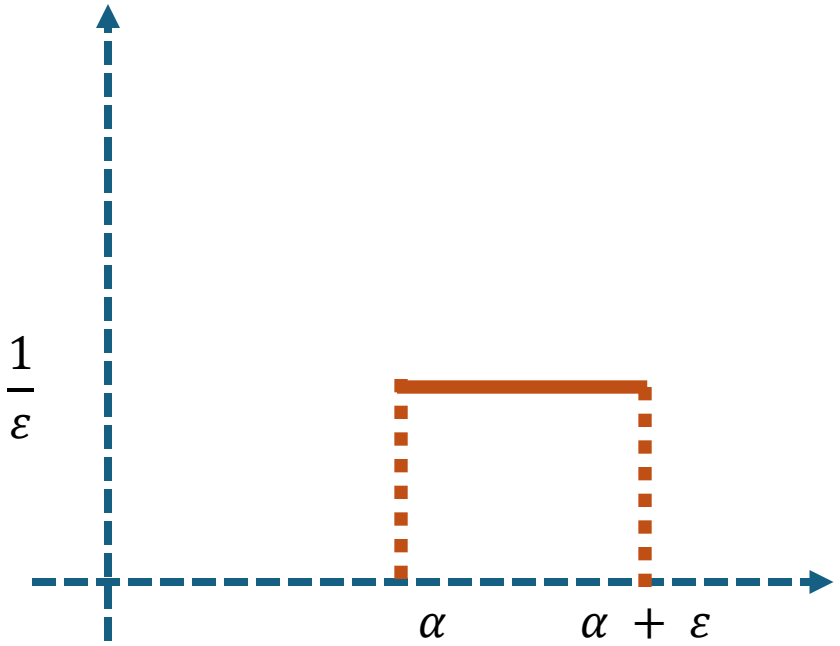
Theoretical Neuroscience 1

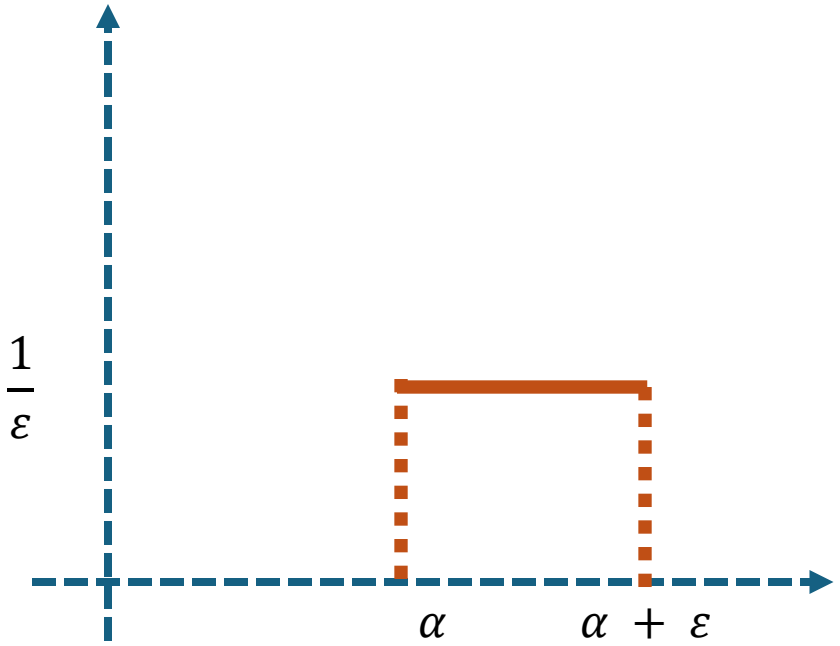
Dirac Delta

Dirac "Function" Idea



$$\delta_{a,\varepsilon}(t) = \begin{cases} \frac{1}{\varepsilon}, & t \in (a, a+\varepsilon) \\ 0, & \text{else} \end{cases}$$

Dirac Integral

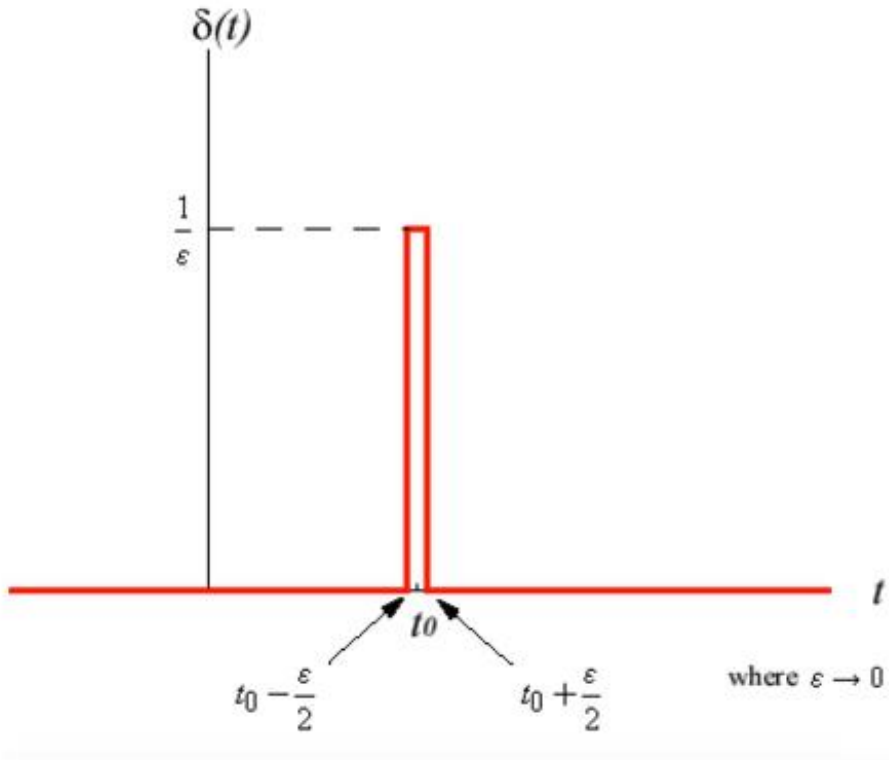


$$\int_0^\infty \delta_{a,\varepsilon}(t)\, dt = 1$$

Definitions

Continuous

- Dirac delta (unit pulse) $\delta(x) = \lim_{b \to 0} \frac{1}{|b|\sqrt{\pi}} e^{-(x/b)^2}$
  - Alternative definition: $\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ipx}\, dp$

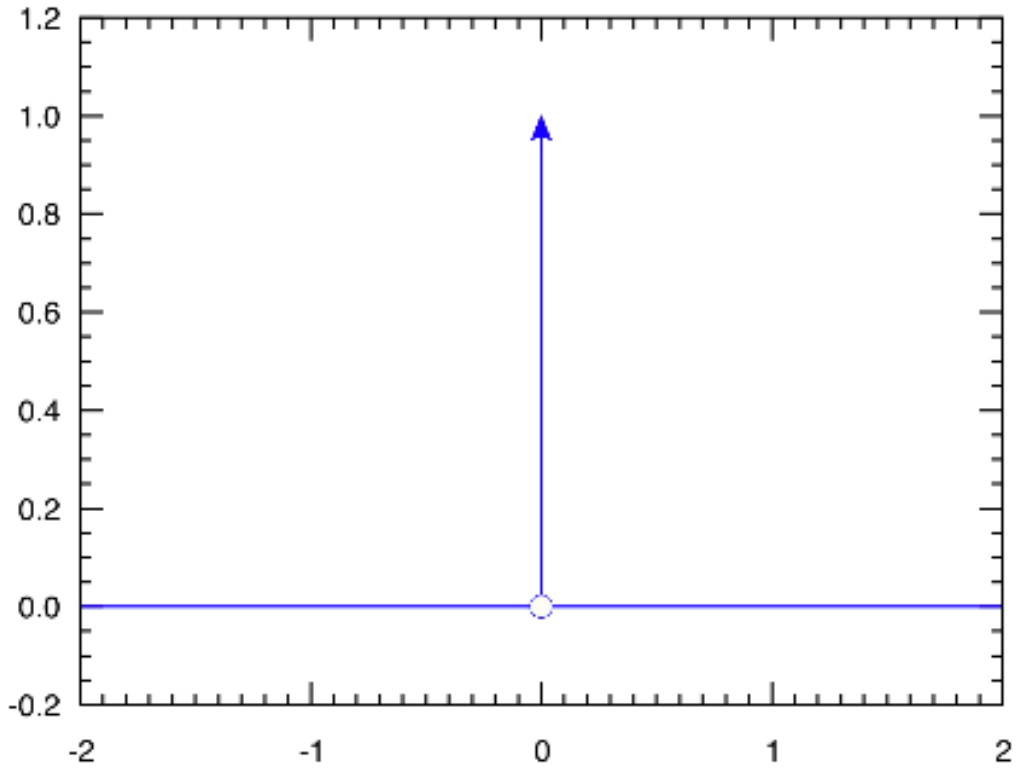


Discrete

- Kronecker delta $\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$

(Plot from wikipedia)

Dirac Sifting Property



(Plot from wikipedia)

$$\int_{a-\epsilon}^{a+\epsilon} f(x)\delta(x-a)dx = f(a)$$