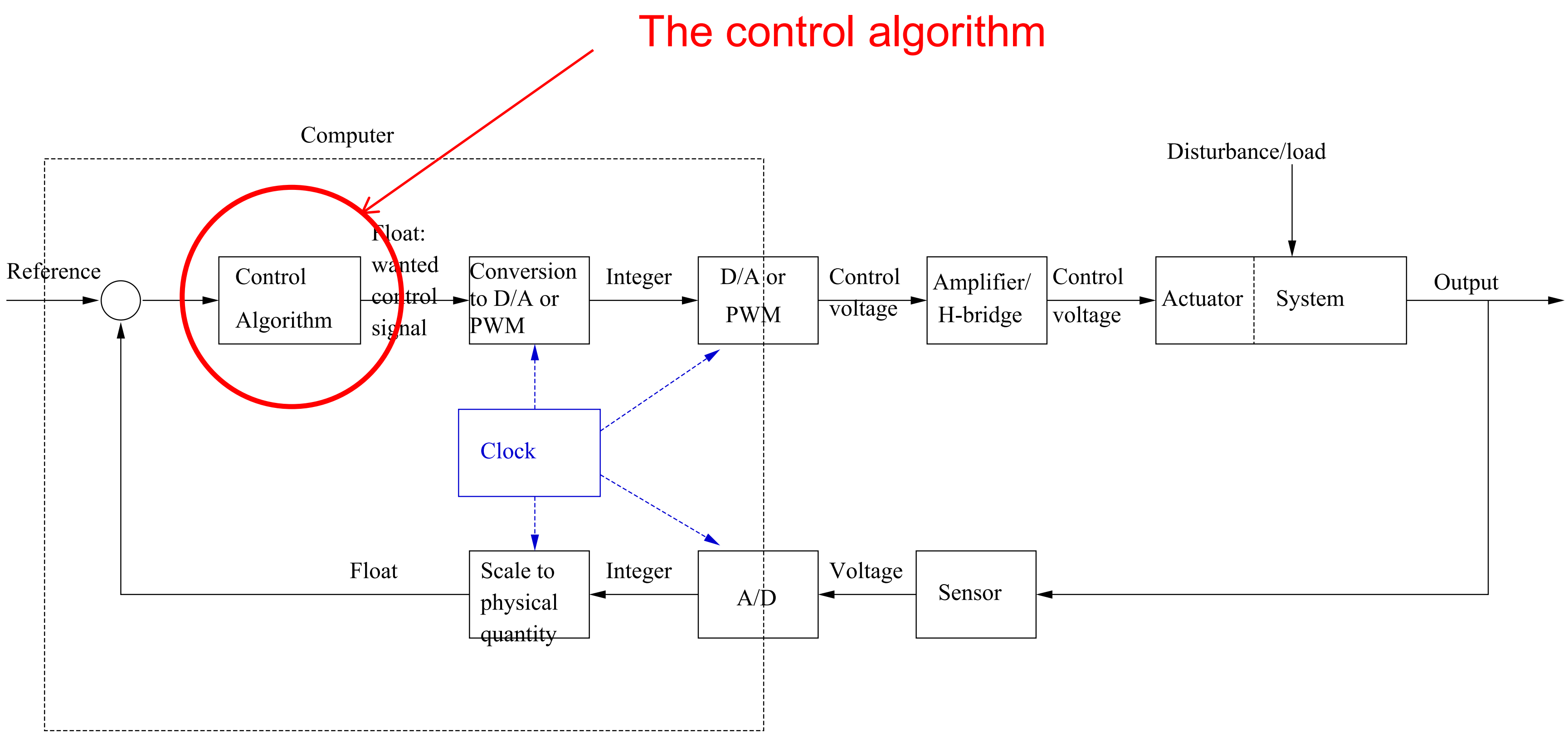
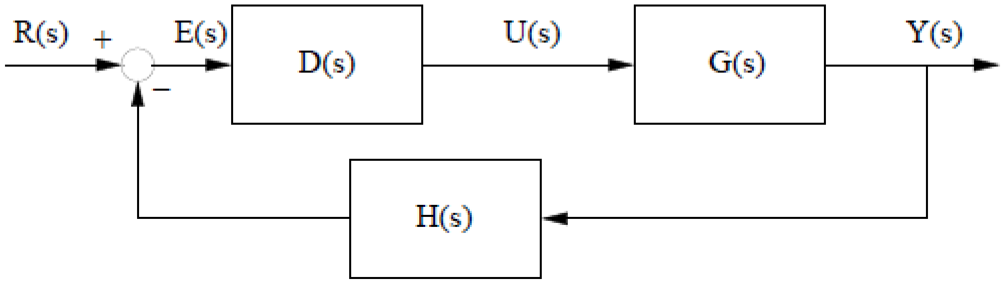
LEAD LAG CONTROL CONTROLLER DESIGN USING BODE PLOT

Standard control system



Standard set-up



$$Closed\quad loop: \qquad T(s) = \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)}$$

$$Open\quad loop: \qquad L(s) = D(s)G(s)H(s)$$

$$Direct\quad term: \qquad D(s)G(s)$$

$$closed\quad loop = \frac{direct\quad term}{1 + open\quad loop}$$

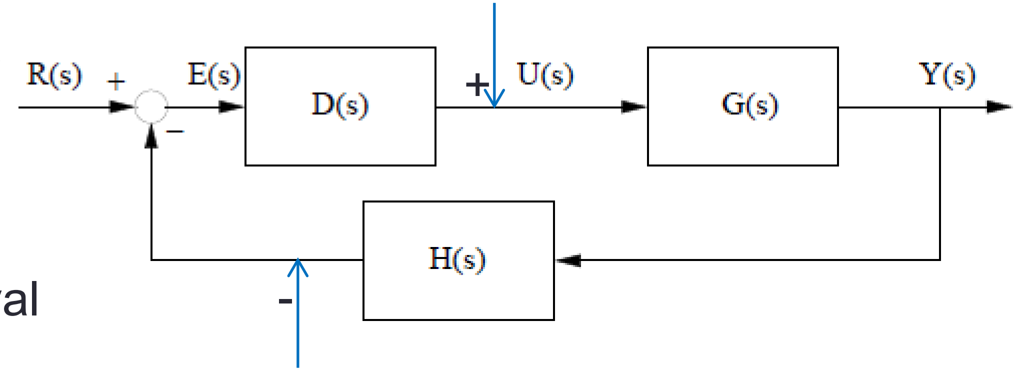
Operating point   small and large signals

Frequency design demands linear systems

Most systems are linear near an operating point

Use the resolution in the computer for the active interval

Actuator value to keep output in the operating point



Operating point

Add the control value to keep the system in the operating point after the controller
Subtract the operating point after the sensor

Stability conditions

Oscillating:

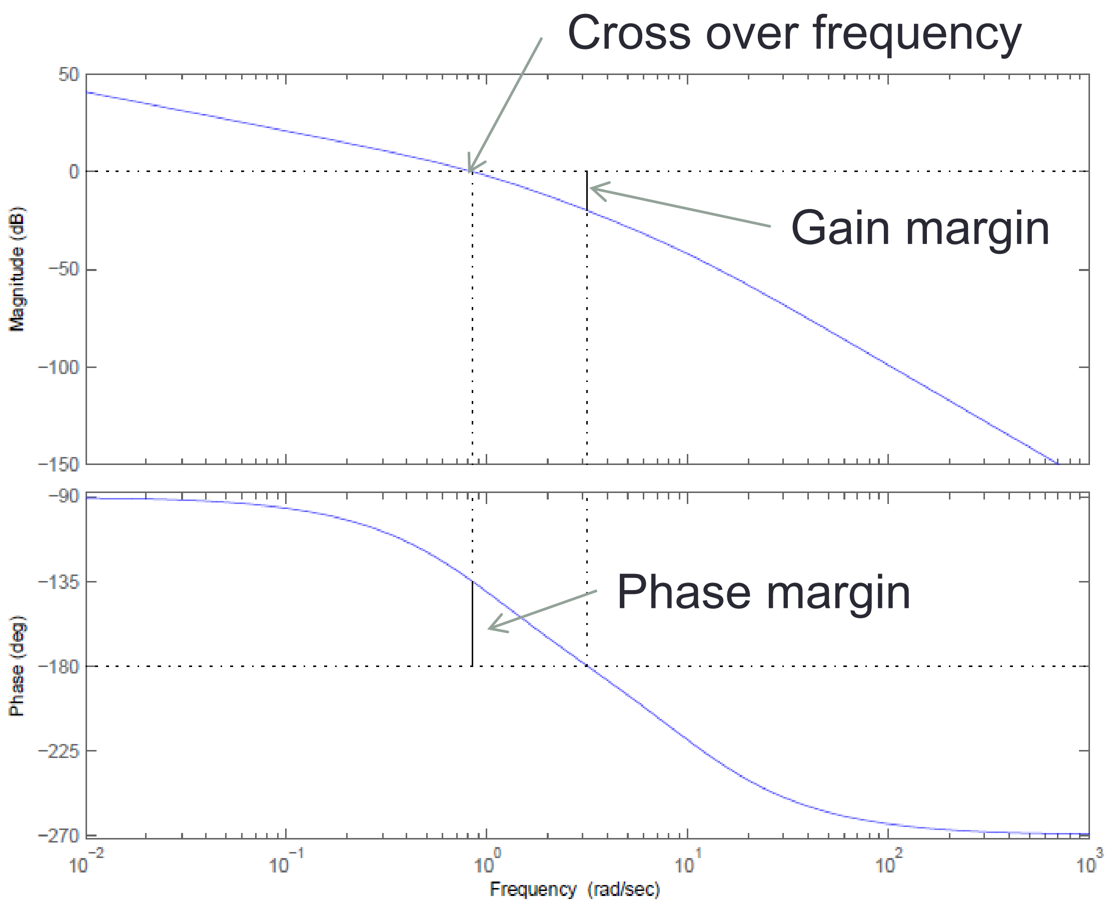
$$D(s)G(s)H(s) = -1 = \ 1 \ \angle -180$$

Unstable:

$$D(s)G(s)H(s) = \ >1 \ \angle -180$$

If there is a frequency ω_1 where the phase $\angle D(\omega_1)G(\omega_1)H(\omega_1)$ is -180 then the gain $|D(\omega_1)G(\omega_1)H(\omega_1)|$ must be smaller than 1 (0 dB) for stability

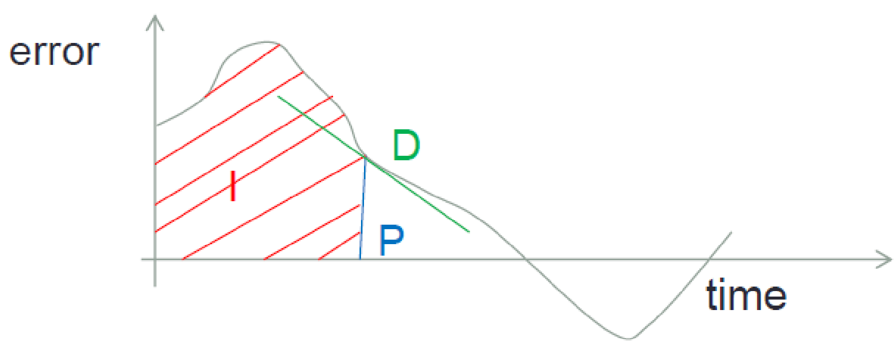
Frequency domain specifications



PID controller

A standard controller is:

$$
\begin{aligned}
D(s) &= K_p\left(1+\frac{1}{T_i\cdot s}+T_d\cdot s\right)\\
&= Proportional(1+Integral+Differential)\\
&= PID-controller
\end{aligned}
$$

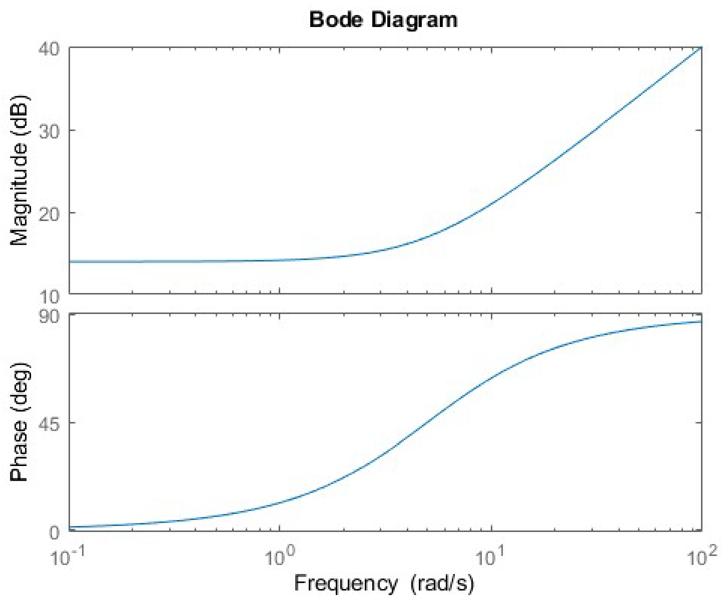


Used in combinations

P control	K
I control	K/s
PI control	$K(T_i s+1)/T_d s$
PD control	$K(T_d s+1)$
PID control	$K(T_i T_d s^2+T_i s+1)/T_i s$

PD control

$PD\ controller\!: \ 1(s+5)$ zero=-5 dc gain = 5



You get a higher phase and gain for high frequencies

Can be used to obtain a higher cross over frequency and a better phase margin

PD example on blackboard

$$G(s) = \frac{10}{(s + 0.1)(s + 10)s}$$

We want
A stable system
A faster system (higher cross over),
Less overshoot => a larger phase margin
no stationary error

Standard PID control

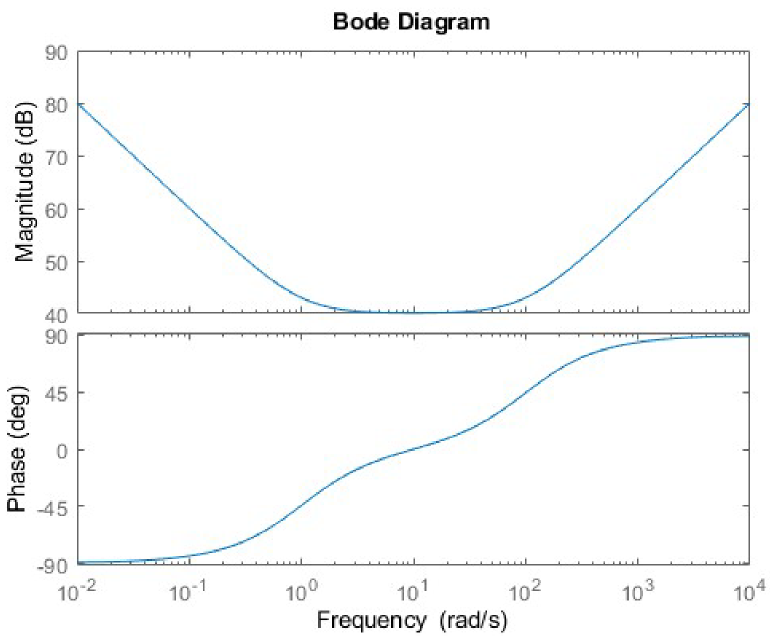
PID controller: *Proportional(1+Integral+Differential)*

$$D(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$D(s) = K_p \frac{T_i s + 1 + T_d T_i s^2}{T_i s} = K\frac{(s+a)(s+b)}{s}$$

$$D(s) = \frac{(s+1)(s+100)}{s}$$

$\frac{1}{s}$ gives good stationary properties
Zeroes can give a faster system and better stability



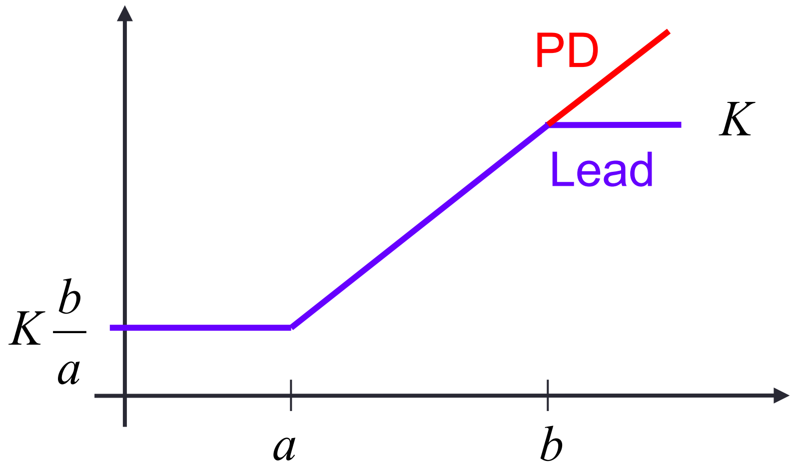
Dynamic Compensation

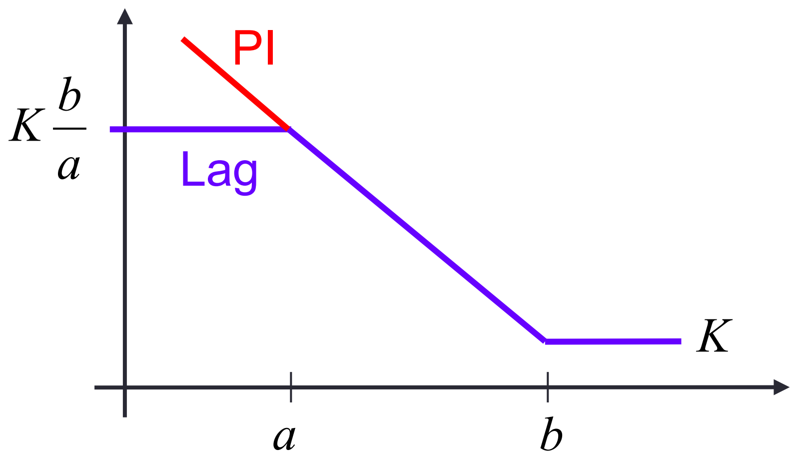
$$D(s) = K\frac{s+a}{s+b} = K\frac{b}{a}\frac{(s/a+1)}{(s/b+1)}$$

Lead compensation $\quad a < b$

Lag compensation $\quad a > b$

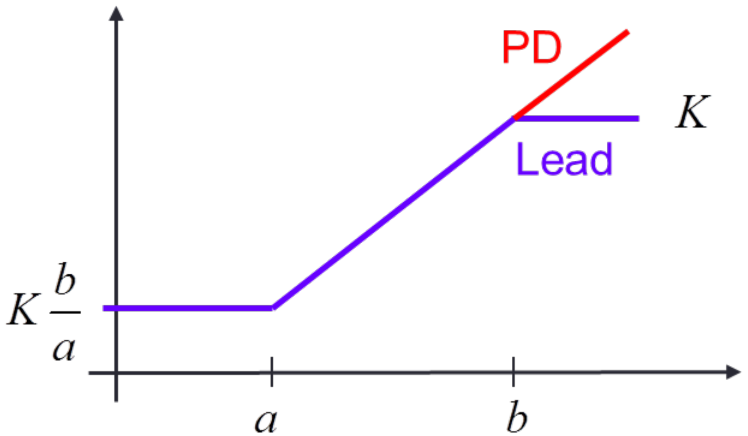
We want to avoid infinity gain





lead compensator

In a PD controller the magnitude of the compensator continuously grows with the increase in frequency. it is undesirable because it amplifies high frequency noise that is typically present in any real system.



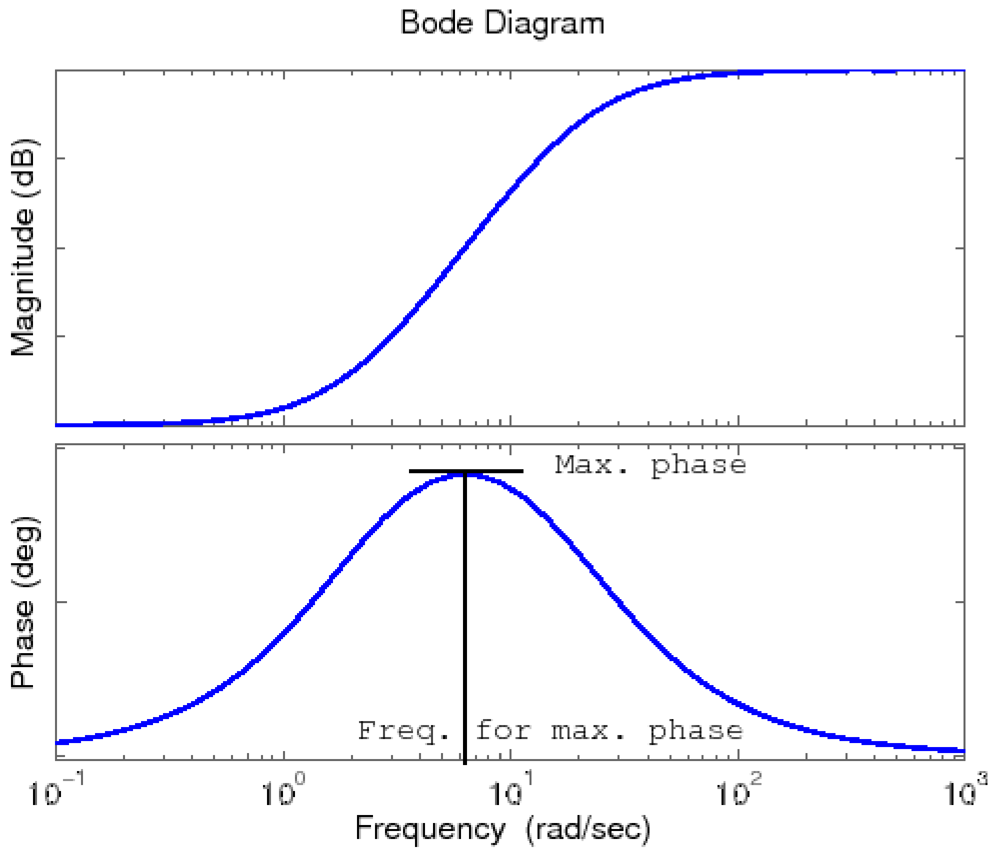
In lead compensator, a first order pole is added to the denominator of the PD controller at frequencies well higher that the corner frequency of the PD controller.

- $D(s) = K\frac{s+a}{s+b} = \frac{b}{a}\frac{(\frac{s}{a}+1)}{(\frac{s}{b}+1)}, a < b$
- $D(s) = K\frac{\tau s+1}{\alpha\tau s+1},\ \alpha < 1$
- $^1/_\alpha$ is the ratio between the pole zero break point (corner) frequencies.

- How is the phase plot for a lead-controller
- What can you gain

Lead compensator
Bode plot

Max phase 90



a significant amount of phase is still provided with much less amplitude at high frequencies.

Lead compensator

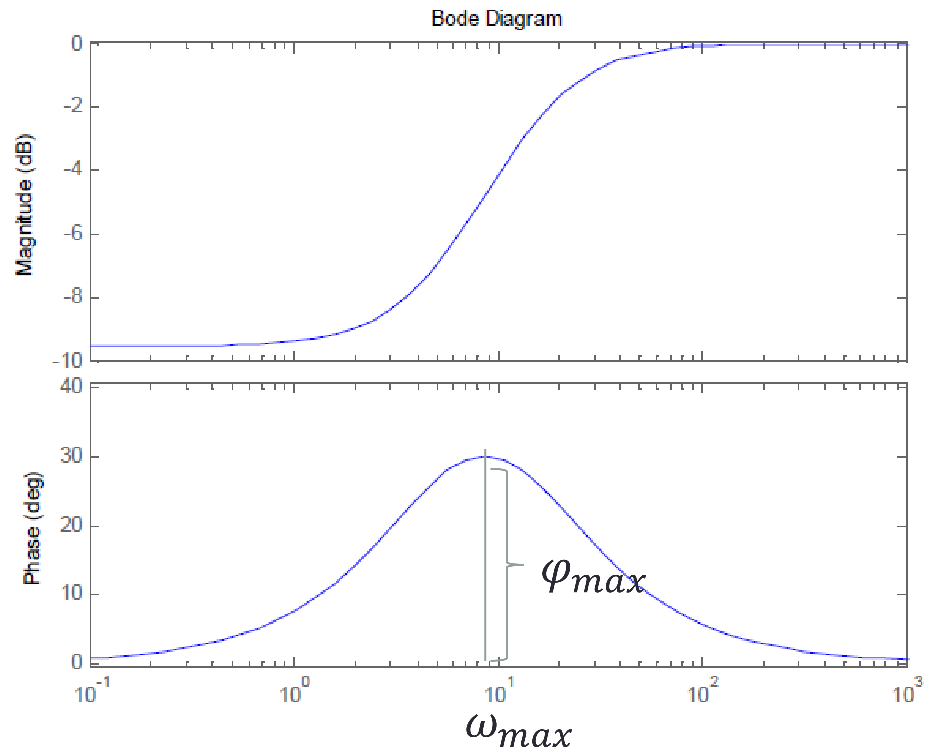
Lead add phase

$$D(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

It can be shown that the phase where the phase is max is

$$\omega_{max} = \frac{1}{T_D\sqrt{\alpha}}$$

The maximum fase occures at the frequency that lies midway between the two corner breakpoint frequencies on the logaritmic axis



The maximum phase contribution approx

$$\sin(\varphi_{max}) = \frac{1-\alpha}{1+\alpha}$$

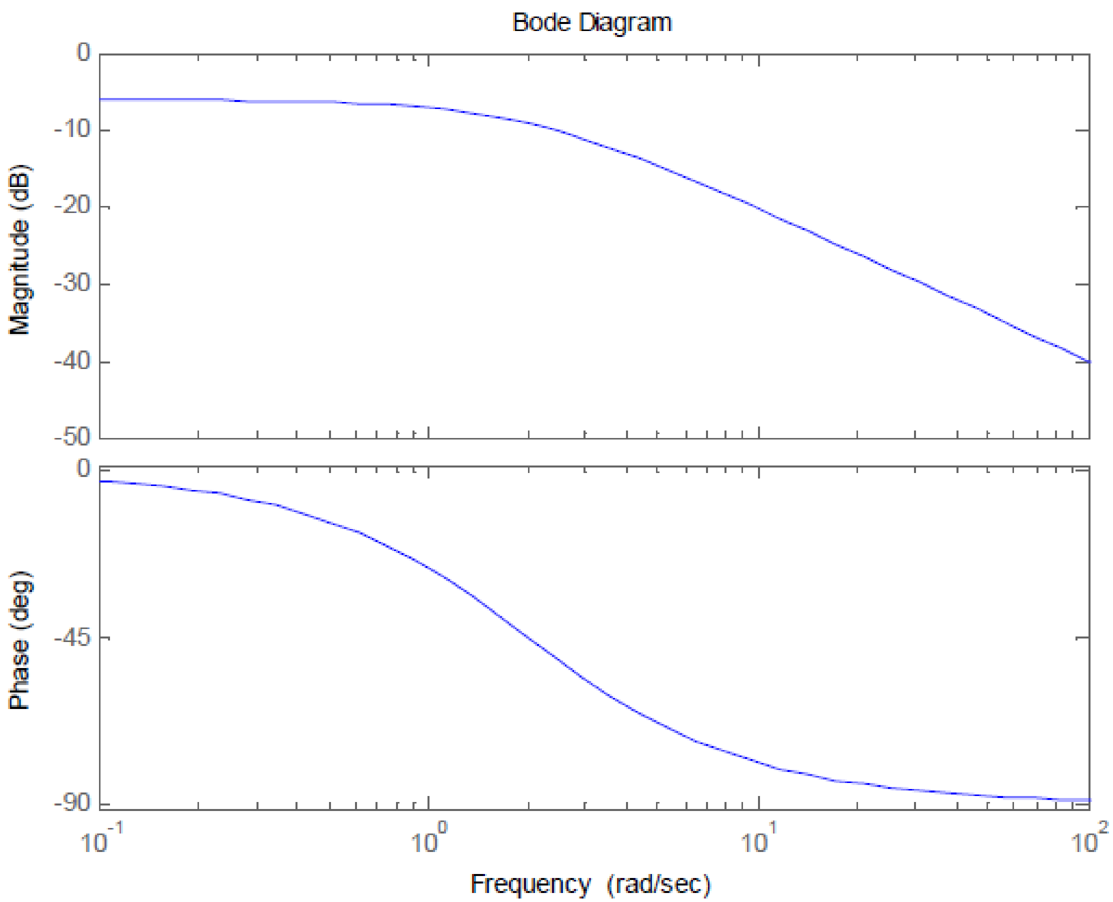
Large $\alpha \Rightarrow \varphi_{max} \rightarrow 90^o$ (approx. $\alpha = 100$)

Example: design a controller for the system $G(s) = \frac{1}{s+2}$

Demands:
No stationary error for step input
Cross over frequency 8 rad/sec

Step 1:
Draw the bode plot for G(s) by hand or by Matlab

No crossover frequency
No integration => stationary error for step



Step 2: eliminiate the stationary error

An integration will eliminate the stationary error

$$D(s) = \frac{1}{s}$$

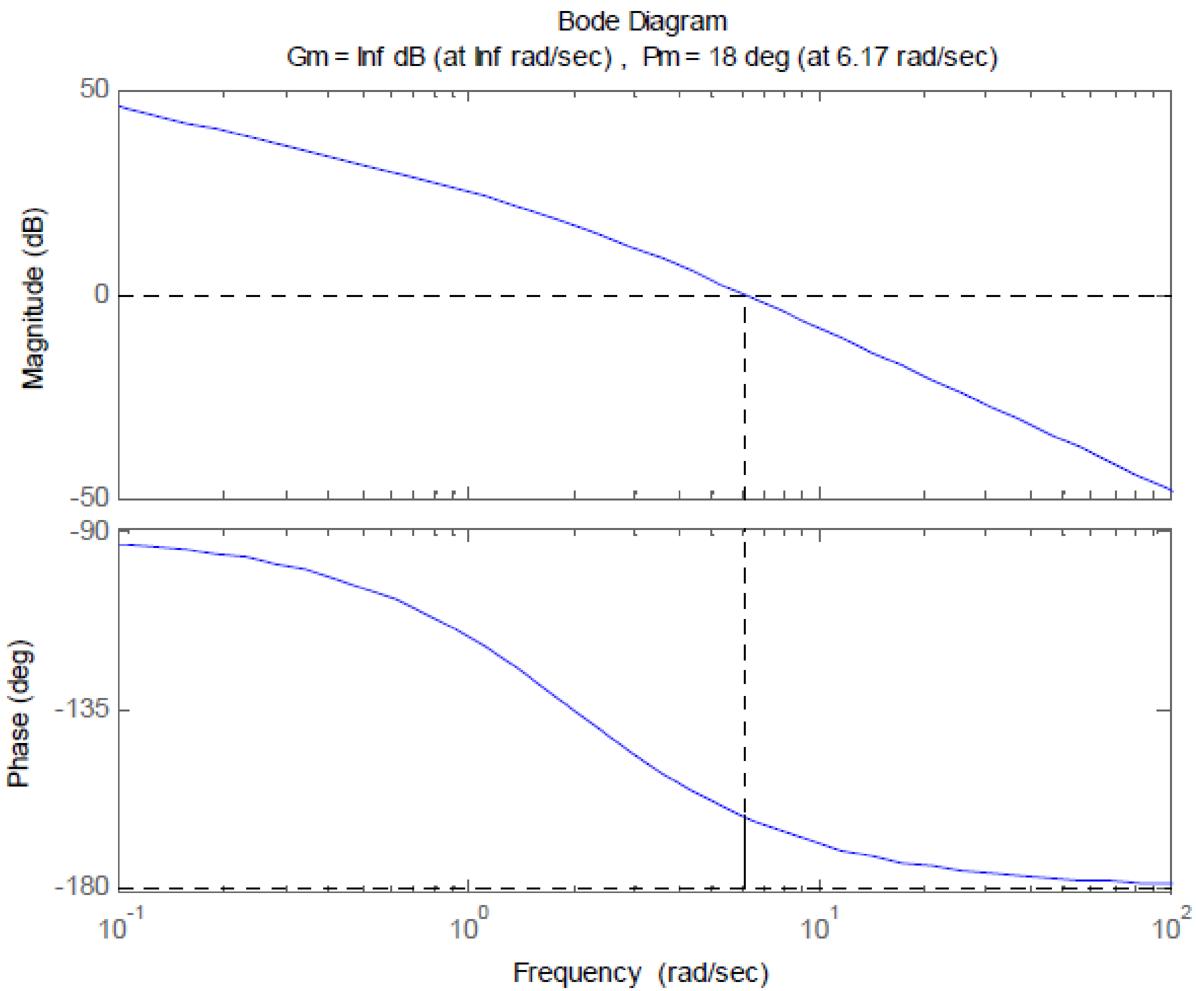
Make a bode plot for the new open loop system

$$D(s)G(s)=\frac{1}{s(s+2)}$$

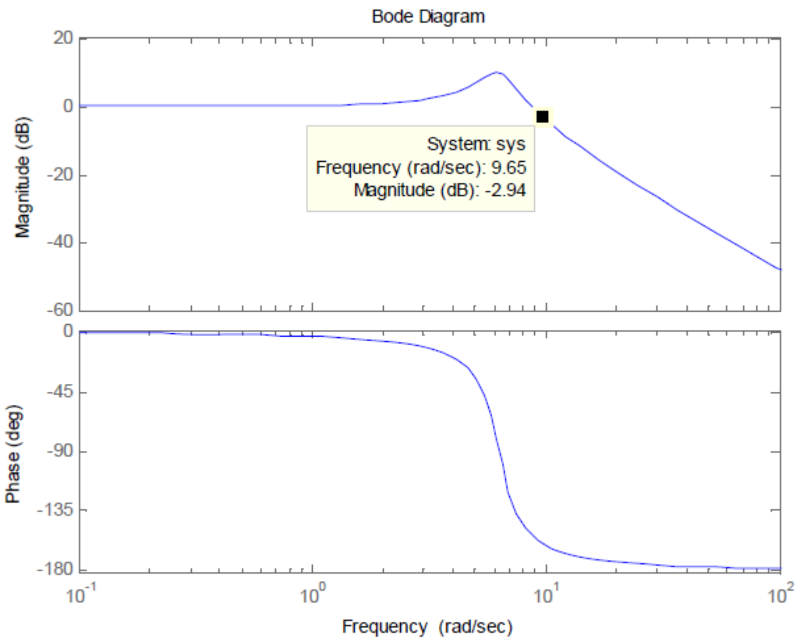
The cross over frequency is 0.4 rad/sec
The missing gain is 37 dB

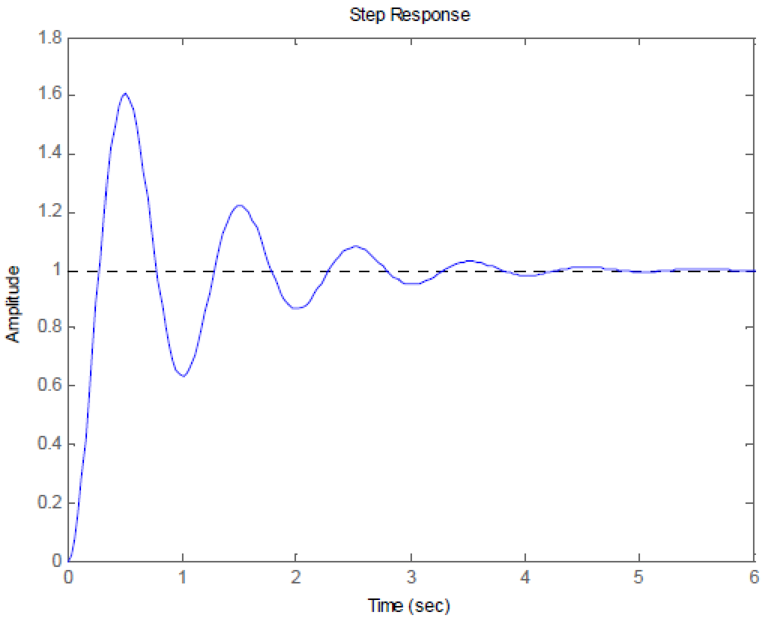
Adjust the gain 37 dB to obtain the cross over frequency

Notich we have a small phase margin



Closed loop bode plot and step response





We want less overshoot

What will happen if you use a lead compensator

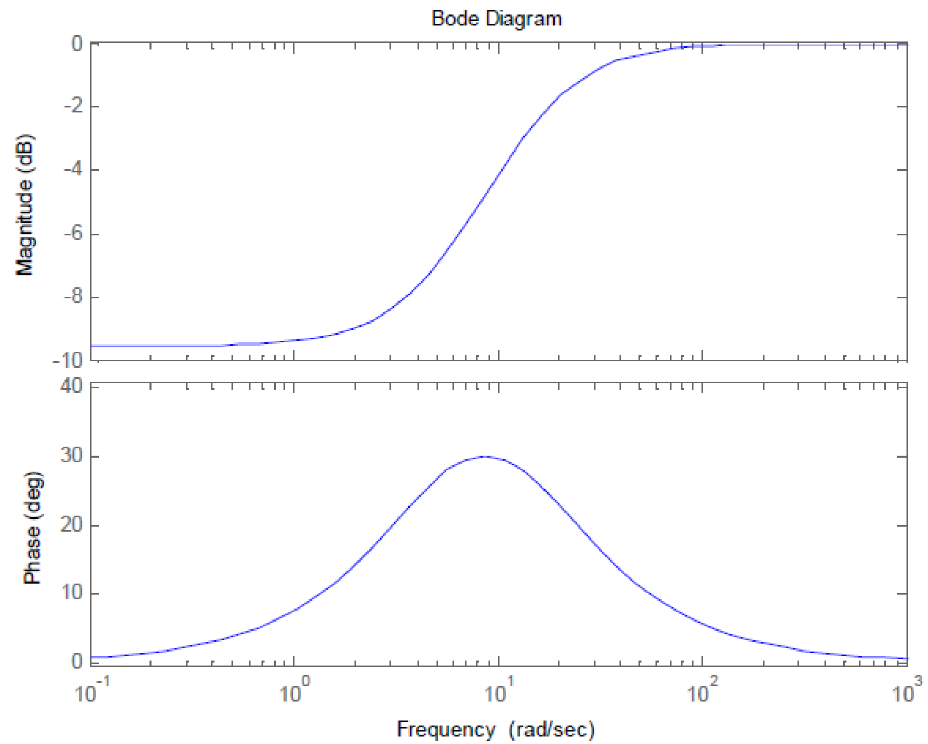
Lead compensator

We add phase
Good for the phase margin around 8 rad/sec
Limited high freqency gain

$$D(s) = \frac{s+5}{s+15}$$

$$D(s) = \frac{15}{5}\frac{{}^{s}/_{5}+1}{{}^{s}/_{15}+1}$$

$$\tau = {}^{1}/_{5}$$
$$\alpha = 3$$



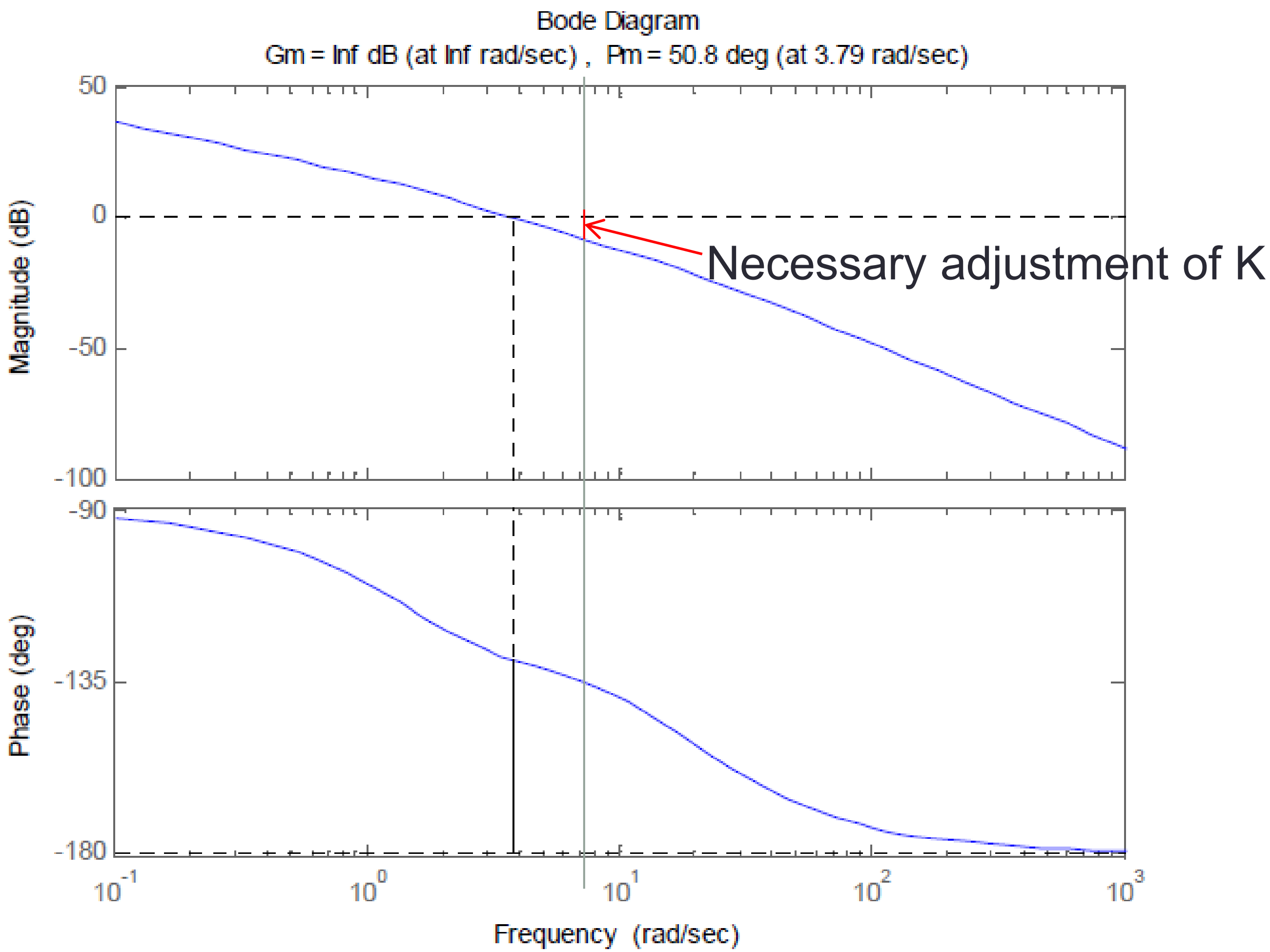
System with lead compensation

Bodeplot for

$$OL: \frac{s+5}{s+15}\frac{1}{s}\frac{1}{s+2}$$

notice: the controller gain is left out

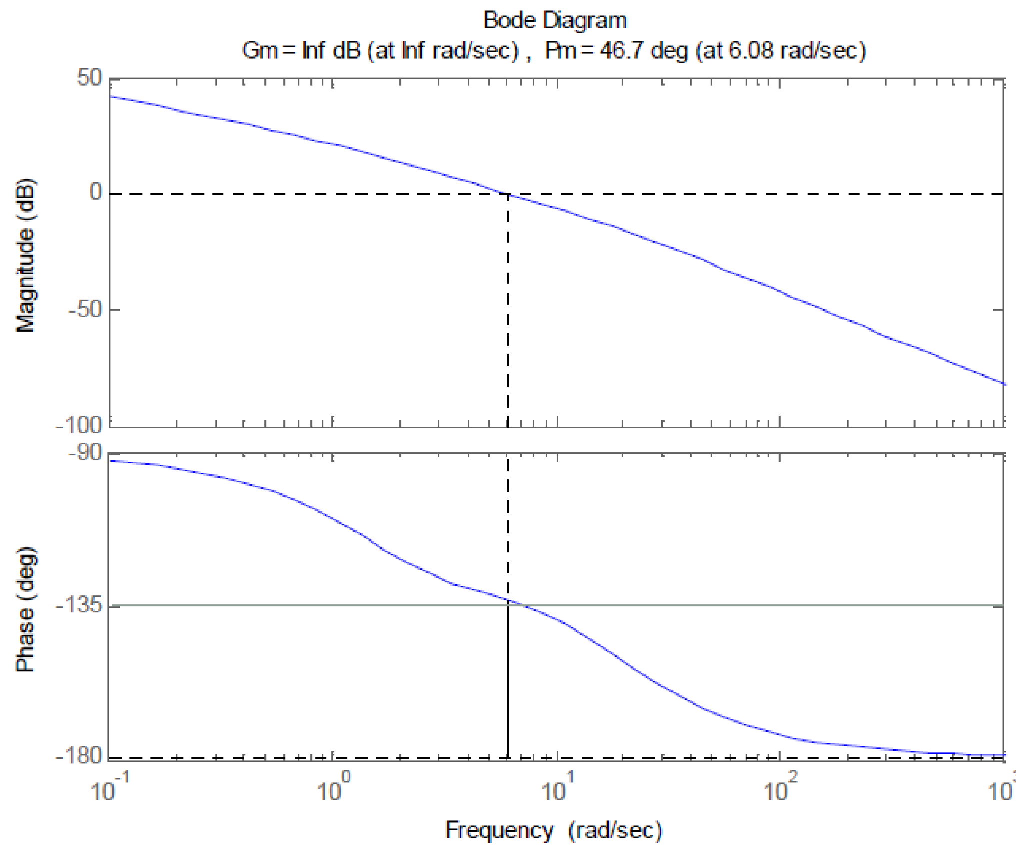
The phase margin is better
The cross over has to be changed by a new K



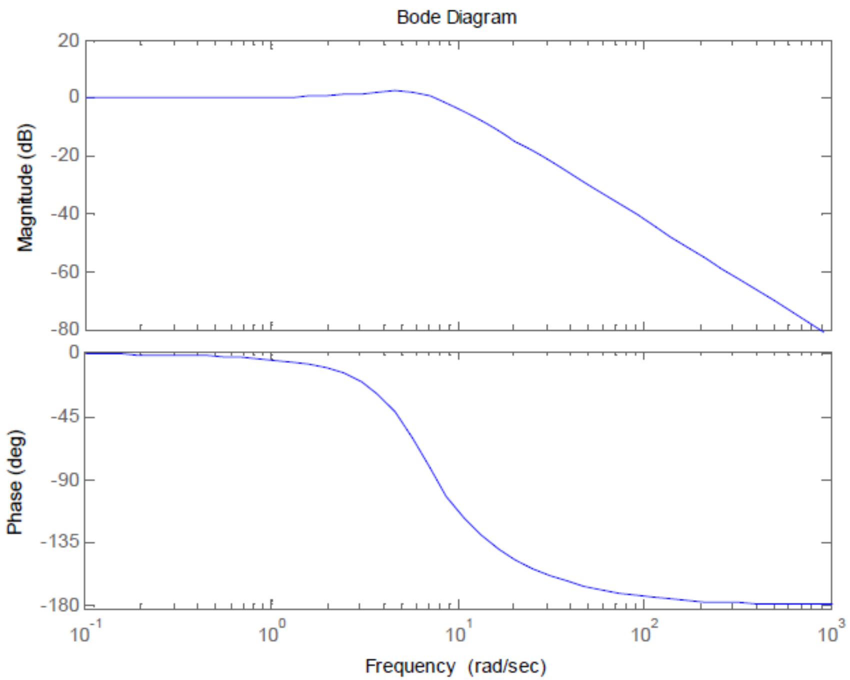
Lead and gain 2

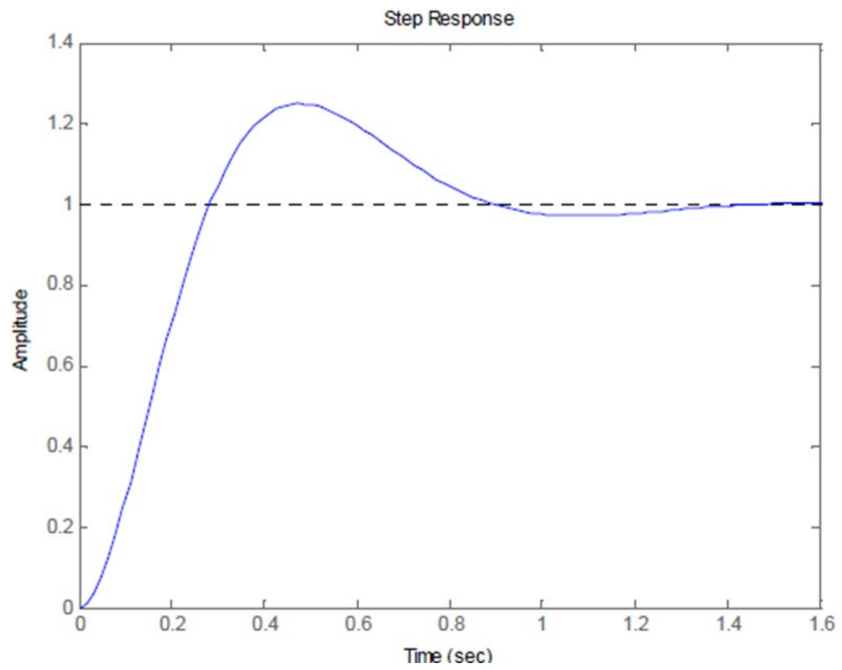
Adjusted gain to obtain the wanted cross over frequency

The phase margin is now 45



Closed loop bode plot and step response





Advantages of lead compensators

1. Phase margin is usually improved.
2. This implies more damping and stability.
3. Bandwidth of open and closed loop systems increases.
4. Reduced overshoot.
5. Steady state error is not affected, since the application is at high frequencies. The gain added at low frequencies is zero

Disadvantages of lead compensators

1. If the original open loop system is unstable, then one may require a large α which can amplify noise also. Or we may need a series of lead compensators.

2. When the original system has already low stability then either the phase has crossed -180 or is rapidly going toward -180. A lead compensator will not perform well in such situations.

3. The phase will rapidly approach -180 if the open loop Transfer function has 2 or more poles close(real or complex) to each other and near the gain cross over.

break

Lag Controller

A lag compensator provides an increased low frequency gain, thus decreasing the steady state error, without changing the transient response significantly.

For frequency response design it is convenient to use the following transfer function of a lag compensator.

$$D(s) = \frac{s+a}{s+b} = \frac{b}{a}\frac{(\frac{s}{a}+1)}{(\frac{s}{b}+1)}, a > b$$

$$D(s) = \alpha\frac{\tau s+1}{\alpha\tau s+1}, \qquad \alpha > 1$$

When s->0, D(s) -> α
When s -> ∞, D(s) -> 1

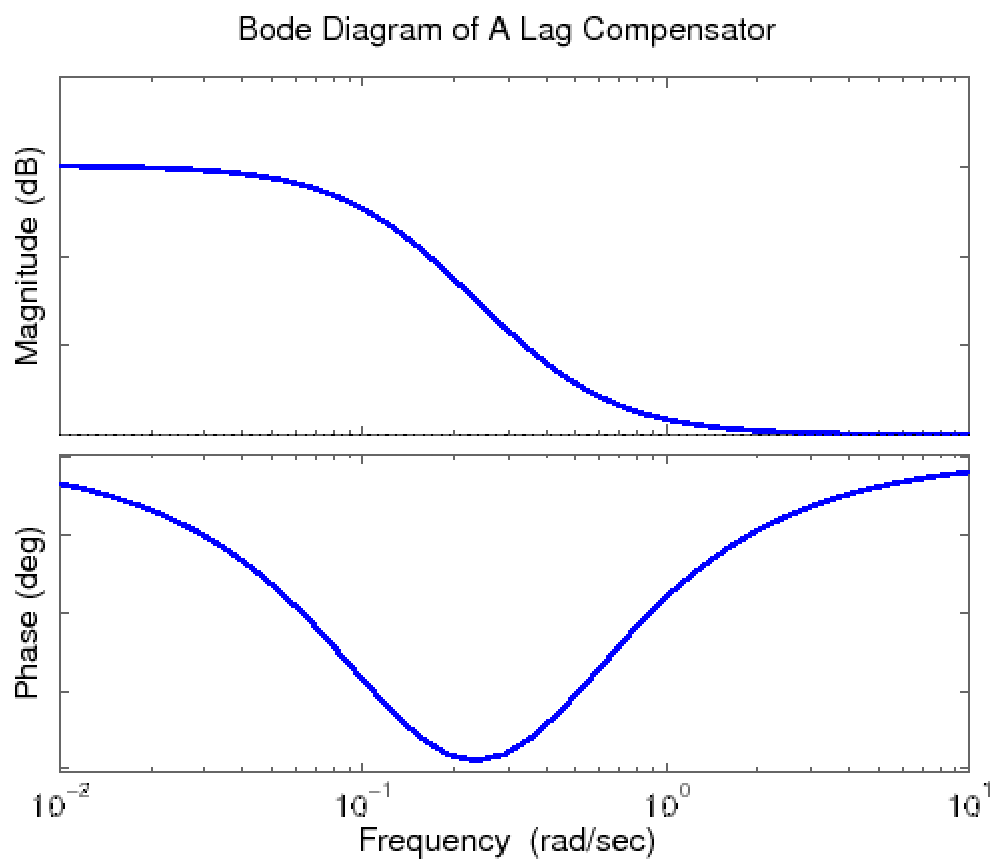
The expression can be adjusted by a gain. The overall lag controller is

$$D(s) = K\alpha\frac{\tau s+1}{\alpha\tau s+1}$$

Lag control

Provide an additional gain of α in the low frequency region and to leave the system sufficient phase margin.

The lag compensator provide the maximum lag near the two corner frequencies, to maintain the PM of the system, the zero of the compensator should be chosen such that $\omega = 1/\tau$ is much lower than the gain crossover frequency of the uncompensated system.



τ is designed such that $1/\tau$ is at least one decade below the gain crossover frequency of the uncompensated system.

Advantages and disadvantages of lag compensators

1. Lag compensator moves the gain cross over to a lower frequency while keeping the phase
curve unchanged.
2. Bandwidth of open and closed loop systems decreases.
3. Rise time increase and system is slower.
4. Since phase curve is untouched, lowering the gain crossover can improve phase margin and
gain margin.

Lead Lag Compensator

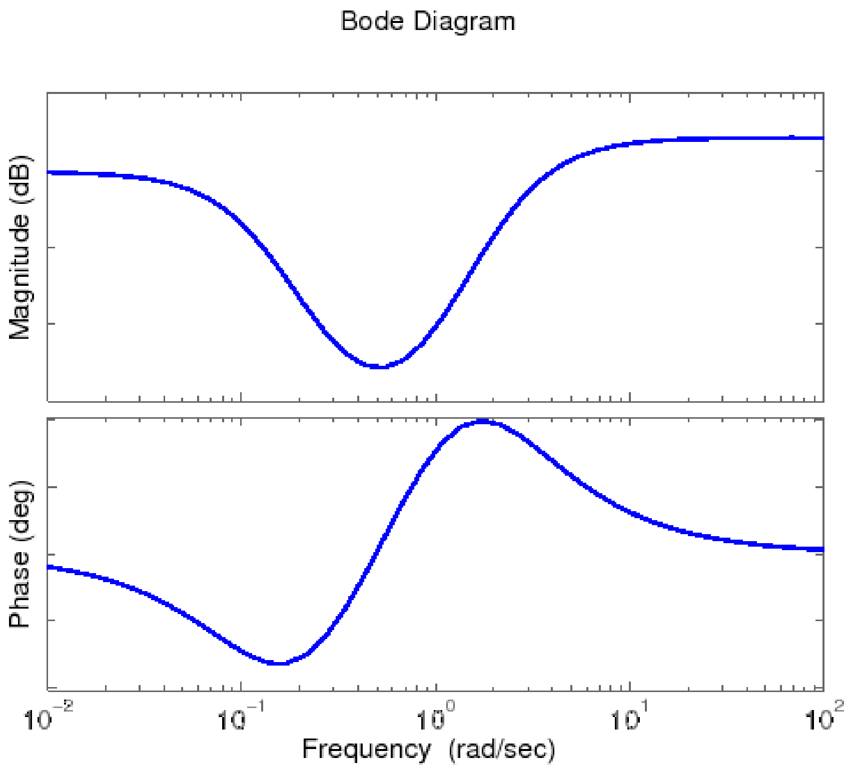
When a single lead or lag compensator cannot guarantee the specified design criteria, a lag-lead compensator can be used.

$$D(s) = K\frac{1+\tau_1 s}{1+\alpha_1\tau_1 s}\frac{1+\tau_2 s}{1+\alpha_2\tau_2}$$

The corner frequencies are

$$\frac{1}{\tau_1}, \frac{1}{\alpha_1\tau_1}, \frac{1}{\tau_2}, \frac{1}{\alpha_2\tau_2}$$

The frequency response is :



Design advise

- If it is not specified which type of compensator has to be designed, one should first check the PM and BW of the uncompensated system with adjustable gain K.
- If PM is small go for lead
- If the BW is smaller than the acceptable BW go for lead. If the BW is large, lead may not be useful since it provides high frequency amplification.
- Go for a lag compensator when BW is large provided the open loop system is stable.
- If the lag compensator results in a too low BW (slow speed of response), a lag-lead compensator may be used.

Example

Consider the following system with transfer function

$$G(s) = \frac{1}{s(0.1s+1)(0.2s+1)}$$

Design a lag-lead compensator C(s) such that

- the phase margin is at least 45°
- the gain crossover frequency around 10 rad/sec
- the velocity error constant K_v is 30.

Bode plot of G(s)

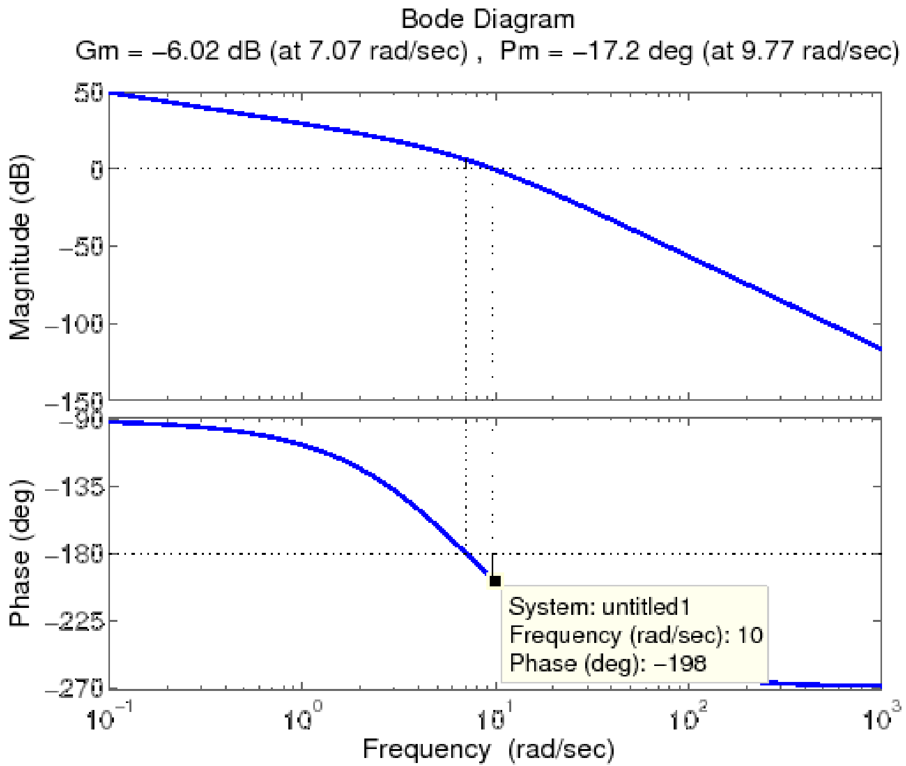
The PM of the uncompensated system with K is negative

The cross over frequency is close to 10

Kv (G)=lim(s->0) $s\dfrac{1}{s(1+0.1s)(1+0.2s)}$

$$=\frac{1}{1*1}=\frac{1}{1}=1$$

A lead compensator can increase the gain crossover frequency and the phase margin.



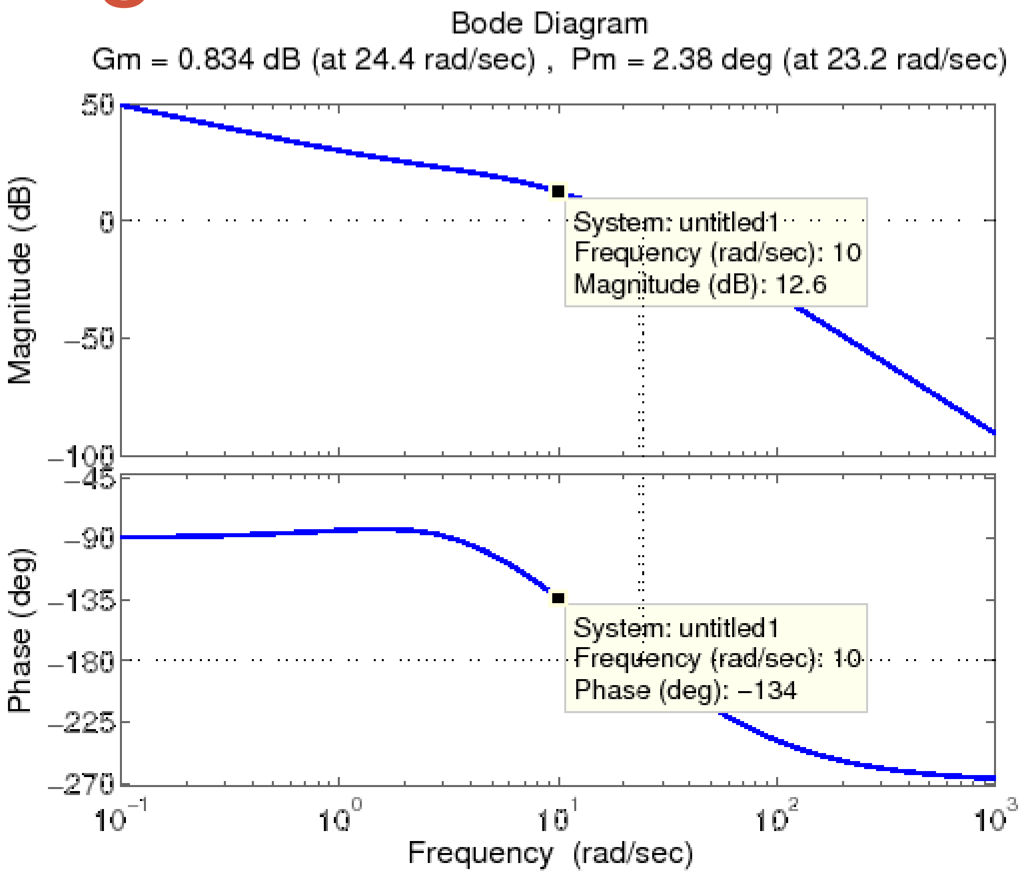
Bodeplot including lead

We want a bette phase margin in w=10

$$D(s) = K\frac{1+4s}{1+17s}$$

The gain crossover frequency is increased to 23.2 rad/sec.
At 10 rad/sec, the phase angle is -134° and gain is 12.6 dB.

We will use the lag part to adjust the gain / the crossover frequency
and
to ajust Kv



Lag

- To make this as the actual gain crossover frequency, the lag part should provide an attenuation of -12.6 dB at high frequencies.

- At high frequencies the magnitude of the lag compensator part is ${}^{1}/_{\alpha_1}$. Thus ,

$$20\ log_{10}\frac{1+\tau_1 s}{1+\alpha_1\tau_1 s} \rightarrow 20\ log_{10}\frac{1}{\alpha_1} = 12.6$$

which gives $\alpha_1 = 4.27$

Now, ${}^{1}/_{\tau_1}$ should be placed much below the new gain crossover frequency to retain the desired PM. Let ${}^{1}/_{\tau_1}$ be 0.25. Thus $\tau_1 = 4$

K and Kv

The lag-lead compensator given by $D(s) = K\frac{1+\tau_1 s}{1+\alpha_1\tau_1 s}\frac{1+\tau_2 s}{1+\alpha_2\tau_2}$, $\alpha_1 > 1$, $\alpha_2 < 1$

$$D(s) = K\frac{1+4s}{1+17s}\frac{1+0.45s}{1+0.023s}$$

K is determined from the demands to Kv

Kv= lim s-> 0 (sD(s)G(s))

$$\lim s \to 0(s * K\frac{(1+4s)}{(1+17s)}\frac{(1+0.45s)}{(1+0.0225)}\frac{1}{s(1+0.1s)(1+0.2s)}) = K$$

Kv = K = 30

-

Calculation af Kv

- Kv can be found as
- lim s-> 0 (sD(s)G(s))
- $\lim s \rightarrow 0(s * K \frac{(1+4s)}{(1+17s)} \frac{(1+0.45s)}{(1+0.0225)} \frac{1}{s(1+0.1s)(1+0.2s)}) = K$
- Kv = K = 30

The lead lag compensated system

$$D(s)G(s) = 30\frac{(1+4s)}{(1+17s)}\frac{(1+0.45s)}{(1+0.0225)}\frac{1}{s(1+0.1s)(1+0.2s)}$$