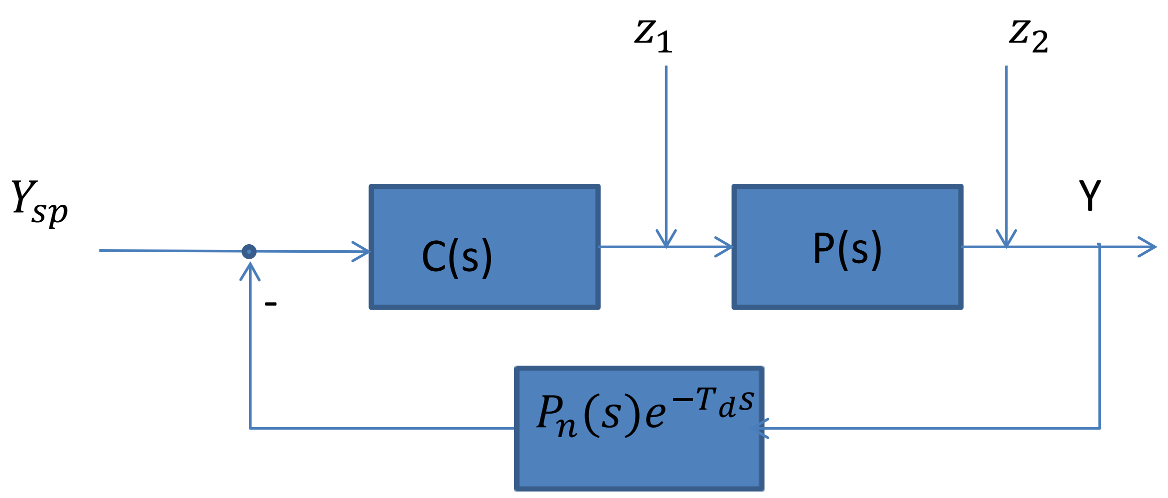
Feedforward design

Outline

- Feedback for robustness and good disturbance rejection
- Feedforward for fast response to set-point changes
- Feedforward for reduction of measurable disturbances

Typical feedback loop



$$\frac{Y(s)}{Y_{sp(s)}} = \frac{C(s)P(s)}{1 + C(s)P(s)P_n(s)e^{-T_d s}}$$

$$\frac{Y}{z_2} = \frac{1}{1 + P_n(s)e^{-T_d s}C(s)P(s)}$$

$$\frac{Y}{z_1} = \frac{P(s)}{1 + P(s)P_n(s)e^{-T_d s}C(s)}$$

problems

- It can be expensive
- Response time to setpoint changes
- noise effect on the output
- effect of the time delay

What do we use feedforward for

- Systems with known transfer functions and few disturbances - open loop control

Feedforward in combination with feedback

- Set-point response
- Reduce the effect of
  - measurable disturbances
  - Known delays

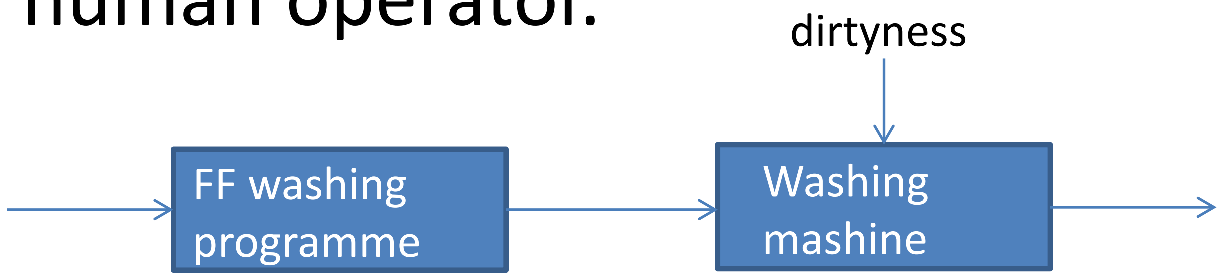
Feedforward because of price

Feedforward/open loop control is used in many processes because of

- simplicity and low cost.

Typical example:

- a conventional washing machine, for which the length of machine wash time is entirely dependent on the judgment and estimation of the human operator.

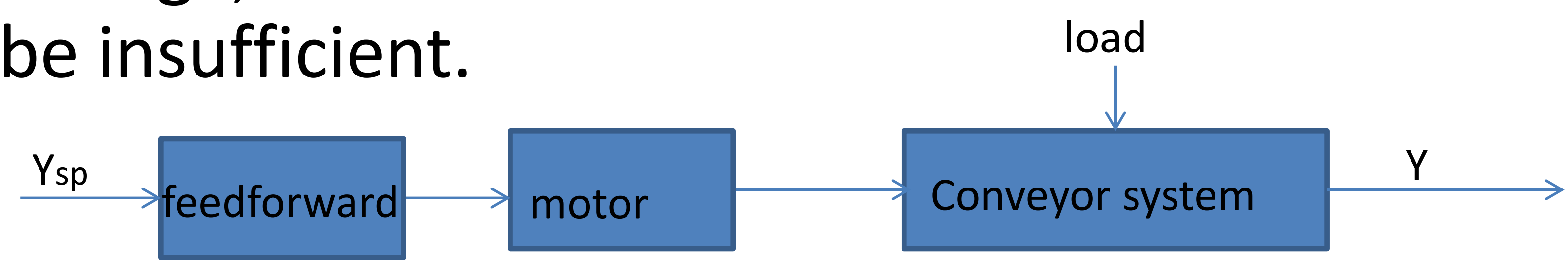


Feedforward control

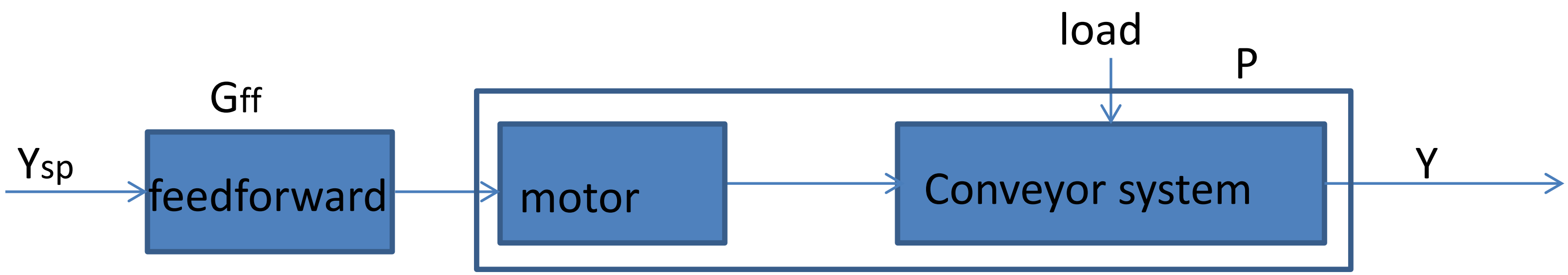
Feedforward control is useful for well-defined systems where the relationship between input and the resultant state can be modeled by a mathematical formula.

Example:

- the voltage fed to a motor driving a constant load to achieve a desired speed.
- If the load is unpredictable, the motor's speed varies as a function of the load as well as of the voltage, therefore a feedforward controller would be insufficient.



Feedforward control



- To obtain a perfect control: $G_{ff} = P^{-1}$, this is not always possible
- Example

$$P(s) = \frac{s-1}{s+2} \Rightarrow P^{-1}(s) = \frac{s+2}{s-1}$$

this system is unstable

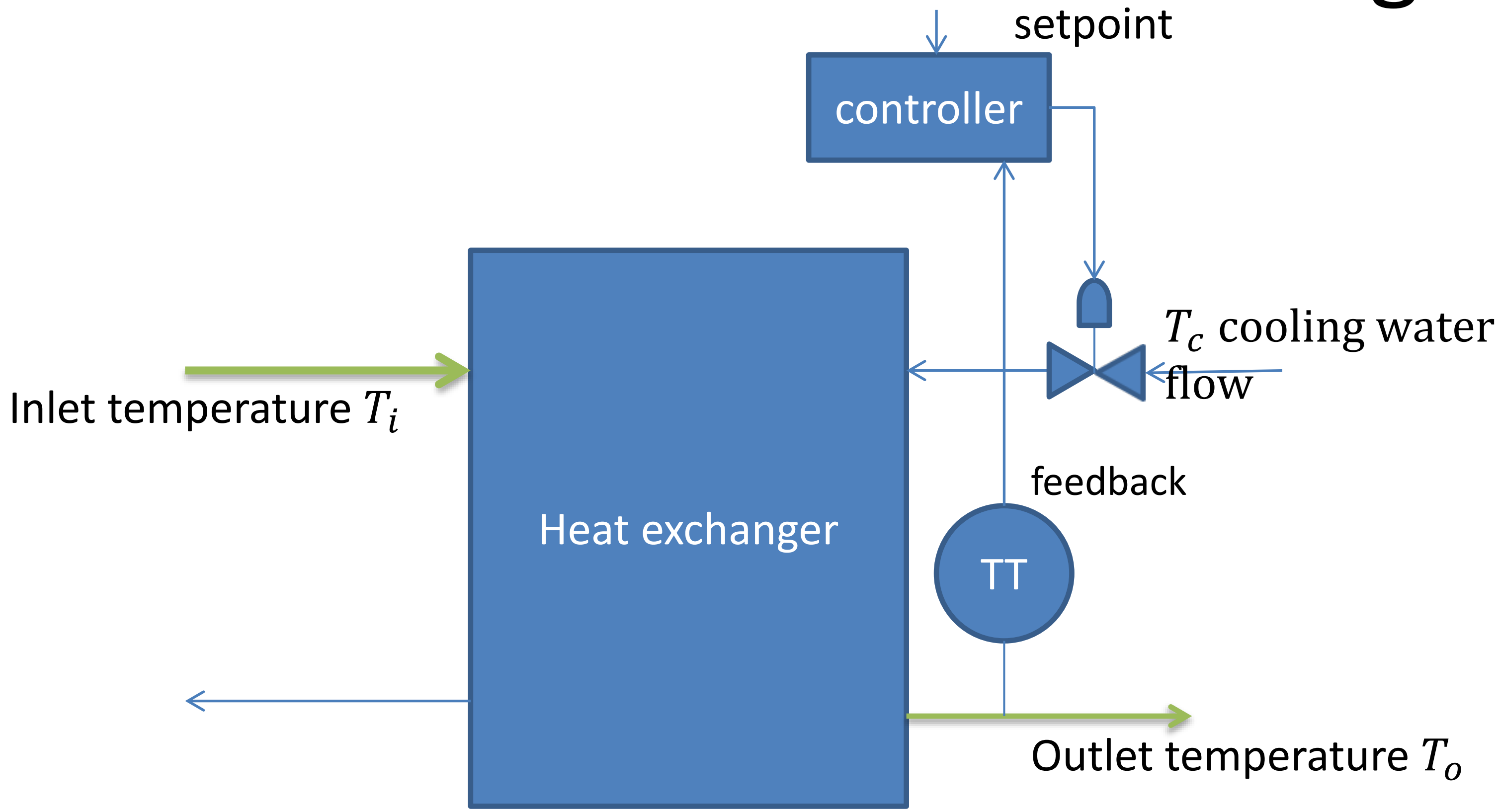
Instead use an approximation $G_{ff} = P^{-1}(0)$

The dynamics are not totaly compensated

Feedback/feedforward

- First: Feedback is design to give robustness and good disturbance rejection
- Second: Feedforward is designed to give good and fast response to set-point changes and rejection of measurable disturbances

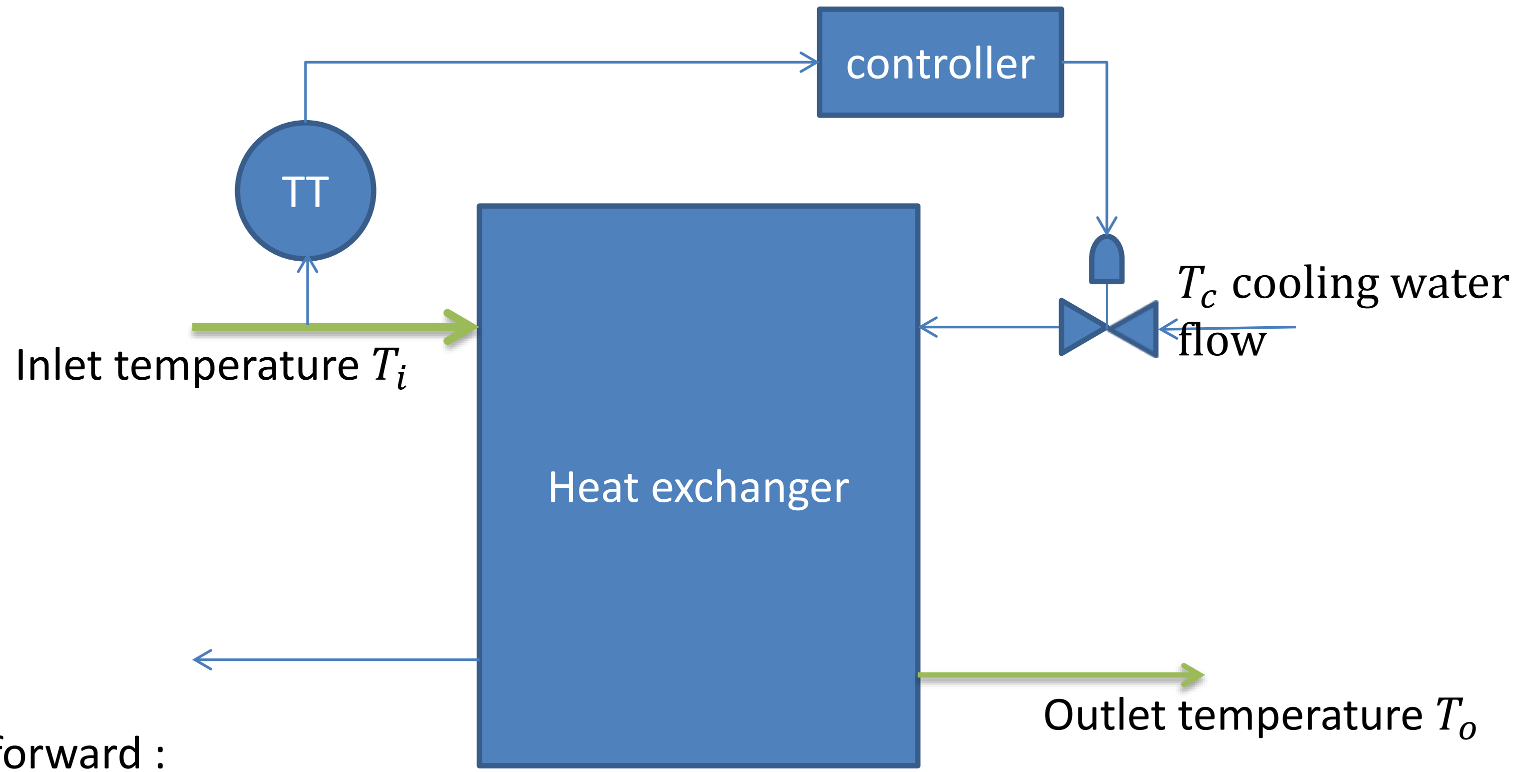
Example
Feedback control of heat exchanger



Feedback :
Control the stationary outlet temperature T_o by adjusting the cooling water flow – there will be a time delay in the system

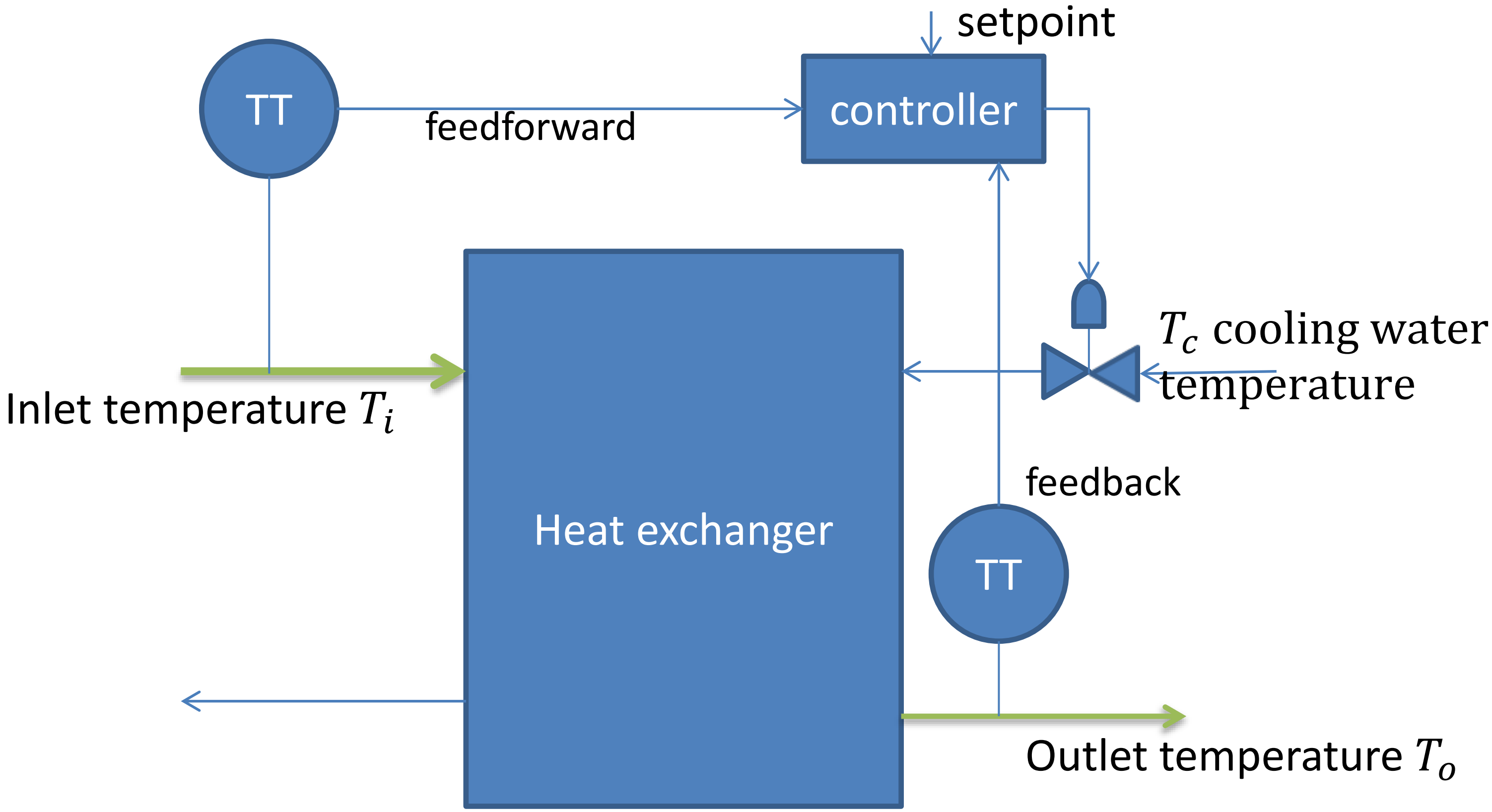
Intuition: We can get better control by measuring T_i

Example
Feedforward control of heat exchanger



Feedforward :
Assumption: T_i has the most dominant variations.
The outlet temperature T_o is controlled by adjusting the cooling water flow by measuring T_i
Note: The model must be correct
The controller doesn't account for disturbances from cooling water temperature, pressure etc.

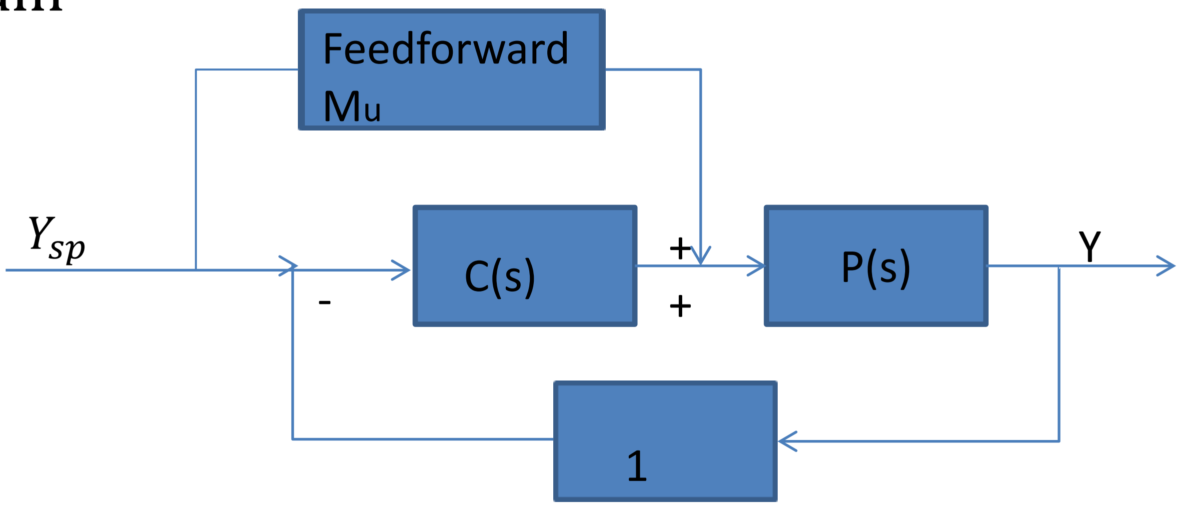
Example
Combined feedforward and feedback



Feedback : control the stationary outlet temperature T_o
Feedforward: fast compensation of T_i disturbaces

Feedback loop + feedforward

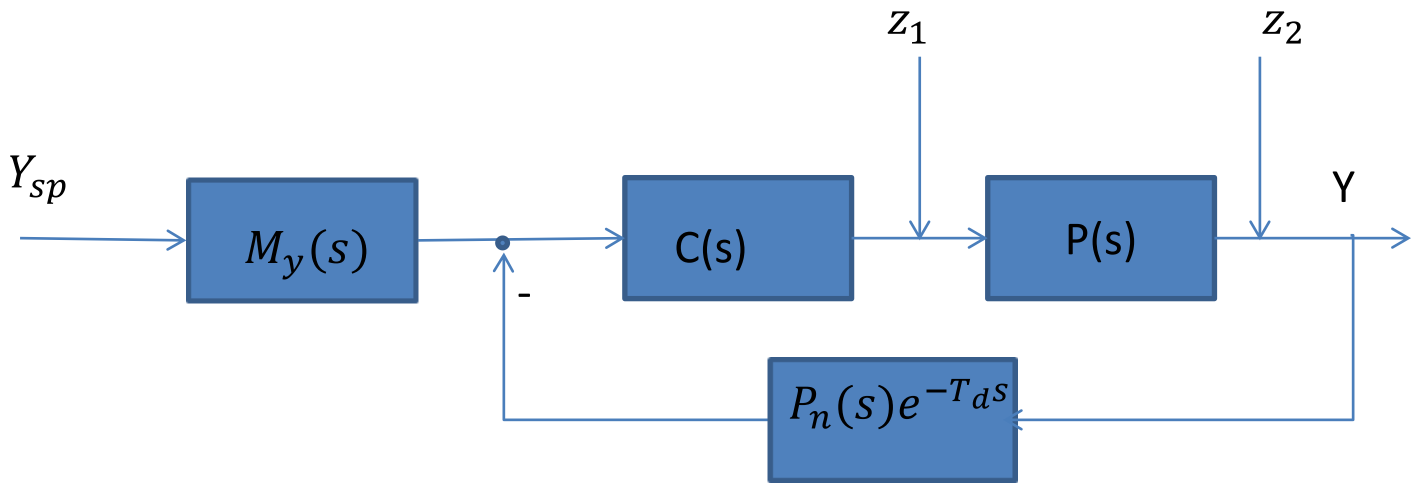
Typical structure to obtain good set point reponse



$$\frac{Y}{Y_{sp}} = \frac{P(C + M_u)}{1 + PC} = \frac{PC + PM_u}{1 + Pc}$$

If $PM_u = 1$
we have a perfect transferfunction ,
but $M_u = P^{-1}$ is to always possible
to implement

feedback loop + feedforward



$$\frac{Y(s)}{Y_{sp(s)}} = M_y(s)\frac{C(s)P(s)}{1 + C(s)P(s)P_n(s)e^{-T_d s}}$$

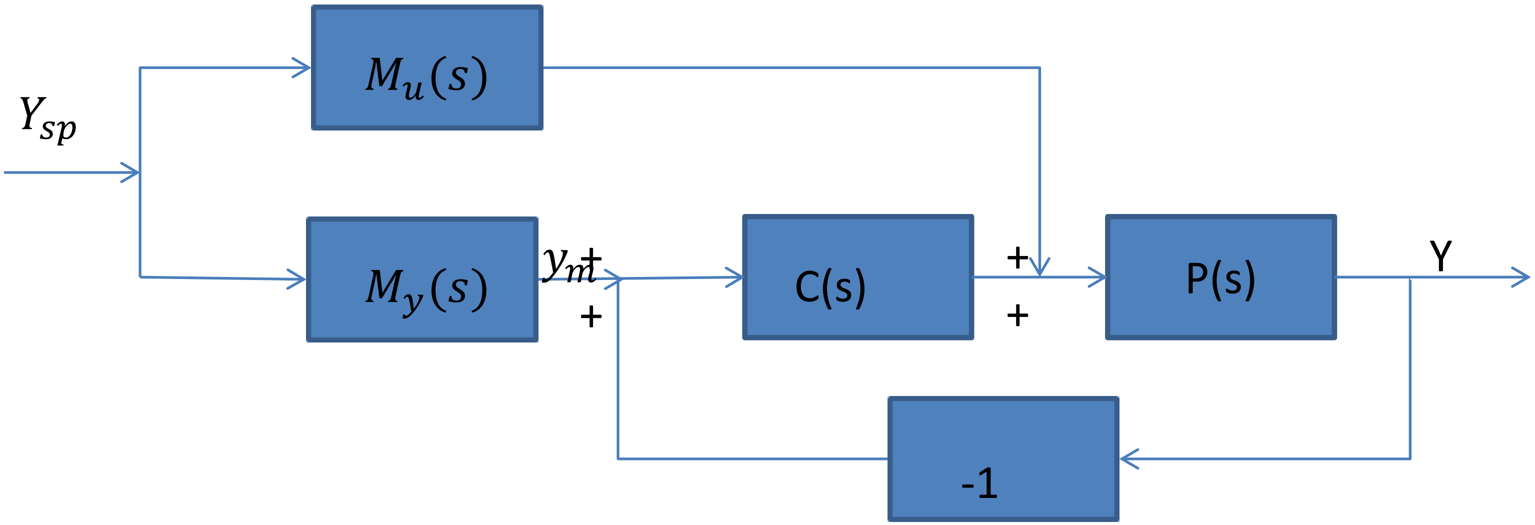
$$\frac{Y}{z_2} = \frac{1}{1 + P_n(s)e^{-T_d s}C(s)P(s)}$$

$$\frac{Y}{z_1} = \frac{P(s)}{1 + P(s)P_n(s)e^{-T_d s}C(s)}$$

$M_y(s)$ gives the wanted output, it acts out side the feedback loop
It has no effect on the noise transfer functions.

Can feedforward help us to reduce the effect of $z_1, z_2, and\ T_d$

Two degree freedom feedforward

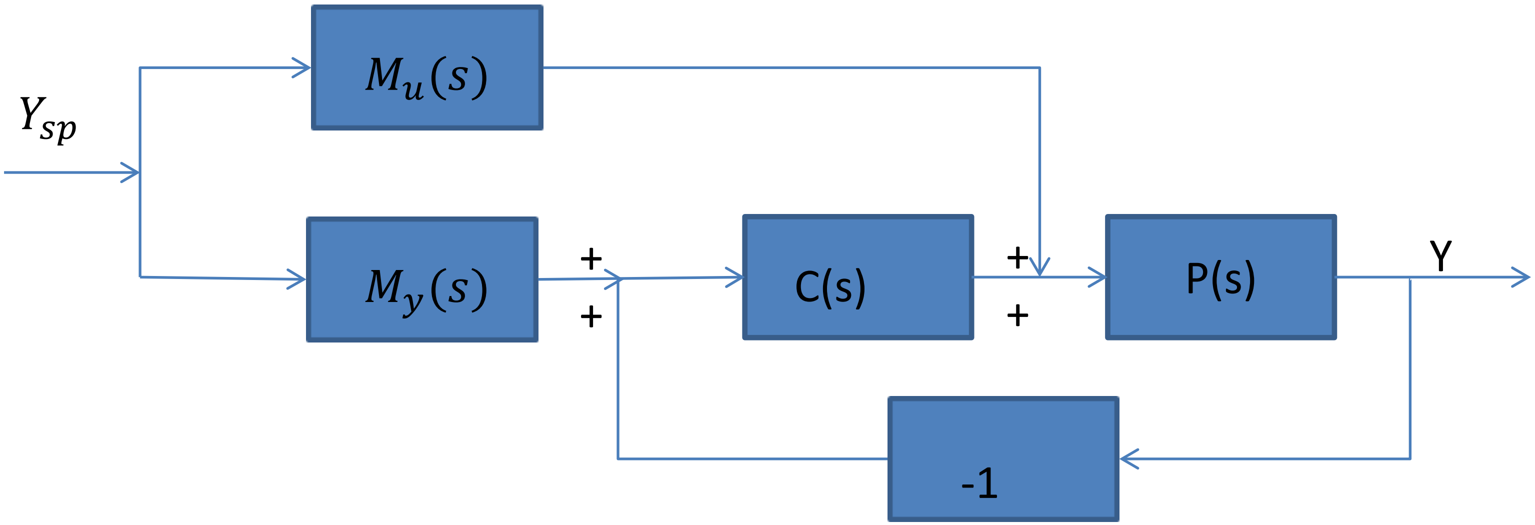


M_y *gives the desired output* $=$ *the* setpoint y_m for the feedback loop $\Leftrightarrow$
M_y is the desired transfer function.

When Y_{sp} is changed, M_u gives a signal, which gives the desired output.

Idealy $y = y_m \Rightarrow$ the feedback loop remains constant.
The feedback loop handles disturbance and noise

Two degree freedom feedforward



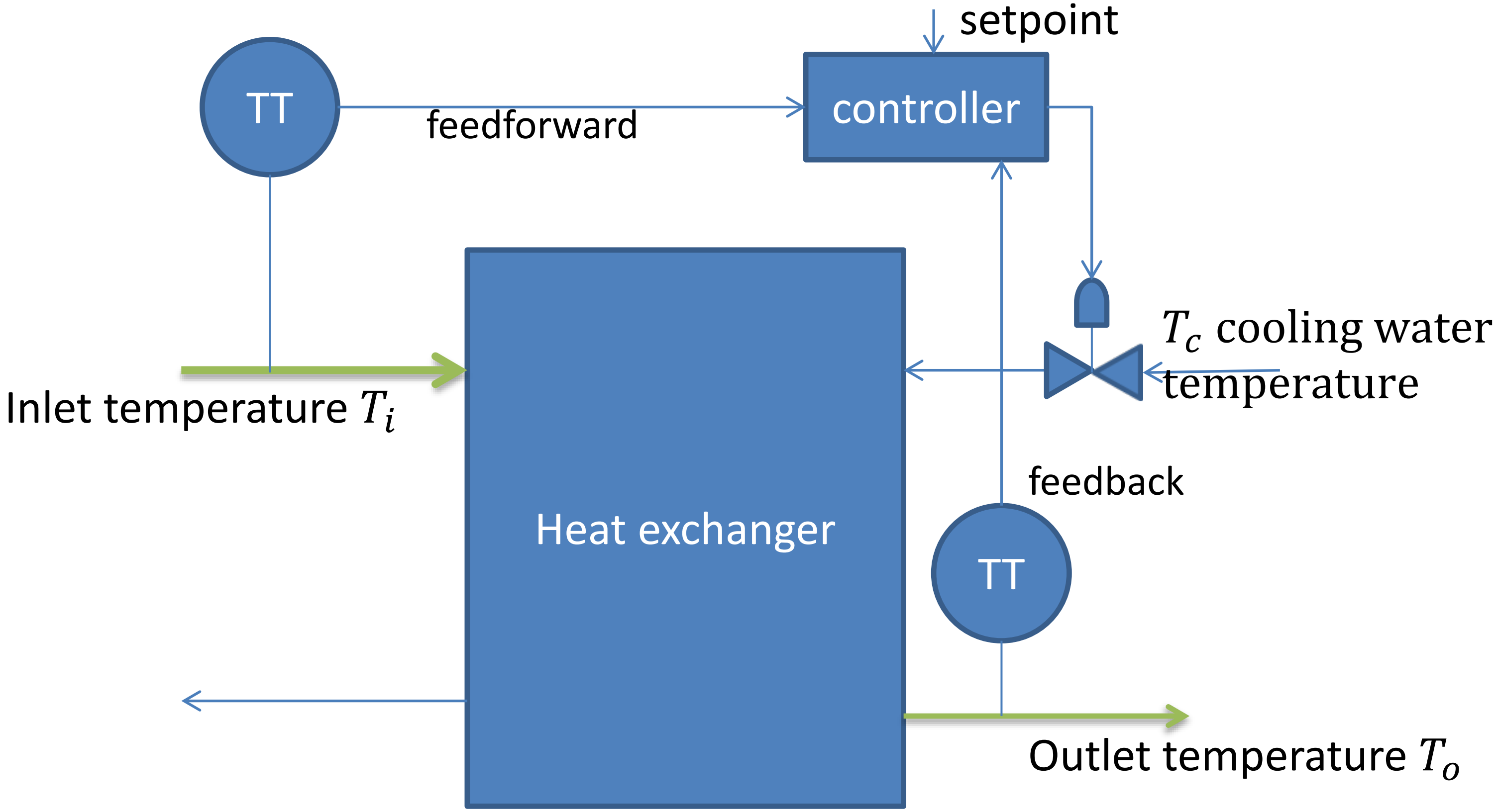
$$\frac{Y(s)}{Y_{sp}(s)} = \frac{P(CM_y + M_u)}{1 + PC} = M_y + \frac{PM_u - M_y}{1 + PC}$$

M_y *is the desired transfer* function

$\frac{PM_u - M_y}{1+PC}$ *must be small*

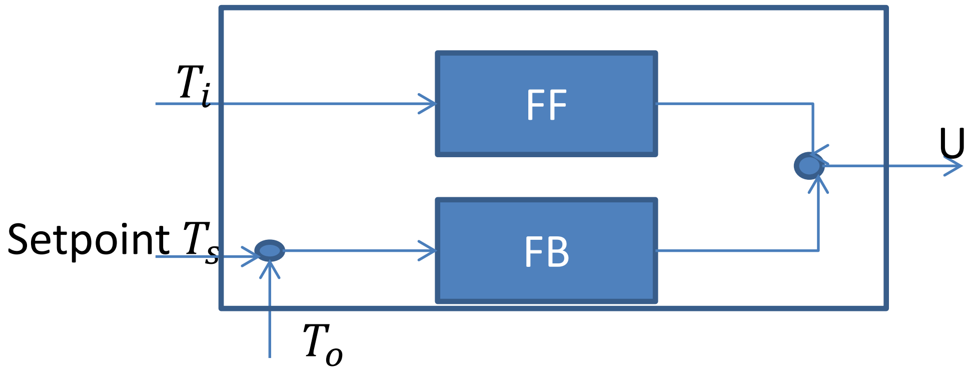
Ideal feedforward for $M_y = PM_u$. This requires good knowledge of P

Example
Combined feedforward and feedback



Feedback : control the stationary outlet temperature T_o
Feedforward: fast compensation of T_i disturbaces

Feedforward design (trail and error)

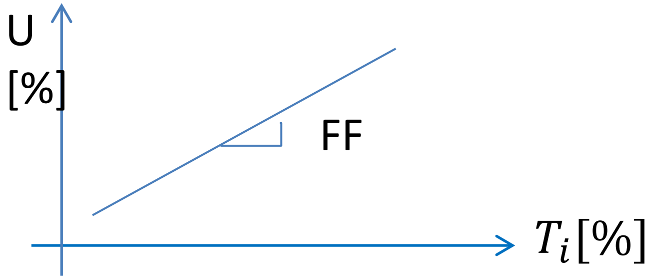


U(s)= $FF * T_i + FB * T_s$

FB : PI controller => we don't need to adress the normal output
FF can be determined by experiments.

1. Trial and error.
   Small values of FF results in temporary errors for variations in T_i.
   Large values of FF will result in error " on the wrong side" e.g. large FF will result in to much cooling when T_i increases.

2. Measure and plot the relation between different values of T_i and U
   FF = the slope of this plot



System inverse

- The ideal feedforward

$$M_y = PM_u \Rightarrow M_u = P^{-1}M_y$$

- The inverse of the proces model can cause problems
- Example

$$P(s) = \frac{1}{\tau s + 1} e^{-Ls} \Rightarrow P^{-1}(s) = (1 + \tau s) e^{sL}$$

Non causal

  - e^{sL} is a prediction
  - $(1 + \tau s)$ requires an ideal derivative

System inverse

- Example

$$P(s) = \frac{s-1}{s+2} \Rightarrow P^{-1}(s) = \frac{s+2}{s-1}$$

this system is unstable

- The cancled poles and zeroes must be stable and fast to ensure stability

Demands for M_y

- $M_u = P^{-1}M_y$
- $Time\ delay\ in\ M_y \geq time\ delay\ in\ P$
- $M_y\ and\ P\ has\ the\ same\ right\ half\ plane\ zeros$
- $The\ excess\ of\ poles\ in\ M_y\ must\ be\ \geq$
  $the\ excess\ of\ poles\ in\ P.$

Solution

Approximate P by a simple function and choose the same structure for M_y

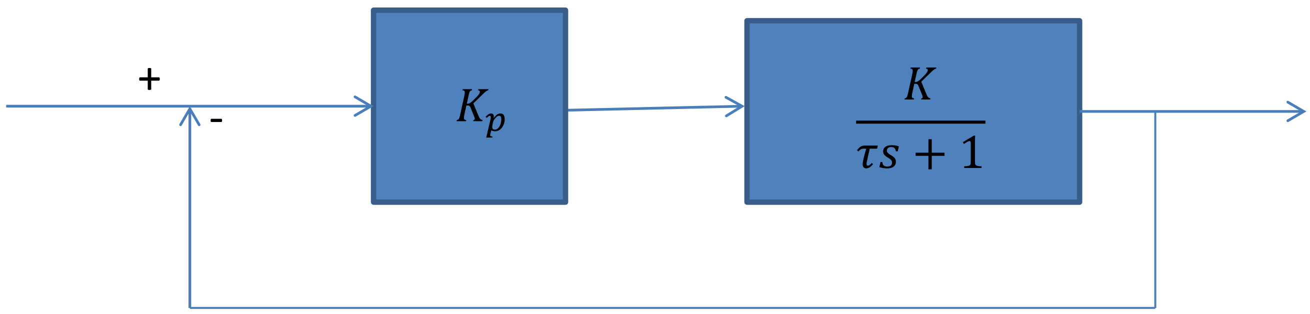
Approximation of P

- P^* is the approximation of P^{-1}
- Most common – neglect the dynamics

$$P^*(s) = P(0)^{-1}$$

First order system with P-control

Given a first order system with a P-controller



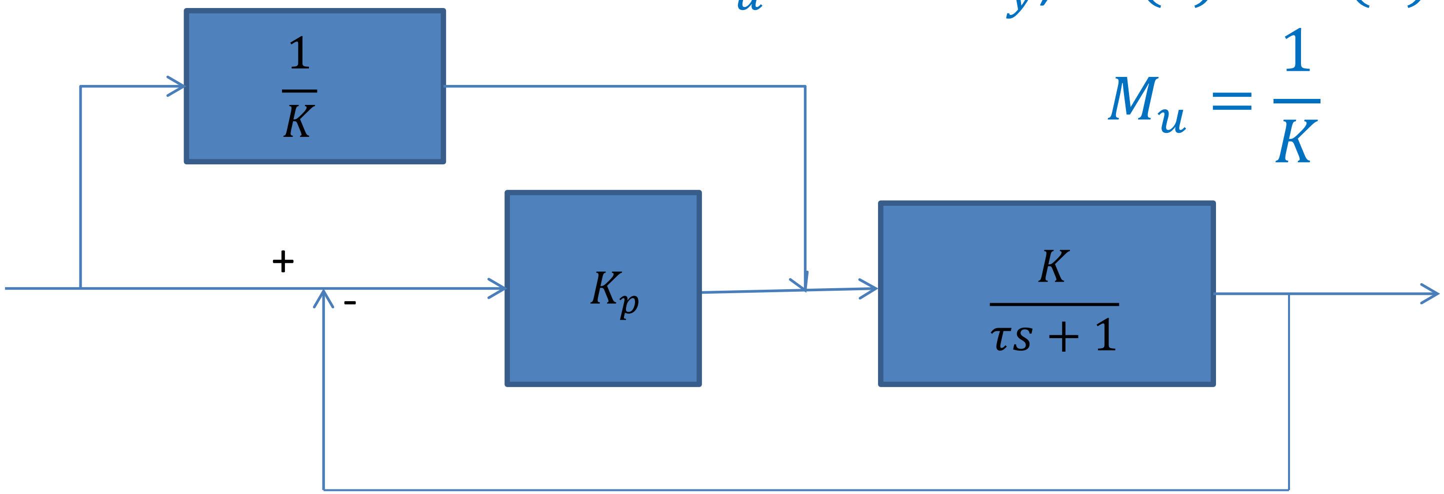
We get a stationary error

$$\frac{Y(s)}{U(s)} = \frac{\frac{K_pK}{\tau s + 1}}{1 + \frac{K_pK}{\tau s + 1}} = \frac{K_pK}{\tau s + 1 + K_pK} = \frac{\frac{K_pK}{1 + K_pK}}{\frac{\tau}{1 + K_pK}s + 1}$$

First order system with P-control and feedforward

$$M_u = P^{-1}M_y,\ P^*(s) = P(0)^{-1} = K$$

$$M_u = \frac{1}{K}$$

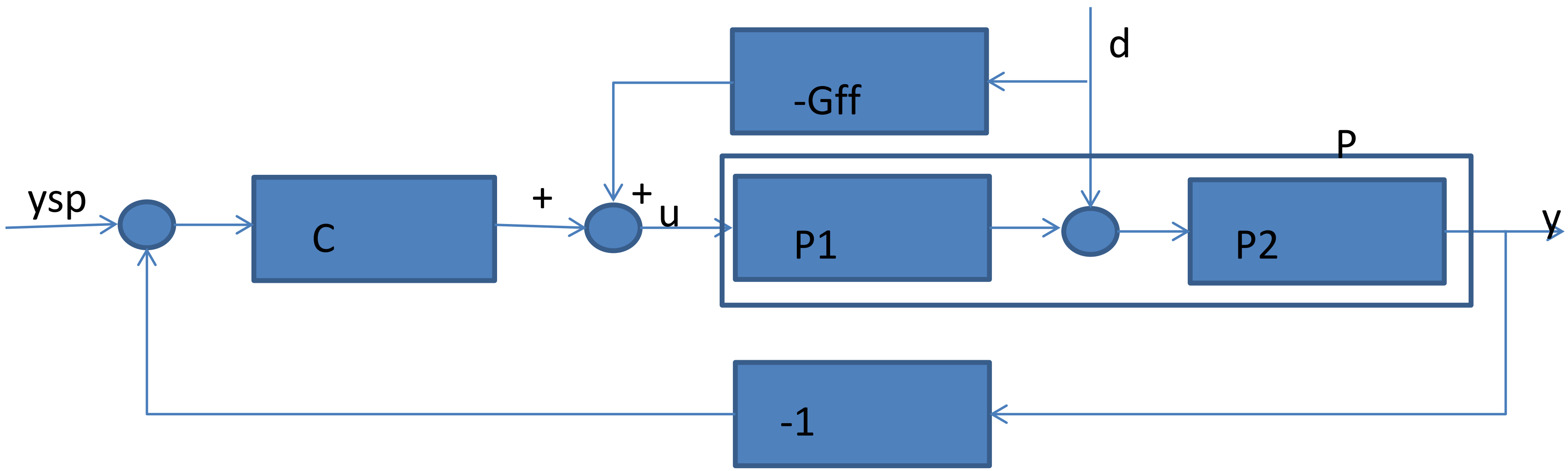


No stationary error

$$\frac{Y(s)}{Y_{sp(s)}} = \frac{P(CM_y+M_u)}{1+PC} = \frac{\frac{K}{\tau s+1}(K_p+\frac{1}{K})}{1+\frac{KK_p}{\tau s+1}} = \frac{KK_p+1}{\tau s+1+KK_p} = \frac{1}{\frac{\tau}{1+KK_p}s+1}$$

The time constant is the same

Disturbance attenuation measurable disturbance



The transfer function from the noise d to the output y is given as:

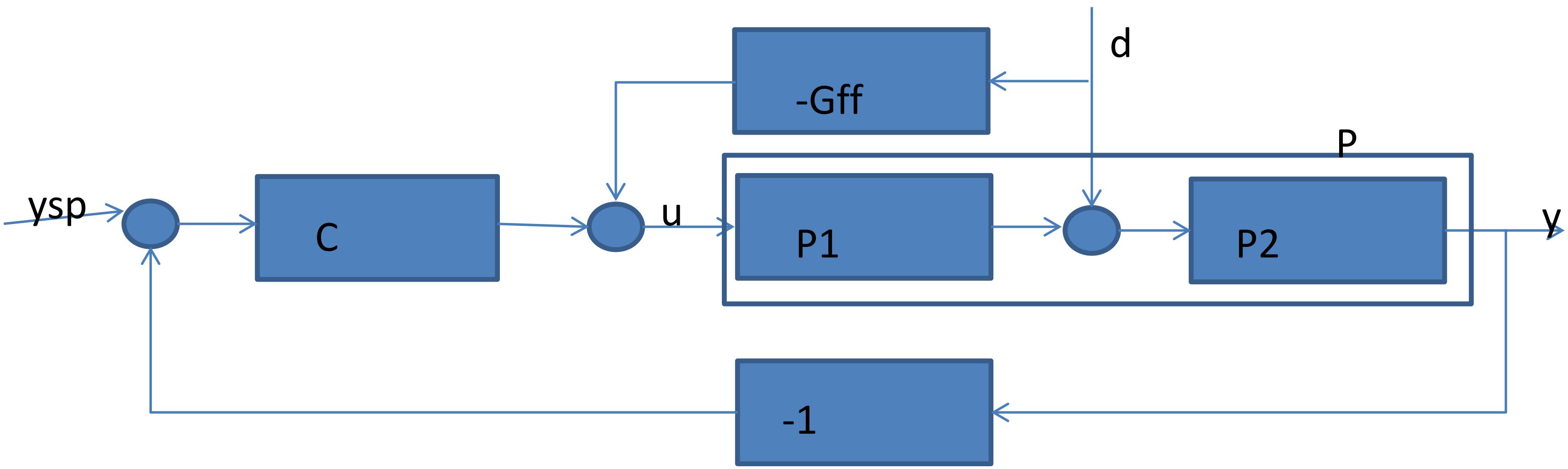
$$G_{yd} = \frac{-G_{ff}P_1P_2 + P_2}{1 + P_1P_2C} = \frac{P_2(1 - P_1G_{ff})}{1 + PC}$$

The disturbance can be reduced in two ways

$(1 - P_1G_{ff})$ can be small -> by a good choise of Gff - feedforward

$(1 + PC)$ can be large -> by feedback design

Disturbance attenuation - feedforward



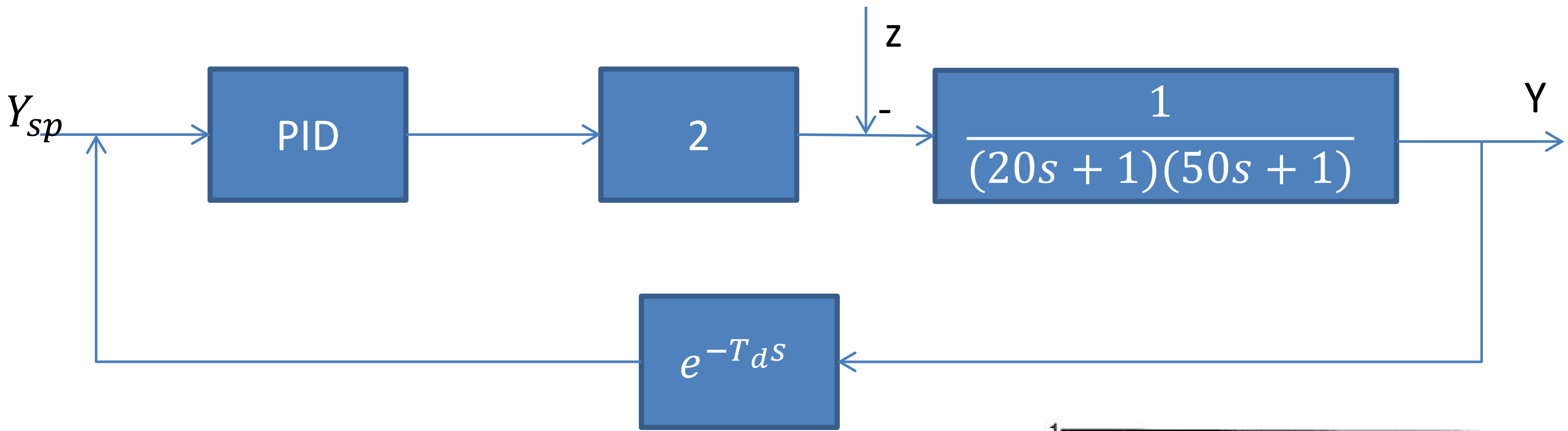
The ideal feedforward is given as $G_{ff} = {P_1}^{-1} = \frac{P_{d \to y}}{P_{u-y}}$

This is not always realizable, then an approximation must be used eg. $P_1(0)^{-1}$

If $P_1 = 1$ then $P_2 = P$ the disturbance can be eliminated

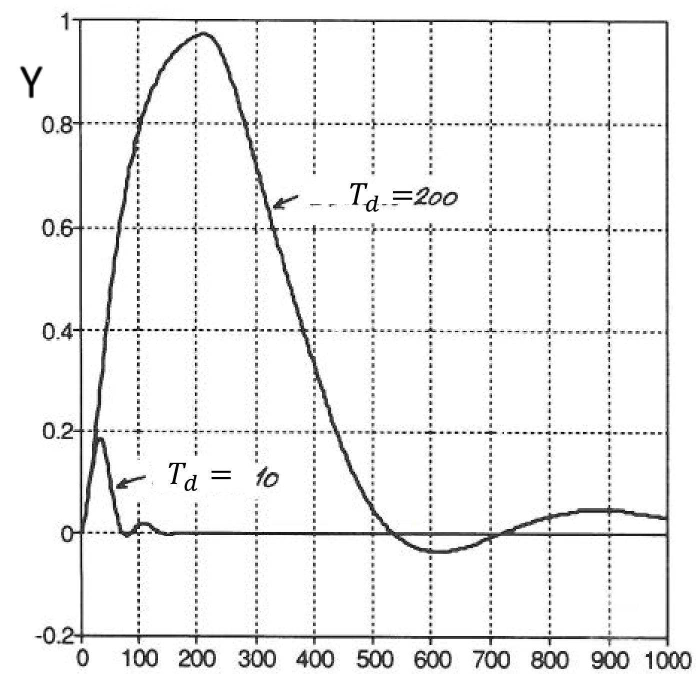
If P1=P then the effects of d are seen in y at the same time as it is seen in the feedforward, then there are no advantage of using feedforward

Control systems with delay disturbance reduction



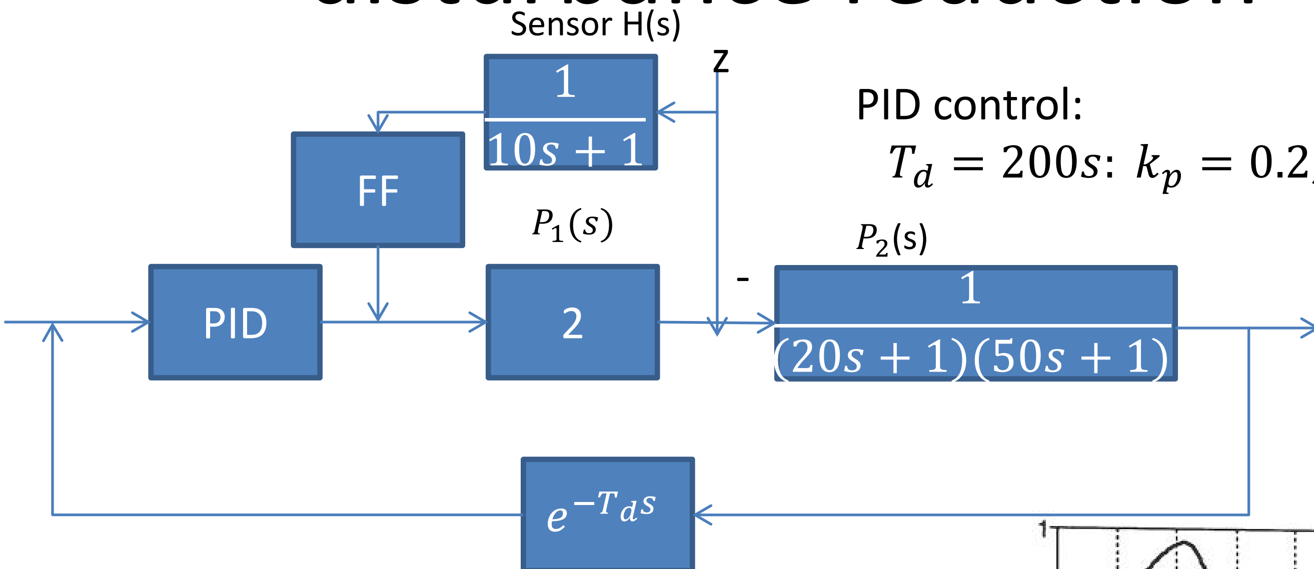
Delays will cause lower stability and have to be included in the controller design

$$T_d = 10\ s:\quad k_p = 2\,, \tau_i = 30, \tau_d = 16$$
$$T_d = 200s:\ k_p = 0.2, \tau_i = 30, \tau_d = 30$$



Response to step change in the load/disturbance z

Control systems with delay disturbance reduction



PID control:

$T_d = 200s$: $k_p = 0.2, \tau_i = 30, \tau_d = 30$

c

FF can be determined so Y and z are independent.

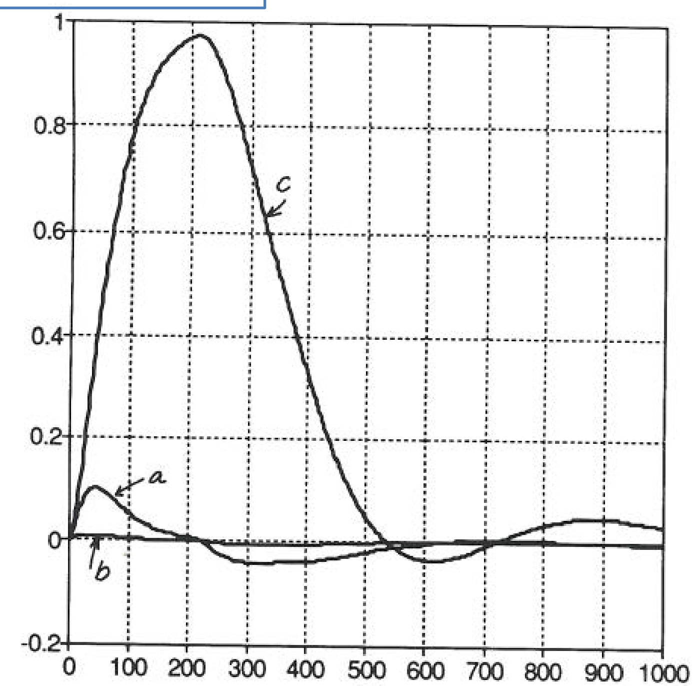
$z(H(s) * FF * P_1(s) - 1)P_2(s)$ = 0

$$FF = \frac{1}{H(s)P_1(s)}$$

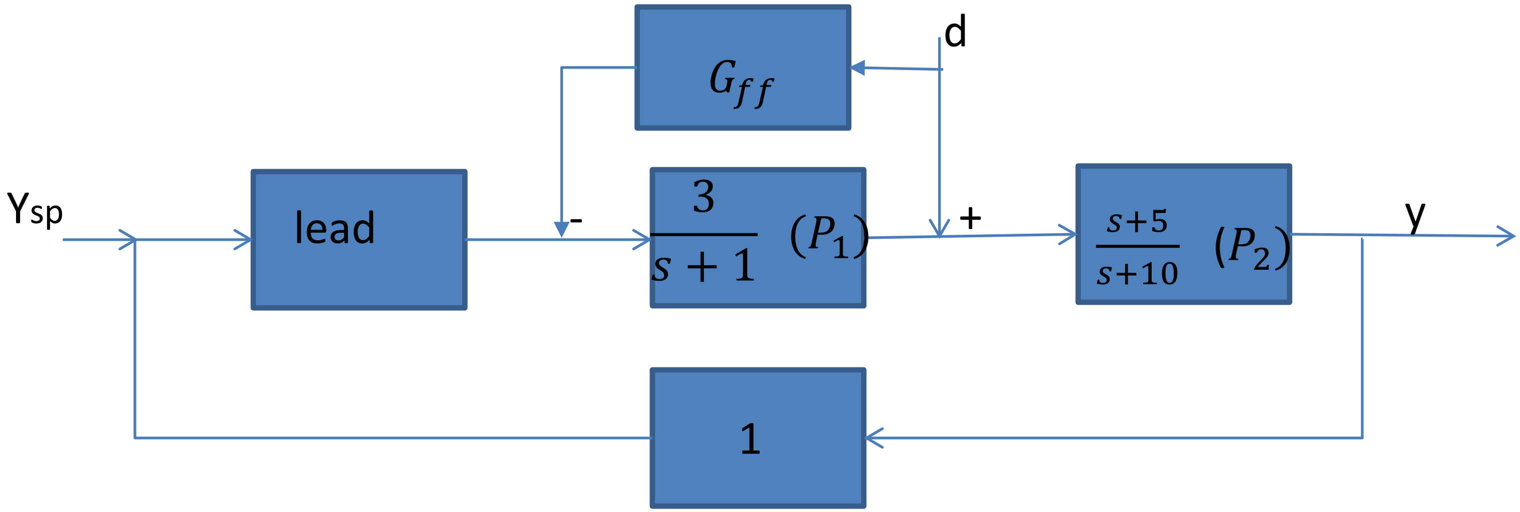
Here:

$FF = \frac{1}{\frac{1}{10s+1}2} = 0.5(1 + 10s)$ PD feedforward (b)

Static FF => FF = 0.5 (a)



Feed forward example

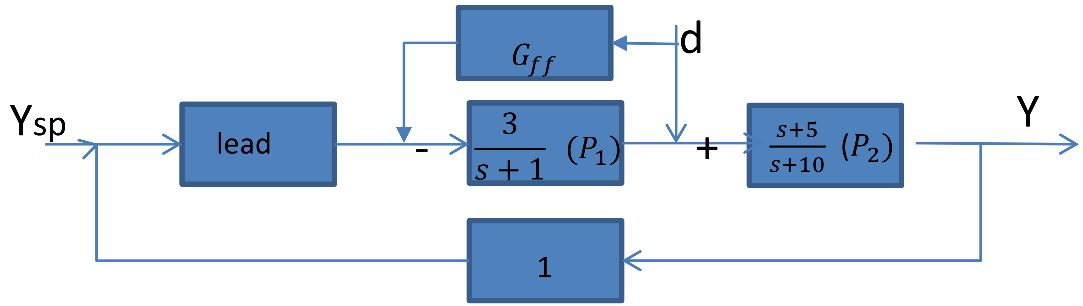


Lead: $5\frac{s+1}{s+20}$ cancels the original pole and add bandwith

Feed forward : $G_{ff}(s) = {P_1}^{-1}(s) = \frac{P_{d\to y}}{P_{u-y}} = \frac{s+1}{3} = \frac{1}{3}(s+1)$

Or leaving the dynamics

$$G_{ff}(0) = \frac{1}{3}$$



Closed loop $\frac{Y}{Y_{sp}} = CL(s) = \dfrac{\frac{5(s+1)}{(s+20)}\frac{3}{(s+1)}\frac{s+5}{s+10}}{1+\frac{(s+1)}{(s+20)}\frac{3}{(s+1)}\frac{s+5}{(s+10)}} =$ CL(0)=0.36 stationary error

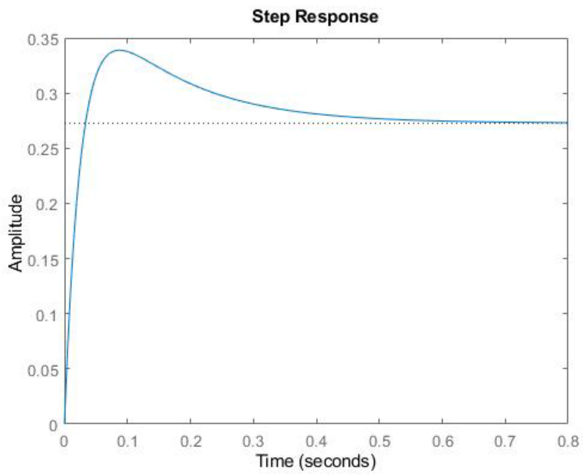
From noise

$(G_{ff}=0)$ $\frac{Y}{d} = YD(s) = \dfrac{\frac{(s+5)}{s+10}}{1+\frac{5(s+1)}{(s+20)}\frac{3}{(s+1)}\frac{s+5}{(s+10)}}$ $YD(0) = 0.36$ (stationary error)

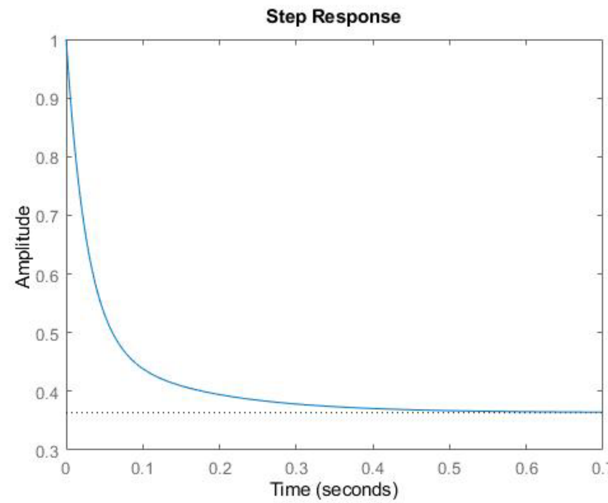
$\left(G_{ff} = (s+1)\right)$ $YD(s) = \dfrac{\left(1-\frac{3}{s+1}\frac{s+1}{3}\right)\frac{s+5}{s+10}}{1+\frac{5(s+1)}{(s+20)}\frac{3}{(s+1)}\frac{s+5}{(s+10)}}$ $YD(0) = 0$ (no stationary error)

$\left(G_{ff} = 1\right)$ $YD1(s) = \dfrac{\left(1-\frac{3}{s+1}\frac{1}{3}\right)\frac{5+s}{s+1}}{1+\frac{5(s+1)}{(s+20)}\frac{3}{(s+1)}\frac{s+5}{(s+10)}}$ $YD1(0) = 0$ (no stationary error)

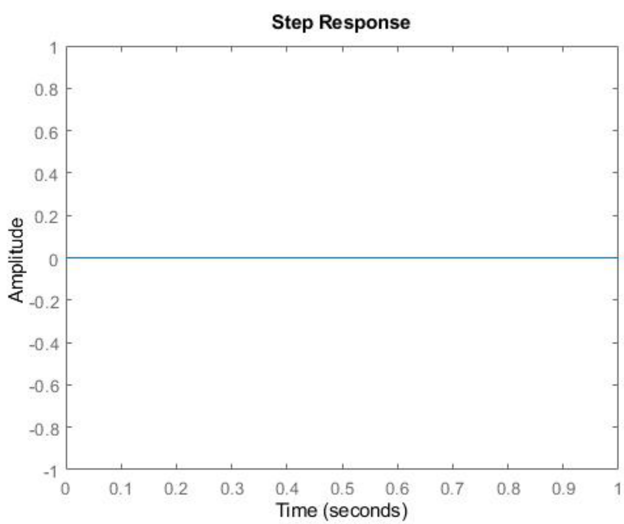
$$\frac{Y}{Y_{sp}}$$



$$\frac{Y}{d} \quad G_{ff} = 0$$



$$\frac{Y}{d} \quad G_{ff} = P_1^{-1} = (s+1)$$



$$\frac{Y}{d} \quad G_{ff} = P_1^{-1}(0) = 1$$