Rouths Stability Criterion

Routh's Stability Criterion
fast check of stability

The closed loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)}$$

The characteristic equation of a system is:

$$1 + D(s)G(s)H(s) = 0$$

It can be written in polynomiaform

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} + a_n$$

Routh’s Stability Criterion

The characteristic equation of a system is:

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} + a_n$$

necessary condition for stability:

- all roots have negative real parts

Routh: (proof in the book)

A necessary and sufficient condition for stability is that all elements in the first column of the Routh array are positive.

It doesn’t indicate the stability margins

Routh’s Array

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} + a_n$$

The coefficients of the characteristic polynomial are arranged in two rows followed by calculated elements

$$\begin{array}{lllll}
s^n & 1 & a_2 & a_4 & \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & \cdots \\
 & & & \vdots & \\
\mathrm{s}^2 & * & * & & \\
\mathrm{s} & * & & & \\
\mathrm{s}^0 & * & & &
\end{array}$$

Routh Array elements

- $b_1 = \frac{-det\begin{bmatrix} 1 & a_2 \\ a_1 & a_3 \end{bmatrix}}{a_1}$
- $b_2 = \frac{-det\begin{bmatrix} 1 & a_4 \\ a_1 & a_5 \end{bmatrix}}{a_1}$
- $b_3 = \frac{-det\begin{bmatrix} 1 & a_6 \\ a_1 & a_7 \end{bmatrix}}{a_1}$

- $c_1 = \frac{-det\begin{bmatrix} a_1 & a_3 \\ b_1 & b_2 \end{bmatrix}}{b_1}$
- $c_2 = \frac{-det\begin{bmatrix} a_1 & a_5 \\ b_1 & b_3 \end{bmatrix}}{b_1}$
- $c_3 = \frac{-det\begin{bmatrix} a_1 & a_7 \\ b_1 & b_4 \end{bmatrix}}{b_1}$

Example

- Given the characteristic polynomial
- $s^3 + 2s^2 + 4s + 6$
- The Routh array are:
- s^3: $1 \quad 4 \quad 0$
- s^2: $2 \quad 6 \quad 0$
- s^1: $\left(\frac{-(1*6-2*4)}{2} = 1\right) \left(\frac{-(1*0-2*0)}{2} = 0\right)$
- s^0: $\left(\frac{-(2*0-1*6)}{1} = 6\right) \left(\frac{-(2*0-1*0)}{1} = 0\right)$
- The system is stable since all the elements in the first row are positive.

Example

Given the system $D(s) = K, \quad G(s) = \frac{s+1}{s(s-1)(s+6)}, \; H(s) = 1$

$$\text{notice}: \textit{there is a positive pole}$$

The characteristic equation is

$$1 + K\frac{s+1}{s(s-1)(s+6)} = 0$$

or

$$s^3 + 5s^2 + (K-6)s + K = 0$$

Calculate the Routh array

Find values of K making the system stable

Example

The closed loop characteristic equation is

$$s^3 + 5s^2 + (K - 6)s + K = 0$$

The Routh array is:

$$\begin{array}{lll} s^3: & 1 & K-6 \\ s^2: & 5 & K \\ s^1: & \left(\frac{4K-30}{5}\right) & 0 \\ s^0: & (K) & 0 \end{array}$$

Example

What is the limits for stabilization

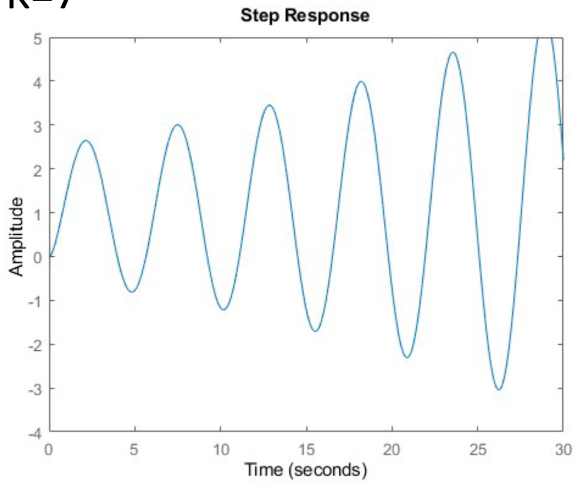
$$\frac{4K - 30}{5} > 0 \; and \;\; K > 0$$

or

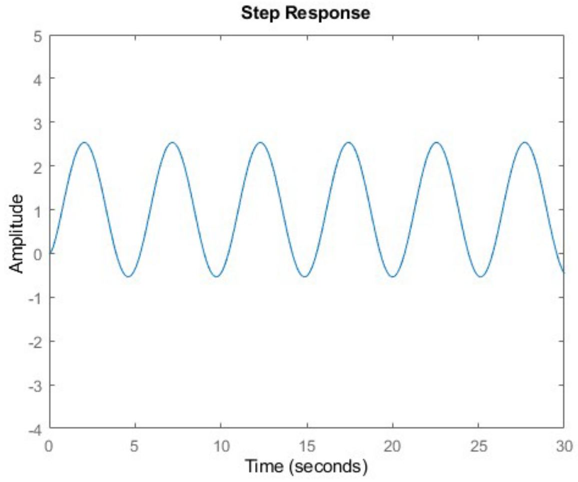
$$K > 7.5 \;\; and \; K > 0$$

Closed loop step responses $\quad G(s) = \frac{s+1}{s(s-1)(s+6)}, \;\; H(s) = 1$

K=7



K=7.5



K=8