ESD5 – Fall 2024
Problem Set 7

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Problem 1 – From the Analog to the Discrete World

Consider the signals:

$$x(t) = A_x \left(\sin(2\pi 4000 t) + \cos(2\pi a_x t)\right) \tag{1}$$

$$y(t) = A_y \left(\sin(2\pi a_x t) + \cos(2\pi b_y t)\right), \tag{2}$$

where $0 \leq a_x \leq 8$ kHz and $0 \leq b_x \leq 16$ kHz. What are the Nyquist rate (sampling frequency) and sampling period for:

(a) $x(t)$?

(b) $y(t)$?

(c) $x(t) + y(t)$?

(d) $x(t)y(t)$?

Let us focus now on $x(t)$. Assume $A_x = 1$ and $a_x = 8$ kHz. Imagine that we observe the signal for 1 ms and we start to observe the signal at $T = 0$. Answer the following:

(e) Using the result from (a), get the number of samples required to reconstruct $x(t)$ for $t \in [0, 1 \cdot 10^{-3}]$ according to the Nyquist rate.

(f) Using Matlab or Python, plot the signal $x(t)$ over this 1 ms and the respective sampled points. What are the values of $x(t)$ in the sampled points?

(g) The last step to the analog signal $x(t)$ become a completely discrete signal is to discretize its amplitude value. Read more about quantization on `https://en.wikipedia.org/wiki/Quantization_(signal_processing)`[1]. If you had access to a 2-bit resolution quantizer, how would you do the quantization of $x(t)$? After quantizing, can you get $x(t)$ back perfectly from the quantized version?

(optional) This optional exercise is about estimating the quantization error. Define the quantization error as $e_n = x[nT_s] - \hat{x}[nT_s]$, where $n = 1, 2, \ldots, N$ with N obtained in (e), T_s being the sampling rate, and $\hat{x}$ being the quantized version. Compute the average quantization error according to the:

$$\mathrm{MSE} = \frac{1}{N}\sum_{n=1}^{N} e_n^2, \tag{3}$$

where MSE is the mean-squared error.

Problem 2 – Conventional AM modulation

Let $m(t) = 0.5\cos(200\pi t) + \cos(400\pi t)$ be the AM baseband modulating signal. Calculate the AM modulated signal and plot its spectrum given amplitude sensitivity $k_a = 0.5$, carrier frequency of $f_c = 2$ kHz, and carrier amplitude of $A_c = 10$ V.

Problem 3 – 4-PAM

Consider an $M = 4$ PAM modulation. Answer the following:

(a) Enumerate the binary symbols that this modulation can represent.

(b) Assume $A_1 = -2, A_2 = -1, A_3 = 1, A_4 = 2$ and $g_T(t) = \cos(2\pi t)$. Draw the signal waveforms and associate them with a symbol enumerated in (a). Please associate the according A_x to the symbol that represents x in the binary numeral system.

(c) Assume that the transmission period of a signal waveform is $T = 1$ s and $f_c = 1$ Hz. Draw how the transmission of "00101101110010" occurs over time. How long does the transmission of such a sequence take?

(d) Assume that at the receiver side, the symbol "00" transmitted by the transmitter suffered from a noise component of $n = 0.5$. What is the value of r after the signal demodulator?

[1] For a more in-depth and formal treatment of Quantization, consider the book: Proakis, J. G. & Manolakis, D. K. (2006), Digital Signal Processing (4th Edition), Prentice Hall. You can easily find it on Google.

(e) Based on the maximum-a-posterior principle, how would the signal detector decide for the following sequence received demodulated signals: $[-1.20, -2.40, 1.49, 2.10]$. Assume that the sequence of true transmitted symbols was: $[00, 00, 11, 11]$. What is the percentage of error committed by the receiver?