Feedback control, Nyquist

Solution

1. The open loop transfer function $KGH(s) = \frac{K}{s(\tau s+1)}$.

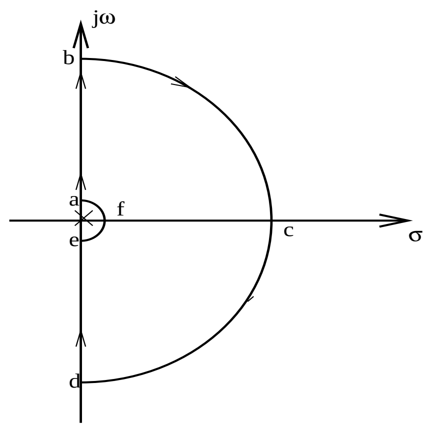

Figure 1: *Nyquist contour*

ab:	$s = j\omega$	$0 < \omega < \infty$
bcd:	$s = lim_{R\rightarrow\infty} Re^{j\Theta}$	$+90 \geq \Theta \geq -90$
de:	$s = j\omega$	$-\infty < \omega < 0$
efa:	$s = \lim_{r\rightarrow 0} re^{j\Theta}$	$-90 \leq \Theta \leq +90$

$\underline{\underline{\text{ab:}}}$ $s = j\omega$

$$KGH(j\omega) = \frac{K}{j\omega(\tau j\omega + 1)} \quad 0 < \omega < \infty$$

$\omega \rightarrow 0 \Rightarrow KGH(j\omega) \rightarrow \frac{K}{j\omega} = -\frac{K}{\omega} j \rightarrow -\infty j$
$\omega \rightarrow \infty \Rightarrow KGH(j\omega) \rightarrow -\frac{K}{\tau\omega^2} \rightarrow 0_-$
For a value between $\omega = 0$ and $\omega = \infty$ the phase is between -90 and -180, meaning that the map is in 3. quadrant.

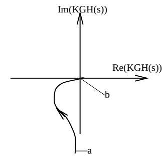

Figure 2: *Map of ab*

bcd: $s = \lim_{R\to\infty} Re^{j\Theta} \qquad +90 \geq \Theta \geq -90.$

$$\lim_{R\to\infty} KGH(Re^{j\Theta}) = \lim_{R\to\infty} \frac{K}{Re^{j\Theta}(\tau Re^{j\Theta}+1)} \to 0$$

bcd is maped in zero.
de: Correspond to ab, though reflected on the first axis.
efa: $s = \lim_{r\to 0} re^{j\Theta} \qquad -90 \leq \Theta \leq +90.$

$$\lim_{r\to 0} KGH(re^{j\Theta}) = \lim_{r\to 0} \frac{K}{re^{j\Theta}(\tau re^{j\Theta}+1)} = \lim_{r\to 0} \frac{K}{re^{j\Theta}} = \lim_{r\to 0} \frac{K}{r} e^{-j\Theta}$$

This is a semicircle with inft. radius from +90 through 0 and ending in -90 deg., because $-90 \leq \Theta \leq +90$.

Combining the maps we find the following Nyquist plot:

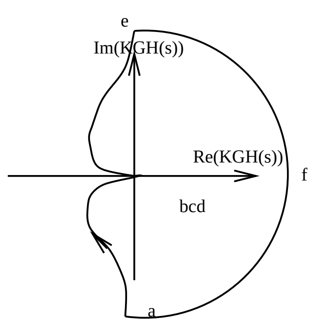

Figure 3: *Nyquist plot*

Because P=0 (we have no poles in the Nyquist contour) and N=0 because the Nyquist plot do not encircle the point -1, we find that N+P=0 meaning that the system is stable.

2. The open loop is $KGH(s) = \frac{K}{s^3}$.
The Nyquist contour is the same as in problem 1.
$\underline{\underline{\text{ab}}}$: $s = j\omega$

$$KGH(j\omega) = \frac{K}{(j\omega)^3} \quad (0 < \omega < \infty)$$

$$KGH(j\omega) = \frac{K}{(j\omega)^3} = -\frac{K}{j\omega^3} = j\frac{K}{\omega^3}$$

this means that ab is mapped on the positive imag. axis from $+\infty j$ to 0.

$\underline{\underline{\text{bcd}}}$: $s = \lim_{R\to\infty} Re^{j\Theta} \qquad +90 \geq \Theta \geq -90.$

$$\lim_{R\to\infty} KGH(Re^{j\Theta}) = \lim_{R\to\infty} \frac{K}{(Re^{j\Theta})^3} \to 0$$

bcd is maped in zero.

$\underline{\underline{\text{de}}}$: The same as ab, but reflected, meaning that the map is on the negative imag. axis.

$\underline{\underline{\text{efa}}}$: $s = \lim_{r\to 0} re^{j\Theta} \qquad -90 \leq \Theta \leq +90.$

$$\lim_{r\to 0} KGH(re^{j\Theta}) = \lim_{r\to 0} \frac{K}{(re^{j\Theta})^3} = \lim_{r\to 0} \frac{K}{r^3} e^{-3j\Theta}$$

The radius is ∞ and the map is going from $3 \cdot 90 = 270^o$ and to $-3 \cdot 90 = -270^o$. $(270 \to 180 \to 90 \to 0 \to -90 \to -180 \to -270)$.

The Nyquist plot is found combining the individual maps

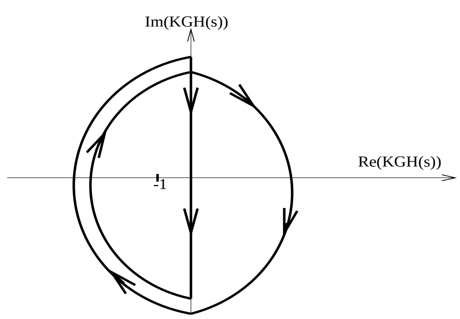

Figure 4: *Nyquist plot*

N (the number of clockwise encirclements of -1) is 2.
P (the number of poles inside the contour) is 0.
This gives Z=N+P=2, where Z is the number of closed loop poles in the right half plane. The system is unstable.

3. The open loop is given by $\frac{1000s^3}{(s+1)^3}$. It must be investigated in a closed contour starting i zero (a) following the imaginary axis to ∞j (b) enclosing the right half plane, (here is the point ∞ (c)) going to (d) and following the negative imaginary axis back to zero.
In this contour the open loop do not contain poles and P=0.

The Nyquist plot is hand sketched:
For the line ab: $s = \omega j$ where ω goes from 0 to ∞:

$$(a) \quad \omega \to 0 \Rightarrow \frac{-1000\omega^3 j}{(\omega j+1)^3} \to \frac{-1000\omega^3 j}{(0+1)^3} \quad [\to -\omega j_{(langs\ neg\ imag\ akse)}] \quad \to 0$$

$$(b) \quad \omega \to \infty \Rightarrow \frac{-1000\omega^3 j}{(\omega j+1)^3} \to \frac{-1000\omega^3 j}{-(\omega)^3 j} = 1000$$

To sketch the Nyquist plot between (a) and (b) the bode plot as shown in figure 5 is used. It is seen that for ω between 0 and 0.6 [rad/sek] the Nyquist plot is in the third quadrant, at 0.6 [rad/sek] the Nyquist plot will come in the second quadrant because 180 degrees is passed at a gain found to approx. 40 dB (100). For ω approx. 2 [rad/sek] the plot will be in the first quadrant and it ends at 60 dB equivalent to 1000. The Nyquist plot for the line ab is in figure 6.

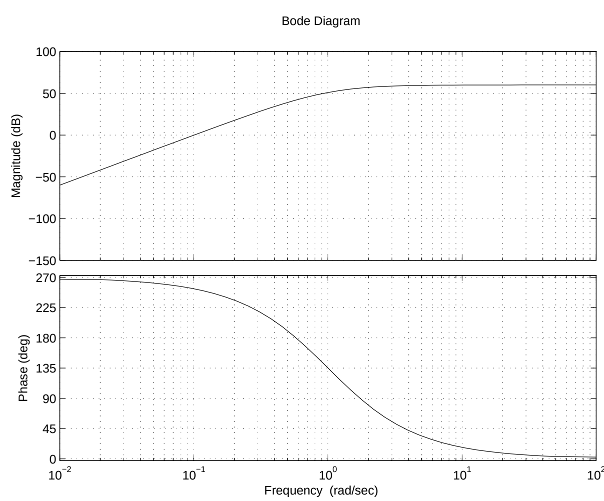

Figure 5: *Bode plot for the open loop*

For the line bcd: $s = \lim_{R\to\infty} Re^{\theta j}$.

$$(bcd) \quad \lim_{R\to\infty} \frac{1000(Re^{\theta j})^3}{(Re^{\theta j}+1)^3} \to \frac{1000(Re^{\theta j})^3}{(Re^{\theta j})^3} = 1000$$

All in all the Nyquist plot is shown in figure 7.

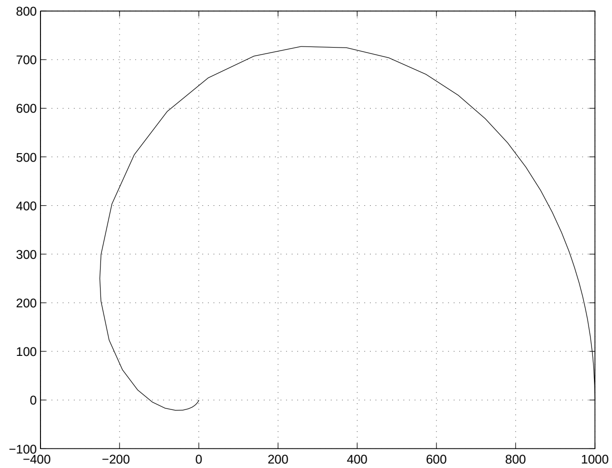

Figure 6: *Nyquist plot for the line ab*

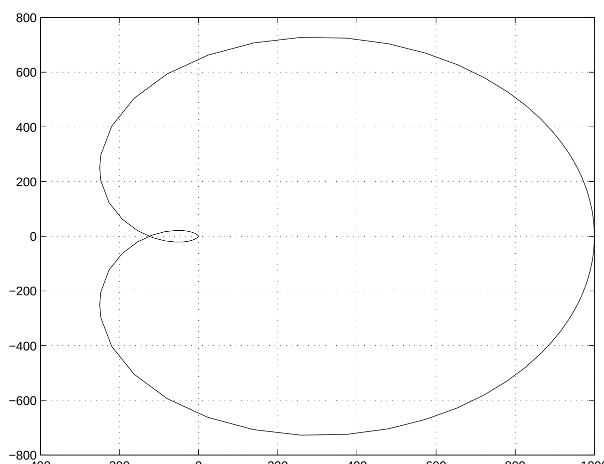

Figure 7: *Nyquist plot*

From the Nyquist plot N=2, meaning that Z=P+N=2 and the system is unstable

4. No solution