Theoretical Neuroscience – WS 2025/2026

Problem Set #3

The problem set is due on Tuesday, November 11th, 11:59 pm. Turn in your solution as a .pdf file via moodle.

1. **Receptive fields** *(2 points)*
   Explain the different definitions of a receptive field and give an example for each of them form different sensory systems (e.g. auditory and visual).

2. **Data analysis** *(4 points)*
   The file `c2p3.mat`[1] contains the responses of a cat LGN cell to two-dimensional visual images[2]. In the file, the variable `counts` is a vector containing the number of spikes in each 15.6 ms bin, and the variable `stim` contains the 32767 images of size 16×16 that were presented at the corresponding times. Specifically, $\text{stim}(x, y, t)$ is the stimulus presented at the coordinate (x, y) at time-step t. Note that stim is an int8 array that must to be converted into a higher precision format before further calculations.

   (a) Calculate the spike-triggered average images for each of the 12 time steps before each spike. Note that in this example, the time bins can contain more than one spike, so the spike-triggered average must be computed by weighting each stimulus by the number of spikes in the corresponding time bin, rather than weighting it by either 1 or 0 depending on whether a spike is present or not.

   (b) Now plot the STA for each of the time steps.

   (c) Describe the spike-triggered average images and how they change over time.

   (d) Take the sum of the images across one spatial dimension (e.g. y) and show the time-development of this one-dimensional variable in a figure (similar to figure 2.25 (c) in Dayan&Abbott, p. 78).

3. **Lagrange Multipliers** *(4 points)*
   In neuroscience many problems involve optimizing a function under certain biological or physical constraints. For example a neuron might aim to maximize signal transmission while minimizing energy consumption. For this, Lagrange Multipliers provide a powerful mathematical framework for finding the optimal solution to a problem subject to constraints.
   In the following we present a step-by-step instruction for optimizing a function $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$:

   - Define the Lagrangian of the system: $\mathcal{L} = f(x, y, z) + \lambda[g(x, y, z) - k]$ where λ is the Lagrange Multiplier.
     (Note: You can add more constraints with λ_1, λ_2...)
   - Calculate the derivatives: $\frac{\partial \mathcal{L}}{\partial x}, \frac{\partial \mathcal{L}}{\partial y}, \frac{\partial \mathcal{L}}{\partial z}, \frac{\partial \mathcal{L}}{\partial \lambda}$
   - Set the four equations from above to zero (at an optimum the derivative vanishes) and solve for x, y, z and λ.

   As an exercise, optimize the following system using the above step-by-step instruction:

$$f(x, y, z) = xyz, \quad \text{with: } x^2 + y^2 + z^2 = 1 \tag{1}$$

[1] see `scipy.io.loadmat` for loading

[2] these data are described in Kara, P, Reinagel, P, & Reid, RC (2000) 'Low response variability in simultaneously recorded retinal, thalamic, and cortical neurons'. Neuron 30:803-817 and were kindly provided by Clay Reid