Communication in Electronic Systems

Lecture 12: Signal Waveforms and Digital Transmission

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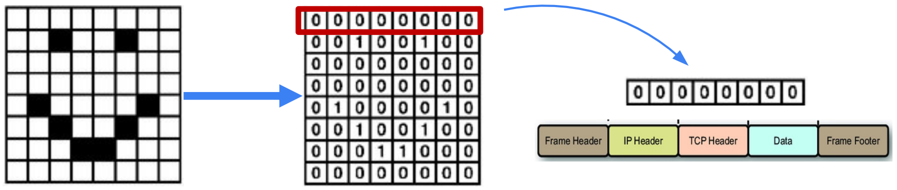
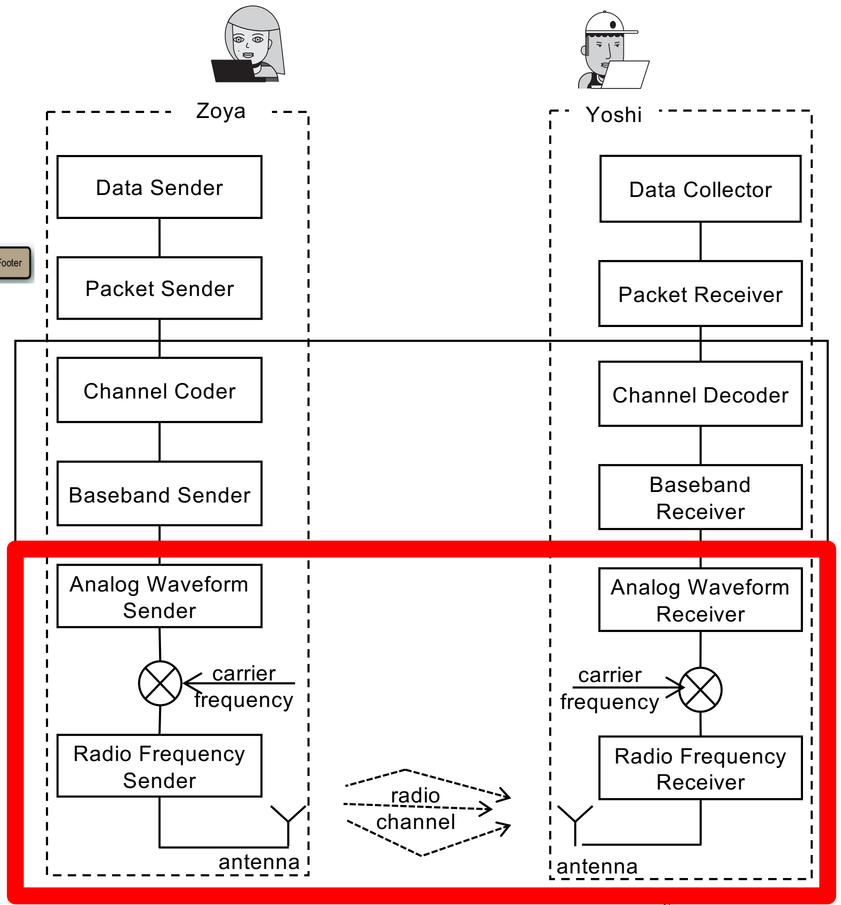
Connectivity

Course Overview: Part 2. Communication and Networking

- MM5: Introduction to Communication Systems
- MM6: Simple Multiuser Systems and Layered System Design
- MM7: Network Topology and Architecture
- MM8: Networking and Transport Layers
- MM9: Introduction to Security
- Guest lecture
- MM11: Coding and Digital Modulation
- **MM12: Signal Waveforms and Digital Transmission**
- MM13: Workshop on Modulation and Link Operation

recap (1): packet

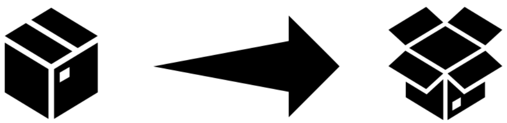




this was abstracted away

before (!!!)

let's open the black box



recap (2): baseband signals

three steps

Baseband

1. map bits to complex-valued baseband symbols (2 dimensions)

example: with two complex values, the symbols can be $1-j$ to represent 0 and $1+j$ for 1

The system transmits one symbol each T seconds: symbol period

$$0110101\ldots \longrightarrow X_0, X_1, X_2, \ldots$$

2. assign a pulse waveform (pulse shape) to each symbol

$$X_0, X_1, X_2, \ldots \longrightarrow \sum_k X_k p(t-kT)$$

3. modulate to a high frequency carrier

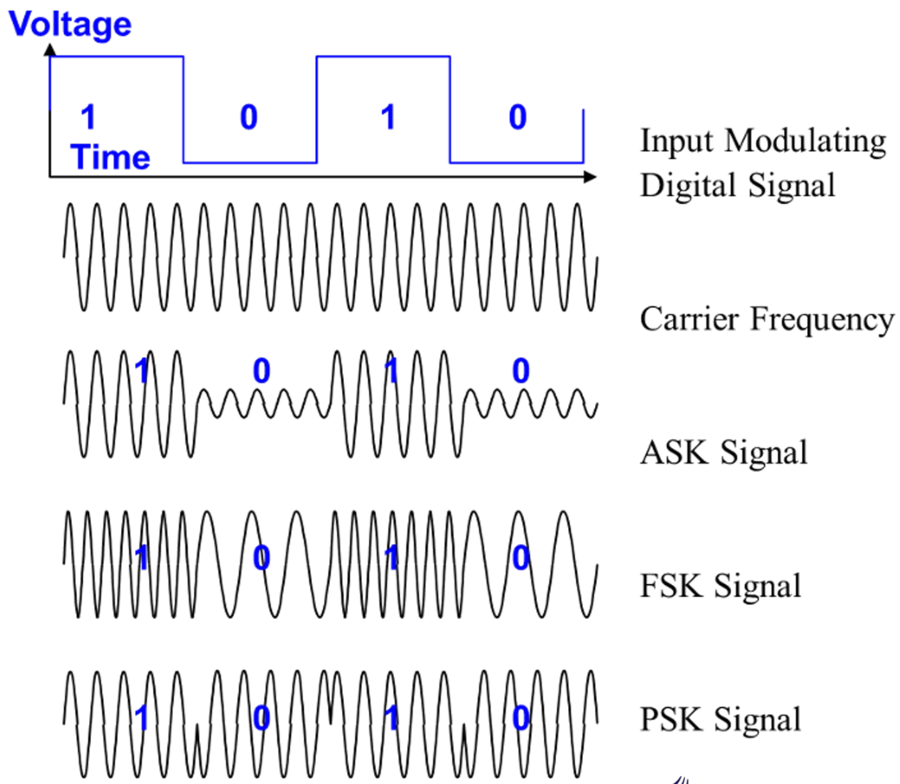
recap (3): digital modulations

amplitude-shift keying (ASK)

phase-shift keying (PSK)

frequency-shift keying (FSK)

quadrature amplitude modulation (QAM)



outline for today

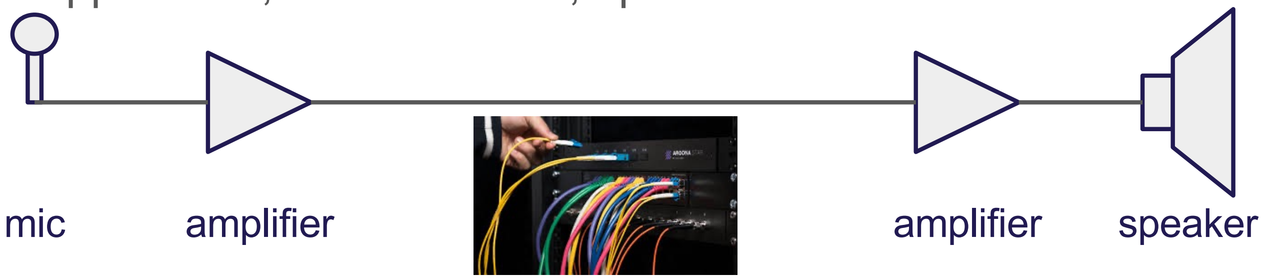
- back to the analog domain
  - how do we get digital signals?
  - from analog-to-digital signals
- signal waveforms
  - one dimension
  - two dimensions
- channel with Gaussian noise (Additive White Gaussian Noise – AWGN)
  - correlation-type demodulator
  - matched-filter demodulator

preliminaries

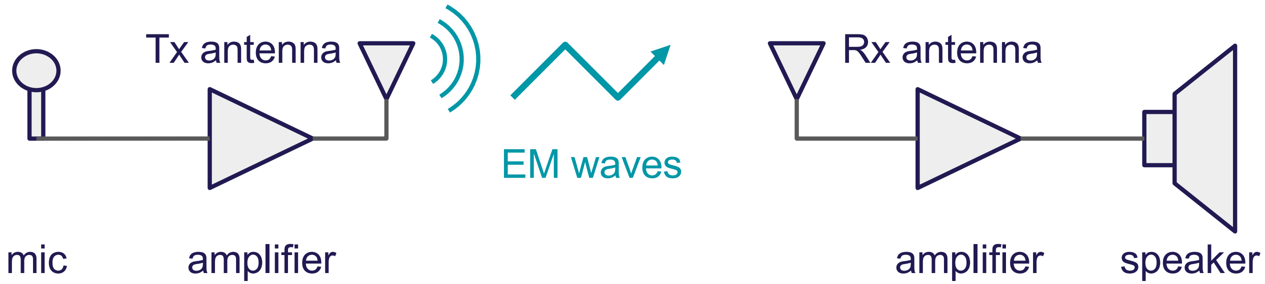
elements of a communication system

transmitter (Tx), receiver (Rx), and a medium (the channel)

wired medium: copper lines, coaxial cables, optic fiber



wireless medium: whatever is between the antennas



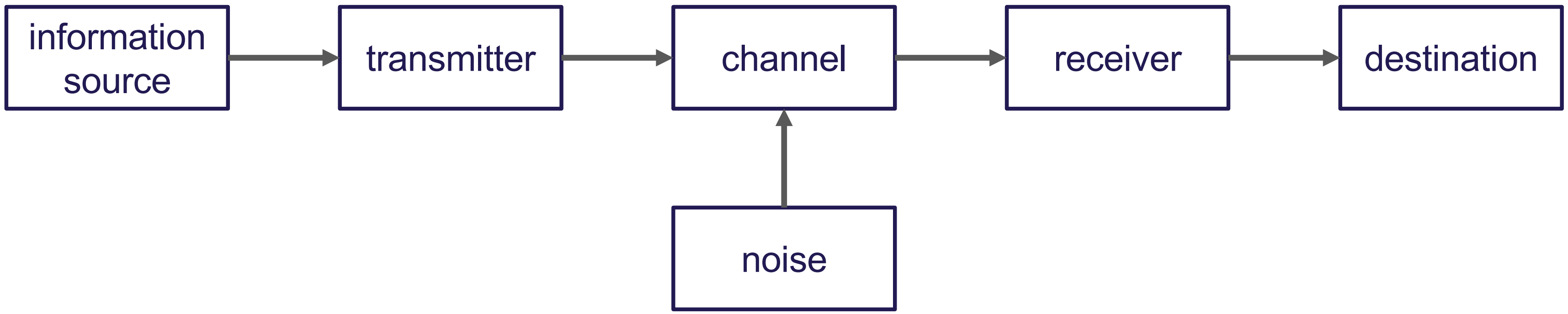
it’s all about the information

recall: the purpose of a communication system is **to convey information**

this information must be **converted into** an appropriate format for the transmission medium

signal: carries information

energy must be spent to create a structured signal



from time to frequency and *vice versa*

time domain: how the properties change over time

frequency domain: how the properties change over frequency

Fourier transform: from time to frequency domain

$$W(f) = \mathcal{F}\left[w(t)\right] = \int_{-\infty}^{\infty} w(t)e^{-j2\pi ft}\mathrm{d}t$$

inverse Fourier transform: back from frequency to time domain

$$w(t) = \mathcal{F}^{-1}\left[W(f)\right] = \int_{-\infty}^{\infty} W(f)e^{j2\pi ft}\mathrm{d}f$$

a reminder of Euler's formula

link between sine/cosine and $e^{j2\pi ft} = e^{j\omega t}$

$$e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$$

$$\cos(2\pi ft) = \frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2}$$

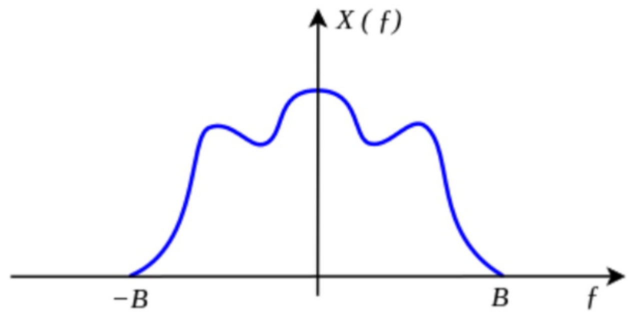
$$e^{-j2\pi ft} = \cos(2\pi ft) - j\sin(2\pi ft)$$

$$\sin(2\pi ft) = \frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j}$$

transmitted power and channel bandwidth

two of the main resources in wireless communications

expensive, so these must be used efficiently



transmitted power

average power of the transmitted signal

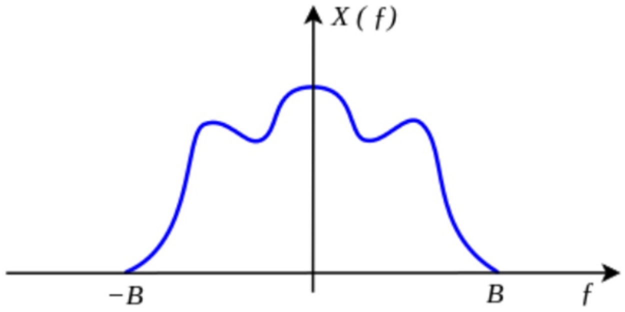
channel bandwidth

range of frequencies allocated to the transmission of the message

baseband and bandpass signals

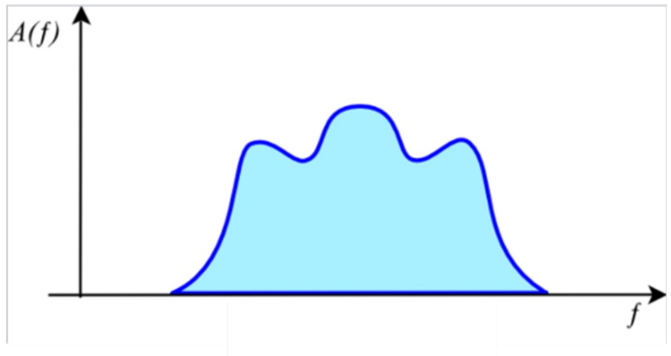
baseband signal

occupies frequencies from 0 to a maximum B



bandpass signal

occupies frequencies from a non-zero value to another maximum non-zero value



from analog to digital

types of information sources

analog

continuous in time

continuous in amplitude

Examples: music, voice, temperature, CO_2

digital

discrete in time

discrete in amplitude

examples: music files, recordings, emails, text files, digital images

informal definition

discrete-time sequence of symbols

these are drawn from a finite alphabet

from analog to digital

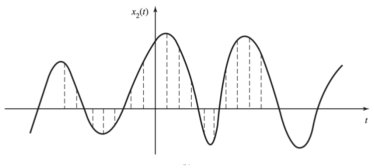
digitalization: analog-to-digital conversion (ADC)

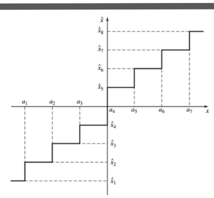
from continuous time and amplitude to discrete time and amplitude

Step 1: sampling: make the signal discrete in time

Step 2: quantization: make the signal discrete in amplitude

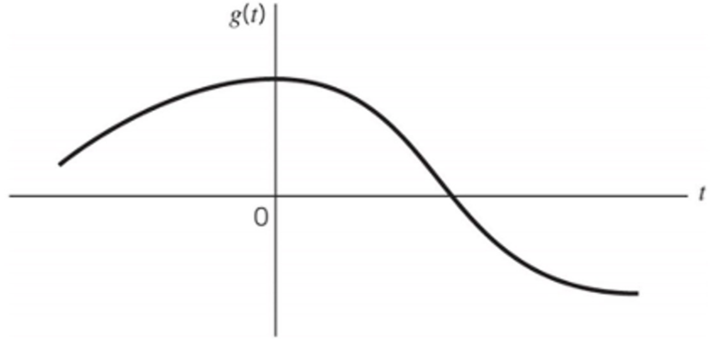
$$x(t) \xrightarrow{\text{sampling}} x(nT_s) \xrightarrow{\text{quantization}} \text{...0110101001001010… (bits)}$$



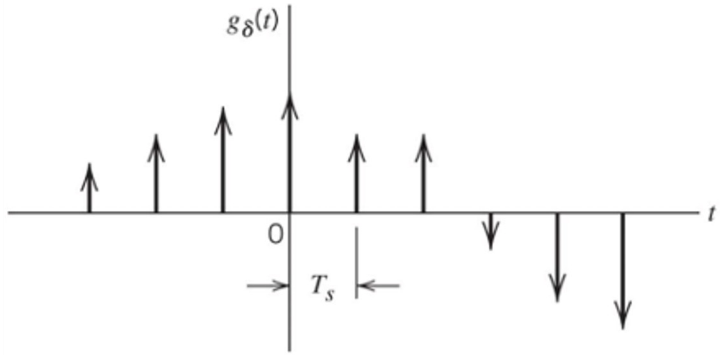


the sampling process

analog signal



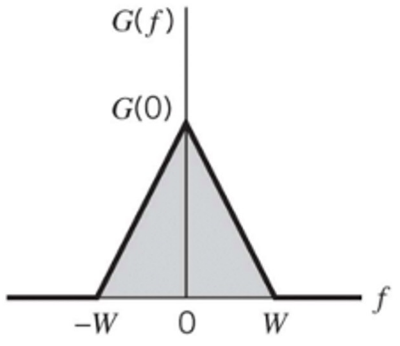
instantaneous sampled version of the signal



in math $$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

how often do I need to sample?

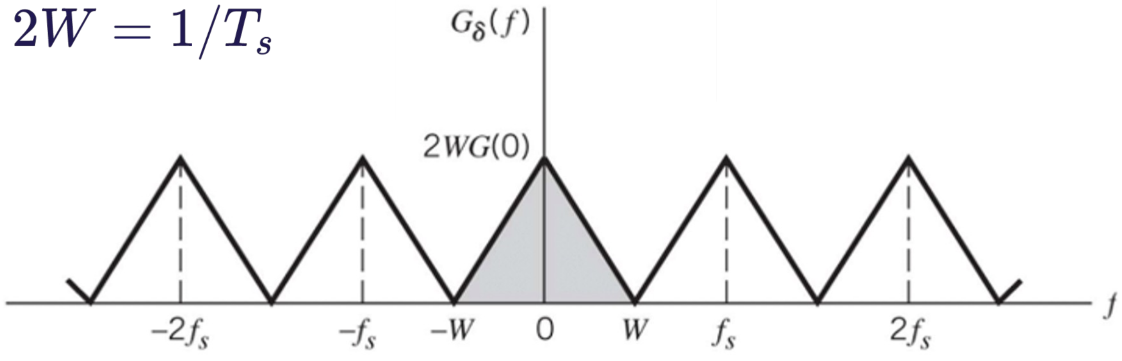
spectrum of band-limited signal $g(t)$



spectrum of $g(t)$ sampled at $f_s = 2W = 1/T_s$

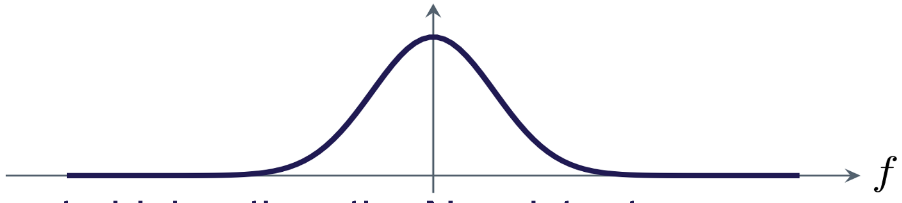
this is the **Nyquist rate**

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)e^{-j2\pi n f T_s}$$

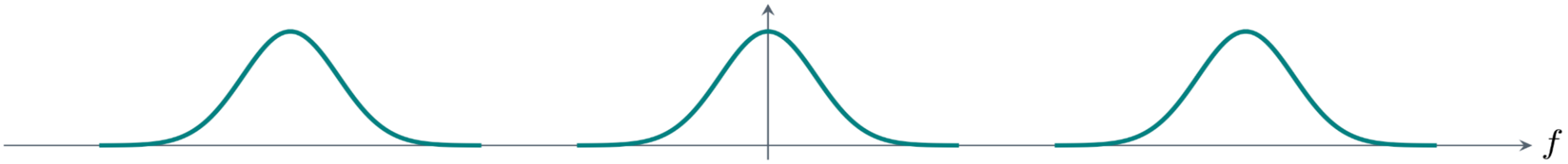


recovering the signal

1. before sampling, use a low pass filter to limit the bandwidth of the signal



2. sample the signal at a rate higher than the Nyquist rate

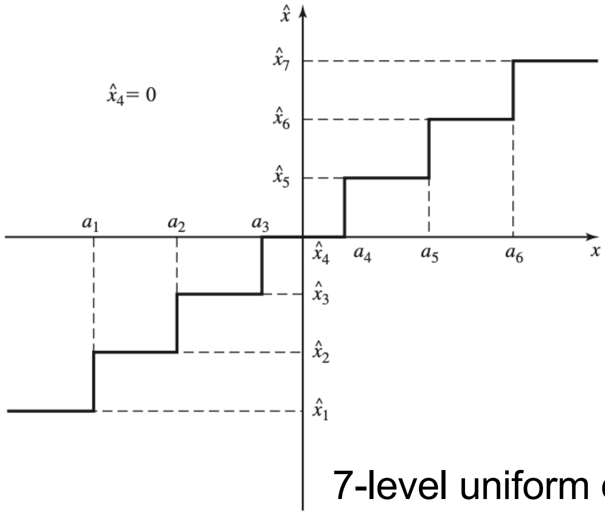


3. how do we recover the signal? **interpolation methods**

after sampling, quantize!

quantization

map voltage values from a continuous set into a smaller and countable set that will represent the voltage as levels



7-level uniform quantizer.

- the interval is proportional to the resolution that we can represent and after reconstruct the analog signal

- always some level of distortion will be introduced

signal waveforms

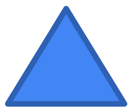
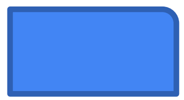
now what?

- we have a digital signal, what should we do?

$$x(t) \xrightarrow{\text{sampling}} x(nT_s) \xrightarrow{\text{quantization}} \ldots 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0$$

- now, we need to introduce signal waveforms that represents the symbols (group of bits) that are going to be transmitted through the channel medium over time

- the idea is that each waveform is mapped to a symbol

- e.g., we have two waveforms  $= 0$  $= 1$

some notation

- let M be the number of waveforms
- denote the m-th waveform as $s_m(t)$
- the number of different binary symbols that can be represented by M is $k = \log_2 M$
- for example,

$M = 2$

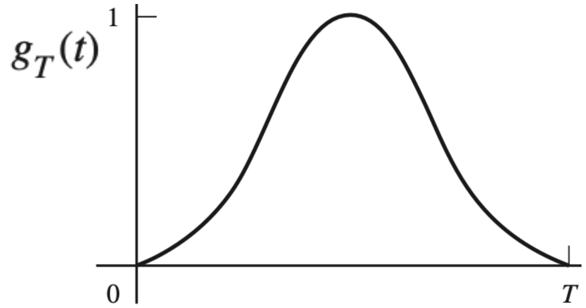
$s_1(t) \Rightarrow 0 \quad s_2(t) \Rightarrow 1$

$M = 4$

$s_1(t) \Rightarrow 00 \quad s_2(t) \Rightarrow 01$
$s_3(t) \Rightarrow 10 \quad s_4(t) \Rightarrow 11$

one-dimension: PAM

M-ary pulse amplitude modulation (PAM)



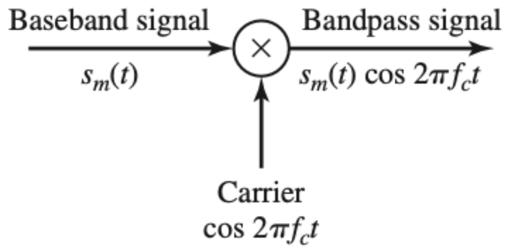
baseband

$$s_m(t) = A_m g_T(t)$$

A_m - amplitude, $g_T(t)$ – is a pulse of some arbitrary shape

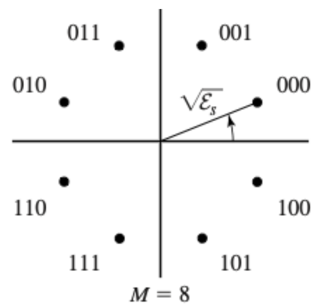
bandpass

$$u_m(t) = A_m g_T(t) \cos 2\pi f_c t$$



two-dimensions: QPSK

M-ary quadrature phase-shift keying
assumptions: points have same energy

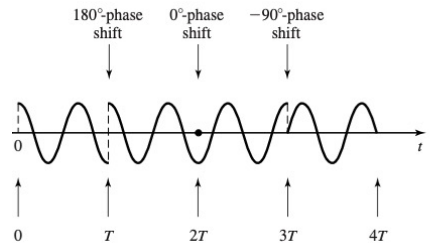


baseband
$M = 8$ signal-point constellation in two dimensions

bandpass

$$u_m(t) = s_t(t)\cos 2\pi f_c t$$

$$u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$$



$M = 4$

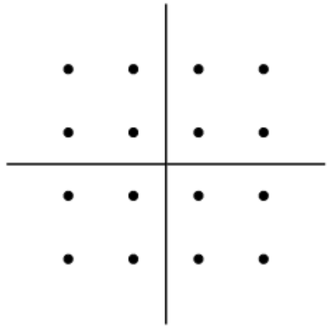
two-dimensions: QAM

M-ary quadrature amplitude modulation

assumptions: points can have different energy

baseband

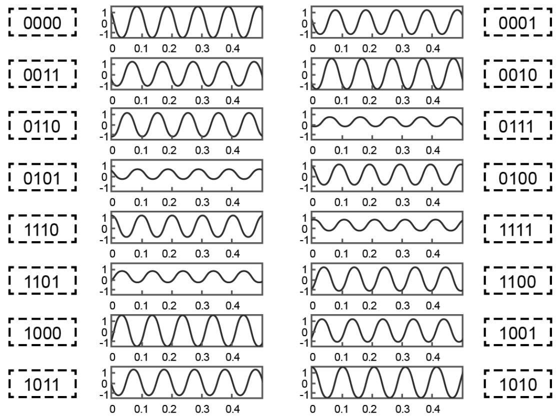
$M = 16$ signal-point constellation in two dimensions



bandpass

$u_{mn}(t) = A_m g_T(t) \cos(2\pi f_c t + \theta_n)$,

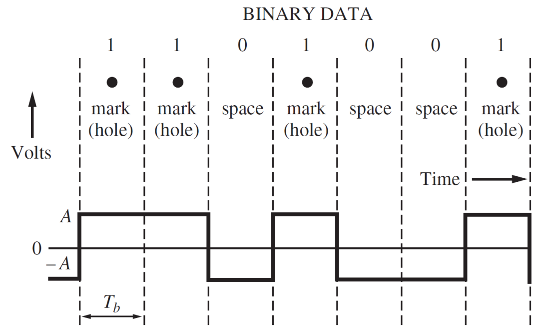
for $m = 1, \ldots, M_1, n = 1, \ldots, M_2, M = M_1 + M_2$



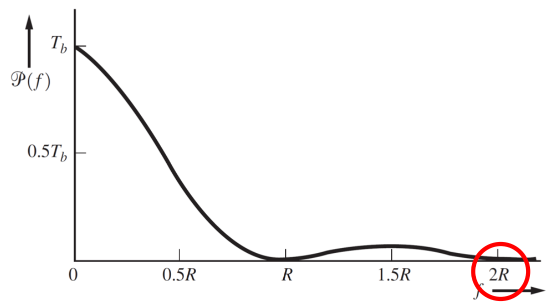
bandwidth of a baseband waveform (1)

- power spectral density (PSD) – power content in the freq. domain

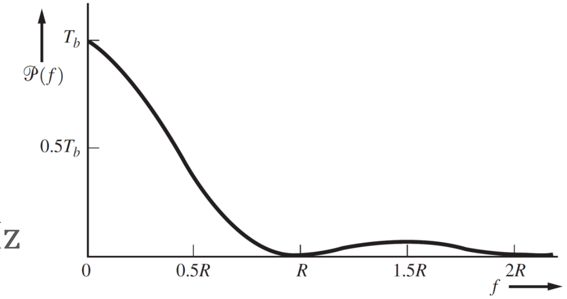
$$P_w(f) = \lim_{T\to\infty}\left(\frac{|W_T(f)|}{T}\right)$$



- BPSK waveform with bitrate $R = 1/T_b$
- worst case: rectangular wave of freq. $R/2$
- equivalent PSD: $P(f) = A^2 T_b \left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2$
- bandwidth definition based on the PSD second null – $2R$



bandwidth of a baseband waveform (2)



example 1

- an RS-232 link operates at a rate of 230,400 baud
- what is the bandwidth of the communication signal?
  - $230{,}400 \text{ baud} \equiv 230{,}400 \text{ bit/s} \quad \Rightarrow \quad B = 2\times 230{,}400 = 460.8 \text{ kHz}$

example 2

- a BPSK baseband signal has a bandwidth of 20 MHz
- 5% of the bits contain link control information
- what is the system throughput?
  - bitrate: $R = \frac{B}{2} = \frac{20\times 10^6}{2} = 10 \text{ Mbit/s}$
  - throughput: $T = (100\% - 5\%)R = 95\%\times 10\times 10^6 = 9.5 \text{ Mbit/s}$

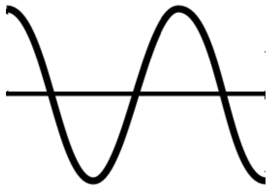
channel with Gaussian noise (AWGN)

transmitting the bits

- now we have the M-ary signal waveforms, how do we transmit a given sequence of bits

1. break the sequence of bits into k-bit blocks or symbols
2. associate each symbol to its corresponding waveform
3. transmit this waveform for a symbol (signaling) interval of T seconds

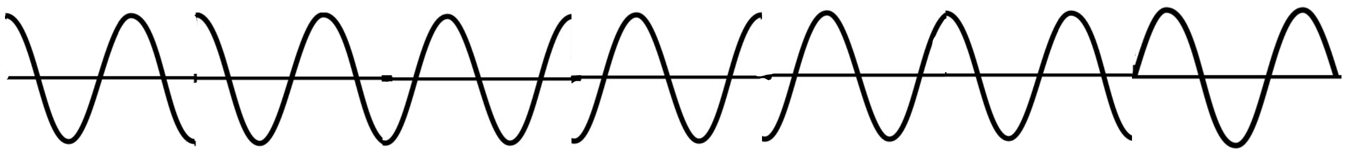
an example

- assume a 4-QPSK with waveform $g(t)$: 

we have the following sequence of bits:

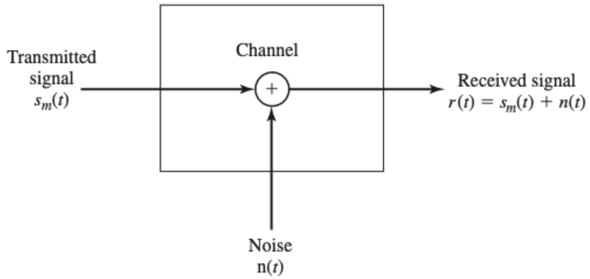
00 10 0101010111

00 10 01 01 01 00 11



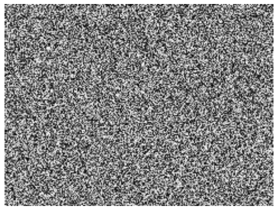
additive white Gaussian noise channels

assuming baseband, we consider the following model:



$$r(t) = s_m(t) + n(t), 0 \leq t \leq T$$

- what is white Gaussian noise:



white noise image

  - model the imperfections introduced by the channel
    -- things that we are uncertain about
  - ”white” comes from the fact that
    the power of the noise is constant over time and frequency
    $S_n(f) = \frac{N_0}{2}$ W/Hz, where N_0 is the noise power
  - Gaussian comes from a theorem from statistics that is known as the central limit theorem

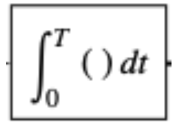
how do we received data?

we have defined very well the transmitter side, but now how do we receive data?

we divide the functions of the received into two parts:

- **signal demodulator:** converts the received waveform $r(t)$ into a vector with length equal to the dimension of the modulation
- **signal detector:** from the vector, guess which of the M waveforms was received

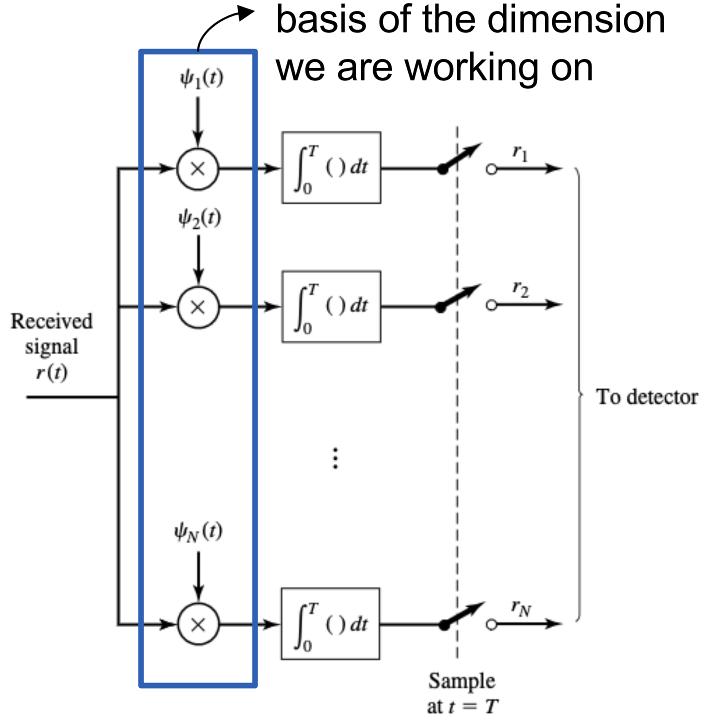
signal demodulator



integrator block

correlation-type demodulator

basis of the dimension
we are working on



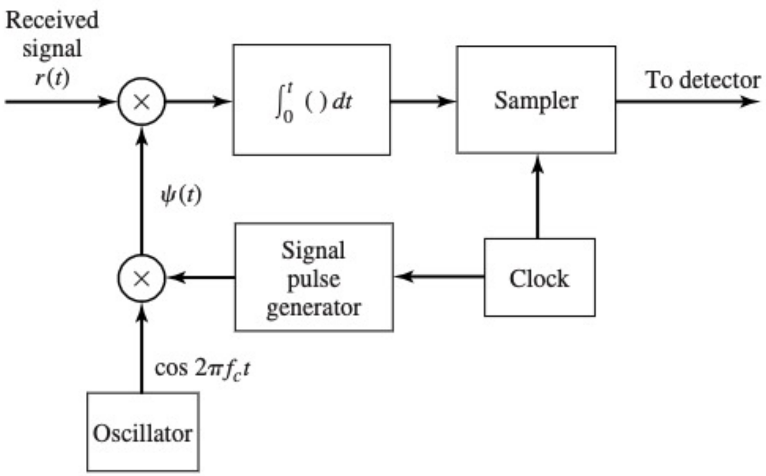
intuition

1. first you correlate the received signal in time to break into one of the components that represents the dimension of the signal waveform space
2. then you integrate over time to extract the energy received over that dimension
3. finally, you sample and get a vector representation

signal demodulator: PAM example

PAM case

we just have one-dimension



transmitted signal

$$u_m(t) = A_m g_T(t) \cos 2\pi f_c t, \quad 0 \le t \le T$$

received signal

$$r(t) = A_m g_T(t) \cos 2\pi f_c t + n(t), \quad 0 \le t \le T$$

noise signal

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

integration

$$\int_0^T r(t)\psi(t)\,dt = A_m \sqrt{\frac{2}{\mathcal{E}_g}} \int_0^T g_T^2(t) \cos^2 2\pi f_c t\,dt + \int_0^T n(t)\psi(t)\,dt$$

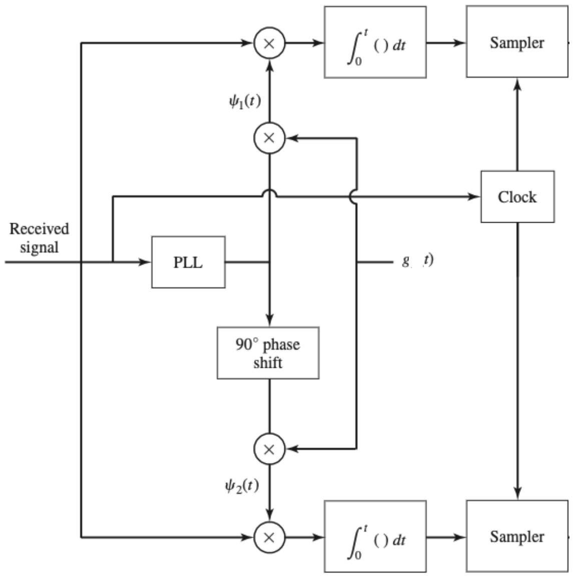
$$= A_m \sqrt{\mathcal{E}_g/2} + n$$

energy of g_T

signal demodulator: QAM example

QAM case

we have two-dimensions



received signal

$$r(t) = A_{mc} g_T(t) \cos(2\pi f_c t + \phi) + A_{ms} g_T(t) \sin(2\pi f_c t + \phi) + n(t)$$

basis

$$\psi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \cos(2\pi f_c t + \hat{\phi})$$

$$\psi_2(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g_T(t) \sin(2\pi f_c t + \hat{\phi})$$

integration

$$r_1 = A_{mc}\sqrt{\mathcal{E}_s} \cos(\phi - \hat{\phi}) + A_{ms}\sqrt{\mathcal{E}_s} \sin(\phi - \hat{\phi}) + n_c \sin\hat{\phi} - n_s \cos\hat{\phi}$$
$$r_2 = A_{mc}\sqrt{\mathcal{E}_s} \sin(\phi - \hat{\phi}) + A_{ms}\sqrt{\mathcal{E}_s} \cos(\phi - \hat{\phi}) + n_c \sin\hat{\phi} - n_s \cos\hat{\phi}$$

signal detector

maximum-a-posteriori principle

- after the detector, we end up with a vector $\boldsymbol{r} = [r_1, r_2, \dots, r_N]$ where N is the number of dimensions used to represent the waveforms

- from $\boldsymbol{r}$ we need to decide which $s_m(t)$ was transmitted

- one way to do so is by using a decision rule based on posterior probabilities:

$$P(signal\ s_m(t)\ was\ transmitted|\boldsymbol{r})\ for\ m = 1, 2, \dots, M$$

intuition: find the $s_m(t)$ with the largest probability of being transmitted given that $\boldsymbol{r}$ was received

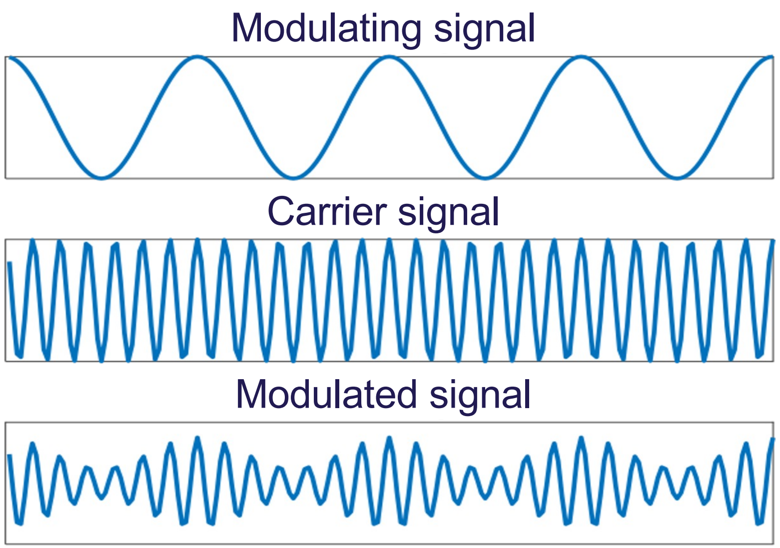
analog modulation

analog and digital modulation

analog modulation

modulating signal is **continuous**

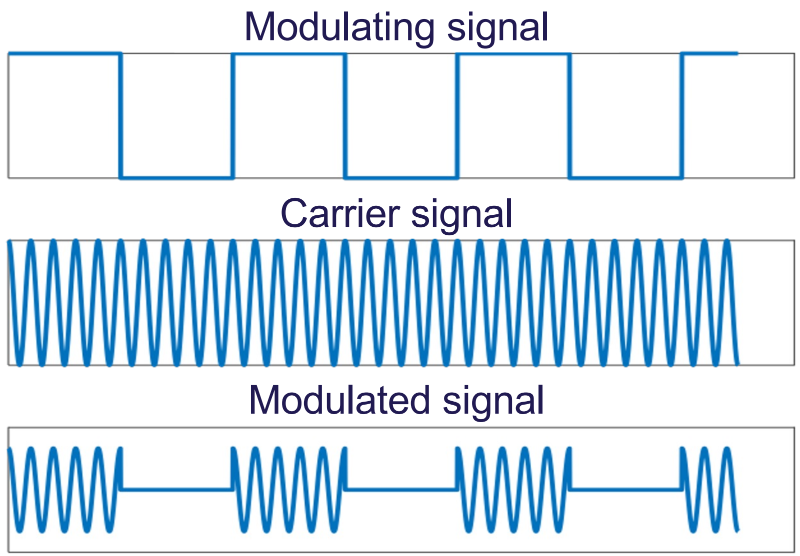
first implemented: AM and FM radio



digital modulation

modulating signal is **discrete**

used in modern communication systems



analog modulation: amplitude, frequency, and phase

$$s(t) = A\cos(\omega_c t + \phi)$$

amplitude

phase

carrier angular frequency

- amplitude modulation (AM)
- angle modulation/phase modulation (PM)
- frequency modulation (FM)

amplitude modulation

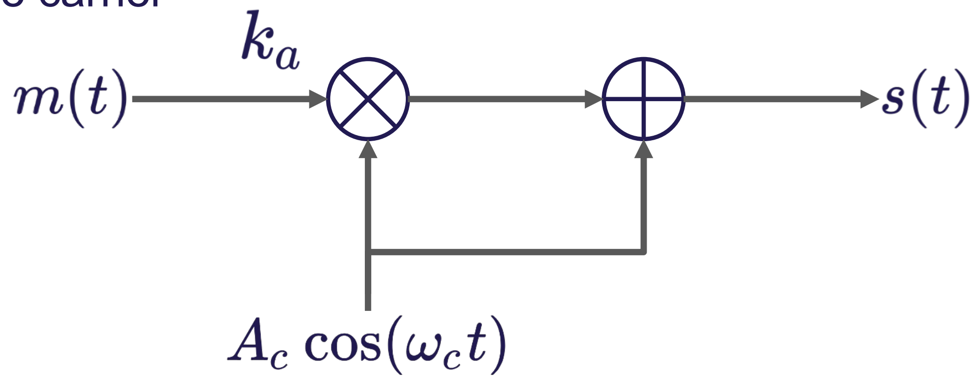
amplitude modulation (AM)

$m(t)$: the baseband signal that carries the information

$s(t)$: the modulated signal with angular carrier frequency $\omega_c = 2\pi f_c$

classical AM

1. multiply the baseband signal by a sinusoid carrier signal
2. add the carrier



k_a: amplitude sensitivity

AM applications and principle

applications
broadcast and radio (540 to 1600 kHz). **cheap user equipment**

principle
the information is carried by the amplitude of the modulated signal

why adding the carrier? Equivalent to a DC shift

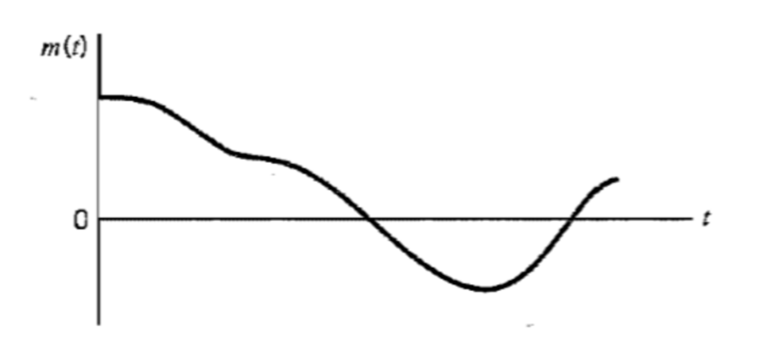
makes the message easy to decode: envelope – DC shift

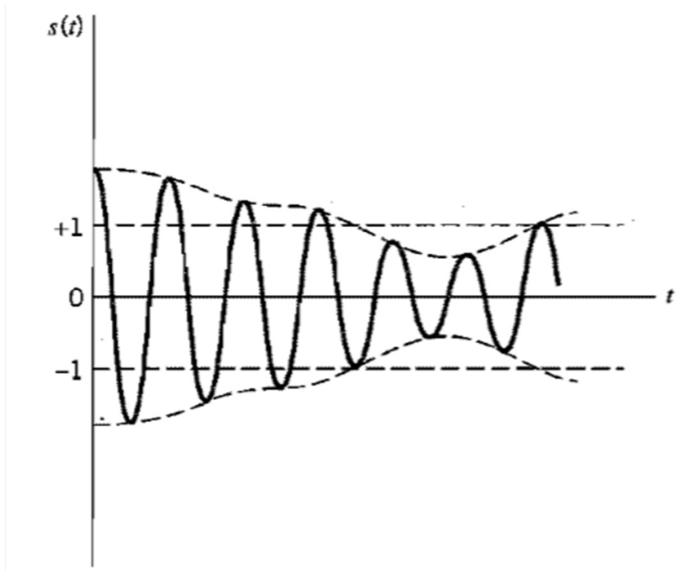
time domain: the signal modulates the amplitude of the carrier

frequency domain: the carrier shifts the frequency of the signal by ω_c

AM signal in time

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(\omega_c t)$$

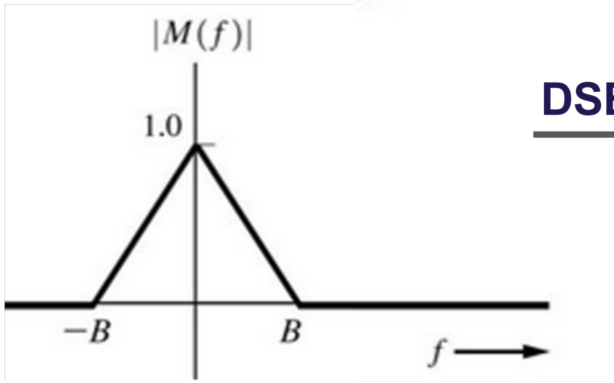




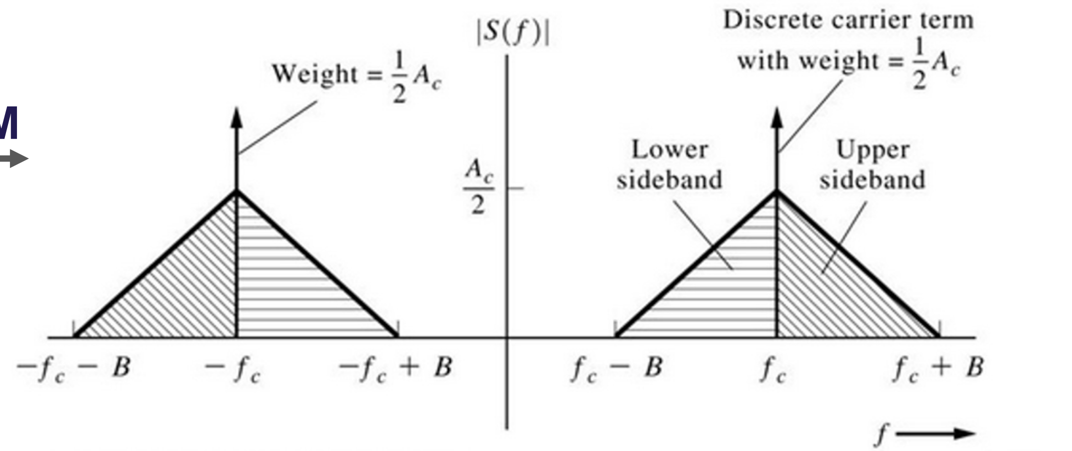
AM signal in frequency (spectrum)

traditional AM is actually double sideband with carrier AM (DSB-AM)

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



DSB-AM



example: single-tone modulation

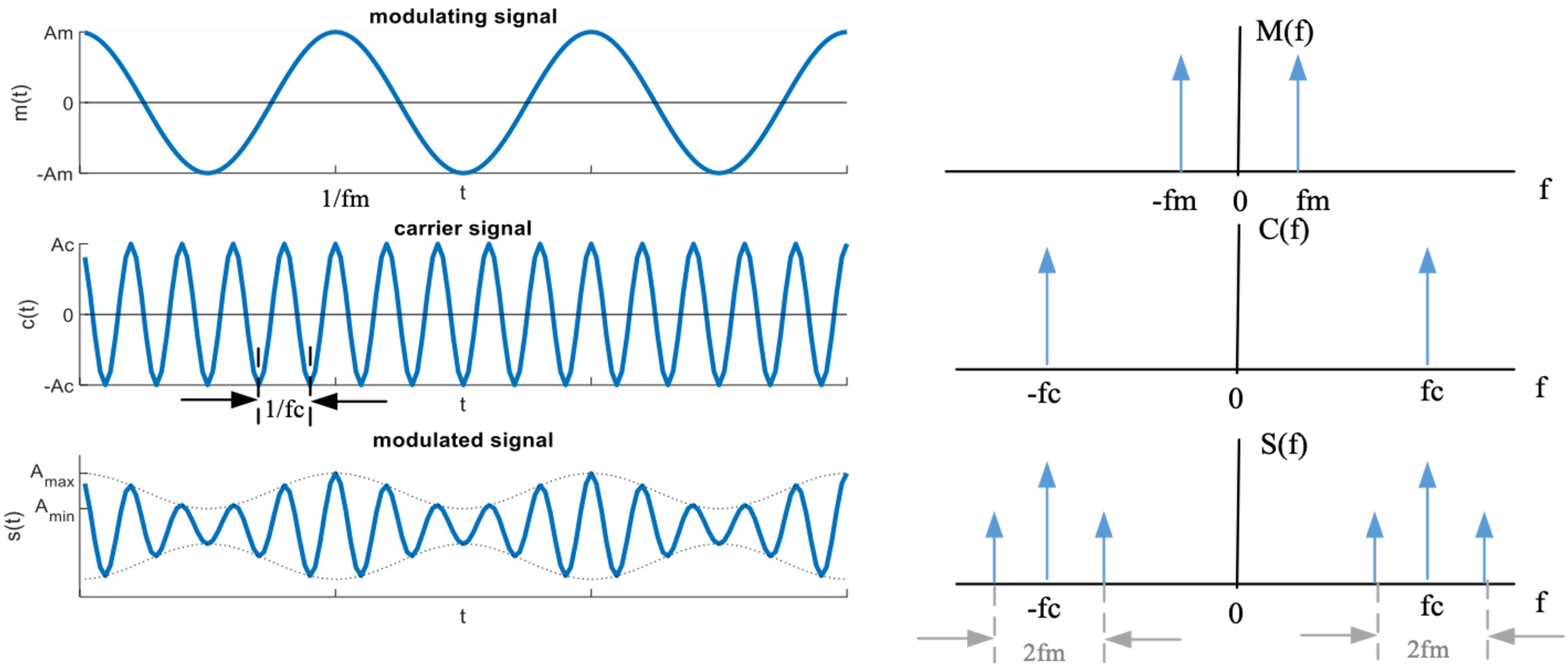
single-tone modulating signal: $m(t) = A_m \cos(\omega_m t)$

modulated signal

$$\begin{aligned} s(t) &= A_c \left[1 + k_a m(t)\right] \cos(\omega_c t) \\ &= A_c \left[1 + k_a A_m \cos(\omega_m t)\right] \cos(\omega_c t) \end{aligned}$$

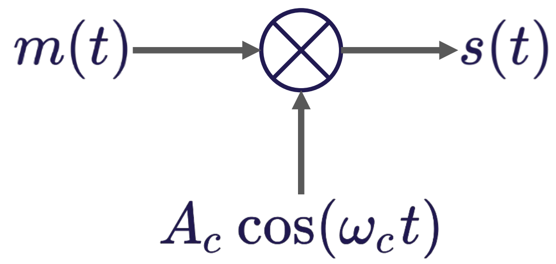
modulation factor $\mu = k_a A_m$

example: single-tone modulation

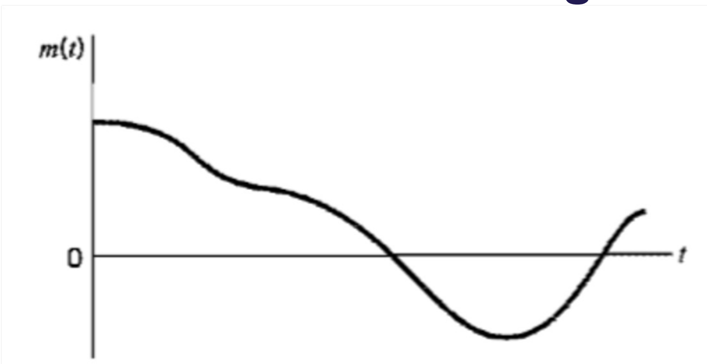


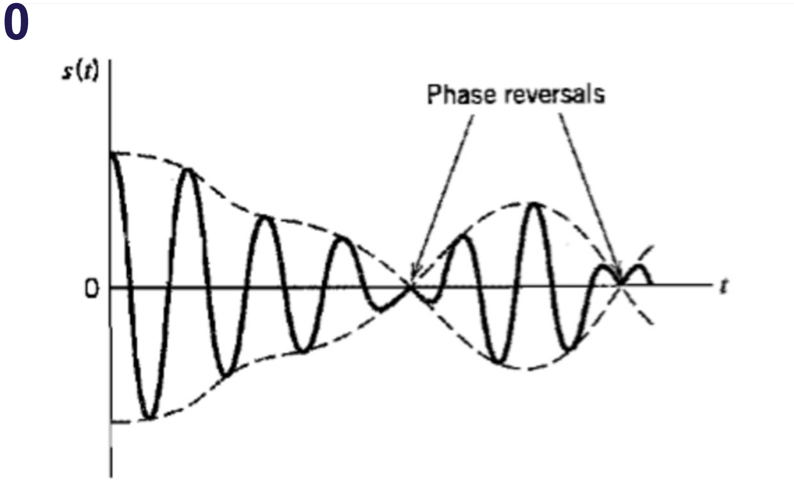
double-sideband suppressed carrier (DSB-SC)

multiply the signal by the carrier but skip its addition



phase reversal whenever the signal crosses 0

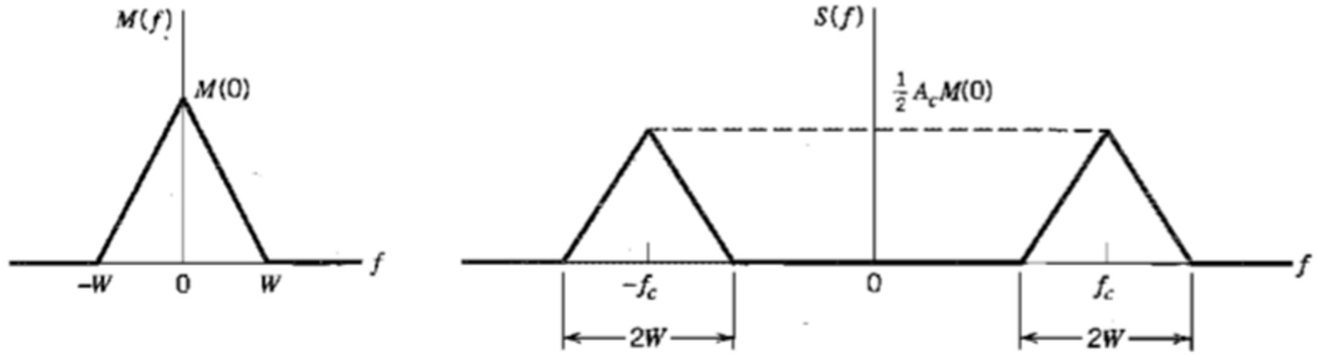




double-sideband suppressed carrier (DSB-SC)

spectrum

$$S(f) = \frac{1}{2}A_c\Big(M\left(f - f_c\right) + M\left(f + f_c\right)\Big)$$



single sideband (SSB)

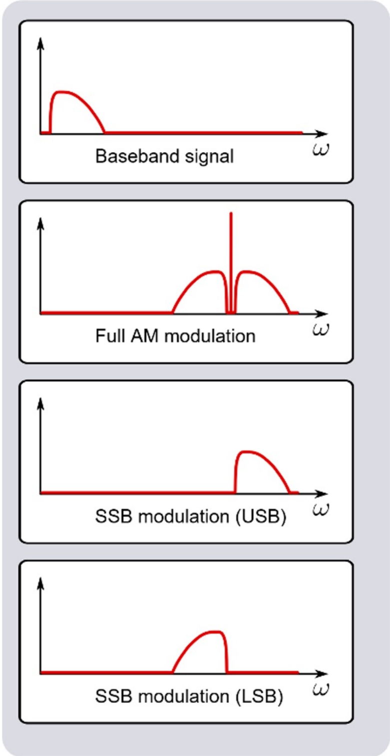
why occupying double the bandwidth if both sides carry the same information?

SSB

similar to DSB, just change settings of band pass filter

pro: only uses half the bandwidth

con: recovering the original signal

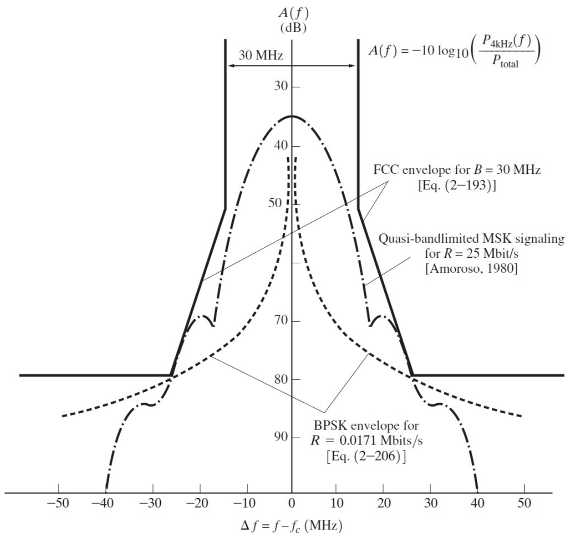


Source: Wikipedia

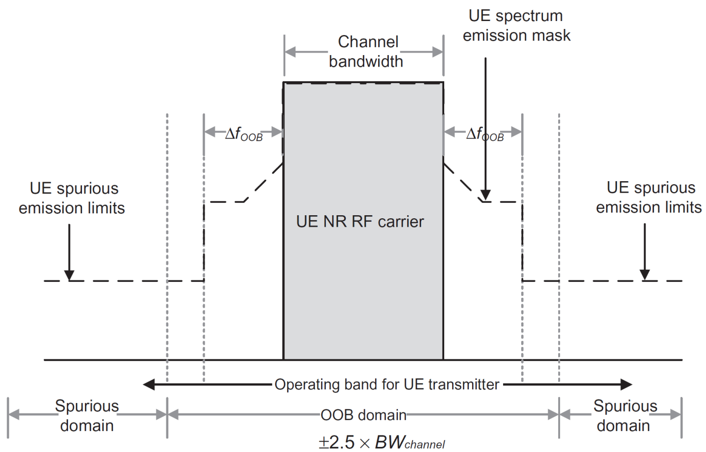
spectrum emission mask (SEM)

- regulatory agencies control how bands of the radio spectrum are used
- spectral masks are defined to limit interference at neighboring bands

FCC SEM for freqs. below $15\ \mathrm{GHz}$:



SEM for 5G devices:



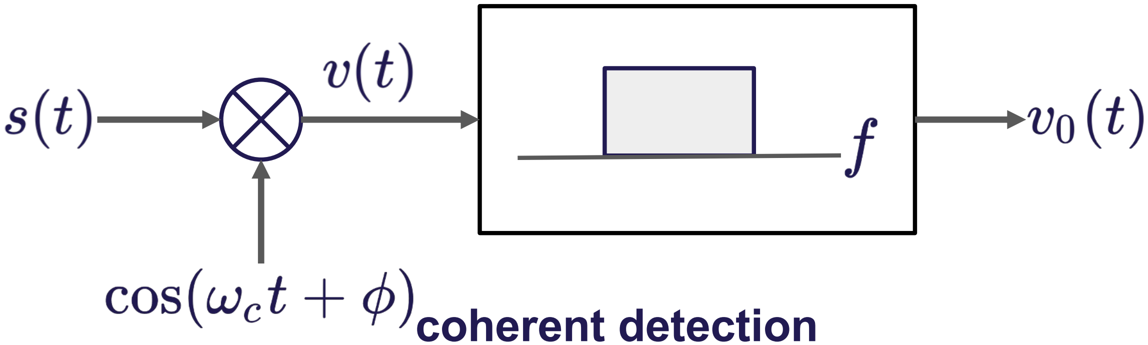
amplitude demodulation

synchronous demodulation

recover the signal by multiplying by the carrier and then using a low pass filter

$$s(t) = m(t)\cos(\omega_c t)$$

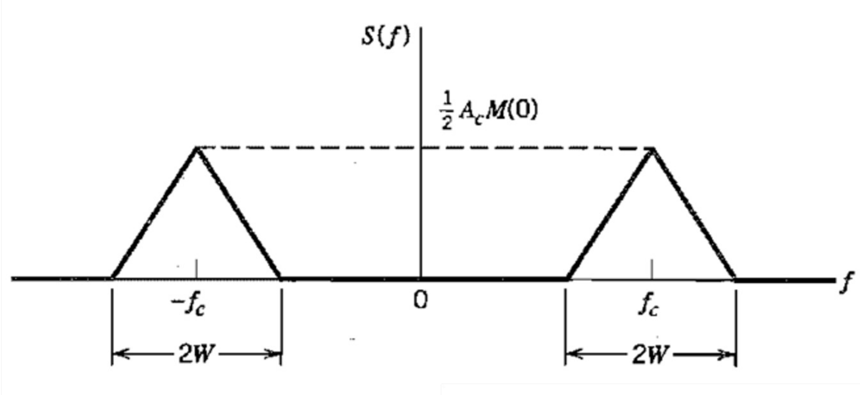
$$v(t) = s(t)\cos(\omega_c t) = m(t)\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right]$$



coherent detection

problem: the receiver must know the carrier perfectly

synchronous demodulation



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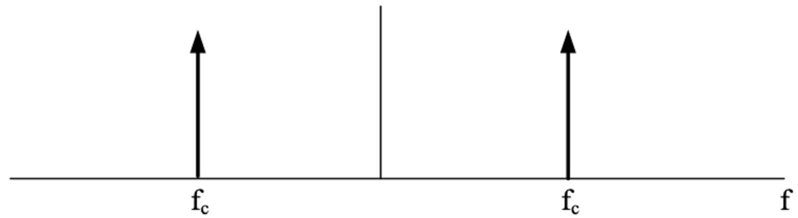

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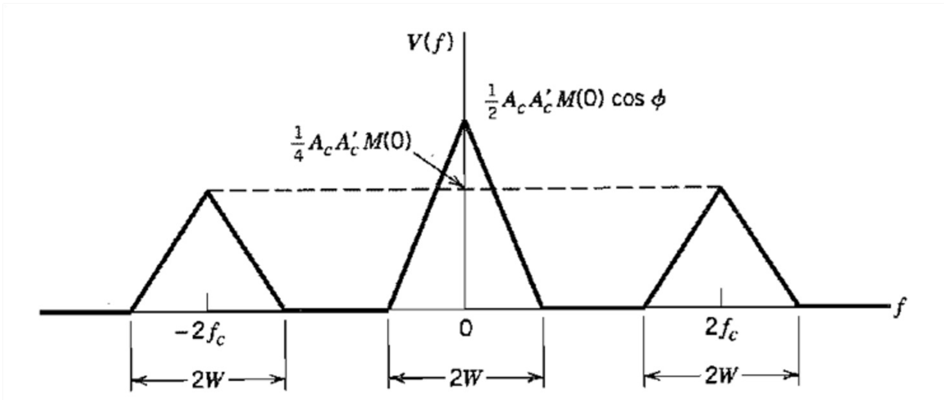
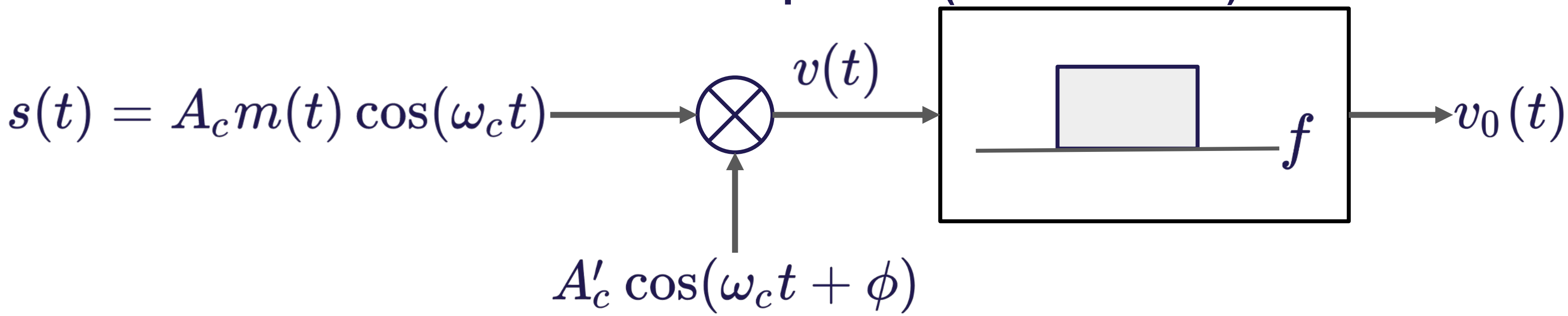


synchronous demodulation

what if the receiver does not know the phase? (non-coherent)



$$
\begin{aligned}
v(t) &= s(t) A_c' \cos(\omega_c t + \phi) \\
&= m(t) A_c A_c' \cos(\omega_c t)\cos(\omega_c t + \phi) \\
&= \frac{m(t)}{2} A_c A_c' \Big(\cos(2\omega_c t + \phi) + \cos(\phi) \Big)
\end{aligned}
$$

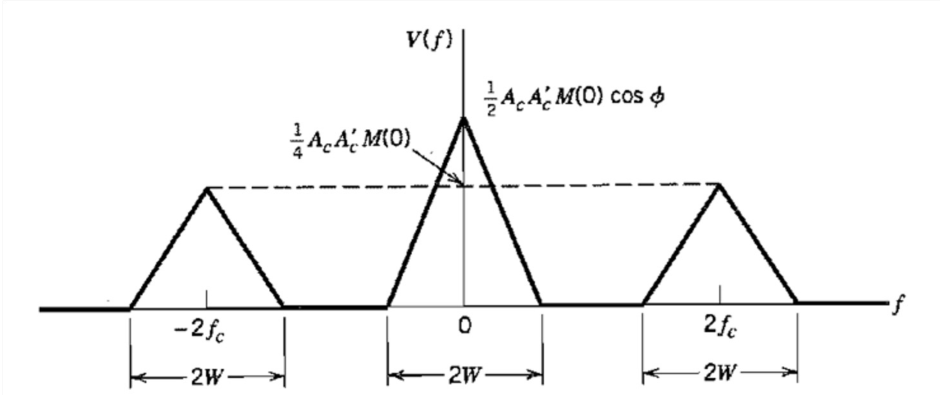
remember:

$2\cos(\theta)\cos(\phi) = \cos(\theta - \phi) + \cos(\theta + \phi)$

synchronous demodulation

after low-pass filter

$$v_0(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t)$$



if the phase is constant and $\neq \pm\pi/2$

$v_0(t)$ is proportional to $m(t)$

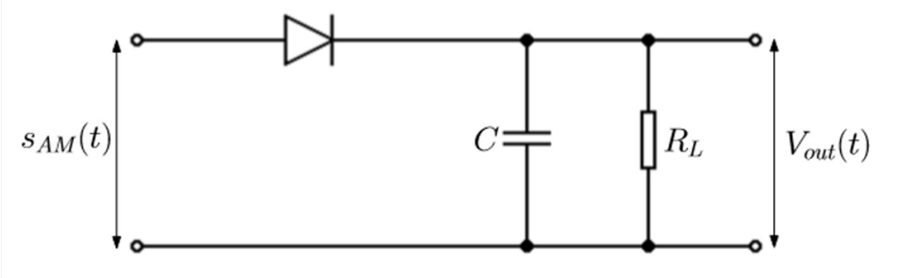
if the phase is $= \pm\pi/2$

$v_0(t) = 0$

envelope detector

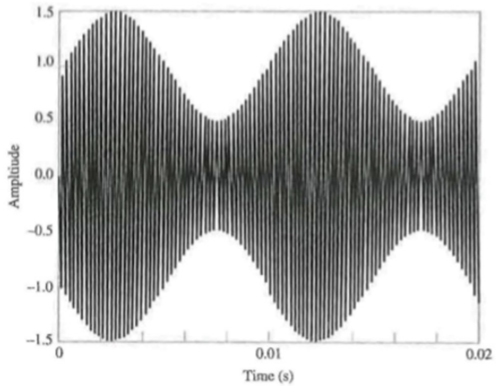
non-coherent detection

the capacitor charges in the positive half-cycle up to the peak value of the input

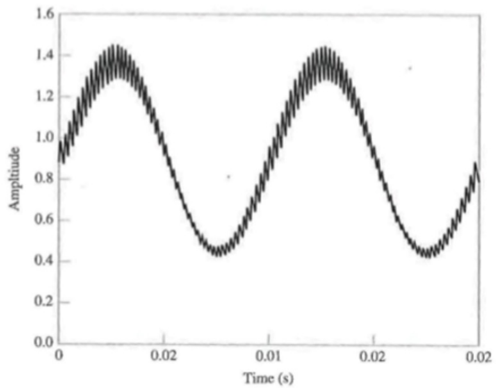


the capacitor discharges slowly in the negative half-cycle

cheap receiver that requires $k_a < 1$



AM wave input



Envelope detector output

demodulation in AM: wrap-up

coherent (synchronous) detection

- the receiver uses a local carrier of the same frequency and phase to detect the signal
- cross-correlation of replica signals at the receiver

non-coherent detection

- no replica signals required
- does not exploit phase reference information
- **pro:** lower complexity at the receiver
- **con:** worse performance

phase and frequency modulation

phase modulation (PM) principle

modulated signal

$$s(t) = A_c \cos(\theta_i(t))$$

A_c : **constant amplitude** of the modulated signal

$\theta_i(t)$: **variable instantaneous angle** of the modulated signal (fluctuates with the message)

$\omega_c t = 2\pi f_c t$: basic angle of the carrier at time t

with no modulating signal $s(t) = A_c \cos(2\pi f_c t)$

phase modulation (PM)

the instantaneous **angle** $\theta_i(t)$ changes linearly as a function of $m(t)$

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

where k_p is the **phase sensitivity** of the modulator

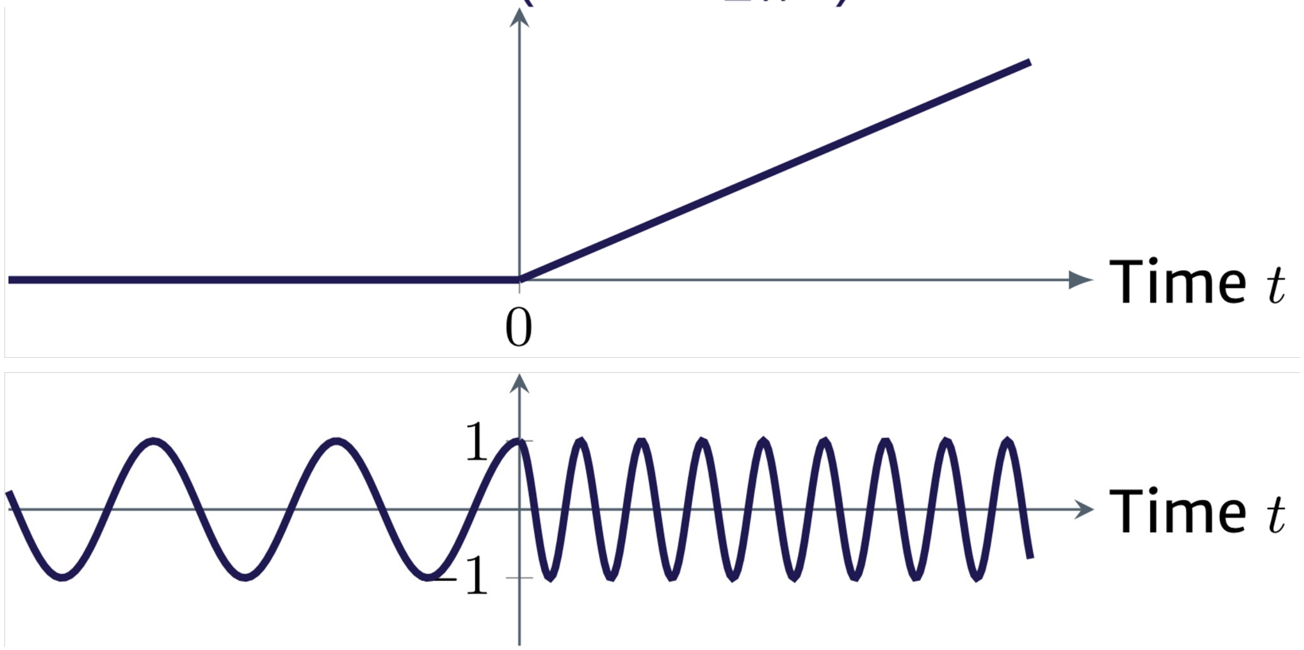
the modulated signal is $s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$

if $m(t)$ is a first-order function, the modulated signal will be shifted from f_c to $f_c + f_m$

$$\begin{aligned}\theta_i(t) &= 2\pi f_c t + k_p a t \\ &= 2\pi \left(f_c + \frac{k_p a}{2\pi}\right) t = 2\pi \left(f_c + f_m\right) t\end{aligned}$$

phase modulation (PM)

$$\theta_i(t) = 2\pi f_c t + k_p a t = 2\pi\left(f_c + \frac{k_p a}{2\pi}\right)t = 2\pi\left(f_c + f_m\right)t$$



frequency modulation (FM) principle

the instantaneous **frequency** $f_i(t)$ changes linearly as a function of $m(t)$

$$f_i(t) = f_c + k_f m(t)$$

where k_f is the **frequency sensitivity** of the modulator

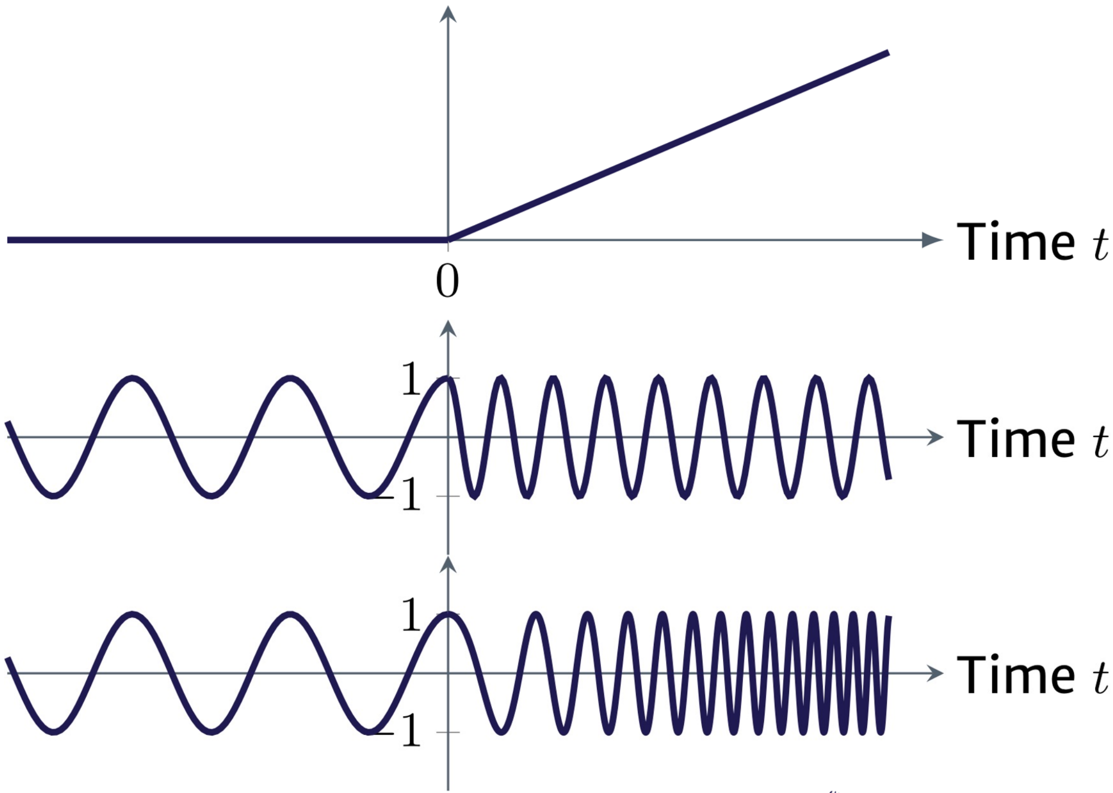
the angle is

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)\mathrm{d}\tau = 2\pi \left(f_c t + k_f \int_0^t m(\tau)\mathrm{d}\tau \right)$$

the modulated signal is

$$s(t) = A_c \cos\left[2\pi \left(f_c t + k_f \int_0^t m(\tau)\mathrm{d}\tau \right) \right]$$

frequency modulation (FM)



$$\theta_i(t) = 2\pi\,(f_c + f_m)\,t$$

$$\theta_i(t) = 2\pi\left(f_c t + k_f \int_0^t m(\tau)\mathrm{d}\tau\right)$$

angle modulation: PM vs. FM

		Phase modulation (PM)	Frequency modulation (FM)
instantaneous phase	$\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi\left(f_c t + k_f \int_0^t m(\tau)\mathrm{d}\tau\right)$
instantaneous frequency	$f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$	$f_c + k_f m(t)$
modulated wave	$s(t)$	$A_c \cos\left(2\pi f_c t + k_p m(t)\right)$	$A_c \cos\left[2\pi\left(f_c t + k_f \int_0^t m(\tau)\mathrm{d}\tau\right)\right]$

k_p: phase-sensitivity factor; k_f: frequency-sensitivity factor

properties of angle modulation

constant terms in the transmitted wave

the amplitude of the PM and FM waves A_c

the average transmitted power $\bar{P} = \frac{A_c}{2}$

non-linearity of the modulation process

$$m(t) = m_1(t) + m_2(t) \qquad s(t) = A_c \cos[2\pi f_c t + k_p\,(m_1(t) + m_2(t))]$$

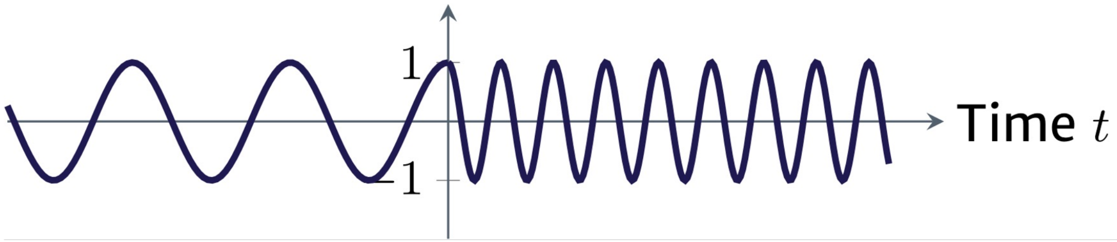
$$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)] \qquad s_2(t) = A_c \cos[2\pi f_c t + k_p m_2(t)]$$

$$\boxed{s(t) \neq s_1(t) + s_2(t)}$$

properties of angle modulation

irregularity of zero-crossing

zero-crossing: points in time where the waveform amplitude changes sign



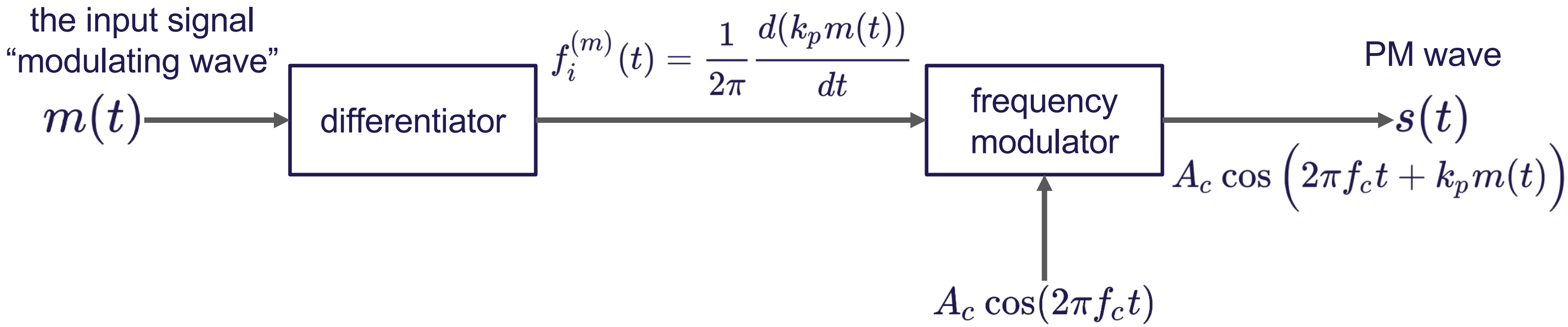
the zero-crossings of the modulated wave carry the information of the signal $m(t)$

visualization difficulty: attributed to the non-linear nature

trade-off: increased transmission bandwidth for improved noise performance

less sensitive to additive noise obtained at the expense of increased Tx. bandwidth

modulation process in PM



$f_i^{(m)}(t) = \frac{1}{2\pi}\frac{d(k_p m(t))}{dt}$ is the instantaneous frequency of the modulating wave

summary and outlook

- we have arrived down to the physical layer and waveforms that transmit information
- mapping of digital modulations to waveforms
- relevance of the Fourier analysis and bandwidth
- overview of the analog modulation methods