Control
E5

Kirsten
kmn@es.aau.dk
Jakob
jakob@es.aau.dk

Outline for today

- Introduction to the course and examination
- The feedback control idea.
- Elements in a general feedback loop
- Block diagrams
- Transfer functions

The Course

- 5 ECTS
- 15 (approximately) lectures including exercises
- Exercises can be Lab-exercise – Measurement reports must be made – they are part of the examination
- Exercises in the group room after each lecture
  - One writing at the blackboard the other helping

Examination

The examination is oral and based on the solution of the exercises in the course.

You must bring your results on paper.

Course content

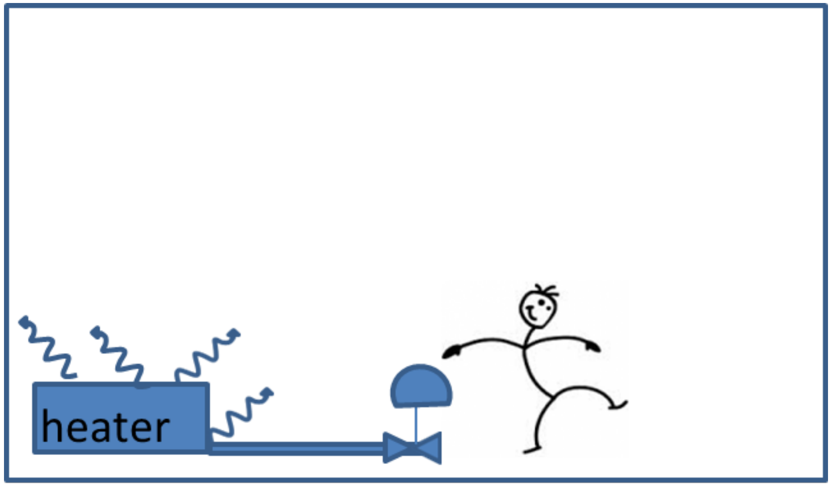
The elements you need to design a feedback control loop

- Introduction – block diagrams and transfer functions
- Time domain analysis
- Frequency analysis
- Controller design
  - Using bode plots
  - Using root locus
- MIMO systems
- Implementation
- State space control

Example: Manual control of indoor temperature

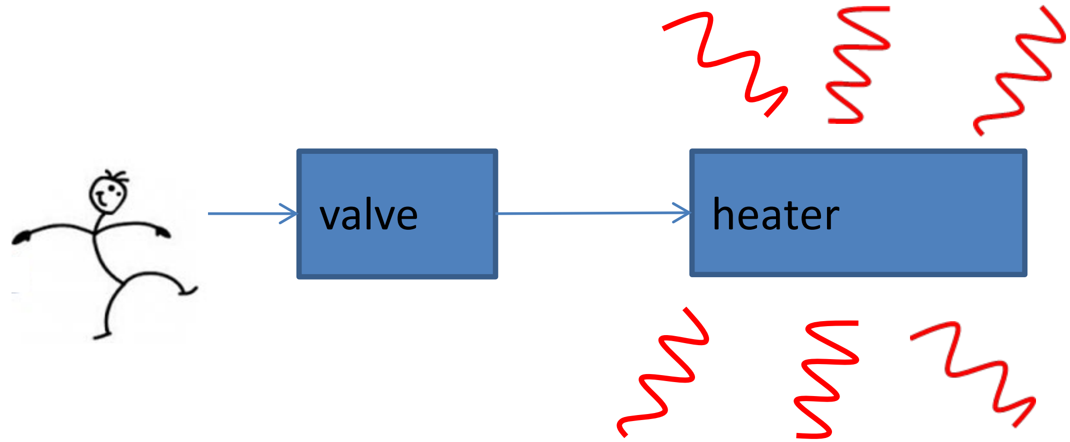
Manual control :

Feedforward/ open loop
– the person feels it is too hot- he adjust the valve to the heater based on experience

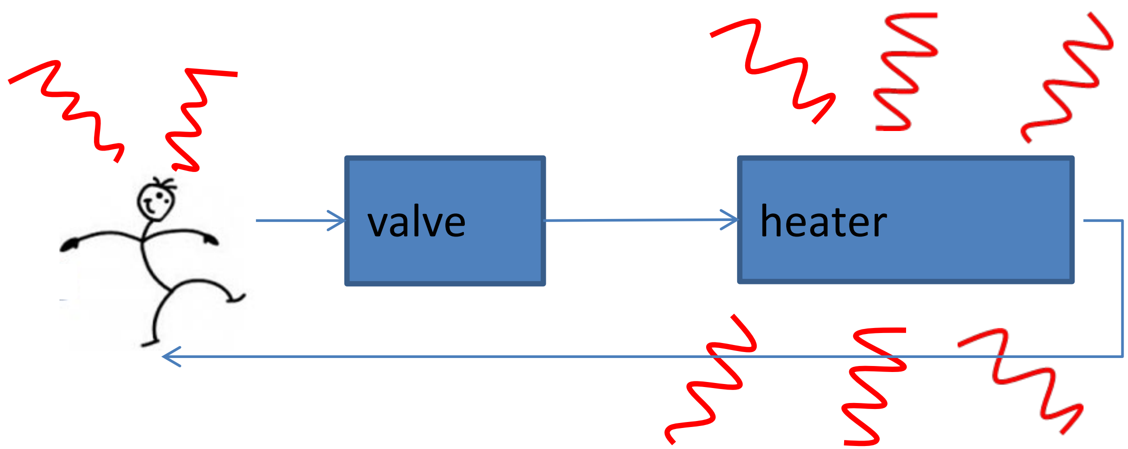


Feedback/ closed loop
- The person adjust the valve position depending on his comfort temperature
- If it is too hot, he closes the valves a bit
- If it is too cold he opens the valve a bit
- He continues until he reach the comfort temperature

Functional block diagram of manual control



The person knows where to set the valve to obtain a good temperature in the room – feed forward control



The person feels how hot it is and adjust the valve
Feed back control

Example: Automatic control of indoor temperature

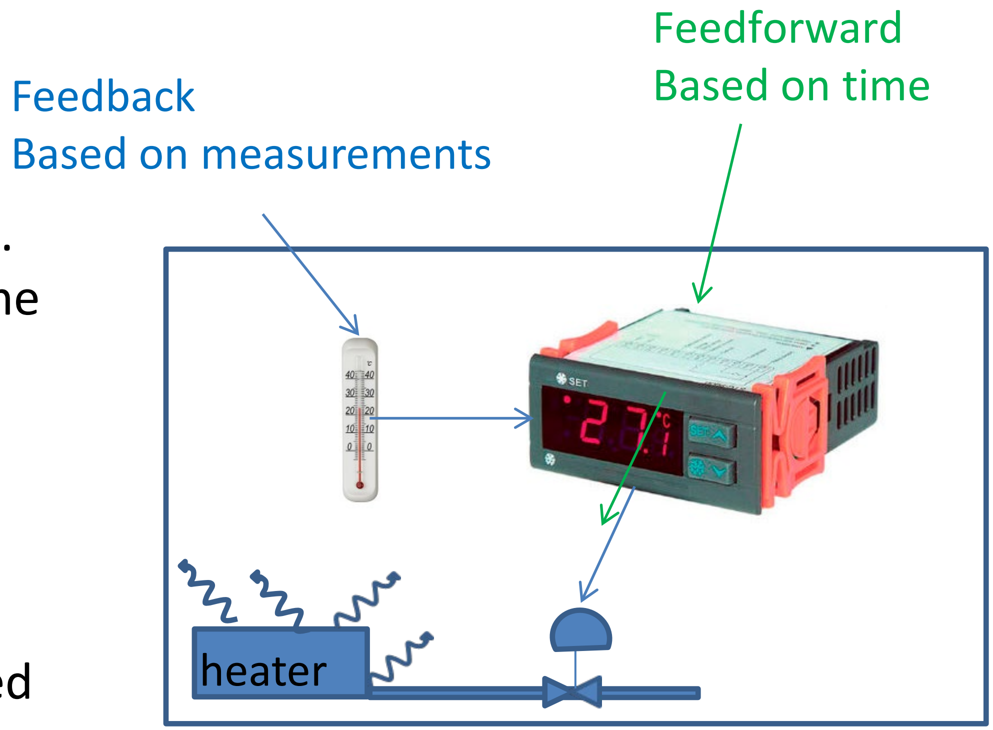
Automatic control

Feedforward/open loop

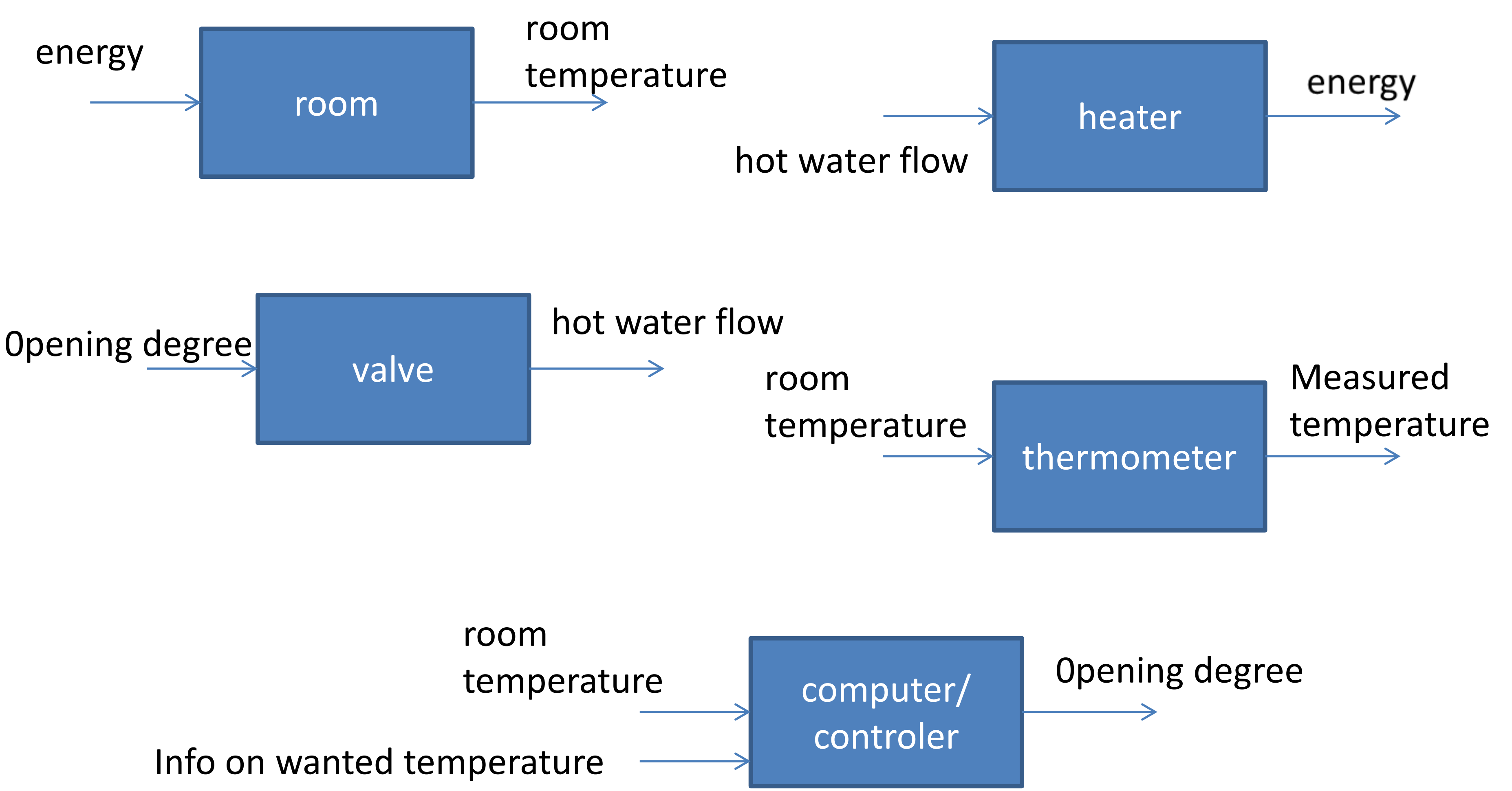
A controller adjust the valve position e.g. based on the time of the day and the time of the year

Feedback/closed loop

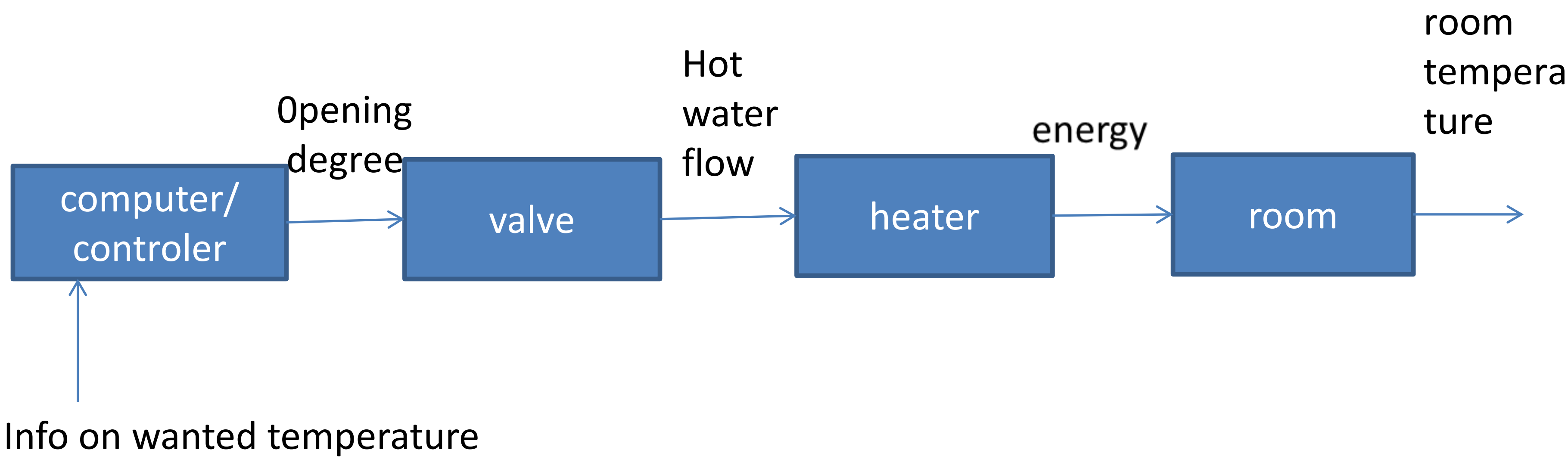
- the room temperature is measured by the thermometer – the controller calculates the difference from the wanted temperature. The valve position is calculated from the error and knowledge of the room dynamics.



Elements in control of indoor temperature

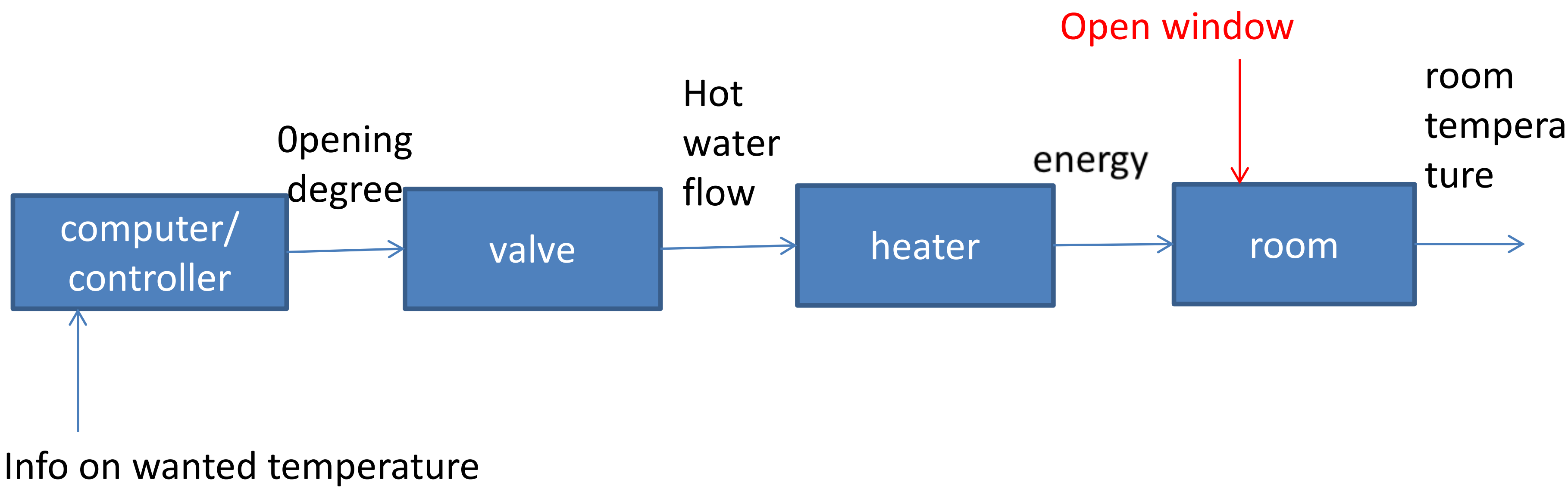


Functional block diagram for feed forward control of indoor temperature

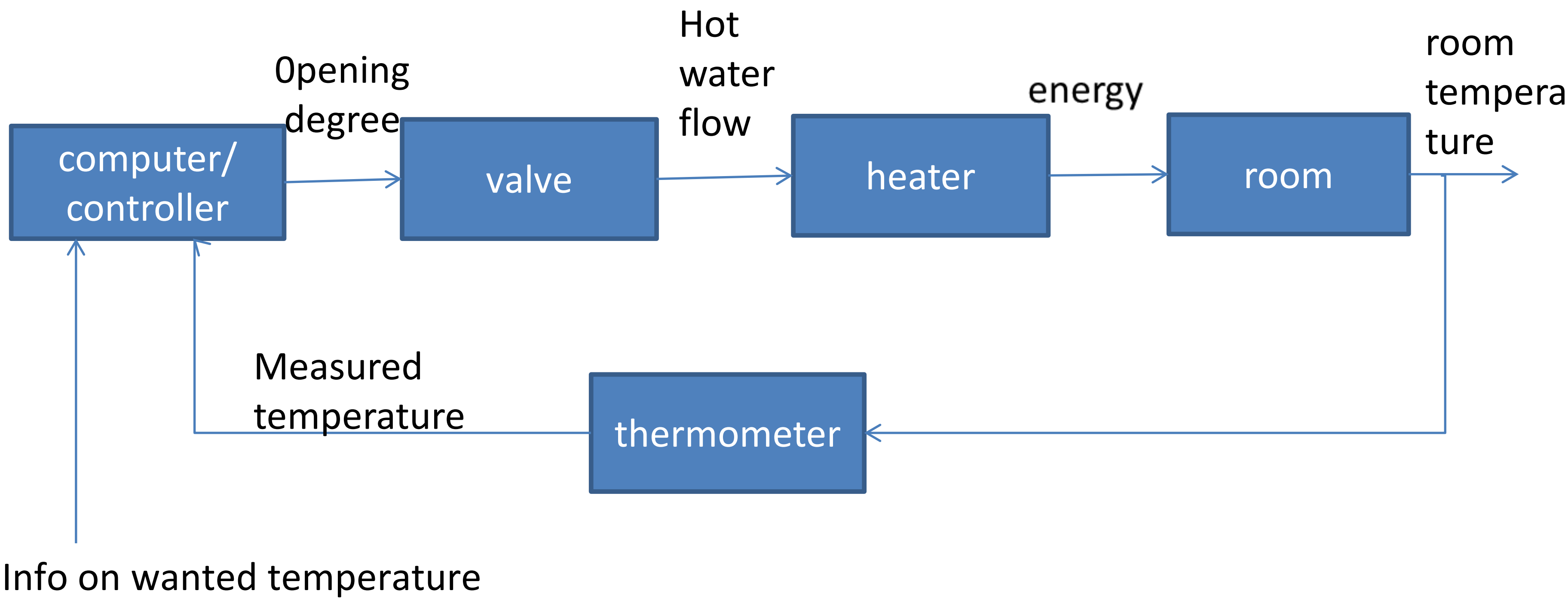


To get the exact temperature the thermodynamic properties have to be known very accurately

Functional block diagram for feed forward control of indoor temperature with disturbances

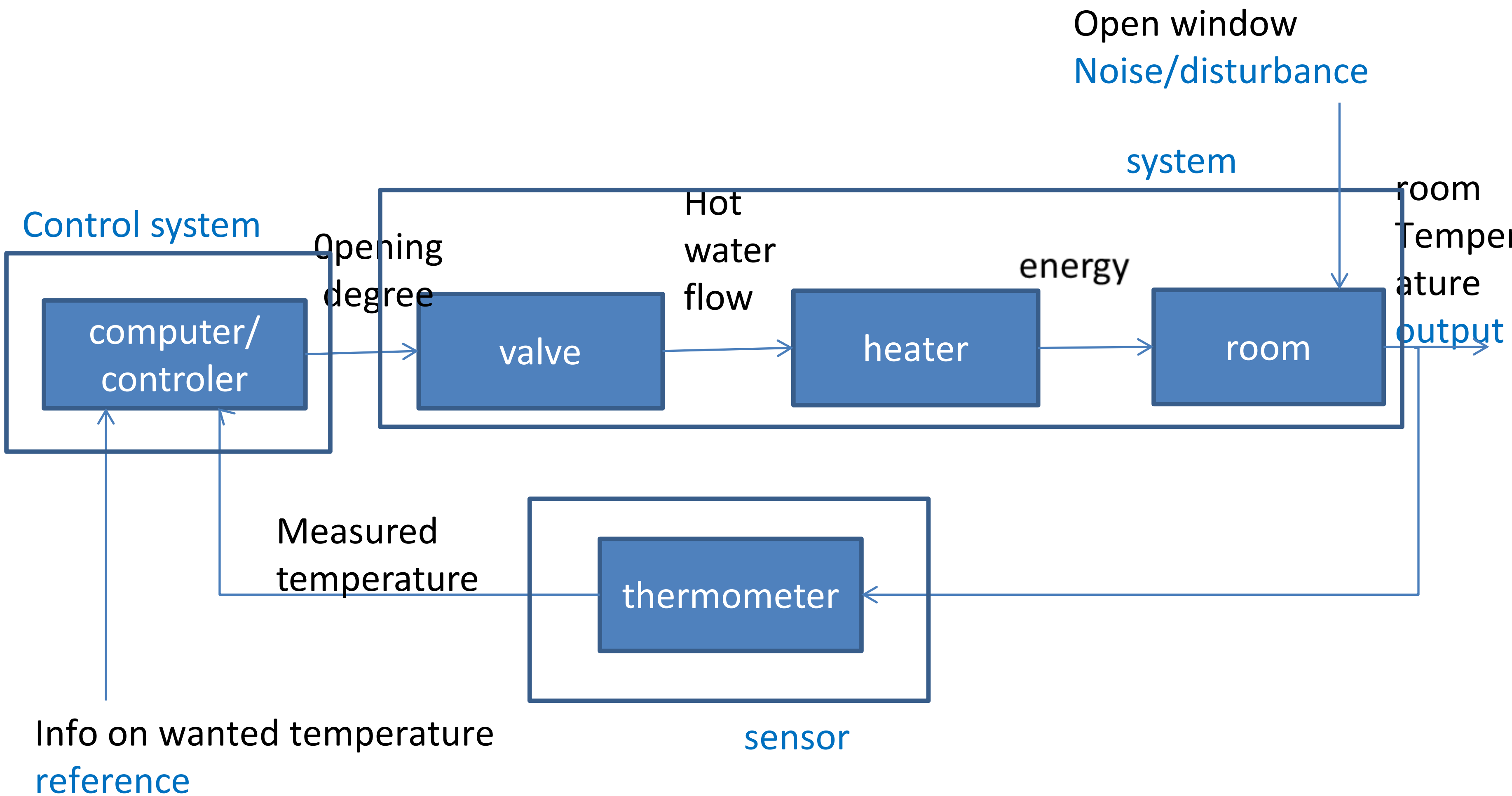


Functional block diagram for feedback control of indoor temperature

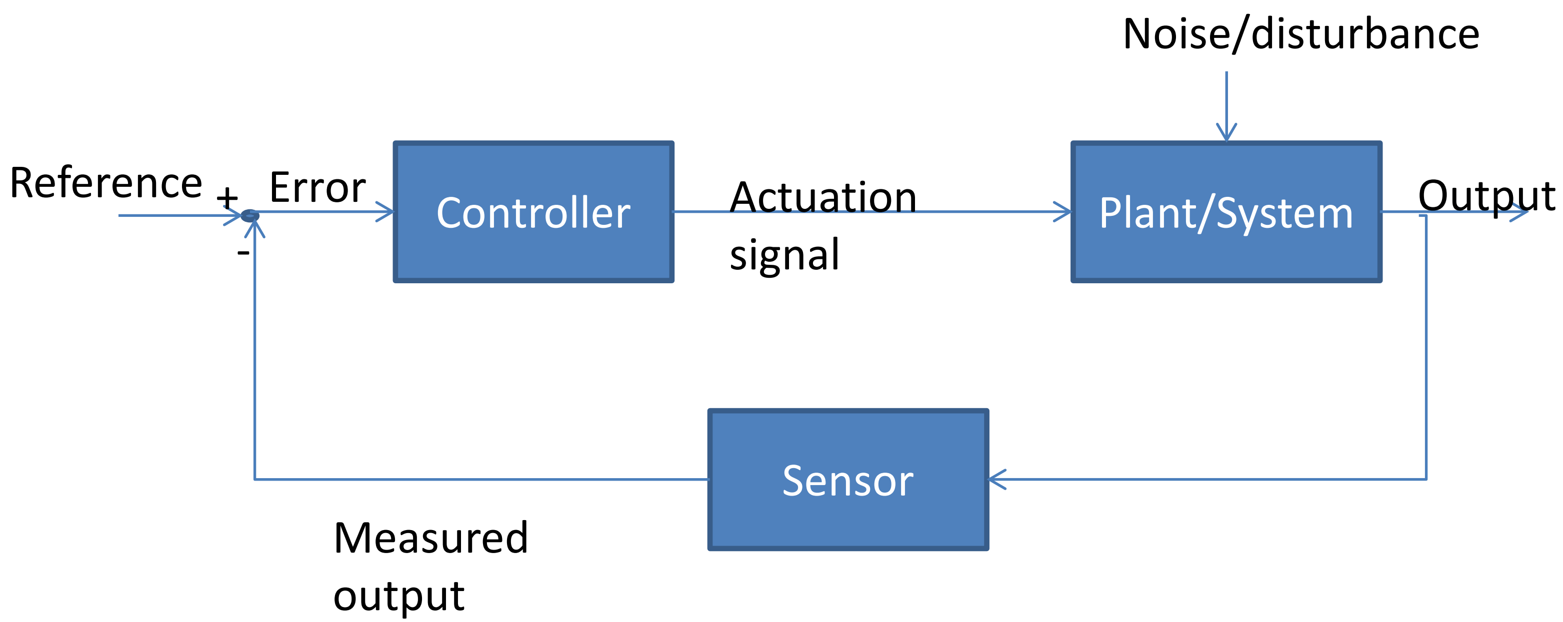


Feed back loop

Functional block diagram for feed back control of indoor temperature



Standard set-up feedback



Arrows are signals

Blocks are system models describing the relation between input and output to the block

Purposes : to follow a reference
to dampen disturbances

Why is transfer functions relevant in general

In creation of the universe it was decided that all relations between input and output with fair approximation can be described by an ordinary linear differential equation

Cite: John Nørgaard Nielsen

In this course we focus on linear systems or approximate to linear systems – you have learned linearization

Calculations are much easier if we use the Laplace transform of the differential equations ~ the transfer functions

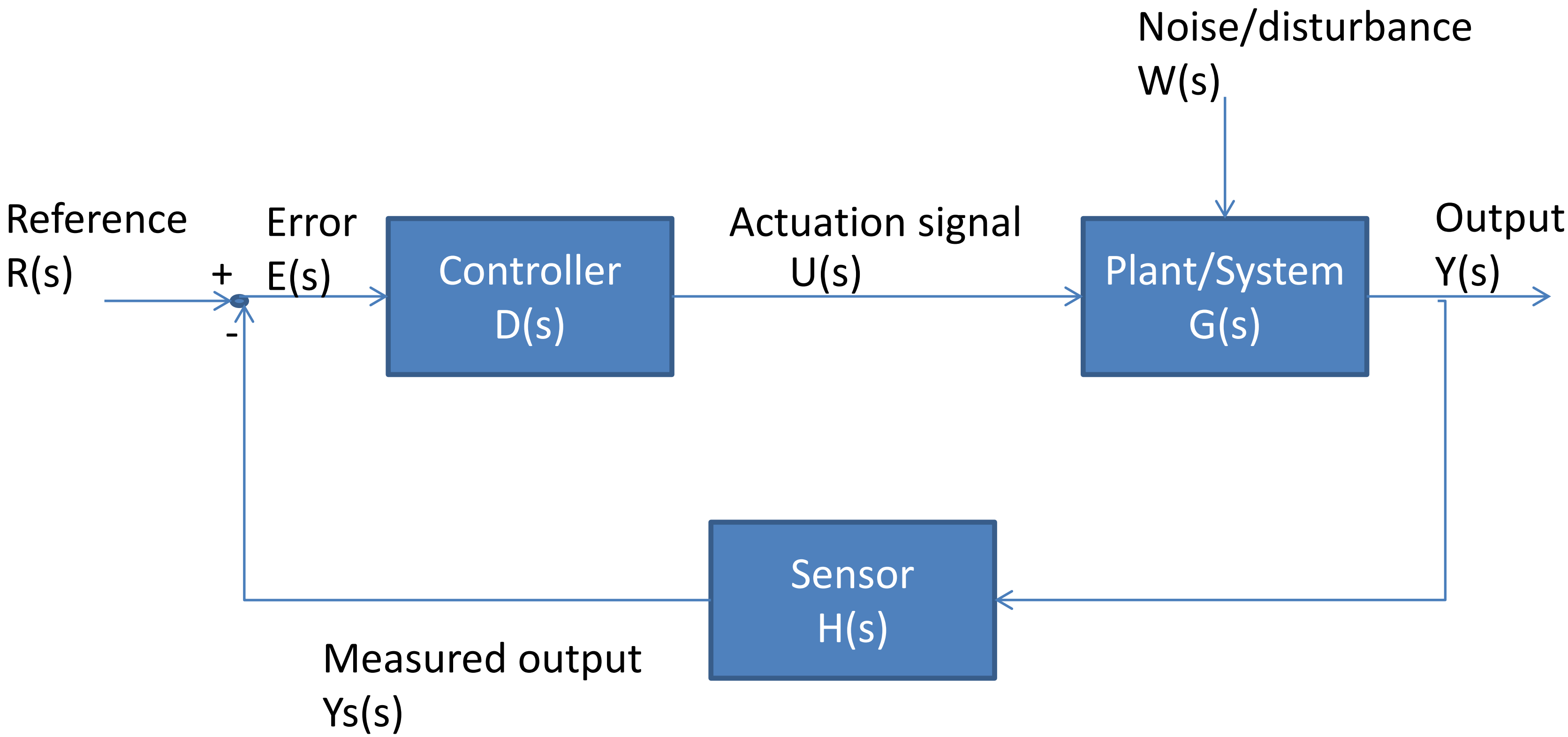
We will look at it in the end of this lecture

Modelling – description of the blocks

To make a mathematical description of the dynamic relation between the input to a system and the output from a system.

- The models can be based on physics e.g. Newtons laws or measurements
- The linear dynamics can be described as differential equations
- The transfer functions is found from the Laplace transform of differential equations

Standard set-up



Arrows are signals
Blocks are system models described as transfer functions = the Laplace transform of linear differential equation describing the relation between input and output

In this course we mainly focus on feed back controller design

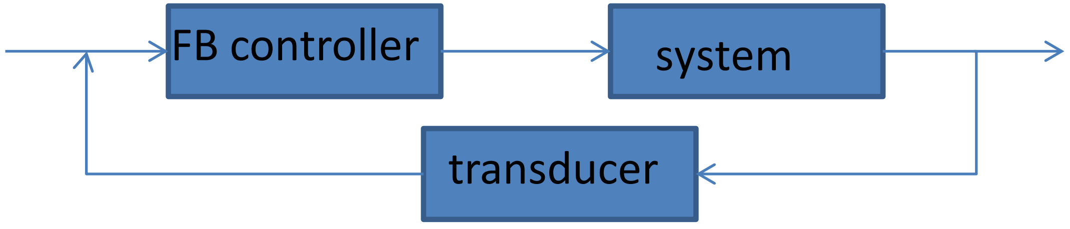
Break

Controller design

To design an algorithm that can calculate the necessary input to a system when a specified output is wanted.

In open loop control/feedforward the calculations are based on a model of the plant.

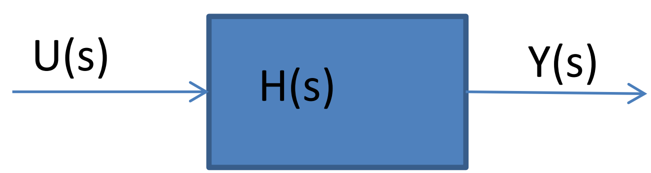
In feedback control the calculations are based on a model and measurements of the system output.



This course will mainly focus on feedback

Block diagrams/ structuring the system

- Blocks are transfer functions between the input to the block and the output from the block = a mathematical model.



$$\frac{Y(s)}{U(s)} = H(s)$$

Blocks can be connected in networks of simple transfer functions
Block diagrams creates a nice overview

Transfer functions

A transfer function is the relation between the input and output of a system described in the La Place domain

$$k\ f(t) = T\ \frac{d\ x(t)}{dt} + x(t)$$

$$kF(s) = TsX(s) + X(s)$$

$$k\ F(s) = [sT + 1]\mathrm{X}(\mathrm{s})$$

$$X(s) = \frac{k}{sT + 1} F(s)$$

La Place transform of input

La Place transform of output

Transfer function

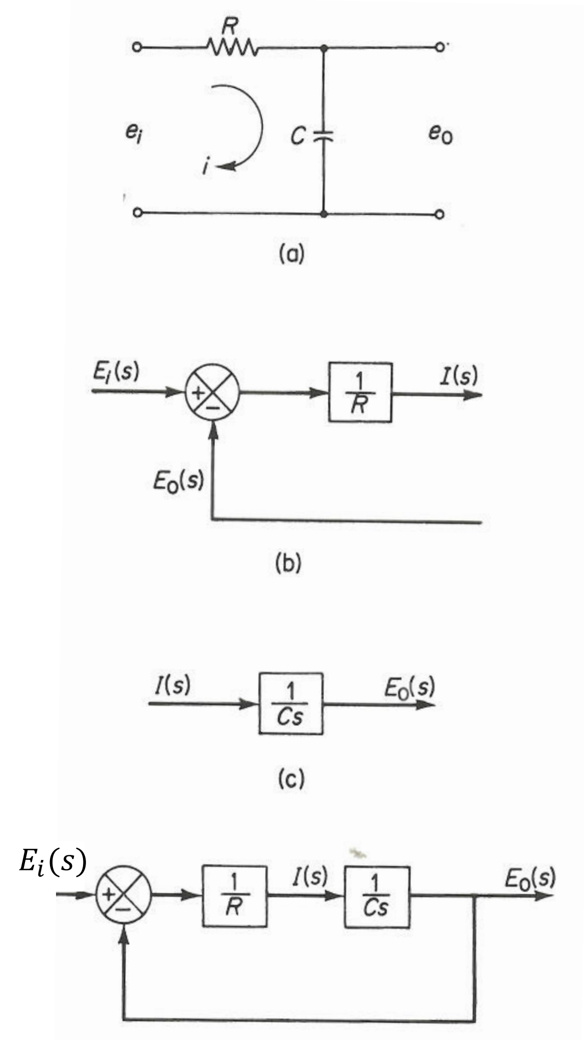
RC circuit –a well known example

$$i(t) = \frac{e_i(t)\ -\ e_o(t)}{R}$$

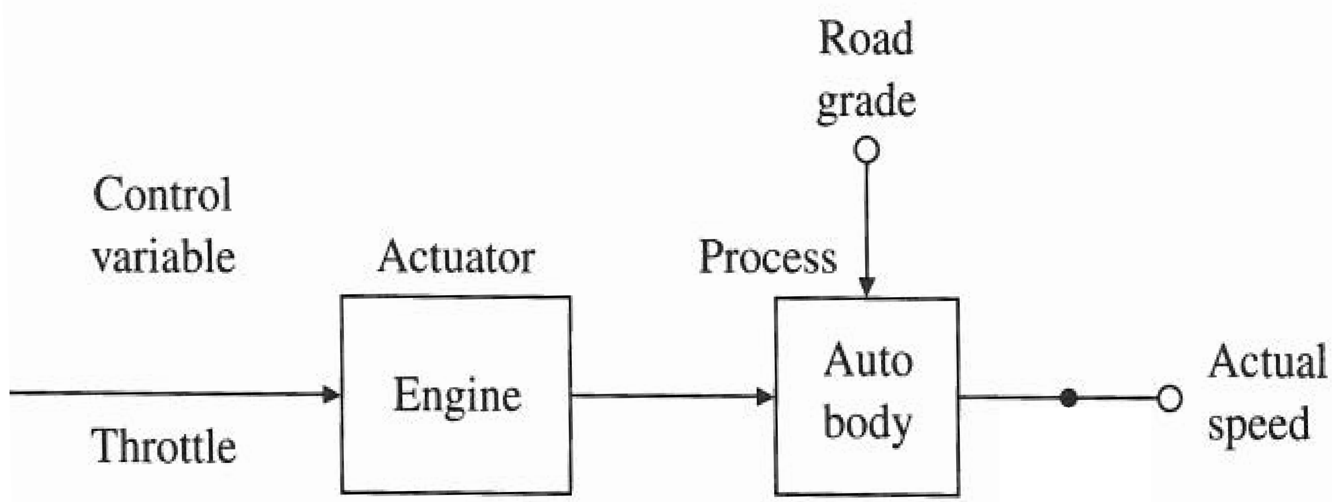
$$e_o = \frac{\int idt}{C}$$

$$I(s) = \frac{E_i(s) - E_o(s)}{R}$$

$$E_0(s) = \frac{1}{Cs} I(s)$$



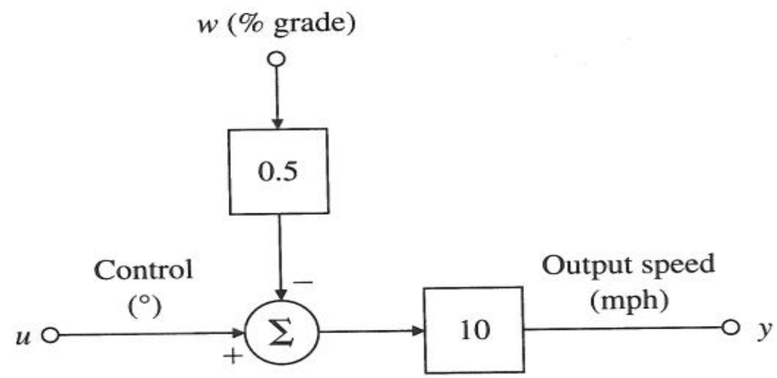
Automobile cruise control example



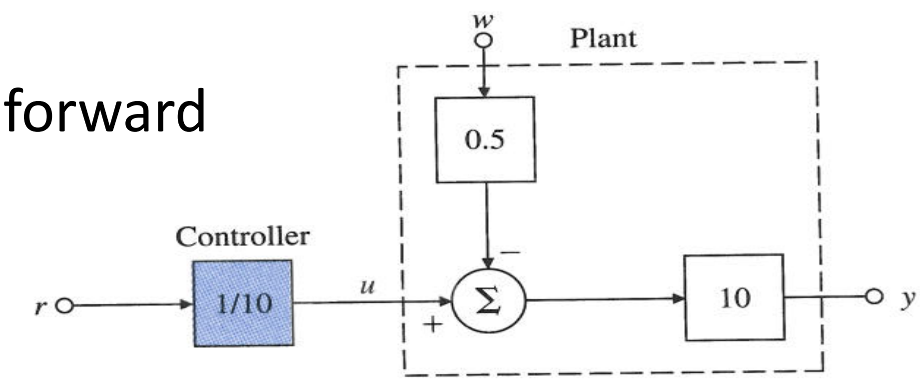
Open loop
Problem: there is no compensation for the road grade

Cruise control

plant

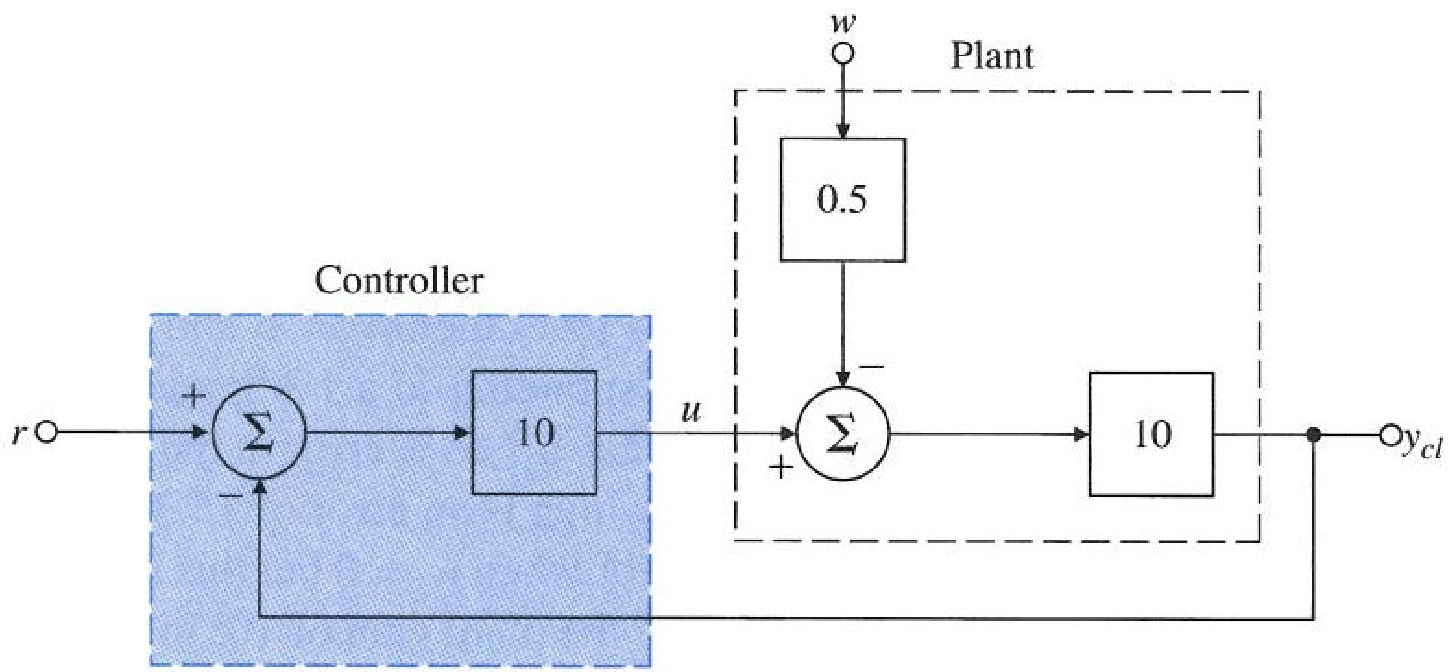


Open loop/ feed forward



Average slope is 0.5 It is not taken into accout in the open loop design

Closed loop cruise control



Room temperature control system

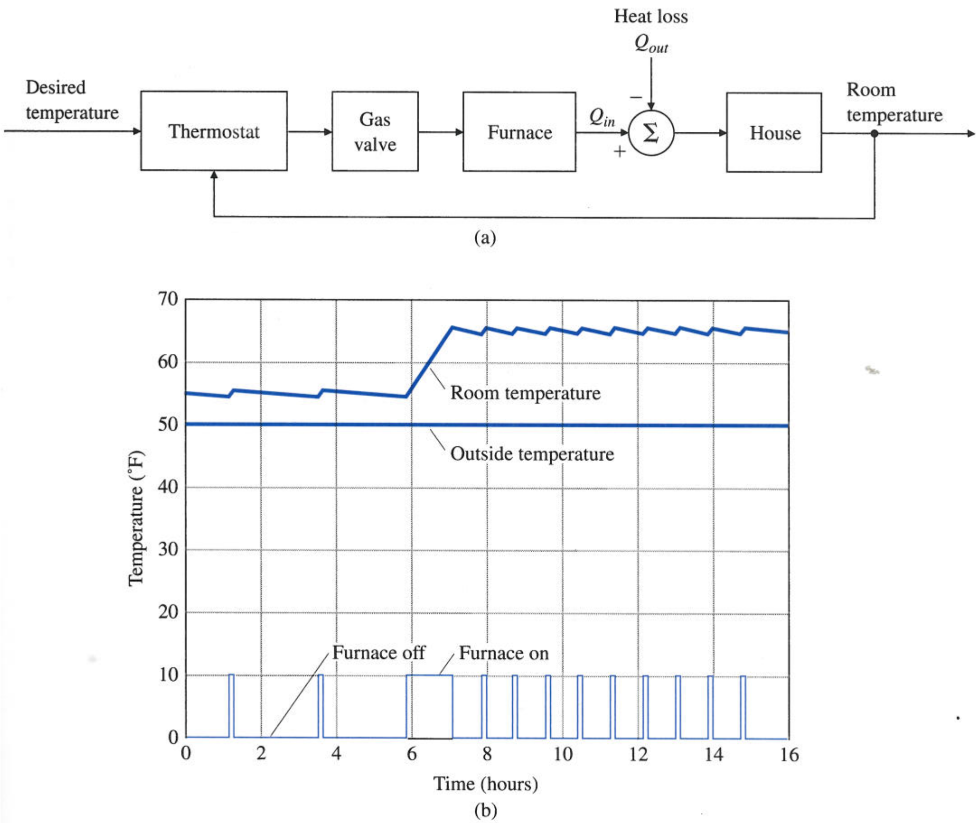
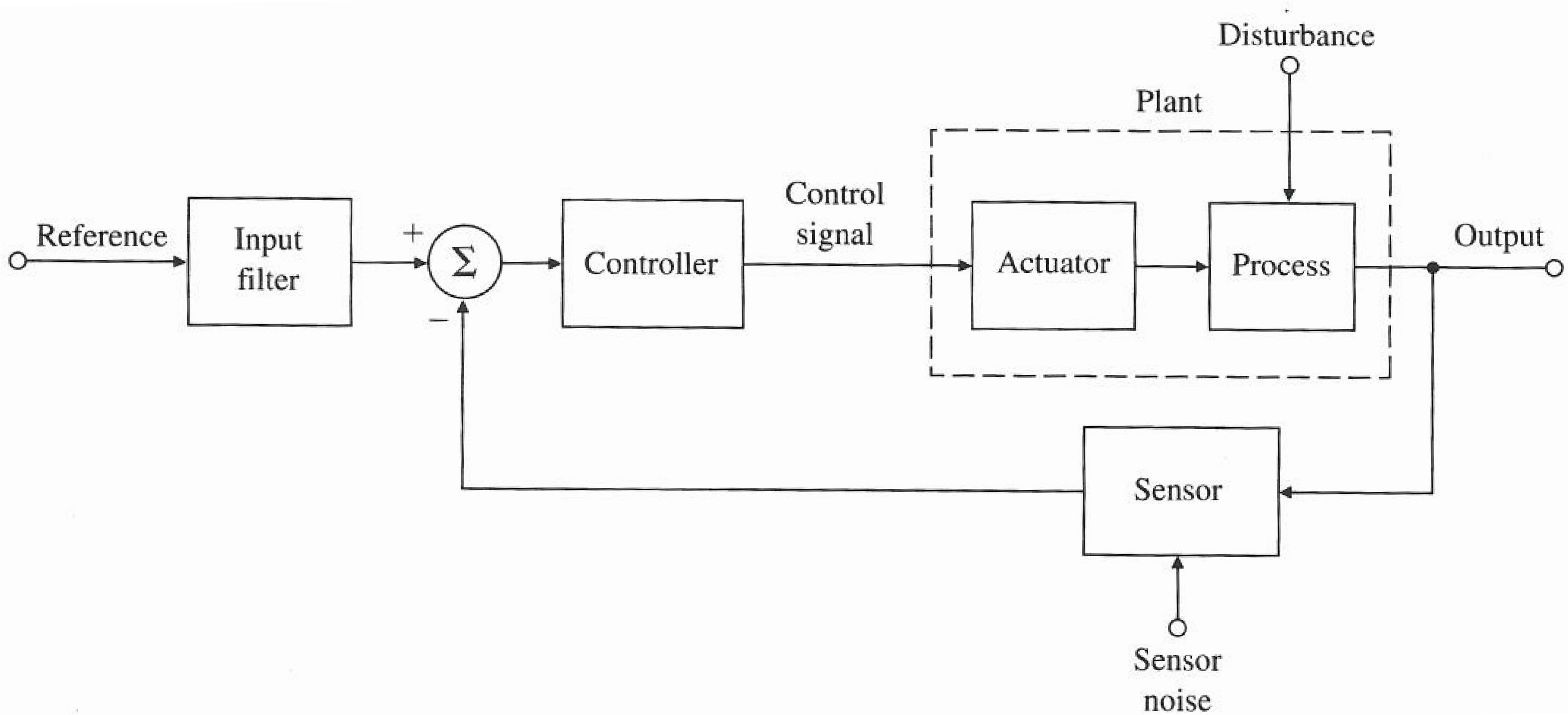


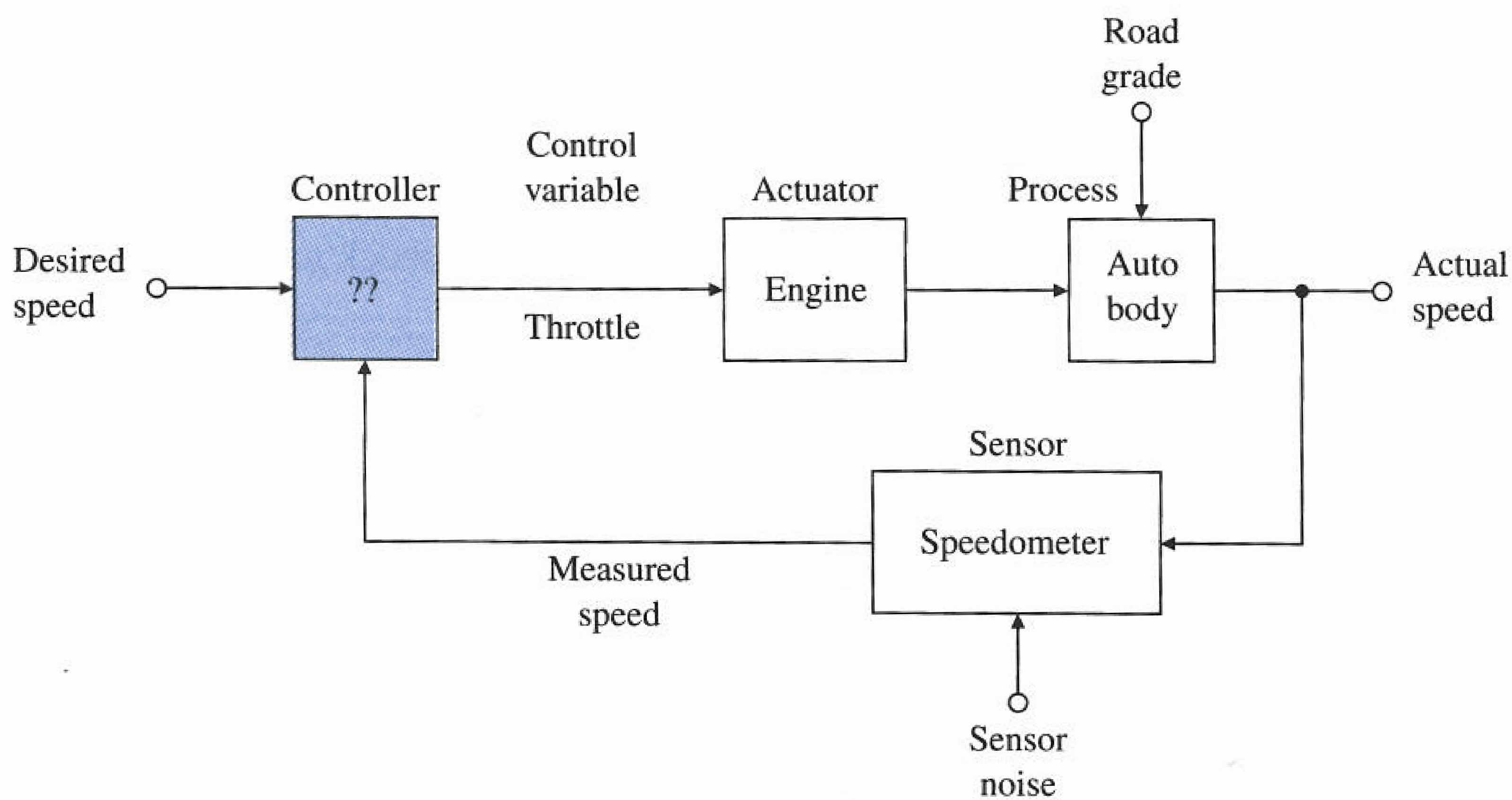
Figure 1.1

(a) Component block diagram of a room temperature control system; (b) plot of room temperature and furnace action

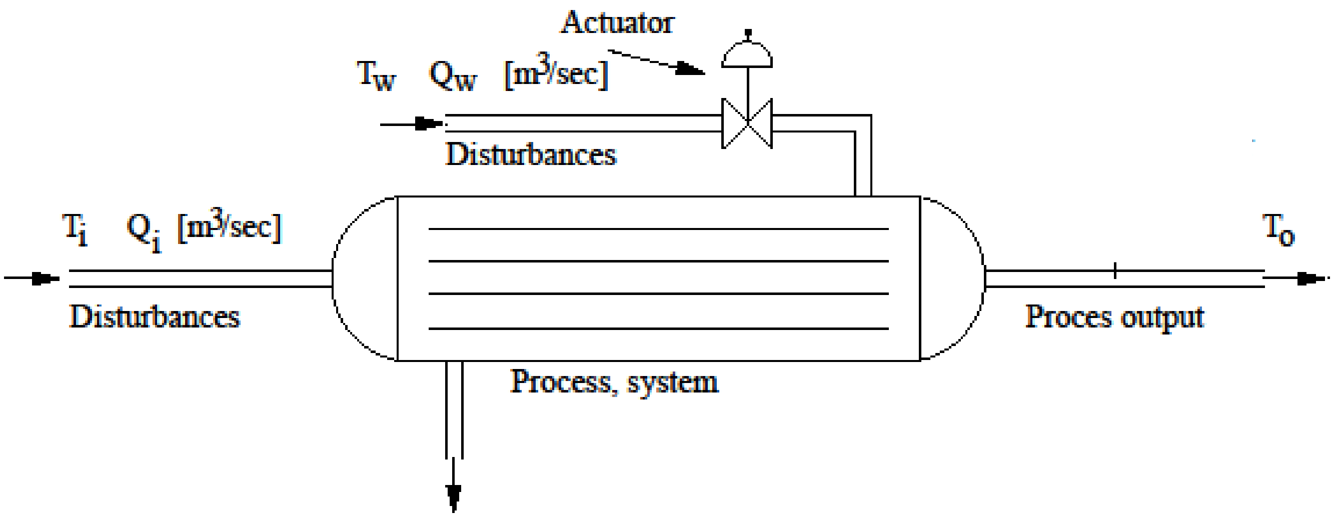
General feedback control



Automobile cruise control

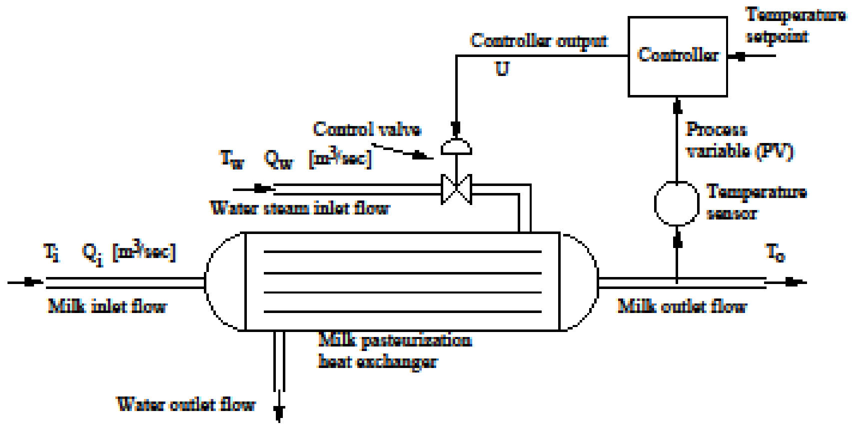


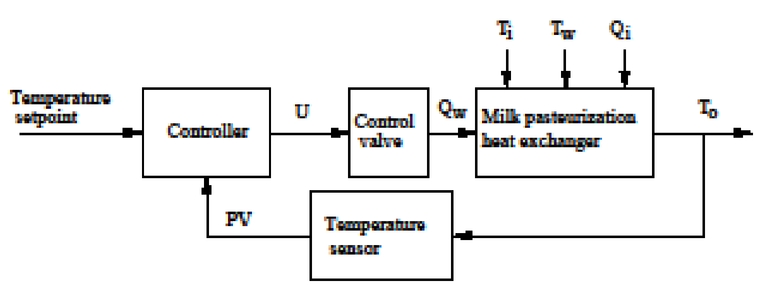
Example: Milk Pasteurization



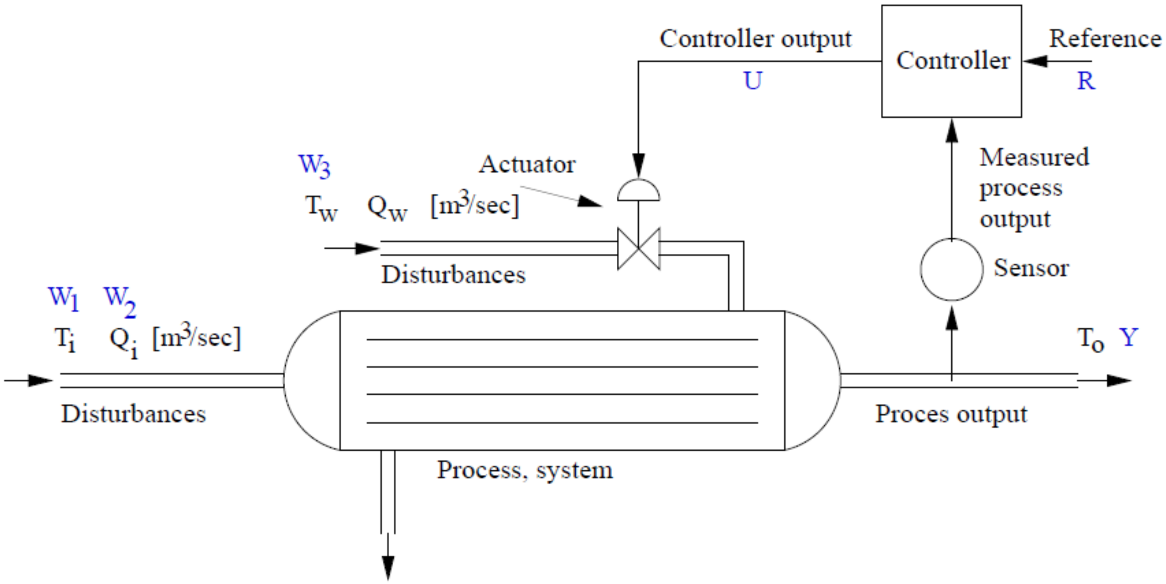
What is input output and disturbances ???

Feedback structure



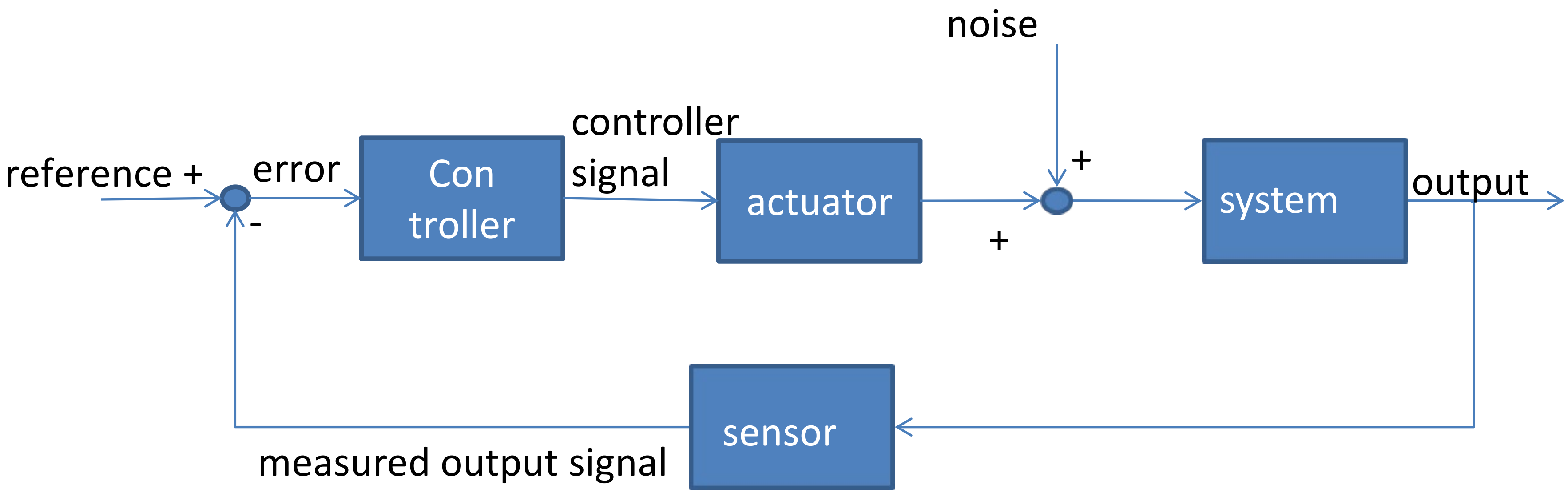


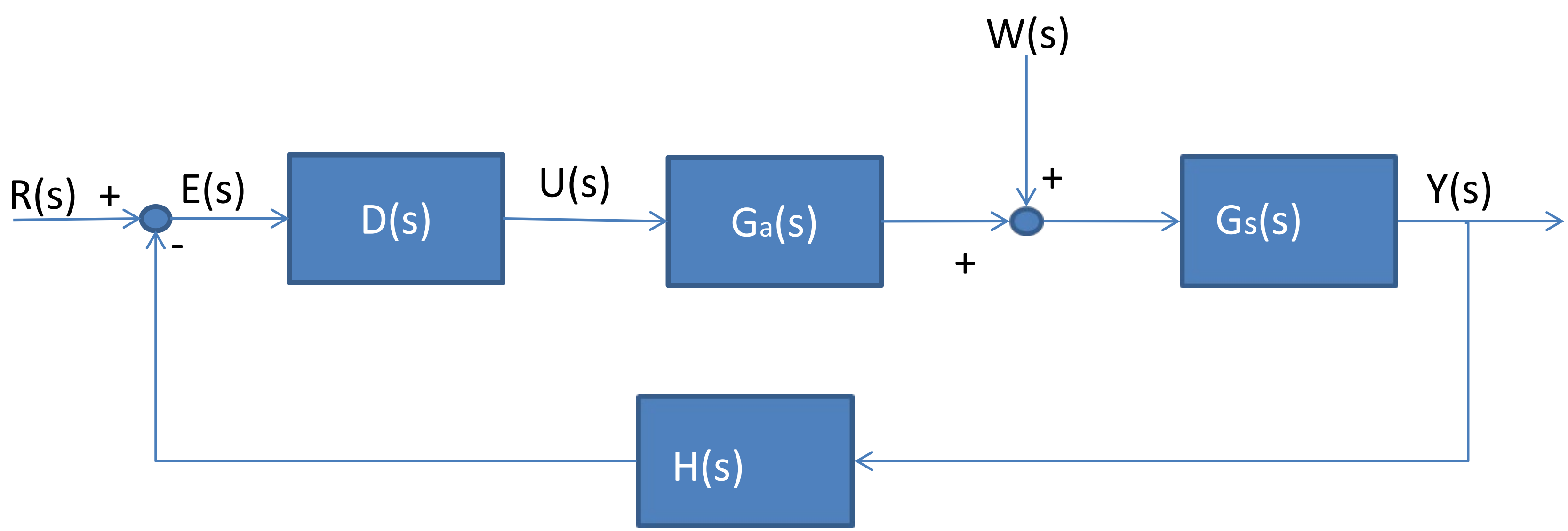
Notation



Controller output
U
Controller
Reference
R
Actuator
W_3
T_w Q_w [m^3/sec]
Measured
process
output
Disturbances
Sensor
W_1 W_2
T_i Q_i [m^3/sec]
T_o Y
Disturbances
Proces output
Process, system

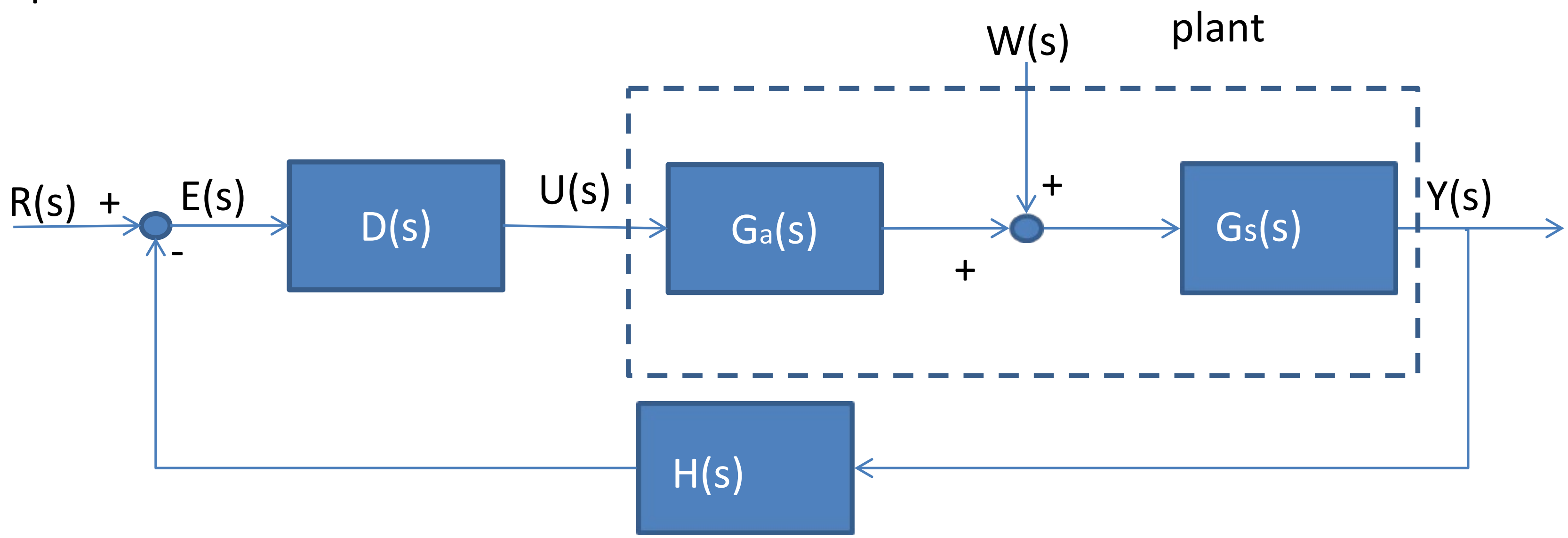
The control loop



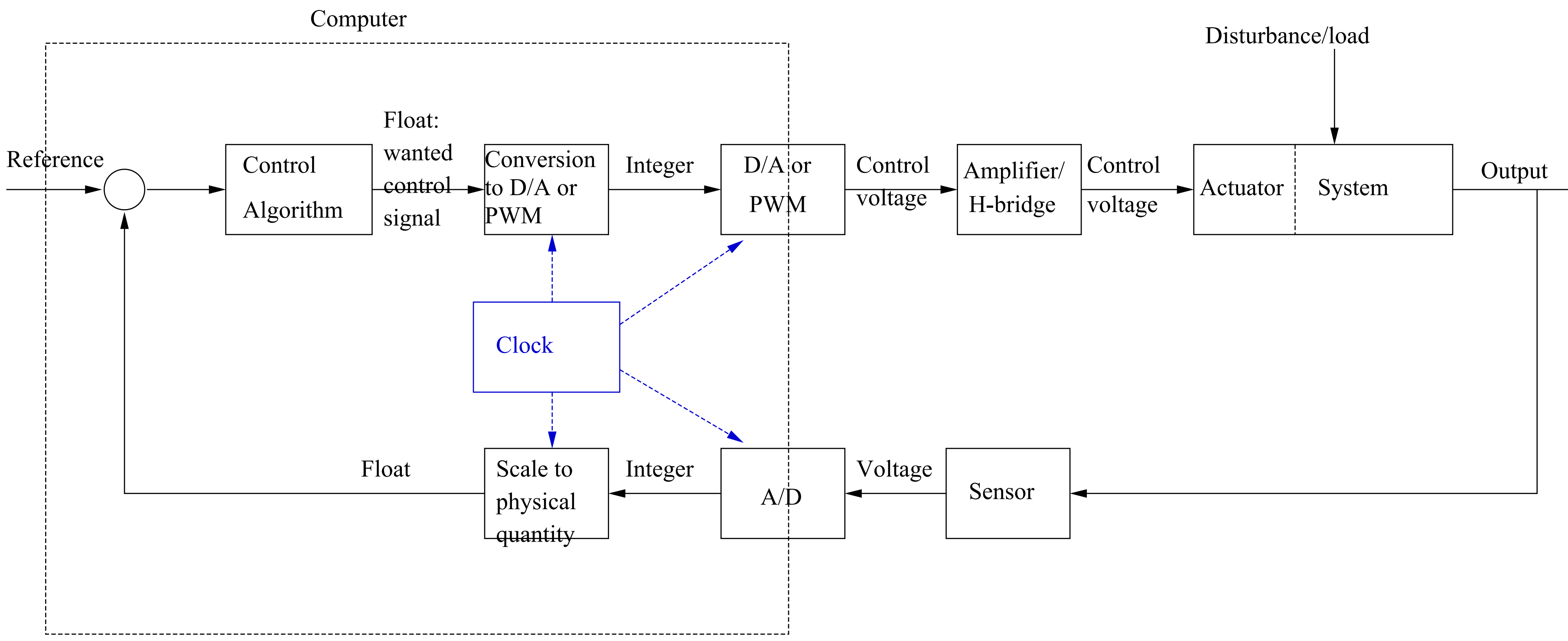


Tasks in the control loop

Find the structure of the control problem (block diagram)
Find differential equations (models) describing G_a, G_s and H
(fx Ga is a motor including PWM, Gs is a robot, H is a tachometer)
Linearize the equations
Find the Lapalce transform
Evaluate the models in the time domain and the frequency domain
Choose the controller type
Design the controller
Simulate
Implement



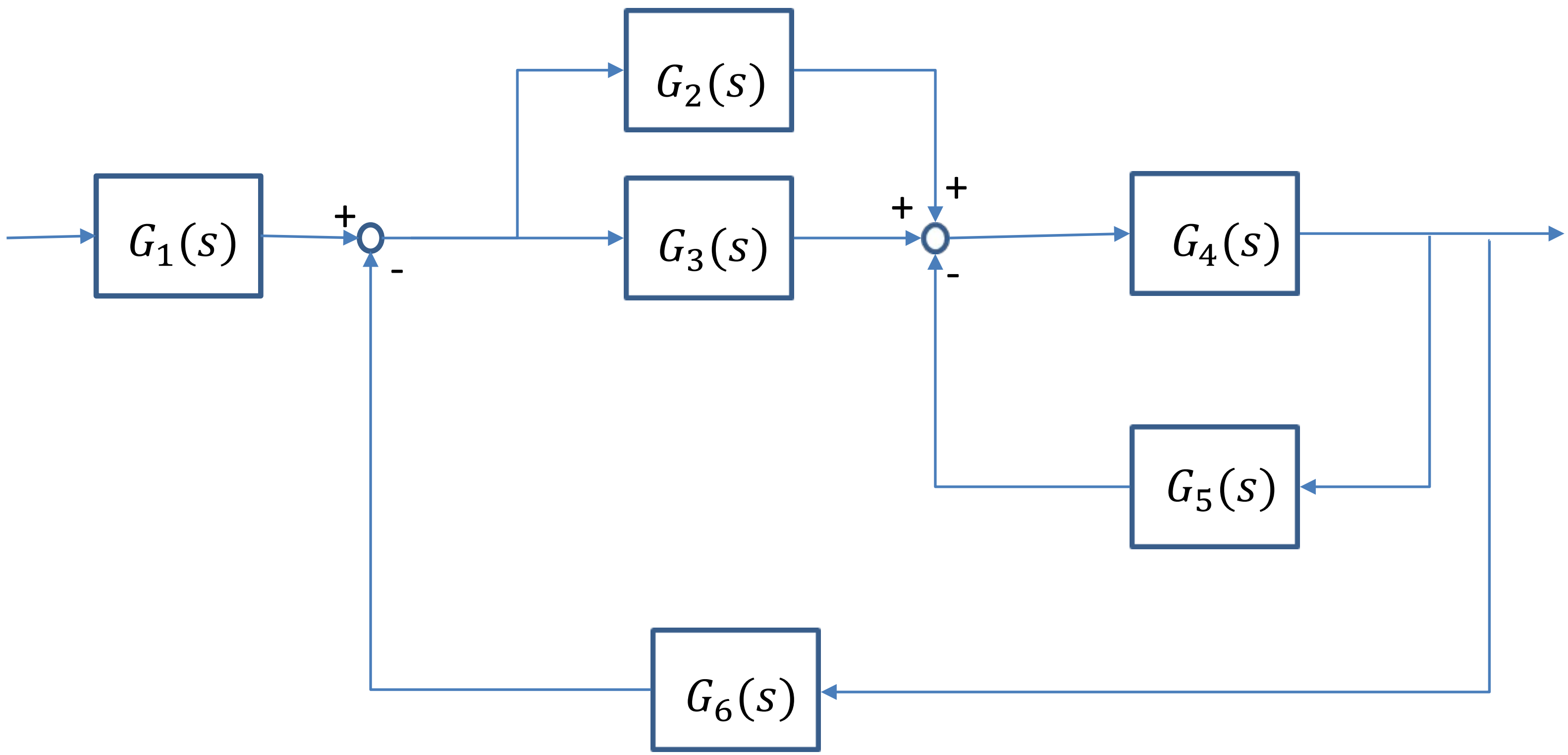
Detailed Functional Block diagram for a feed back loop



The elements will be handled in this course

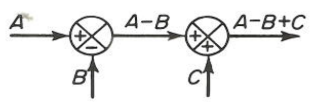
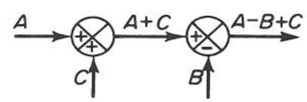
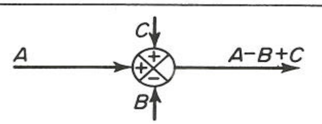
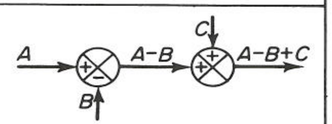
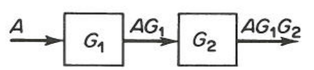
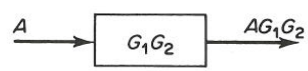
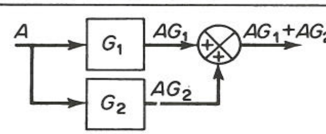
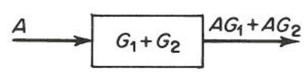
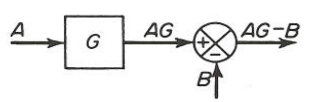
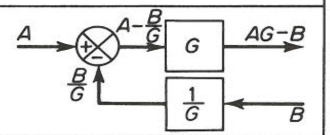
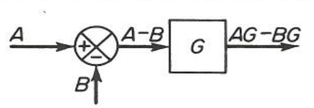
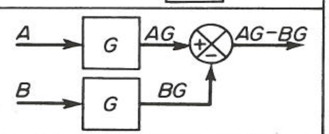
- BREAK

Block diagram reduction

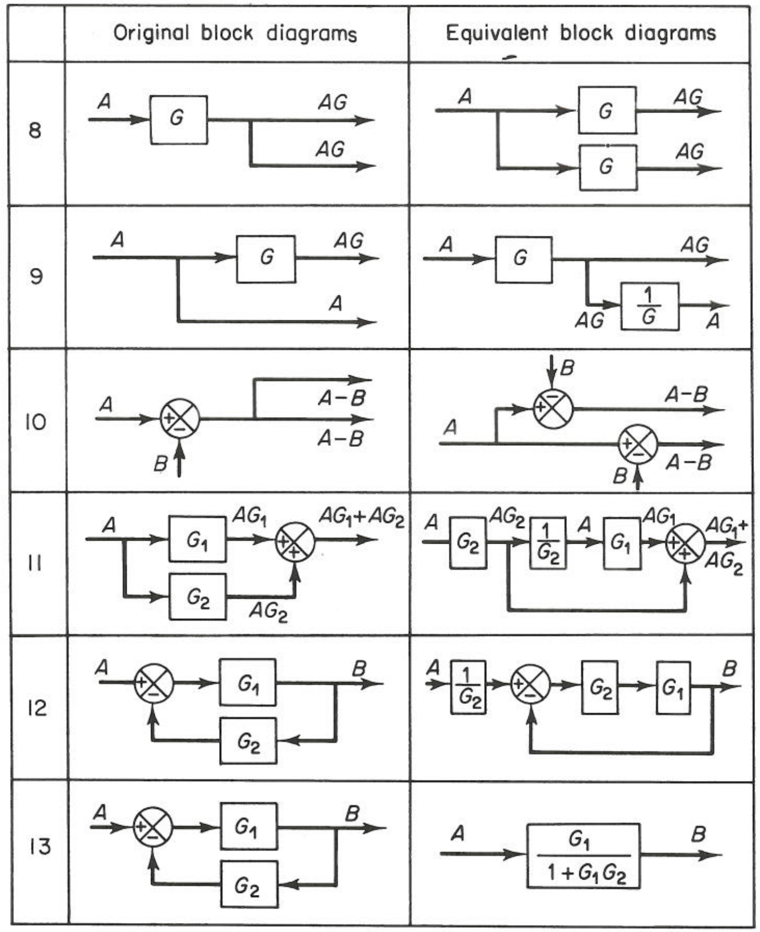


Block diagram algebra

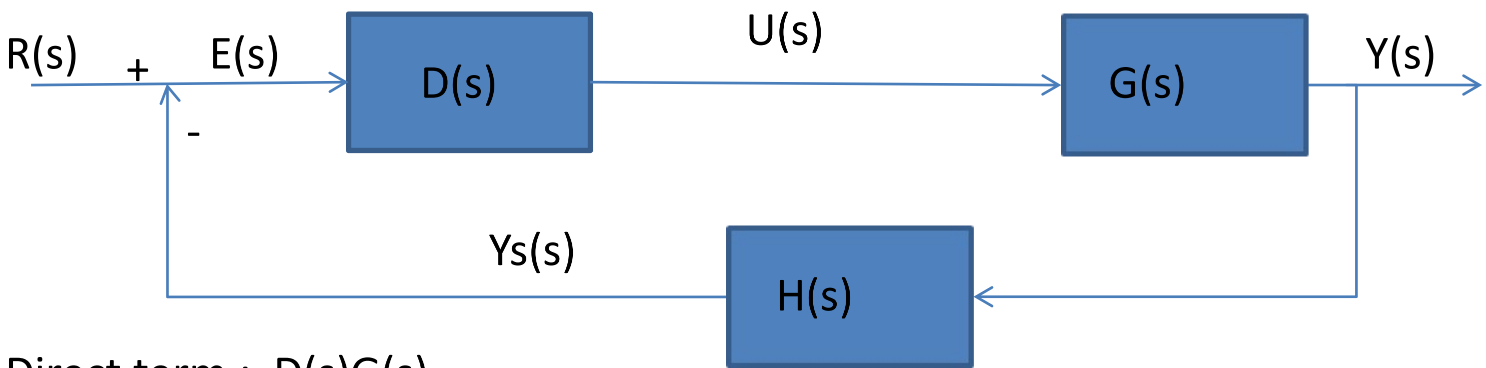
Table 4-3. Rules of Block Diagram Algebra

	Original block diagrams	Equivalent block diagrams
1		
2		
3		
4		
5		
6		
7		

Block diagram algebra



Most important transfer function in this course



Direct term : D(s)G(s)

Open loop : D(s)G(s)H(s)

Closed loop : Y(s)= G(s)U(s) = G(s)D(s)E(s)

E(s)= R(s)-Ys(s)

Ys(s)= H(s)Y(s) =>

Y(s) = D(s)G(s)(R(s)-H(s)Y(s))

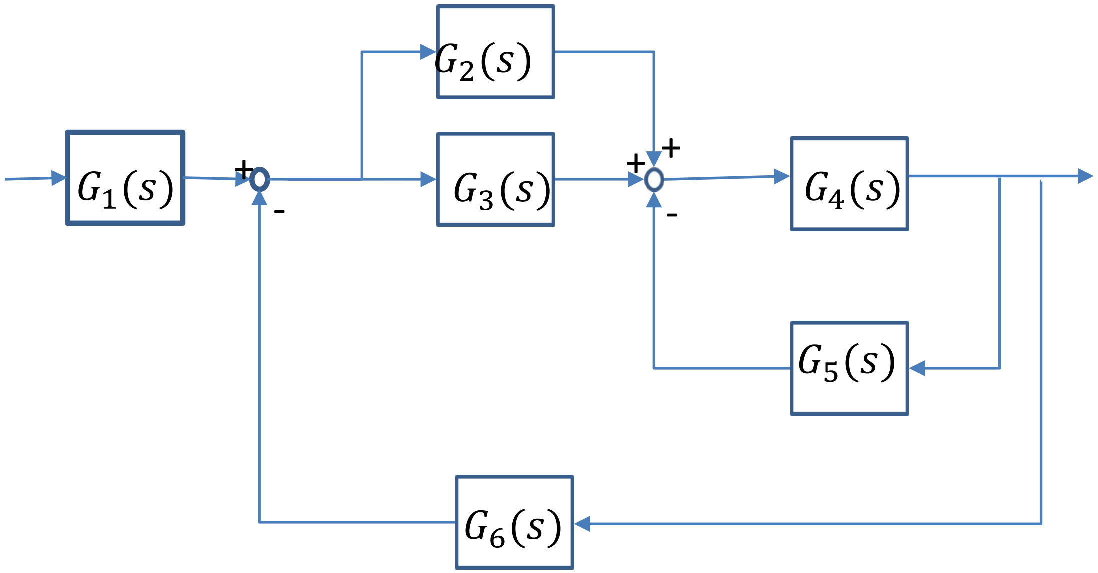
Y(s) = D(s)G(s)R(s) – D(s)G(s)H(s)Y(s)

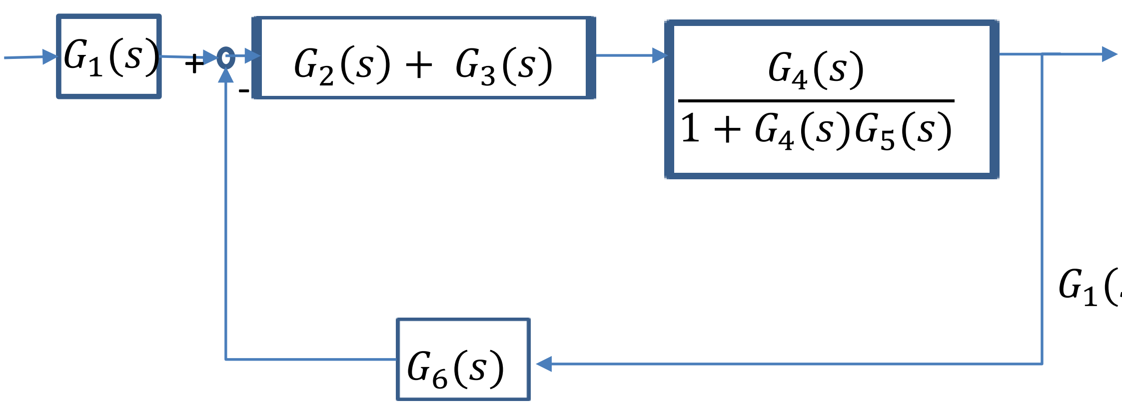
Y(s)(1+D(s)G(S)H(s)) = D(s)G(s)R(s)

$$\boxed{\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)} = \frac{direct\ term}{1 + open\ loop}}$$

Sometimes we like to call this the golden formula

Block diagram reduction





$$G_1(s)\frac{(G_2(s) + G_3(s))\frac{G_4(s)}{1 + G_4(s)G_5(s)}}{1 + G_6(s)(G_2(s) + G_3(s))\frac{G_4(s)}{1 + G_4(s)G_5(s)}}$$

Block diagram reduction RC-circuit

$$i(t) = \frac{e_i(t)\ -\ e_o(t)}{R}$$

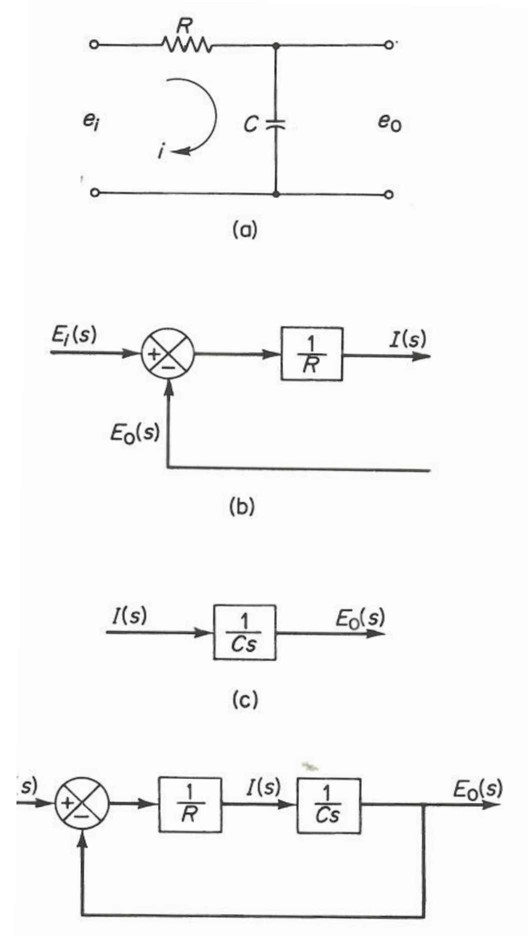
$$e_o = \frac{\int idt}{C}$$

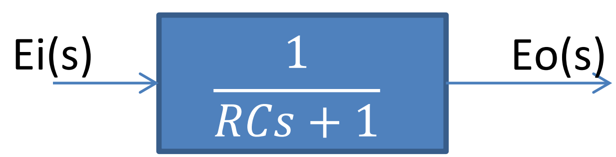
$$I(s) = \frac{E_i(s) - E_o(s)}{R}$$

$$E_0(s) = \frac{1}{Cs} I(s)$$

$$E_0(s) = \frac{1}{Cs}\frac{E_i(s) - E_o(s)}{R} \Rightarrow \left(1 + \frac{1}{RCs}\right) E_o(s) = \frac{1}{RCs} E_i(s)$$

$$\frac{E_o(s)}{E_I(s)} = \frac{\frac{1}{R}\frac{1}{Cs}}{1 + \frac{1}{R}\frac{1}{Cs}} = \frac{1}{RCs + 1}$$





Block diagram reduction RC-circuit

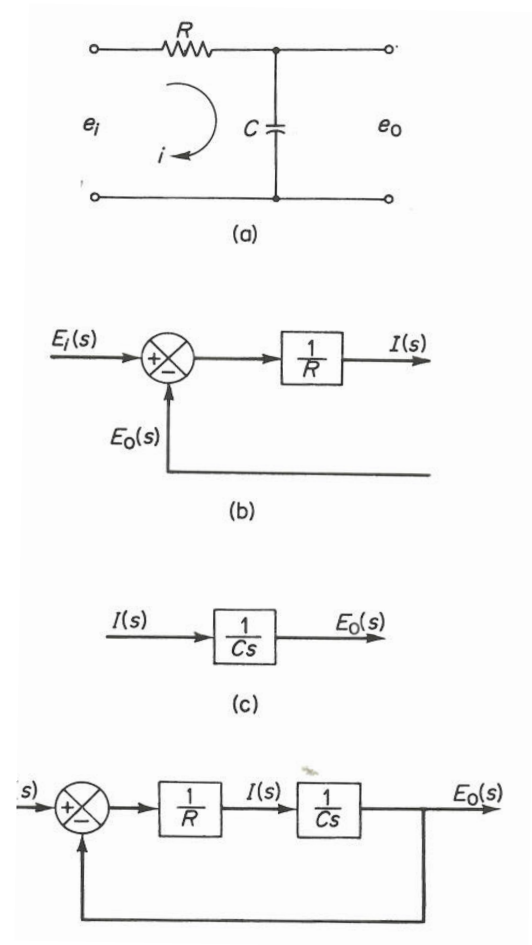
$$i(t) = \frac{e_i(t) \; - \; e_o(t)}{R}$$

$$e_o = \frac{\int idt}{C}$$

$$I(s) = \frac{E_i(s) - E_o(s)}{R}$$

$$E_0(s) = \frac{1}{Cs} I(s)$$

$$E_0(s) = \frac{1}{Cs} \frac{E_i(s) - E_o(s)}{R} \Rightarrow \left(1 + \frac{1}{RCs}\right) E_o(s) = \frac{1}{RCs} E_i(s)$$



$$\boxed{\frac{E_o(s)}{E_I(s)} = \frac{\frac{1}{R}\frac{1}{Cs}}{1 + \frac{1}{R}\frac{1}{Cs}} = \frac{1}{RCs + 1}}$$

$$\boxed{\frac{E_o(s)}{E_I(s)} = \frac{direct\ term}{1 + open\ loop}}$$