Problem 3.1

$$(a)\quad D_0 = 4N\left(\frac{d}{\lambda}\right), eq.\,6.49\ or\ table\ 6.8$$

$$20 = 10\log_{10}\left(D_0\right) \Rightarrow D_0 = 100$$

$$100 = 4N\left(\frac{\lambda}{4\lambda}\right) = N \Rightarrow N = 100$$

$$(b)\quad L = 99\left(\frac{\lambda}{4}\right) = 24.75\lambda$$

Problem 3.1

$$(c) \quad \theta_{3dB} = \theta_h = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right), tab.\,6.4$$

$$\theta_{3dB} \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi\left(\frac{\lambda}{4}\right)100}\right) \simeq 2\cos^{-1}\left(1 - \frac{1.391\cdot 4}{\pi\cdot 100}\right)$$

$$\theta_{3dB} \simeq 2\cos^{-1}\left(1 - 0.01771\right) \simeq 2\cdot 10.799° \simeq 21.6°$$

Problem 3.1

$$(d) \quad sidelobe(dB) \simeq -13.5dB$$ (Sinc function)

$$(e) \quad \beta = \pm kd = \pm \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pm \frac{\pi}{2} = \pm 90°, eq. 6.20$$

Problem 3.2

$$f_r = 2.441GHz$$

$$\varepsilon_r = 2.2\left(pp - plastic\right)$$

$$h = 3mm$$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$W = \frac{\lambda}{2}\sqrt{\frac{2}{2.2+1}} \simeq 48.6mm, eq.14.6$$

Problem 3.2

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + \frac{12h}{W}}} \simeq 2.005,\ eq.\,14.1$$

$$\Delta L = h \cdot 0.412 \frac{\left(\varepsilon_{reff} + 0.3\right)\left(\frac{W}{h} + 0.264\right)}{\left(\varepsilon_{reff} - 0.258\right)\left(\frac{W}{h} + 0.8\right)} \simeq 1.569mm,\ eq.\,14.2$$

Problem 3.2

$$L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}}\cdot\sqrt{\mu_0\varepsilon_0}} = \frac{c}{2f_r\sqrt{\varepsilon_{reff}}} = \frac{\lambda_0}{2\sqrt{\varepsilon_{reff}}} = 42.87mm,\ eq.14.4$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$L_{eff} = L - 2\Delta L = 39.73mm,\ eq.14.3$$

$R_{in0} \simeq 240\Omega$ Feed point $for\ 50\Omega \Rightarrow y_0 = 13.88mm, eq.14.20a$

Problem 3.2

$$Z_c = \frac{60}{\sqrt{\varepsilon_{reff}}} \ln\left[\frac{8h}{W_0} + \frac{W_0}{4h}\right], \ for \ \frac{W_0}{h} \le 1, eq. 14.19a$$

$$e^{\left(\frac{Z_c \sqrt{\varepsilon_{reff}}}{60}\right)} = \left[\frac{8h}{W_0} + \frac{W_0}{4h}\right] \Rightarrow$$

Problem 3.2

$$\Rightarrow 8hW_0^{-1} + \frac{1}{4h}W_0 = e^{\left(\frac{Z_c\sqrt{\varepsilon_{reff}}}{60}\right)} \Rightarrow$$

$$\Rightarrow \frac{1}{4h}W_0^{\,2} - e^{\left(\frac{50\sqrt{2.05}}{60}\right)}W_0 + 24 = 0 \Rightarrow$$

$$\Rightarrow \frac{1}{12}W_0^{\,2} - 3.36W_0 + 24 = 0 \Rightarrow$$

Problem 3.2

$$W_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3.3 \pm \sqrt{3.3^2 - 4 \cdot \frac{1}{12} \cdot 24}}{2 \cdot \frac{1}{12}} = 9.6mm \vee 30mm$$

$$wrong\ \ formula! \frac{W}{h}\ is\ not \le 1!$$

Problem 3.2

$$Z_c = \frac{120\pi}{\sqrt{\varepsilon_{reff}}\left[\frac{W_0}{h}+1.393+0.667\ln\left(\frac{W_0}{h}+1.444\right)\right]},\ for\ \frac{W_0}{h}>1,\ eq.14.19b$$

$$Z_c\sqrt{\varepsilon_{reff}}\,\frac{1}{3}\cdot\ln\left(\frac{W_0}{h}+1.444\right)=\eta-\frac{Z_c\sqrt{\varepsilon_{reff}}W_0}{h}-Z_c\sqrt{\varepsilon_{reff}}\cdot 1.393\Rightarrow$$

Problem 3.2

$$\Rightarrow c_1 \ln\left(\frac{W_0}{h} + 1.444\right) + c_2 W_0 + c_3 = 0$$

optimize numerically or try

$$W_0 = 9.3mm \Rightarrow Z_c = 50\Omega, OK!$$

- See matlab program Imp.m