Exercise solutions for “Basic physics and media changes”

Part I, Problem 1

Plane progressive wave in air with frequency $f = 100\ [Hz]$ and amplitude $P_{max} = 3\ [Pa]$

(a) Intensity

$$I = \frac{P_{max}^2}{2\rho_0\ c} = \frac{3^2}{2 \cdot 415} = 1.08 \times 10^{-2}\ [W/m^2] \tag{1}$$

(b) Intensity level

$$L_I = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{1.08 \times 10^{-2}}{10^{-12}}\right) = 100\ [dB\ re.10^{-12}W/m^2] \tag{2}$$

(c) Energy density

$$\mathscr{E} = \frac{1}{2} \cdot \frac{P_{max}^2}{\rho_0\ c^2} = \frac{3^2}{2 \cdot 415 \cdot 343} = 3.16 \times 10^{-5}\ [J/m^3] \tag{3}$$

(d) Particle speed amplitude

$$|\vec{u}| = \frac{P_{max}}{\rho_0\ c} = 7.23 \times 10^{-3}\ [m/s] \tag{4}$$

(e) Particle displacement peak amplitude

$$\xi = \int \vec{u}\ dt = \frac{\vec{u}}{j\omega} \Rightarrow |\xi| = \frac{u}{\omega} = 1.15 \times 10^{-5}\ [m] \tag{5}$$

(f) wavelength

$$\lambda = \frac{c}{f} = \frac{343}{100} = 3.43\ [m] \tag{6}$$

(g) Effective sound pressure (RMS)

$$P_e = \frac{P_{max}}{\sqrt{2}} = 2.12\ [Pa] = 21.2\ [\mu bar] \tag{7}$$

(h) Sound pressure level

$$SPL = 20\log_{10}\left(\frac{P_e}{P_{max}}\right) = 20\log_{10}\left(\frac{2.12\ Pa}{20\ \mu Pa}\right) = 101\ [dB\ re.20\ \mu Pa] \tag{8}$$

Part II, Problem 1

Exercise 6.2.1 from the book (p. 167): Plane wave $P_e = 50\ [Pa]$ (RMS) incident normal at a water-air boundary. From Appendix 10 p. 527-528 we have:

$$\begin{aligned} \text{fresh water at } 20^\circ[C] & \quad r_1 = \rho_1 c_1 = 1.48 \times 10^6\ [Pa \cdot s/m] & (9) \\ \text{air at } 20^\circ[C] & \quad r_2 = \rho_2 c_2 = 415\ [Pa \cdot s/m] & (10) \\ \text{Pressure transmission coefficient Eq. (6.2.9):} & \quad T = \frac{2}{1 + r_1/r_2} & (11) \end{aligned}$$

(a) Effective transmitted pressure:

$$P_{e,t} = P_{e,i} \times T = \frac{2 \cdot 50}{1 + (1.48 \times 10^6/415)} = 2.8 \times 10^{-2}\ [Pa] \tag{12}$$

(b) Intensity of incident and transmitted waves:

$$I_i = \frac{P_{e,i}^2}{r_1} = \frac{50^2}{1.48 \times 10^6} = 1.69 \times 10^{-3}\ [W/m^2] \tag{13}$$

$$I_t = \frac{P_{e,t}^2}{r_2} = \frac{(2.8 \times 10^{-2})^2}{415} = 1.89 \times 10^{-6}\ [W/m^2] \tag{14}$$

(c) Level difference between incident and transmitted intensities:

$$L_{\Delta I} = 10 \cdot log_{10}\left(\frac{I_i}{I_t}\right) = -29.5\ [dB] \tag{15}$$

(d) Same three questions but with ice instead of air: $r_2 = 2.95 \times 10^6\ [Pa \cdot s/m]$

$$P_{e,t} = \frac{2 \cdot 50}{1 + (1.48 \times 10^6/2.95 \times 10^6)} = 66.59\ [Pa] \tag{16}$$

$$I_i = 1.69 \times 10^{-3}\ [W/m^2] \tag{17}$$

$$I_t = \frac{(66.59)^2}{2.95 \times 10^6} = 1.5 \times 10^{-3}\ [W/m^2] \tag{18}$$

$$L_{\Delta I} = -0.507\ [dB] \tag{19}$$

(e) Power reflection coefficient. From Eq. (6.2.10), for ice as the second media:

$$R_\Pi = R_I = |R|^2 = \left(\frac{1 - r_1/r_2}{1 + r_1/r_2}\right)^2 = \left(\frac{1 - 1.48 \times 10^6/2.95 \times 10^6}{1 + 1.48 \times 10^6/2.95 \times 10^6}\right)^2 = 0.1101 \tag{20}$$