7.7 Frequency sampling method

The frequency sampling method allows us to design nonrecursive FIR filters for both standard frequency selective filters (lowpass, highpass, bandpass filters) and filters with arbitrary frequency response. A unique attraction of the frequency sampling method is that it also allows recursive implementation of FIR filters, leading to computationally efficient filters. With some restrictions, recursive FIR filters whose coefficients are simple integers may be designed, which is attractive when only primitive arithmetic operations are possible, as in systems implemented with standard microprocessors.

7.7.1 Nonrecursive frequency sampling filters

Suppose we wish to obtain the FIR coefficients of the filter whose frequency response is depicted in Figure 7.16(a). We could start by taking N samples of the frequency response at intervals of kF_s/N, $k = 0, 1, \ldots, N-1$. The filter coefficients $h(n)$ can be obtained as the inverse DFT of the frequency samples:

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k)\,\mathrm{e}^{\mathrm{j}(2\pi/N)nk} \tag{7.16}$$

where $H(k)$, $k = 0, 1, \ldots, N-1$, are samples of the ideal or target frequency response.

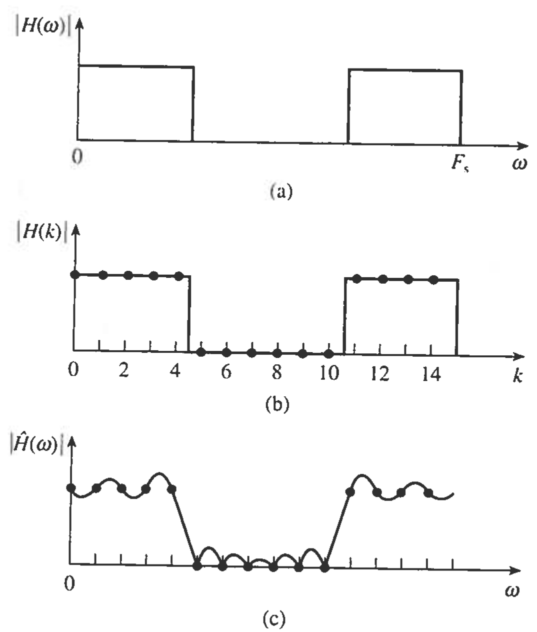

Figure 7.16 Concept of frequency sampling. (a) Frequency response of an ideal lowpass filter. (b) Samples of the ideal lowpass filter. (c) Frequency response of lowpass filter derived from the frequency samples of (b).

It can be shown (see Example 7.9) that for li symmetrical impulse response, we can write (for N

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] +\right.$$

where $\alpha = (N-1)/2$. For N odd, the upper limit ir resulting filter will have a frequency response that response at the sampling instants. However, betwee may be significantly different (Figure 7.16(c)). To desired frequency response, clearly we must take samples.

An alternative frequency sampling filter, kno frequency samples at intervals of

$$f_k = (k + 1/2)F_s/N, \quad k = 0, 1, \ldots, N-1$$

Figure 7.17 compares the sampling grids for b schemes. For a given filter specification, both meth frequency responses. The designer needs to decid his or her needs.

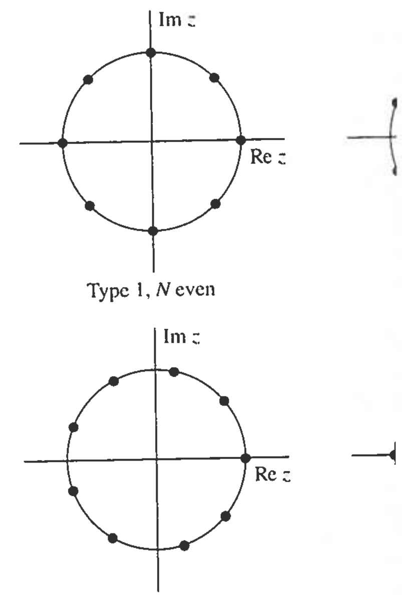

Figure 7.17 The four possible z-plane sampling grid sampling filters.

sampling method allows us to design nonrecursive FIR filters for both ncy selective filters (lowpass, highpass, bandpass filters) and filters with ency response. A unique attraction of the frequency sampling method is ws recursive implementation of FIR filters, leading to computationally With some restrictions, recursive FIR filters whose coefficients are may be designed, which is attractive when only primitive arithmetic possible, as in systems implemented with standard microprocessors.

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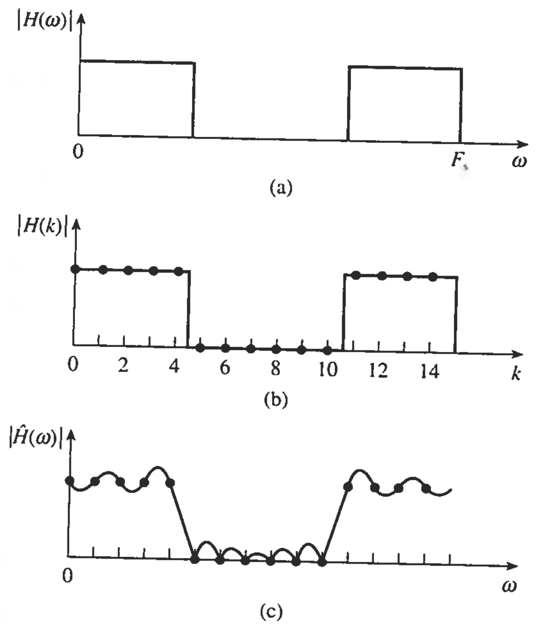
$$\sum_{k=0}^{N-1} H(k)\, e^{j(2\pi/N)nk} \qquad (7.16)$$

$0, 1, \ldots, N-1$, are samples of the ideal or target frequency response.



ept of frequency sampling. (a) Frequency response of an ideal lowpass (b) Samples of the ideal lowpass filter. (c) Frequency response of ass filter derived from the frequency samples of (b).

It can be shown (see Example 7.9) that for linear phase filters, with positive symmetrical impulse response, we can write (for N even),

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] + H(0)\right] \qquad (7.17)$$

where $\alpha = (N-1)/2$. For N odd, the upper limit in the summation is $(N-1)/2$. The resulting filter will have a frequency response that is exactly the same as the original response at the sampling instants. However, between the sample instants, the response may be significantly different (Figure 7.16(c)). To obtain a good approximation to the desired frequency response, clearly we must take a sufficient number of frequency samples.

An alternative frequency sampling filter, known as type 2, results if we take frequency samples at intervals of

$$f_k = (k + 1/2)F_s/N, \quad k = 0, 1, \ldots, N-1 \qquad (7.18)$$

Figure 7.17 compares the sampling grids for both types of frequency sampling schemes. For a given filter specification, both methods will lead to somewhat different frequency responses. The designer needs to decide which of the two types best suits his or her needs.

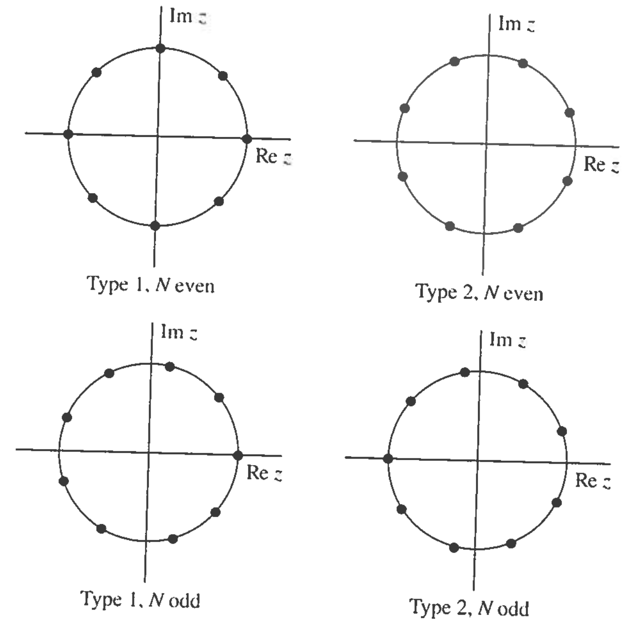


Figure 7.17 The four possible z-plane sampling grids for the two types of frequency sampling filters.

Example 7.9

(1) Show that the impulse response coefficients of a linear phase FIR filter with positive symmetry, for N even, can be expressed as

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] + H(0)\right]$$

where $\alpha = (N-1)/2$, and $H(k)$ are the samples of the frequency response of the filter taken at intervals of kF_s/N.

(2) A requirement exists for a lowpass FIR filter satisfying the following specifications:

passband	0–5 kHz
sampling frequency	18 kHz
filter length	9

Obtain the filter coefficients using the frequency sampling method.

Solution

(1)

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k)\,e^{j(2\pi/N)nk} \tag{7.19}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\,e^{-j2\pi\alpha k/N}\,e^{j2\pi kn/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\,e^{j2\pi k(n-\alpha)/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\cos[2\pi k(n-\alpha)/N] + j\sin[2\pi k(n-\alpha)/N]$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\cos[2\pi k(n-\alpha)/N] \tag{7.20}$$

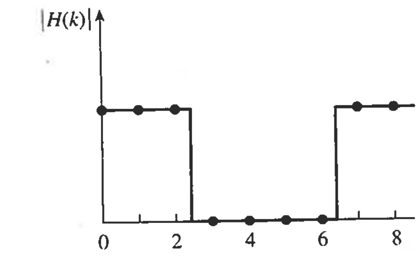
since $h(n)$ is entirely real. For the important case of linear phase $h(n)$ will be symmetrical and so we can write

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] + H(0)\right] \tag{7.21}$$

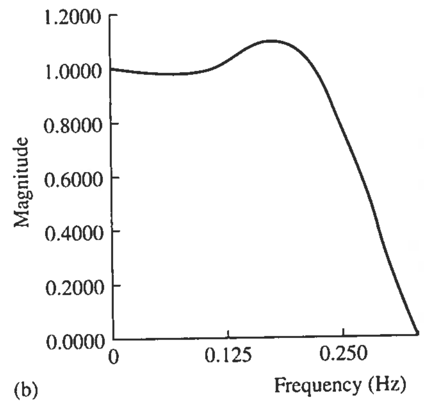
For N odd, the upper limit in the summation is $(N-1)/2$.

(2) The ideal frequency response is depicted in Figure 7.18(a). The frequency samples are taken at intervals of kF_s/N, that is at intervals of $18/9 = 2$ kHz. Thus the frequency samples are given by

$$|H(k)| = 1 \quad k = 0, 1, 2$$
$$0 \quad k = 3, 4$$



(a)



(b)

Figure 7.18 (a) Ideal frequency response showing sampl of frequency sampling filter.

Table 7.10 Nonrecursive coefficients for Example 7.9.

h[0] =	7.2522627e−02
h[1] =	−1.1111111e−01
h[2] =	−5.9120987e−02
h[3] =	3.1993169e−01
h[4] =	5.5555556e−01

Using Equation 7.21 with a limit of $(N-1)/2$ obtain the impulse response coefficients (see T

The CD in the companion handbook (see th compute the FIR coefficients given the values frequency response for the filter is shown in Fi filter has a poor amplitude response, caused by passband (where $|H(k)| = 1$) to the stopband (

at the impulse response coefficients of a linear phase FIR filter with symmetry, for N even, can be expressed as

$$n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] + H(0)\right]$$

$= (N-1)/2$, and $H(k)$ are the samples of the frequency response of taken at intervals of kF_s/N.

ement exists for a lowpass FIR filter satisfying the following itions:

ssband	0–5 kHz
npling frequency	18 kHz
er length	9

ne filter coefficients using the frequency sampling method.

$$\begin{aligned} \imath) &= \frac{1}{N}\sum_{k=0}^{N-1} H(k)\,e^{j(2\pi/N)nk} && (7.19) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\,e^{-j2\pi\alpha k/N}\,e^{j2\pi kn/N} \\ &= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\,e^{j2\pi k(n-\alpha)/N} \\ &= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\cos[2\pi k(n-\alpha)/N] + j\sin[2\pi k(n-\alpha)/N] \\ &= \frac{1}{N}\sum_{k=0}^{N-1} |H(k)|\cos[2\pi k(n-\alpha)/N] && (7.20) \end{aligned}$$

:) is entirely real. For the important case of linear phase $h(n)$ will be ical and so we can write

$$\imath) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} 2|H(k)|\cos[2\pi k(n-\alpha)/N] + H(0)\right] \qquad (7.21)$$

d, the upper limit in the summation is $(N-1)/2$.

l frequency response is depicted in Figure 7.18(a). The frequency are taken at intervals of kF_s/N, that is at intervals of $18/9 = 2$ kHz. frequency samples are given by

$(k)| = 1 \quad k = 0, 1, 2$

$\quad 0 \quad k = 3, 4$

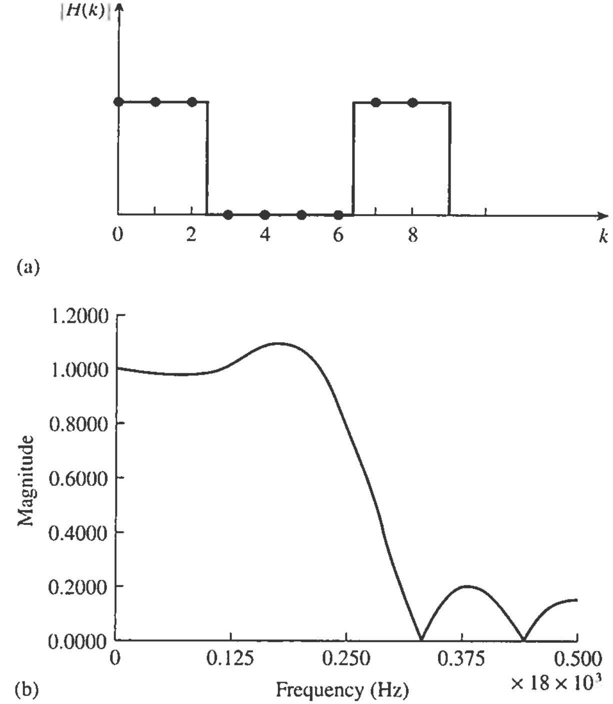


Figure 7.18 (a) Ideal frequency response showing sampling points. (b) Frequency response of frequency sampling filter.

Table 7.10 Nonrecursive coefficients for the FIR filter of Example 7.9.

h[0] =	7.2522627e−02	= h[8]
h[1] =	−1.1111111e−01	= h[7]
h[2] =	−5.9120987e−02	= h[6]
h[3] =	3.1993169e−01	= h[5]
h[4] =	5.5555556e−01	= h[4]

Using Equation 7.21 with a limit of $(N-1)/2$ and the frequency samples we obtain the impulse response coefficients (see Table 7.10).

The CD in the companion handbook (see the Preface) contains a program to compute the FIR coefficients given the values of the frequency samples. The frequency response for the filter is shown in Figure 7.18(b). It is seen that the filter has a poor amplitude response, caused by the abrupt change from the passband (where $|H(k)| = 1$) to the stopband (where $|H(k)| = 0$).

7.7.1.1 Optimizing the amplitude response

The problem above is akin to that of the rectangular window. We recall that, in the case of the window method, we can trade off wider transition width for improved amplitude response. To improve the amplitude response of frequency sampling filters, at the expense of wider transition, we can introduce frequency samples in the transition band. Figure 7.19 illustrates a typical specification for a lowpass filter with three transition band frequency samples. For a lowpass filter, the stopband attenuation increases, approximately, by 20 dB for each transition band frequency sample (Rabiner *et al.*, 1970), with a corresponding increase in the transition width:

approximate stopband attenuation	$(25 + 20M)$ dB
approximate transition width	$(M + 1)F_s/N$

where M is the number of transition band frequency samples and N is the filter length.

The values of the transition band frequency samples that will give the optimum stopband attenuation are determined by an optimization process (Rabiner *et al.*, 1970). A useful optimization objective is to find values of the transition band frequency samples, $T_1, T_2, \ldots, T_M$ that minimize the peak stopband ripple (that is, they maximize the stopband attenuation). Mathematically, this may be stated as:

$$\underset{\{T_1, T_2, \ldots, T_M\}}{\text{minimize}} \begin{bmatrix} \max|W[H_D(\omega) - H(\omega)]| \\ \{\omega \text{ in the stopband}\} \end{bmatrix} \tag{7.22}$$

where $H_D(\omega)$ and $H(\omega)$ are, respectively, the ideal and actual frequency responses of the filter; W is a weighting factor.

Rabiner *et al.* (1970) have provided a table of optimal (in the sense of Equation 7.22) values of transition band frequency samples which is widely used. A sample of the optimal values of transition band frequency samples is given in Table 7.11 for

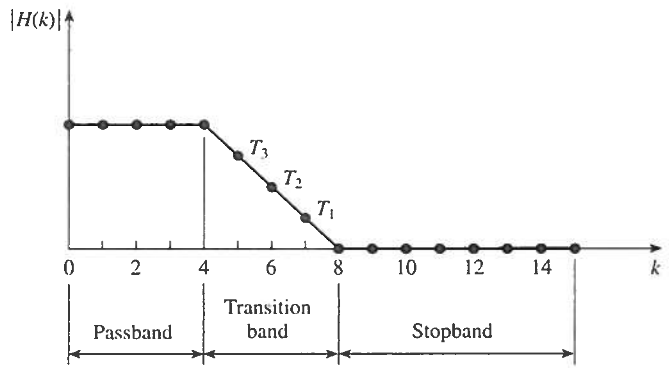


Figure 7.19 Lowpass filter frequency samples including three transition band samples. Note: because of the symmetry in the amplitude response only one half of the filter response is shown.

Table 7.11 Optimum transition band frequency samp sampling filters for $N = 15$ (adapted from

BW	***Stopband attenuation* (dB)**	T_1
One transition band frequency sample, N = 15		
1	42.309 322 83	0.433 782 9
2	41.262 992 86	0.417 938 2
3	41.253 337 86	0.410 473 6
4	41.949 077 13	0.404 058 8
5	44.371 245 38	0.392 681 8
6	56.014 165 88	0.357 665 2
Two transition band frequency samples, N = 15		
1	70.605 405 85	0.095 001 2
2	69.261 681 56	0.103 198 2
3	69.919 734 95	0.100 836 1
4	75.511 722 56	0.084 074 9
5	103.460 783 00	0.051 802 0
Three transition band frequency samples, N = 15		
1	94.611 661 91	0.014 550 7
2	104.998 130 80	0.010 009 7
3	114.907 193 18	0.008 734 1
4	157.292 575 84	0.003 787 9

BW refers to the number of frequency samples in the p

$N = 15$. In the table, the bandwidth refers to the n passband of the filter.

In most cases, the values of the transition band the following ranges: for one transition frequency

$$0.250 < T_1 < 0.450$$

for two transition frequency samples,

$$0.040 < T_1 < 0.150$$
$$0.450 < T_2 < 0.650$$

for three transition frequency samples,

$$0.003 < T_1 < 0.035$$
$$0.100 < T_2 < 0.300$$
$$0.550 < T_3 < 0.750$$

The lower values are for filters with wide band attenuation.

mplitude response

e is akin to that of the rectangular window. We recall that, in the w method, we can trade off wider transition width for improved :. To improve the amplitude response of frequency sampling filters, wider transition, we can introduce frequency samples in the gure 7.19 illustrates a typical specification for a lowpass filter with nd frequency samples. For a lowpass filter, the stopband attenua- proximately, by 20 dB for each transition band frequency sample 70), with a corresponding increase in the transition width:

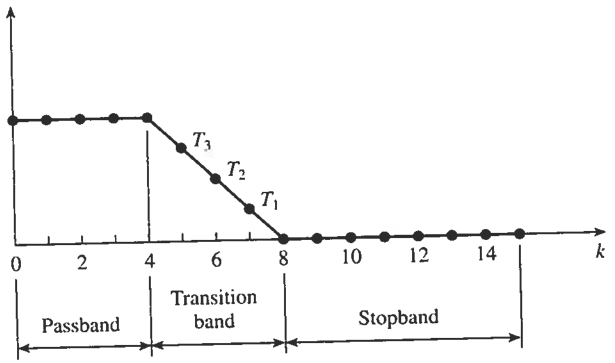
: stopband attenuation $(25 + 20M)$ dB
: transition width $(M + 1)F_s/N$

mber of transition band frequency samples and N is the filter length. the transition band frequency samples that will give the optimum on are determined by an optimization process (Rabiner *et al.*, 1970). ation objective is to find values of the transition band frequency $\ldots, T_M$ that minimize the peak stopband ripple (that is, they band attenuation). Mathematically, this may be stated as:

$$\begin{bmatrix} \max|W[H_D(\omega) - H(\omega)]| \\ \{\omega \text{ in the stopband}\} \end{bmatrix} \tag{7.22}$$

$H(\omega)$ are, respectively, the ideal and actual frequency responses of weighting factor.

(1970) have provided a table of optimal (in the sense of Equation ransition band frequency samples which is widely used. A sample lues of transition band frequency samples is given in Table 7.11 for



wpass filter frequency samples including three transition band samples. te: because of the symmetry in the amplitude response only one half of the er response is shown.

Table 7.11 Optimum transition band frequency samples for type 1 lowpass frequency sampling filters for $N = 15$ (adapted from Rabiner *et al.*, 1970).

BW	*Stopband attenuation* (dB)	T_1	T_2	T_3
One transition band frequency sample, N = 15				
1	42.309 322 83	0.433 782 96		
2	41.262 992 86	0.417 938 23		
3	41.253 337 86	0.410 473 63		
4	41.949 077 13	0.404 058 84		
5	44.371 245 38	0.392 681 89		
6	56.014 165 88	0.357 665 25		
Two transition band frequency samples, N = 15				
1	70.605 405 85	0.095 001 22	0.589 954 18	
2	69.261 681 56	0.103 198 24	0.593 571 18	
3	69.919 734 95	0.100 836 18	0.589 432 70	
4	75.511 722 56	0.084 074 93	0.557 153 12	
5	103.460 783 00	0.051 802 06	0.499 174 24	
Three transition band frequency samples, N = 15				
1	94.611 661 91	0.014 550 78	0.184 578 82	0.668 976 13
2	104.998 130 80	0.010 009 77	0.173 607 13	0.659 515 26
3	114.907 193 18	0.008 734 13	0.163 973 10	0.647 112 64
4	157.292 575 84	0.003 787 99	0.123 939 63	0.601 811 54

BW refers to the number of frequency samples in the passband.

$N = 15$. In the table, the bandwidth refers to the number of frequency samples in the passband of the filter.

In most cases, the values of the transition band frequency samples normally lie in the following ranges: for one transition frequency sample,

$$0.250 < T_1 < 0.450$$

for two transition frequency samples,

$$0.040 < T_1 < 0.150$$

$$0.450 < T_2 < 0.650$$

for three transition frequency samples,

$$0.003 < T_1 < 0.035$$

$$0.100 < T_2 < 0.300$$

$$0.550 < T_3 < 0.750$$

The lower values are for filters with wide bandwidth and lead to more stopband attenuation.

Example 7.10

(1) A linear phase 15-point FIR filter is characterized by the following frequency samples:

$$|H(k)| = 1 \quad k = 0, 1, 2, 3$$
$$0 \quad k = 4, 5, 6, 7$$

Assuming a sampling frequency of 2 kHz, obtain its frequency response.

(2) Compare the frequency response of the filter if (a) one transition band frequency sample is used, (b) two transition band frequency samples are used, and (c) three transition band frequency samples are used.

Solution

(1) With the frequency samples as input to the design program fresamp.c (see the appendix), the coefficients of the filter are given in column 2 of Table 7.12. The corresponding frequency response is given in Figure 7.20(a).

(2) For case (a) the value of the transition band frequency sample, from Table 7.11, is 0.4041. Thus, the frequency samples for the filter are:

$$|H(k)| = 1 \quad k = 0, 1, 2, 3$$
$$0.404\,06 \quad k = 4$$
$$0 \quad k = 5, 6, 7$$

With these frequency samples as input to the design program, the coefficients of the filter were computed and are summarized in Table 7.12. The corresponding frequency response is given in Figure 7.20(b).

Table 7.12 Nonrecursive filter coefficients for various transition bands frequency samples.

	No transition samples	*One transition sample*	*Two transition samples*	*Three transition samples*
h[0] =	−4.9815884e−02	−1.3766696e−02	−5.7195305e−03	−4.2282741e−03
h[1] =	4.1202267e−02	−2.3832554e−03	−7.6781827e−03	−7.6031627e−03
h[2] =	6.6666666e−02	3.9729333e−02	2.3920000e−02	1.8793332e−02
h[3] =	−3.6487877e−02	1.2729081e−02	2.5763613e−02	2.8145113e−02
h[4] =	−1.0786893e−01	−9.1220745e−02	−7.3701817e−02	−6.6396840e−02
h[5] =	3.4078020e−02	−1.8619356e−02	−4.4185450e−02	−5.2511978e−02
h[6] =	3.1889241e−01	3.1326097e−01	3.0552137e−01	3.0183514e−01
h[7] =	4.6666667e−01	5.2054133e−01	5.5216000e−01	5.6393334e−01

Because of symmetry, only the first half of the coefficients are listed here.

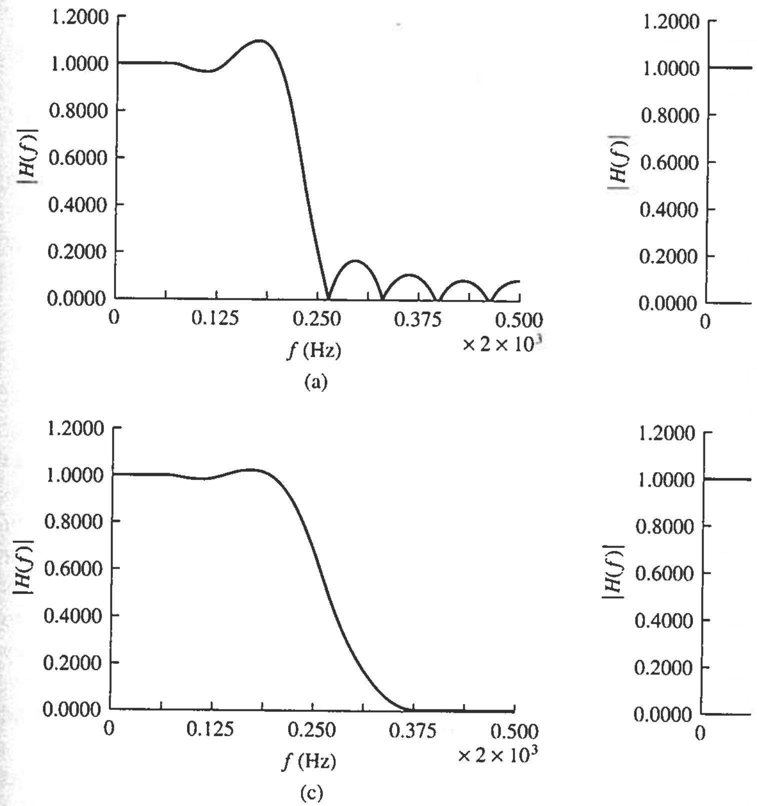


Figure 7.20 Frequency response of frequency sampling filter with (a) no tra transition band frequency sample; (c) two transition band freque band frequency samples.

For cases (b) and (c), the frequency sam

$$|H(k)| = 1 \quad k = 0, 1, 2, 3$$
$$0.5571 \quad k = 4$$
$$0.0841 \quad k = 5$$
$$0 \quad k = 6, 7$$
$$|H(k)| = 1 \quad k = 0, 1, 2, 3$$
$$0.6018 \quad k = 4$$
$$0.1239 \quad k = 5$$
$$0.0038 \quad k = 6$$
$$0 \quad k = 7$$

e 15-point FIR filter is characterized by the following frequency

= 1 $k = 0, 1, 2, 3$

0 $k = 4, 5, 6, 7$

ampling frequency of 2 kHz, obtain its frequency response.

frequency response of the filter if (a) one transition band
nple is used, (b) two transition band frequency samples are used,
transition band frequency samples are used.

uency samples as input to the design program fresamp.c (see the
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= 1 $k = 0, 1, 2, 3$

0.404 06 $k = 4$

0 $k = 5, 6, 7$

equency samples as input to the design program, the coefficients
ere computed and are summarized in Table 7.12. The
g frequency response is given in Figure 7.20(b).

ursive filter coefficients for various transition bands frequency samples.

ition es	One transition sample	Two transition samples	Three transition samples
4e-02	−1.3766696e−02	−5.7195305e−03	−4.2282741e−03
7e-02	−2.3832554e−03	−7.6781827e−03	−7.6031627e−03
6e-02	3.9729333e−02	2.3920000e−02	1.8793332e−02
7e-02	1.2729081e−02	2.5763613e−02	2.8145113e−02
3e-01	−9.1220745e−02	−7.3701817e−02	−6.6396840e−02
0e-02	−1.8619356e−02	−4.4185450e−02	−5.2511978e−02
1e-01	3.1326097e−01	3.0552137e−01	3.0183514e−01
7e-01	5.2054133e−01	5.5216000e−01	5.6393334e−01

y, only the first half of the coefficients are listed here.

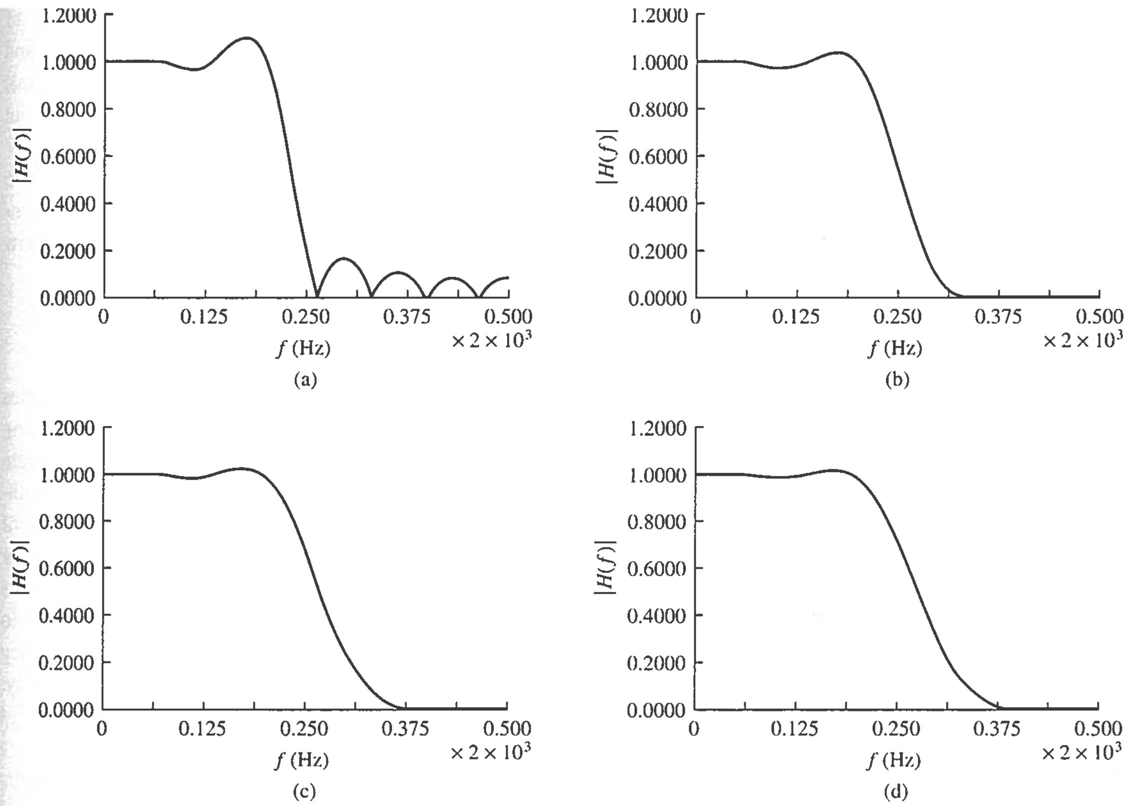


Figure 7.20 Frequency response of frequency sampling filter with (a) no transition band frequency samples; (b) one transition band frequency sample; (c) two transition band frequency samples and (d) three transition band frequency samples.

For cases (b) and (c), the frequency samples are defined respectively as

$$|H(k)| = \begin{cases} 1 & k = 0, 1, 2, 3 \\ 0.5571 & k = 4 \\ 0.0841 & k = 5 \\ 0 & k = 6, 7 \end{cases}$$

$$|H(k)| = \begin{cases} 1 & k = 0, 1, 2, 3 \\ 0.6018 & k = 4 \\ 0.1239 & k = 5 \\ 0.0038 & k = 6 \\ 0 & k = 7 \end{cases}$$

The coefficients for these cases are summarized in the fourth and fifth columns of Table 7.12. The corresponding frequency responses are given in Figures 7.20(c) and 7.20(d). It is seen that, as the number of transition band frequency samples increases, the amplitude response (in terms of the passband and stopband ripples) improves, but at the expense of increasing transition width or roll-off.

An alternative approach that may be used to improve the amplitude response is to obtain a large number of frequency samples, by sampling at closer intervals, to compute the impulse response using Equation 7.21, and then to apply one of the window functions discussed earlier to reduce the filter to the desired length.

7.7.1.2 Automatic design of frequency sampling filters

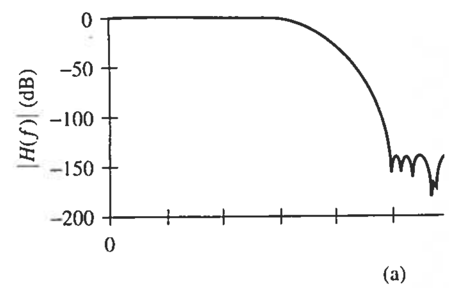
As stated before, tables of optimum values of the transition band frequency samples are available in the literature (Rabiner *et al.*, 1970) and are widely used for designing frequency sampling filters. If the designer wants a filter not tabulated, approximate values of the transition band frequency samples may be obtained by linear interpolation, but this is not always possible especially if the design involves a large number of transition band samples. Further, the information in the tables is not in a form filter designers are familiar with; for example, bandedge frequencies and passband ripples are not given. A general purpose computer program has recently been developed to automate many aspects of the design of nonrecursive and recursive frequency sampling filters (Ifeachor and Harris, 1993; Harris and Ifeachor, 1998). Essentially, in the program the values of the transition band samples are optimized by a hybrid genetic algorithm (GA) approach to give maximum attenuation in the stopband for a given set of filter specifications. The approach has been tested against tabulated results in the literature and was found to equal or improve on them in every case. It also allows filters not tabulated to be designed.

Example 7.11 Find the optimum transition band frequency samples and the corresponding filter coefficients for a lowpass filter meeting the following specifications:

passband edge frequency	0.143 (normalized)
stopband edge frequency	0.245 (normalized)
number of filter coefficients	49

Solution From the specifications, the number of frequency samples, $N = 49$. The sample numbers corresponding to the passband and stopband edge frequencies are 6 and 12, respectively. The number of transition band samples, $M = 5$. Thus the frequency samples for the ideal magnitude–frequency response are given by:

$$
\begin{aligned}
|H(k)| &= 1, \qquad && k = 0, 1, \ldots, 6 \\
&T_{k-6}, && k = 7, \ldots, 11 \\
&0 && k = 12, \ldots, 24
\end{aligned}
$$



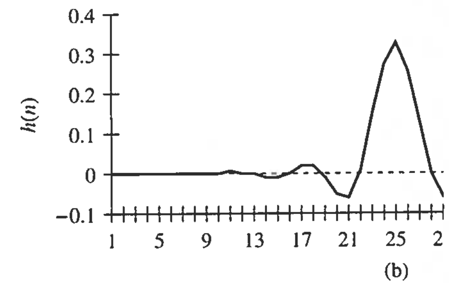


Figure 7.21 (a) The interpolated frequency response; Passband ripple: 0.046 dB; stopband atte passband width: 0.15; five transition sam Transition sample values: 0.855 456, 0.4 0.000 644.

The values of T_1 to T_5 are unspecified and are hybrid GA program. The results of the optimizati ure 7.21.

Although the hybrid GA approach produces res in the literature, its main strength lies in the fact cients for filters that are not tabulated which are mu interpolation. It is also able to design filters with r

7.7.2 Recursive frequency sampling filters

Recursive forms of the frequency sampling filt advantages over the nonrecursive forms if a large zero valued. It can be shown (see Example 7.12) filter, $H(z)$, can be expressed in a recursive form:

$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi k/N} z^{-1}} = H_1(z)H$$

or these cases are summarized in the fourth and fifth columns of orresponding frequency responses are given in Figures 7.20(c) and that, as the number of transition band frequency samples increases, onse (in terms of the passband and stopband ripples) improves, but increasing transition width or roll-off.

approach that may be used to improve the amplitude response is to nber of frequency samples, by sampling at closer intervals, to com- esponse using Equation 7.21, and then to apply one of the window d earlier to reduce the filter to the desired length.

n of frequency sampling filters

tables of optimum values of the transition band frequency samples e literature (Rabiner *et al.*, 1970) and are widely used for designing ng filters. If the designer wants a filter not tabulated, approximate sition band frequency samples may be obtained by linear interpola- t always possible especially if the design involves a large number of mples. Further, the information in the tables is not in a form filter iliar with; for example, bandedge frequencies and passband ripples general purpose computer program has recently been developed to aspects of the design of nonrecursive and recursive frequency Ifeachor and Harris, 1993; Harris and Ifeachor, 1998). Essentially, e values of the transition band samples are optimized by a hybrid (GA) approach to give maximum attenuation in the stopband for a specifications. The approach has been tested against tabulated results nd was found to equal or improve on them in every case. It also abulated to be designed.

n transition band frequency samples and the corresponding filter lowpass filter meeting the following specifications:

lge frequency	0.143 (normalized)
lge frequency	0.245 (normalized)
ilter coefficients	49

cations, the number of frequency samples, $N = 49$. The sample nding to the passband and stopband edge frequencies are 6 and 12, number of transition band samples, $M = 5$. Thus the frequency leal magnitude–frequency response are given by:

$k = 0, 1, \ldots, 6$

-6, $k = 7, \ldots, 11$

$k = 12, \ldots, 24$

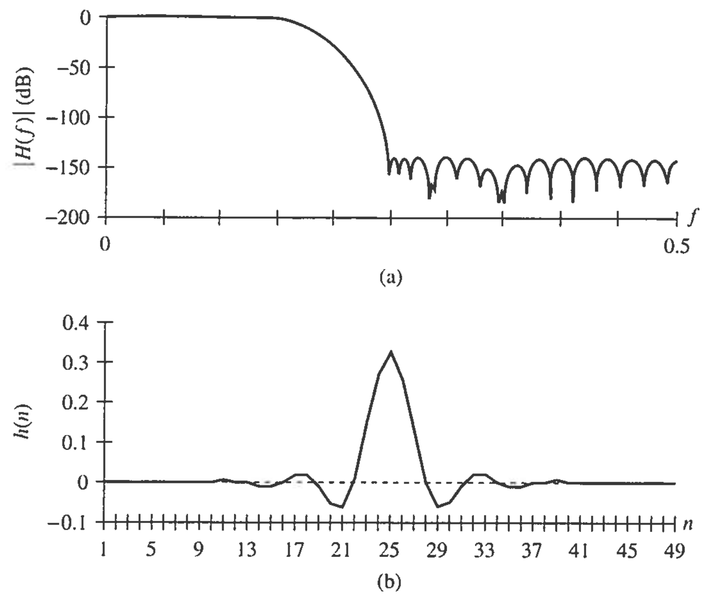


Figure 7.21 (a) The interpolated frequency response; (b) the filter coefficients. Passband ripple: 0.046 dB; stopband attenuation: 139.64 dB; passband width: 0.15; five transition samples; 49 filter coefficients. Transition sample values: 0.855 456, 0.485 507, 0.148 961, 0.019 693, 0.000 644.

The values of T_1 to T_5 are unspecified and are found by optimization using the hybrid GA program. The results of the optimization process are summarized in Figure 7.21.

Although the hybrid GA approach produces results that improve slightly on those in the literature, its main strength lies in the fact that it can quickly produce coefficients for filters that are not tabulated which are much more useful than those found by interpolation. It is also able to design filters with more transition samples.

7.7.2 Recursive frequency sampling filters

Recursive forms of the frequency sampling filter offer significant computational advantages over the nonrecursive forms if a large number of frequency samples are zero valued. It can be shown (see Example 7.12) that the transfer function of an FIR filter, $H(z)$, can be expressed in a recursive form:

$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{j2\pi k/N} z^{-1}} = H_1(z)H_2(z) \tag{7.23}$$