Chapter 6

REFLECTION AND TRANSMISSION

6.1 CHANGES IN MEDIA

When an acoustic wave traveling in one medium encounters the boundary of a second medium, reflected and transmitted waves are generated. Discussion of this phenomenon is greatly simplified if it is assumed that both the incident wave and the boundary between the media are planar and that all media are fluids. The complications that arise when one of the media is a solid will be left to Section 6.6. However, it is worthwhile to note that for normal incidence many solids obey the same equations developed for fluids. The only modification needed is that the speed of sound in the solid must be the *bulk speed of sound*, based on both the *bulk* and *shear* moduli since, unlike the bar of Chapter 3, an extended solid is not free to change its transverse dimensions. See Appendix A11. Values of the bulk speeds of sound in some solids are listed in Appendix A10.

The ratios of the pressure amplitudes and intensities of the reflected and transmitted waves to those of the incident wave depend on the characteristic acoustic impedances and speeds of sound in the two media and on the angle the incident wave makes with the interface. Let the incident and reflected waves travel in a fluid of characteristic acoustic impedance $r_1 = \rho_1 c_1$, where ρ_1 is the equilibrium density of the fluid (the subscript "0" has been suppressed for economy of notation) and c_1 the speed of sound in the fluid. Let the transmitted wave travel in a fluid of characteristic acoustic impedance $r_2 = \rho_2 c_2$. If the complex pressure amplitude of the incident wave is $\mathbf{P}_i$, that of the reflected wave $\mathbf{P}_r$, and that of the transmitted wave $\mathbf{P}_t$, then we can define the *pressure transmission and reflection coefficients*:

$$\mathbf{T} = \mathbf{P}_t / \mathbf{P}_i \tag{6.1.1}$$

$$\mathbf{R} = \mathbf{P}_r / \mathbf{P}_i \tag{6.1.2}$$

Since the intensity of a harmonic plane progressive wave is $P^2/2r$, the *intensity transmission and reflection coefficients* are real and are defined by

$$T_I = I_t / I_i = (r_1 / r_2)|\mathbf{T}|^2 \tag{6.1.3}$$

$$R_I = I_r/I_i = |\mathbf{R}|^2 \tag{6.1.4}$$

Most real situations have beams of sound with *finite* cross-sectional area. As we have seen, a beam can be described locally by nearly parallel rays and thus can be approximated by a plane wave of finite extent. While there can be some anomalies resulting from diffraction at the edges of the beam, if the cross-sectional area is sufficiently great compared to a wavelength, they can be ignored and the equations developed in this chapter applied.

The power carried by a beam of sound is the acoustic intensity multiplied by the cross-sectional area of the beam. If an incident beam of cross-sectional area A_i is obliquely incident on a boundary, the cross-sectional area A_t of the transmitted beam generally is not the same as that of the incident beam. It will be shown later that the cross-sectional areas of the incident and reflected beams are equal under all circumstances. The *power transmission and reflection coefficients* are defined by

$$T_\Pi = (A_t/A_i)T_I = (A_t/A_i)(r_1/r_2)|\mathbf{T}|^2 \tag{6.1.5}$$

$$R_\Pi = R_I = |\mathbf{R}|^2 \tag{6.1.6}$$

From conservation of energy, the power in the incident beam must be shared between reflected and transmitted beams so that

$$R_\Pi + T_\Pi = 1 \tag{6.1.7}$$

Cases more complicated than those included in this chapter are available in specialized textbooks.[1]

6.2 TRANSMISSION FROM ONE FLUID TO ANOTHER: NORMAL INCIDENCE

As indicated in Fig. 6.2.1, let the plane $x = 0$ be the boundary between fluid 1 of characteristic acoustic impedance r_1 and fluid 2 of characteristic acoustic impedance r_2. Let there be an incident wave traveling in the $+x$ direction,

$$\mathbf{p}_i = \mathbf{P}_i e^{j(\omega t - k_1 x)} \tag{6.2.1}$$

which, when striking the boundary, generates a reflected wave

$$\mathbf{p}_r = \mathbf{P}_r e^{j(\omega t + k_1 x)} \tag{6.2.2}$$

and a transmitted wave

$$\mathbf{p}_t = \mathbf{P}_t e^{j(\omega t - k_2 x)} \tag{6.2.3}$$

All the waves must have the same frequency, but, since the speeds c_1 and c_2 are different, the wave numbers $k_1 = \omega/c_1$ in fluid 1 and $k_2 = \omega/c_2$ in fluid 2 are different.

[1]Officer, *Introduction to the Theory of Sound Transmission*, McGraw-Hill (1958). Ewing, Jardetzky, and Press, *Elastic Waves in Layered Media*, McGraw-Hill (1957). Brekhovskikh, *Waves in Layered Media*, Academic Press (1960).

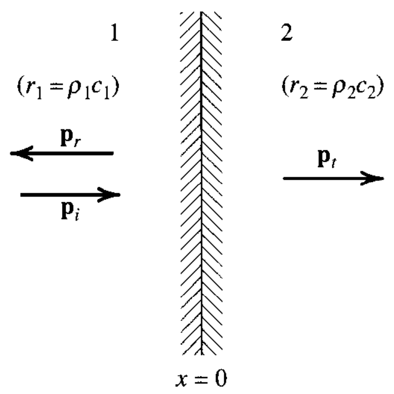

Figure 6.2.1 Reflection and transmission of a plane wave normally incident on the planar boundary between fluids with different characteristic impedances.

There are two boundary conditions to be satisfied for all times at all points on the boundary: (1) the acoustic pressures on both sides of the boundary must be equal and (2) the normal components of the particle velocities on both sides of the boundary must be equal. The first condition, *continuity of pressure*, means that there can be no net force on the (massless) plane separating the fluids. The second condition, *continuity of the normal component of velocity*, requires that the fluids remain in contact. The pressure and normal particle velocity in fluid 1 are $\mathbf{p}_i + \mathbf{p}_r$ and $(\mathbf{u}_i + \mathbf{u}_r)\hat{x}$ so that the boundary conditions are

$$\mathbf{p}_i + \mathbf{p}_r = \mathbf{p}_t \qquad \text{at } x = 0 \tag{6.2.4}$$

$$\mathbf{u}_i + \mathbf{u}_r = \mathbf{u}_t \qquad \text{at } x = 0 \tag{6.2.5}$$

Division of (6.2.4) by (6.2.5) yields

$$\frac{\mathbf{p}_i + \mathbf{p}_r}{\mathbf{u}_i + \mathbf{u}_r} = \frac{\mathbf{p}_t}{\mathbf{u}_t} \qquad \text{at } x = 0 \tag{6.2.6}$$

which is a statement of the *continuity of normal specific acoustic impedance* across the boundary.

Since a plane wave has $\mathbf{p}/\mathbf{u} = \pm r$, the sign depending on the direction of propagation, (6.2.6) becomes

$$r_1 \frac{\mathbf{p}_i + \mathbf{p}_r}{\mathbf{p}_i - \mathbf{p}_r} = r_2 \tag{6.2.7}$$

which leads directly to the reflection coefficient

$$\mathbf{R} = \frac{r_2 - r_1}{r_2 + r_1} = \frac{r_2/r_1 - 1}{r_2/r_1 + 1} \tag{6.2.8}$$

Then, since (6.2.4) is equivalent to $1 + \mathbf{R} = \mathbf{T}$, we have

$$\mathbf{T} = \frac{2r_2}{r_2 + r_1} = \frac{2r_2/r_1}{r_2/r_1 + 1} \tag{6.2.9}$$

The intensity reflection and transmission coefficients follow directly from (6.1.3) and (6.1.4),

$$R_I = \left(\frac{r_2 - r_1}{r_2 + r_1}\right)^2 = \left(\frac{r_2/r_1 - 1}{r_2/r_1 + 1}\right)^2 \tag{6.2.10}$$

and

$$T_I = \frac{4r_2r_1}{(r_2 + r_1)^2} = \frac{4r_2/r_1}{(r_2/r_1 + 1)^2} \tag{6.2.11}$$

Since the cross-sectional areas of all the beams are equal, the power coefficients in (6.1.5) and (6.1.6) are equal to the intensity coefficients.

From (6.2.8), $\mathbf{R}$ is always real. It is positive when $r_1 < r_2$ and negative when $r_1 > r_2$. Consequently, at the boundary the acoustic pressure of the reflected wave is either in phase or 180° out of phase with that of the incident wave. When the characteristic acoustic impedance of fluid 2 is greater than that of fluid 1 (a wave in air incident on the air–water interface), a positive pressure in the incident wave is reflected as a positive pressure. On the other hand, if $r_1 > r_2$ (a wave in water incident on the water–air interface), a positive pressure is reflected as a negative pressure. Note that when $r_1 = r_2$ then $\mathbf{R} = 0$, and there is complete transmission.

From (6.2.9), it is seen that $\mathbf{T}$ is real and positive regardless of the relative magnitudes of r_1 and r_2. Consequently, at the boundary the acoustic pressure of the transmitted wave is *always* in phase with that of the incident wave. Study of (6.2.11) reveals that whenever r_1 and r_2 have strongly dissimilar values, the intensity transmission coefficient is small. In addition, from the symmetries of (6.2.10) and (6.2.11), it is apparent that the intensity reflection and transmission coefficients are *independent* of the direction of the wave. For example, they are the same from water into air as from air into water. This is a special case of *acoustic reciprocity*.

In the limit $r_1/r_2 \to 0$, the wave is reflected with no reduction in amplitude and no change in phase. The transmitted wave has a pressure amplitude twice that of the incident wave, and the normal particle velocity at the boundary is zero. Because of this latter fact, the boundary is termed *rigid*.

For $r_1/r_2 \to \infty$, the amplitude of the reflected wave is again equal to that of the incident wave, and the transmitted wave has zero pressure amplitude. Since the acoustic pressure at the boundary is zero, the boundary is termed *pressure release*.

6.3 TRANSMISSION THROUGH A FLUID LAYER: NORMAL INCIDENCE

Assume that a plane fluid layer of uniform thickness L lies between two dissimilar fluids and that a plane wave is normally incident on its boundary, as indicated in Fig. 6.3.1. Let the characteristic impedances of the fluids be r_1, r_2, and r_3, respectively.

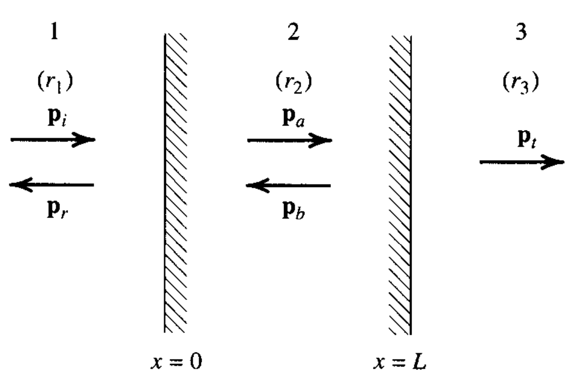

Figure 6.3.1 Reflection and transmission of a plane wave normally incident on a layer of uniform thickness.

When an incident signal in fluid 1 arrives at the boundary between fluids 1 and 2, some of the energy is reflected and some transmitted into the second fluid. The portion of the wave transmitted will proceed through fluid 2 to interact with the boundary between fluids 2 and 3, where again some of the energy is reflected and some transmitted. The reflected wave travels back to the boundary between fluids 1 and 2, and the whole process is repeated. If the duration of the incident signal is less than $2L/c_2$, an observer in either fluid 1 or 3 will see a series of echoes separated in time by $2L/c_2$ whose amplitudes can be calculated by applying the results of the previous section the appropriate number of times. Otherwise, if the incident wave train has a monofrequency carrier and has duration much greater than $2L/c_2$, it can be assumed to be

$$\mathbf{p}_i = \mathbf{P}_i e^{j(\omega t - k_1 x)} \tag{6.3.1}$$

The various transmitted and reflected waves now combine so that in the steady state the wave reflected back into fluid 1 is

$$\mathbf{p}_r = \mathbf{P}_r e^{j(\omega t + k_1 x)} \tag{6.3.2}$$

the transmitted and reflected waves in fluid 2 are

$$\mathbf{p}_a = \mathbf{A} e^{j(\omega t - k_2 x)} \tag{6.3.3}$$

$$\mathbf{p}_b = \mathbf{B} e^{j(\omega t + k_2 x)} \tag{6.3.4}$$

and the wave transmitted into fluid 3 is

$$\mathbf{p}_t = \mathbf{P}_t e^{j(\omega t - k_3 x)} \tag{6.3.5}$$

Continuity of the normal specific acoustic impedance at $x = 0$ and at $x = L$ gives

$$\frac{\mathbf{P}_i + \mathbf{P}_r}{\mathbf{P}_i - \mathbf{P}_r} = \frac{r_2}{r_1}\frac{\mathbf{A} + \mathbf{B}}{\mathbf{A} - \mathbf{B}} \qquad \frac{\mathbf{A}e^{-jk_2L} + \mathbf{B}e^{jk_2L}}{\mathbf{A}e^{-jk_2L} - \mathbf{B}e^{jk_2L}} = \frac{r_3}{r_2} \tag{6.3.6}$$

and algebraic manipulation yields the pressure reflection coefficient

$$\mathbf{R} = \frac{(1 - r_1/r_3)\cos k_2L + j(r_2/r_3 - r_1/r_2)\sin k_2L}{(1 + r_1/r_3)\cos k_2L + j(r_2/r_3 + r_1/r_2)\sin k_2L} \tag{6.3.7}$$

The intensity transmission coefficient is found by using (6.1.3)–(6.1.7) and noting that $A_t = A_i$:

$$T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3)\cos^2 k_2L + (r_2^2/r_1r_3 + r_1r_3/r_2^2)\sin^2 k_2L} \tag{6.3.8}$$

A few special forms of (6.3.8) are of particular interest.

1. If the final fluid is the same as the initial fluid, $r_1 = r_3$,

$$T_I = \frac{1}{1 + \frac{1}{4}(r_2/r_1 - r_1/r_2)^2 \sin^2 k_2L} \tag{6.3.9}$$

If, in addition, $r_2 \gg r_1$, (6.3.9) further simplifies to

$$T_I = \frac{1}{1 + \frac{1}{4}(r_2/r_1)^2 \sin^2 k_2 L} \tag{6.3.10}$$

This latter situation applies, for example, to the transmission of sound from air in one room through a solid wall into air in an adjacent room. The solid materials forming the walls of rooms have such large characteristic impedances relative to air that $(r_2/r_1)\sin k_2L \gg 2$ for all reasonable frequencies and thicknesses of walls. Therefore, when the fluid medium is air, (6.3.10) reduces to

$$T_I \approx \left(\frac{2r_1}{r_2 \sin k_2 L}\right)^2 \tag{6.3.11}$$

Finally, for all situations except those of high frequencies and very thick walls, $k_2L \ll 1$ and $\sin k_2L \approx k_2L$ so that (6.3.11) becomes

$$T_I = \left(\frac{2}{k_2 L}\frac{r_1}{r_2}\right)^2 \tag{6.3.12}$$

(At 1 kHz the value of k_2L for a 0.1 m thick concrete wall is $2\pi \times 1000 \times 0.1/3100 = 0.20$.) Note that the transmitted pressure is inversely proportional to the thickness L and, therefore, also inversely proportional to the mass per unit area of the wall. This behavior is observed to be approximately true for certain kinds of commonly encountered walls. In the case of solid panels in water, both terms occurring in the denominator of (6.3.10) usually are significant, so that the complete equation must be used. However, for either thin panels or low frequencies such that $(r_2/r_1)\sin k_2L \ll 1$, (6.3.10) simplifies to

$$T_I \approx 1 \tag{6.3.13}$$

This behavior is used in the design of free-flooding streamlined domes for sonar transducers.

2. Another special form of (6.3.8) is obtained by assuming that the intermediate fluid has a larger characteristic impedance than either fluid 1 or fluid 3 but such small thickness that $r_2 \sin k_2L \ll 1$ and $\cos k_2L \approx 1$. Then, (6.3.8) reduces to

$$T_I = \frac{4r_3r_1}{(r_3 + r_1)^2} \tag{6.3.14}$$

This is equivalent to (6.2.11), which gives the intensity transmission coefficient for a wave moving directly from fluid 1 into fluid 3. Thus, a thin membrane of solid material of appropriate characteristic impedance may be used in preventing two gases or two liquids from mixing and yet not interfere with sound transmission between them. In particular, note that if $r_1 = r_3$ then there is total transmission from fluid 1 to fluid 3 as if fluid 2 did not exist.

3. Returning to the general form of T_I in (6.3.8), we see that if $k_2L = n\pi$, (6.3.8) reduces to (6.3.14) for frequencies

$$f \approx nc_2/2L \tag{6.3.15}$$

For these frequencies, $L \approx n\lambda_2/2$ and the intermediate layer is an integral number of half-wavelengths thick. Again, it is as if fluid 2 did not exist.

4. Finally, if $k_2L \approx (n - \frac{1}{2})\pi$, where n is any integer, then we have $L \approx (2n - 1)\lambda_2/4$ so that $\cos k_2L \approx 0$ and $\sin k_2L \approx 1$, and (6.3.8) becomes

$$T_I = \frac{4r_1r_3}{(r_2 + r_1r_3/r_2)^2} \tag{6.3.16}$$

for frequencies very close to $f = (n - \frac{1}{2})c_2/2L$. As an interesting special case, note that (6.3.16) yields $T_I \approx 1$ when $r_2 = \sqrt{r_1r_3}$. It is therefore possible to obtain total transmission of acoustic power from one medium to another through the use of an intermediate medium whose characteristic impedance is the geometric mean of the other two. However, this action is selective, since it occurs only for narrow bands of frequencies centered about these particular values. This technique of obtaining complete transmission of acoustic power through the use of a quarter-wavelength intermediate layer is similar to the method of making nonreflective glass lenses by coating them with a quarter-wavelength layer of some suitable material. Another example is the use of quarter-wavelength sections to match an antenna to an electrical transmission line.

The impedance $\mathbf{z}_2$ presented to fluid 1 by any number of sequential layers can be expressed in terms of the pressure reflection coefficient. The boundary between fluid 1 and fluid 2 corresponds to an impedance given by

$$\mathbf{z}_2 = \left.\frac{\mathbf{p}_i + \mathbf{p}_r}{\mathbf{u}_i + \mathbf{u}_r}\right|_{x=0} \tag{6.3.17}$$

Division of numerator and denominator by $\mathbf{p}_i$ and use of the relation $\mathbf{p}_\pm = \pm r\mathbf{u}_\pm$ results in

$$\mathbf{z}_2 = r_1\frac{1 + \mathbf{R}}{1 - \mathbf{R}} \tag{6.3.18}$$

In this way a multilayered fluid boundary to the right of fluid 1 can be replaced by a single boundary at $x = 0$ whose impedance may have real and imaginary components.

6.4 TRANSMISSION FROM ONE FLUID TO ANOTHER: OBLIQUE INCIDENCE

Assume that the boundary separating two fluids is the plane $x = 0$ and that the incident, reflected, and transmitted waves make the respective angles θ_i, θ_r, and θ_t with the x axis, as shown in Fig. 6.4.1. For propagation vectors lying in the x-y plane these waves can be written as

$$\mathbf{p}_i = \mathbf{P}_i e^{j(\omega t - k_1 x \cos\theta_i - k_1 y \sin\theta_i)} \tag{6.4.1}$$

$$\mathbf{p}_r = \mathbf{P}_r e^{j(\omega t + k_1 x \cos\theta_r - k_1 y \sin\theta_r)} \tag{6.4.2}$$

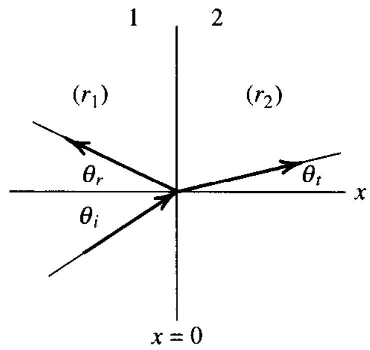

Figure 6.4.1 Reflection and transmission of a plane wave obliquely incident on the planar boundary between fluids with different characteristic impedances.

and

$$\mathbf{p}_t = \mathbf{P}_t e^{j(\omega t - k_2 x \cos \boldsymbol{\theta}_t - k_2 y \sin \boldsymbol{\theta}_t)} \tag{6.4.3}$$

The reason for writing $\boldsymbol{\theta}_t$ as a complex quantity will emerge shortly.

Applying continuity of pressure at the boundary $x = 0$ yields

$$\mathbf{P}_i e^{-jk_1 y \sin\theta_i} + \mathbf{P}_r e^{-jk_1 y \sin\theta_r} = \mathbf{P}_t e^{-jk_2 y \sin \boldsymbol{\theta}_t} \tag{6.4.4}$$

Since this must be true for all y, the exponents must all be equal. This means that

$$\boxed{\sin\theta_i = \sin\theta_r} \tag{6.4.5}$$

so that the angle of incidence is equal to the angle of reflection, and

$$\boxed{\frac{\sin\theta_i}{c_1} = \frac{\sin\boldsymbol{\theta}_t}{c_2}} \tag{6.4.6}$$

a statement of Snell's law. The presence of sines rather than cosines results from the convention of measuring the angle with respect to the normal to the boundary when dealing with reflection and transmission in air. For ray tracing in underwater and atmospheric acoustics, the convention in Section 5.14 is usually adopted. Since the exponents in (6.4.4) are all equal, this equation reduces to

$$1 + \mathbf{R} = \mathbf{T} \tag{6.4.7}$$

Continuity of the normal component of particle velocity at the boundary gives

$$\mathbf{u}_i \cos\theta_i + \mathbf{u}_r \cos\theta_r = \mathbf{u}_t \cos\boldsymbol{\theta}_t \tag{6.4.8}$$

Replacing each $\mathbf{u}$ with the appropriate value of $\pm\mathbf{p}/r$, and recalling that $\theta_i = \theta_r$ we have

$$1 - \mathbf{R} = \frac{r_1}{r_2}\frac{\cos\boldsymbol{\theta}_t}{\cos\theta_i}\mathbf{T} \tag{6.4.9}$$

Equations (6.4.7) and (6.4.9) can be combined to eliminate **T**, giving

$$R = \frac{r_2/r_1 - \cos\boldsymbol{\theta}_t/\cos\theta_i}{r_2/r_1 + \cos\boldsymbol{\theta}_t/\cos\theta_i} = \frac{r_2/\cos\boldsymbol{\theta}_t - r_1/\cos\theta_i}{r_2/\cos\boldsymbol{\theta}_t + r_1/\cos\theta_i} \tag{6.4.10}$$

where Snell's law reveals

$$\cos\boldsymbol{\theta}_t = (1 - \sin^2\boldsymbol{\theta}_t)^{1/2} = [1 - (c_2/c_1)^2 \sin^2\theta_i]^{1/2} \tag{6.4.11}$$

Equation (6.4.10) is known as the *Rayleigh reflection coefficient*. It is important to note three consequences of this equation.

1. If $c_1 > c_2$, the angle of transmission $\boldsymbol{\theta}_t$ is *real* and *less* than the angle of incidence. The transmitted beam is bent *toward* the normal for all angles of incidence.
2. If $c_1 < c_2$, and $\theta_i < \theta_c$, where the *critical angle* θ_c is defined by

$$\boxed{\sin\theta_c = c_1/c_2} \tag{6.4.12}$$

   the angle of transmission is again *real* but *greater* than the angle of incidence; the transmitted beam is bent *away* from the normal for all angles of incidence less than the critical angle.
3. If $c_1 < c_2$, and $\theta_i > \theta_c$, the transmitted wave assumes a peculiar form. From (6.4.11), we see that $\sin\boldsymbol{\theta}_t$ is real and greater than unity, so that $\cos\boldsymbol{\theta}_t$ is now pure imaginary,

$$\cos\boldsymbol{\theta}_t = -j[(c_2/c_1)^2 \sin^2\theta_i - 1]^{1/2} \tag{6.4.13}$$

   Examination of (6.4.3) then reveals that the transmitted pressure is

$$\begin{aligned} \mathbf{p}_t &= \mathbf{P}_t e^{-\gamma x} e^{j(\omega t - k_1 y \sin\theta_i)} \\ \gamma &= k_2[(c_2/c_1)^2 \sin^2\theta_i - 1]^{1/2} \end{aligned} \tag{6.4.14}$$

   The transmitted wave propagates in the y direction, *parallel* to the boundary, and has an amplitude that decays *perpendicular* to the boundary. [Had we chosen the *positive* imaginary root in (6.4.13), γ would have been negative, and the amplitude would have *increased* exponentially with increasing x, a physical impossibility.] Because $\boldsymbol{\theta}_t$ is pure imaginary, the numerator of **R** in (6.4.10) is the complex conjugate of the denominator. Both have the same magnitude, but they have opposite phase. Solving for the phase angle of the ratio and using (6.4.13) to express $\cos\boldsymbol{\theta}_t$ in terms of the angle of incidence and the critical angle gives

$$\begin{aligned} \mathbf{R} &= e^{j\phi} \\ \phi &= 2\tan^{-1}[(\rho_1/\rho_2)\sqrt{(\cos\theta_c/\cos\theta_i)^2 - 1}] \end{aligned} \tag{6.4.15}$$

   under the restriction $\theta_i > \theta_c$. For all angles greater than the critical angle, the reflected wave has the same amplitude as the incident wave. The incident wave is totally reflected and in the steady state no energy propagates away from the boundary into the second medium. The transmitted wave possesses energy, but its propagation vector is parallel to the boundary so that the wave "clings" to the interface. For angles of incidence just exceeding critical, ϕ is close to zero, the reflection coefficient is $+1$, and the interface resembles a rigid boundary. As θ_i increases toward extreme grazing, ϕ approaches π, the reflection coefficient approaches -1, and the interface resembles a pressure release boundary.

If we return to the general case, Snell's law shows that the reflected and incident beams have the same cross-sectional area, as asserted earlier. The power transmission coefficient (6.1.5) can be most readily computed from (6.1.7), giving

$$T_{\Pi} = \left(4\frac{r_2}{r_1}\frac{\cos\boldsymbol{\theta}_t}{\cos\theta_i}\right) \Bigg/ \left(\frac{r_2}{r_1} + \frac{\cos\boldsymbol{\theta}_t}{\cos\theta_i}\right)^2 \qquad \boldsymbol{\theta}_t \text{ real} \qquad (6.4.16)$$

$$T_{\Pi} = 0 \qquad \boldsymbol{\theta}_t \text{ imaginary} \qquad (6.4.17)$$

The first equality (6.4.16) applies when either $c_1 > c_2$ or $\theta_i < \theta_c$. When $r_2/r_1 = \cos\boldsymbol{\theta}_t/\cos\theta_i$, the power reflection coefficient is zero and all the incident power is transmitted. If this condition is combined with (6.4.6) to eliminate $\boldsymbol{\theta}_t$, then

$$\sin\theta_I = \left(\frac{(r_2/r_1)^2 - 1}{(r_2/r_1)^2 - (c_2/c_1)^2}\right)^{1/2} = \left(\frac{1-(r_1/r_2)^2}{1-(\rho_1/\rho_2)^2}\right)^{1/2} \qquad (6.4.18)$$

defines the *angle of intromission* θ_I, the angle of incidence for which there is no reflection and, therefore, complete transmission. This angle can exist under only two circumstances: (1) $r_1 < r_2$ and $c_2 < c_1$ or (2) $r_1 > r_2$ and $c_2 > c_1$. In this second circumstance there is a critical angle, and it is greater than the angle of intromission.

At *grazing incidence*, $\theta_i \to 90°$, $\cos\theta_i \to 0$ and (6.4.10) is reduced to $R \approx -1$. Consequently, at grazing incidence there is complete reflection of the incident acoustic energy irrespective of the relative characteristic acoustic impedances of the two fluids.

Figures 6.4.2–6.4.5 show typical behaviors for the reflection coefficients for all possible conditions.

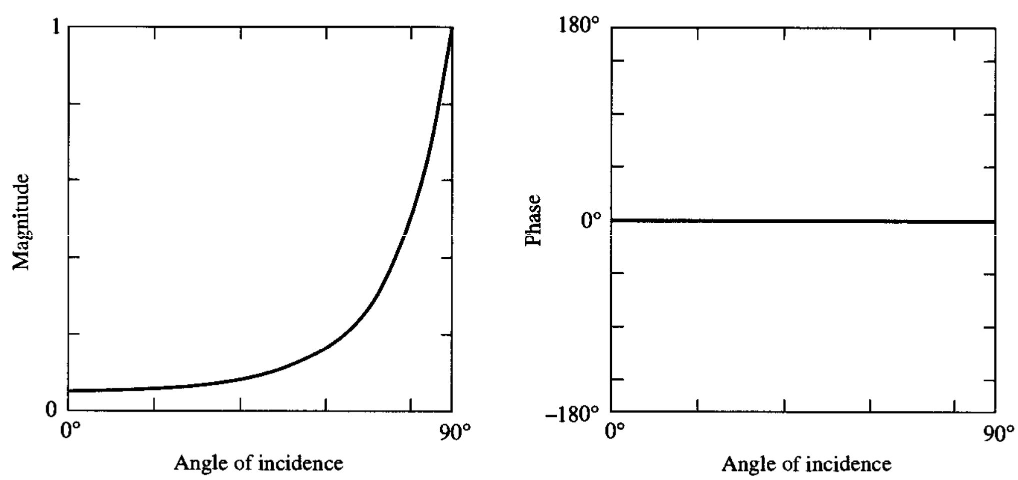

Figure 6.4.2 Magnitude and phase of the reflection coefficient for reflection from a slow bottom with $c_2/c_1 = 0.9$ and $r_2/r_1 = 0.9$.

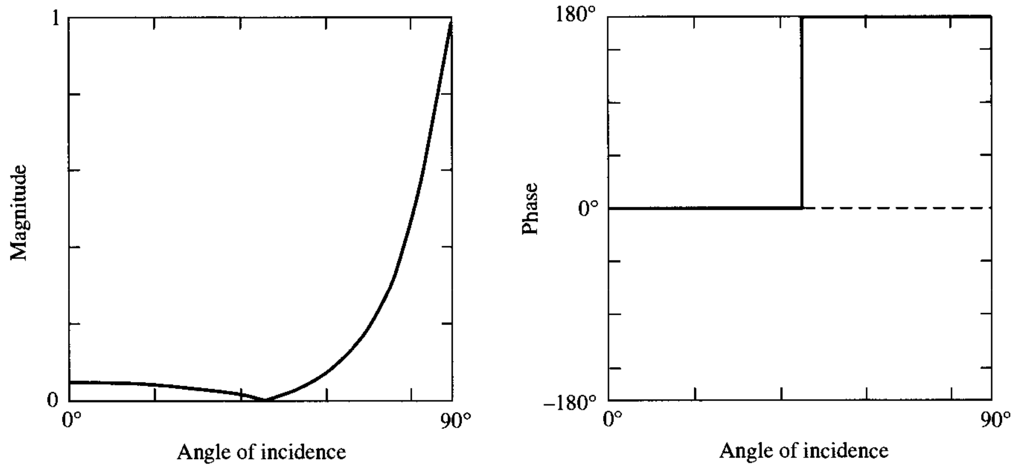

Figure 6.4.3 Magnitude and phase of the reflection coefficient for reflection from a slow bottom with $c_2/c_1 = 0.9$ and $r_2/r_1 = 1.1$. Note angle of intromission at 46.4°.

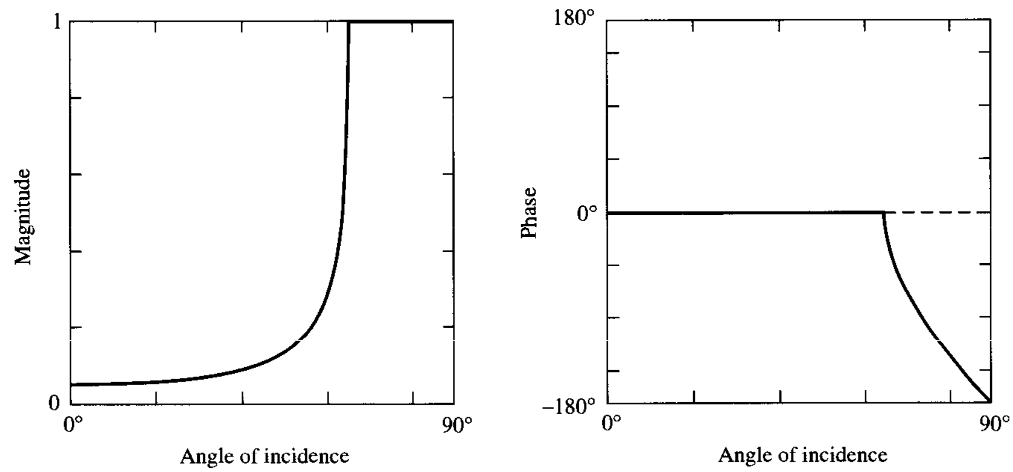

Figure 6.4.4 Magnitude and phase of the reflection coefficient for reflection from a fast bottom with $c_2/c_1 = 1.1$ and $r_2/r_1 = 1.1$. Note critical angle at 65.6°.

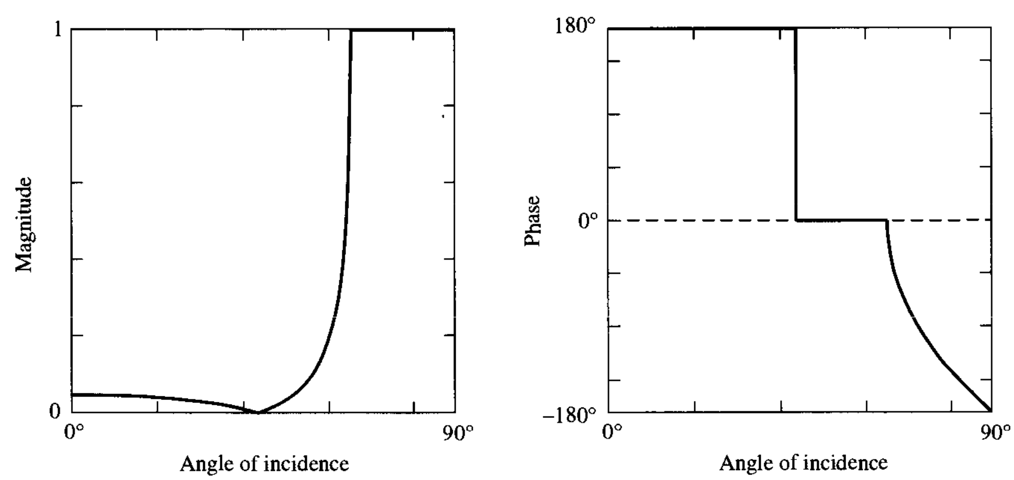

Figure 6.4.5 Magnitude and phase of the reflection coefficient for reflection from a fast bottom with $c_2/c_1 = 1.1$ and $r_2/r_1 = 0.9$. Note angle of intromission at 43.2° and critical angle at 65.6°.

The reflection that takes place in seawater from a sand or silt bottom is a good example of reflection associated with two fluids in contact. Such behavior is to be expected since the saturated sand or silt is more like a fluid than a solid in its inability to transmit shear waves. As a first approximation, (6.4.10) may be used for computing a reflection coefficient. Measured values of ρ_2 and c_2 for sand and silt yield $\rho_2/\rho_1 = 1.5$ to 2.0 and $c_2/c_1 = 0.9$ to 1.1, where ρ_1 and c_1 are the values for seawater.

*6.5 NORMAL SPECIFIC ACOUSTIC IMPEDANCE

Satisfying the boundary conditions at the interface of two fluids amounts to requiring continuity of pressure and continuity of the normal component of particle velocity across the boundary. This is equivalent to requiring continuity of the *normal specific acoustic impedance* $\mathbf{z}_n$:

$$\mathbf{z}_n = \frac{\mathbf{p}}{\vec{\mathbf{u}} \cdot \hat{n}} = \frac{\mathbf{p}}{\mathbf{u} \cos\theta_i} \tag{6.5.1}$$

where $\hat{n}$ is the unit vector perpendicular to the interface and θ_i is the appropriate angle. The normal specific acoustic impedance at the boundary can be expressed in terms of the properties of the incident and reflected waves at the boundary,

$$\mathbf{z}_n = \frac{r_1}{\cos\theta_i} \frac{1 + \mathbf{R}}{1 - \mathbf{R}} \tag{6.5.2}$$

and this solved for the pressure reflection coefficient gives

$$\mathbf{R} = \frac{\mathbf{z}_n - r_1/\cos\theta_i}{\mathbf{z}_n + r_1/\cos\theta_i} \tag{6.5.3}$$

Note that for normal incidence $\mathbf{z}_n = r_2$ and $\cos\theta_i = 1$, so this equation reduces to (6.2.8). For oblique incidence $\mathbf{z}_n = r_2/\cos\boldsymbol{\theta}_t$ and (6.5.3) is identical with (6.4.10). Because the incident and reflected pressures are not always exactly in or out of phase, the normal specific acoustic impedance may be a complex quantity:

$$\mathbf{z}_n = r_n + jx_n \tag{6.5.4}$$

where r_n and x_n are the normal specific acoustic resistance and reactance, respectively. The reflected wave at the boundary may either lead or lag the incident wave by angles ranging from 0° to 180°.

*6.6 REFLECTION FROM THE SURFACE OF A SOLID

Solids can support two types of elastic waves—longitudinal and shear. In an isotropic solid (amorphous materials like glass, hardened clays, concrete, and polycrystalline substances) of transverse dimensions much larger than the wavelength of the acoustic wave, the appropriate phase speed for the longitudinal waves is not the *bar speed* $\sqrt{Y/\rho_0}$, but rather the *bulk speed*

$$c^2 = \left(\mathcal{B} + \tfrac{4}{3}\mathcal{G}\right)/\rho_0 \tag{6.6.1}$$

where $\mathcal{B}$ and $\mathcal{G}$ are the *bulk* and *shear moduli* of the solid and ρ_0 its density. (See Appendix A11. Values of the bulk speed for various solids are given in Appendix A10.) The bulk speed for each material is always higher than that for longitudinal waves in thin bars.

(a) Normal Incidence

For this case, we have $\cos\theta_i = 1$ and (6.5.3) becomes

$$\mathbf{R} = \frac{(r_n - r_1) + jx_n}{(r_n + r_1) + jx_n} \tag{6.6.2}$$

The intensity reflection coefficient is

$$R_I = \frac{(r_n - r_1)^2 + x_n^2}{(r_n + r_1)^2 + x_n^2} \tag{6.6.3}$$

and the intensity transmission coefficient is

$$T_I = \frac{4r_n r_1}{(r_n + r_1)^2 + x_n^2} \tag{6.6.4}$$

(b) Oblique Incidence

No single simple method is available for analyzing the reflection of plane waves obliquely incident on the surface of a solid. Because of the differences in the porosity and internal elastic structure of various solids, the nature of the process varies. For instance, the wave transmitted into the solid may be refracted (1) so that it is propagated effectively only perpendicular to the surface, (2) in a manner similar to plane waves entering a second fluid, or (3) into two waves, a longitudinal wave traveling in one direction and a transverse (shear) wave traveling at a lower speed in a different direction.

1. The first type of refraction occurs for *normally-reacting* or *locally acting* surfaces. One example of this occurs in *anisotropic* solids, where waves propagated parallel to the surface travel with a much lower speed than those propagated perpendicular to the surface. This is typical of solids having a honeycomb structure in which the speed of compressional waves through the fluid contained in capillary pores perpendicular to the surface is much higher than that from pore to pore through the solid material of the structure. This type of refraction also will occur in an *isotropic* solid when the speed of longitudinal wave propagation in the solid is small compared with that in the adjacent fluid. Many sound-absorbing materials used in buildings (acoustic tile, perforated panels, etc.) behave as normally-reacting surfaces. When $c_2 \ll c_1$, Snell's law requires that $\theta_t \ll \theta_i$, and a reasonable approximation is to set $\cos\theta_t = 1$. This yields (6.5.3), which can be rewritten as

$$\mathbf{R} = \frac{(r_n - r_1/\cos\theta_i) + jx_n}{(r_n + r_1/\cos\theta_i) + jx_n} \tag{6.6.5}$$

This has the same form as (6.6.2) but with r_1 replaced by $(r_1/\cos\theta_i)$. The intensity reflection and transmission coefficients consequently are given by (6.6.3) and (6.6.4), respectively, with the same replacement. See Problem 6.6.4.

For most solid materials $r_n > r_1$, so that as θ_i increases an angle will be reached where $r_n = (r_1/\cos\theta_i)$. When this occurs the power reflection coefficient $R_\Pi = R_I$ will be near its minimum value. In particular, if x_n were zero, R_Π would be zero and T_Π would be unity. For $\theta_i \to 90°$, R_Π approaches unity. Plotted in Fig. 6.6.1 are curves for the reflection coefficient R_Π as a function of the angle of incidence θ_i for a few assumed values of the nondimensional parameters r_n/r_1 and x_n/r_1.

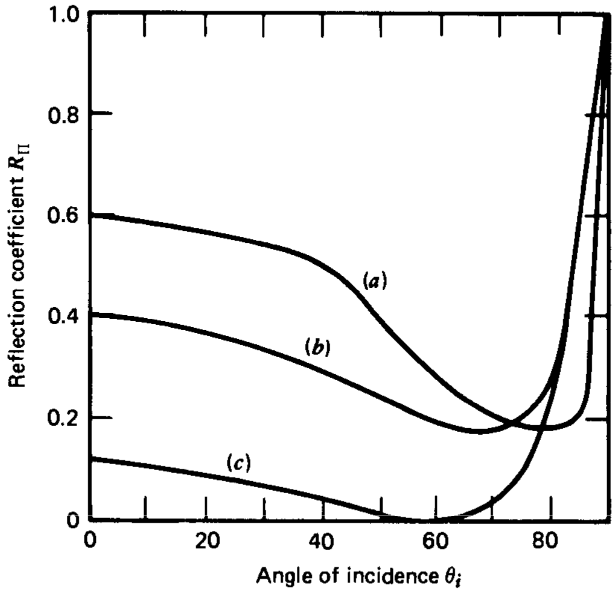

Figure 6.6.1 The reflection coefficient for a typical normally-reacting solid. (*a*) $r_n/r_1 = x_n/r_1 = 4$. (*b*) $r_n/r_1 = x_n/r_1 = 2$. (*c*) $r_n/r_1 = 2$ and $x_n/r_1 = 0$.

2. The second type of refraction is similar to the reflection and refraction occurring between two fluids as discussed in Section 6.4.
3. The third type of refraction occurs for rigid elastic solids. A detailed discussion requires consideration of the coupling of acoustic energy from the incident wave into both shear and longitudinal waves in the solid. Interested readers are referred to the sources listed at the beginning of this chapter.

*6.7 TRANSMISSION THROUGH A THIN PARTITION: THE MASS LAW

A case of practical importance in architectural acoustics is the transmission of sound through a thin partition between two enclosures, as found in many office or temporary working spaces. The partition is often a material whose motion is normal to the interface regardless of the angle of incidence of the sound, and whose thickness L is much smaller than a wavelength ($k_2L \ll 1$) for the frequency range of interest. Since both media 1 and 3 are the same, by Snell's law any wave transmitted into 3 must have the same direction of propagation as the incident wave in 1. The angles are the same, so that continuity of the normal component of particle velocity is equivalent to

$$\mathbf{u}_i + \mathbf{u}_r = \mathbf{u}_t \tag{6.7.1}$$

If the intervening layer 2 is thin and completely flexible with surface density ρ_S, the layer can be treated as an interface possessing mass so that the difference in pressures across the interface equals the product of the surface density ρ_S of the interface and its acceleration,

$$\mathbf{p}_i + \mathbf{p}_r - \mathbf{p}_t = j\omega\rho_S\mathbf{u}_t\cos\theta \tag{6.7.2}$$

After (6.7.1) is multiplied by r_1 and converted into pressures, and both (6.7.1) and (6.7.2) are divided by the incident pressure amplitude, these equations yield

$$\begin{aligned} 1 - \mathbf{R} &= \mathbf{T} \\ 1 + \mathbf{R} &= \mathbf{T} + j\frac{\omega\rho_S}{r_1}\mathbf{T}\cos\theta \end{aligned} \tag{6.7.3}$$

Solution for the power transmission coefficient results in

$$T_{\Pi}(\theta) = |\mathbf{T}(\theta)|^2 = \frac{1}{1 + \left[(\omega \rho_s / 2r_1)\cos\theta\right]^2} \tag{6.7.4}$$

For an incident wave falling normally on the surface, (6.7.4) reduces to the equivalent case given by (6.3.10) with fluids 1 and 3 having the same characteristic impedances and fluid 2 a thin layer with $r_2 \gg r_1$. (See Problem 6.7.1.)

In most practical situations in air for moderate frequencies the quantity $\omega \rho_S / r_1$ is relatively large. For example, a light partition between two work spaces made out of a sheet or two of gypsum or thick plywood will have a nominal surface density $\rho_S \sim 10\ \mathrm{kg/m^2}$. For frequencies above about 60 Hz, $\omega \rho_S / r_1 > 9$. For this case, (6.7.4) can be approximated by

$$T_{\Pi}(\theta) \sim (2r_1/\omega \rho_S \cos\theta)^2 \tag{6.7.5}$$

as long as θ does not exceed about 70°. This approximation, which fails for near-grazing incidence, expresses a form of the *mass law*: the power transmission coefficient is reduced fourfold for each doubling of the surface density. Further properties of the transmission of sound from a space containing a *diffuse sound field* through a partition to another space, and the effects of *coincidence* on the transmission of acoustic power through an elastic partition, will be deferred until Chapter 13.

6.8 METHOD OF IMAGES

Up to now, we have discussed the reflection and transmission of *plane* waves at plane interfaces. In this section we will investigate the reflection of *spherical* waves at plane boundaries, beginning with boundaries that are perfectly reflecting. (An approximation of such a boundary is the air–water interface.) This problem is amenable to the *method of images*. This approach is also used in electrostatics and in optics. A familiar example of application of the method of images in optics is the analysis of the interference resulting from the reflection of a light source from a single mirror (*Lloyd's mirror*).

In the case of a single plane boundary, fluid 2 is replaced by fluid 1 and an *image* is introduced whose strength and location are selected to satisfy the boundary conditions on the plane of the former interface. Because a solution of the wave equation is unique if the boundary conditions are satisfied, the acoustic field in the real fluid 1 is the same as that for the original situation. The acoustic field in the space containing fluid 2 will *not* be correctly represented.

(a) Rigid Boundary

Let a source of spherical waves be placed in a fluid of medium 1, which extends throughout all space. If this source is located on the z axis a distance $+d$ from the origin, as shown in Fig. 6.8.1, a spherical wave exists in all space given by

$$\begin{aligned} \mathbf{p}_i &= \frac{A}{r_-} e^{j(\omega t - k r_-)} \\ r_- &= [(z-d)^2 + y^2 + x^2]^{1/2} \end{aligned} \tag{6.8.1}$$

where r_- is the distance from the point (0, 0, d). If a second source, the *image*, of equal strength, frequency, and initial phase angle is placed at (0, 0, $-d$),

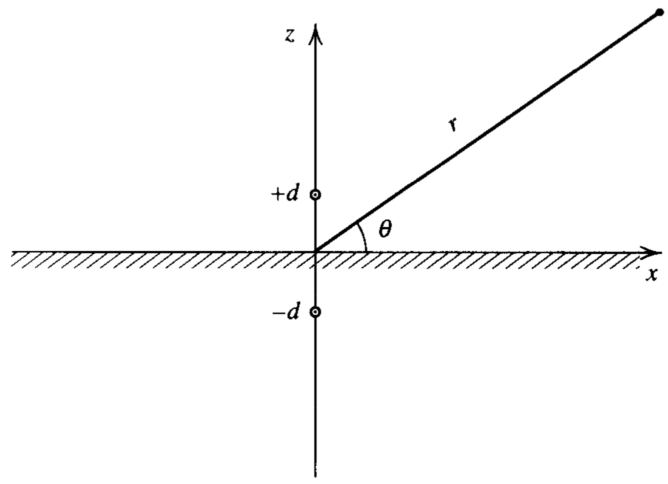

Figure 6.8.1 The use of image theory for calculating the acoustic field of a source of spherical waves near a rigid planar boundary. The source is located at $(0, 0, +d)$ and an image of equal strength and the same phase is located at $(0, 0, -d)$. The field point is located at (r, θ).

$$\begin{aligned} \mathbf{p}_r &= \frac{A}{r_+} e^{j(\omega t - kr_+)} \\ r_+ &= [(z+d)^2 + y^2 + x^2]^{1/2} \end{aligned} \tag{6.8.2}$$

it is easy to show that the normal component of the particle velocity vanishes on the x-y plane. Therefore, the fluid on the negative side of the x-y plane can be replaced by a rigid boundary at $z = 0$. (The components of the particle velocity parallel to the x-y plane do not cancel, so there is a velocity parallel to the boundary. Including the effects of viscosity would introduce some very small acoustic losses at the boundary, but these are negligible for our purposes here.)

The pressure in the region $z > 0$ is given by the sum of (6.8.1) and (6.8.2):

$$\mathbf{p} = \mathbf{p}_i + \mathbf{p}_r = A\left(\frac{1}{r_-} e^{-jkr_-} + \frac{1}{r_+} e^{-jkr_+}\right) e^{j\omega t} \tag{6.8.3}$$

While it is instructive to plot the pressure amplitude in the region of fluid 1 without making any approximations (see Problem 6.8.1C), greater insight can be gained by looking for an analytical solution for distances $r \gg d \cos\theta$, where θ is the *grazing* angle with respect to the boundary. In this approximation, a little geometry shows

$$\begin{aligned} \Delta r &\approx d \sin\theta \\ r_- &\approx r - \Delta r \\ r_+ &\approx r + \Delta r \end{aligned} \tag{6.8.4}$$

so that (6.8.3) becomes

$$\mathbf{p}(r, \theta, t) \approx \frac{A}{r} e^{j(\omega t - kr)} \left(\frac{e^{jk\Delta r}}{1 - \Delta r/r} + \frac{e^{-jk\Delta r}}{1 + \Delta r/r} \right) \tag{6.8.5}$$

The term $\Delta r/r$ gives the differences in amplitudes resulting from the slightly different distances to the field point from the source and its image. They yield minor contributions as long as $r \gg \Delta r$. The $k\Delta r$ in the exponent is another matter. Unless $k\Delta r \ll 1$, there will be significant phase interference between the pressures received from the source and its image. Discarding the terms $\Delta r/r$ in

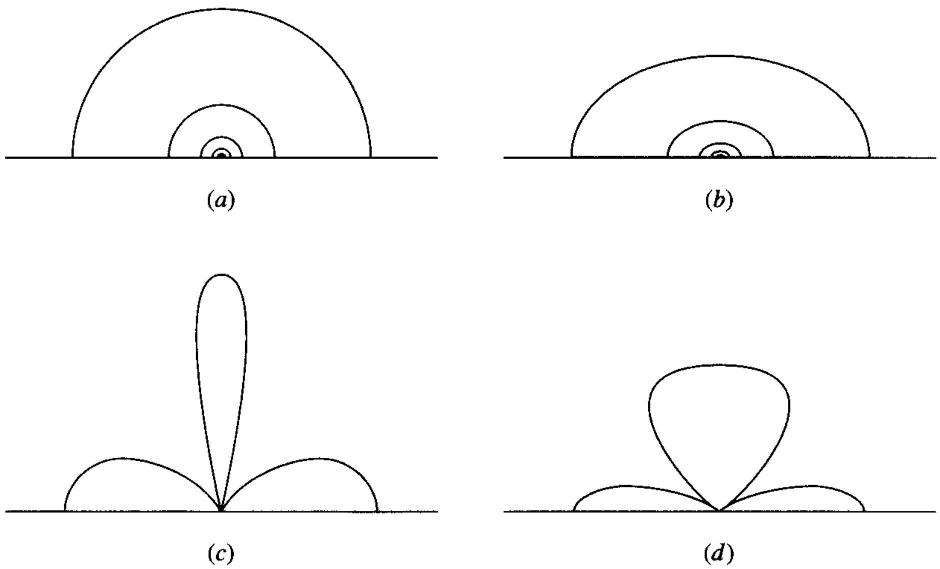

Figure 6.8.2 Contours of equal pressure amplitude for a source of spherical waves with wave number k at a distance d from a planar rigid surface. (*a*) $kd = 0.40$. (*b*) $kd = 0.80$. (*c*) $kd = 1.60$. (*d*) $kd = 3.20$.

the denominators in (6.8.5) and using standard exponential and trigonometric relationships gives

$$\mathbf{p}(r, \theta, t) \approx \frac{2A}{r} \cos(kd \sin\theta) e^{j(\omega t - kr)} \qquad \text{(rigid boundary)} \qquad (6.8.6)$$

The pressure field is an outgoing spherical wave with the amplitude depending on the angle θ. The pressure field for reflection of a spherical wave from a rigid boundary is sketched in Fig. 6.8.2 for several values of kd.

(b) Pressure Release Boundary

The pressure field for reflection of a spherical wave from a pressure release surface is found by using an image of the same amplitude as the source, but of *opposite* phase. The proof of this and the derivation of the result,

$$\mathbf{p}(r, \theta, t) \approx j\frac{2A}{r} \sin(kd \sin\theta) e^{j(\omega t - kr)} \qquad \text{(pressure release boundary)} \qquad (6.8.7)$$

are left as exercises.

(c) Extensions

There are a number of extensions to these simple examples that can be made easily.

1. The method does not depend on the source vibrating at a single frequency. If the source emits a spherical wave

$$\mathbf{p}_i = \frac{1}{r_-} f(ct - r_-) \qquad (6.8.8)$$

the pressure field of the image would be the same function with r_+ replacing r_- and multiplied by ± 1 depending on whether the boundary is rigid or pressure release. The resultant total acoustic pressure is

$$\mathbf{p} = \frac{1}{r_-} f(ct - r_-) \pm \frac{1}{r_+} f(ct - r_+) \tag{6.8.9}$$

2. There may be several elements comprising the source (as an array of point sources or a loudspeaker). Elementary application of superposition shows that the image will be the mirror reflection of the source with amplitude multiplied by ± 1 depending on the boundary condition.
3. If the boundary is not rigid or pressure release, then the method of images can be used as a reasonably good approximation if the source is many wavelengths away from the boundary, so that at the boundary the radii of curvature of the wave fronts are much greater than a wavelength. Under this condition, the waves incident on the boundary look locally like plane waves and the local reflection coefficient will be very similar to that for an incident plane wave. Then, for example, the spherical monofrequency source radiating the pressure field (6.8.1) will generate a total field

$$\mathbf{p} = \mathbf{p}_i + \mathbf{p}_r = A\left(\frac{1}{r_-} e^{-jkr_-} + \frac{\mathbf{R}(\theta)}{r_+} e^{-jkr_+}\right) e^{j\omega t} \tag{6.8.10}$$

   where the reflection coefficient is evaluated at the angle θ defined for the *specular reflection* ($\theta_r = \theta_i$) between source and field point. The resulting field will be missing certain features that a more exact, and considerably more complicated, analysis would provide, but under the geometrical restriction stated above these effects will be relatively small. This restriction must also be applied to the location of the field point, as explained below.
4. More than one reflecting surface can be present (a hall of mirrors). Under the same geometrical limitation as in extension 3, each boundary behaves like a mirror with reflectivity determined along each possible path from the source to the field point.

All applications of the method of images for rigid or pressure release plane boundaries exhibit a very important feature. If we have a point source at $(0, 0, d)$ and a point receiver at (x, y, z), examination of the expressions for r_- and r_+ shows that exchanging the positions of source and receiver does not change the value of the acoustic pressure at the receiver. The pressure fields in the medium for the two different geometries may have different interference patterns, but the signal observed by the receiver will be unaffected by the exchange. This means, for example, that the geometrical restrictions made in extension 3 concerning d/λ to obtain a simple approximation must also be applied to the distance between the field point (receiver) and the boundary.

For directional sources and receivers, and for other than rigid and pressure release boundaries, the conditions of the exchange and the relative orientations of the source and receiver must be handled more carefully, but similar results can be obtained. This will be dealt with further in Chapter 7 when we develop a more general expression of *acoustic reciprocity*.

PROBLEMS

6.2.1. A 1 kHz plane wave in water of 50 Pa effective (rms) pressure is incident normally on the water–air boundary. (*a*) What is the effective pressure of the plane wave transmitted into the air? (*b*) What is the intensity of the incident wave in the water and of the wave transmitted into the air? (*c*) Express, as a decibel reduction, the ratio of the intensity of the transmitted wave in air to that of the incident wave in water. (*d*) Answer the same three questions for the above sound wave incident on a thick layer of ice. (*e*) What is the power reflection coefficient from the layer of ice?

6.2.2. If a plane wave is reflected from the ocean floor at normal incidence with a level 20 dB below that of the incident wave, what are the possible values of the specific acoustic impedance of the fluid bottom material?

6.2.3. (*a*) A plane wave in seawater is normally incident on the water–air interface. Find the pressure and intensity transmission coefficients. (*b*) Repeat (*a*) for a wave in air normally incident on the air-water interface. (*c*) For (*a*) and (*b*) find the change in pressure and intensity levels if P_{ref} and I_{ref} are the same in both media.

6.2.4. Assume a reflection coefficient $\mathbf{R} = 0.5$ for a normally incident wave in air at 500 Hz and pressure amplitude 2 Pa. (*a*) What are the intensities of the incident, reflected, and transmitted waves? (*b*) Calculate the intensity of the total pressure field in fluid 1. (*c*) Write the total field in fluid 1 as the sum of a traveling and a standing wave. (*d*) Calculate the intensities for each of the two waves in part (*c*). (*e*) Are your results consistent with conservation of energy?

6.2.5C. For Problem 6.2.4, plot the total pressure amplitude in fluid 1 as a function of the distance from the boundary. Derive an equation that relates the ratio of the pressure amplitude at the antinodes to that at the nodes in terms of the pressure amplitudes of the incident and reflected waves and compare to your graph.

6.2.6C. Plot the pressure reflection and transmission coefficients and the intensity reflection and transmission coefficients for normal incidence of a plane wave on a fluid–fluid boundary for $0 < r_1/r_2 < 10$. Comment on the results for $r_1/r_2 = 0$, $r_1/r_2 = 1$, and $r_1/r_2 \to \infty$.

6.3.1. Show that when $r_2 = r_3$ the pressure reflection, intensity reflection, and intensity transmission coefficients all reduce to those for Section 6.2.

6.3.2. (*a*) What must be the thickness of, and the speed of sound in, a plastic layer having a density of 1500 kg/m^3 if it is to transmit plane waves at 20 kHz from water into steel with no reflection? (*b*) What would be the intensity reflection coefficient back into water for normally incident waves impinging on an infinitely thick layer of this plastic?

6.3.3. For a 2 kHz plane wave in water impinging normally on a steel plate of 1.5 cm thickness, (*a*) what is the transmission loss, expressed in dB, through the steel plate into water on the opposite side? (*b*) What is the power reflection coefficient of this plate? (*c*) Repeat (*a*) and (*b*) for a 1.5 cm thick slab of sponge rubber having a density of 500 kg/m^3 and a longitudinal wave speed of 1000 m/s.

6.3.4. Given the task of maximizing the transmission of sound waves from water into steel, (*a*) what is the optimum characteristic impedance of the material to be placed between the water and the steel? (*b*) What must be the density of, and sound speed in, a layer of 1 cm thickness that will produce 100% transmission at 20 kHz?

6.3.5. For normal incidence on a layer between fluids 1 and 3 and with $r_2 = r_1$, (*a*) show that the magnitude of the pressure reflection coefficient reduces to that in Section 6.2 for the appropriate fluids. (*b*) Interpret the phase angle of the reflection coefficient in terms of the time of flight of the signal in the layer.

6.3.6. Assume a layer of fluid 2 separates fluid 1 from fluid 3. (*a*) Compare the pressure reflection coefficient for a plane wave traveling in fluid 1 that reflects normally from the layer with that for a plane wave traveling in fluid 3 that reflects normally from the layer. (*b*) Are the pressure reflection coefficients the same for the two cases? (*c*) Repeat (*a*) and (*b*) for the power reflection and transmission coefficients. (*d*) What do these results suggest in terms of energy transmission?

6.3.7C. Plot the intensity transmission coefficient as a function of the scaled layer thickness k_2L for $r_1/r_2 = 2$ and $1 < r_1/r_3 < 9$. Comment on the conditions required to obtain minimum and maximum transmissions.

6.3.8. A plane wave pulse consisting of 10 cycles of 10 kHz carrier is normally incident from water onto a 50 m thick layer of red clay overlaying a thick sedimentary bottom with $c_3 = 2300$ m/s and $\rho_3 = 2210$ kg/m^3. For the first two reflections received back in the water, calculate (*a*) the time interval between arrivals, (*b*) the amplitudes relative to the incident pulse, and (*c*) the relative phase between the arrivals assuming they were continuous waves rather than pulses.

6.4.1. (*a*) As a function of θ_i, plot the amplitude and phase of the pressure reflection coefficient for the case $c_2 = c_1$ and $\rho_2 > \rho_1$ and identify any significant features. (*b*) Plot the amplitude and phase of the pressure reflection coefficient for the case $\rho_2 = \rho_1$ and $c_2 > c_1$ and identify any significant features.

6.4.2. For plane wave reflection from a fluid–fluid interface it is observed that at normal incidence the pressure amplitude of the reflected wave is one-half that of the incident wave (no phase information is recorded). As the angle of incidence is increased, the amplitude of the reflected wave first decreases to zero and then increases until at 30° the reflected wave is as strong as the incident wave. Find the density and sound speed in the second medium if the first medium is water.

6.4.3. A plane wave traveling from air into hydrogen gas through a thin separating membrane is refracted by 40° from its original direction. (*a*) What is the angle of incidence in the air? (*b*) What is the sound power transmission coefficient?

6.4.4. A plane wave in water of 100 Pa peak pressure amplitude is incident at 45°on a mud bottom having $\rho_2 = 2000$ kg/m^3 and $c_2 = 1000$ m/s. Compute (*a*) the angle of the ray transmitted into the mud, (*b*) the peak pressure amplitude of the transmitted ray, (*c*) the peak pressure amplitude of the reflected ray, and (*d*) the sound power reflection coefficient.

6.4.5. Plane waves in water of 100 Pa effective (rms) pressure are incident normally on a sand bottom. The sand has density 2000 kg/m^3 and sound speed 2000 m/s. (*a*) What is the effective pressure of the wave reflected back into the water? (*b*) What is the effective pressure of the wave transmitted into the sand? (*c*) What is the power reflection coefficient? (*d*) What is the smallest angle of incidence at which all of the incident energy will be reflected?

6.4.6. A sand bottom in seawater is characterized by $\rho_2 = 1700$ kg/m^3 and $c_2 = 1600$ m/s. (*a*) What is the critical angle of incidence corresponding to total reflection? (*b*) For what angle of incidence is the power reflection coefficient equal to 0.25? (*c*) What is the power reflection coefficient for normal incidence?

6.4.7C. For the oblique reflection of a plane wave at a fluid–fluid interface, plot the magnitude and phase of the pressure reflection coefficient as a function of θ_i for (*a*) $r_2/r_1 = 0.5$ and $c_2/c_1 = 0.5, 1, 1.5$; (*b*) $r_2/r_1 = 1$ and $c_2/c_1 = 0.5, 1, 1.5$; and (*c*) $r_2/r_1 = 1.5$ and $c_2/c_1 = 0.5, 1, 1.5$.

6.4.8. Derive the approximate behavior (lowest order in small angle) of **R** (amplitude and phase) (*a*) in terms of the *grazing* angle $\beta = (\pi/2 - \theta_i)$ for near-grazing incidence, (*b*) in terms of $(\theta_c - \theta_i)$ for θ_i slightly less than θ_c, and (*c*) in terms of $(\theta_i - \theta_c)$ for θ_i

slightly more than θ_c. (*d*) With the help of the approximation $\exp(\delta) \approx 1 + \delta$ for small δ, approximate the results through lowest order for these cases in exponential form.

6.6.1. An acoustic tile panel is characterized by a normal specific acoustic impedance of $900 - j1200$ Pa·s/m. (*a*) For what angle of incidence in air will the power reflection coefficient be a minimum? (*b*) What is the power reflection coefficient for an angle of incidence of 80°? (*c*) What is the power reflection coefficient for normal incidence?

6.6.2. A wall reflects plane waves like a normally-reacting surface of normal specific acoustic impedance $\mathbf{z}_n = r_1 + j\omega\rho_S$, where r_1 is the characteristic impedance of the air and ρ_S is the area density of the wall (kg/m^2). Derive a general equation for the power reflection coefficient as a function of the incident angle θ_i. For $\rho_S = 2$ kg/m^2, compute and plot the power reflection coefficient at 100 Hz as a function of θ_i.

6.6.3C. Plot the magnitude and phase of the pressure reflection coefficient as a function of incident angle for a normally-reacting solid for (*a*) $r_n/r_1 = 2$ and $x_n/r_1 = 0$, (*b*) $r_n/r_1 = x_n/r_1 = 2$, and (*c*) $r_n/r_1 = x_n/r_1 = 4$. Comment on the conditions for minimum reflection coefficient.

6.6.4. Starting with (6.6.5), show that the power reflection and transmission coefficients for oblique reflection from a normally-reacting solid are

$$R_\Pi = \frac{(r_n \cos\theta_i - r_1)^2 + (x_n \cos\theta_i)^2}{(r_n \cos\theta_i + r_1)^2 + (x_n \cos\theta_i)^2}$$

$$T_\Pi = \frac{4r_1 r_n \cos\theta_i}{(r_n \cos\theta_i + r_1)^2 + (x_n \cos\theta_i)^2}$$

6.7.1. (*a*) With the help of the speed of sound c_2 for the material of the partition, show that $\omega\rho_S/r_1 = (r_2/r_1)k_2L$, where L is the thickness of the partition. (*b*) For normal incidence, find the inequality necessary for (6.3.10) to reduce to (6.7.4). (*c*) Is the inequality of (*b*) consistent with the approximations made in obtaining (6.3.12)?

6.8.1C. A source of spherical waves of wave number k is a distance d above an infinite, plane, rigid boundary. (*a*) For $kd = 0.1$, plot contours of equal pressure amplitude for the exact solution (6.8.3) and the approximation (6.8.6). Comment on the differences, if any. (*b*) Repeat for $kd = 10$.

6.8.2. (*a*) Show that in the limit $kd \ll 1$ the pressure field (6.8.6) reduces to

$$\mathbf{p}(r, \theta, t) \approx \frac{2A}{r} e^{j(\omega t - kr)}$$

(*b*) What effect does the presence of the boundary have on the sound pressure level observed (many wavelengths away) for the same source in the absence of the boundary?

6.8.3. (*a*) Show from (6.8.3) that the use of an image source of the same amplitude but opposite phase satisfies the pressure release boundary condition on the plane halfway between them. (*b*) Show that for $r \gg d\cos\theta$ the pressure above a pressure release surface is given by (6.8.7) and (*c*) obtain an expression for the locations of the pressure nodal surfaces.

6.8.4. In the limit $kd \ll 1$ show that (6.8.7) reduces to

$$\mathbf{p}(r, \theta, t) \approx -j\frac{2Akd}{r} \sin\theta \, e^{j(\omega t - kr)}$$

6.8.5. (*a*) A source of spherical waves of frequency f and pressure amplitude A at 1 meter is a distance d above a plane, rigid boundary. Calculate the amplitude of the pressure on the boundary as a function of d and r. (*b*) For the same source at the same distance above a plane, pressure release boundary, calculate the normal component of the particle velocity on the boundary. (*c*) Make plots of (*a*) as a function of R, the distance along the boundary measured from the point closest to the source, for various values of d. (*d*) Repeat (*c*) for (*b*).

6.8.6C. (*a*) A source of spherical waves of frequency f and pressure amplitude A at 1 meter is located in water a distance d above a flat, quartz sand bottom. For $kd = 20\pi$, plot the amplitude of the pressure at the same distance d above the boundary as a function of kr, where r is the distance between source and receiver. (*b*) Repeat for a red clay bottom.

6.8.7C. A source of monofrequency spherical waves is located in air midway between parallel rigid surfaces a distance H apart. (*a*) Assuming incoherent summation, design a program to calculate the pressure at a receiver also located at the midpoint for distances greater than $10d$ away from the source. (*b*) Determine if the pressure approaches an asymptotic functional dependence proportional to $1/\sqrt{r}$ as the number of images is increased. (*c*) How well do your results for larger numbers of images satisfy the relation $SPL(1) - SPL(r) = 10\log r + 10\log(H/\pi)$?

6.8.8. Assume that the near-shore ocean can be modeled by two nonparallel planes: a horizontal pressure release top and a sloping rigid bottom. (*a*) Sketch the position and indicate the phases of the images representing the paths from a source in the layer that reflect once off the top, once off the bottom, first off the top then off the bottom, and first off the bottom and then off the top. (*b*) Show that all images lie on a circle passing through the source and centered at the shore.

6.8.9C. A horizontal plane, pressure release surface makes an angle of 20° with a plane, rigid bottom. A source of spherical waves of frequency f and pressure amplitude A at 1 meter is located at mid-depth in this wedge at a distance R from the vertex. (*a*) Assuming incoherent summation, calculate the pressure amplitude as a function of distance along the mid-depth of the wedge. Cover the distance from the vertex out to at least twice R (to avoid overflow, omit distances in the immediate vicinity of the source). (*b*) Repeat assuming coherent summation. Note that there are a total of 17 images in addition to the source.

6.8.10C. Repeat Problem 6.8.9C for a line passing through the source and parallel to the vertex. Cover distances out to at least $2R$ from the source.