# Lecture 12 –Time Series Analysis

## Forecasting Time Series

Information Systems for Sustainable Society (is3) | WiSo Faculty | Univ.-Prof. Dr. Wolfgang Ketter | 1/11/23

# Agenda

- Properties of Time Series
- Applications and Examples
- Descriptive Models
- Forecasting Models

# Goals

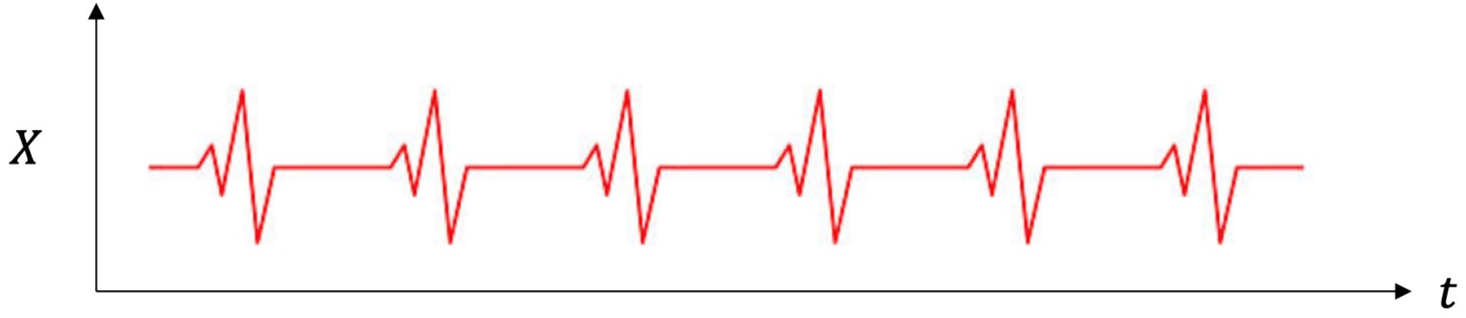
After this lecture, you will be able to:

- Explain key properties of time series data
- Describe, measure, and remove **trend** and **seasonality** from a time series
- Understand the concept of **stationarity**
- Create and interpret **autocorrelation function** (acf) plots
- Understand **ARIMA** models for forecasting
- Create your own time series forecast

# What is a time series?

A time series is a sequence of observations over time.

ECG graph measuring heart activity

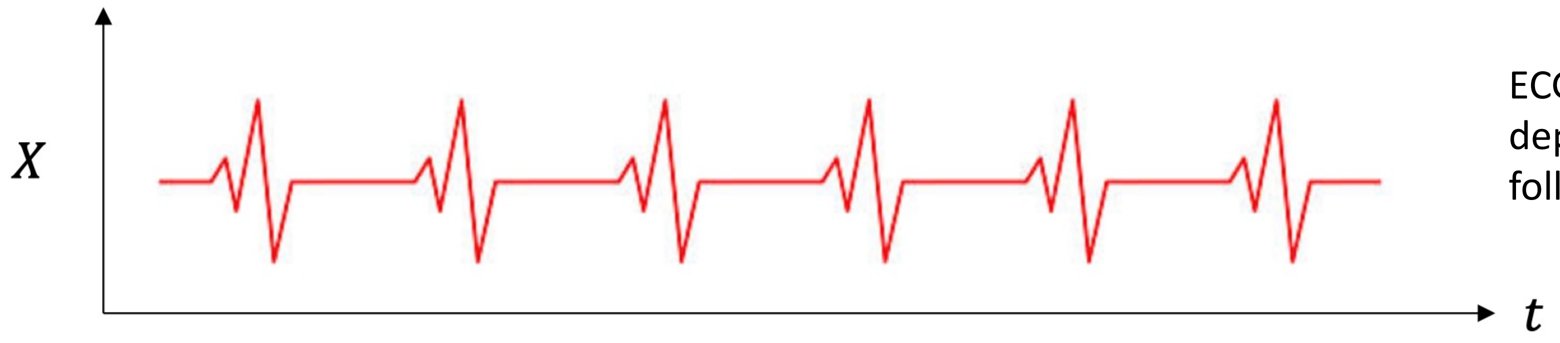



Notation: We have observations $X_1$ , … , $X_n$ , where $X_t$ denotes the observation at time $t$

In this lecture, we will consider time series with observations at equally-spaced times (not always the case, e.g., point processes, bus arrival, rain fall ).

# Dependent Observations

Each observation in a time series is **dependent** on all other observations.




ECG graph has clear dependence: peaks followed by valleys

Why is this important? Most statistical models assume that individual observations are independent.

But this assumption does not hold for time series data.

Analysis of time series data must take into account the time order of the data.

# Time Series Components

Generally, Time Series have four components:

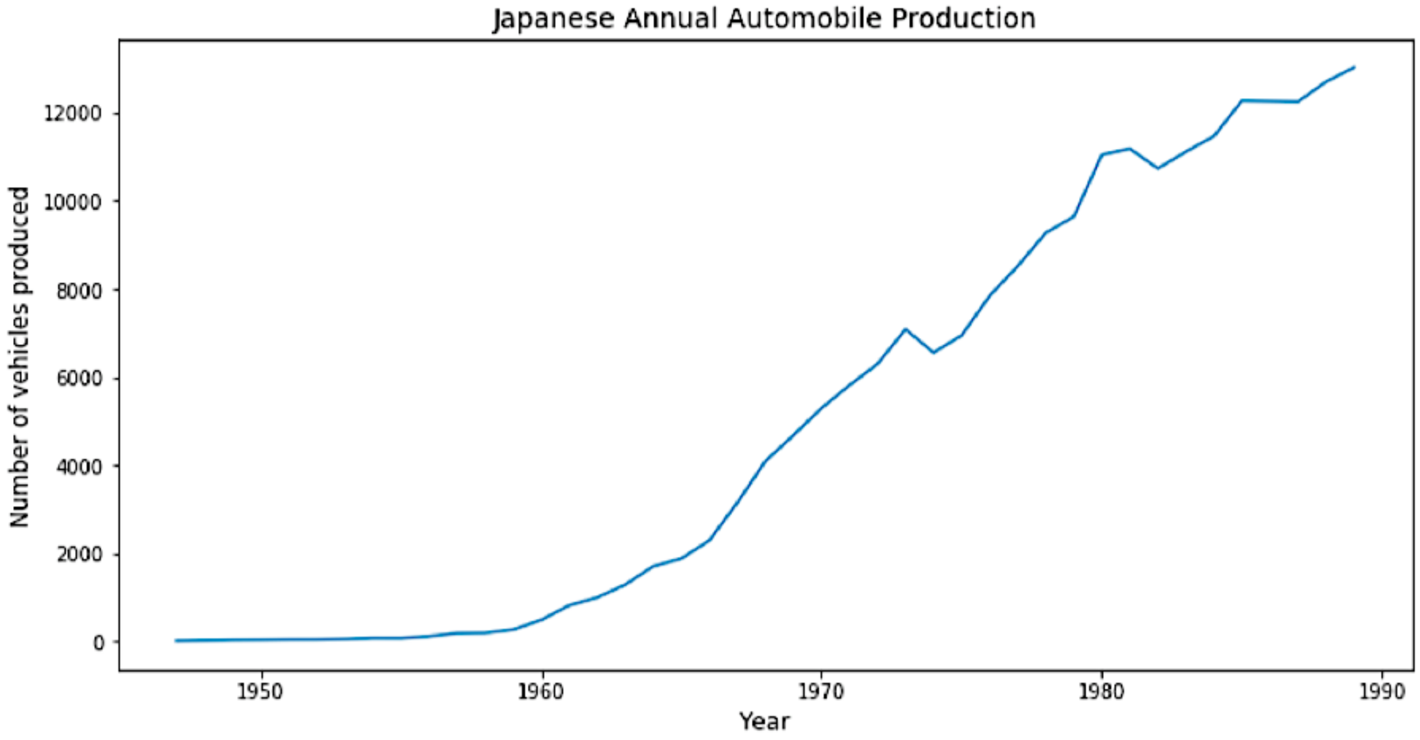
- ***Level:*** describes the average value of the series
- ***Trend:*** is the change in the series from one period to the next
- ***Seasonality:*** describes the short-term cyclical behavior of the series that is observed several times within the series
- ***Noise:*** is the random variations in the series

The first three are considered to be invisible, as they characterize the underlying series, which we only observe with added noise.

# Trend and Seasonality

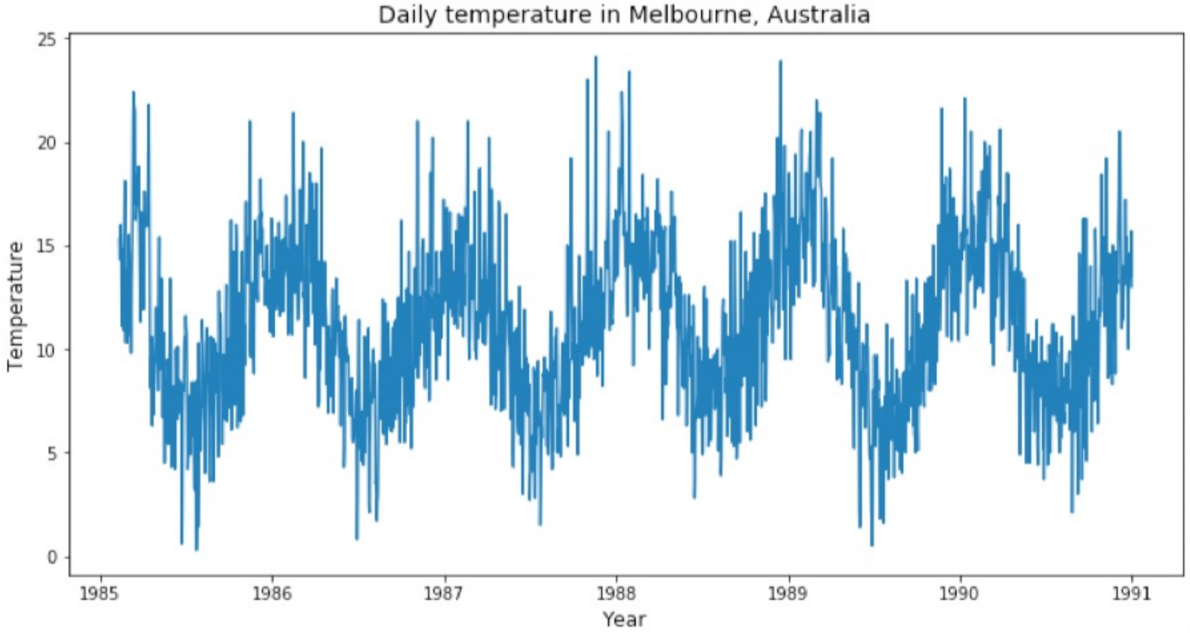
Many time series display trends and seasonal effects.

A **trend** is a change in the long term mean of the series.



# Trend and Seasonality

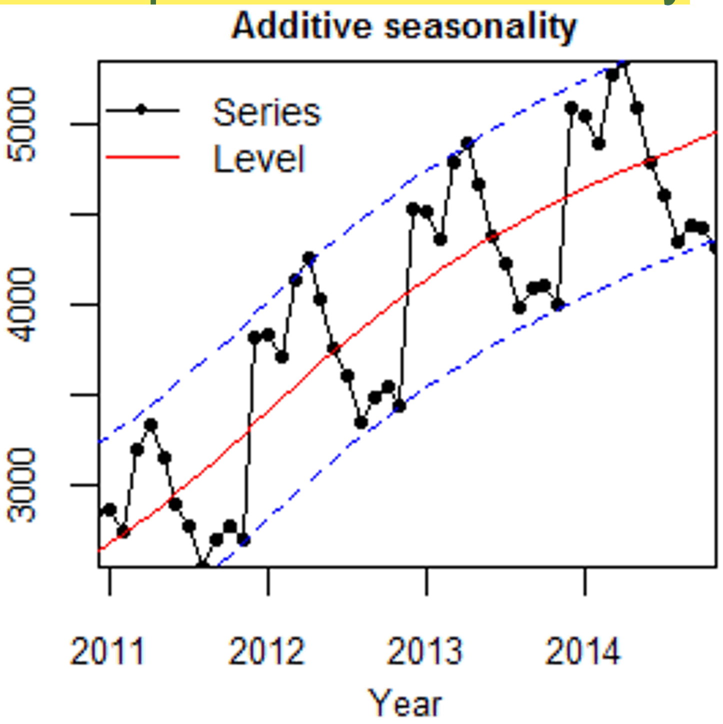
A **seasonal effect** is a cyclic pattern of a *fixed period* present in the series.

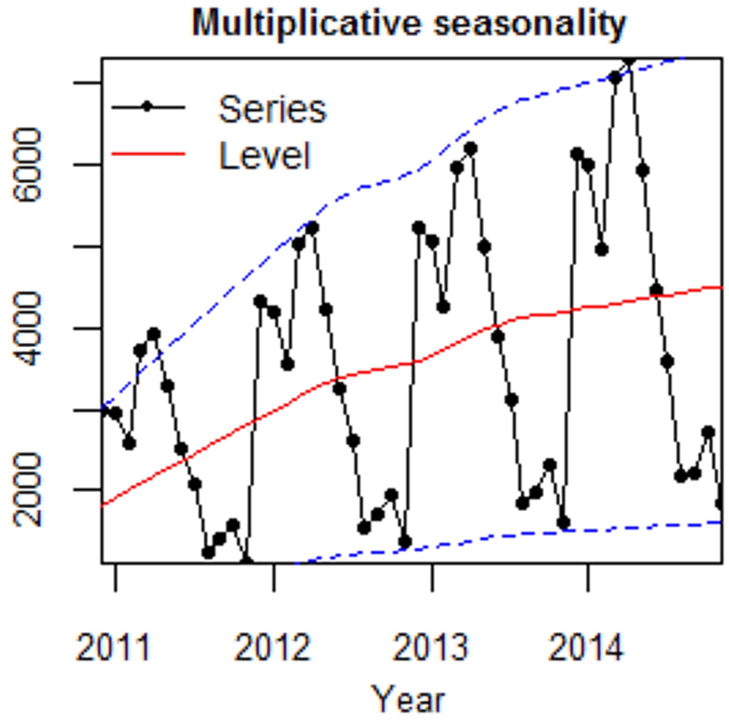



The season (or period) is the length of the cycle (e.g. an annual season).

Seasonal effect can be additive (constant over time) or multiplicative (increasing over time).

# Additive vs. Multiplicative seasonality

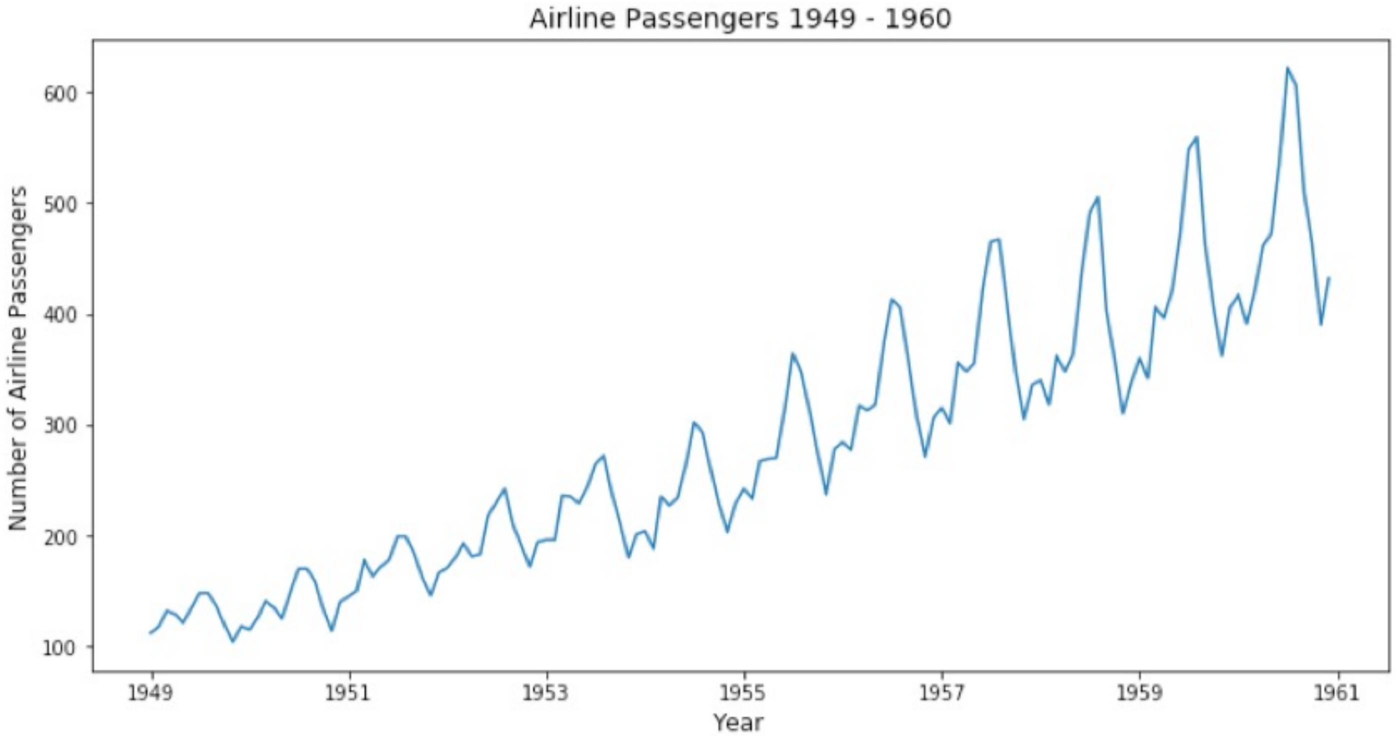



- Seasonality is a common characteristic of time series. It can appear in two forms: additive and multiplicative. In the former case the amplitude of the seasonal variation is independent of the level, whereas in the latter it is connected.
- Note that in the example of multiplicative seasonality the season is becoming “wider”. Obviously if the level was decreasing the seasonal amplitude of the multiplicative case would decrease as well.

# Trend and Seasonality

A series can have both a trend and a seasonal effect.




- Is there a trend in this data?
- Is there a seasonality effect in this data? Which type?

# Trend and Seasonality


A fun example: seasonal patterns are quite common.

This elevation while running around Park seems to have a seasonal effect!

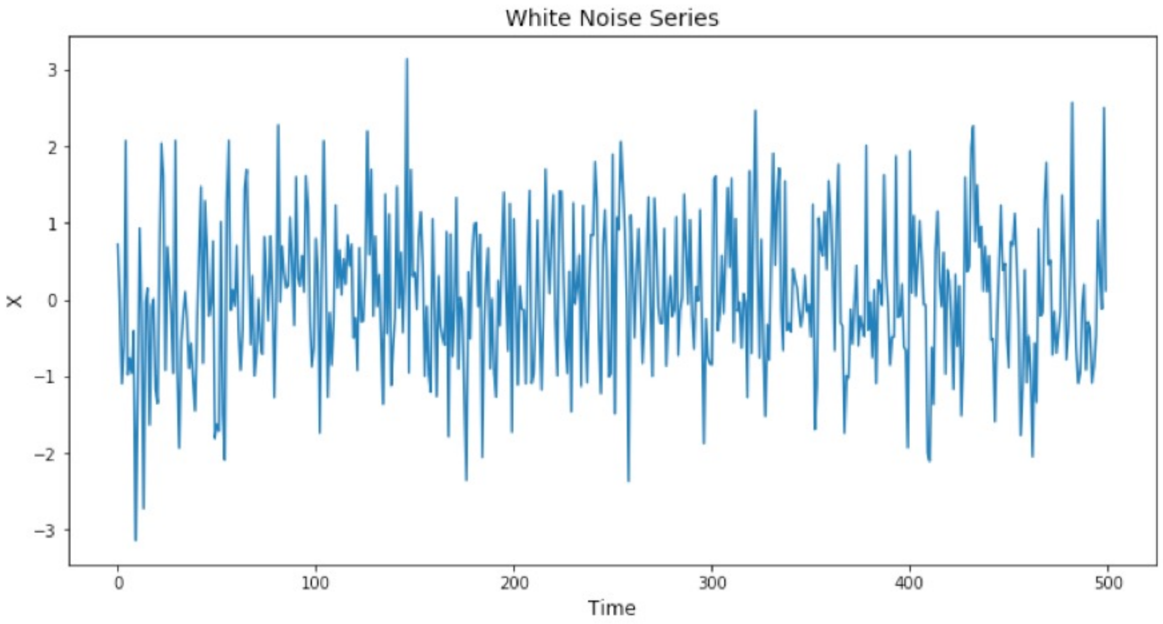
(Makes sense because running the same loop repeatedly).

Is there a seasonality effect in this graph? What is the reason? How about trend?

# Stationarity

A time series is called **stationary** if one section of the data looks like any other section of the data, in terms of its distribution.

A white noise series (sequence of random numbers) is stationary.



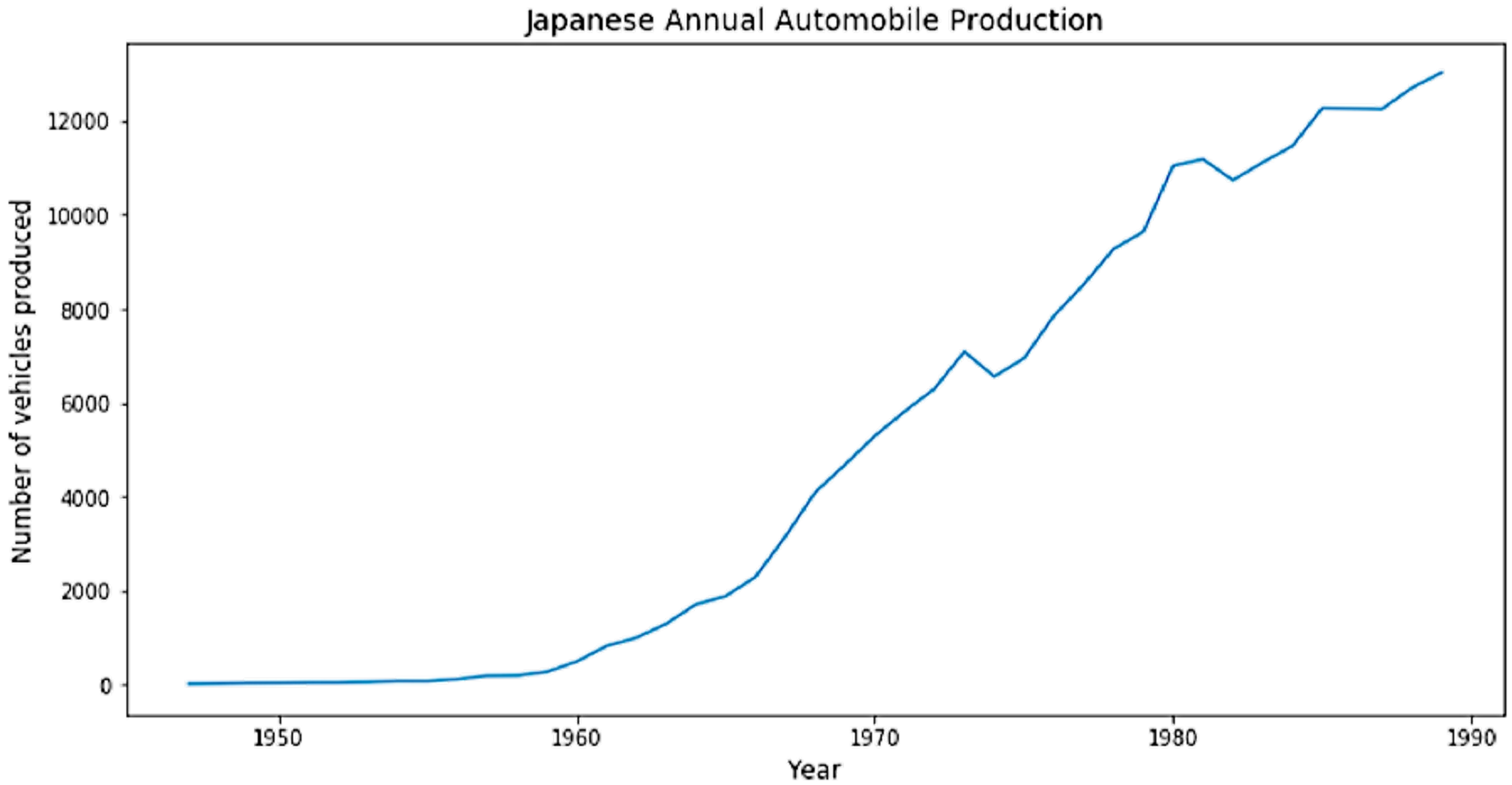

More formally, a time series is stationary if $X_{1:k}$ and $X_{t+k-1}$ have the same distribution, for all $k$ and $t$. (Every section of length $k$ has the same distribution of values).

# What is a White Noise Time Series?

- A time series may be white noise.
- A time series is white noise if the variables are independent and identically distributed with a mean of zero.
- This means that all variables have the same variance ($\sigma^2$) and each value has a zero correlation with all other values in the series.
- White noise is an important concept in time series analysis and forecasting. It is important for two main reasons:
  - **Predictability**: If your time series is white noise, then, by definition, it is random. You cannot reasonably model it and make predictions.
  - **Model Diagnostics**: The series of errors from a time series forecast model should ideally be white noise.
- When forecast errors are white noise, it means that all of the signal information in the time series has been harnessed by the model in order to make predictions. All that is left is the random fluctuations that cannot be modelled.

# Stationarity

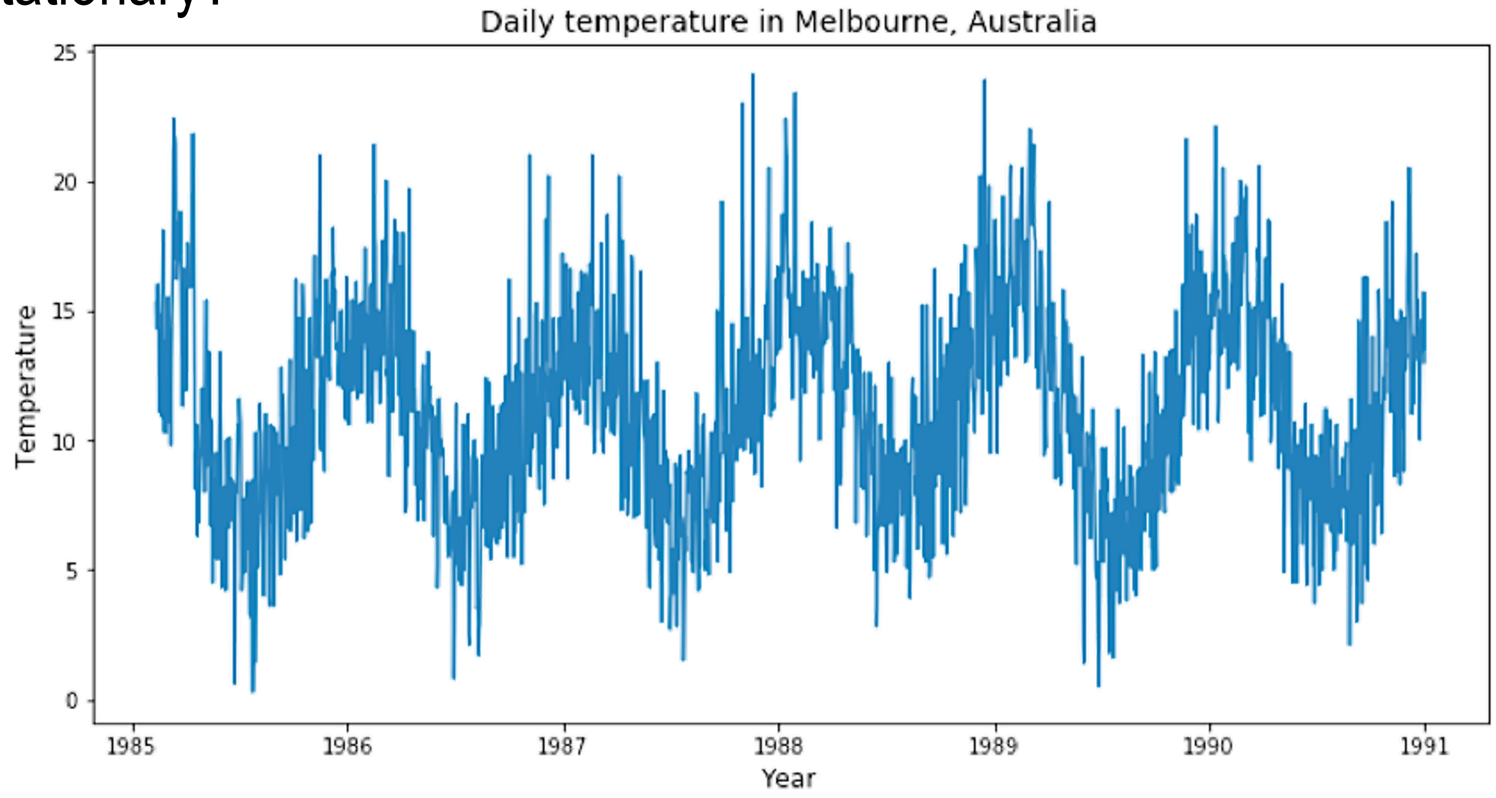
Is this time series stationary?




No, a series with a trend is non-stationary.

# Stationarity

Is this time series stationary?




No, a series with seasonality is non-stationary.

# Stationary Series

**Definition**: A series $x_t$ is said to be **(weakly) stationary** if it satisfies the following properties:

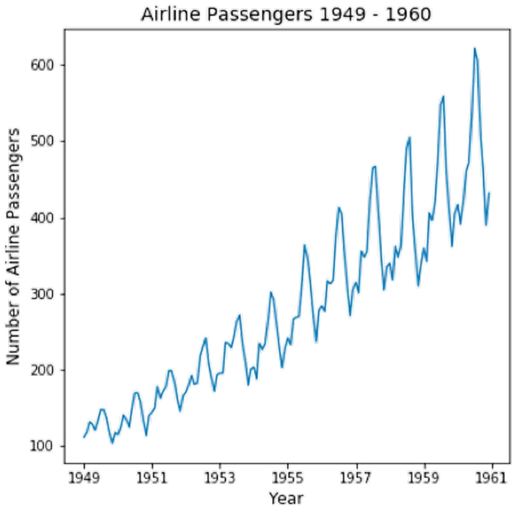
- The mean $E(x_t)$ is the same for all $t$.
- The variance of $x_t$ is the same for all $t$.
- The covariance (and also correlation) between $x_t$ and $x_{t-h}$ is the same for all $t$.

**Definition**: Let $x_t$ denote the value of a time series at time $t$. The Auto Correlation Function (ACF) of the series gives correlations between $x_t$ and $x_{t-h}$ for h = 1, 2, 3, etc. Theoretically, the autocorrelation between $x_t$ and $x_{t-h}$ equals

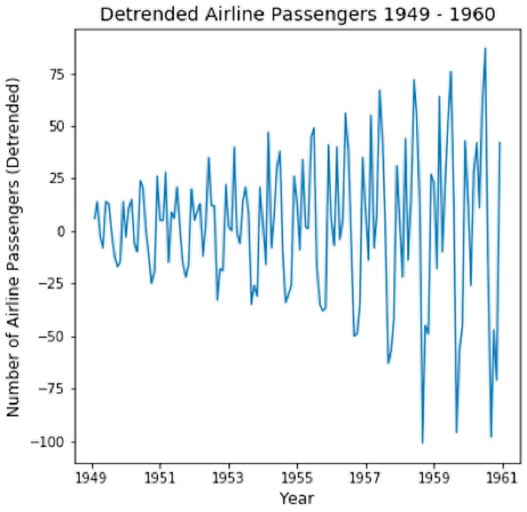
$$\frac{\text{Covariance}(x_t, x_{t-h})}{\text{Std.Dev.}(x_t)\text{Std.Dev.}(x_{t-h})} = \frac{\text{Covariance}(x_t, x_{t-h})}{\text{Variance}(x_t)}$$

# Stationarity

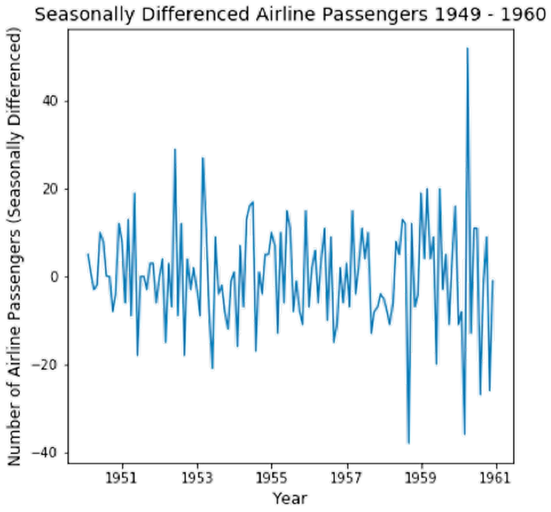
It’s often useful to transform a non-stationary series into a stationary series for modelling.




Original series




Removing trend
(First-order differencing)




Removing seasonality
(Seasonal differencing)

This is stationary

For removing the trend, we can fit a line to the data and we subtract the series values from that line. The result is the detrend residuals. But these residuals have seasonality effect. Once we remove the seasonality effect, we would have stationary series.

# Agenda

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- Applications and Examples
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- Forecasting Models

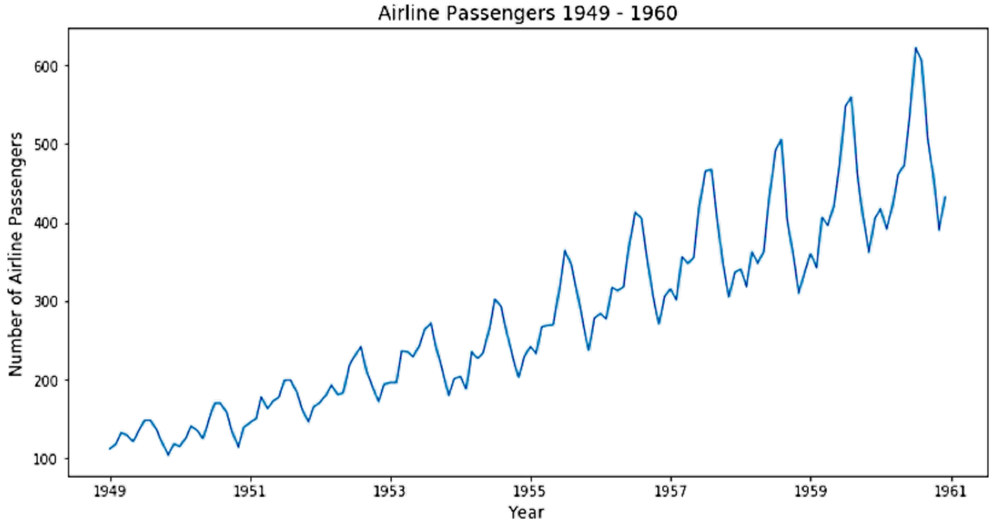
# Applications of Time Series

A few applications of time series data:

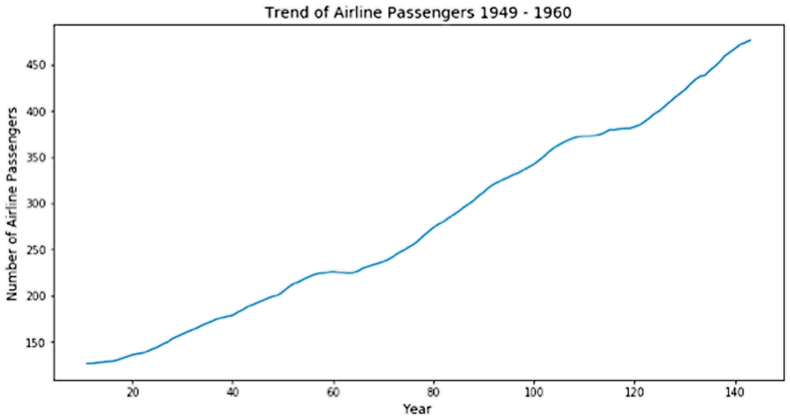
- Description
- Explanation
- Control
- Forecasting

# Application: description

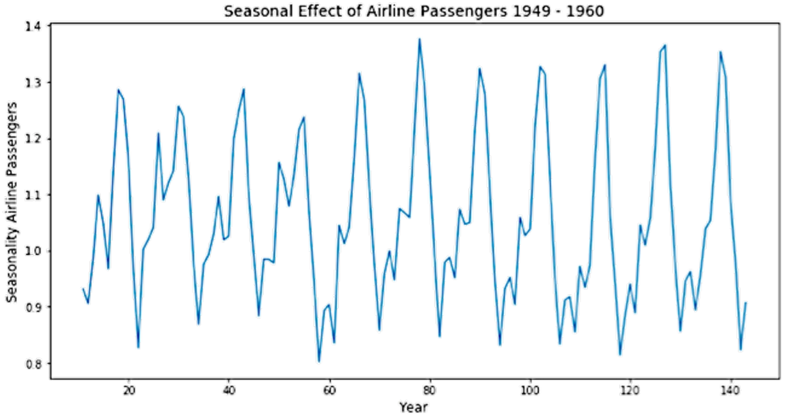
Can we identify and measure the trends, seasonal effects, and outliers in the series?




Original Series



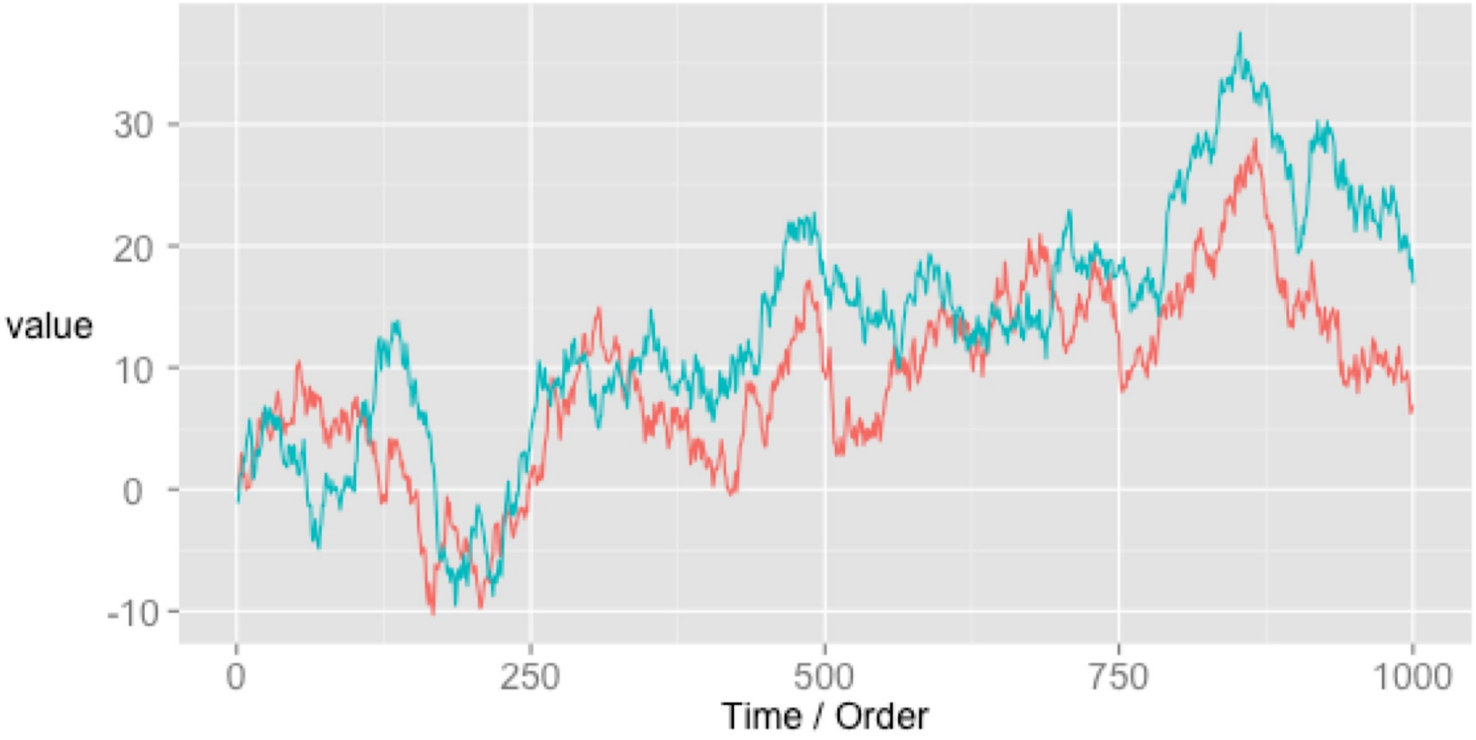

Trend component




Seasonal component

# Application: explanation

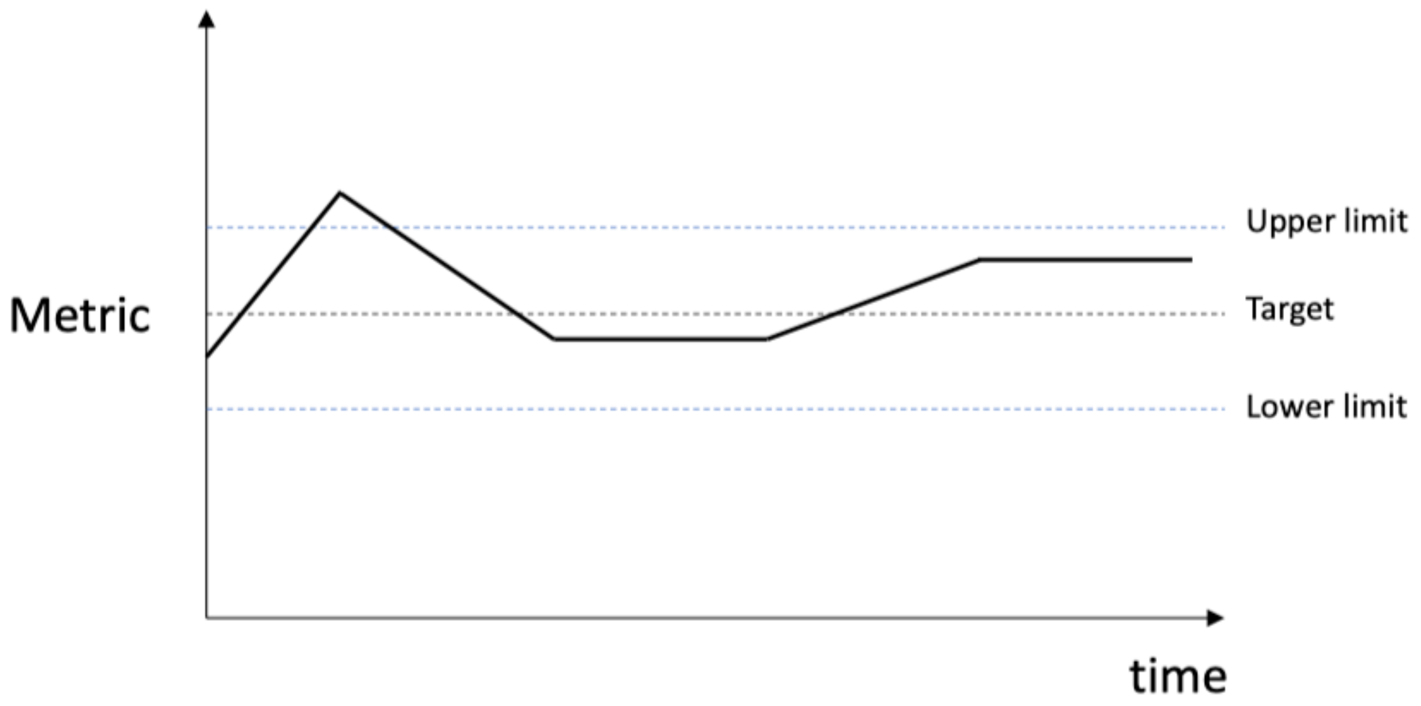
Can we use one time series to explain/predict values in another series?




Model using linear systems: convert one series to another using linear operations.

# Application: control

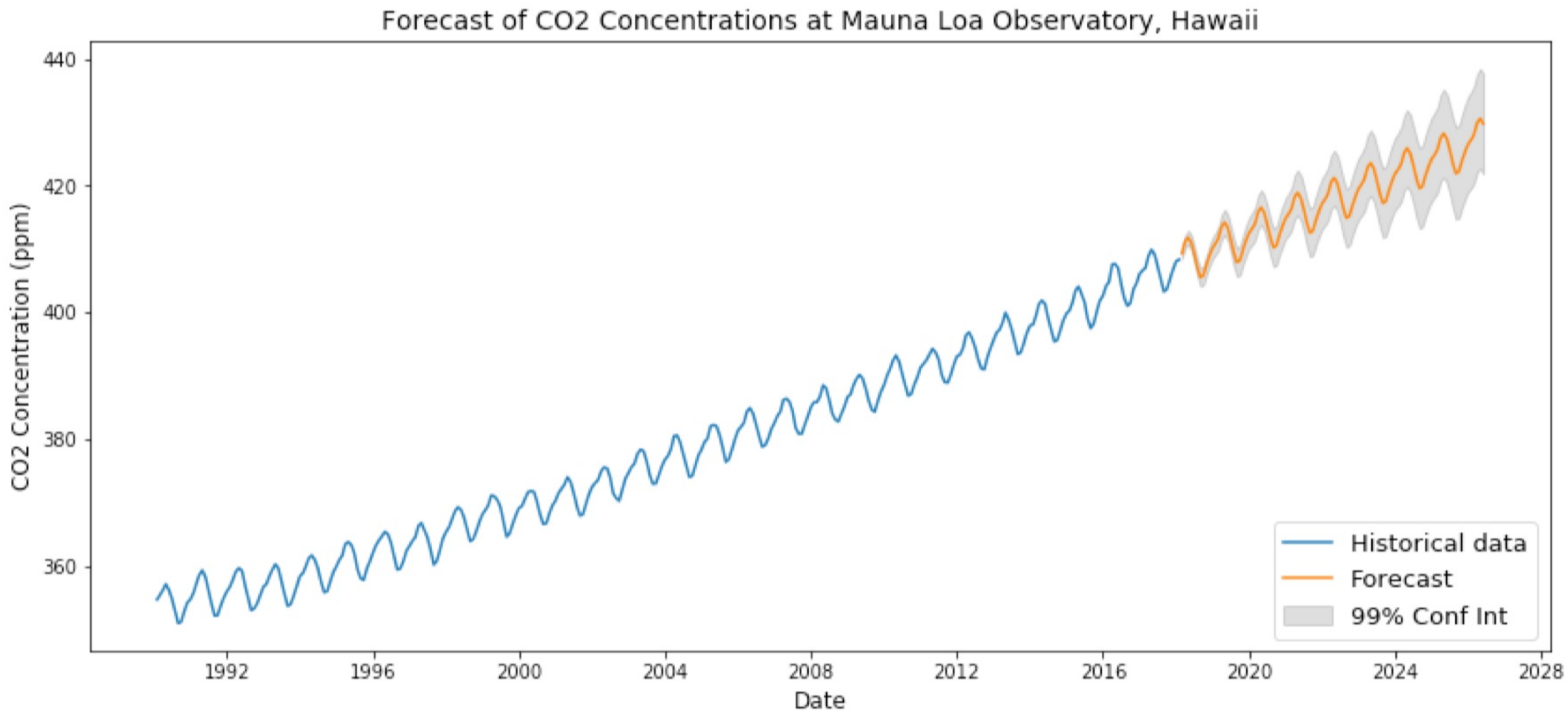
Can we identify when a time series is deviating away from a target?




Example: Manufacturing quality control

# Application: forecasting

Using observed values, can we predict future values of the series?



# Applications of Time Series

In this lecture:

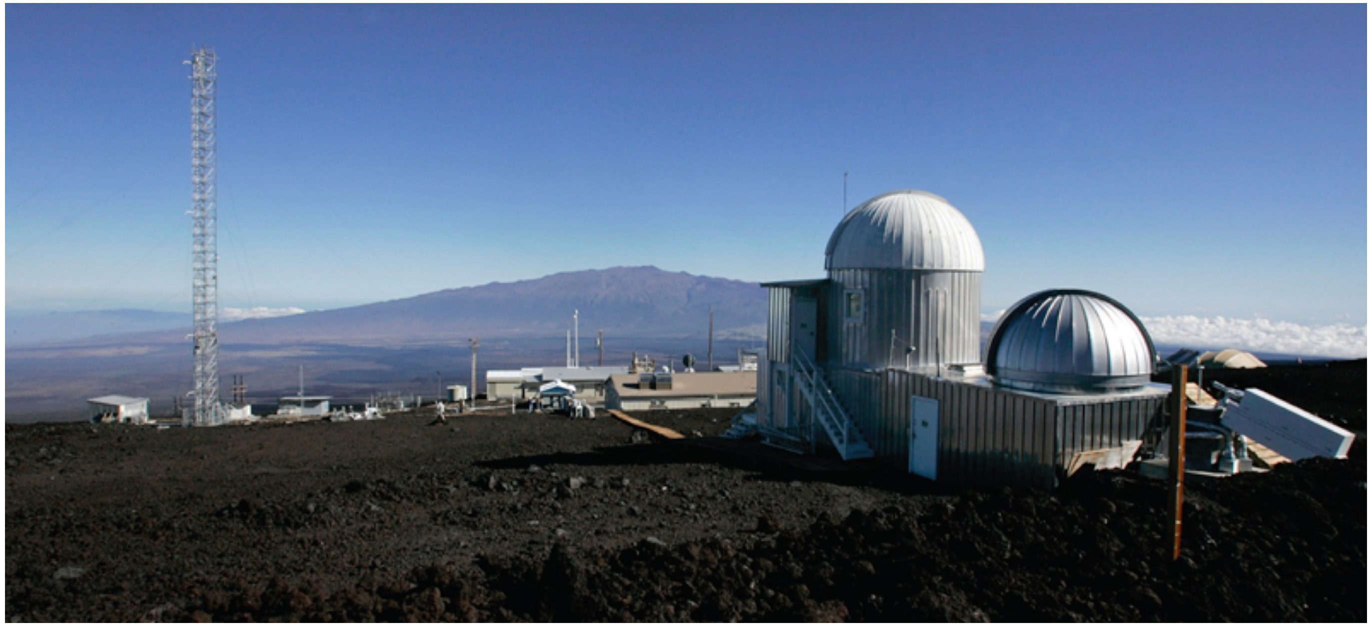
- Description

  Can we identify and measure the trends, seasonal effects, and outliers in the series?
- Explanation
- Control
- Forecasting

  Using observed values, can we predict future values of the series?

# Example: Keeling Curve

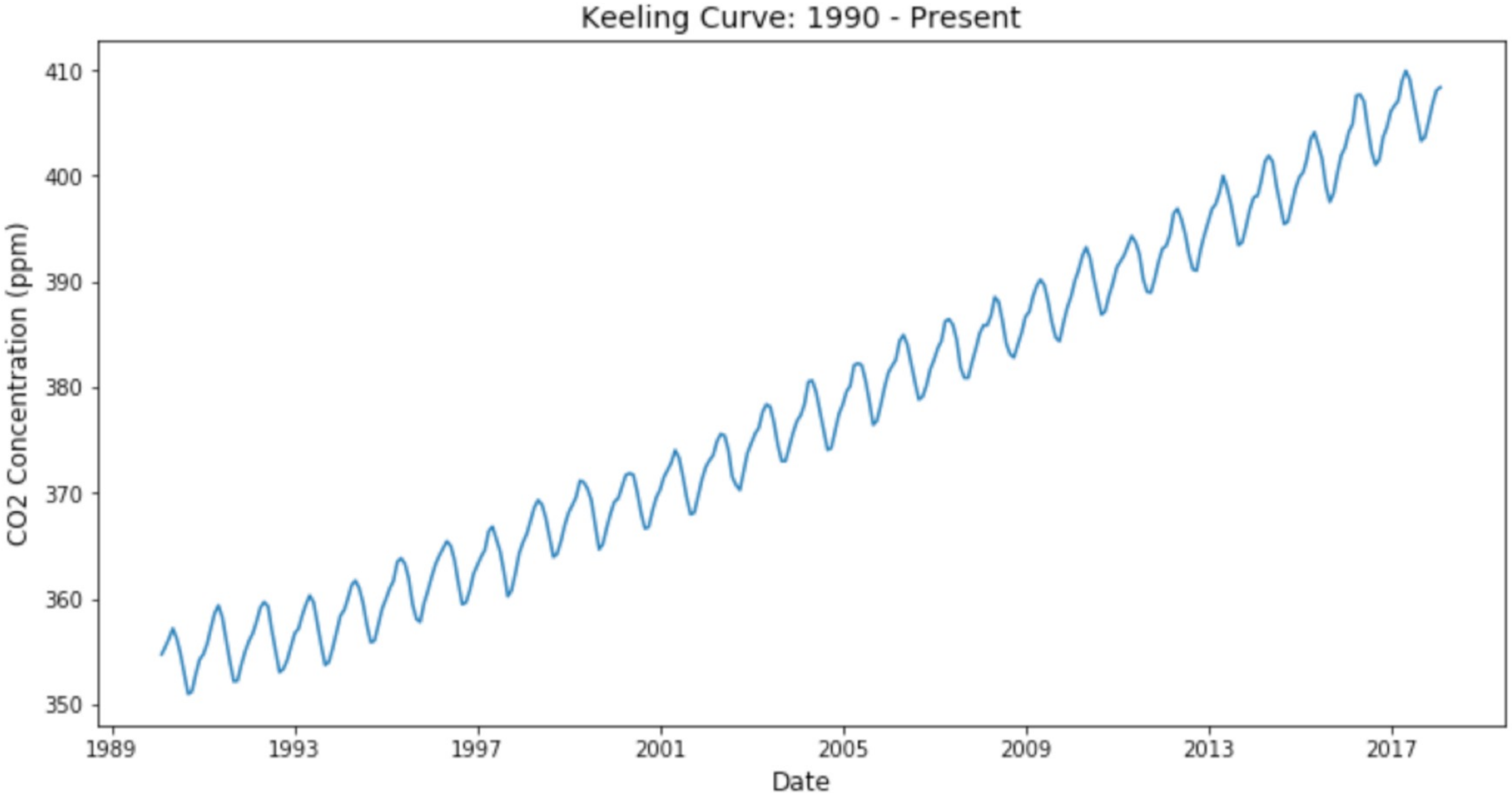
The Keeling Curve is the foundation of modern climate change research.



Daily observations of atmospheric $CO_2$ concentrations since 1958 at the Mauna Loa Observatory in Hawaii.

# Example: Keeling Curve

# Example: Keeling Curve




Why is there an annual season? Plants grow in spring, die in fall

Why is there a trend? Climate change

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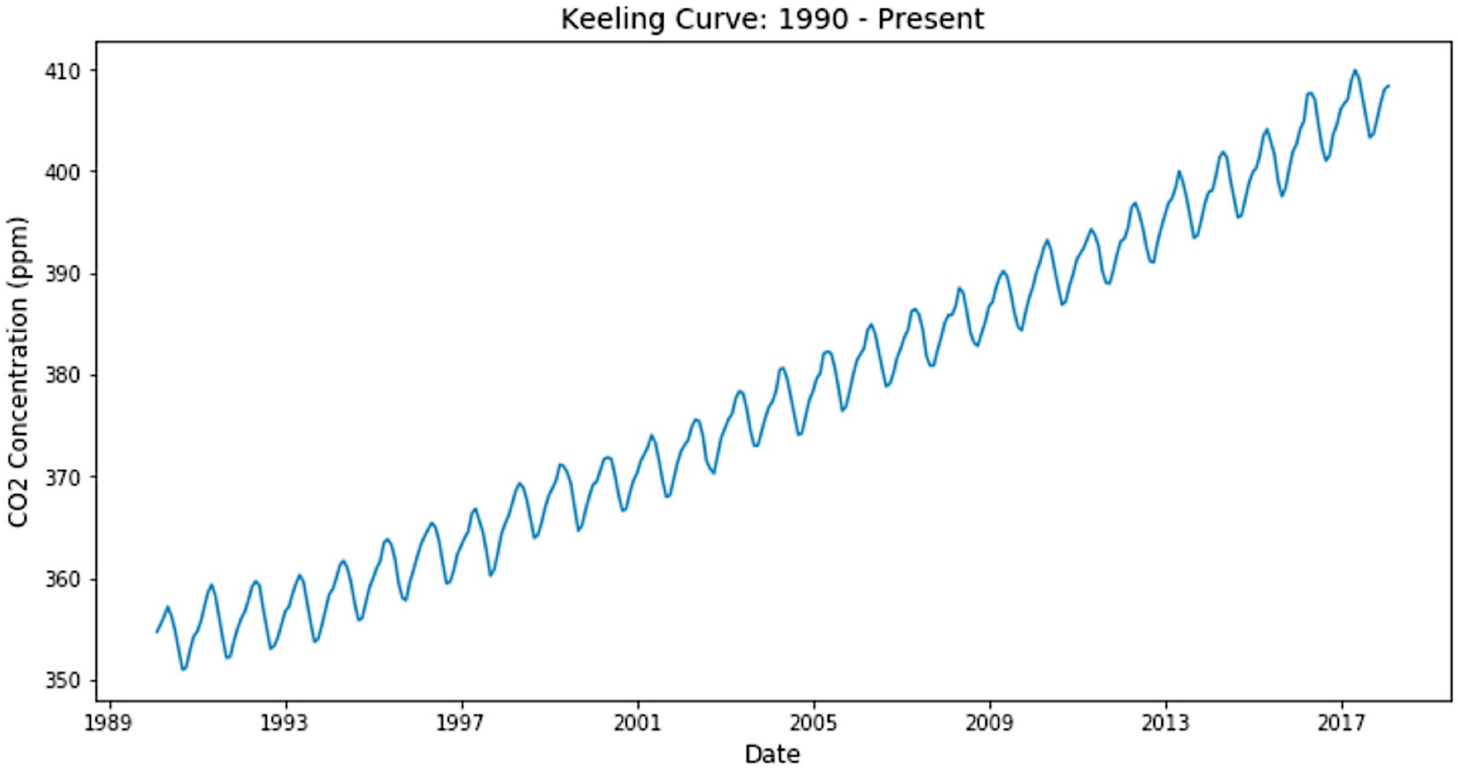
# Time plot

The first thing you should do in any time series analysis is plot the data.




Plotting helps us identify salient properties of the series:

- Trend
- Seasonality
- Outliers
- Missing data

# Measuring the trend

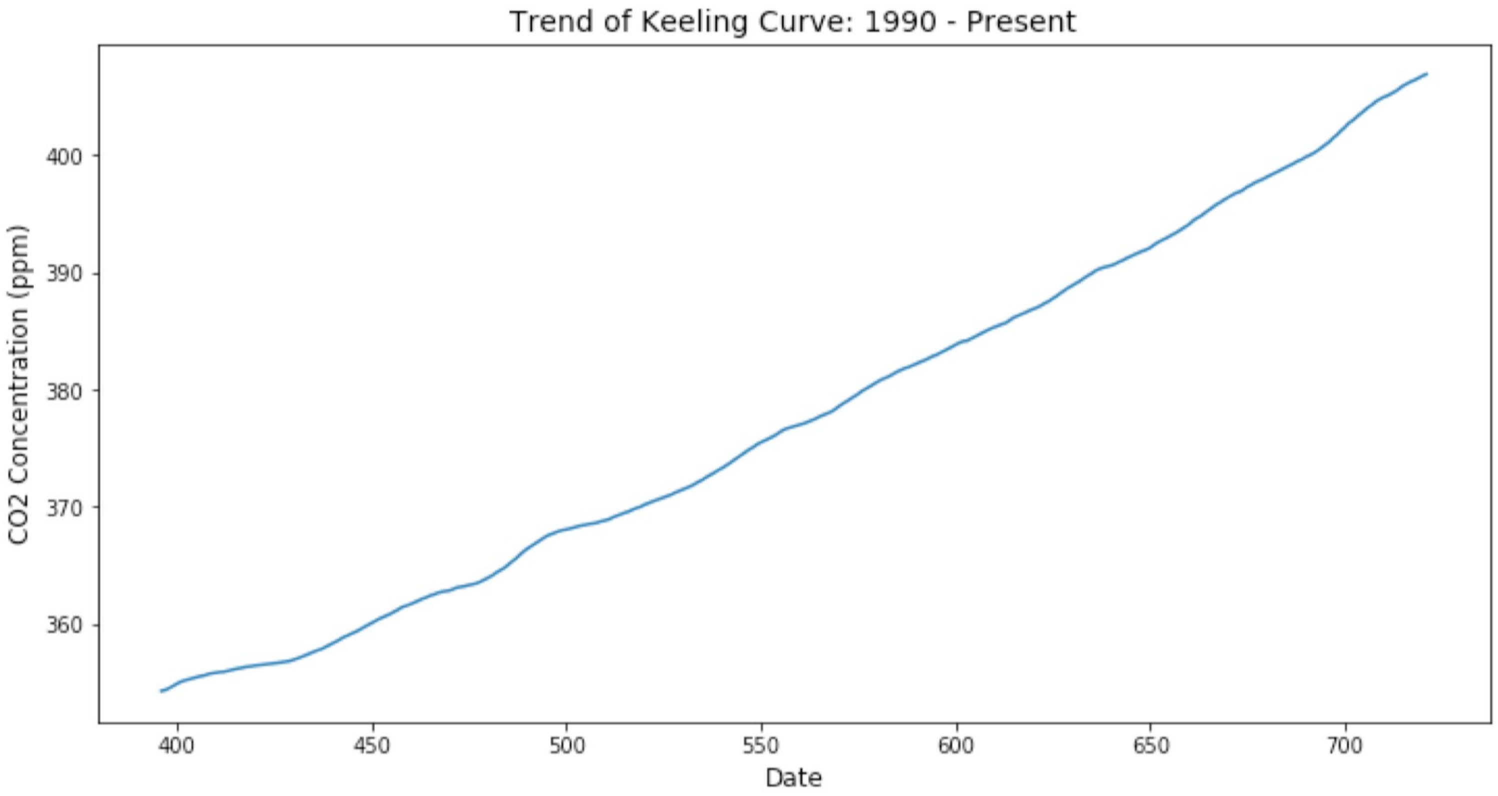
Next, we can take a more systematic approach in measuring the trend of the series.

We can estimate a trend by using a moving average.

$$X_t = \frac{1}{2k+1} \sum_{i=-k}^{k} X_{t+i}$$

# Measuring the trend

Implementing the moving average is easy.



# Removing the trend

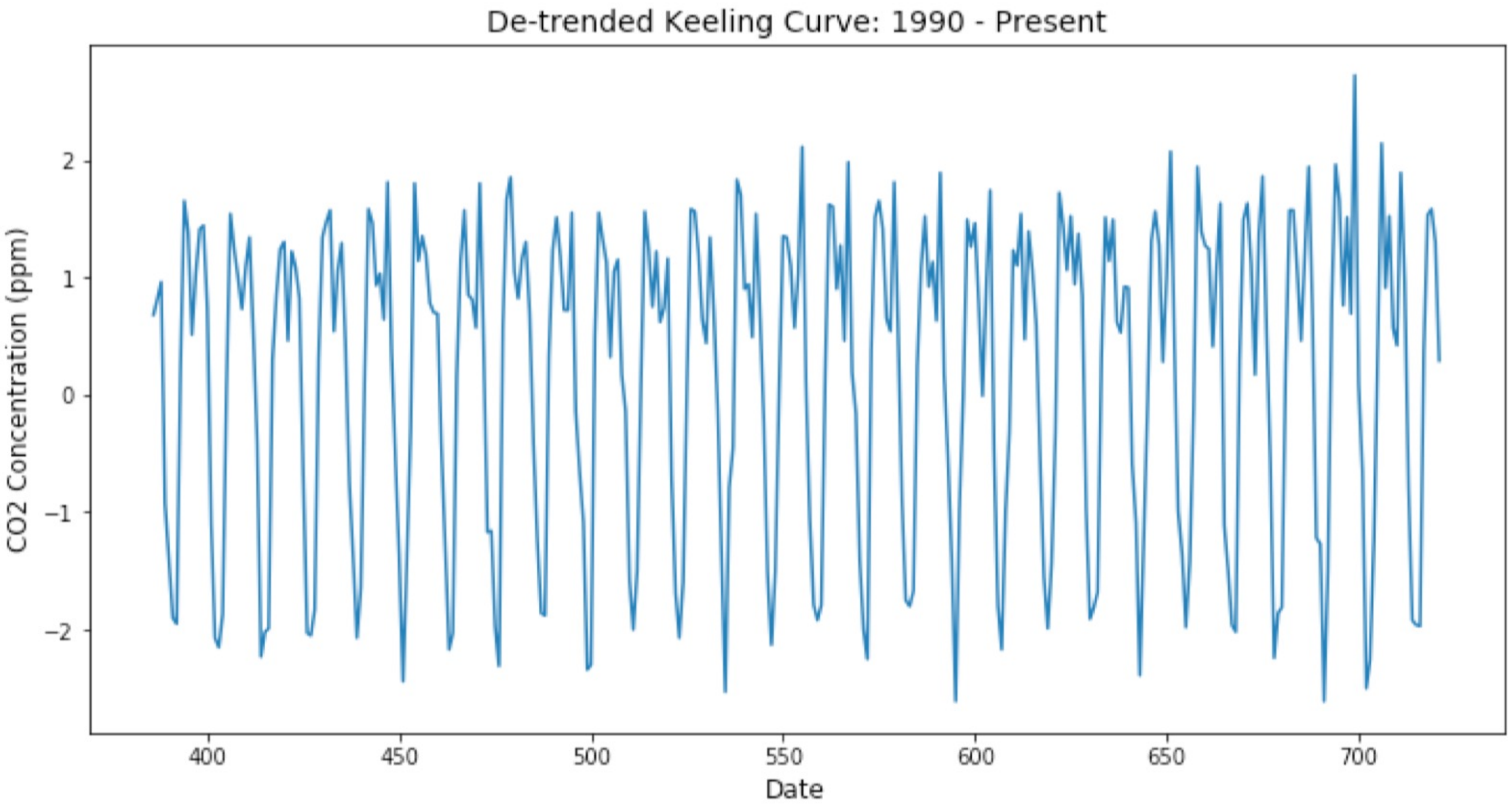
We can also remove the trend by first-order differencing.

$$X'_t = X_t - X_{t-1}$$

$X'_t$ will be a de-trended series.

# Removing the trend

Implementing first-order differencing.



# Removing seasonality

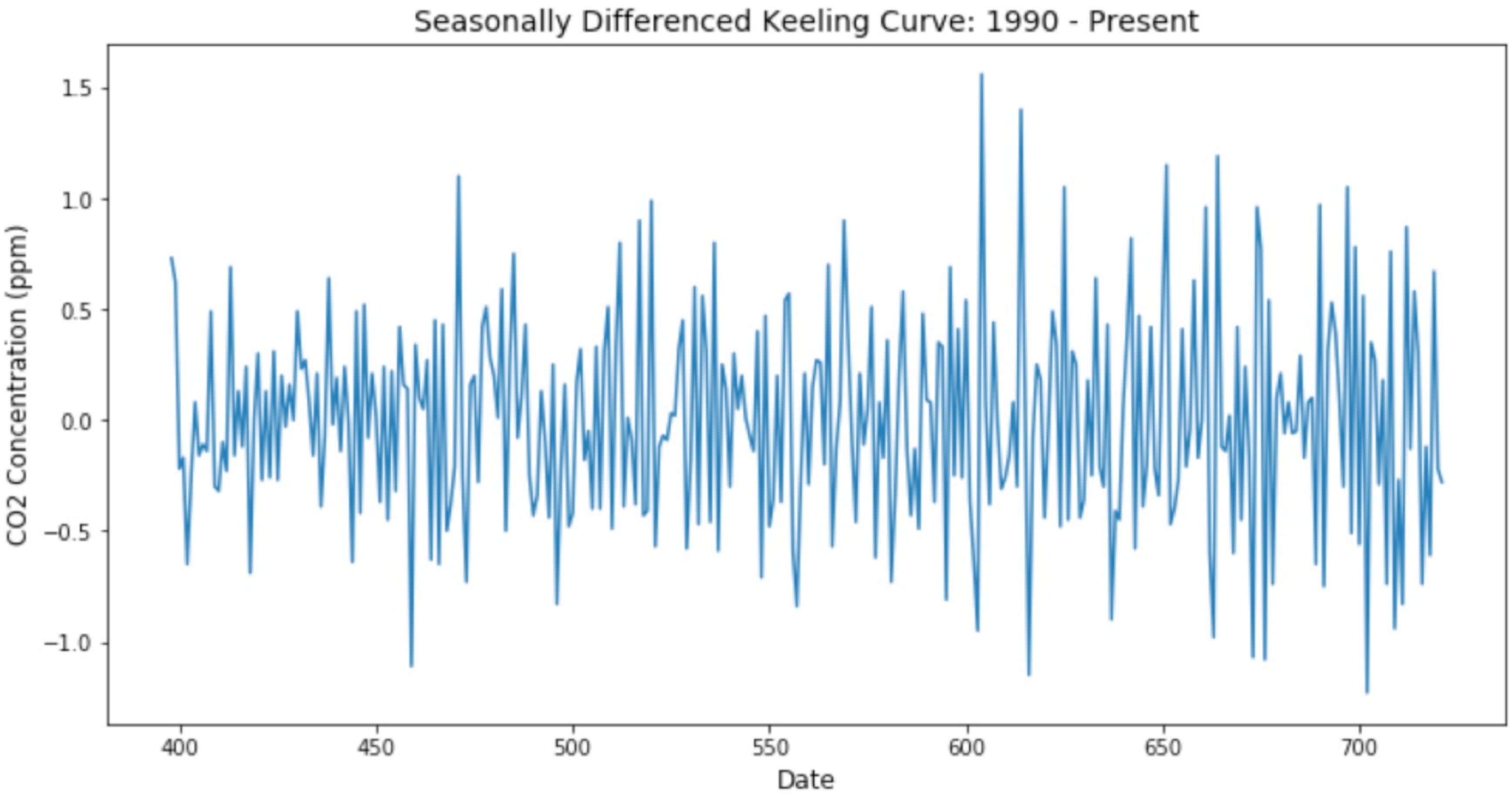
We can also remove the seasonality through seasonal differencing.

$$X'_t = X_t - X_{t-m}$$

where *m* is the length of the season

$X'_t$ will be a de-seasonalized series

# Removing seasonality



# Agenda

- Properties of Time Series
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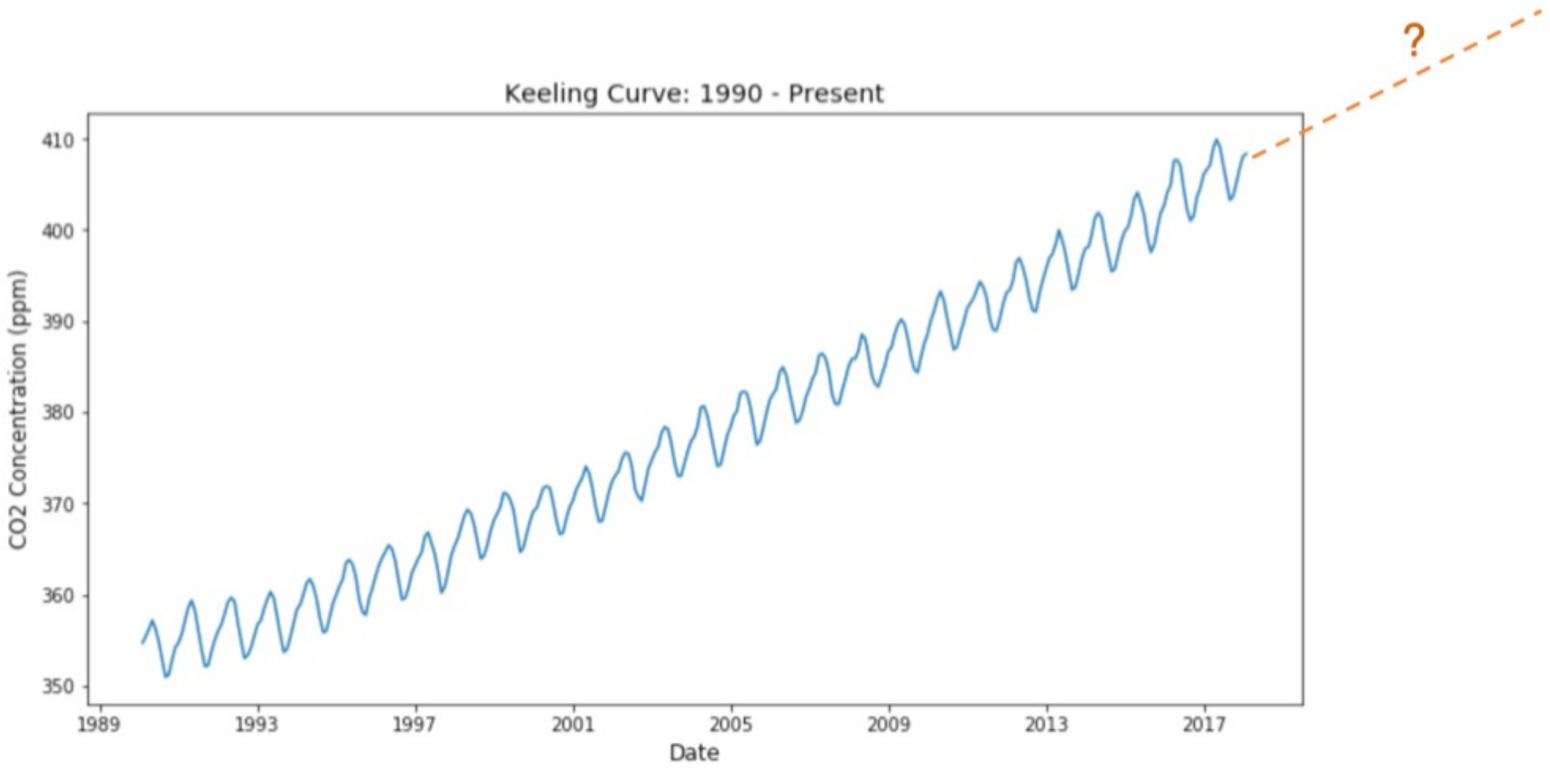
# Important Characteristics to Consider First

Some important questions to first consider when first looking at a time series are:

- Is there a **trend**, meaning that, on average, the measurements tend to increase (or decrease) over time?
- Is there **seasonality**, meaning that there is a regularly repeating pattern of highs and lows related to calendar time such as seasons, quarters, months, days of the week, and so on?
- Are their **outliers**? In regression, outliers are far away from your line. With time series data, your outliers are far away from your other data.
- Is there a **long-run cycle** or period unrelated to seasonality factors?
- Is there **constant variance** over time, or is the variance non-constant?
- Are there any **abrupt changes** to either the level of the series or the variance?

# Forecasting




Can we predict future values of the Keeling curve using observed values?

# Types of Models

There are two basic types of “time domain” models.

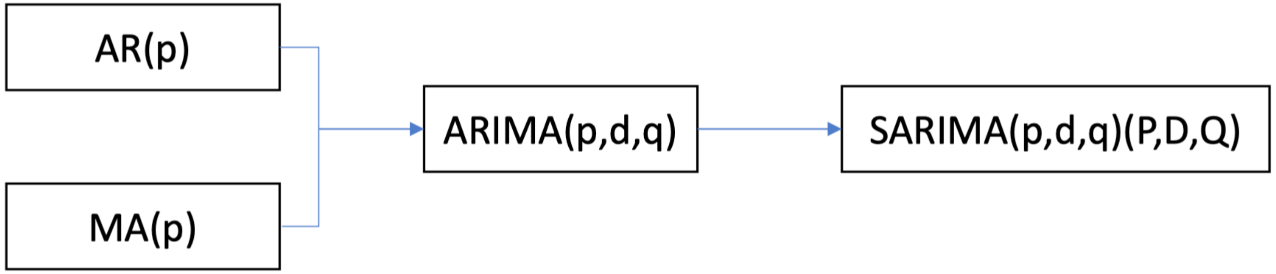
- Models that relate the present value of a series to past values and past prediction errors - these are called ARIMA models (for Autoregressive Integrated Moving Average).

- Ordinary regression models that use time indices as x-variables.  These can be helpful for an initial description of the data and form the basis of several simple forecasting methods.

# Forecasting

Now, we will introduce a class of linear models called the ARIMA models, which can be used for time series forecasting.

There are several variants of ARIMA models, and they build on each other.




ARIMA models work by modelling the autocorrelations (correlations between successive observations) in the data.

# Autoregressive Model: AR

An autoregressive model predicts the response $X_t$ & using a linear combination of past values of the variable. Parameterized by $p$, (the number of past values to include).

$$\boldsymbol{X_t = \theta_0 + \theta_1 X_{t-1} + \theta_2 X_{t-2} + \cdots + \theta_p X_{t-p}}$$

This is the same as doing linear regression with lagged features. For example, this is how you would set up your dataset to fit an autoregressive model with $p$ = 2:

| t | $X_t$ |
|---|---|
| 1 | 400 |
| 2 | 500 |
| 3 | 300 |
| 4 | 100 |
| 5 | 200 |

$\longrightarrow$

| $X_{t-2}$ | $X_{t-1}$ | $X_t$ |
|---|---|---|
| 400 | 500 | 300 |
| 500 | 300 | 100 |
| 300 | 100 | 200 |

# Moving Average Model: MA

A moving average model predicts the response $X_t$ using a linear combination of past forecast errors.

$$\boldsymbol{X_t = \beta_0 + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \cdots + \beta_q \epsilon_{t-q}}$$

where $\epsilon_i$ is normally distributed white noise (mean zero, variance one)

Parameterized by $q$, the number of past errors to include. The predictions $X_t$ can be the weighted moving average of past forecast errors.

# Auto Regressive Integrated Moving Average Model: ARIMA

Combining an autoregressive (AR) and moving average (MA) model, we get the ARIMA model.

$$X'_t = \theta_0 + \theta_1 X_{t-1} + \theta_2 X_{t-2} + \cdots + \theta_p X_{t-p}$$

$$+ \beta_0 + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \cdots + \beta_q \epsilon_{t-q}$$

Note that now we are regressing on $X'_t$, which is the differenced series $X_t$ . The order of difference is determined by the the parameter $d$. For example, if $d$ = 1:

$$X'_t = X_t - X_{t-1} \; for \; t = 2,3, \ldots, N$$

So the ARIMA model is parameterized by p (order of the AR part), q (order of the MA part), and d (degree of differencing).

# Seasonal ARIMA: SARIMA

Extension of ARIMA to model seasonal data.

Includes a non-seasonal part (same as ARIMA) and a seasonal part. The seasonal part is similar to ARIMA, but involves backshifts of the seasonal period.

In total, 6 parameters:

- (p, d, q) for non-seasonal part
- $(P, D, Q)_s$ for seasonal part, where s is the length of season