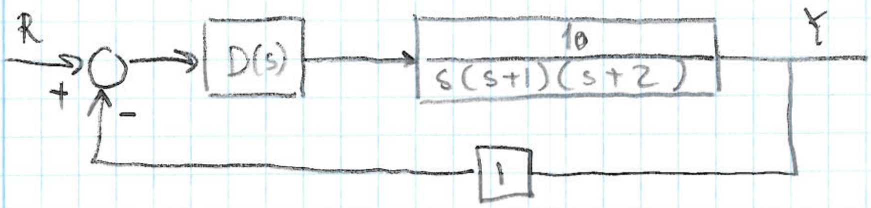
Lead-Lag Control

unity feedback system

$$G(s) = \frac{10}{s(s+1)(s+2)}$$



First we will ensure the dynamics of the system ~ phase and Gain margin.

We make a bodeplot for $G(s)H(s) = G(s)\cdot 1 = G(s)$

It is done by hand and by Matlab on the next page.

The phase margin and gain margin are found in the plot

phase margin$(G(s))$ - find the point when the magnitude is 0 dB - determin the phase

aprox $180 - 195 = -15$ (the handmade plot)

aprox $180 - 200 = -20$ (the Matlab mad plot)

Gain margin : find the point where the phase $= -180°$

determin the Gain -

aprox -10 dB (handmade)

aprox -15 dB (matlab)

Bodeplot for $G(s) = \frac{10}{s(s+1)(s+2)}$

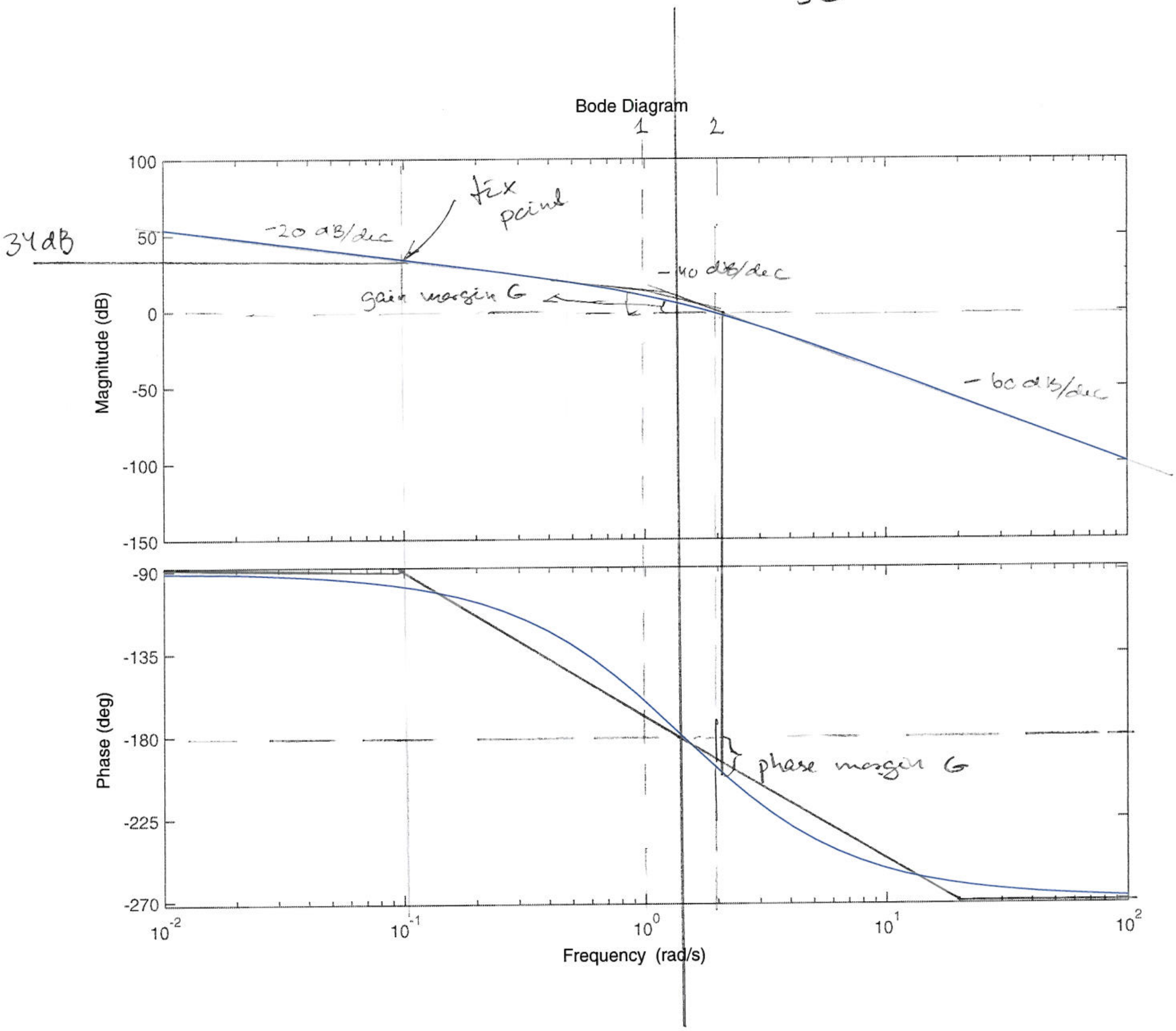
by Matlab and by hand

to fix the handmade bodeplot a point is needed

we choose $\omega = 0.1$ - a decade from a pole

$$|G(j\cdot 0.1)| = \left|\frac{10}{\sqrt{0.1^2}\sqrt{0.1^2+1^2}\sqrt{0.1^2+2^2}}\right| \approx \left|\frac{10}{0.1\cdot 1\cdot 2}\right| = \left|\frac{10}{0.2}\right| = 50$$

$50 = 34$ dB



phase margin G (handmade) ~ $-15°$

phase margin G (Matlab) ~ $-20°$

Gain margin G (hand made) ~ 15 dB

—"— G (Matlab) ~ 10 dB

Demand for the controller

phase margin $= 45°$

Gain margin ≥ 10 dB

To obtain this we need (worst case)

phase : $45 + 20 = 65°$ (Matlab)

Gain : $10\text{dB} + 15\text{dB} = 25\text{dB}$ (handmade)

A PD controller or a lead controller can give more phase

We will use a lead compensator as we don't want high gain at high frequencies

lead : $K \frac{(s+a)}{(s+b)}$ $a < b$

The frequency where we get most phase is between a and b (in the middle on a logaritmic axis)

In this case we choose to raise the phase for $\omega = 2$.

In the plots, you can see results for

$$D_{lead}(s) = \frac{s+0.2}{s+20}$$

$$\frac{s+0.5}{s+10}$$

$$\frac{s+0.8}{s+4}$$

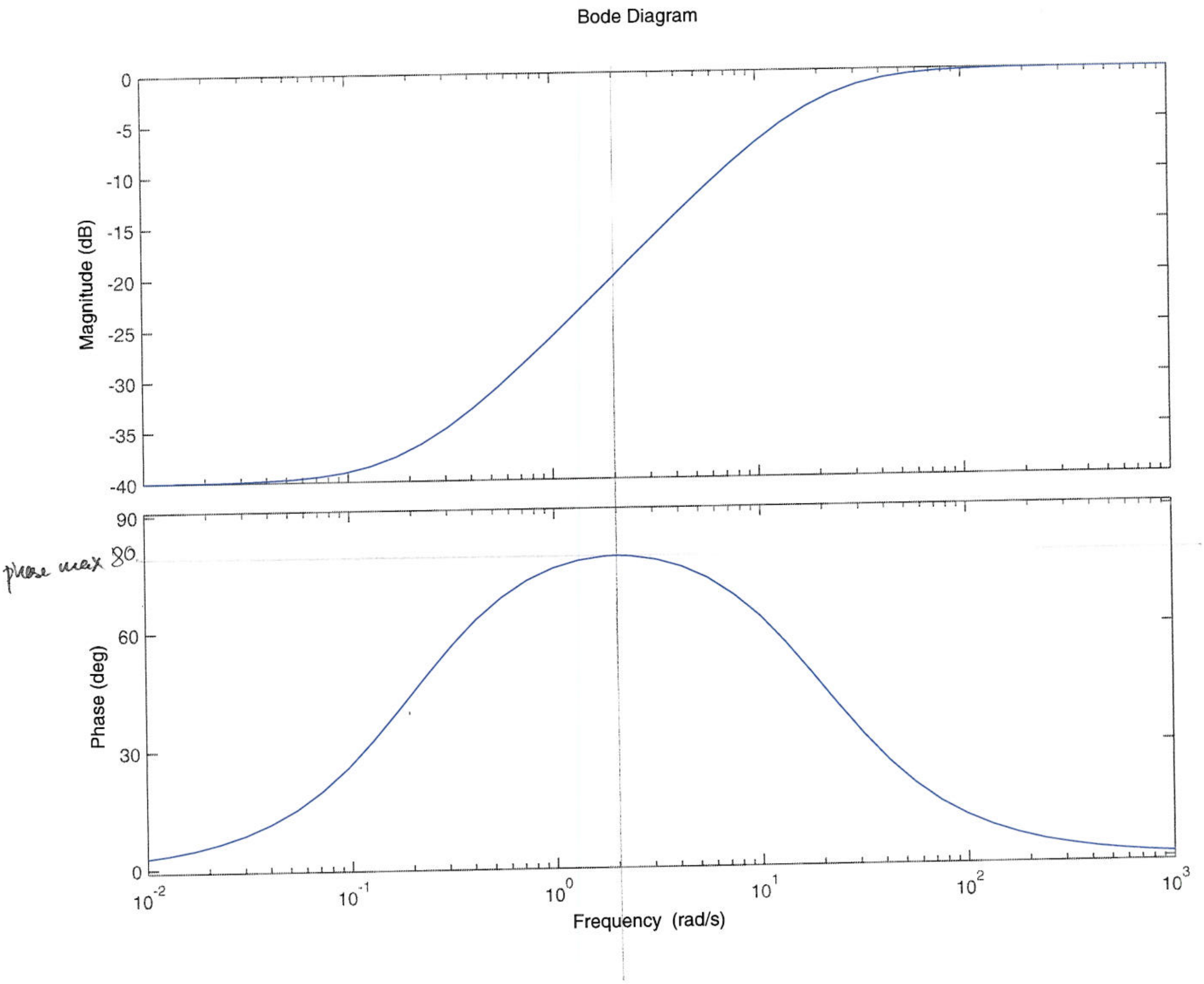
they all fullfill the gain and phase margin demands.

One way to choose between the controllers can be to make a closed loop stepresponse and choose the most desired response

In this case I choose $\frac{s+0.5}{s+10}$ it gives a fast response without overshoot.

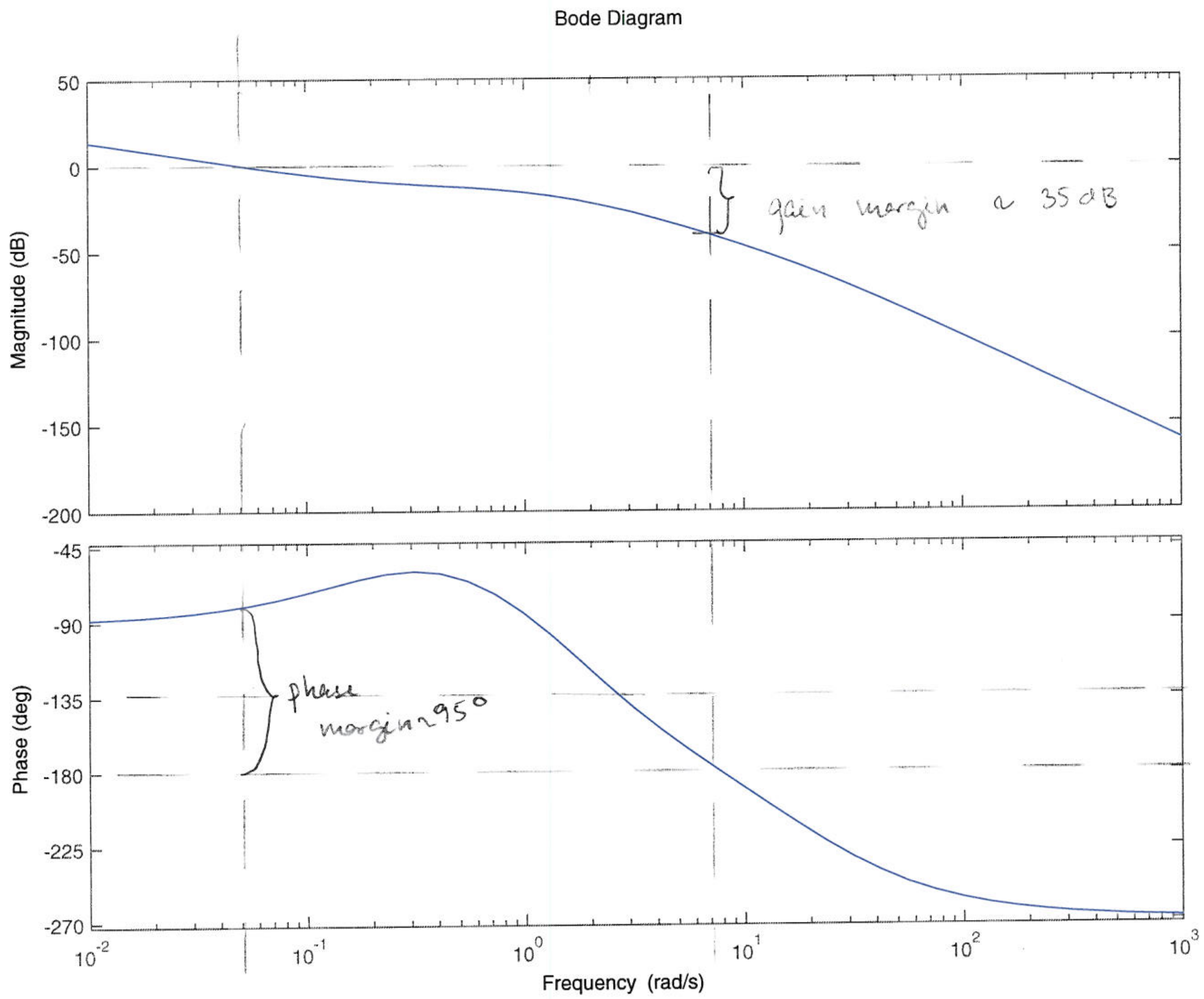
2A

$$D_{lead}(s) = \frac{s+0.2}{s+20}$$



2B

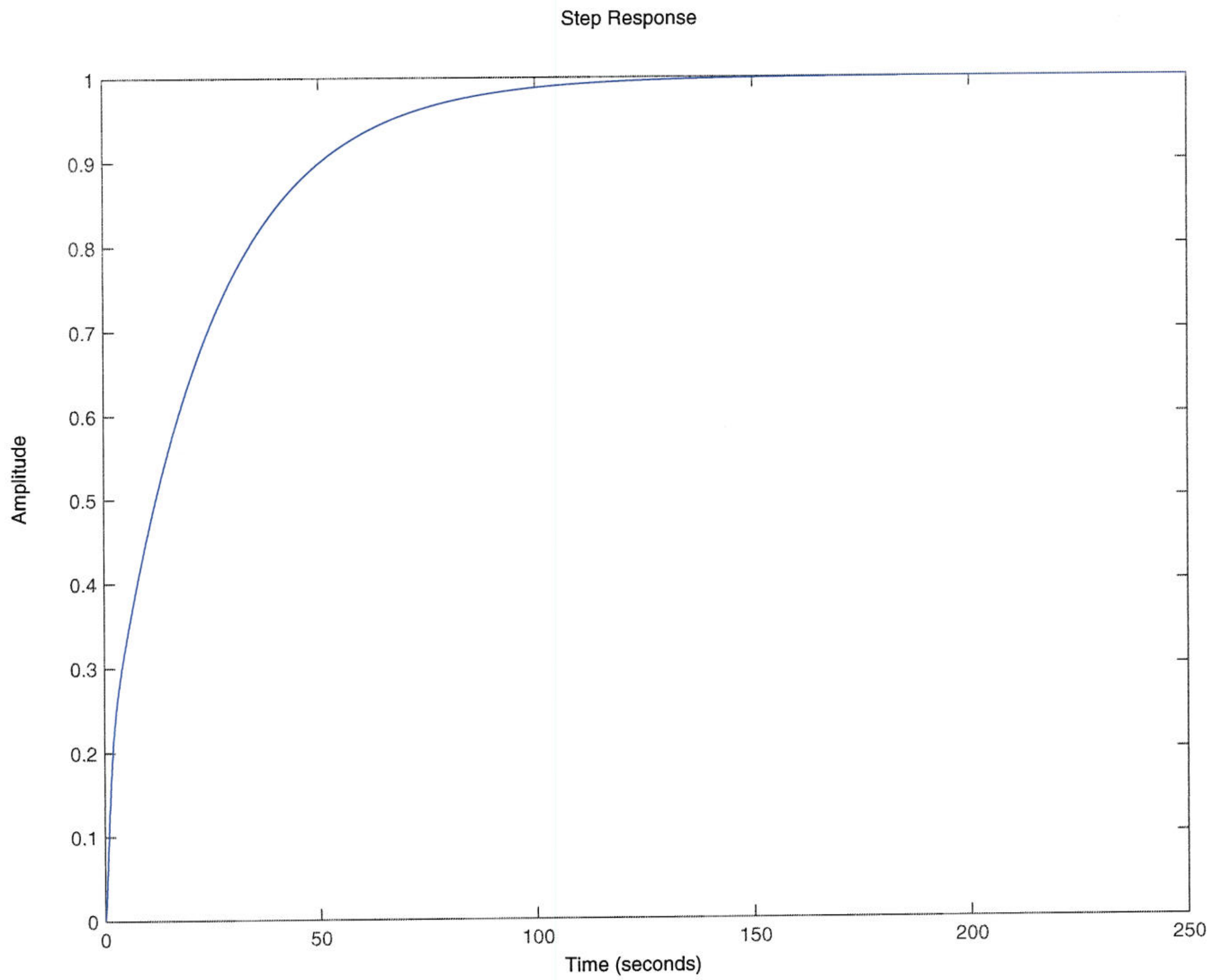
Bode $D_{lead} \cdot G = \frac{s+0.2}{s+20} \frac{16}{s(s+1)(s+2)}$



2c

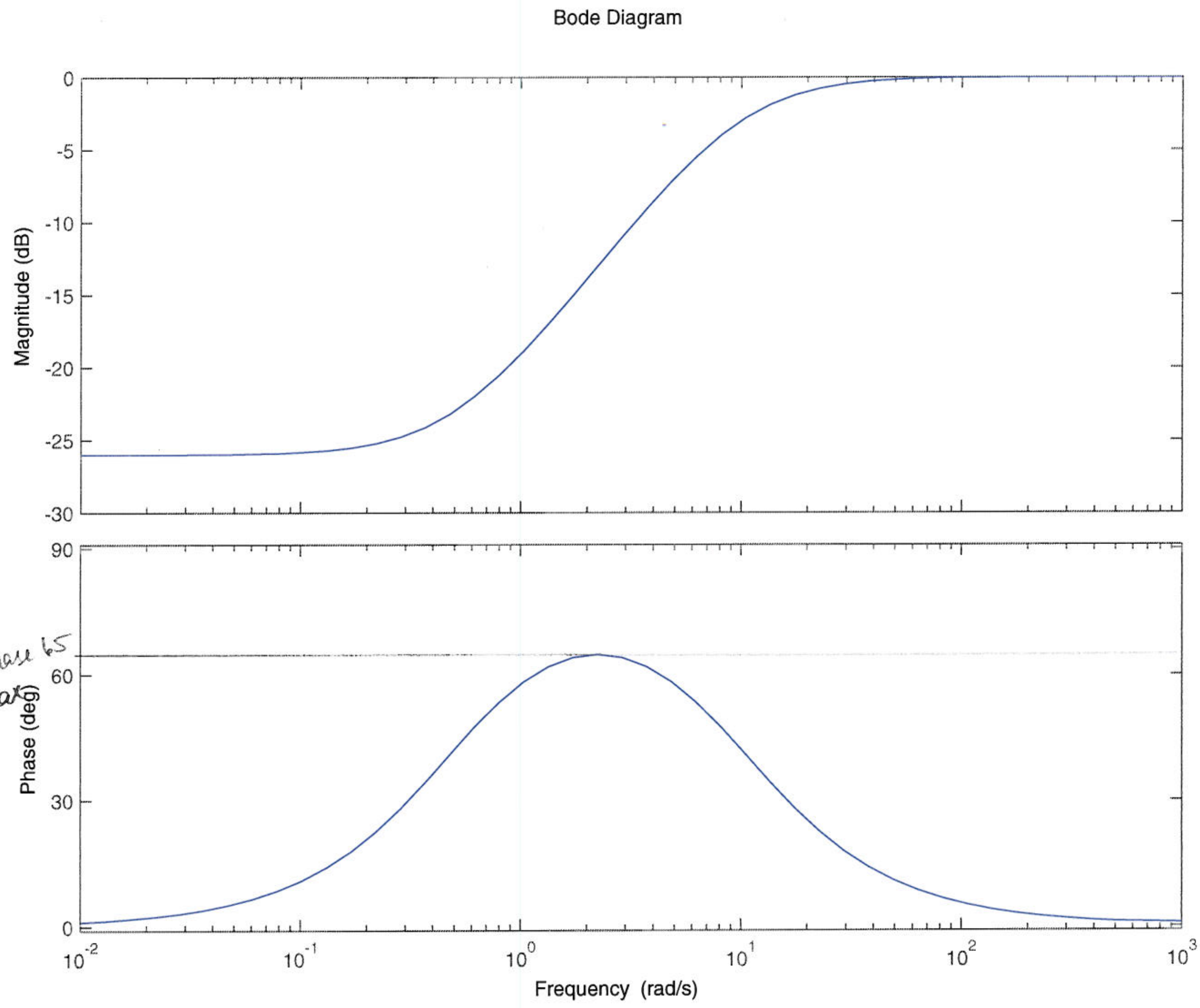
closed loop step response

$$OL: \frac{s+0.2}{s+20} \cdot \frac{10}{s(s+1)(s+2)}$$



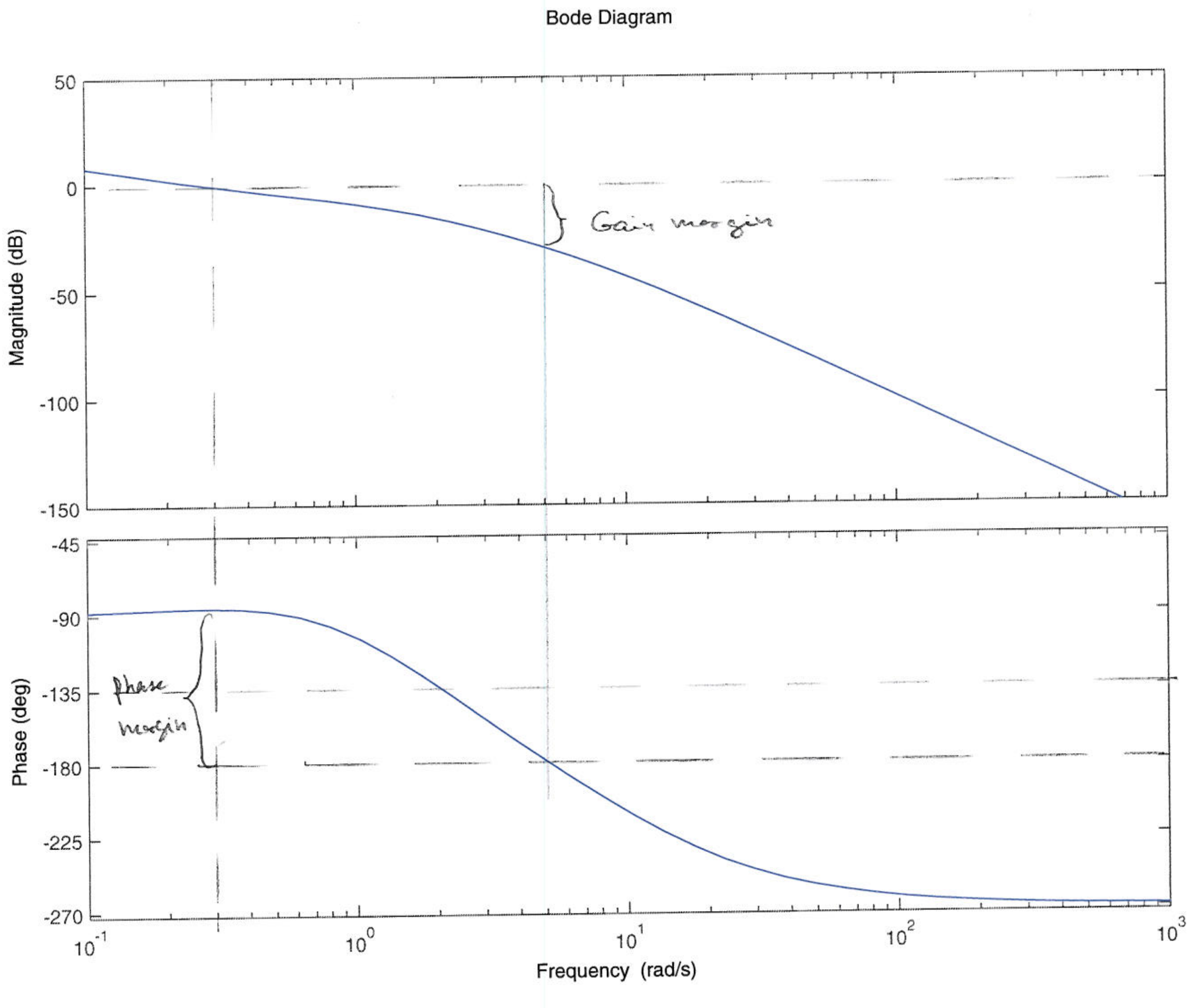
3A

$$D_{lead} = \frac{s+6.5}{s+10}$$



3B

$$D_{lead} \cdot G = \frac{s+0.5}{s+10} \; \frac{10}{s(s+1)(s+2)}$$



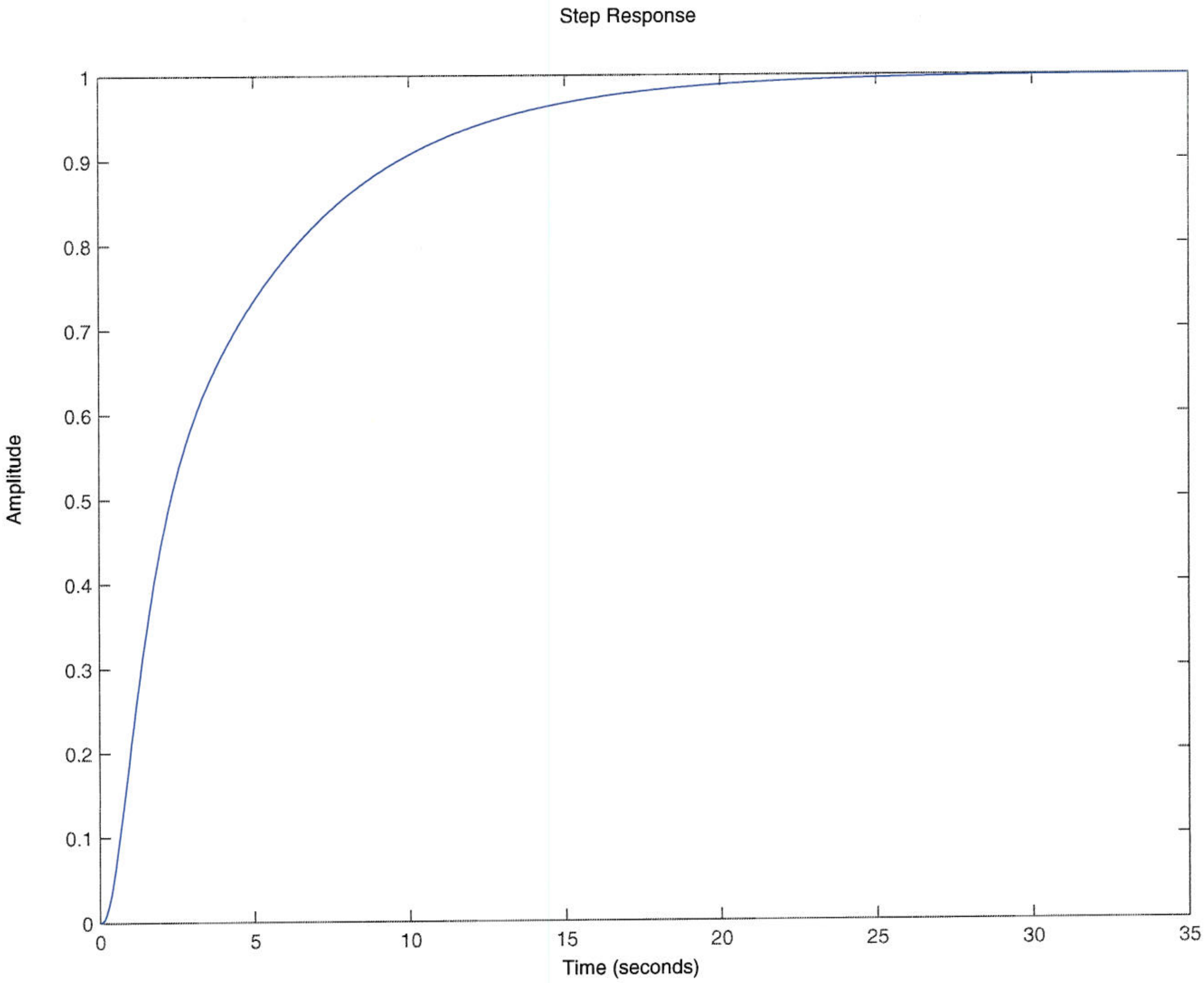
phase margin ~ 96

gain margin ~ 30 dB

3c

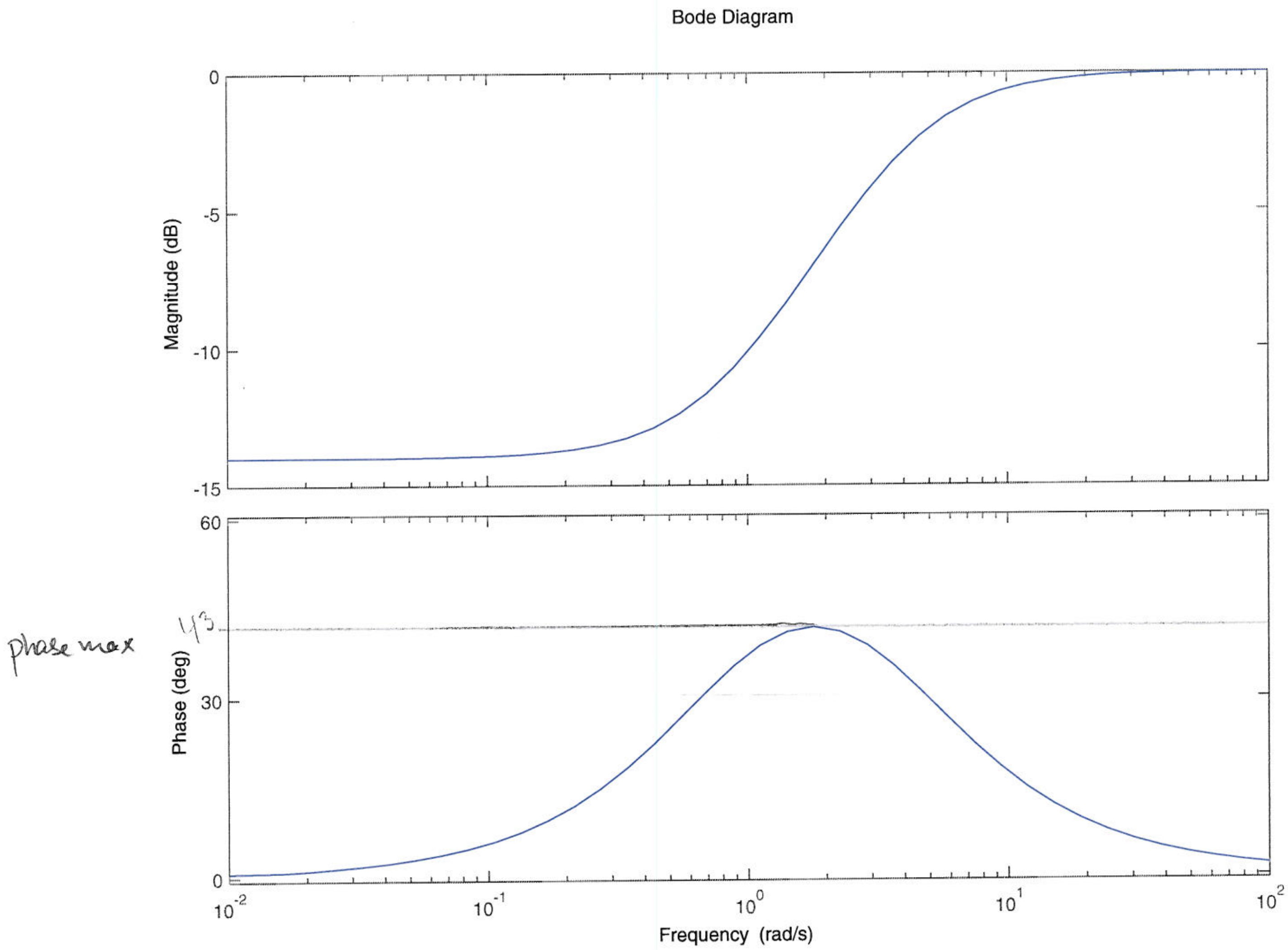
closed loop step response

$$\frac{s+0.5}{s+10} \quad \frac{10}{s(s+1)(s+2)}$$



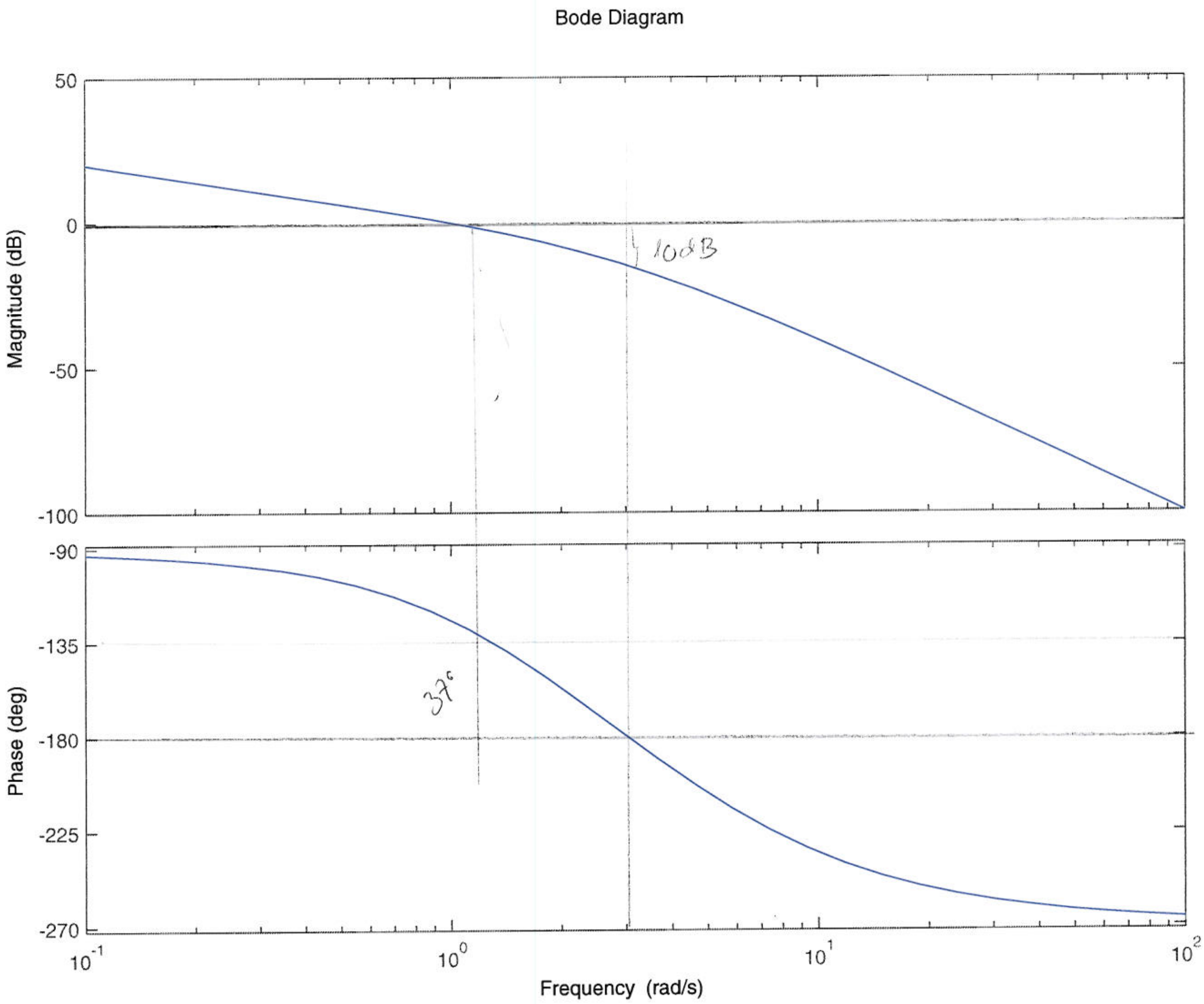
4A

$$D_{lead} = \frac{s+0.8}{s+4}$$



4B

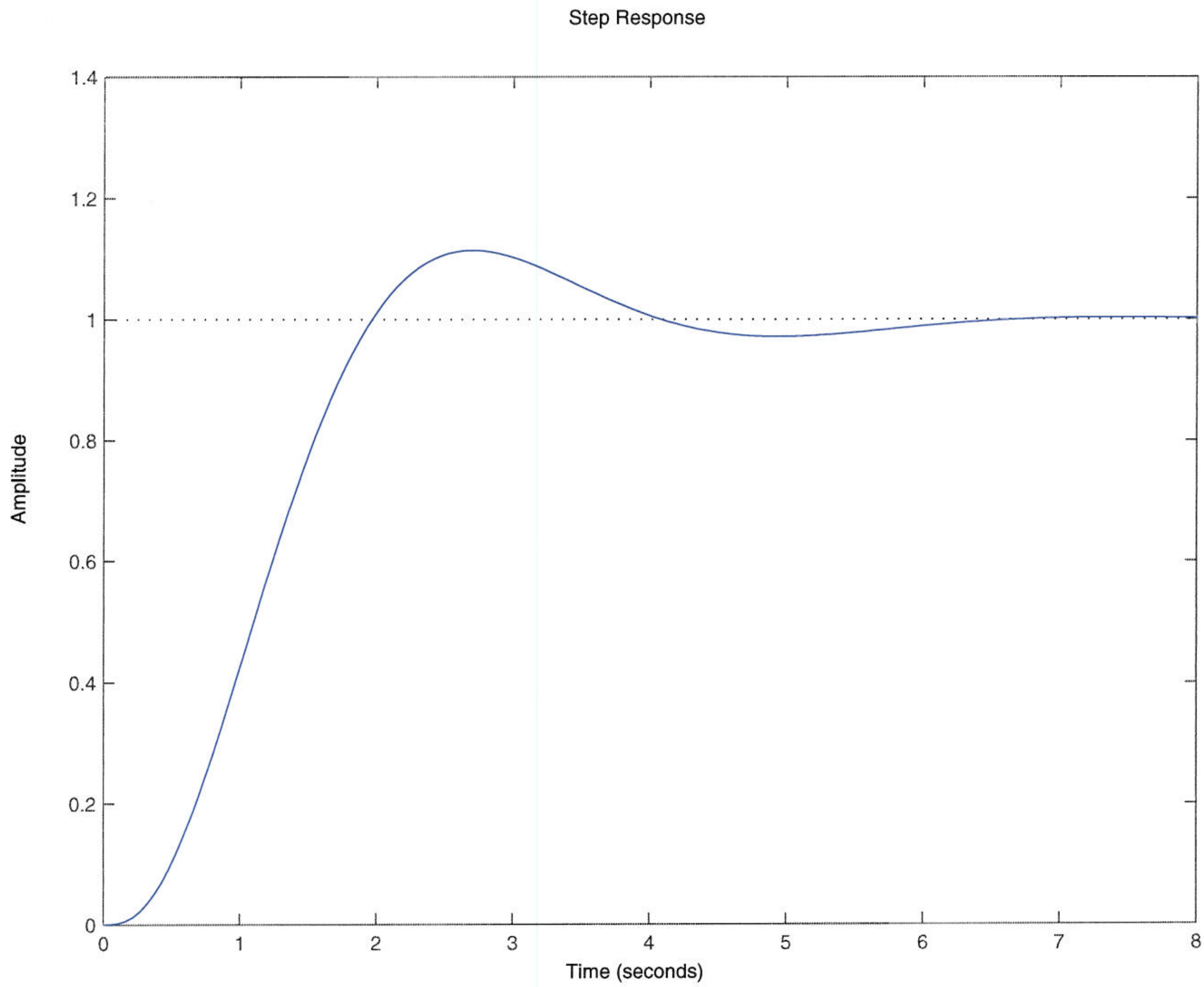
$$G \cdot D_{lead} = \frac{s+0.8}{s+4} \quad \frac{10}{s(s+1)(s+2)}$$



4c

closed loop stepresponse

$$OL: \quad \frac{s+0.8}{s+4} \quad \frac{10}{s(s+1)(s+2)}$$



Calculate K_v

$$K_v = \lim_{s \to 0} s\, D(s)\, G(s)$$

$$D_{lead} = \frac{s+0.5}{s+10}$$

$$K_{v_{lead+G}} = \lim_{s \to 0} s \frac{s+0.5}{s+10} \frac{10}{s(s+1)(s+2)}$$

$$= \lim_{s \to 0} \frac{10(s+0.5)}{(s+10)(s+1)(s+2)} = \frac{10 \cdot 0.5}{10 \cdot 1 \cdot 2} = \frac{5}{20}$$

$$= \frac{1}{4}$$

K_v can be improved by a lag controller

$$D_{lag}(s) = \frac{s+a}{s+b}$$

$$K_{v\,lag} = \lim_{s \to 0} D_{lag}(s) = \lim_{s \to 0} \frac{s+a}{s+b} = \frac{a}{b} \qquad b < a$$

$$K_{v_{total}} = K_{v_{lag}} \cdot K_{v_{lead \cdot G}} = \frac{a}{b} \cdot \frac{1}{4}$$

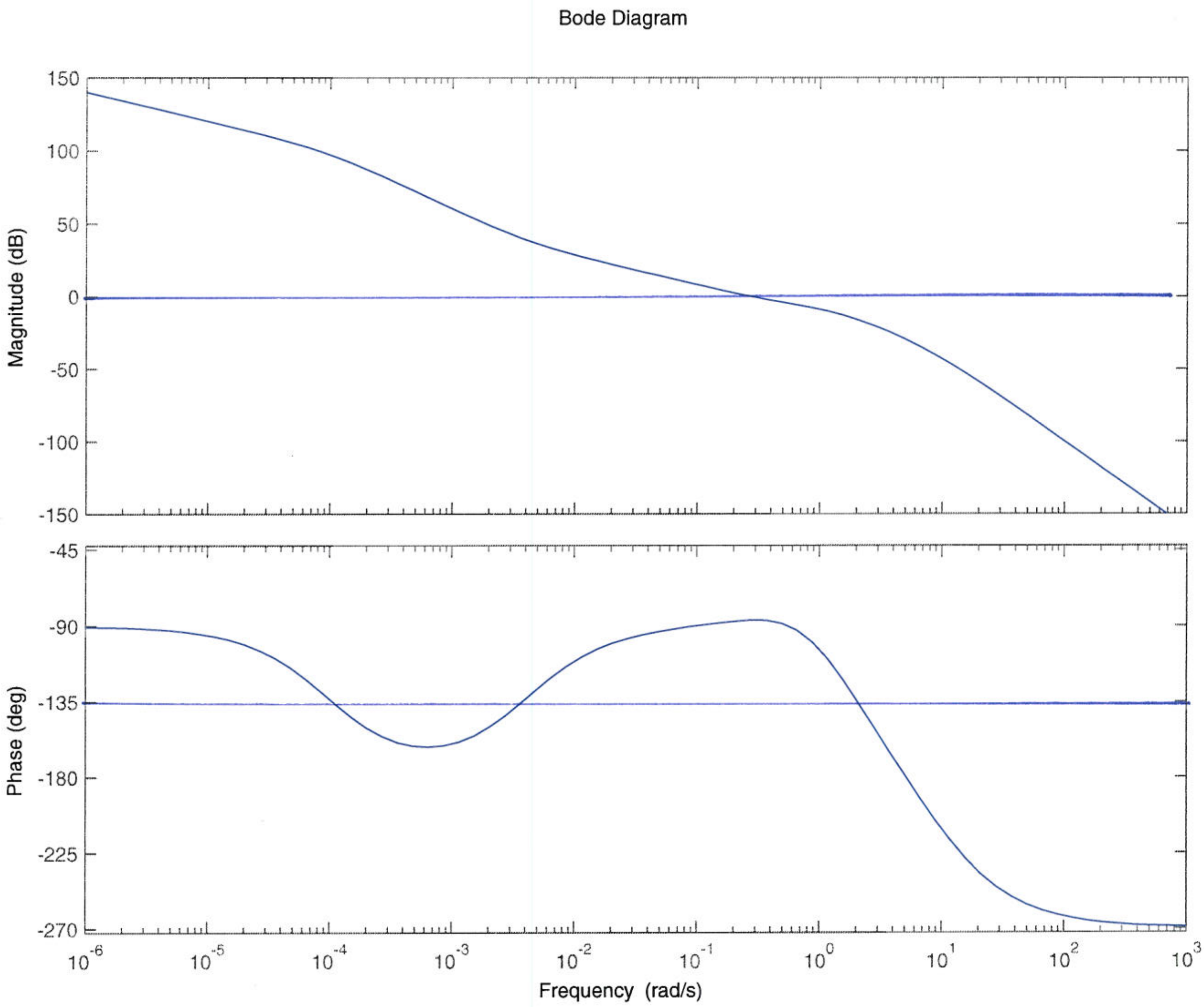
$$K_{v\,tal} = 10 \Rightarrow 10 = \frac{a}{b} \cdot \frac{1}{4} \Rightarrow \frac{a}{b} = 40 \Rightarrow a = 40 \cdot b$$

a and b must be choosen far away from the other poles and zeroes

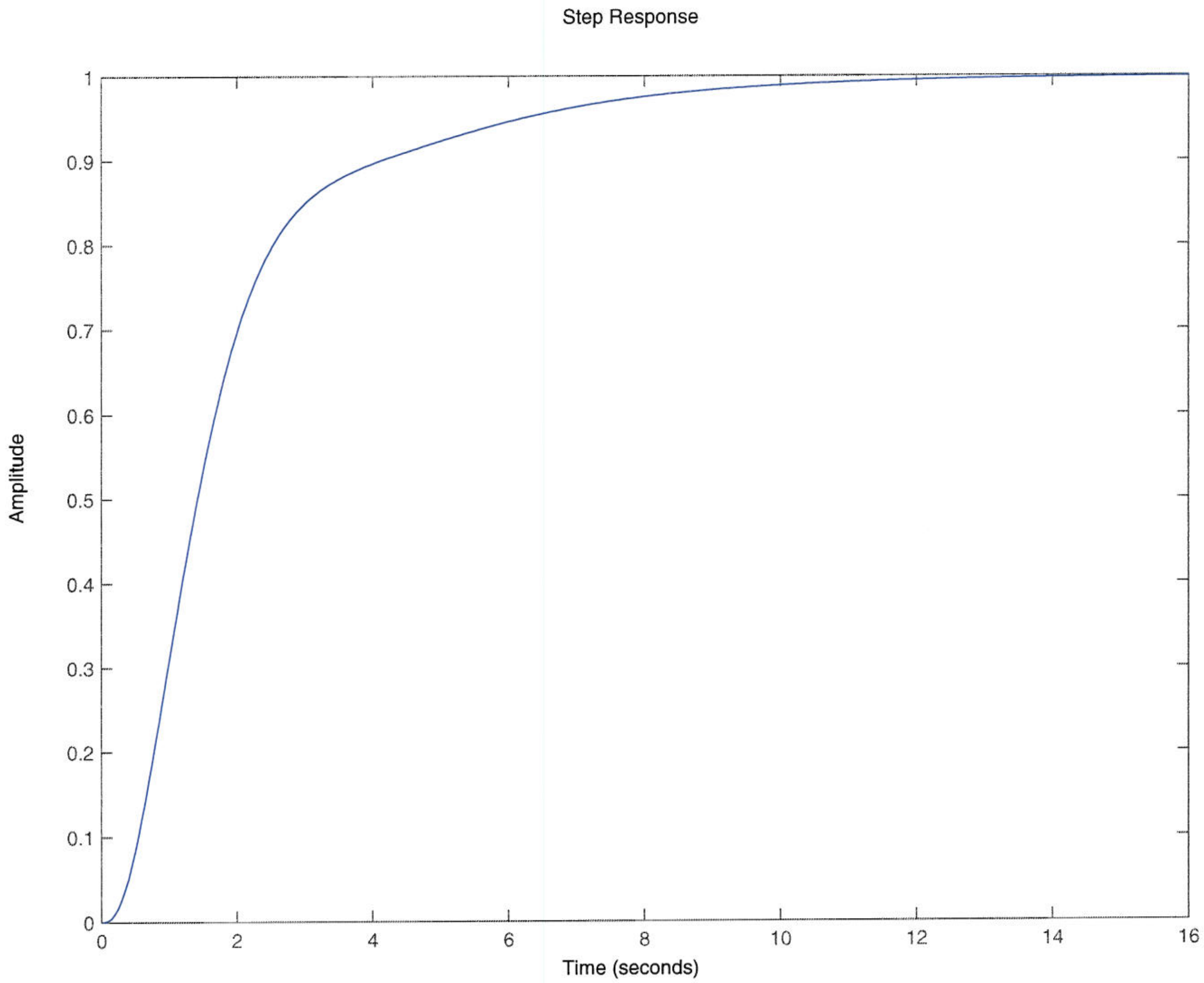
We use

$b = 0.0001 \Rightarrow$ ~~a = [illegible]~~ $a = 0.004$

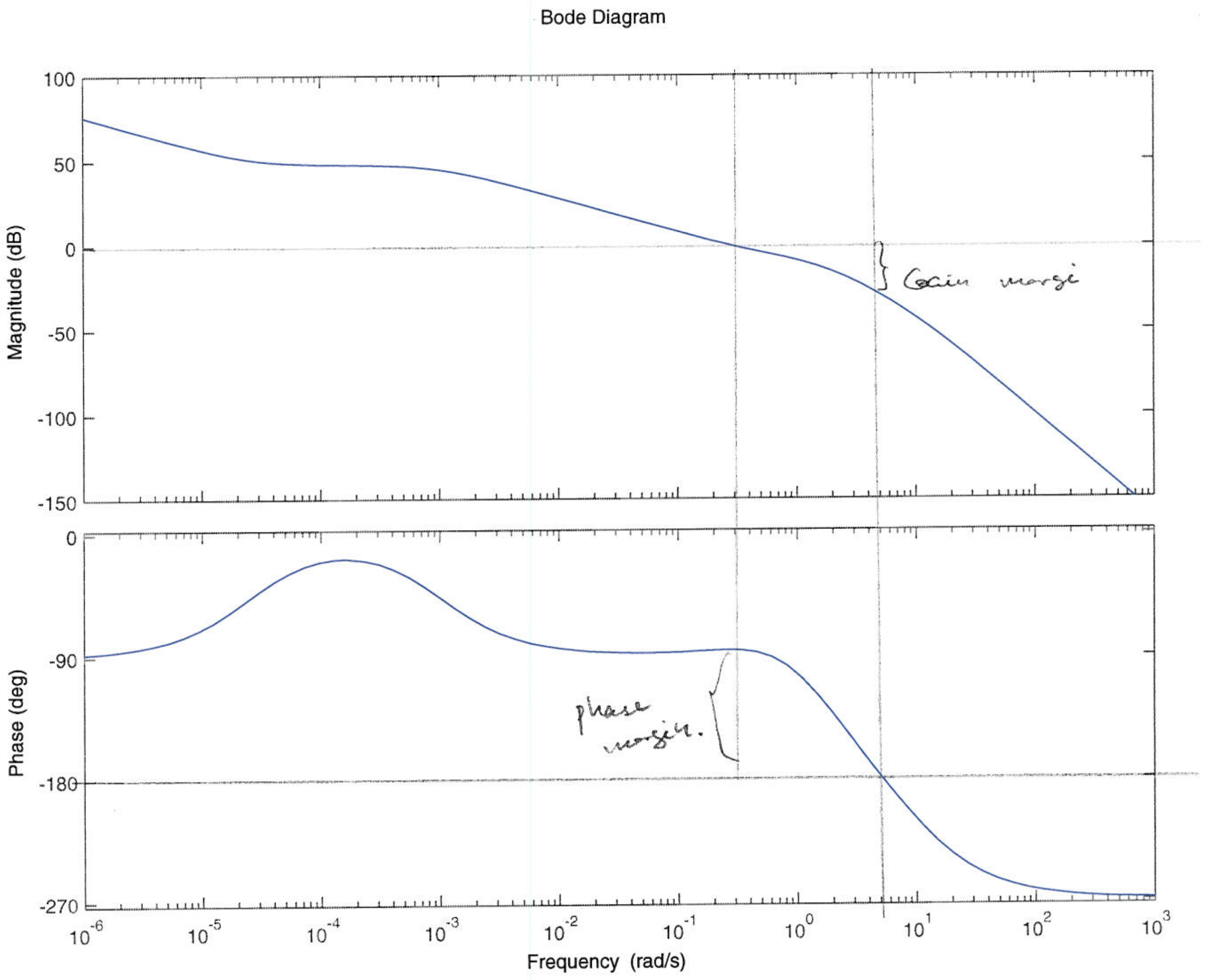
$$\frac{s+0.004}{s+0.0001} \quad \frac{s+0.5}{s+10} \quad \frac{10}{s(s+1)(s+2)}$$



$$\frac{s + 0.04}{s + 0.0001} \qquad \frac{s+0.5}{s+10} \qquad \frac{10}{s(s+1)(s+2)}$$



Lead-lag :
$$\frac{s+0.000025}{s+0.0001} \quad \frac{s+0.5}{s+10} \quad \frac{10}{s(s+1)(s+2)}$$



Lead Lag controlled system

$$\frac{s+0.00025}{s+0.0001} \quad \frac{s+0.5}{s+10} \quad \frac{10}{s(s+1)(s+2)}$$

step response - closed loop