manipulated and controlled variables) is best is not effective. What is important is the ability of the control system to keep the process at setpoint in the face of load disturbances. Thus, load rejection is the most important criterion for deciding what variables to pair and what controller structure is best.

The RGA is useful for avoiding poor pairings. If the diagonal element in the RGA is negative, the system may show integral instability—the same situation that we discussed in the context of the Niederlinski index. Very large values of the RGA indicate that the system can be quite sensitive to changes in the parameter values.

12.3.2 Decoupling

Some of the earliest work in multivariable control involved the use of decouplers to remove the interaction between the loops. Figure 12.7 gives the basic structure of the system. The decoupling matrix $\underline{\underline{D}}_{(s)}$ is chosen such that each loop does not affect the other. Figure 12.8 shows the details of a 2×2 system. The decoupling element D_{ij} can be selected in a number of ways. One of the most straightforward is to set $D_{11} = D_{22} = 1$ and design the D_{12} and D_{21} elements so that they cancel (in a feedforward way) the effect of each manipulated variable in the other loop. For example, suppose Y_1 is not at its setpoint but Y_2 is. The G_{C1} controller changes m_1 to drive Y_1 back to Y_1^{set}. But the change in m_1 disturbs Y_2 through the G_{M21} transfer function.

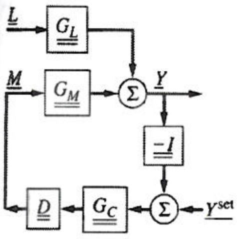

FIGURE 12.7
Decoupling.

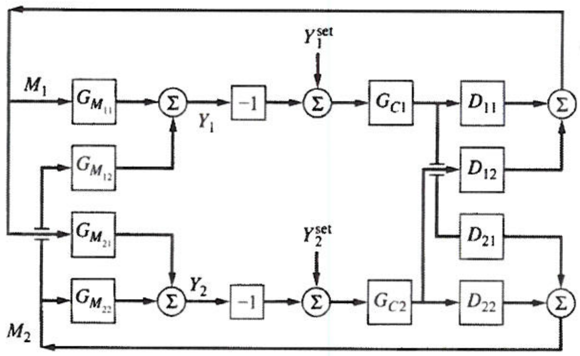

FIGURE 12.8
Block diagram of 2×2 system with decouplers.

If, however, the D_{21} decoupler element is set equal to $(-G_{M21}/G_{M22})$, there is a change in m_2 that comes through the G_{M22} transfer function and cancels out the effect of the change in m_1 on Y_2.

$$D_{21} = \frac{-G_{M21}}{G_{M22}} \tag{12.61}$$

Using the same arguments for the other loop, the D_{12} decoupler could be set equal to

$$D_{12} = \frac{-G_{M12}}{G_{M11}} \tag{12.62}$$

This "simplified decoupling" splits the two loops so that they can be independently tuned. Note, however, that the closedloop characteristic equations for the two loops are *not* $1 + G_{M11}G_{C1} = 0$ and $1 + G_{M22}G_{C2} = 0$. The presence of the decouplers changes the closedloop characteristic equations to

$$1 + G_{C1}\frac{G_{M11}G_{M22} - G_{M12}G_{M21}}{G_{M22}} = 0 \tag{12.63}$$

$$1 + G_{C2}\frac{G_{M11}G_{M22} - G_{M12}G_{M21}}{G_{M11}} = 0 \tag{12.64}$$

Other choices of decouplers are also possible. However, since decoupling may degrade the load rejection capability of the system, the use of decouplers is not recommended except in those cases where setpoint changes are the major disturbances.

12.4 CONCLUSION

The notation used for multivariable systems was reviewed in this chapter, and some important concepts were developed. The most important topic is the derivation of the characteristic equation for a closedloop multivariable process, which permits us to determine if the system is stable or unstable.

PROBLEMS

12.1. Wardle and Wood (*Inst. Chem. Eng. Symp. Ser.* *32*:1, 1969) give the following transfer function matrix for an industrial distillation column:

$$\underline{\underline{G_M}} = \begin{bmatrix} \dfrac{0.126e^{-6s}}{60s+1} & \dfrac{-0.101e^{-12s}}{(48s+1)(45s+1)} \\ \dfrac{0.094e^{-8s}}{38s+1} & \dfrac{-0.12e^{-8s}}{35s+1} \end{bmatrix}$$