Exercise Sheet 2

Literature:

G.F. Franklin, J.D. Powell and A. Emami-Naeini: *Feedback Control of Dynamic Systems*, 5th edition, pp. 455-458, pp. 471-487.

Exercise 1

Determine for each of the following systems whether it is controllable:

$$(1): \quad \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \qquad (2): \quad \dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$(3): \quad \dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \qquad (4): \quad \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$(5): \quad \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u, \qquad (6): \quad \dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

Exercise 2

We consider the following (controllable) system:

$$\dot{x} = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u \tag{1}$$

1. Write (1) in *controllable canonical form*, and find the corresponding state space transformation
2. Find by virtue of the controllable canonical form a state feedback law $u = Fx$ for (1), such that the characteristic polynomium for the closed loop system becomes $s^2 + 3s + 2$. NOTE: the state feedback law must feed back the *original* states. Hence, the feedback law derived for the canonical form must be transformed back.
3. Verify by computing

$$A + BF = \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} f_1 & f_2 \end{pmatrix} = \begin{pmatrix} 7 + 4f_1 & -9 + 4f_2 \\ 6 + 3f_1 & -8 + 3f_2 \end{pmatrix}$$

   (the nice result is coincidential - usually the matrix will be 'full').

Exercise 3

The figure illustrates two bodies connected via a spring, moving on top of a third body. This third

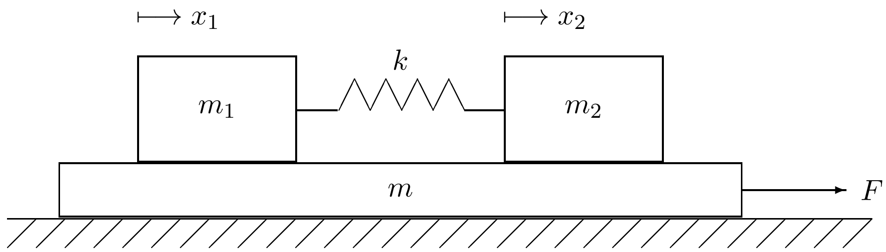

Figure 1: Coupled carts

body is subjected to an external force F, which is considered to be a control signal. It is assumed that the two top bodies have masses m_1 resp. m_2, and that the friction forces acting on these two bodies are viscous with the same coefficients of friction.

1. Specify (without doing any algebra) the condition for the system to be controllable.

2. The system might be simplified slightly by considering the induced velocity v on the lower body to be the control signal. This leads to the following state space model ($v_1 = \dot{x}_1$ og $v_2 = \dot{x}_2$):

$$\overset{\cdot}{\begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix}} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{c}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{c}{m_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{c}{m_1} \\ \frac{c}{m_2} \end{pmatrix} v$$

   Consider the correctness of this model, and verify the heuristic result from 1, for instance by choosing a couple of sets of numerical values for the parameters involved, using MATLAB$^{\text{TM}}$.