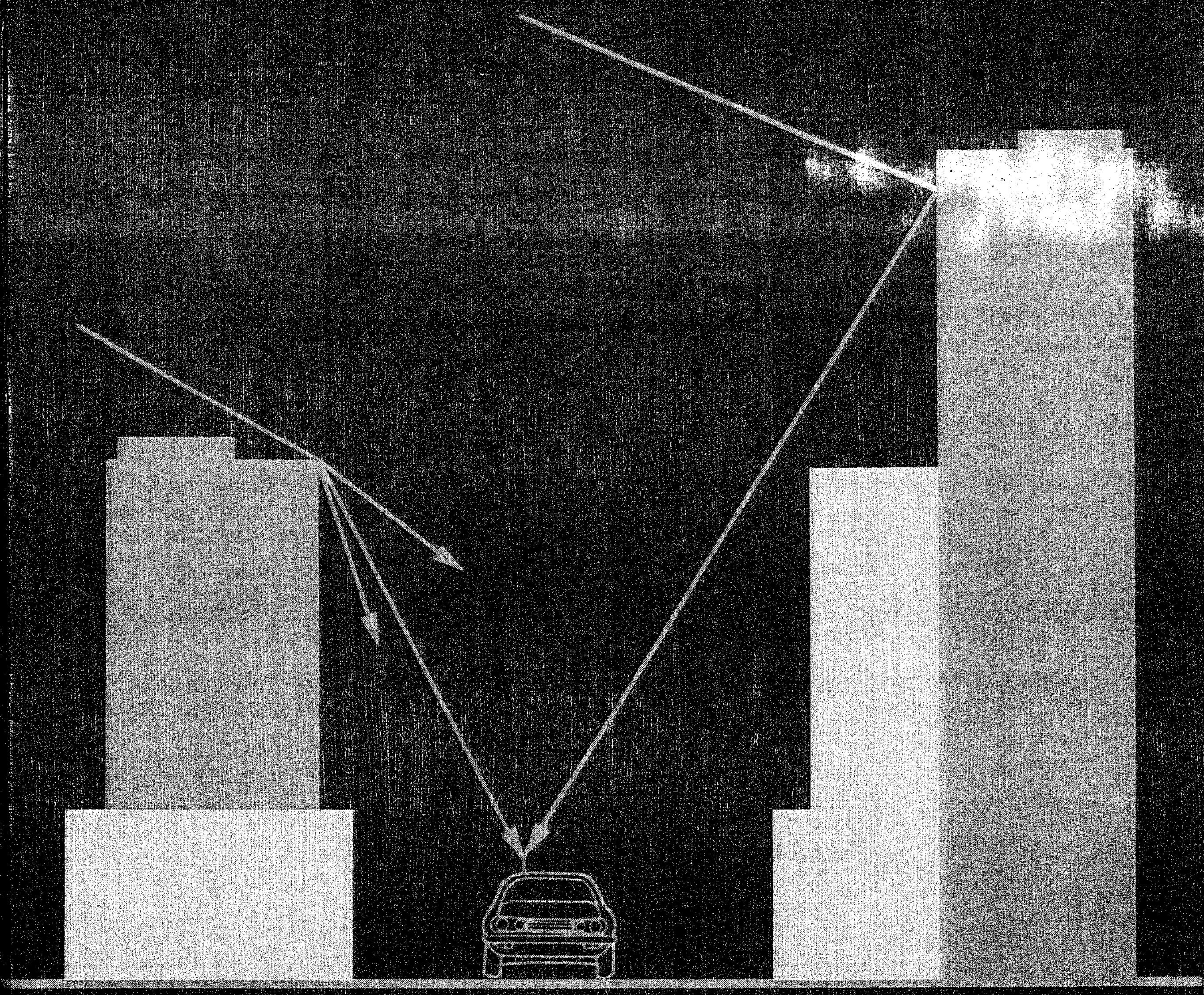
The Mobile Radio Propagation Channel



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Finally, we note that the maximum useful power that can be delivered to the terminals of a matched receiver is

$$P = \frac{E^2 A}{\eta} = \frac{E^2}{120\pi} \cdot \frac{\lambda^2 G_R}{4\pi} = \left(\frac{E\lambda}{2\pi}\right)^2 \frac{G_R}{120} \tag{2.8}$$

2.3 PROPAGATION OVER A REFLECTING SURFACE

The free-space propagation equation applies only under very restricted conditions; in practical situations there are almost always obstructions in or near the propagation path or surfaces from which the radio waves can be reflected. A very simple case, but one of practical interest, is that of propagation between two elevated antennas within line of sight of each other, above the surface of the earth. We will consider two cases, firstly that of propagation over a spherical reflecting surface and secondly when the distance between the antennas is small enough for us to neglect curvature and assume the reflecting surface to be flat. In these cases, illustrated in Figs. 2.1 and 2.4, the received signal is made up of a combination of direct and ground-reflected waves. In order to determine the resultant we need to know the reflection coefficient.

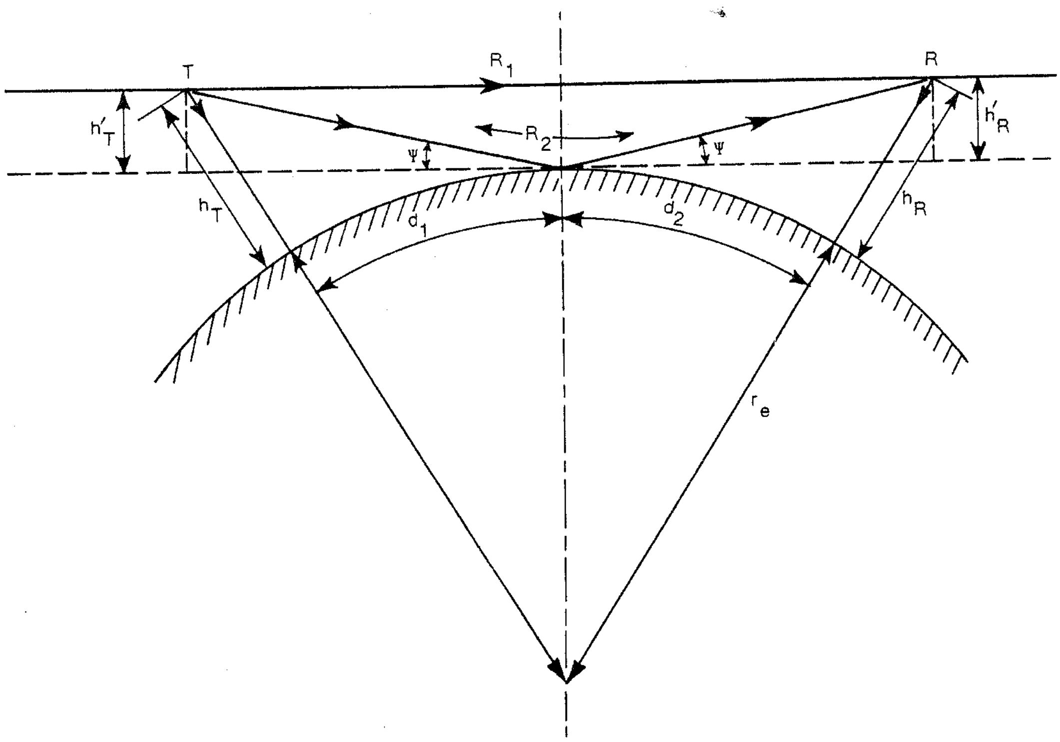

Fig. 2.1 Two mutually-visible attennas located above a smooth, spherical earth of effective radius r_e

2.3.1 The reflection coefficient of the earth

The amplitude and phase of the ground-reflected wave depends on the reflection coefficient of the earth at the point of reflection and differs for horizontal and vertical polarisation. In practice the earth is neither a perfect conductor nor a perfect dielectric so the reflection coefficient depends on the ground-constants, in particular and dielectric constant ε and the conductivity σ.

For a horizontally-polarised wave incident on the surface of the earth (assumed to be perfectly smooth) the reflection coefficient is given by [1, Chap. 16]

$$\rho_h = \frac{\sin\psi - \sqrt{(\varepsilon/\varepsilon_0 - j\sigma/\omega\varepsilon_0) - \cos^2\psi}}{\sin\psi + \sqrt{(\varepsilon/\varepsilon_0 - j\sigma/\omega\varepsilon_0) - \cos^2\psi}}$$

where ω is the angular frequency of the transmission and ε_o is the dielectric constant of free space. Writing ε_r as the relative dielectric constant of the earth yields

$$\rho_h = \frac{\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}{\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}} \tag{2.9}$$

where

$$x = \frac{\sigma}{\omega\varepsilon_0} = \frac{18 \times 10^9 \sigma}{f}$$

For vertical polarisation the corresponding expression is

$$\rho_v = \frac{(\varepsilon_r - jx)\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}{(\varepsilon_r - jx)\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}} \tag{2.10}$$

It is apparent that the reflection coefficients ρ_h and ρ_v are complex and the reflected wave will therefore differ in both magnitude and phase from the incident wave. Examination of eqns. (2.9) and (2.10) reveals some quite interesting differences. For horizontal polarisation the relative phase of the incident and reflected waves is nearly 180° for all angles of incidence. For very small values of ψ (near-grazing incidence) eqn. (2.9) shows that the reflected wave is equal in magnitude and 180° out of phase with the incident wave for all frequencies and all ground conductivities. In other words, for grazing incidence

$$\rho_h = |\rho_h| \angle \theta = 1 \angle 180° = -1 \tag{2.11}$$

As the angle of incidence is increased then $|\rho_h|$ and θ change, but only by relatively small amounts. The change is greatest at higher frequencies and when the ground conductivity is poor.

For vertical polarisation the results are quite different. At grazing incidence, there is no difference between horizontal and vertical polarisation and eqn. (2.11) still applies. However as ψ is increased, substantial differences appear.

The magnitude and relative phase of the reflected wave both decrease rapidly as ψ increases and at an angle known as the pseudo-Brewster angle the magnitude becomes a minimum and the phase reaches $-90°$. At values of ψ greater than the Brewster angle, $|\rho_v|$ increase again and the phase tends towards zero. The very sharp changes that occur in these circumstances are illustrated by Fig. 2.2 which shows the values of $|\rho_v|$ and θ as functions of the angle of incidence ψ. It can be seen that the pseudo-Brewster angle is about 15° at frequencies that are of interest for mobile communications ($x \ll \varepsilon_r$) although at lower frequencies and higher conductivities it becomes smaller, approaching zero if $x \gg \varepsilon_r$.

Table 2.1 shows typical values for the ground constants that affect the value of ρ.

It can be seen that the conductivity of flat, good ground is much higher than for the poorer ground found in mountainous areas, whilst the dielectric constant, typically 15, can be as low as 4 or as high as 30. It should be noted that over lakes or seas, the reflection properties are quite different because

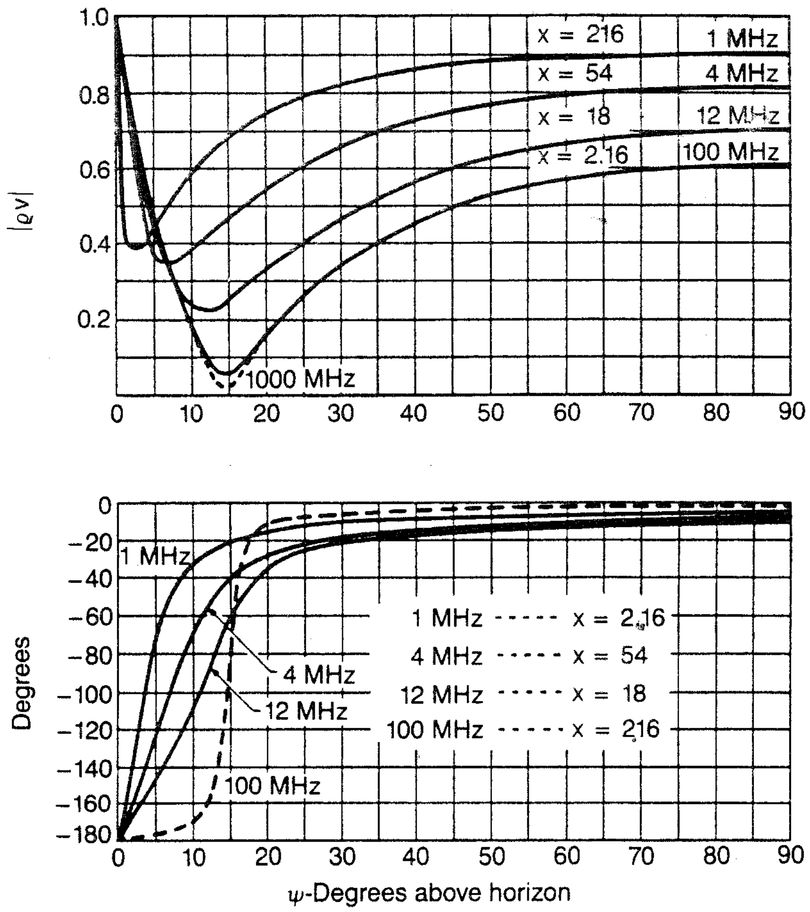

Fig. 2.2 Magnitude and phase of the plane wave reflection coefficient for vertical polarisation. Curves drawn for $\sigma = 12 \times 10^{-3}$, $\varepsilon_r = 15$. *Approximate results for other frequencies and conductivities can be obtained by calculating the value of* x *as* $18 \times 10^3 \sigma / f_{\mathrm{MHz}}$

Table 2.1 TYPICAL VALUES OF GROUND CONSTANTS

Surface	*Conductivity* σ *(siemens)*	*Dielectric constant* ε_r
Poor ground (dry)	10^{-3}	4–7
Average ground	5×10^{-3}	15
Good ground (wet)	2×10^{-2}	25–30
Sea water	5	81
Fresh water	10^{-2}	81

of the high values of both σ and ε_r. Eqn. (2.11) applies for horizontal polarisation, particularly over seawater, but ρ may be significantly different from -1 for vertical polarisation.

2.3.2 Propagation over a curved reflecting surface

The situation of two mutually-visible antennas sited on a smooth earth of effective radius r_e is shown in Fig. 2.1. The heights of the antennas above the earth's surface are h_T and h_R and above the tangent plane through the point of reflection the heights are h'_T and h'_R. Simply geometry gives,

$$d_1^2=[r_e+(h_T-h'_T)]^2-r_e^2=(h_T-h'_T)^2+2r_e(h_T-h'_T)\simeq 2r_e(h_T-h'_T) \quad (2.12)$$

and similarly

$$d_2^2\simeq 2r_e(h_R-h'_R) \quad (2.13)$$

Using eqns. (2.12) and (2.13) we obtain

$$h'_T=h_T-\frac{d_1^2}{2r_e} \quad \text{and} \quad h'_R=h_R-\frac{d_2^2}{2r_e} \quad (2.14)$$

The reflecting point, where the two angles marked ψ are equal, can be determined by noting that, providing $d_1, d_2 \gg h_T, h_R$, the angle ψ in radians, is given by

$$\psi=\frac{h'_T}{d_1}=\frac{h'_R}{d_2}$$

Hence,

$$\frac{h'_T}{h'_R}\simeq\frac{d_1}{d_2} \quad (2.15)$$

Using the obvious relationship $d=d_1+d_2$ together with eqns. (2.14) and (2.15) allows us to formulate a cubic equation in d_1,

$$2d_1^3-3dd_1^2+[d^2-2r_e(h_T+h_R)]d_1+2r_eh_Td=0 \quad (2.16)$$

The appropriate root of this equation can be found by standard methods starting from the rough approximation

$$d_1 \simeq \frac{d}{1 + h_T/h_R}$$

In order to calculate the field strength at a receiving point it is normally assumed that the difference in path length between the direct and ground reflected waves is negligible as far as attenuation is concerned, but it cannot be neglected with regard to the phase difference along the two paths. The length of the direct path is

$$R_1 = d\left[1 + \frac{(h'_T - h'_R)^2}{d^2}\right]^{1/2}$$

whilst the length of the reflected path is

$$R_2 = d\left[1 + \frac{(h'_T + h'_R)^2}{d^2}\right]^{1/2}$$

The difference $\Delta R = R_2 - R_1$ is

$$\Delta R = d\left\{\left[1 + \frac{(h'_T + h'_R)^2}{d^2}\right]^{1/2} - \left[1 + \frac{(h'_T - h'_R)^2}{d^2}\right]^{1/2}\right\}$$

and if $d \gg h'_T$, h'_R this reduces to

$$\Delta R = \frac{2h'_T h'_R}{d} \tag{2.17}$$

The corresponding phase difference is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta R = \frac{4\pi h'_T h'_R}{\lambda d} \tag{2.18}$$

If the field strength at the receiving antenna due to the direct wave is E_d, then the total received field E is

$$E = E_d[1 + \rho\exp(-j\Delta\phi)]$$

where ρ is the reflection coefficient of the earth, and $\rho = |\rho|\exp(j\theta)$. Thus,

$$E = E_d\{1 + |\rho|\exp[-j(\Delta\phi - \theta)]\} \tag{2.19}$$

This equation can be used to calculate the received field strength at any location, but it should be noted that the curvature of the spherical earth produces a certain amount of divergence of the ground-reflected wave as shown in Fig. 2.3. This effect can be taken into account by using, in eqn. (2.19) a value of ρ which is different from that derived in Section (2.3.1) for reflection from a plane surface. The appropriate modification consists of multiplying the value of ρ for a plane surface by a divergence factor D, given by [3],

$$D \simeq \left[1 + \frac{2d_1 d_2}{r_e(h'_T + h'_R)}\right]^{-1/2} \tag{2.20}$$

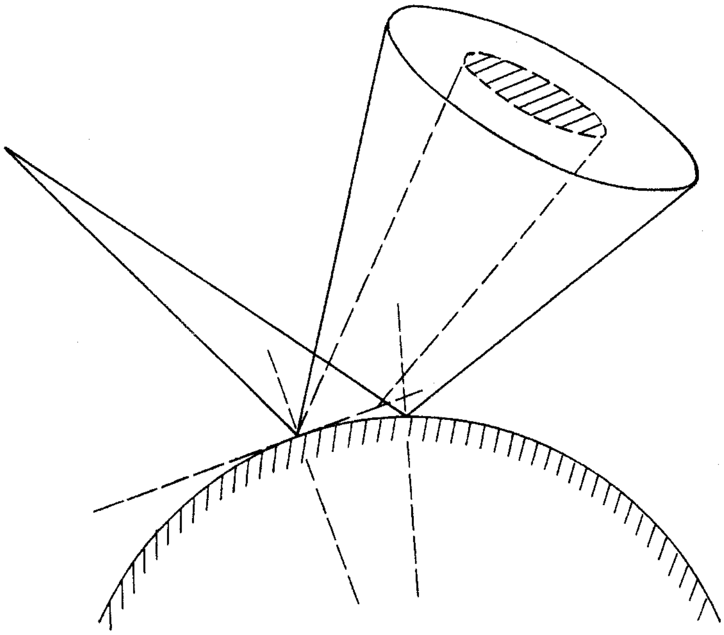

Fig. 2.3 Divergence of reflected rays from a spherical surface

The value of D can be of the order of 0.5, so the effect of the ground-reflected wave is considerably reduced.

2.3.3 Propagation over a plane reflecting surface

For distances less than a few tens of kilometres, it is often permissible to neglect earth curvature and assume the surface to be smooth and flat as in Fig. 2.4. If, in addition, we assume grazing incidence so that $\rho = -1$, then eqn. (2.19) becomes

$$E = E_d[1 - \exp(-j\Delta\phi)]$$
$$= E_d[1 - \cos\Delta\phi + j\sin\Delta\phi]$$

Thus,

$$|E| = |E_d|\,[1 + \cos^2\Delta\phi - 2\cos\Delta\phi + \sin^2\Delta\phi]^{1/2}$$
$$= 2|E_d|\sin\frac{\Delta\phi}{2}$$

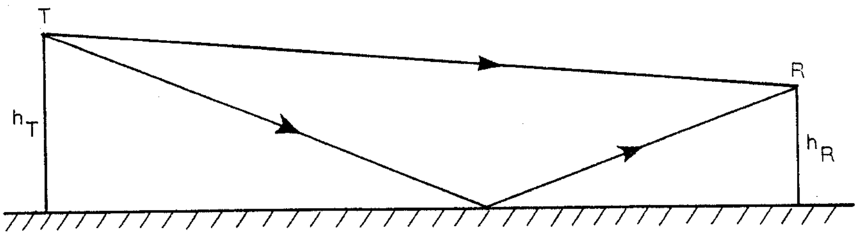

Fig. 2.4 Propagation over a plane earth

and using eqn. (2.18), with $h'_T = h_T$ and $h'_R = h_R$

$$|E| = 2|E_d| \sin\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

The received power P_R is proportional to E^2 so

$$P_R = 4|E_d|\, sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

$$= 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right) \tag{2.21}$$

If $d \gg h_T$, h_R, this becomes

$$\frac{P_R}{P_T} = G_T G_R \left(\frac{h_T h_R}{d^2}\right)^2 \tag{2.22}$$

Eqn. (2.22) is known as the plane-earth propagation equation. It differs from the free-space relationship, eqn. (2.3), in two important ways. First, as a consequence of the assumption that $d \gg h_T$, h_R the angle $\Delta\phi$ is small and λ cancels out of eqn. (2.22) leaving it frequency-independent. Secondly, it shows an inverse fourth-power law with range rather than the inverse square law of eqn. (2.3). This means a far more rapid decrease in received power with range, 12 dB for each doubling of distance in this case.

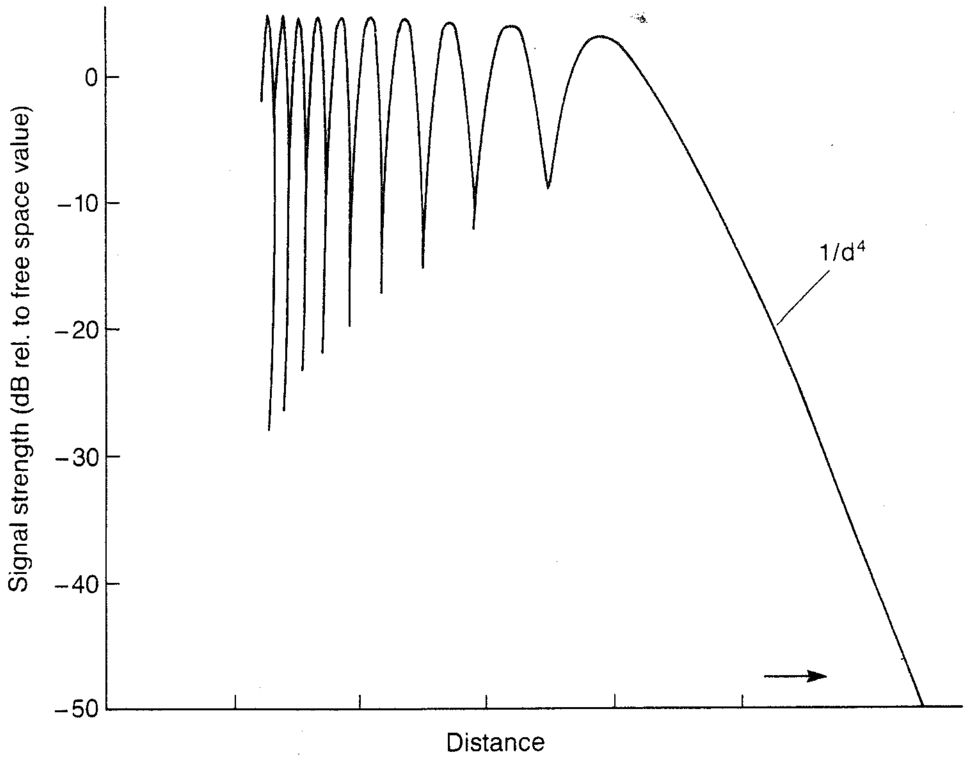

Fig. 2.5 Variation of signal strength with distance in the presence of a specular reflection.

It should be emphasised that eqn. (2.22) only applies at ranges where the assumption $d \gg h_T$, h_R is valid. Close to the transmitter eqn. (2.21) must be used and this gives alternate maxima and minima in the signal strength as shown in Fig. 2.5.

In convenient logarithmic form eqn. (2.22) can be written

$$L_P = 10\log_{10}G_T + 10\log_{10}G_R + 20\log_{10}h_T + 20\log_{10}h_R - 40\log_{10}d \quad (2.23)$$

and by comparison with eqn. (2.6) we can write a 'basic loss' between isotropic antennas as

$$L_B = 20\log_{10}h_T + 20\log_{10}h_R - 40\log_{10}d \quad (2.24)$$

2.4 GROUND ROUGHNESS

The discussion in the previous section pre-supposed that the reflecting surface is smooth and the analysis was therefore based on the assumption that a specular reflection takes places at the point where the transmitted wave is incident on the earth's surface. When the surface is rough the specular reflection assumption is no longer realistic since a rough surface presents many facets to the incident wave. A diffuse reflection therefore occurs and the mechanism is more akin to scattering. In these conditions characterisation by a single, complex reflection coefficient is not appropriate since the random nature of the surface results in an unpredictable situation. Only a small fraction of the incident energy may be scattered in the direction of the receiving antenna, and the 'ground-reflected' wave may therefore make a negligible contribution to the received signal.

The question therefore arises as to what constitutes a 'rough' as opposed to a smooth surface. Clearly a surface that might be considered rough at some frequencies and angles of incidence may approach a smooth surface if these parameters are changed. A measure of roughness is needed to quantify the problem and the criterion normally used is known as the Rayleigh criterion. The problem is illustrated in Fig. 2.6 (a) and an idealised rough surface profile is shown in Fig. 2.6 (b).

Consider the two rays A and B shown in Fig. 2.6 (b). Ray A is reflected from the upper part of the rough surface and ray B from the lower part. Relative to the wavefront AA′ shown, the difference in path length of the two rays when they reach the points C and C′ after reflection is

$$\begin{aligned}\Delta l &= (AB+BC)-(A'B'+B'C') \\ &= \frac{d}{\sin\psi}(1-\cos 2\psi) \\ &= 2d\sin\psi \end{aligned} \quad (2.25)$$

The phase difference between C and C′ is therefore

$$\Delta\theta = \frac{2\pi}{\lambda}\Delta l = \frac{4\pi d\sin\psi}{\lambda} \quad (2.26)$$