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Answers

Problem 3

The radiation intensity of a given antenna is : $U = |\cos(2\theta)|$ and the intensity exists in the region defined by $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq \varphi \leq 2\pi$.

3.a. Directivity :

To find the total radiated power the radiation intensity is integrated over its closed surface S.

$$S = \begin{cases} 0 \leq \theta \leq \frac{\pi}{2} \\ 0 \leq \varphi \leq 2\pi \\ 0 \text{ elsewhere} \end{cases}$$

$$P_{rad} = \oiint_S U\, dS = \oint_0^{2\pi} \oint_0^{\frac{\pi}{2}} U \sin\theta\, d\theta\, d\varphi = \oint_0^{2\pi} d\varphi \oint_0^{\frac{\pi}{2}} |\cos(2\theta)| \sin\theta\, d\theta$$

$$P_{rad} = \oint_0^{2\pi} d\varphi \oint_0^{\frac{\pi}{2}} |1 - 2\sin^2\theta| \sin\theta\, d\theta = \oint_0^{2\pi} d\varphi \oint_0^{\frac{\pi}{2}} |\sin\theta - 2\sin^3\theta|\, d\theta \;\; (\text{because } \sin\theta \geq 0 \text{ here})$$

$$\boldsymbol{P_{rad}} = [1]_0^{2\pi} * abs\left([-\cos\theta]_0^{\frac{\pi}{2}} + \left[\frac{2}{3}\sin^2\theta\cos\theta + \frac{4}{3}\cos\theta\right]_0^{\frac{\pi}{2}} \right) = 2\pi * abs\left(1 + \left(-\frac{4}{3}\right)\right) = \boldsymbol{\frac{2}{3}\pi}$$

Note : Power must be positive.

The maximum radiation is directed along $\theta = 0$. Thus, $U_{max} = 1$.

The maximum directivity is equal to :

$$\boldsymbol{D_0} = \frac{4\pi\, U_{max}}{P_{rad}} = \boldsymbol{6}$$

$$\boldsymbol{D_0(dBi)} = 10 * \log 10(6) = \boldsymbol{7{,}8\, dBi}$$

3.b. HPBW :

The half-power beamwidth can be found with the function equal to half of $U_{max}(\theta)$.

Thus, $\frac{1}{2}\, U_{max}(\theta) = \frac{1}{2} \Rightarrow \cos(2\theta_h) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ = 0,5236 rad

Because U is symmetric around $\theta = 0$, **HPBW(el)** = 2*0,5236 = **1,0472 rad= 60°**

Because U doesn't depend on φ , **HPBW(az)** = 2*pi = **6,2832 rad= 360°**

Problem 4

Data:

$f = 2500MHz \Rightarrow \lambda = \frac{c}{f} = 0.12\ m$
$P_r = 50.10^{-12}\ W\ , P_t = 0\ dBm = 1mW$
$A = 2{,}50.10^{-3}m^2$

4.a. Free space propagation :
Friis transmission equation can be applied assuming reflection and polarization – matched lossless antennas:

$$\frac{P_r}{P_t} = \lambda^2 \frac{D_t * D_r}{(4\pi R)^2}$$

Where $\boldsymbol{D_t} = \frac{4\pi A}{\lambda^2} = \mathbf{2,181}$ and $\boldsymbol{D_r} = \mathbf{1,643}$ for the half wave dipole.

The maximum distance of communication in free space is **R = 80,85m**

4.b. The maximum distance in free space is achieved under the following conditions :

- Perfect impedance matching
- Perfect polarization matching
- Lossless antennas

4.c. In this case the reader uses circular polarization resulting in a mismatched polarization and thus losses. The polarization mismatch is 50 % with one circularly polarized antenna.

Friis law gives $R^2 = \frac{P_t}{P_r}\frac{D_t * D_r * \lambda^2}{(4\pi)^2} * \frac{1}{2}$. **R = 57,17 m**

4.d. Perfect reflecting surface:
In the case where a LOS plus a ground reflection exist the following equation applies for lossless antennas:

$$\frac{P_r}{P_t} = D_t * D_r \left(\frac{h_t h_r}{R^2}\right)^2,$$

and is valid for $R \geq \frac{4h_t h_r}{\lambda} = 33{,}33m\ (h_t = h_r = 1m)$

Note: This equation, which replaces the Friis' law for R>33m, implies that the received power is independent of frequency and increases with the square of the height for both the MS and the BS.

The maximum distance for polarization matched antennas is **R = 92,01 m.**