EIT5 solutions to extra exercises

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Exercise 1

We are given $m(t) = 2\cos(2\pi 1000 t)$, $f_c = 100$ MHz, and $\Delta f = 10$ kHz. Therefore:

- $A_m = 2$ V
- $f_m = 1000$ Hz
- $P_c = 50$ W and $P_c = A_c^2/2$. Hence, $A_c = \sqrt{P_c/2} = 5\,\text{V}$
- $k_f = 10/2 = 5$ kHz/V
- $\beta = \Delta f / f_m = 10$

a) The FM waveform is given as

$$s_{\text{FM}}(t) = A_c \cos\left[2\pi\left(f_c t + k_f \int_0^t m(\tau)d\tau\right)\right] \tag{1}$$

Then, we calculate the integral

$$\int_0^t m(\tau)d\tau = \int_0^t 2\cos(2\pi 1000\tau)d\tau = \frac{\sin(2\pi 1000 t)}{\pi 1000} \tag{2}$$

And so, we get the FM waveform

$$s_{\text{FM}}(t) = 5\cos\left[2\pi\left(10^7 t + \frac{5\sin(2\pi 1000 t)}{\pi 1000}\right)\right] \tag{3}$$

b) The bandwidth of an FM signal can be approximated using the Carson's rule as

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* \tag{4}$$

Since we have a single modulating carrier $f_m^* = f_m$ and we get

$$B_{\text{FM}} \approx 2\times 10^4 + 2\times 10^3 = 22\text{kHz} \tag{5}$$

Since $\beta = 10$, we have a wideband FM signal

c) We are given $f_{space} = 99.95$ MHz and $f_{mark} = 100.05$ MHz and $R_b = 3$ kbps.

The bandwidth of a BFSK signal is approximated as

$$B_{\text{BFSK}} \approx f_{mark} - f_{space} + 2R_{sym} \tag{6}$$

So, we need to find $R_{sym} = R_b/\log_2 M$. Since the signal is binary (BFSK), the number of symbols $M = 2$ and we get $R_{sym} = 3\,\text{kbauds}$.

$$B_{\text{BFSK}} \approx 100.05\cdot 10^6 - 99.95\cdot 10^6 + 2\cdot 3\cdot 10^3 = 106\,\text{kHz} \tag{7}$$

EXERCISE 2

We are given $m(t) = 2\cos(1\pi 100t) - \cos(2\pi 500t)$.
a) The carrier is $c(t) = 10\cos(2\pi 1.5 \cdot 10^6 t)$ and the amplitude sensitivity is $k_a = 0.2$.
The traditional AM (DSBAM) modulated signal is

$$\begin{aligned} v_{\text{AM}}(t) &= A_c\left(1 + k_a m(t)\right)\cos(2\pi f_c t) \\ &= 10\Big(1 + 0.4\cos(2\pi 100t) - 0.2\cos(2\pi 500t)\Big) \\ &\quad \times \cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \tag{8}$$

And, hence, its spectrum is

$$\begin{aligned} V_{\text{AM}}(f) = 5 &\left(\delta\left(f - 1.5 \cdot 10^6\right) + \delta\left(f + 1.5 \cdot 10^6\right)\right) \\ &+ \left(\delta\left(f - 1.5 \cdot 10^6 - 100\right) + \delta\left(f + 1.5 \cdot 10^6 + 100\right)\right) \\ &+ \left(\delta\left(f - 1.5 \cdot 10^6 + 100\right) + \delta\left(f + 1.5 \cdot 10^6 - 100\right)\right) \\ &-0.5\left(\delta\left(f - 1.5 \cdot 10^6 - 500\right) + \delta\left(f + 1.5 \cdot 10^6 + 500\right)\right) \\ &-0.5\left(\delta\left(f - 1.5 \cdot 10^6 + 500\right) + \delta\left(f + 1.5 \cdot 10^6 - 500\right)\right) \end{aligned} \tag{9}$$

b) Now the amplitude of the carrier is $A_c = 2$ V
The DSB-SC AM signal is

$$\begin{aligned} v_{\text{DSB-SC}}(t) &= A_c k_a m(t)\cos(2\pi f_c t) \\ &= \Big(0.8\cos(2\pi 100t) - 0.4\cos(2\pi 500t)\Big)\cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \tag{10}$$

An its spectrum is

$$\begin{aligned} V_{\text{DSB-SC}}(f) = 0.2 &\left(\delta\left(f - 1.5 \cdot 10^6 - 100\right) + \delta\left(f + 1.5 \cdot 10^6 + 100\right)\right) \\ &+0.2\left(\delta\left(f - 1.5 \cdot 10^6 + 100\right) + \delta\left(f + 1.5 \cdot 10^6 - 100\right)\right) \\ &-0.1\left(\delta\left(f - 1.5 \cdot 10^6 - 500\right) + \delta\left(f + 1.5 \cdot 10^6 + 500\right)\right) \\ &-0.1\left(\delta\left(f - 1.5 \cdot 10^6 + 500\right) + \delta\left(f + 1.5 \cdot 10^6 - 500\right)\right) \end{aligned} \tag{11}$$

c) DSB-SC is much more power efficient: all the power is in the sidebands (no carrier). On the other hand,

$$\frac{5^2}{5^2 + 1^2 + 1^2 + (-0.5)^2 + (-0.5)^2} = 0.909 \tag{12}$$

of the power is in the carrier for DSBAM. On the downside, the simple envelope detector cannot be used for DSB-SC.

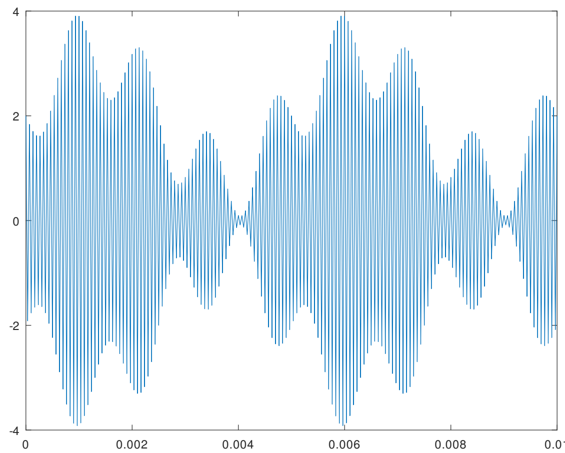

Fig. 1. Signal for exercise 3 with $k_a = 1/5$.

Exercise 3

This type of exercise will not be in the exam

We are given (changed the signal's name) $m(t) = 3\sin(2\pi 200t) - 2\sin(2\pi 800t)$.

The signal is modulated with $c(t) = A_c \cos(2\pi f_c t)$ with power $P_c = 8$ W.

1. As in the first exercise, we get the amplitude of the carrier as $A_c = \sqrt{P_c/2} = 2$ V.

For ordinary AM modulation we need to avoid phase reversals, which occur when $\mu = k_a A_m > 1$. In other words, we need to ensure that

$$|k_a m(t)| < 1 \qquad \text{for all } t \tag{13}$$

We can rewrite $k_a m(t)$ as

$$x(t) = 3k_a \sin(2\pi 200t) - 2k_a \sin(2\pi 800t) \tag{14}$$

We can approximate the maximum of $|x(t)| \approx 4.79$ using numerical methods. Therefore, we have that $k_a \leq 1/4.79$.

2. We will make $k_a = 1/5$. Hence, the modulated signal is given by

$$\begin{aligned} v_{\text{AM}}(t) &= A_c \left(1 + k_a m(t)\right) \cos(2\pi f_c t) \\ &= 2\left(1 + \frac{3}{5}\sin(2\pi 200t) - \frac{2}{5}\sin(2\pi 800t)\right) \\ &\quad \times \cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \tag{15}$$

Then, the spectrum of this signal is

$$\begin{aligned} V_{\text{AM}}(f) = &\left(\delta\left(f - 1.5\cdot 10^6\right) + \delta\left(f + 1.5\cdot 10^6\right)\right) \\ &+ \frac{6}{20j}\left(\delta\left(f - 1.5\cdot 10^6 - 200\right) + \delta\left(f + 1.5\cdot 10^6 + 200\right)\right) \\ &+ \frac{6}{20j}\left(\delta\left(f - 1.5\cdot 10^6 + 200\right) + \delta\left(f + 1.5\cdot 10^6 - 200\right)\right) \\ &- \frac{4}{20j}\left(\delta\left(f - 1.5\cdot 10^6 - 800\right) + \delta\left(f + 1.5\cdot 10^6 + 800\right)\right) \\ &- \frac{4}{20j}\left(\delta\left(f - 1.5\cdot 10^6 + 800\right) + \delta\left(f + 1.5\cdot 10^6 - 800\right)\right) \end{aligned} \tag{16}$$

EXERCISE 4

We are given $f_c = 750$ kHz and $k_f = 15$ kHz/V plus the modulating signal

$$m(t) = 5\cos(2\pi 25 \cdot 10^3 t) \tag{17}$$

and the modulated signal

$$s(t) = A_c \cos\left(2\pi 750 \cdot 10^3 t + \beta \sin(2\pi 25 \cdot 10^3 t)\right) \tag{18}$$

a. The frequency deviation is $\Delta f = k_f A_m = 75$ kHz, the modulation index is $\beta = \Delta f / f_m = 3$. Due to the Carson's rule, 98% of the spectrum is contained within

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* = 150 \cdot 10^3 + 20 \cdot 10^3 = 170\,\text{kHz} \tag{19}$$

centered at frequency $f_c = 750$ kHz.
b) To square the signal we need

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1 \tag{20}$$

Then

$$\cos^2(A) = \frac{\cos(2A) + 1}{2} \tag{21}$$

The term $1/2$ is simply a DC shift.

$$s^2(t) = \frac{A_c}{2}\cos\left(2\pi(2 \cdot 750 \cdot 10^3)t + 2\beta \sin(2\pi 25 \cdot 10^3 t)\right) + \frac{1}{2} \tag{22}$$

We have that $\Delta f = 2\beta f_m = 6 * 25 \cdot 10^3 = 150\,\text{kHz}$ and the modulation index is $\beta' = 2\beta = 6$. The spectrum now has a delta at frequency 0 due to the DC shift.

Exercise 5

We have that

$$R_b = NR_{sym} = R_{sym}\log_2 M = 100\text{kbps} \tag{23}$$

Then, for all the modulations except FSK, the bandwidth is $B = 2R_{sym}$.
a)$M = 2$, so, the bandwidth for BFSK is

$$B_{\text{BFSK}} \approx f_{mark} - f_{space} + 2R_{sym} = 2 \cdot 200 \cdot 10^3 + 2 \cdot 100 \cdot 10^3 = 600\,\text{kHz} \tag{24}$$

b) $M = 2$ so, the bandwidth is

$$B_{\text{OOK}} = 2R_{sym} = \frac{2R_b}{\log_2 M} = 200\,\text{kHz} \tag{25}$$

c) $M = 4$ so, the bandwidth is

$$B_{\text{QPSK}} = 2R_{sym} = \frac{2R_b}{\log_2 4} = 100\,\text{kHz} \tag{26}$$

d) $M = 16$ so, the bandwidth is

$$B_{\text{16-PSK}} = 2R_{sym} = \frac{2R_b}{\log_2 16} = 50\,\text{kHz} \tag{27}$$

e)Same as in d)
f) $M = 512$ so, the bandwidth is

$$B_{\text{512-QAM}} = 2R_{sym} = \frac{2R_b}{\log_2 512} = 22.222\,\text{kHz} \tag{28}$$