

# Communication Systems.

Lecture 3, part 1

Antenna Arrays and Microstrip antennas

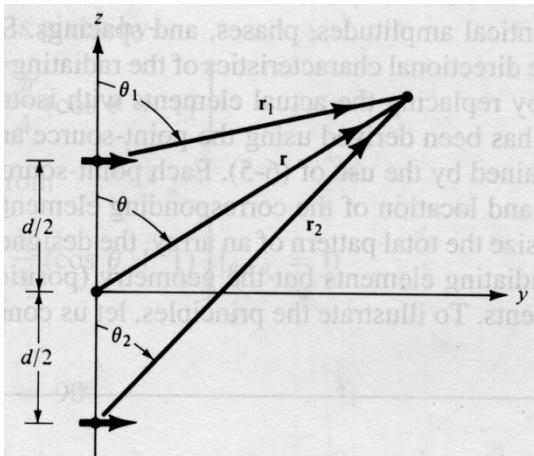
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# Antenna Arrays

In an array of identical elements, there are 6 controls for Shaping the overall radiation pattern of the array:

- 1 The Geometrical configuration of the array (linear, circular, rectangular etc.)
- 2 The relative displacement of the individual elements
- 3 The excitation amplitude of the elements
- 4 The excitation phase of the elements
- 5 The radiation pattern of the elements
- 6 Number of elements

# Two-element Array



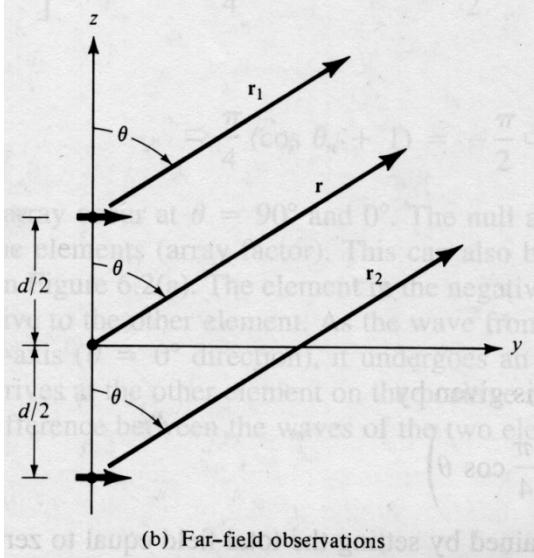
$$\vec{E}_t = \vec{E}_1 + \vec{E}_2 =$$

$$\hat{a}_\theta j \eta \frac{k I_0 l}{4\pi} \left\{ \frac{e^{-j(kr_1 - \frac{\beta}{2})}}{r_1} \cos(\theta_1) + \frac{e^{-j(kr_2 - \frac{\beta}{2})}}{r_2} \cos(\theta_2) \right\}$$

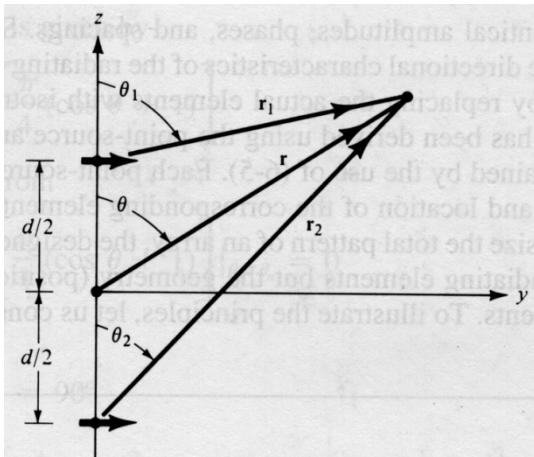
$$\theta_1 \approx \theta_2 \approx \theta$$

$$\left. \begin{aligned} r_1 &\approx r - \frac{d}{2} \cos(\theta) \\ r_2 &\approx r + \frac{d}{2} \cos(\theta) \end{aligned} \right\} \text{for Phase}$$

$$r_1 \approx r_2 \approx r \} \text{for Amplitude}$$



# Two-element Array



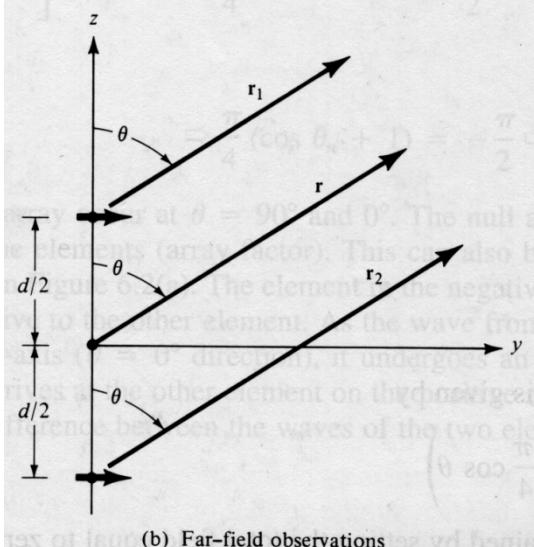
$$\vec{E}_t = \vec{E}_1 + \vec{E}_2 = \hat{a}_\theta j \eta \frac{k I_0 l}{4\pi} \left\{ \frac{e^{-j(kr_1 - \frac{\beta}{2})}}{r_1} \cos(\theta_1) + \frac{e^{-j(kr_2 - \frac{\beta}{2})}}{r_2} \cos(\theta_2) \right\}$$

Assuming Farfield observations =>

$$\vec{E}_t = \hat{a}_\theta j \eta \frac{k I_0 l e^{-jkr}}{4\pi r} \cos(\theta) 2 \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

or

$$\vec{E}_t = \vec{E}_{SingleElement} \times ArrayFactor$$



# Two-element Array

$$\vec{E}_t = \vec{E}_{SingleElement} \times ArrayFactor$$

The array factor (AF) is a function of the geometry of the array and the excitation phase (uniform array). =>  
By varying the separation  $d$ , and/or the phase,  $\beta$ , between the elements the characteristic of the array antenna can be controlled!

# Array general

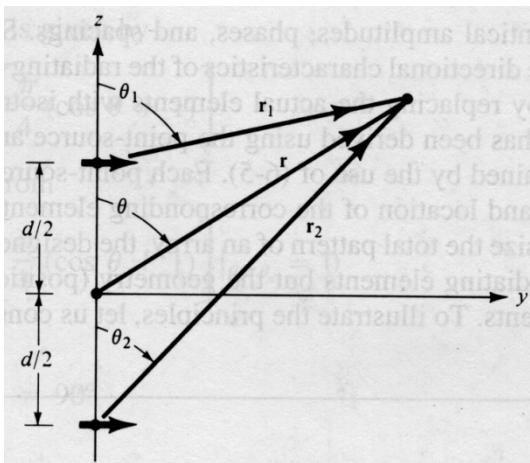
$$\vec{E}_t = \vec{E}_{SingleElement} \times ArrayFactor$$

Each array has its own array factor.

The AF is determinate by N, number of elements, their location, their relative magnitude and phase and the spacing.

The AF is simpler for identical amplitudes, phase and spacing.

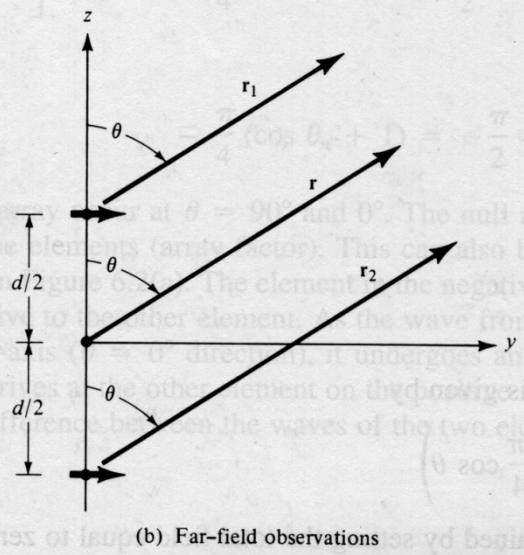
# Two Element array example



(a) Two infinitesimal dipoles

$$\vec{E}_t = \hat{a}_\theta j \eta \frac{k I_0 l e^{-jkr}}{4\pi r} \cos(\theta) 2 \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$



(b) Far-field observations

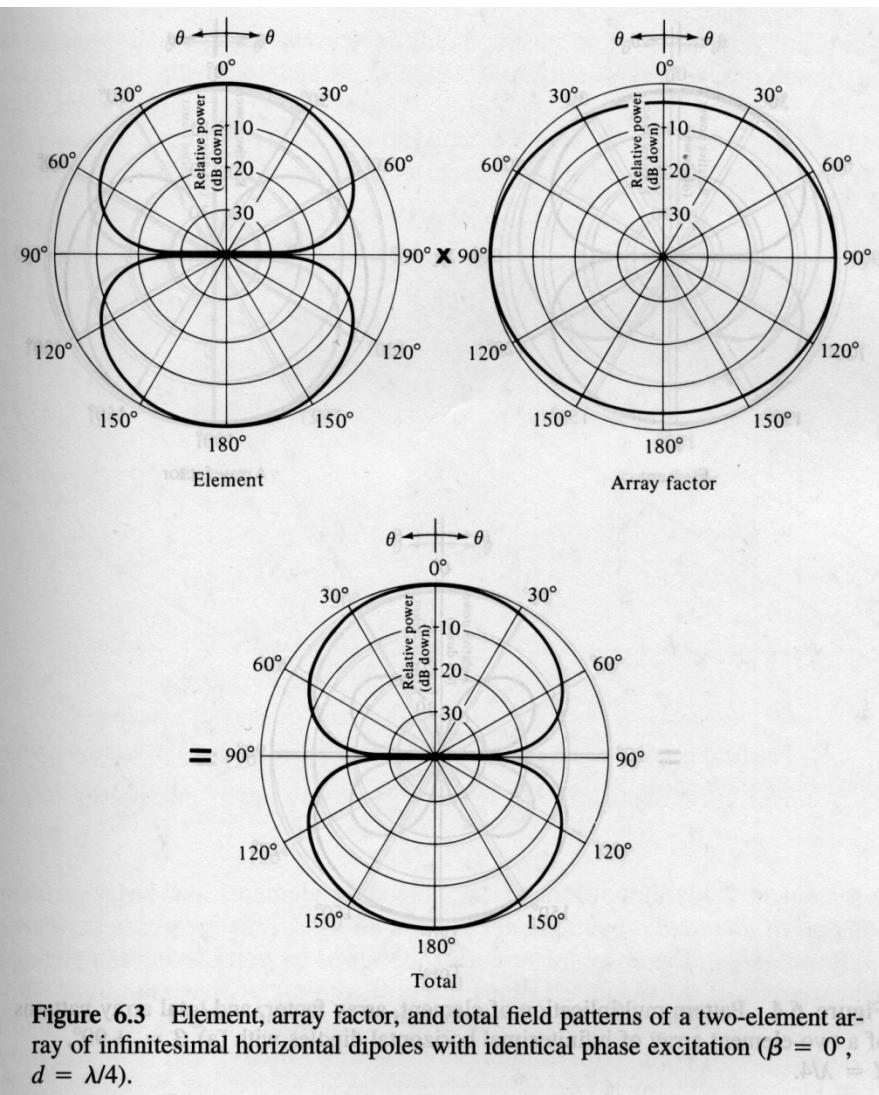
$$\beta = 0, \quad d = \frac{\lambda}{4} \Rightarrow$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4} \cos \theta\right)$$

$$E_{tn} = 0 \Rightarrow$$

$$\theta_n = 90^\circ$$

# Two Element array example



**Figure 6.3** Element, array factor, and total field patterns of a two-element array of infinitesimal horizontal dipoles with identical phase excitation ( $\beta = 0^\circ$ ,  $d = \lambda/4$ ).

$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

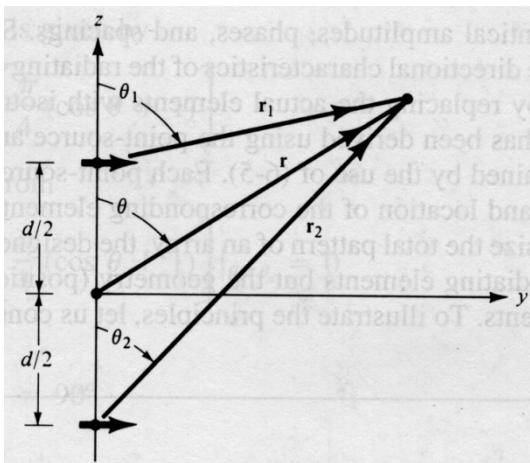
$$\beta = 0, \quad d = \frac{\lambda}{4} \Rightarrow$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4} \cos \theta\right)$$

$$E_{tn} = 0 \Rightarrow$$

$$\theta_n = 90^\circ$$

# Two Element array example



(a) Two infinitesimal dipoles

$$\vec{E}_t = \hat{a}_\theta j \eta \frac{k I_0 l e^{-jkr}}{4\pi r} \cos(\theta) 2 \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

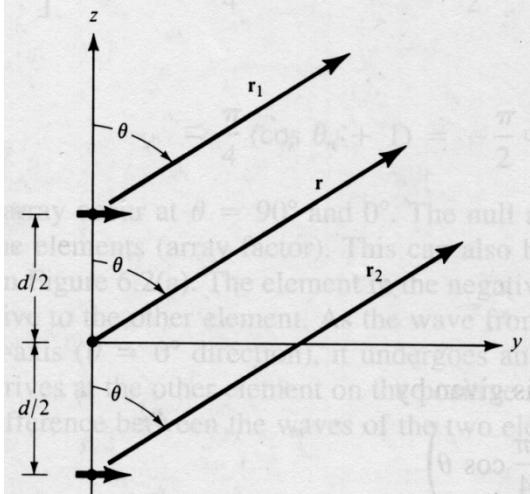
$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

$$\beta = \frac{\pi}{2}, \quad d = \frac{\lambda}{4} \Rightarrow$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4}(\cos \theta + 1)\right)$$

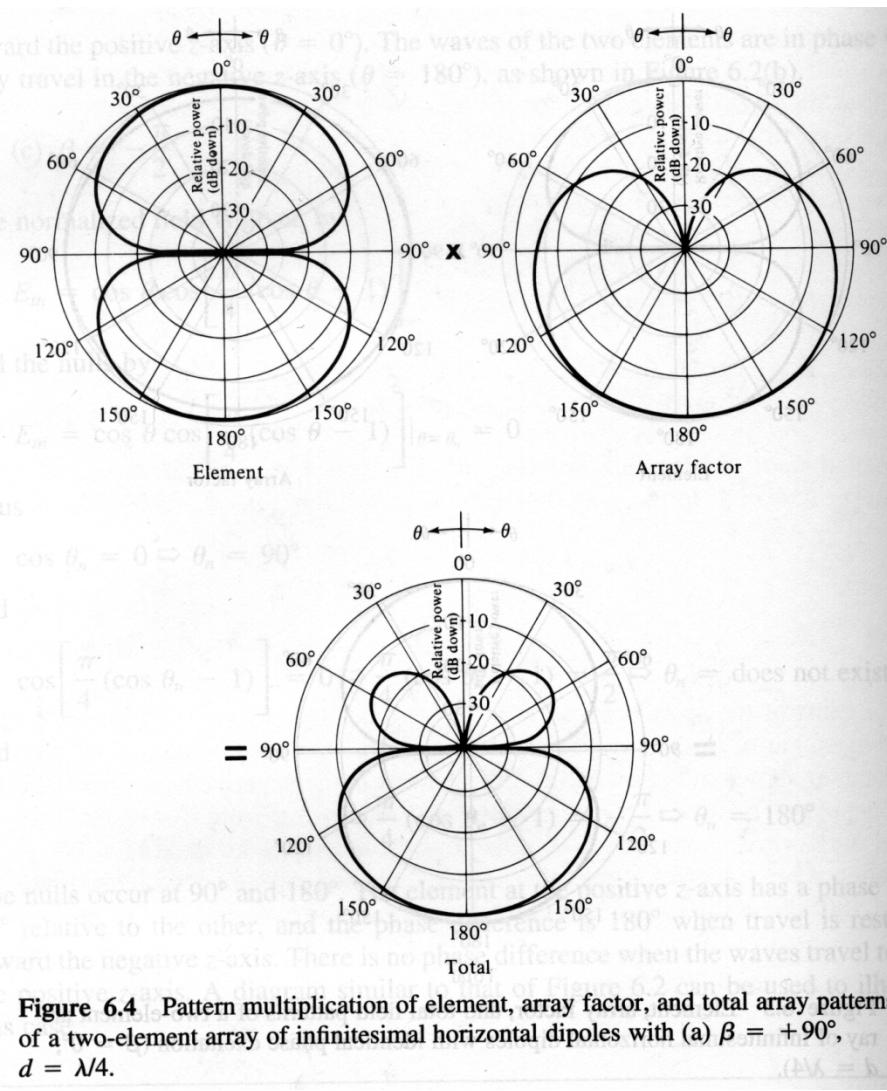
$$E_{tn} = 0 \Rightarrow$$

$$\theta_n = 90^\circ, \text{ and } \theta_n = 0^\circ$$



(b) Far-field observations

# Two Element array example



$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

$$\beta = \frac{\pi}{2}, \quad d = \frac{\lambda}{4} \Rightarrow$$

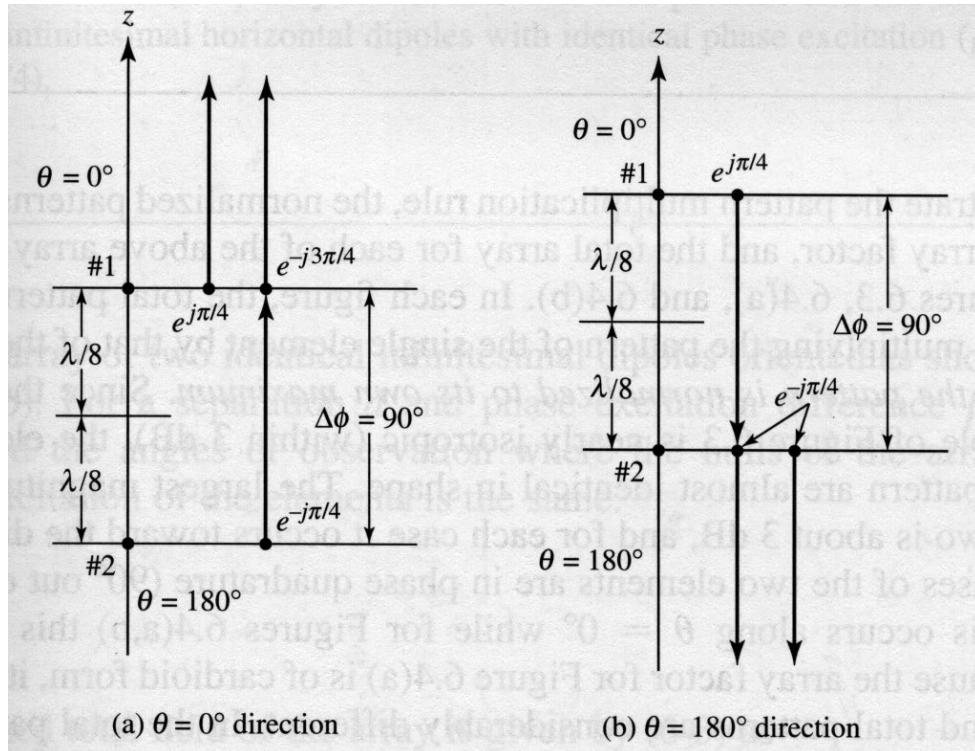
$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4}(\cos\theta + 1)\right)$$

$$E_{tn} = 0 \Rightarrow$$

$$\theta_n = 90^\circ, \text{ and } \theta_n = 0^\circ$$

**Figure 6.4** Pattern multiplication of element, array factor, and total array patterns of a two-element array of infinitesimal horizontal dipoles with (a)  $\beta = +90^\circ$ ,  $d = \lambda/4$ .

# Two Element array example



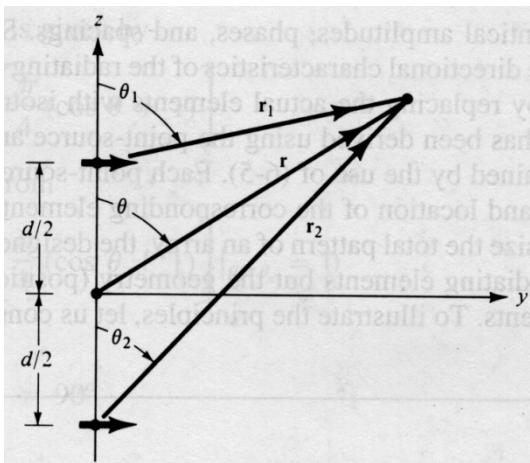
$$\beta = \frac{\pi}{2}, \quad d = \frac{\lambda}{4} \Rightarrow$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4}(\cos\theta + 1)\right)$$

$$E_{tn} = 0 \Rightarrow$$

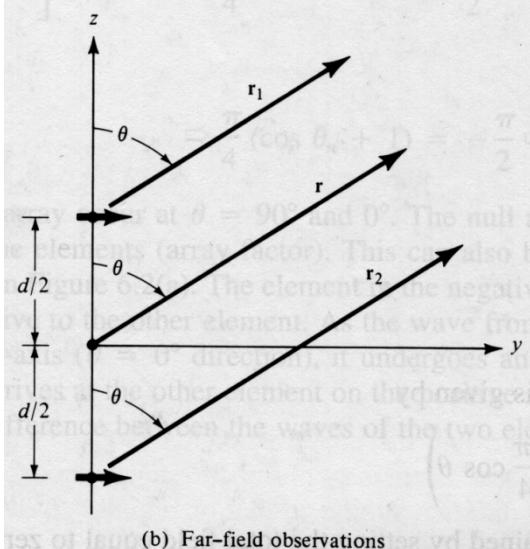
$$\theta_n = 90^\circ, \text{ and } \theta_n = 0^\circ$$

# Two Element array example



$$\vec{E}_t = \hat{a}_\theta j \eta \frac{k I_0 l e^{-jkr}}{4\pi r} \cos(\theta) 2 \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$



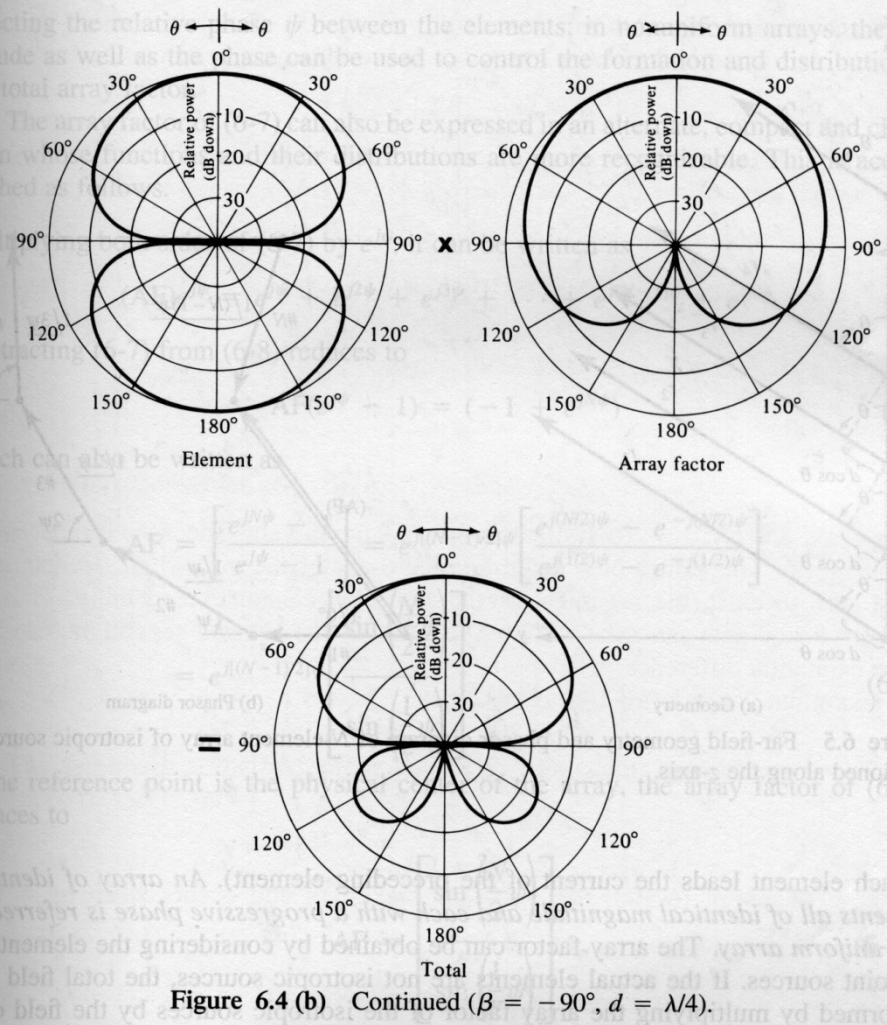
$$\beta = -\frac{\pi}{2}, \quad d = \frac{\lambda}{4} \Rightarrow$$

$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4}(\cos \theta + 1)\right)$$

$$E_{tn} = 0 \Rightarrow$$

$$\theta_n = 90^\circ, \text{ and } \theta_n = 180^\circ$$

# Two Element array example



$$E_{tn} = \cos(\theta) \cos\left(\frac{1}{2}(kd \cos(\theta + \beta))\right)$$

$$\beta = -\frac{\pi}{2}, \quad d = \frac{\lambda}{4} \Rightarrow$$

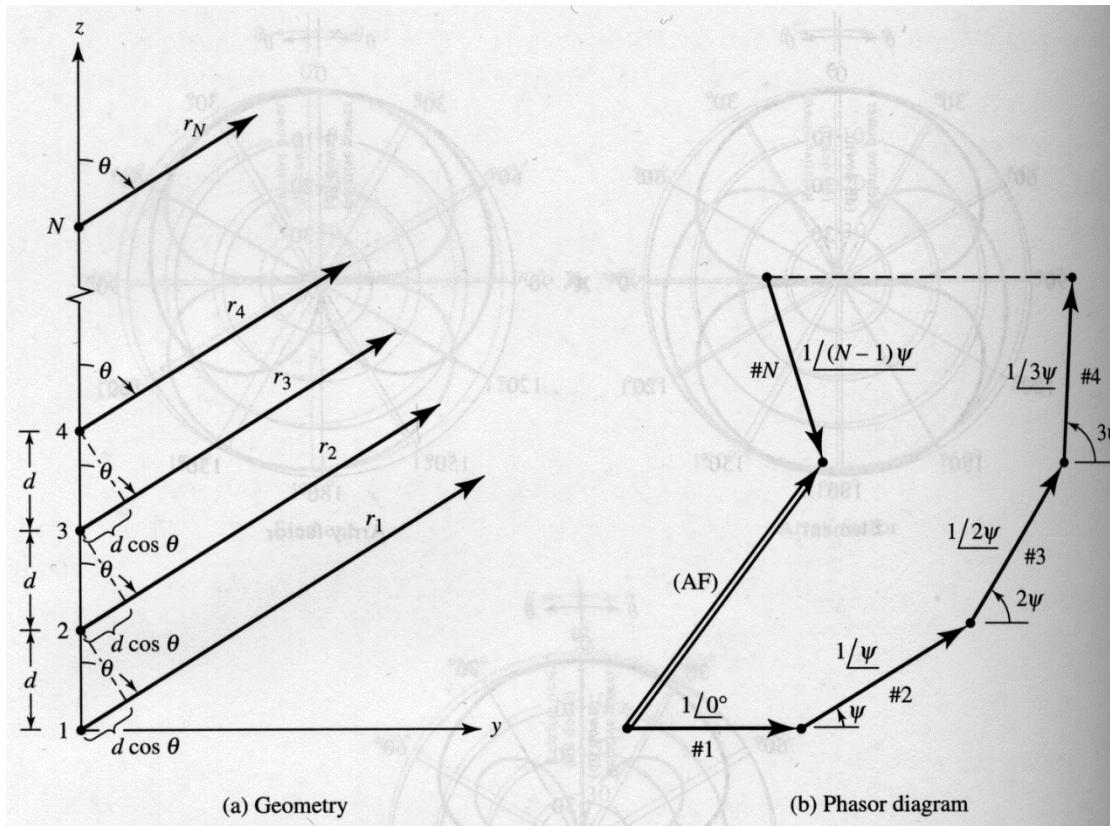
$$E_{tn} = \cos(\theta) \cos\left(\frac{\pi}{4}(\cos\theta + 1)\right)$$

$$E_{tn} = 0 \Rightarrow$$

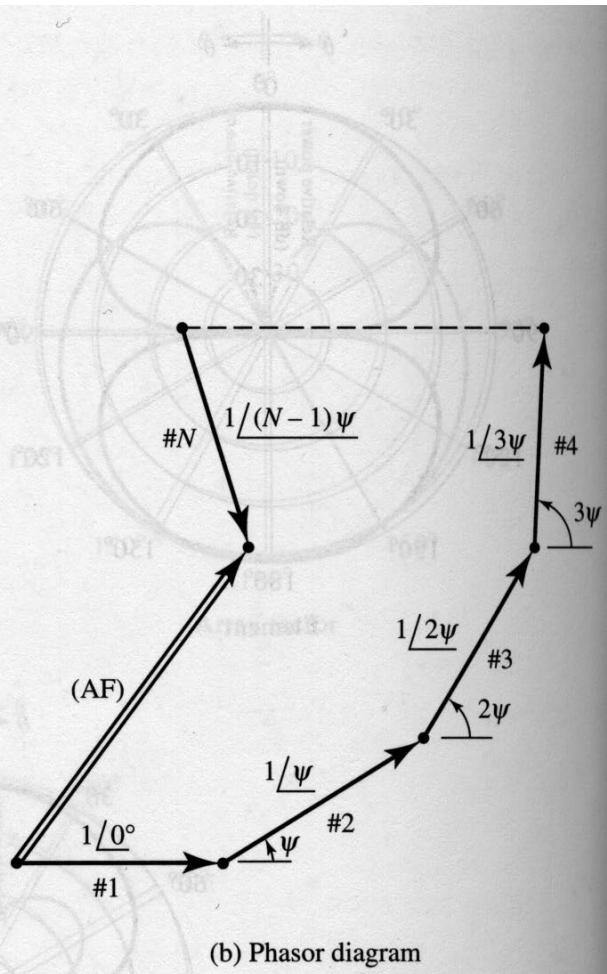
$$\theta_n = 90^\circ, \text{ and } \theta_n = 180^\circ$$

# Uniform Array

An array of identical elements all of identical magnitude and each with a progressive phase is referred to as a uniform array



# Uniform Array



$$AF = \sum_1^N e^{j(n-1)(kd \cos \theta + \beta)}, \text{ or}$$

$$AF = \sum_1^N e^{j(n-1)\Psi}$$

$$\Psi = kd \cos \theta + \beta$$

# Uniform Array

$$AF = \sum_1^N e^{j(n-1)\Psi} \quad \left. \psi = kd \cos \theta + \beta \right\} \Rightarrow$$

$$AF = \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}, \text{ or}$$

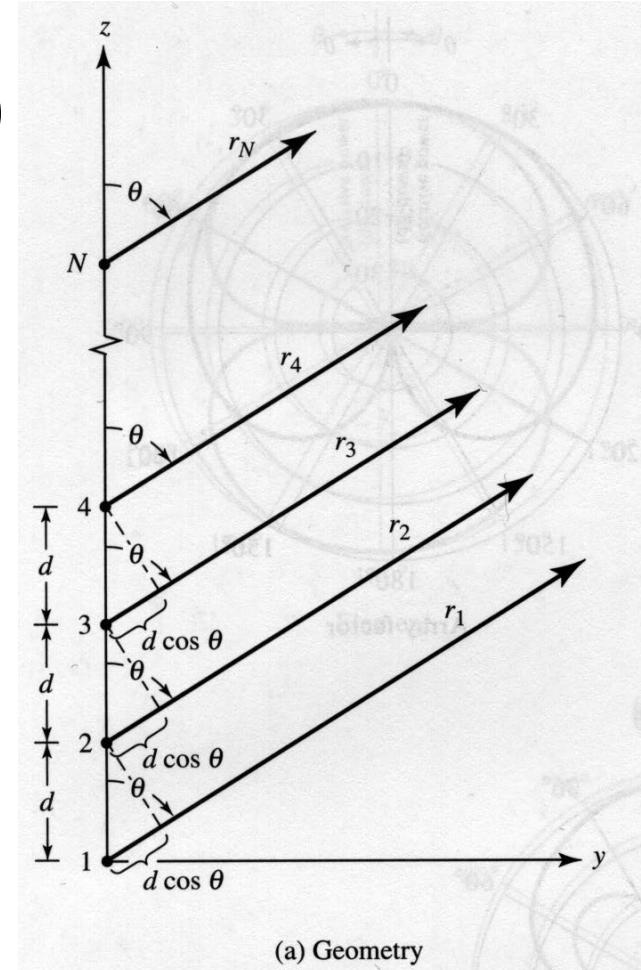
$$AF_n = \frac{1}{N} \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}$$

# Broadside Array

Maximum at  $\theta=90$ : Maximum when  $\psi=0$

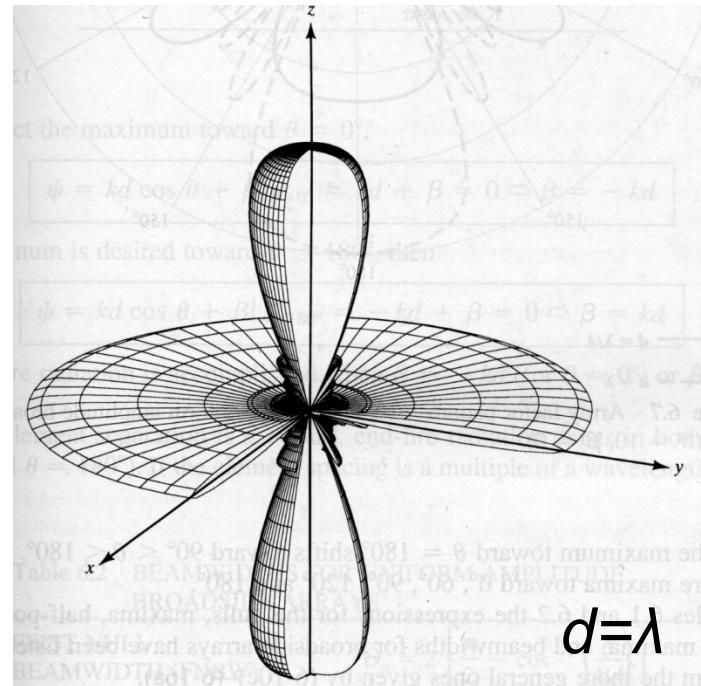
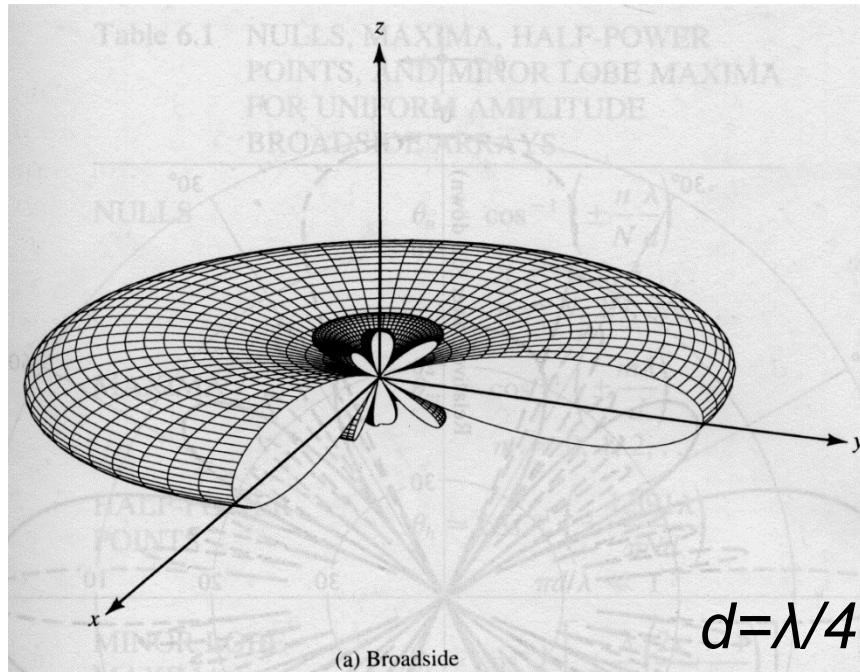
$$\psi = kd \cos \theta + \beta = 0 \Rightarrow$$

$$\beta = 0$$

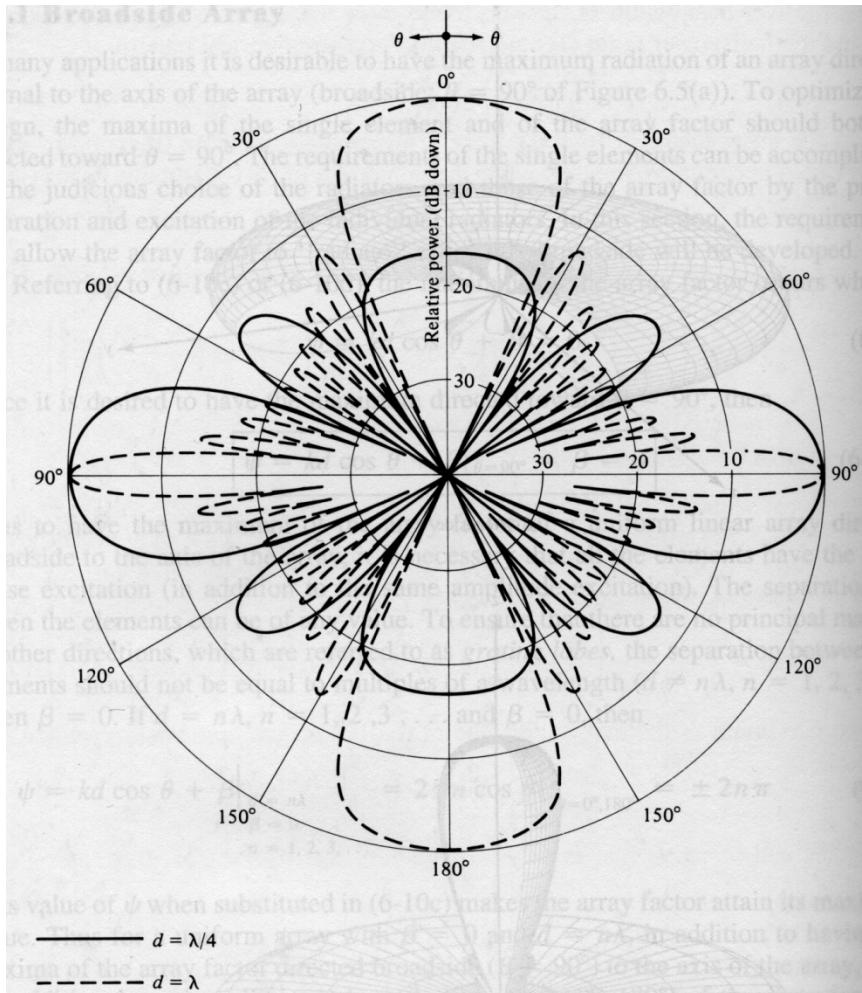


# Broadside Array

To ensure no principal maxima in other directions => separation must be  $d < \lambda$  when  $\beta = 0$



# Broadside Array



Note that the larger the overall length of the antenna the narrower the beamwidth – but other problems arises if the spacing is larger than one wavelength!

# Ordinary End-fire Array

Maximum at  $\theta=0$  or  $180^\circ$ :

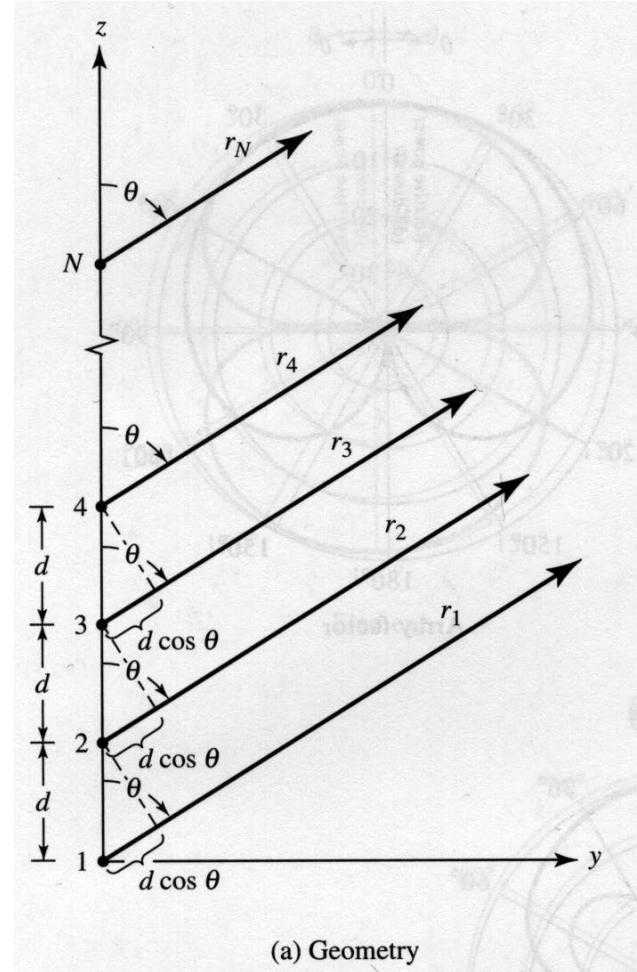
Maximum when  $\psi=0$

$$\psi = kd \cos \theta + \beta = 0 \Rightarrow$$

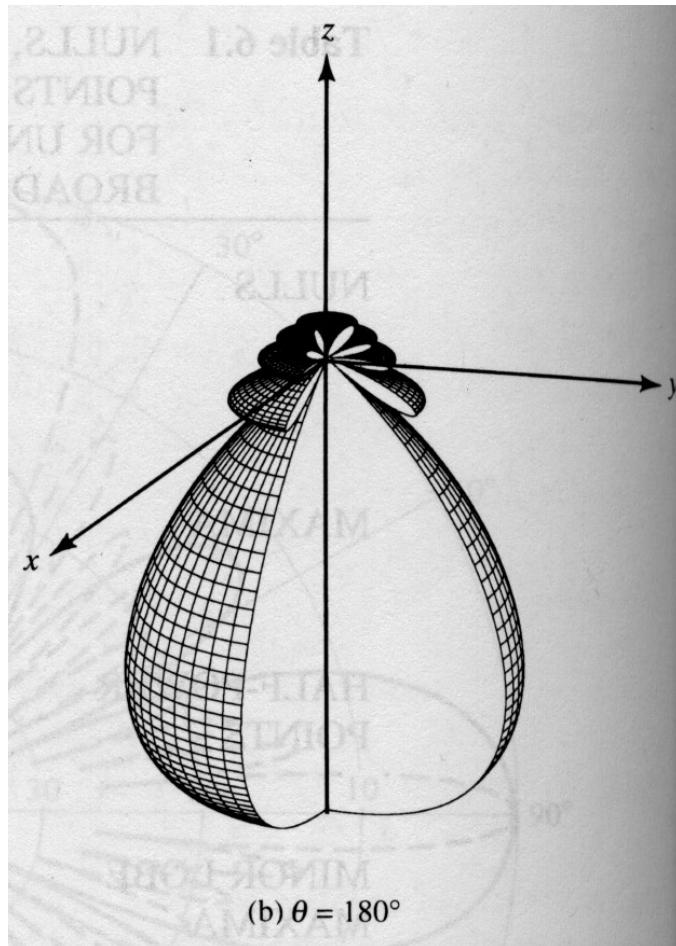
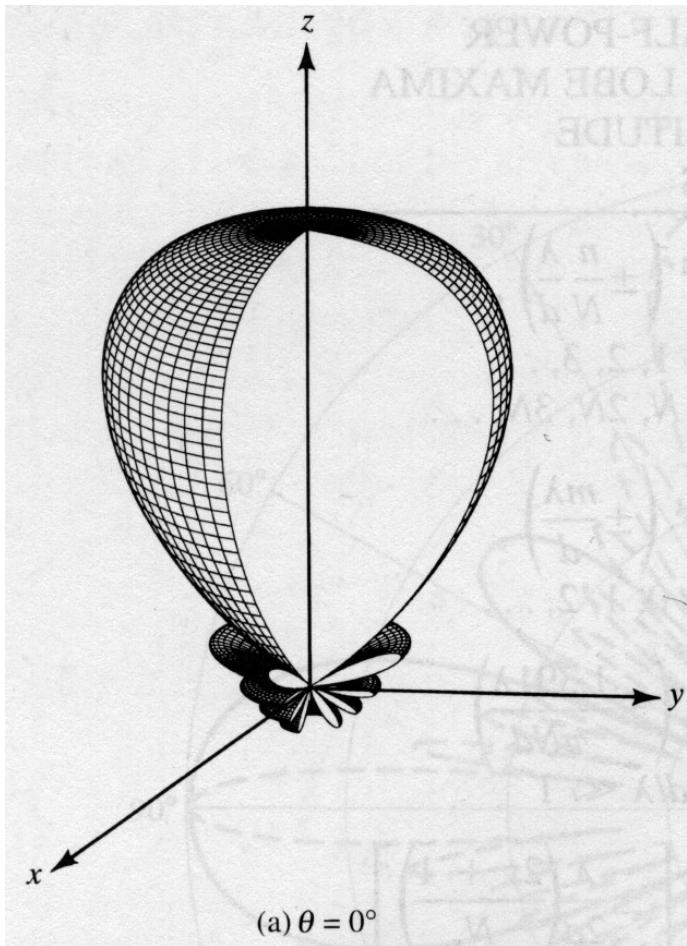
$$\beta = kd$$

$$\psi = kd \cos(180^\circ) + \beta = 0 \Rightarrow$$

$$\beta = -kd$$



# Ordinary End-fire Array



Note, end-fire  
Radiate only  
in one  
direction IF

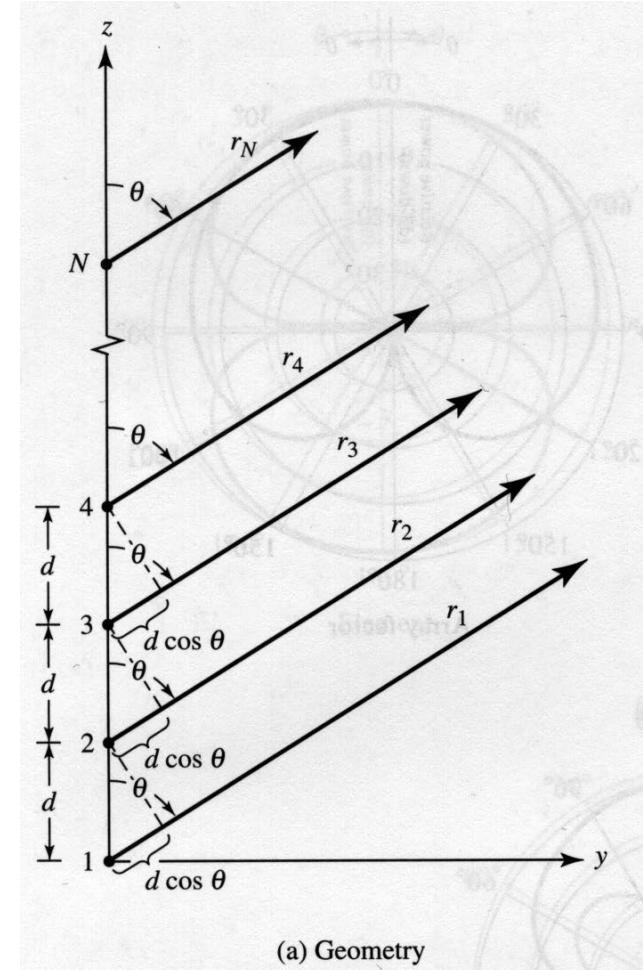
$$d < \frac{\lambda}{2}$$

# Phase ( or Scanning) Array

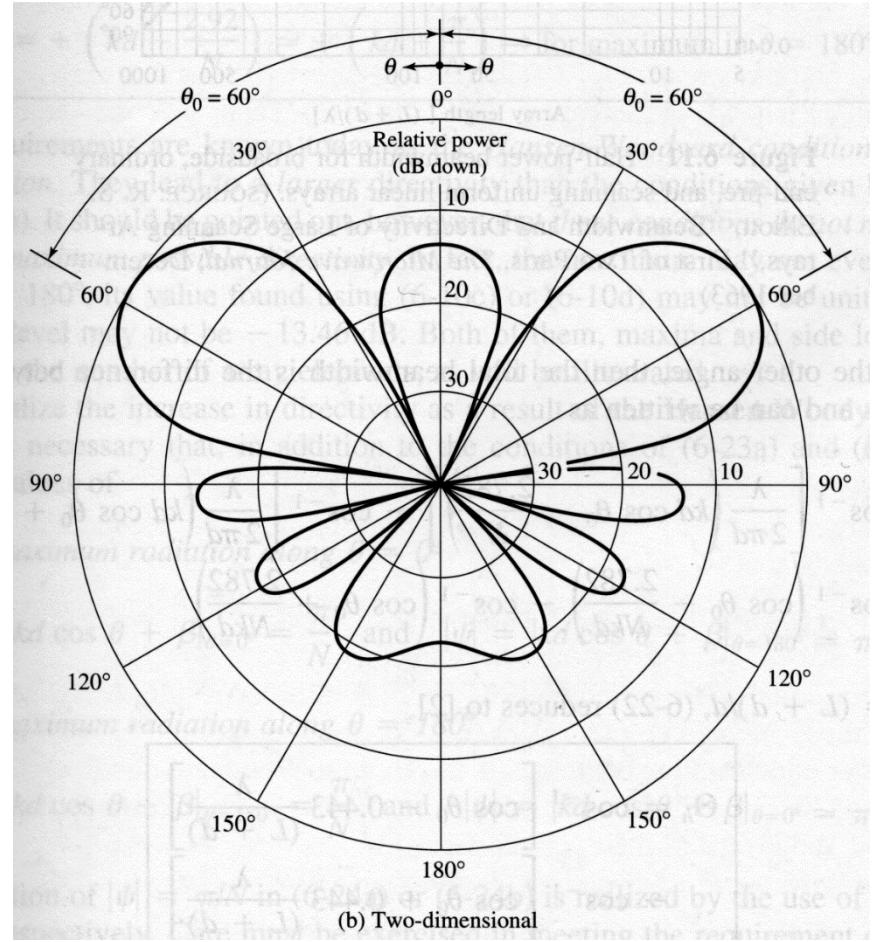
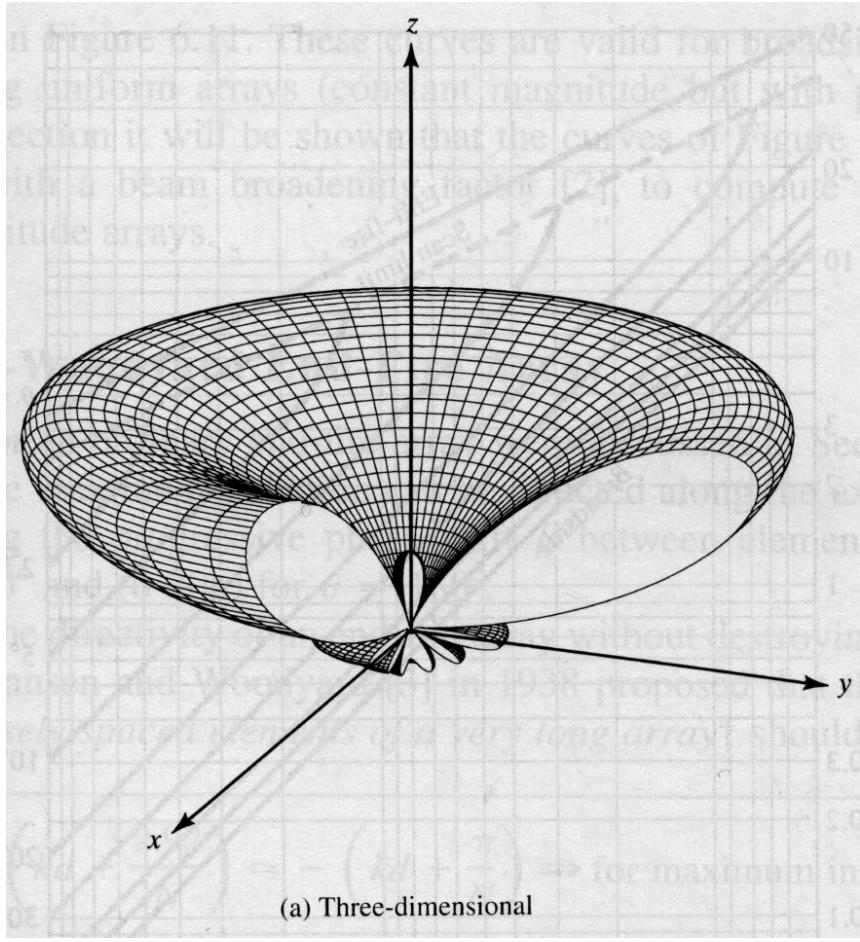
Maximum at  $\theta$ : Maximum when  $\psi=0$

$$\psi = kd \cos \theta + \beta \Big|_{\theta=\theta_0} = 0 \Rightarrow$$

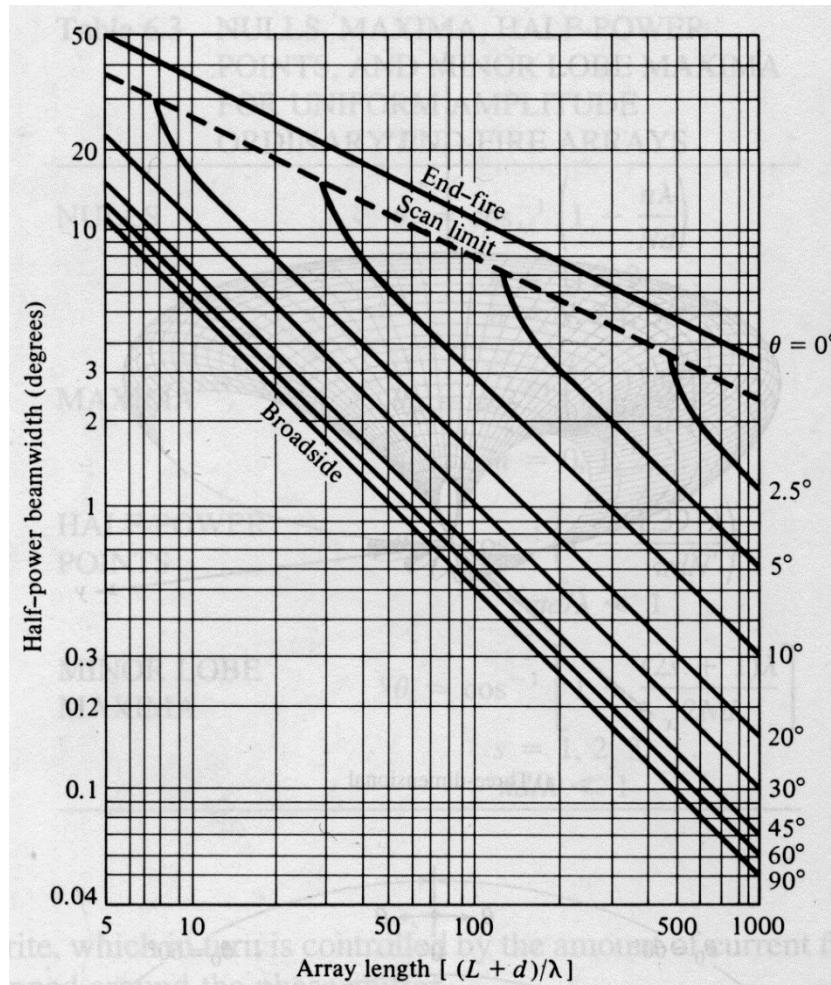
$$\beta = -kd \cos \theta_0$$



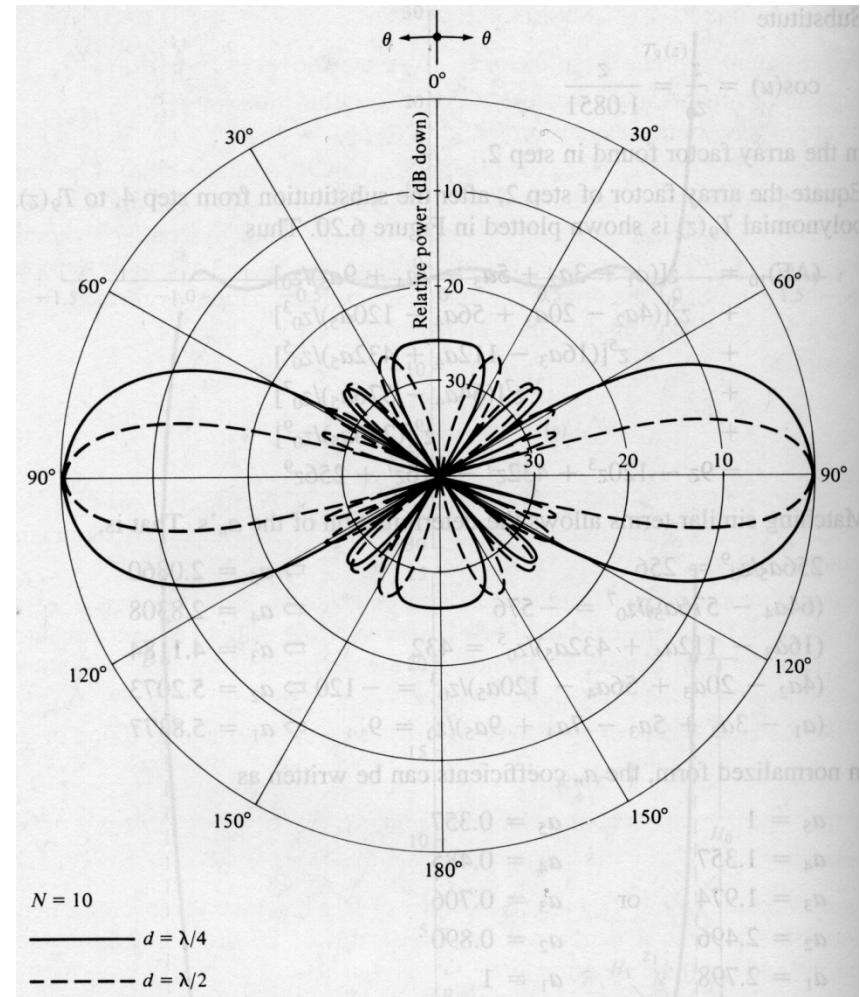
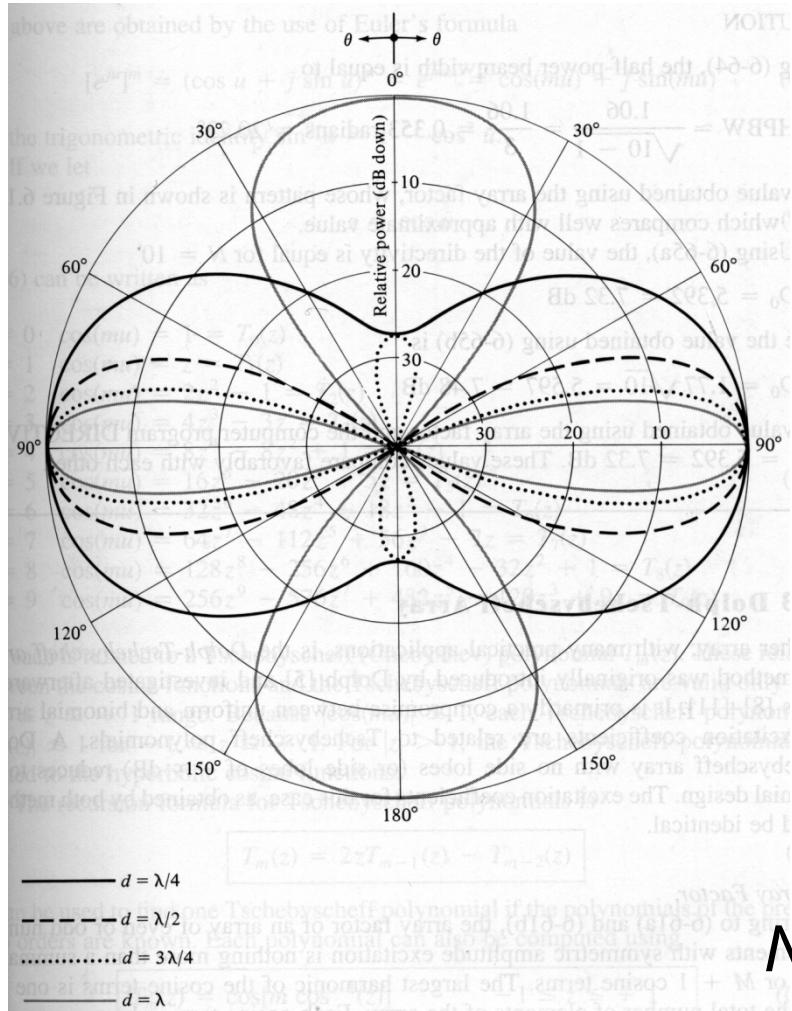
# Phase ( or Scanning) Array



# Directivity of uniform linear Arrays



# Binomial and Tschebyscheff linear Arrays



# Problem 3.1

Design an ordinary end-fire uniform linear array with only maximum so that its directivity is 20 dBi. The spacing between the elements is  $\lambda/4$ , and its length is much greater than the spacing. Determine the:

- a) Number of elements
- b) Overall length of the array (in wavelengths)

# Problem 3.1

- c) Approximate half-power beamwidth (in degrees)
- d) Amplitude level (compared to the maximum of the major lobe) of the first minor lobe in dB.
- e) Progressive phase shift between the elements in degrees.

# Problem 3.2

Design a patch antenna for Bluetooth,(frequency 2.402 to 2.480 GHz).Use PP material with  $\epsilon_r=2.2$  and a thickness of 3 mm.

The impedance at the edge is  $240 \Omega$ , calculate the position for 50 Ohm impedance.

Find the W for a 50 Ohm microstrip.

Make the antenna.