speed up indirect illumination computation by photon mapping
  4. Add a black hole object with gravitational lens distortion
  5. Implement other 'random' generators and compare
  6. Add dispersion to the refraction computations
  7. Add a fish-eye camera model
- Suggest a topic on your own

Add 2D texture mapping support and implement a procedural texture (2D and 3D)

- Add computation of (u,v) coordinates to the sphere intersection (polar)
- Load a texture from a file an map it onto a sphere
  - Nearest pixel or linear interpolation
- Implement a procedural texture in 2D and 3D:
  - (u,v) -> RGB and (x,y,z) -> RGB
  - E.g.: Checkerboard or bricks
  - Map one sphere with the 2D the other with a 3D (solid) texture
- Optional: Add bump mapping to the material

Speed up triangle intersections

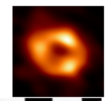
- Build a bounding sphere hierarchy for mesh objects:
  - Group neighboring triangles
  - Compute (non-tight) bounding spheres and store triangle IDs in the first sphere layer
  - Group bounding spheres of the next layer containing the lower layer spheres (and so on …)
  - Implement an intersection test hierarchically traversing down the sphere layers until it (maybe) hits a triangle
- Analyze the number of intersections also compared to the brute force version (and speed-up)

Speed up indirect illumination computation by photon mapping

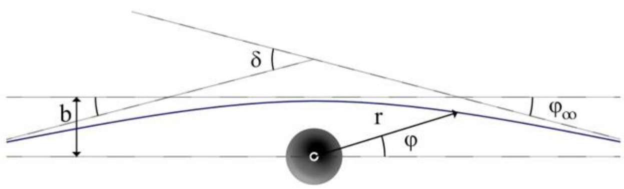
- Start from the light source to send light rays into the scene via path tracing
- Store intersection points in 3D space along with the incoming direction of the photon in a kd-tree
  - Either implement yourself or use a library such as https://github.com/flann-lib/flann
- From the camera now radiosities can be compute by a one pass intersection, and a range query into the kd-tree. Via photons, material, and cached directions compute the indirect lighting.

Add a black hole object with gravitational lens distortion

- Light is 'bend' around heavy gravitational objects
- Use an estimation function to compute the kink of a ray close to a black hole (heavy mass sphere) as approximation.



https://marcel-ritter.com/wp-content/uploads/2016/12/MR.pdf



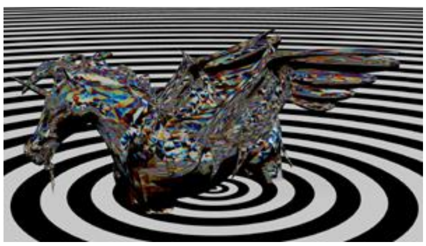
- Optional: solve the geodesic equations numerically (Schwarzschild metric)

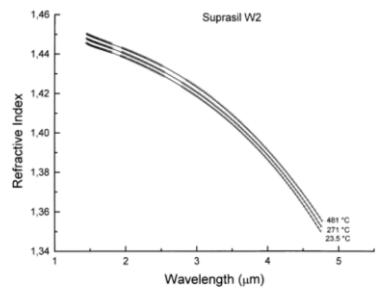
Implement other 'noise' generators and compare

- Add a window region for rendering
- Add other 'random' number generators
  - Halton23
  - Stratified
- Use instead of drand48()
- Compute a high-quality image for comparison (very large number of samples)
- Evaluate the other generators using lower sample counts with respect to the high-quality image. (e.g. visual, difference, snr, run-time)

Add dispersion to the refractions

- Implement a black and white solid texture for the walls
- Compute refractions dependent on the wavelength
  - Send multiple color-component rays per refraction





https://www.researchgate.net/publication/223868835_Temperature_dependence_of_refractive_index_of_glassy_SiO2_in_the_infrared_wavelength_range

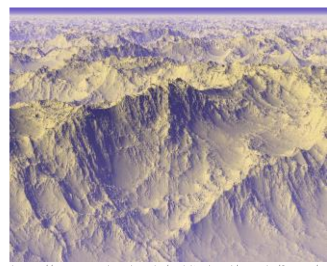
Add fish-eye lens

- Exchange the pinhole camera by a fish-eye camera
- Use a mapping of polar 2D pixel to 3D hemisphere to compute the outgoing camera ray directions
- Add a depth of field effect

Suggest on your own

- E.g. from: https://www.pbr-book.org
  - Other material models (sub surface scattering)
  - Procedural textures (noise and fractals)
  - …

https://tryingtobeananimator.wordpress.com/2017/03/20/subsurface-scattering



https://users.math.yale.edu/public_html/People/frame/Fractals/Panorama/Art/MountainsSim/Romantic/Romantic.html
https://www.classes.cs.uchicago.edu/archive/2015/fall/23700-1/final-project/MusgraveTerrain00.pdf

Practical sessions Schedule

Date	Topic	Remark
13.3.	Introduction	
20.3.	Theory 1 (Radiosity)	Hand-out PA1
27.3.	Programming 1 (Radiosity)	
	Easter Break	
17.4.	Programming 2 (Radiosity)	
24.4.	**Presentation** Assignment 1	*Hand-in PA1,* Hand-out PA2
01.5.	Staatsfeiertag	
08.5.	Programming 3 (Path Tracing)	
15.5.	Programming 4 (Path Tracing)	
22.5.	Theory 2 (Sampling), Support A1	Project
29.5.	*Pfingsten*	
5.6.	**Presentation** Assignment 2 (Meshlab?)	*Hand-in PA2*
12.6.	Theory 3 (ICP, Marching Cubes)	
19.6.	Programming support	
26.6.	**Presentation** final project	*Hand-in project*