# Database Systems

## Relational Algebra

Prof. Dr. Agnès Voisard     Muhammed-Ugur Karagülle

Institute of Computer Science, Databases and Information Systems Group

Fraunhofer FOKUS

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# Notes

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# Outline



1 Introduction

2 Operations

3 Examples

4 Limits of the relational algebra

5 Questions

Collection of operations for manipulating relations.
The result of each operation is a new relation.
2 groups of operations:

- Set operations:
  - Union ($\cup$)
  - Intersection ($\cap$)
  - Difference ($\setminus$)
  - Cartesian product ($\times$)
- Operations developed specifically for relational databases:
  - Projection ($\pi$)
  - Selection ($\sigma$)
  - Join ($\bowtie$)

## Reference relations:

Relation R1

| A | B | C |
|---|---|---|
| a | b | c |
| d | a | f |
| c | b | d |

Relation R2

| D | E | F |
|---|---|---|
| b | g | a |
| d | a | f |

# Reference relations (cont'd)

HOBBY

| PersonName | Age | HobbyName |
|---|---|---|
| Jane | 24 | Fly fishing |
| Melony | 25 | Singing |
| Brian | 29 | Tennis |
| Brian | 29 | Jogging |
| Charlie | 31 | Dancing |
| Steve | 24 | Singing |

# Projection ($\Pi$)

Unary operation.

Elimination of some attributes of a relation.

$\Pi_{<Attribute-List>}$(Relation)

$$\Pi_X(r) = \{ t[X] \mid t \in r \}, X \subseteq R.$$

# Examples of projection



**Example:**

$\Pi_{A,C}(R1)$

| A | C |
|---|---|
| a | c |
| d | f |
| c | d |

# Examples of projection (cont'd)



**Example:**

$\Pi_{PersonName,Age}$ (HOBBY). Duplicate elimination.

| PersonName | Age |
|---|---|
| Jane | 24 |
| Melony | 25 |
| Brian | 29 |
| ~~Brian~~ | ~~29~~ |
| Charlie | 31 |
| Steve | 24 |

Get the tuples of R that satisfy a certain condition C.

$\sigma_{<C>}$(R).

Condition: logical formula made up of clauses of the form:

$A_i \; \theta \; A_j$ HobbyName="Singing"

$A_i \; \theta \; k$

where $A_i$, $A_j$ are attributes of R,

k constant value from the domain of $A_i$,

$\theta \in \{=, <, \leq, >, \geq, \neq\}$.

Clauses connected by the Boolean operators "and" ($\wedge$), "or" ($\vee$).
The negation "not" ($\neg$) can also be used.

$$Cond = (A_i = k),$$
$$\sigma_{A_i=k}(r) = \{t \in r \mid t[A_i] = k\}.$$

# Examples of selections

**Example:**

$\sigma_{B=b}(R1)$.

| A | B | C |
|---|---|---|
| a | b | c |
| c | b | d |

**Example:**

Select the *people who are singing*: in the HOBBY database:

$\sigma_{HobbyName=\text{``Singing''}}$(HOBBY).

| PersonName | Age | HobbyName |
|---|---|---|
| Melony | 25 | Singing |
| Steve | 24 | Singing |

**Example:**

*Who are the 24 year-old people who are singing?*:

$\sigma_{\textit{Age}=24 \wedge \textit{HobbyName}=\text{“}\textit{Singing}''}$ (HOBBY).

| PersonName | Age | HobbyName |
|---|---|---|
| Steve | 24 | Singing |

Selection is **commutative**.

$\sigma_{<cond1>}(\sigma_{<cond2>}$ (R)) = $\sigma_{<cond2>}(\sigma_{<cond1>}$ (R))

Cascade of selections $\equiv$ selection with conjunctive condition (“and”).

$\sigma_{<cond1>}(\sigma_{<cond2>}(\ldots\sigma_{<condn>}))$ (R) = $\sigma_{<cond1>\wedge cond2\wedge\ldots\wedge condn>}$ (R).

Several operations one after the other:

1. One operation at a time and store the intermediate result, or
2. Single **relational algebra expression** by nesting the operations.

# Sequences of operations and renaming (cont'd)



**Example:**

*Name of the people who sing?*

HOBBY-TEMP $\leftarrow \sigma_{HobbyName=''Singing''}$(HOBBY))

RESULT $\leftarrow \Pi_{PersonName}$(HOBBY-TEMP).

**Example:**

$\Pi_{PersonName}(\sigma_{HobbyName=''Singing''}$(HOBBY).

**Renaming** attributes in intermediate and result relations: list the new attributes.

**Example:**

HOBBY-TEMP $\leftarrow \sigma_{HobbyName=''Singing''}$(HOBBY)
RESULT(Name) $\leftarrow \Pi_{PersonName}$(HOBBY-TEMP).

Input: 2 relations:

$R(A_1, \ldots, A_n)$ of arity $n$
$S(B_1, \ldots, B_m)$ of arity $m$.

Cartesian product of $R$ and $S$:

$T = R \times S$.

$R_1 \times R_2 = \{t = t_1 + t_2 \mid t_1 \in R_1, t_2 \in R_2\}$.

Schema of $T$: $T(A_1, \ldots, A_n, B_1, \ldots, B_m)$

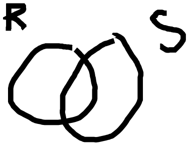
Value of $T$: set of possible $(n + m)$-tuples whose first $n$ components form a tuple in $R$ and last $m$ components form a tuple in $S$.

**Example:**

R1 X R2

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| a | b | c | b | g | a |
| a | b | c | d | a | f |
| d | a | f | b | g | a |
| d | a | f | d | a | f |
| c | b | d | b | g | a |
| c | b | d | d | a | f |

Input: 2 relations:




$R(A_1, \ldots, A_n)$ and
$S(B_1, \ldots, B_m)$

- Same arity
- Compatible domains

$T = R \cup S$.

Schema of $T$: $T(A_1, \ldots, A_n)$ or $T(B_1, \ldots, B_m)$ or other attribute names with same domain.

Value of $T$: $\{t \mid t \in R \text{ or } t \in S\}$,

**Example:**

R1 U R2

| | | |
|---|---|---|
| a | b | c |
| d | a | f |
| c | b | d |
| b | g | a |
| ~~d~~ | ~~a~~ | ~~f~~ |

# Difference (-)



Input: 2 relations:

$R(A_1, \ldots, A_n)$ and
$S(B_1, \ldots, B_m)$

- Same arity
- Compatible domains

$T = R - S$.

Schema of $T$: $T(A_1, \ldots, A_n)$ or $T(B_1, \ldots, B_m)$ or other attribute names with same domain.

Value of $T$: $\{t \mid t \in R \text{ and } t \notin S\}$

**Example:**

R1 - R2

| | | |
|---|---|---|
| a | b | c |
| c | b | d |

# Intersection ($\cap$)



Input: 2 relations:

$R(A_1, \ldots, A_n)$ and
$S(B_1, \ldots, B_m)$

- Same arity
- Compatible domains

$T = R \cap S$

Schema of $T$: $T(A_1, \ldots, A_n)$ or $T(B_1, \ldots, B_m)$ or other attribute names with same domain.

Value of $T$: $\{t \mid t \in R \text{ and } t \in S\}$

**Example:**

R1 ∩ R2

| | | |
|---|---|---|
| d | a | f |

Input: 2 relations with at least one attribute in common.

R1

| A | B | C |
|---|---|---|
| a | b | c |
| d | b | c |
| b | b | f |
| c | a | d |

R2

| B | C | D |
|---|---|---|
| b | c | d |
| b | c | e |
| a | d | b |

R1 $\bowtie$ R2

| A | B | C | D |
|---|---|---|---|
| a | b | c | d |
| a | b | c | e |
| d | b | c | d |
| d | b | c | e |
| c | a | d | b |

$R \bowtie S$:

1 Compute $R \times S$ (schema (A,B,C,B,C,D))

2 For the attributes $A_k$ common to $R$ and $S$ select the tuples with the same value for $R[A_k]$ and $S[A_k]$ (here B,C).

3 For each $A_k$ project out column $S[A_k]$ and keep all other attributes.

$R(A_1, A_2, \ldots, \underline{A_i}, \underline{A_{i+1}})$ and
$S(\underline{A_i}, \underline{A_{i+1}}, \ldots, A_{n-1}, A_n)$
$V$ set of attributes in common.

If $U = \{A_1, A_2, \ldots, A_i, A_{i+1}, \ldots, A_n\}$

Schema of $T$: $T(U) = T(A_1, A_2, \ldots, A_i, A_{i+1}, \ldots A_n)$.

Value of $T$: $\Pi_U(\sigma_{\forall A_k \in V, R[A_k] = S[A_k]}(R \times S))$

Input: 2 relations

$R(A_1, \ldots, A_n)$
$S(B_1, \ldots, B_k$

$R \underset{A_i \theta B_j}{\bowtie} S$:

- Tuples in $R \times S$ such that $A_i \theta B_j$ is true
- $\theta \in \{=, <, \leq, >, \geq, \neq\}$

$T = R \underset{A_i \theta B_j}{\bowtie} S$

Schema of $T : T(A_1, \ldots, A_n, B_1, \ldots, B_k)$
Value of $T = \sigma_{A_i \theta B_j}(R \times S)$

**Example:**

$R \underset{B<D}{\bowtie} S$

R

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

S

| D | E |
|---|---|
| 3 | 1 |
| 6 | 2 |

$R \underset{B<D}{\bowtie} S$

| A | B | C | D | E |
|---|---|---|---|---|
| 1 | 2 | 3 | 3 | 1 |
| 1 | 2 | 3 | 6 | 2 |
| 4 | 5 | 6 | 6 | 2 |

**Example:**

$R \underset{C=D}{\bowtie} S$

R

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

S

| D | E |
|---|---|
| 3 | 1 |
| 6 | 2 |

$R \underset{C=D}{\bowtie} S$

| A | B | C | D | E |
|---|---|---|---|---|
| 1 | 2 | 3 | 3 | 1 |
| 4 | 5 | 6 | 6 | 2 |

## Example:

R

| A | B |
|---|---|
| 1 | a |
| 1 | b |
| 3 | a |

S

| C | D | E |
|---|---|---|
| 1 | b | a |
| 2 | b | c |
| 4 | a | a |

$R \bowtie S$
A =< C

| A | B | C | D | E |
|---|---|---|---|---|

$R \bowtie S$
B = D

| A | B | C | D | E |
|---|---|---|---|---|

Semijoin of $R(A_1, \ldots, A_n)$ by $S$:

$R \propto S$ is the projection of $R \bowtie S$ onto the attributes of $R$:

$$R \propto S = \Pi_{A_1,\ldots,A_n}(R \bowtie S)$$

R1

| A | B | C |
|---|---|---|
| a | b | c |
| d | b | c |
| b | b | f |
| c | a | d |

R2

| B | C | D |
|---|---|---|
| b | c | d |
| b | c | e |
| a | d | b |

R1 $\ltimes$ R2

| A | B | C |
|---|---|---|
| a | b | c |
| d | b | c |
| c | a | d |

The most expensive operation of the relational algebra.

Comparing all pairs of tuples: $O(n^2)$ time (relations of size $n$, number of tuples).

One way to compute equijoin or natural joins: sort both relations and merge the lists.

Time $O(m + 2nlogn)$, $m$ number of tuples of the join (max $n^2$).

=> Avoid doing joins
Transform an algebraic expression into an equivalent one that can be evaluated faster.

=> **Query optimization techniques**.

$R$ of arity $m$,
$S$ of arity $n$.
$m > n$ and $S \neq \emptyset$.

$T = R \div S$: set of $(m-n)$-tuples $a_1, \ldots, a_{m-n}$ such that for all $n$-tuples $a_{m-n+1}, \ldots, a_m$ in $S$, the tuple $a_1, \ldots, a_m$ is in $R$.

R

| A | B | C | D |
|---|---|---|---|
| a | b | c | d |
| a | b | e | f |
| b | c | e | f |
| e | d | c | d |
| e | d | e | f |
| a | b | d | e |

S

| C | D |
|---|---|
| c | d |
| e | f |

R - S

| A | B |
|---|---|
| a | b |
| e | d |

Set of relational algebra operations $\{\sigma, \pi, \cup, -, \times\}$ is a **complete set**: any of the other relational algebra operations can be expressed as a sequence of operations from this set.

Join

$R \bowtie S \equiv \sigma_{cond}(R \times S)$

Intersection not required:

$R \cap S \equiv (R \cup S) - ((R - S) \cup (S - R))$

Relational database schema:

**EMPLOYEE**(<u>EName</u>, Salary, Dept)
**DEPARTMENT**(<u>DName</u>, ManagerName, Location)

Retrieve all employees who work in the sales department.

$\sigma_{Dept=``sales''}$ (EMPLOYEE)

Give the name of all employees.

Give the name of Miller's manager.

Give the name of the employees who work in Berkeley.

How about the query: "Display all managers of Smith"'.

## Aggregate functions

Mathematical aggregate functions on collections on values (tuples).

- SUM
- AVERAGE
- MAXIMUM
- MINIMUM or
- COUNT

## Recursive closure

Applied to **recursive relationship** between tuples of the same type.

**Example:**

Employee relation relates an employee (supervisee) to another one.

**Recursive operation**: Retrieve all supervisees e' of an employee e **at all levels**.

Relational algebra: Ok at a specific level.
=> Retrieve employees e' step-by-step and take the union.
**Problem**: Where to stop? (looping mechanism).

# Questions



# What will come next?



1 Welcome to Database Systems
2 Introduction to Database Systems
3 Entity Relationship Design Diagram (ERM)
4 Relational Model
5 Relational Algebra
6 Structured Query Language (SQL)
7 Relational Database Design - Functional Dependencies
8 Relational Database Design - Normalization
9 Online Analytical Processing + Embedded SQL
10 Data Mining
11 Physical Representation - Storage and File Structure
12 Physical Representation - Indexing and Hashing
13 Transactions
14 Concurrency Control Techniques
15 Recovery Techniques
16 Query Processing and Optimization