Theoretical Neuroscience

Poisson process

The homogeneous Poisson process

- During a very short time interval Δt there is a fixed probability of an event (spike) to happen, independent of what happens previously.
- If r is the rate of a Poisson process, the probability of an event to happen in a short interval Δt is $r\Delta t$.
- The probability of seeing exactly n spikes in a long time interval T is given by the Poisson distribution:

$$P_T[n] = \frac{(\mathrm{r}T)^n}{n!}\exp(-\mathrm{r}T)$$

Derivation

- Divide time T into bins $\Delta t = T/M$
- The probability for a spike in one bin is $r\Delta t$.
- $P_T[n]$ is product of three factors:
  - Probability of generating n spikes within a specified set of M bins.
  - Probability of not generating spikes in the remaining M-n bins.
  - Combinatorial factor equal to the number of ways of putting n spikes into M bins.
- Thus the probability of seeing exactly n spikes in T is:

$$P_T[n] = \lim_{\Delta t \to 0} \frac{M!}{(M-n)!n!} (\mathrm{r}\Delta t)^n (1 - \mathrm{r}\Delta t)^{M-n}$$

Derivation (cont.)

$$P_T[n] = \lim_{\Delta t \to 0} \frac{M!}{(M-n)!n!} (\mathrm{r}\Delta t)^n (1 - \mathrm{r}\Delta t)^{M-n}$$

For Δt->0, $M=T/\Delta t$ grows without bound and thus:

$$\frac{M!}{(M-n)!} = M(M-1)(M-2)\cdots(M-n+1) \approx M^n = \frac{T^n}{\Delta t^n}$$

Further

ln(1-x) = -(x^1/1 * x^2/2 *.....* x^n/n) => first order is enough to approx. for small timesteps

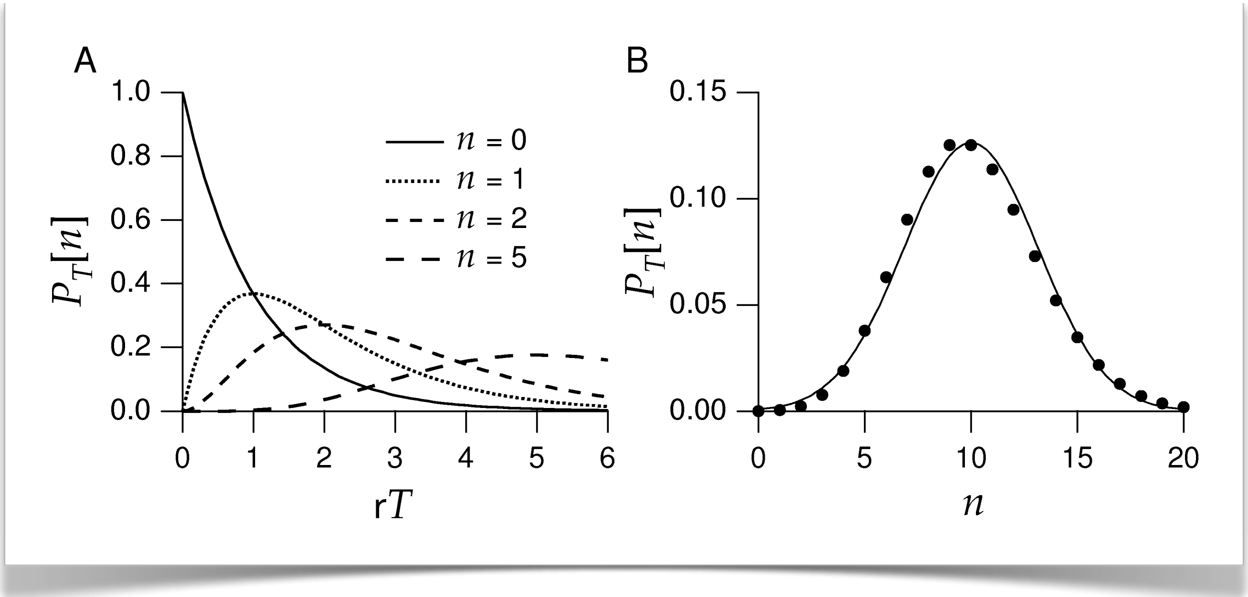
$$\ln(1 - r\Delta t)^{M-n} = (M-n)\ln(1 - r\Delta t) \approx -Mr\Delta t = -Tr$$

And so

$$(1 - r\Delta t)^{M-n} = e^{-Tr}$$

$$P_T[n] = \frac{(\mathrm{r}T)^n}{n!} \exp(-\mathrm{r}T)$$

Properties



$$P_T[n] = \frac{(\mathrm{r}T)^n}{n!} \exp(-\mathrm{r}T)$$

Properties:

- Mean: E[n] = rT
- Variance: E[(n – E[n])²] = rT
- Fano factor: Variance/Mean = 1
- good approximation by Gaussian for large rT