Theoretical Neuroscience – WS 2025/2026

Problem Set #2

The problem set is due on Tuesday, November 3rd, 11:59 pm. Turn in your solution as a .pdf file via moodle.

1. *(3 points)* **Examples of Poisson distributions**
   Explain why you expect the following numbers are Poisson distributed, or not.
   - The number of tax returns processed by a revenue officer per day.
   - The number of meteors hitting the ground of planet earth per week during a meteor shower.
   - The size of these meteors.
   - The number of people entering a given pharmacy on a Wednesday between 18:00 and 18:10 during flu season and outside of flu season.
   - The number of school kids in a class suffering from a cold on a given day.
   - The height in cm of all the students in this class.

2. *(7 points)* **Poisson process sampling**

   (a) Generate a time series of spike times for a neuron that has a constant firing rate of $r = 100\ Hz$ for 20 s using a custom sampler of a Poisson process. Such a sample (Poisson spike generator) can be implemented as follows:
   - Chose a steps of size $\Delta t = 0.001\ s$. These form your time steps.
   - For each of these steps, draw a random number x_{rand} from the uniform distribution $U[0, 1]$.
   - If $r\Delta t > x_{\mathrm{rand}}$ in a given time step, a spike has fired; otherwise no spike has fired.

   Based on these spike time occurences, compute the spike counts for a binning interval (with bin length between 1 and 100 ms). Repeat this calculation for 5 interval lengths.
   Next, compute the Fano factor[1] for these spike counts. What value for the Fano factor did you expect? How does your calculated value compare to your expectation? (Hint: 10 % deviation from the ideal value is within the numerical expectation).

   (b) Based on the spike time occurences, calculate the interspike intervals τ. Plot and interpret a histogram of the distribution of τ. Next, compute the mean $\langle \tau \rangle$ and the standard deviation σ_τ of interspike intervals. Their ratio $C_V = \frac{\sigma_T}{\langle \tau \rangle}$ is called coefficient of variation. A Poisson process has a $C_V = 1$. How well does your simulation approximate this value? What parameters of your simulation can be changed to improve this approximation?

   (c) Which metric does the fano factor describe, which the coefficient of variation? Why does it make sense to evaluate both?

Additional resources
Please see Neuromatch tutorials (Computational Neuroscience course, W0D5) for a basic ideas from probability and statistics. See Section 2.2 for Poisson distributions.

[1] Defined as variance of the spike count in the different bins over the mean spike count