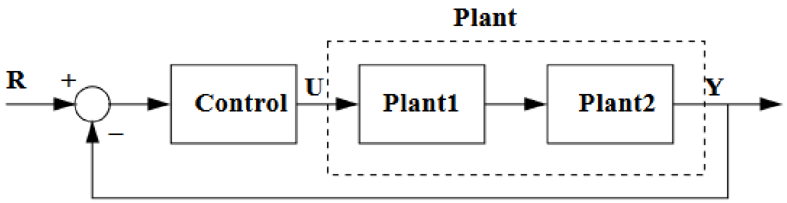
Cascade control
and
Rouths Stability Criterion

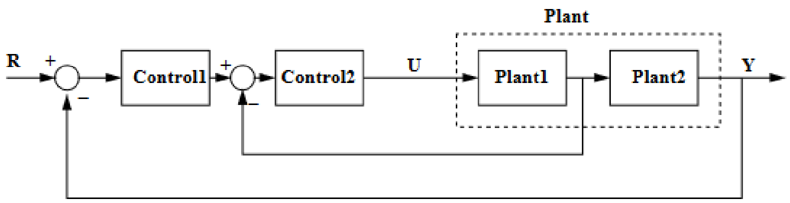
Outline

- Cascading for
  - Faster response / bandwith
  - Dampening of disturbances
  - Linearization
  - stabilization
- Rouths stability criterion

Cascading



Series control.



Cascade control.

Advantage:

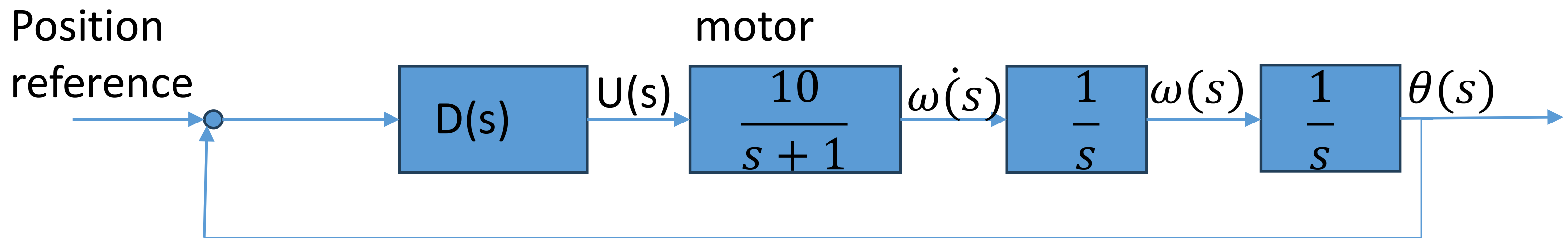
- Better disturbance rejection
- More robust to parameter variations
- Linearization of Plant
- Faster response

Disadvantage:

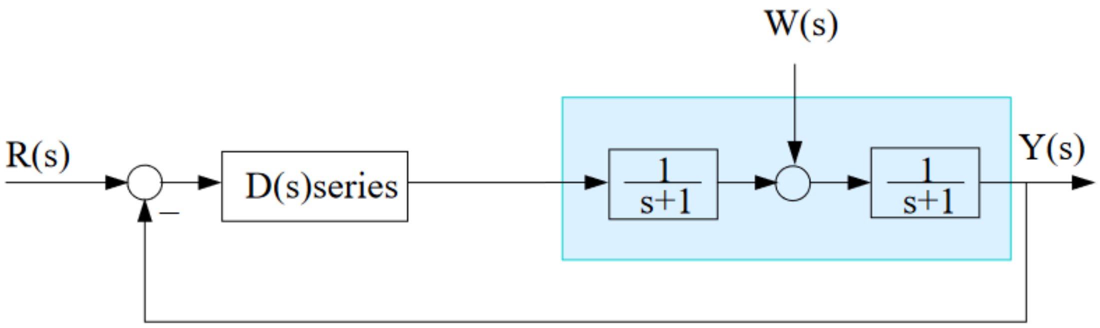
- More measurements

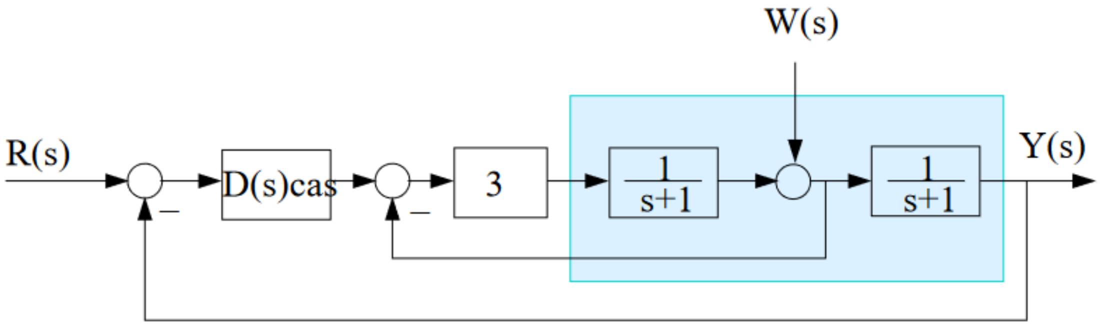
Cascading - example on blackboard

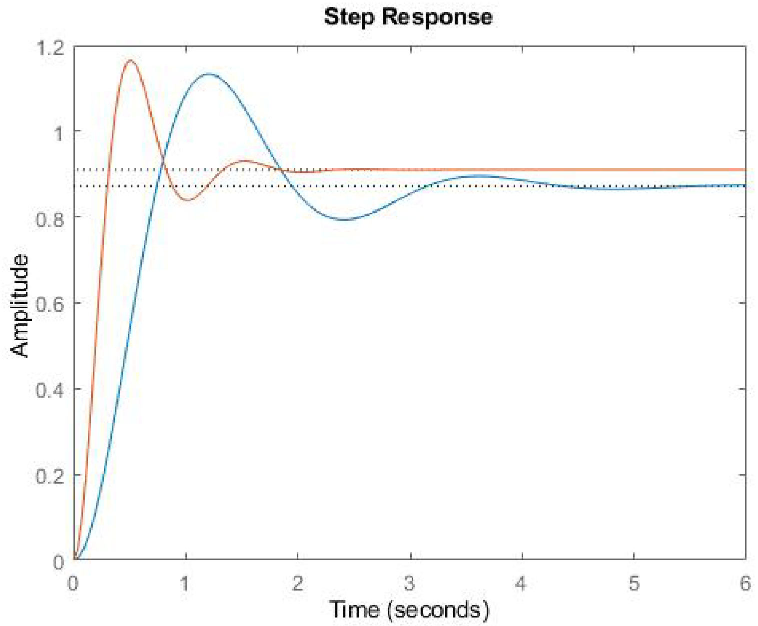
Better stability



Cascading - example

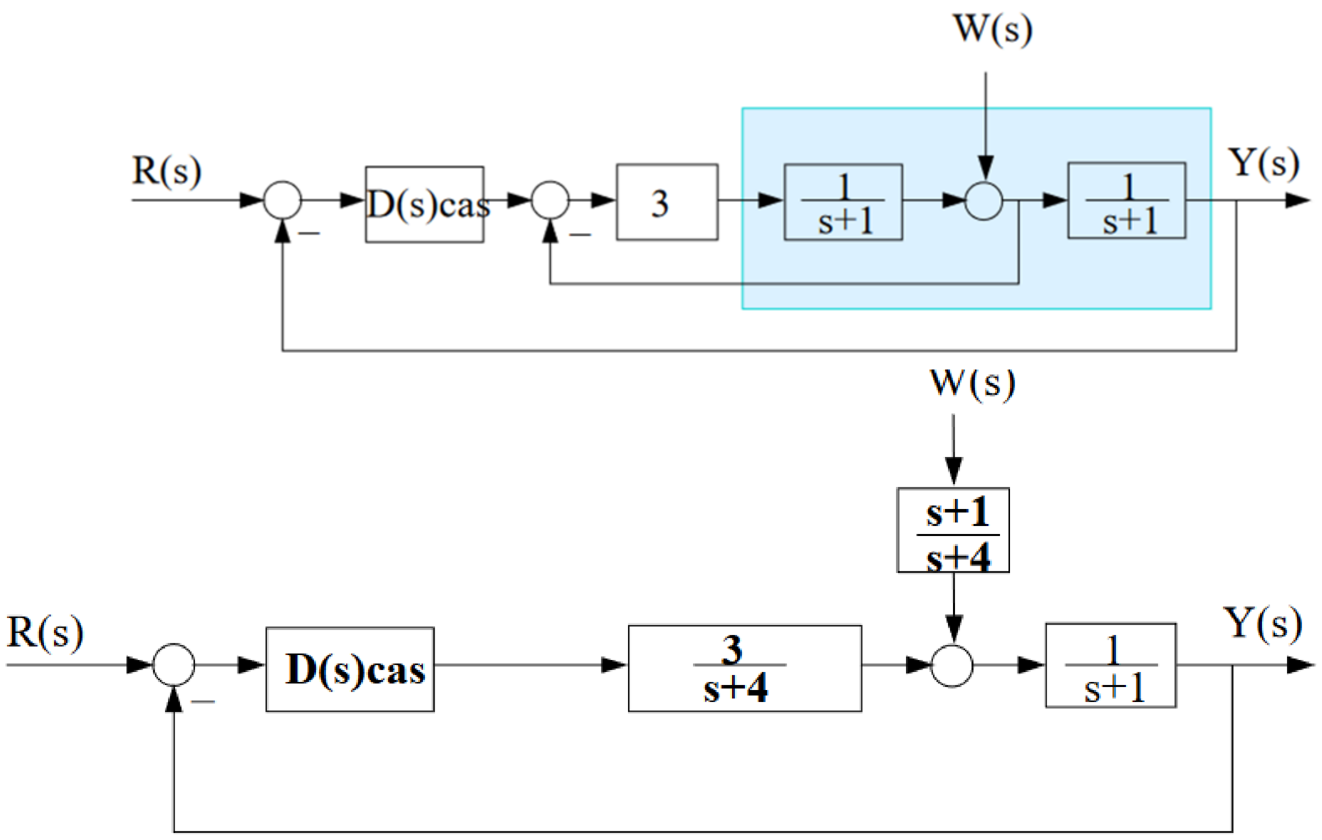






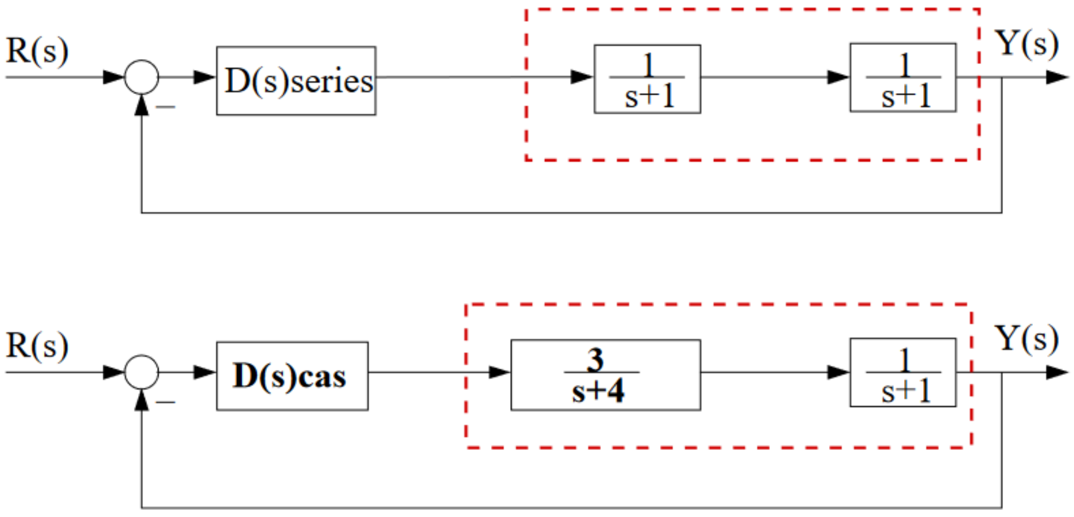
The same plant controlled by a series controller and a cascade controller.
If both controlled systems have a proportional controller and 45 ^{o} phasemargin, $D(s)series = 6.8$ and $D(s)cas = 13.5$.

Cascading example



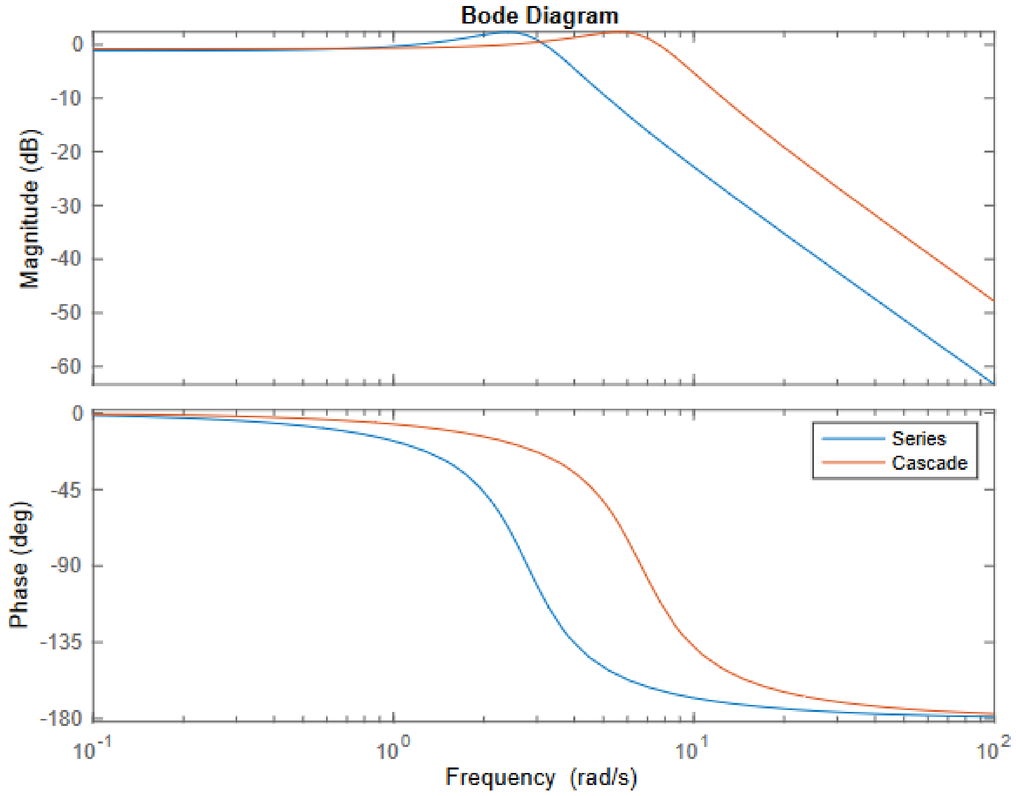
The cascaded system is reformulated using the loop rule.
We will compare the reformulated cascaded system with the series controlled system.

Cascading example - bandwith

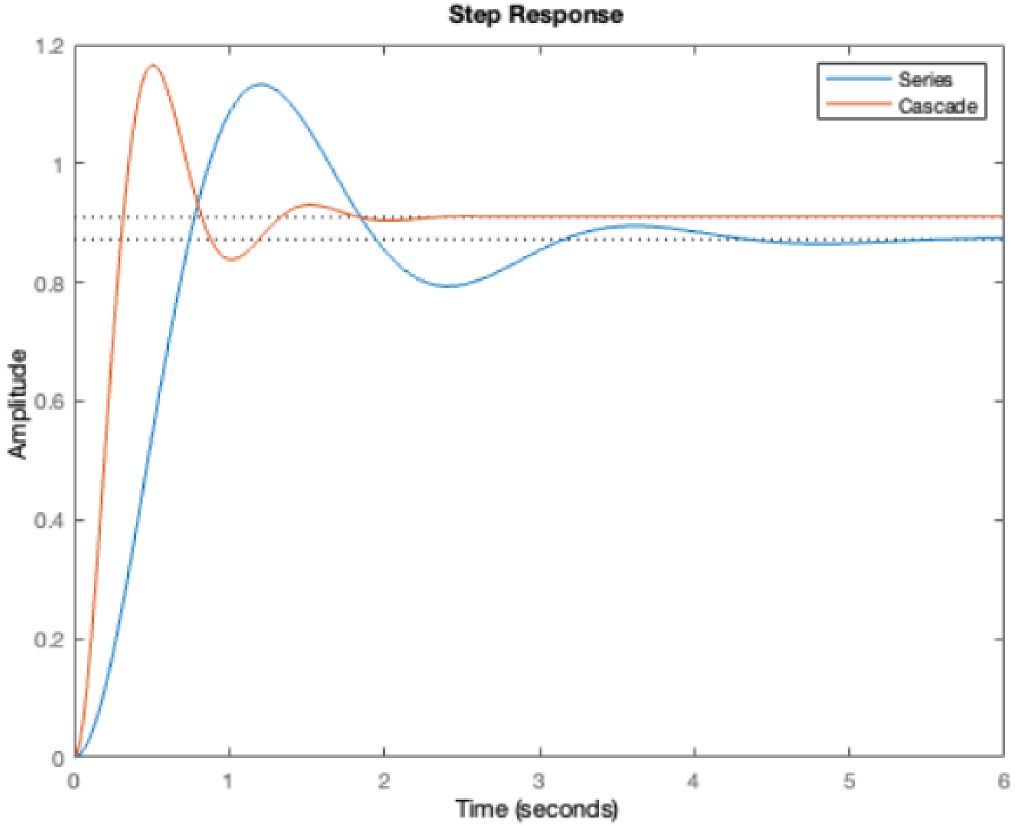


The cascaded system is easier to control than the series controlled
WHY?

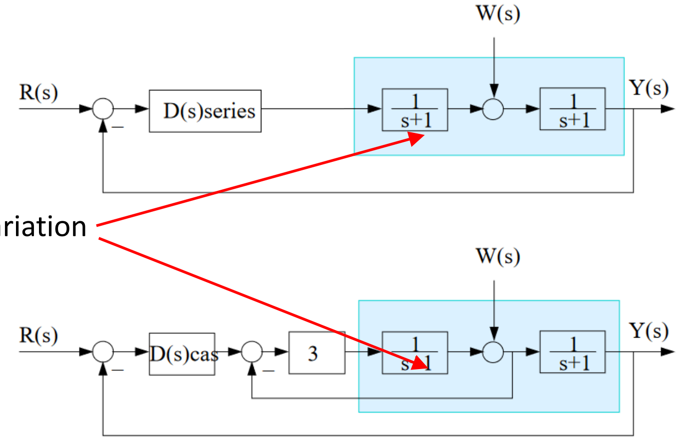
Closed loop frequency response



Closed loop step response

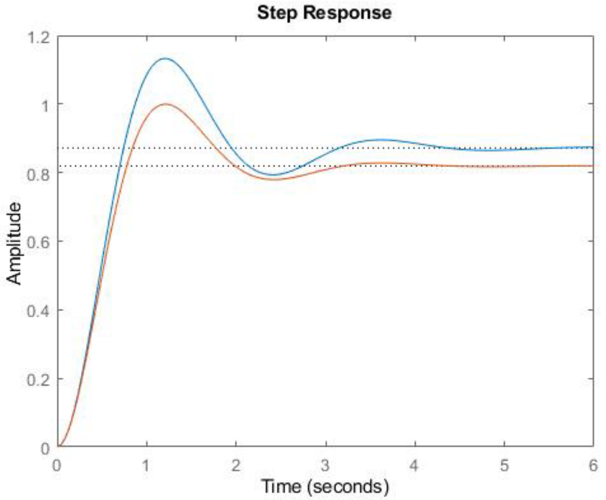


Cascading – example parameter variation

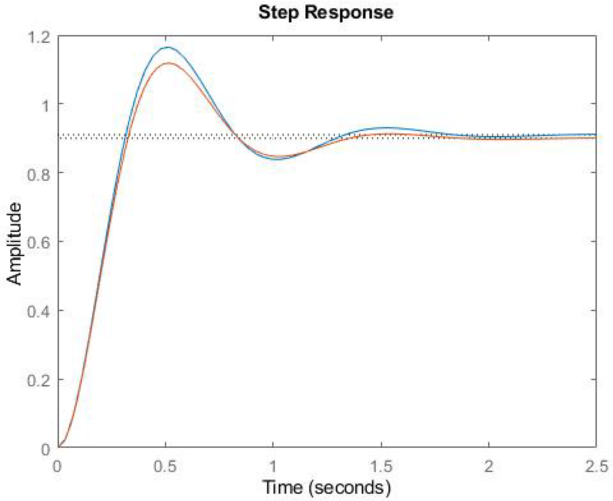


$$D(s)_{series} = 6.8$$
$$D(s)_{cas} = 13.5$$

Series with and without parameter variation



cascading with and without parameter variation



Series

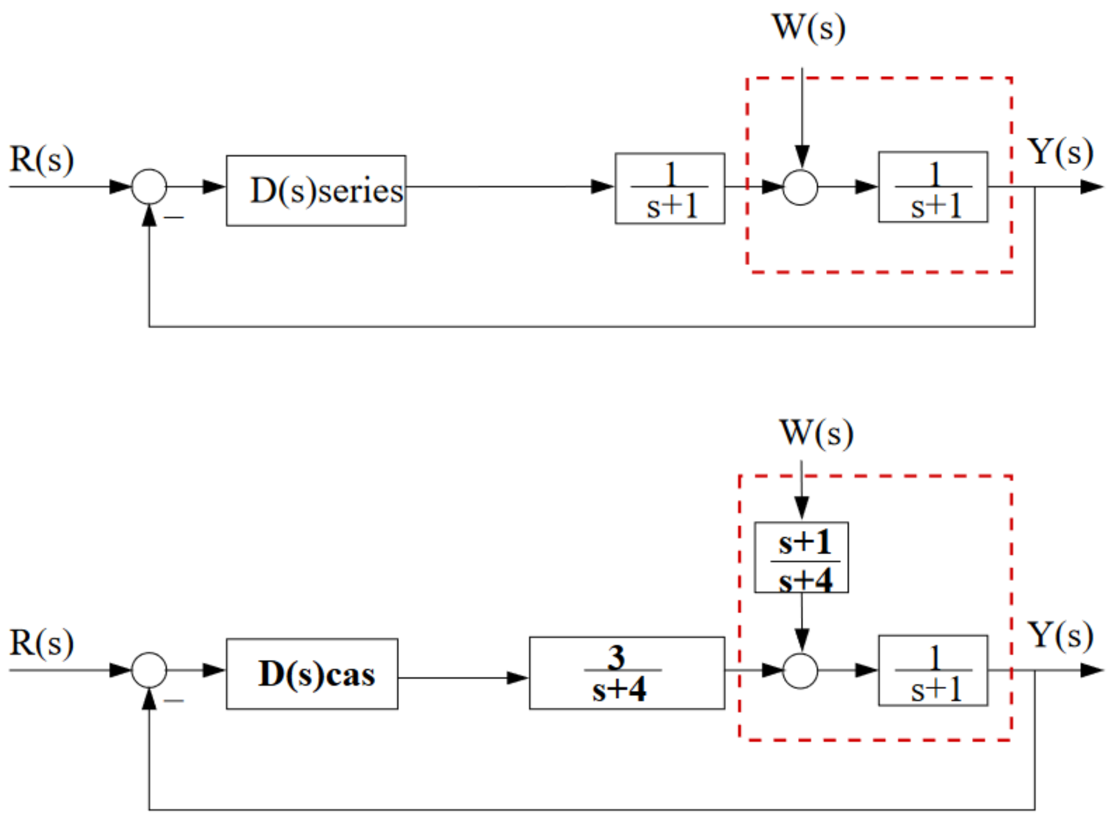
$$\frac{Y(s)}{R(s)} = \frac{\frac{6.8}{(s+1.5)(s+1)}}{1+\frac{6.8}{(s+1.5)(s+1)}} = \frac{6.8}{s^2+2.5s+8.3}$$

Cascade

Inner loop $\frac{\frac{3}{s+1.5}}{1+\frac{3}{s+1.5}} = \frac{3}{s+4.5}$

Outer loop $\frac{\frac{13.5*3}{(s+4.5)(s+1)}}{1+\frac{13.5*3}{(s+4.5)(s+1)}} = \frac{40.5}{(s+4.5)(s+1)+40.5} = \frac{40.5}{s^2+5.5s+45}$

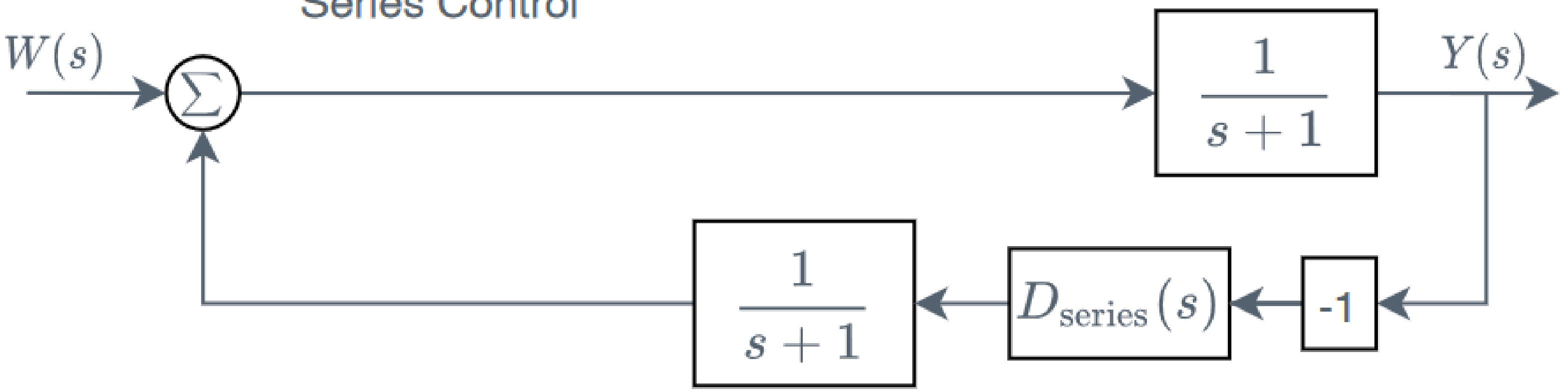
Cascading – example - disturbance



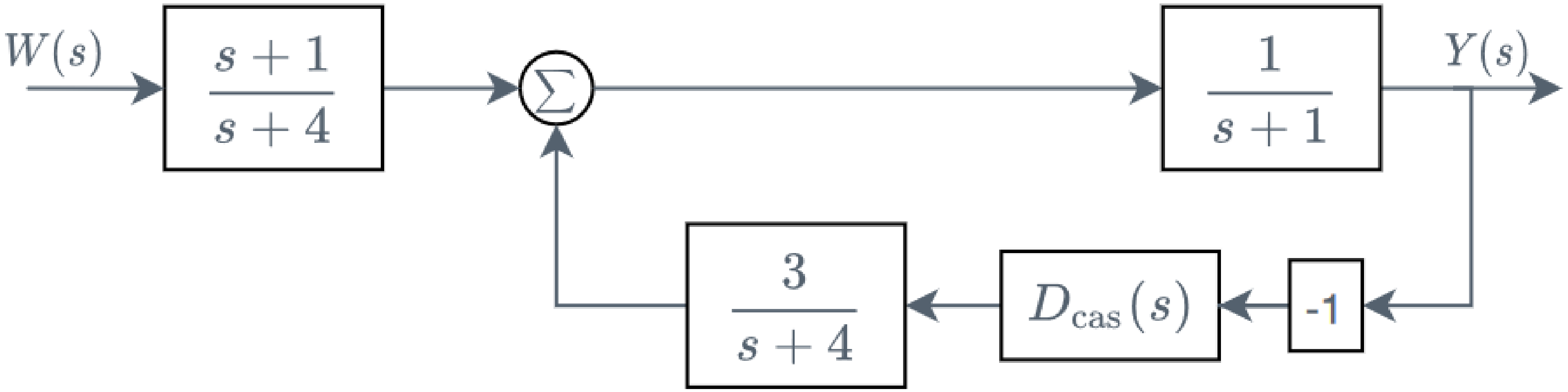
The disturbance condition is better in the cascade.
WHY?

We study the series and cascade controlled systems from disturbance, $W(s)$, to output, $Y(s)$:

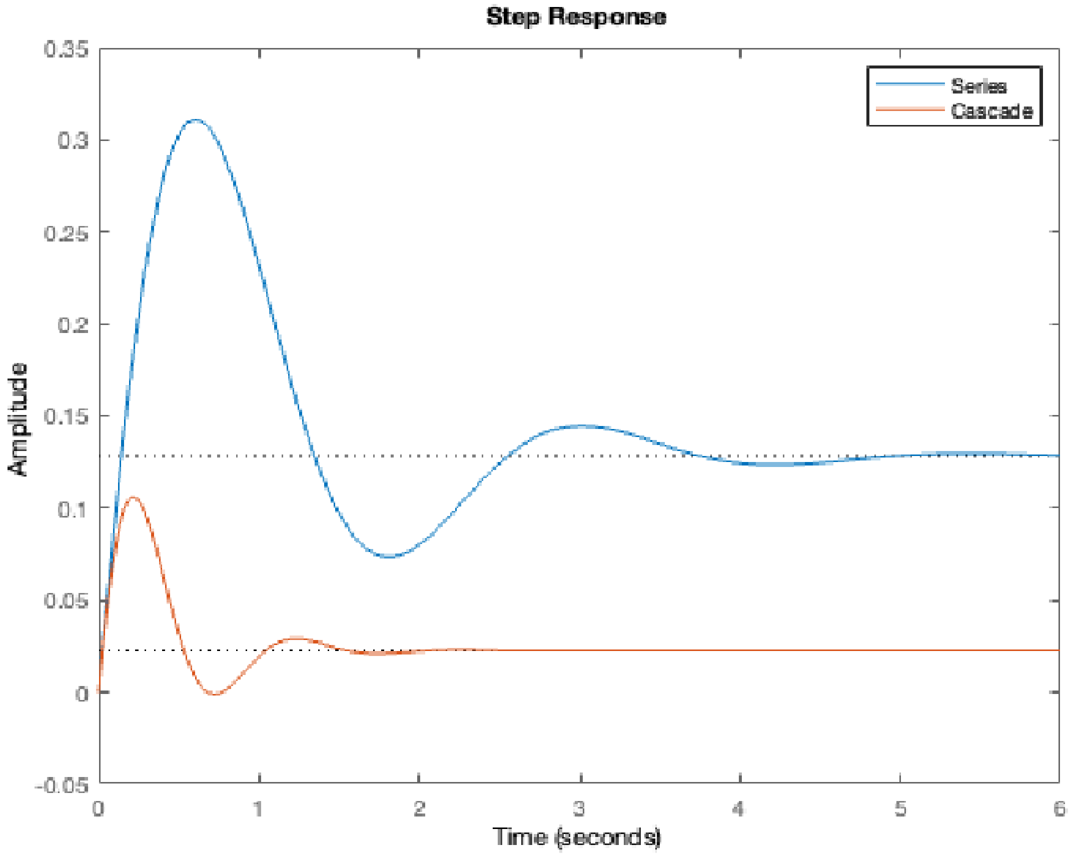
Series Control



Cascade Control

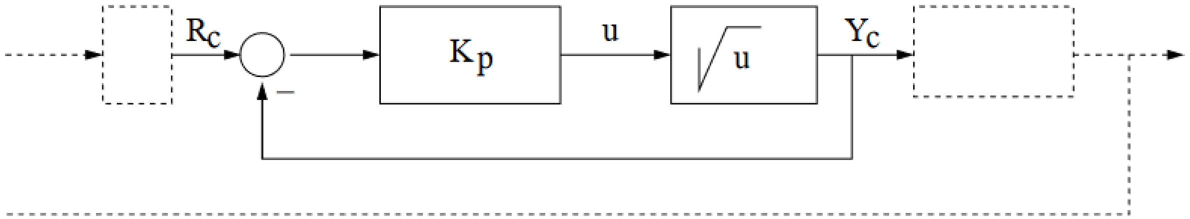


Step responses from disturbance, $W(s)$, to output, $Y(s)$:



Impact of disturbance has been significantly reduced.

Cascading – example - linearity



Linearity:

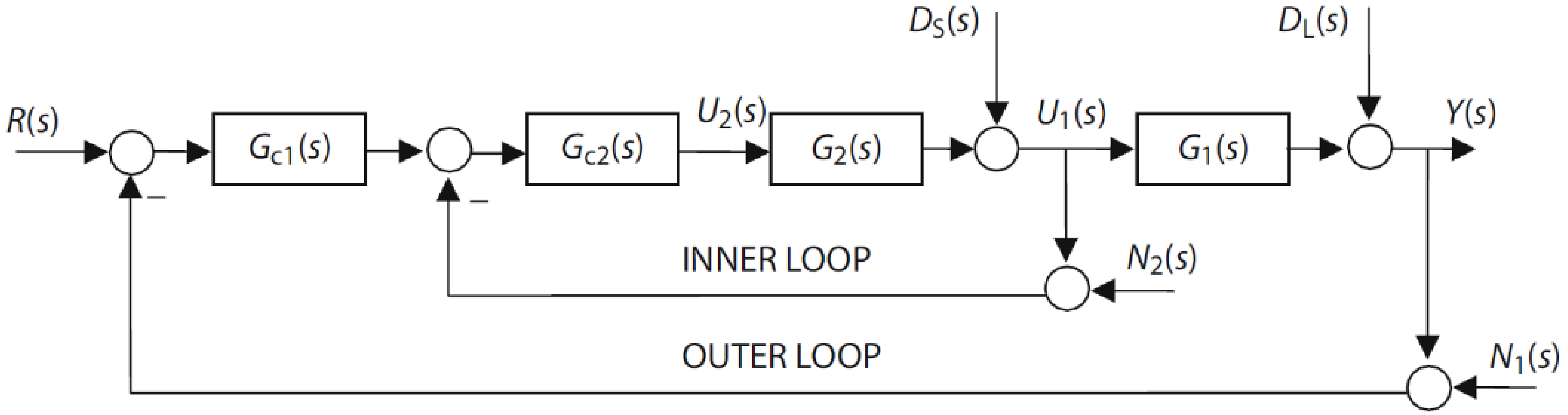
$$Y_c = \sqrt{K_p(R_c - Y_c)} \Rightarrow$$

$$Y_c^2 = K_p(R_c - Y_c) \Rightarrow$$

$$Y_c = R_c - \frac{Y_c^2}{K_p}$$

for $K_p \to \infty$ we have $Y_c \to R_c$

Inner loops in control



Key

$G_1(s)$, Outer process model
$D_L(s)$, System load disturbance
$G_{c1}(s)$, Outer process controller

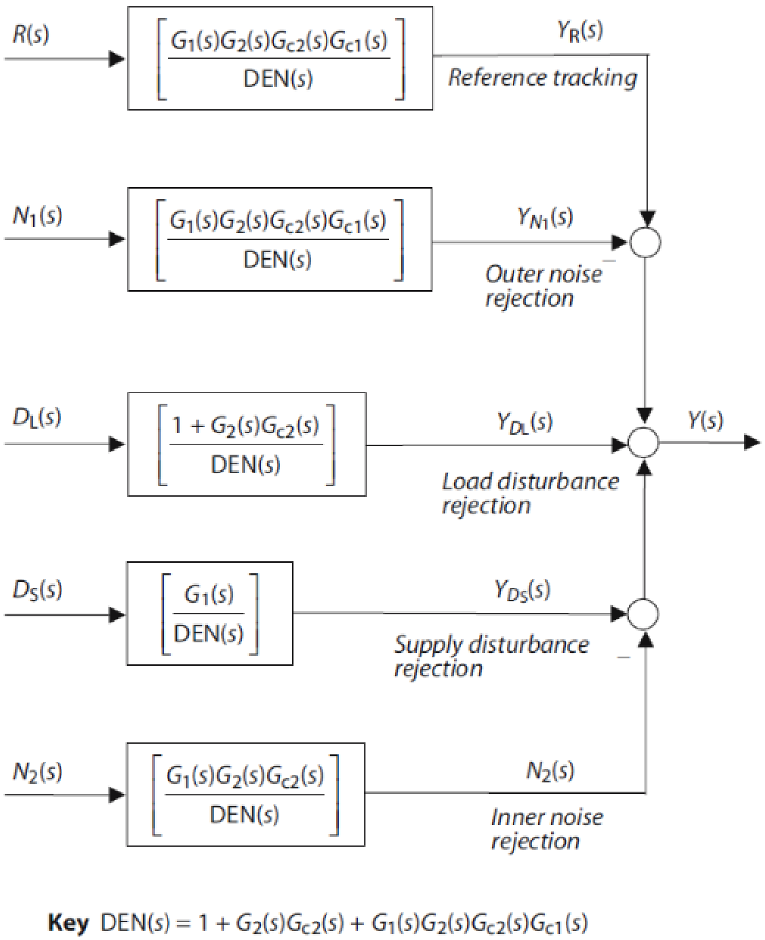
$G_2(s)$, Inner process model
$D_S(s)$, System supply disturbance
$G_{c2}(s)$, Inner process controller

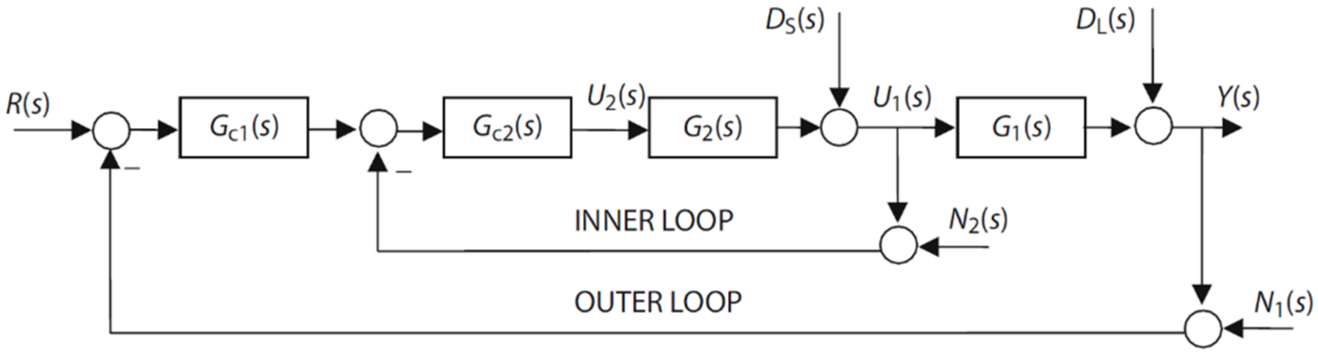
$Y(s)$, System output
$N_1(s)$, Outer process measurement noise
$U_1(s) = Y_2(s)$, Outer process input

$Y_2(s) = U_1(s)$, Inner system output
$N_2(s)$, Inner process measurement noise
$U_2(s)$, Inner process input

$R(s)$, Reference signal

Inner loops in control





Inner loops in control

Table 2.4 Typical cascade control for simple process models.

Outer process	Outer controller forms	Inner process	Inner controller forms
$G_1(s) = \left[\dfrac{K_1}{\tau_1 s + 1}\right]$	$G_{c1}(s) = k_{P1} + \dfrac{k_{I1}}{s}$	$G_2(s) = \left[\dfrac{K_2}{\tau_2 s + 1}\right]$	$G_{c2}(s) = k_{P2} + \dfrac{k_{I2}}{s}$

Performance	**Outer**: $G_{c1}(s)$-P **Inner**: $G_{c2}(s)$-P	**Outer**: $G_{c1}(s)$-P **Inner**: $G_{c2}(s)$-PI	**Outer**: $G_{c1}(s)$-PI **Inner**: $G_{c2}(s)$-P	**Outer**: $G_{c1}(s)$-PI **Inner**: $G_{c2}(s)$-PI
Step ref. tracking	Offset exists	Offset exists	Offset eliminated	Offset eliminated
Outer noise rejection High frequency	–40 dB per decade	–20 dB per decade	–40 dB per decade	–40 dB per decade
Load disturbance rejection Step model	Offset exists	Offset exists	Offset eliminated	Offset eliminated
Supply disturbance rejection Step model	Offset exists	Offset eliminated	Offset eliminated	Offset eliminated
Inner noise rejection High frequency	–40 dB per decade	–40 dB per decade	–40 dB per decade	–40 dB per decade

Inner loops in control

1. Disturbances arising within the inner loop, $D_s(s)$, are corrected the inner- loop control before they can influence the outer loop output, Y(s).
2. Phase lag existing in the inner loop, $G_2(s)$, is reduced measurable by the inner loop. This improves the speed of response of the outer loop.
3. Gain variations in the inner loop (non-linearity's) are overcome within its own loop
4. The closed loop rise time can be chosen to 3-5 times faster than the rise time of $G_2(s)$.