① Exercise 1

$$D(s) = K\frac{(s+2)}{(s+0.5)} = \frac{U(s)}{E(s)}$$

We cross multiply

$$(s+0.5)U(s) = K(s+2)E(s)$$

$$s\cdot U(s) + 0.5\,U(s) = K\cdot E(s)\cdot s + K\cdot 2E(s)$$

We find the differential equation

$$\dot{u}(t) + 0.5\,u(t) = K\dot{e}(t) + 2Ke(t)$$

We use Eulers method for approximation

$$\dot{x}(t) \simeq \frac{x(k+1) - x(k)}{T}$$

$$\frac{u(k+1) - u(k)}{T} + 0.5\,u(k) = K\cdot\frac{e(k+1) - e(k)}{T} + K\cdot 2\,e(k)$$

$$u(k+1) - u(k) + 0.5\cdot u(k)\cdot T = K(e(k+1) - e(k) + TK\cdot 2\,e(k)$$

$$u(k+1) = (1 - 0.5T)u(k) + K(2T-1)e(k) + K\,e(k+1)$$

(2)

Pseudo code

```
X = 0        ; initialization
Define constants
a = 1 - 0.5T
b = K(2T - 1)
Read AD converter to obtain y and r
e = r - y
u = au + be
go back to read when time = T seconds from
    last reading
```

2.

Design a lead controller

$$D(s) = k\frac{s+a}{s+b}$$

For the plant

$$G(s) = \frac{1}{s(s+1)}$$

Assume unity feed back

Demands: Wc = 2 and the phase margin > 45

Find digital controllers to implement the lead controller using the sample rates of 10 Hz, 20 Hz and 40 Hz

Try tustin and zoh

Plot the step response for the continuous system and the digital systems

--- solution -

D(s) is designed in the bodeplot

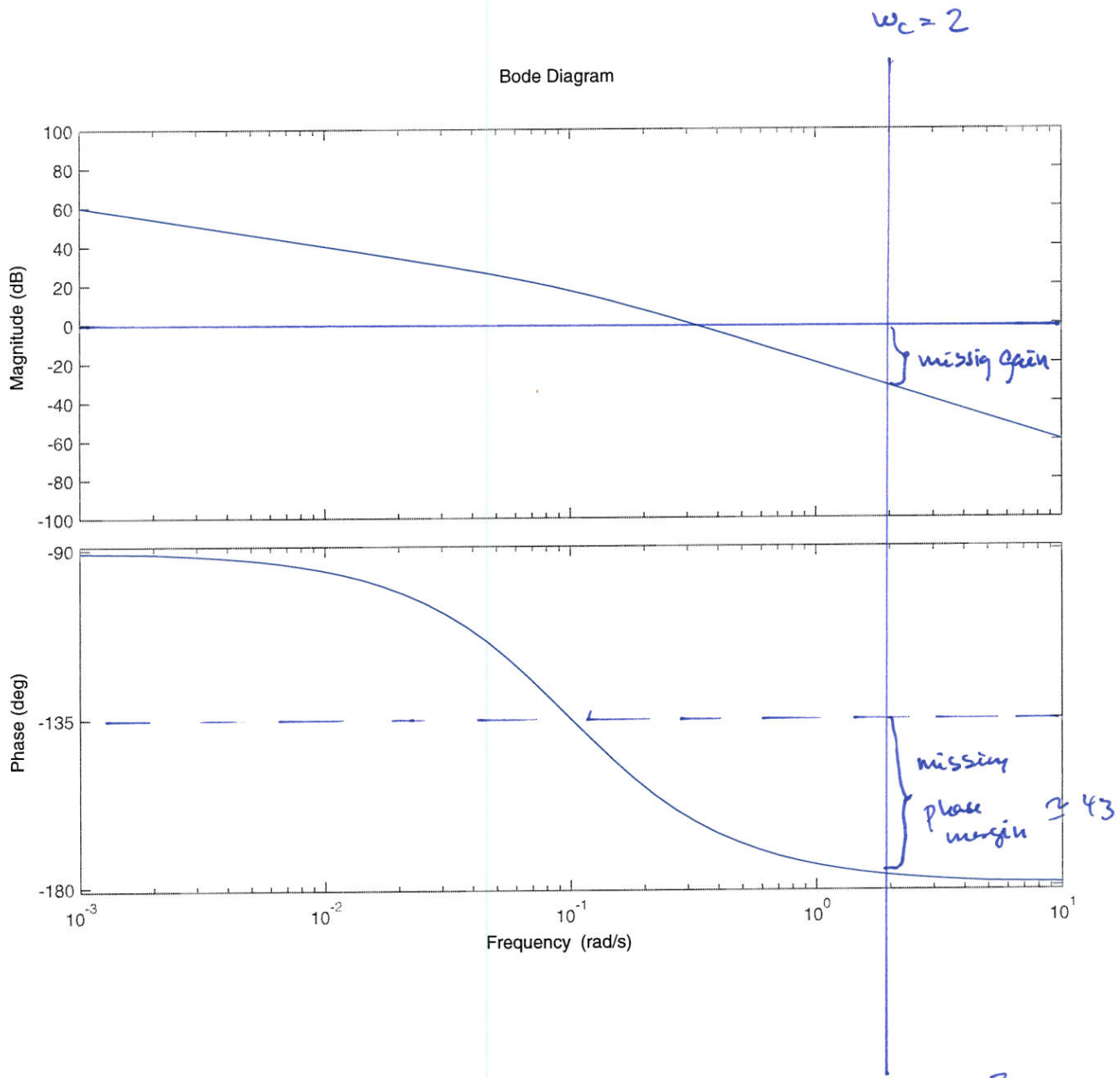
$$D(s) = 100\frac{s+0.6}{s+6}$$

Matlab for investigation

```
s=tf('s')
G=0.1/(s*(s+0.1));
H=1
D=100*(s+0.6)/(s+6)
T=feedback(D*G,H);
Td=c2d(T,0.2,'tustin');
step(T,'-',Td,'--')
```

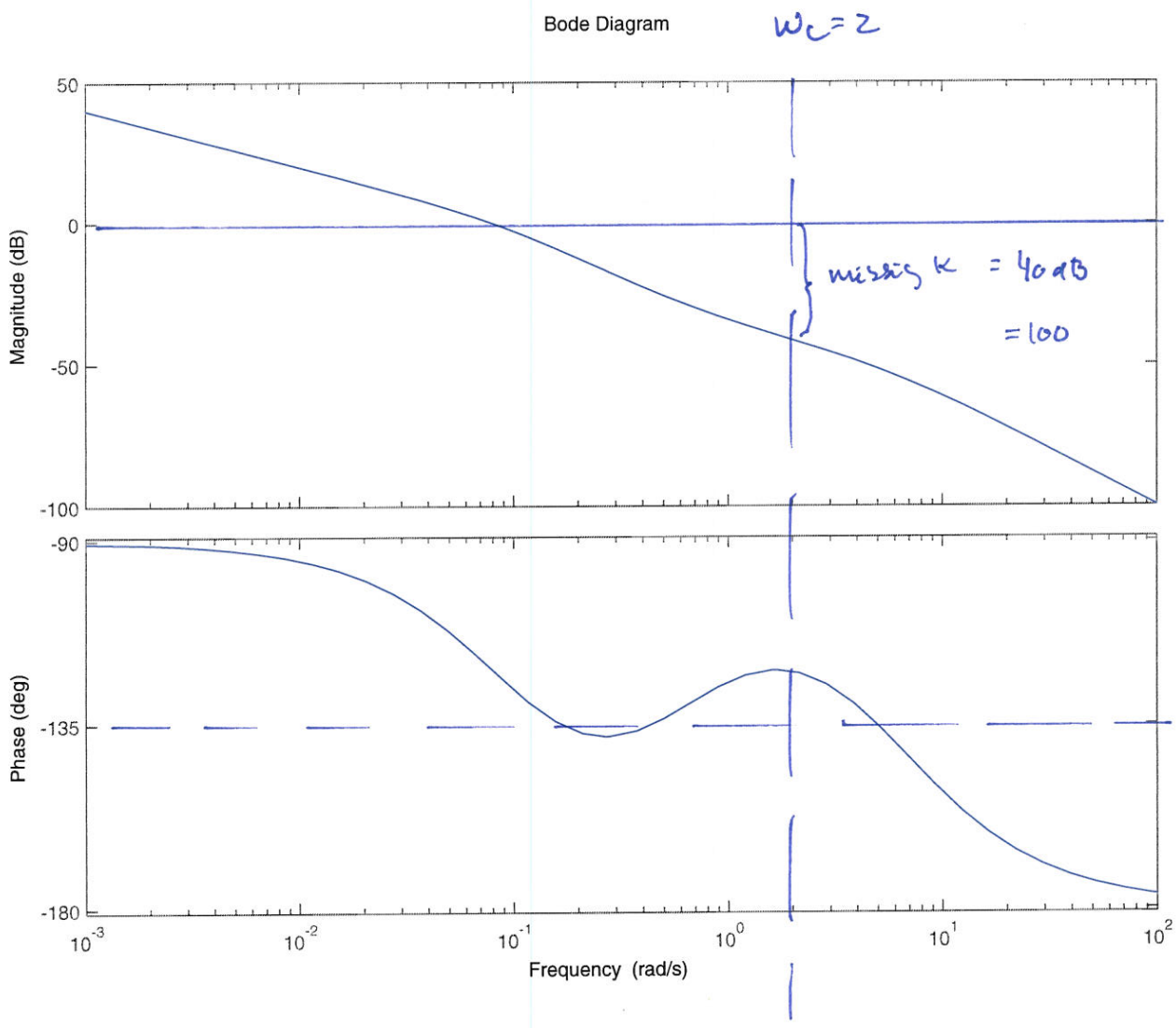
System $= G(s) = \dfrac{0.1}{s(s+0.1)}$

Bode plot



We implement a lead controller centered in 2

ex. $\dfrac{s+0.6}{s+6.}$



$$D(s) = 100 \cdot \frac{s + 0.6}{s + 6}$$