

Theoretical Neuroscience

Poisson process

The homogeneous Poisson process

- During a very short time interval Δt there is a fixed probability of an event (spike) to happen, independent of what happens previously.
- If r is the rate of a Poisson process, the probability of an event to happen in a short interval Δt is $r\Delta t$.
- The probability of seeing exactly n spikes in a long time interval T is given by the Poisson distribution:

$$P_T[n] = \frac{(rT)^n}{n!} \exp(-rT)$$

Derivation

- Divide time T into bins $\Delta t = T/M$
- The probability for a spike in one bin is $r\Delta t$.
- $P_T[n]$ is product of three factors:
 - Probability of generating n spikes within a specified set of M bins.
 - Probability of not generating spikes in the remaining $M-n$ bins.
 - Combinatorial factor equal to the number of ways of putting n spikes into M bins.
- Thus the probability of seeing exactly n spikes in T is:

$$P_T[n] = \lim_{\Delta t \rightarrow 0} \frac{M!}{(M-n)!n!} (r\Delta t)^n (1 - r\Delta t)^{M-n}$$

Derivation (cont.)

$$P_T[n] = \lim_{\Delta t \rightarrow 0} \frac{M!}{(M-n)!n!} (r\Delta t)^n (1 - r\Delta t)^{M-n}$$

For $\Delta t > 0$, $M = T/\Delta t$ grows without bound and thus:

$$\frac{M!}{(M-n)!} = M(M-1)(M-2) \cdots (M-n+1) \approx M^n = \frac{T^n}{\Delta t^n}$$

Further

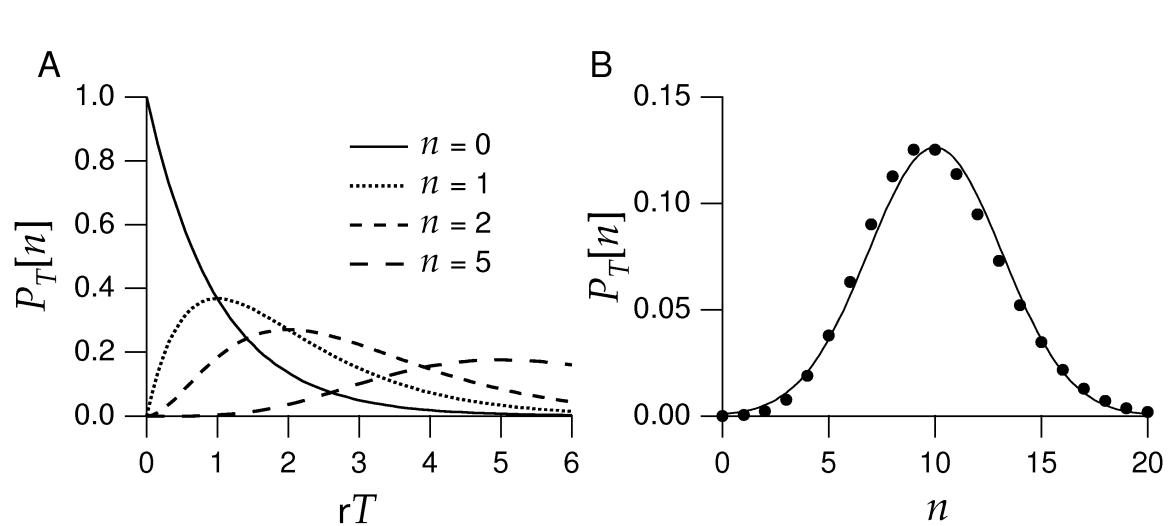
$$\ln(1 - r\Delta t)^{M-n} = (M-n) \ln(1 - r\Delta t) \approx -Mr\Delta t = -Tr$$

And so

$$(1 - r\Delta t)^{M-n} = e^{-Tr}$$

$$P_T[n] = \frac{(rT)^n}{n!} \exp(-rT)$$

Properties



$$P_T[n] = \frac{(rT)^n}{n!} \exp(-rT)$$

Properties:

Mean: $E[n] = rT$

Variance: $E[(n - E[n])^2] = rT$

Fano factor: Variance/Mean = 1

good approximation by Gaussian for large rT