ESD5 – Fall 2024
Lecture Notes – Lecture 6

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Example 1 – TDMA with Periodic Reservation

Time Division Multiple Access (TDMA) enables multiuser communication by allocating dedicated time slots for sending or receiving information to or from a single user at a time. In this example, we will see how TDMA, along with the strategy of periodic reservation, can be applied to solve the problem of 4 users who, with a certain frequency, need to send messages to a receiver using a single channel.

Initially, we introduce the scenario of Fig. 1. We have 4 users, Zoya, Yoshi, Xia, and Walt, who periodically transmit data packets to Basil. Each packet contains 1,000 bits and the users are capable of transmitting at a rate equal to 1 kbit/s. The users can communicate with Basil only via a shared wireless channel, making it impossible for Basil to receive information coming from two or more users transmitting at the same time. To coordinate the users' transmissions, Basil adopts TDMA with the periodic reservation strategy, allocating, in a TDMA frame that lasts 4 s, a 1-second slot for each user. As illustrated in Fig. 1, the TDMA frame pattern repeats indefinitely so that each user has an opportunity to transmit a data packet every 4 s.

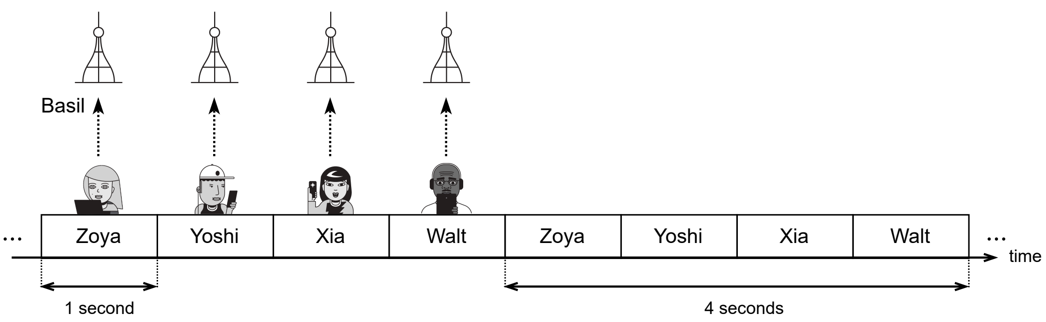

Figure 1: TDMA frame with periodic reservation of equal slots to Zoya, Yoshi, Xia, and Walt send data packets to Basil.

Let us analyze Basil's coordination strategy from Zoya's perspective. Since all users are assigned equal slots, the analysis applies to all of them. During the slot reserved for Zoya, she sends all the 1,000 bits within 1 s, and then remains silent for 3 s until the arrival of the slot reserved for her in the next frame. This means that, for Basil to receive 2,000 bits from Zoya, 2 entire TDMA frames are necessary, which, last 8 s in total. Knowing this, we can compute the effective data rate of Zoya as $\frac{2{,}000}{8} = 0.25$ kbit/s, equivalent to one-fourth of the packet data rate of 1 kbit/s. Hence, as the 4 users evenly share the time to access the wireless channel, their effective data rates are divided by four.

Now, we analyze the coordination strategy from Basil's perspective. During a single TDMA frame, Basil receives 4,000 bits of information, resulting in a throughput of $\frac{4{,}000}{4} =$ 1 kbit/s. Notice that this throughput is valid only if all 4 users always have a data packet to send when the time of their reserved slots arrive. In the presence of one or more idle users, the throughput decreases, as described in the sequence. To verify this case, consider that, within an interval comprising 10 TDMA frames, only Xia had data packets to send. Therefore, the throughput experienced by Basil decreases to $\frac{10{,}000}{80} = 0.25$ kbit/s. Such a result demonstrates the periodic reservation adopted by Basil is very inefficient when only one of the 4 users transmits data packets. The throughput could be kept at 1 kbit/s if Basil had the flexibility of reallocating to Xia the 3 slots initially reserved for the idle users. Such flexibility can be achieved by redesigning the procedure employed by Basil to reserve the slots for the 4 users, obtaining a dynamic reservation strategy like the one shown in the next example.

Example 2 – Dynamic Reservation and the Cost of Overhead

As seen in the previous example, TDMA with periodic reservation might be inefficient when the data traffic oscillates, since slots may be kept reserved for users that have no packets to transmit. To address this weakness, we will develop a dynamic reservation strategy for Basil, allowing him to reserve slots for Zoya, Yoshi, Xia, and Walt transmissions only when they have data packets to send.

Consider again the scenario where 4 users periodically transmit data packets to Basil. But now, Basil employs the dynamic reservation scheme depicted in Fig. 2, divided into 3 phases. The phases are the reservation slots, the allocation packet, and the data slots. During the reservation and data slots, the users transmit messages to Basil, while the opposite happens in the allocation packet phase. Let us define the phases separately.

Reservation slots. There are 4 reservation slots, one for each user connected to Basil. In the reservation slots, the users notify Basil of their intention to transmit a data packet. In Fig. 2, Zoya and Xia have data packets to transmit to Basil, and, to share this information with Basil, they transmit a bit "1" in their respective reservation packets. Since Yoshi and Walt do not have any data packets to send, they transmit a bit "0". In total, the reservation slots represent 4 bits.

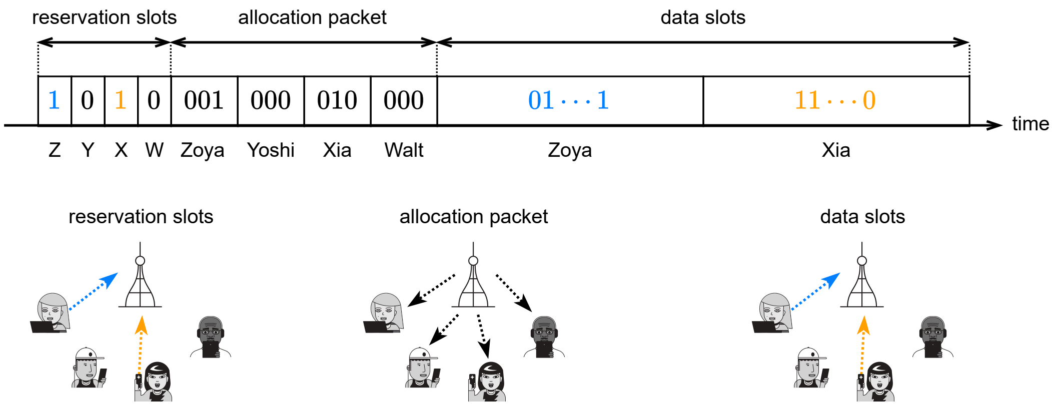

Figure 2: TDMA frame with dynamic reservation of slots to Zoya, Yoshi, Xia, and Walt send data packets to Basil.

Allocation packet. Through the allocation packet, Basil gives instructions to the users about what data slots they can use to transmit their packets. Specifically, the allocation packet comprises 4 groups of 3 bits, each dedicated to a user connected to Basil. Hence, the allocation packet has 12 bits in total. Observing Fig. 2, we can see that Basil filled the bits for Yoshi and Walt with "000", as they did not notify any intention to transmit data in this frame. On the other hand, the bits for Zoya and Xia are filled with "001" and "010", respectively, meaning that the first data slot is reserved for Zoya and the second one is reserved for Xia.

Data slots. The data slots are reserved for the users to transmit their packets. We consider that the number of data slots present in a frame is equal to the number of users that have data to transmit. This means that we can have 1 up to 4 data slots, considering that there is always one user with a data packet ready to send. The user transmits a data packet containing D bits during the slot reserved for them. Therefore, in a frame, there are potentially D, $2D$, $3D$, or $4D$ data bits. In Fig. 2, as indicated by Basil in the allocation packet, Zoya transmits in the first data slot, while Xia does it in the second one.

Now that we have our dynamic reservation strategy, let us compute the throughput and compare it with the result achieved by periodic reservation. Initially, we consider the Basil and the users transmit at a rate of 1 kbit/s, while a user data packet contains $D = 1,000$ bits. Notice that the bits transmitted over the reservation slots (4 bits) and the allocation packet (12 bits) do not carry data, working only as *control information* for Basil to reserve the right amount of slots for the users. This control information receives the name of *communication overhead*. To evaluate the impact of the communication overhead in the reservation strategy, we compute how much time of the TDMA frame is occupied by the control bits, getting $\frac{16}{1{,}000} = 0.016$ s.

We start by calculating the throughput considering that all 4 users have a packet to transmit in the frame. In this case, the number of total bits to be received by Basil is 4,000, meaning that the users will need $\frac{4{,}000}{1{,}000} = 4$ s in total to transmit the data. Hence, the system throughput is $\frac{4{,}000}{0.016+4} = 0.996$ kbit/s. Look that this throughput value is very close to the 1 kbit/s attained by the periodic reservation of Example 1. The difference is that, as the dynamic strategy requires the exchange of control information between Basil and the users, a small fraction of the TDMA frame will be occupied by useless information from the data perspective. In principle, you may think that we developed a complicated strategy for getting no performance improvement. Even worse, we got a throughput degradation compared to the periodic reservation we had before! But remember that we start working on this dynamic strategy to address the weakness of periodic reservation in the presence of idle users. Let us evaluate the throughput in that case.

Consider that only Zoya has a data packet to transmit during the TDMA frame. Therefore, the number of total data bits to be received by Basil is 1000 and, accordingly, $\frac{1{,}000}{1{,}000} = 1$ s is necessary for these bits to be conveyed. Accounting for the communication overhead of 0.016 s, the system throughput becomes $\frac{1{,}000}{0.016+1} = 0.984$ kbit/s. Notice that this throughput is approximately 4 times higher than the one achieved by periodic reservation under the same conditions. The gains of the dynamic strategy come from the fact that there are no more unused reserved data slots, as the reservation process happens only if the users inform they have packets to send. Therefore, we can consider that the developed dynamic reservation strategy has accomplished the mission!

Now, consider that Zoya has been replaced by a sensor that transmits short data packets containing 4 bits. If the sensor is the only one that has a packet within the frame, it would take $\frac{4}{1{,}000} = 0.004$ s to transmit it to Basil. In this case, the system throughput would be $\frac{4}{0.016+0.004} = 0.2$ kbit/s. Look that the throughput degraded significantly by decreasing the data packet size. This happens when the number of bits in the communication overhead becomes comparable to the number of data bits. In this situation, the dynamic reservation strategy we developed becomes inefficient, and other solutions, like random access, can be adopted to improve the throughput.

Example 3 – Framed ALOHA

In this example, we consider the case where Zoya and Yoshi transmit data packets to Basil without coordination. This means that, differently from Examples 1 and 2, Basil does not control what slots can be used by the users to convey their data. In this case, Zoya and Yoshi adopt framed ALOHA, a random access strategy. As depicted in Fig. 3, now the TDMA frame comprises S equal slots that can carry a single data packet. During each frame, Zoya and Yoshi randomly select a single slot to transmit the data packet to Basil. All slots can be selected with the same probability, meaning that the probability of Zoya or Yoshi picking a specific slot is $\frac{1}{S}$. When both users select the same slot in a frame, a collision happens, and Basil cannot receive the data packets sent in that frame. On the other hand, when users select different slots, Basil successfully receives the 2 data packets. We consider that Basil can perfectly detect the occurrence of collisions.

There is a successful transmission of the data packet in a specific slot if any of these two events happen:

- Zoya selects this specific slot for packet transmission, and Yoshi does not.
- Yoshi selects this specific slot for packet transmission, and Zoya does not.

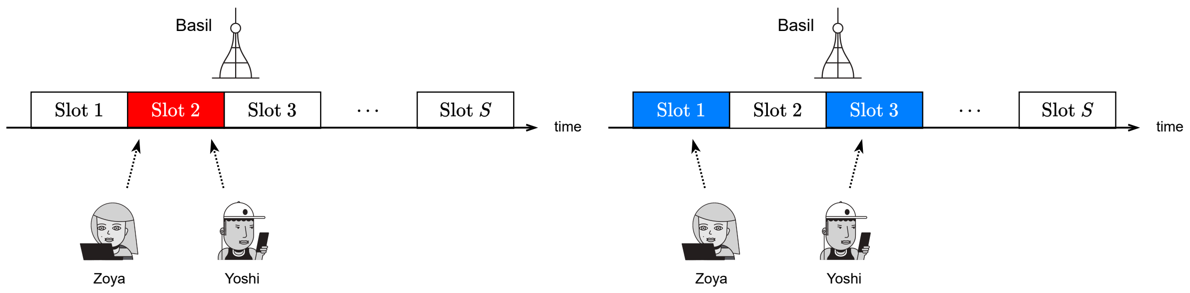

Figure 3: TDMA frame used by Basil for receiving data packets from Zoya and Yoshi using framed ALOHA. (Left) Both Zoya and Yoshi select Slot 2 for transmitting their packets, resulting in a collision and no packet reception at Basil. (Right) No collision occurs, as Zoya and Yoshi select different slots for packet transmission.

These events respectively happen with the probabilities:

$$\underbrace{\frac{1}{S}}_{\text{Pr(Zoya Tx)}} \times \underbrace{\left(1-\frac{1}{S}\right)}_{\text{Pr(Yoshi not Tx)}} \quad \text{and} \quad \underbrace{\left(1-\frac{1}{S}\right)}_{\text{Pr(Zoya not Tx)}} \times \underbrace{\frac{1}{S}}_{\text{Pr(Yoshi Tx)}} . \tag{1}$$

Therefore, the total probability of successful transmission can be derived as:

$$P(S) = \frac{1}{S}\left(1-\frac{1}{S}\right) + \left(1-\frac{1}{S}\right)\frac{1}{S} = \frac{2}{S}\left(1-\frac{1}{S}\right).$$

Notice that this probability is maximized if we pick $S = 2$, obtaining $P(S = 2) = \frac{1}{2}$. This means that to maximize the probability of successful transmission over a slot, we need as many slots as users.

Now, we generalize this result for the case where K users want to transmit data packets to Basil. The probability of successful transmission in a specific packet for a specific user can be defined as:

$$\underbrace{\frac{1}{S}}_{\text{Pr(1 user Tx)}} \times \underbrace{\left(1-\frac{1}{S}\right) \times \cdots \times \left(1-\frac{1}{S}\right)}_{\text{Pr(K-1 users not Tx)}} = \frac{1}{S}\left(1-\frac{1}{S}\right)^{K-1} . \tag{2}$$

Since there are K potential users selecting this slot to send a packet to Basil, the total probability of successful transmission in a slot is:

$$P(S) = \frac{K}{S}\left(1-\frac{1}{S}\right)^{K-1} . \tag{3}$$

As we found previously in the case with 2 users, this probability is maximized when the number of slots is the same as the number of users, *i.e.*, $S = K$. Notice that, when $S = K$ and the number of users increases indefinitely, the probability reaches a well-defined lower bound,

$$\lim_{K\to\infty} P(S = K) = \lim_{K\to\infty}\left(1-\frac{1}{K}\right)^{K-1} = e^{-1}. \tag{4}$$

This result reveals that the probability of successful transmission experienced by an individual user is higher when the users contend for the slots in smaller groups.