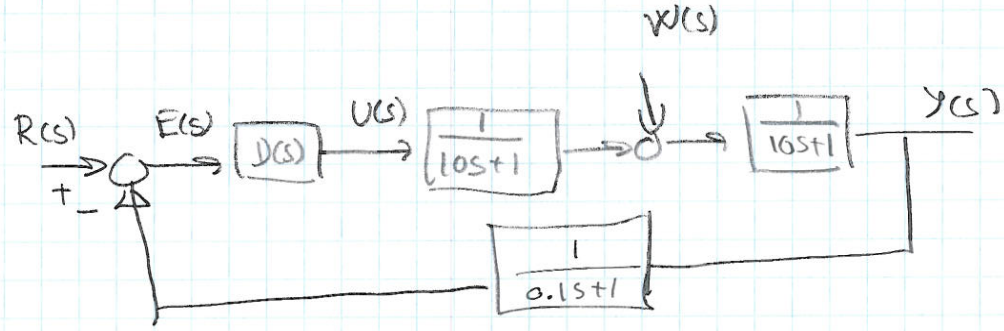
(1)




A: $D(s) = k_p$ B: $D(s) = k\left(1+\frac{1}{T_1 s}\right) = K \frac{T_1 s + 1}{T_1 s}$

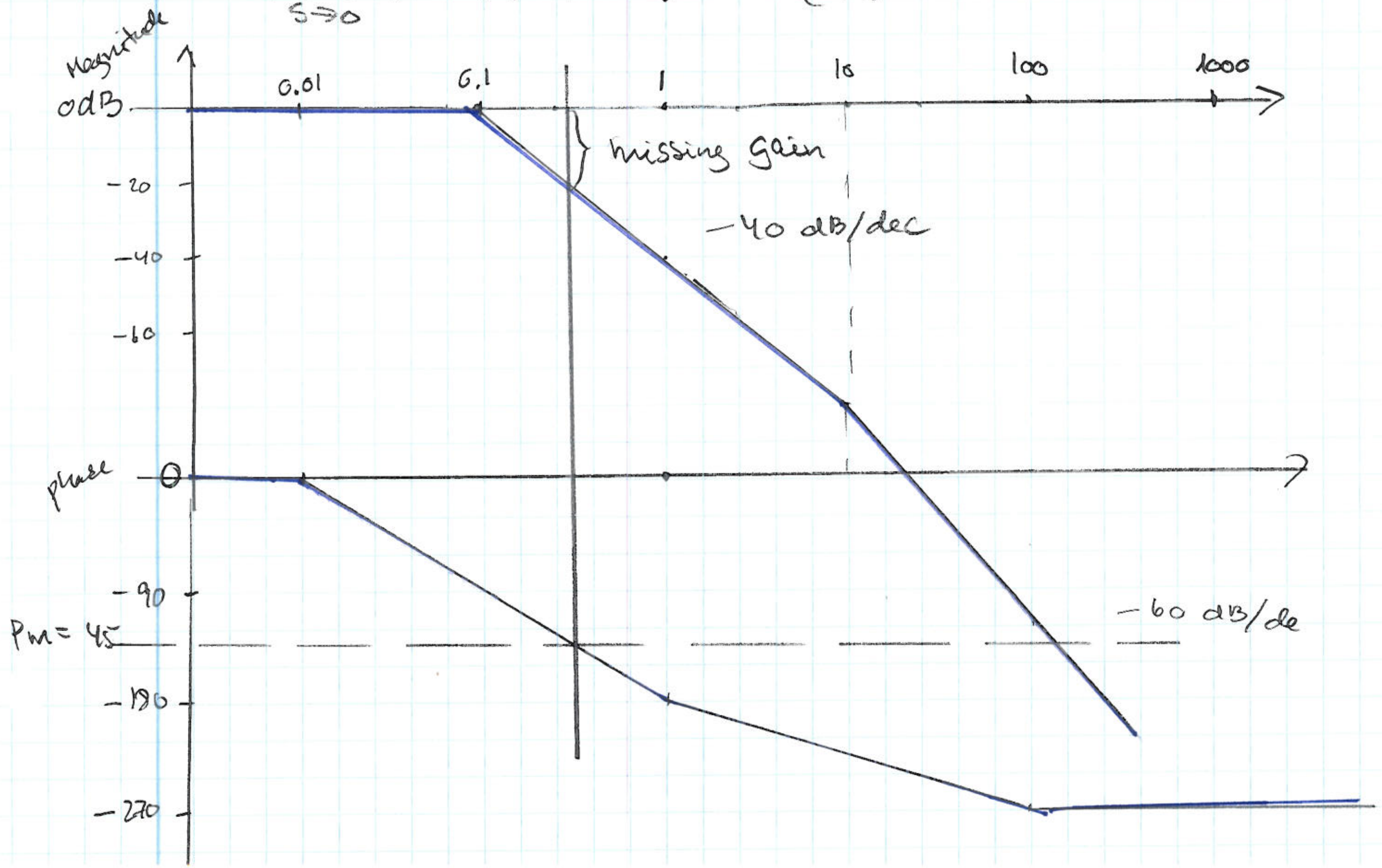
To design a controller we make a bode plot for the uncontrolled system ($D(s) = 1$)

open loop: $1 \cdot \frac{1}{10s+1} \frac{1}{10s+1} \frac{1}{0.1s+1}$

$$= \frac{0.1}{s+0.1} \frac{0.1}{s+0.1} \frac{10}{s+10} = \frac{0.1}{(s+0.1)^2 (s+10)}$$

There is no integratin (pole=0) ⇒ the magnitude has an asyptotic value

$$\lim_{s\to 0} \frac{0.1}{(s+0.1)^2(s+10)} = \frac{0.1}{(0.1)^2 \cdot 10} = 1 = 0\,dB$$

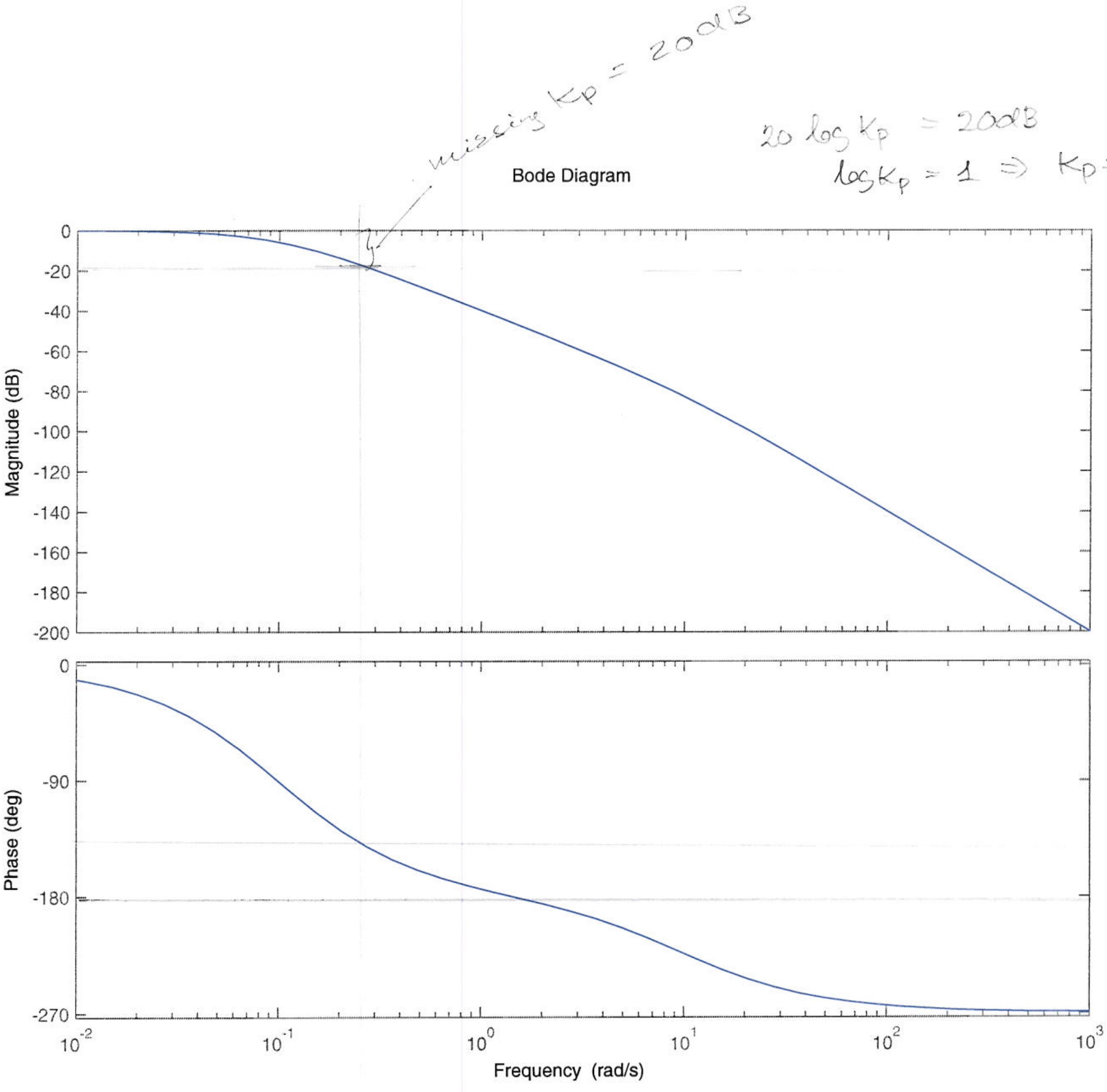


2

$$\frac{0.1}{(s+0.1)^2(s+10)}$$

$20 \log K_p = 20dB$

$\log K_p = 1 \Rightarrow K_p = 10$



(3)

To obtain a phase margin = 45

the phase must be $-180 + 45 = -135$

when the gain is 0 dB

A proportional Controler $K_p$ can only change

the magnitude not the phase $\Rightarrow$

$K_p$ must be choosen, so the magnitude

= 0dB at the frequency when the phase = -135

Looking at the handmade Bodeplot that is

$K_p \approx 20$ dB $\Rightarrow$ $20 \log K_p = 20$ $\Rightarrow$ $\log K_p = 1$

$K_p = 10$

Matlab - Bodeplot : $K_p \sim 20$dB $\Rightarrow$ $K_p = 10$

4

PI - control

$$D(s) = K \frac{Ts+1}{Ts} = K \frac{s+\frac{1}{T}}{s}$$

If we want to cancle a pole in $-0.1$

$\frac{1}{T} = 0.1 \sim T = 10$

To design $K$ in the PI controler we draw a bodeplot for $\frac{s+\frac{1}{T}}{s} \frac{0.1}{(s+0.1)^2(s+10)}$

$$= \frac{s+0.1}{s} \cdot \frac{0.1}{(s+0.1)^2(s+10)} = \frac{0.1}{s(s+0.1)(s+10)}$$

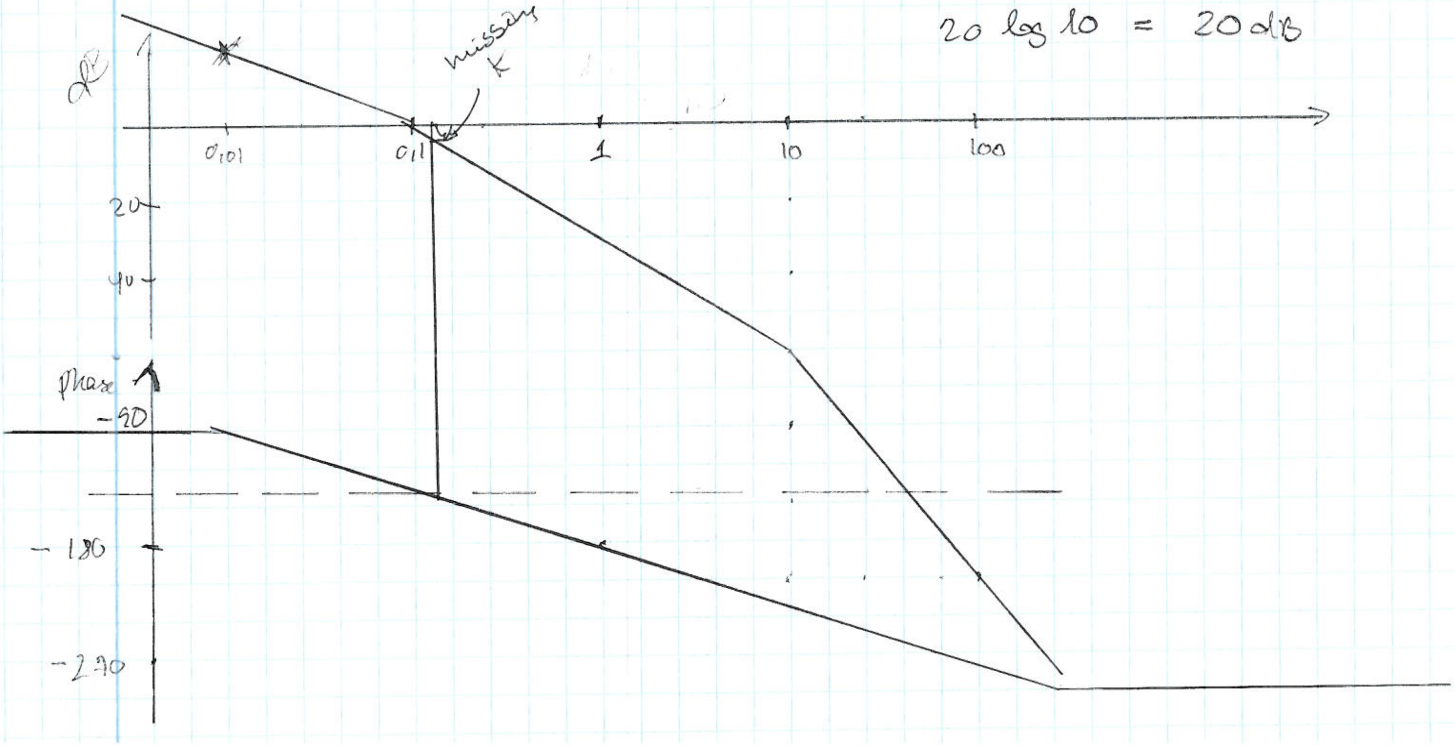
this transferfunktion has a pole i $0 \Rightarrow$ no asymptotic value for the magnitude. Insted we calculate the magnitude i a point more than a decade away from a pole

$\omega = 0.01$

$$\left| \frac{0.1}{(j\omega)(j\omega+0.1)(j\omega+10)} \right|_{\omega=0.01}$$

$$\approx \frac{0.1}{\sqrt{0.01^2}\sqrt{0.01^2+0.1^2}\sqrt{0.01^2+10^2}} = \frac{0.1}{0.01 \cdot 0.1 \cdot 10} = 10$$

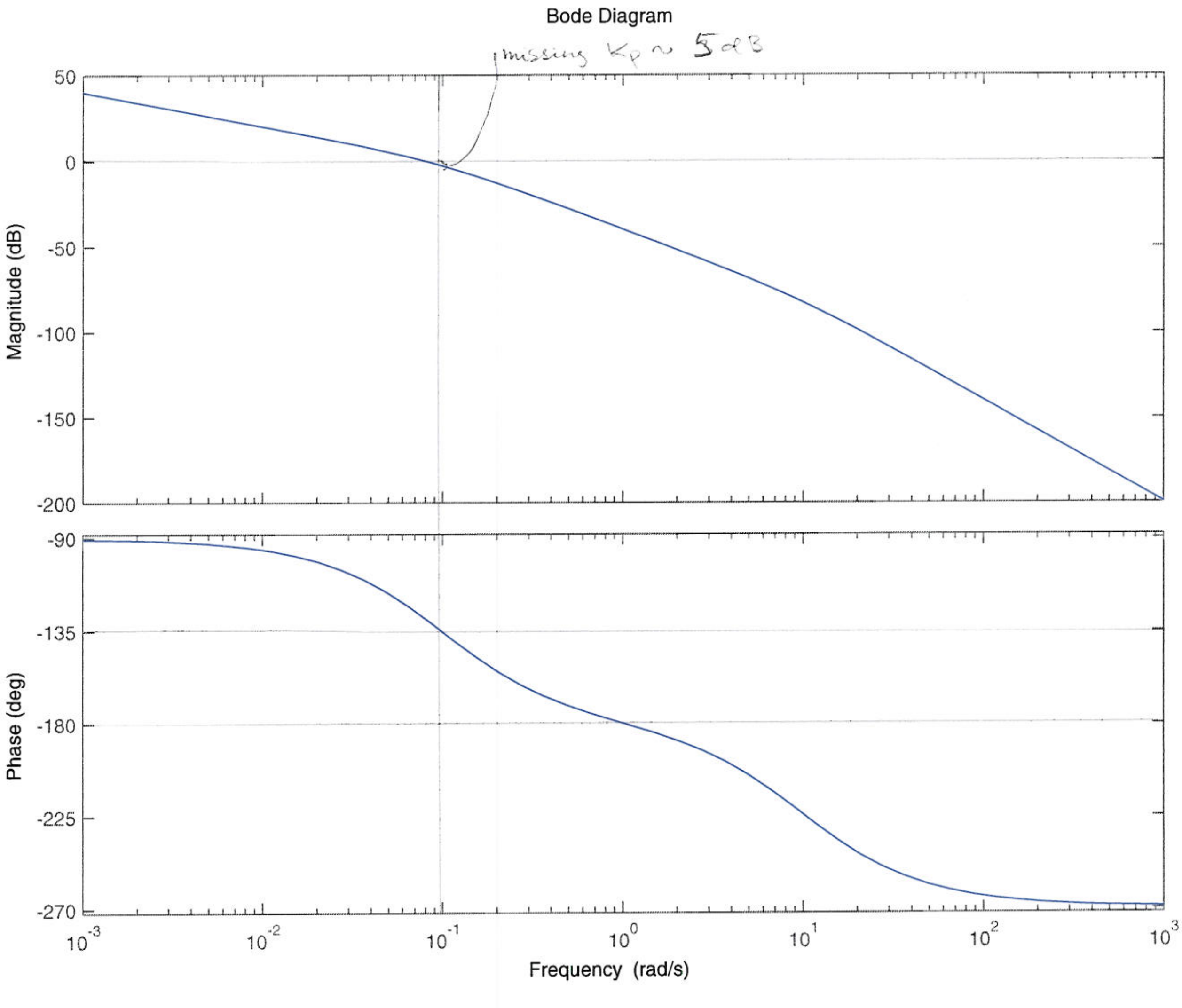
$20 \log 10 = 20\text{dB}$



5

PI - control

$$\frac{0.1}{s(s+0.1)(s+10)}$$



b

The gain K in the PI controlle is found from the bode plot of G(s) in combination with $\frac{s+\frac{1}{T}}{s} = \frac{s+0.1}{s}$

K is not changing the phase the magnitude must be raised by Kp til 0 dB = 1 at the ω value where the pase = $-135^0$

K (handmade) ~ 5 dB ~ 3

Matlab ~ 5 dB ~ 3

The PI - contoolle is $3 \frac{s+0.1}{s}$

(7)

$$Y(s) = T(s)R(s) + F(s)W(s)$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\text{direct term } (R \to Y)}{1 + \text{open loop}} = \frac{D(s)\frac{1}{(10s+1)^2}}{1 + D(s)\frac{1}{(10s+1)^2}\frac{1}{(0.1s+1)}}$$

$$= \frac{D(s)\frac{0.1^2}{(s+0.1)^2}}{1 + D(s)\frac{0.1 \cdot 0.1 \cdot 10}{(s+0.1)^2(s+10)}} = \frac{D(s) \cdot 0.1^2 \cdot (s+10)}{(s+0.1)^2(s+10) + D(s) \cdot 0.}$$

$$F(s) = \frac{Y(s)}{W(s)} = \frac{\text{direct term } (W \to Y)}{1 + \text{open loop}} = \frac{\frac{1}{10s+1}}{1 + \frac{1}{10s+1}\frac{1}{0.1s+}D(s)\frac{1}{10s+1}}$$

$$= \frac{\frac{0.1}{s+0.1}}{1 + \frac{(0.1)^2}{(s+0.1)^2}\frac{10}{(s+10)} \cdot D(s)} = \frac{(s+0.1)(s+10) \cdot 0.1}{(s+0.1)^2(s+10) + 0.1\, D(s)}$$

P - control $\quad D(s) = K_p = 10$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{10 \cdot 0.1^2 \cdot (s+10)}{(s+0.1)^2(s+10) + 10 \cdot 0.1} = \frac{0.1\,(s+10)}{(s+0.1)^2(s+10)+1}$$

$$\lim_{s \to 0} \frac{Y(s)}{R(s)} = \frac{0.1}{(0.1)^2(10)+1} = \frac{0.1}{0.1+1} \approx 0.9$$

$$F(s) = \frac{Y(s)}{W(s)} = \frac{(s+0.1)(s+10) \cdot 0.1}{(s+0.1)^2(s+10) + 0.1 \cdot 10} = \frac{(s+0.1)(s+10) \cdot 0.1}{(s+0.1)^2(s+10)+1}$$

$$\lim_{s \to 0} \frac{Y(s)}{W(s)} = \frac{0.1 \cdot 10 \cdot 0.1}{(0.1)^2(10)+1} = \frac{0.9}{0.1+1} = \frac{0.9}{1.1} = 0.09$$

(8)

PI - control $$D(s) = 3\,\frac{s+0.1}{s}$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{3\,\frac{s+0.1}{s}\cdot 0.1^2 \cdot (s+10)}{(s+0.1)^2(s+10) + 3\,\frac{(s+0.1)}{s}\cdot 0.1}$$

$$= \frac{0.03\,(s+0.1)\,(s+10)}{s(s+0.1)^2(s+10) + 0.3(s+0.1)}$$

$$= \lim_{s\to 0} \frac{Y(s)}{R(s)} = \frac{0.03\cdot 0.1\cdot 10}{0\cdot(0.1)^2(10) + 0.3\cdot 0.1} = \frac{0.03}{0.03} = 1$$

$$F(s) = \frac{Y(s)}{W(s)} = \frac{(s+0.1)(s+10)\cdot 0.1}{(s+0.1)^2(s+10) + 0.1\cdot 3\,\frac{s+0.1}{s}}$$

$$= \frac{s(s+0.1)(s+10)\cdot 0.1}{s(s+0.1)^2(s+10) + 0.3\,(s+0.1)}$$

$$\lim_{s\to 0} \frac{Y(s)}{W(s)} = \frac{0\cdot 0.1\cdot 10\cdot 0.1}{0(0.1)^2\cdot 10 + 0.03\cdot 0.1} = \frac{0}{0.003} = \underline{0}$$

$\Rightarrow$ no stationary error

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The results are confirmed in the closed loop step responses.

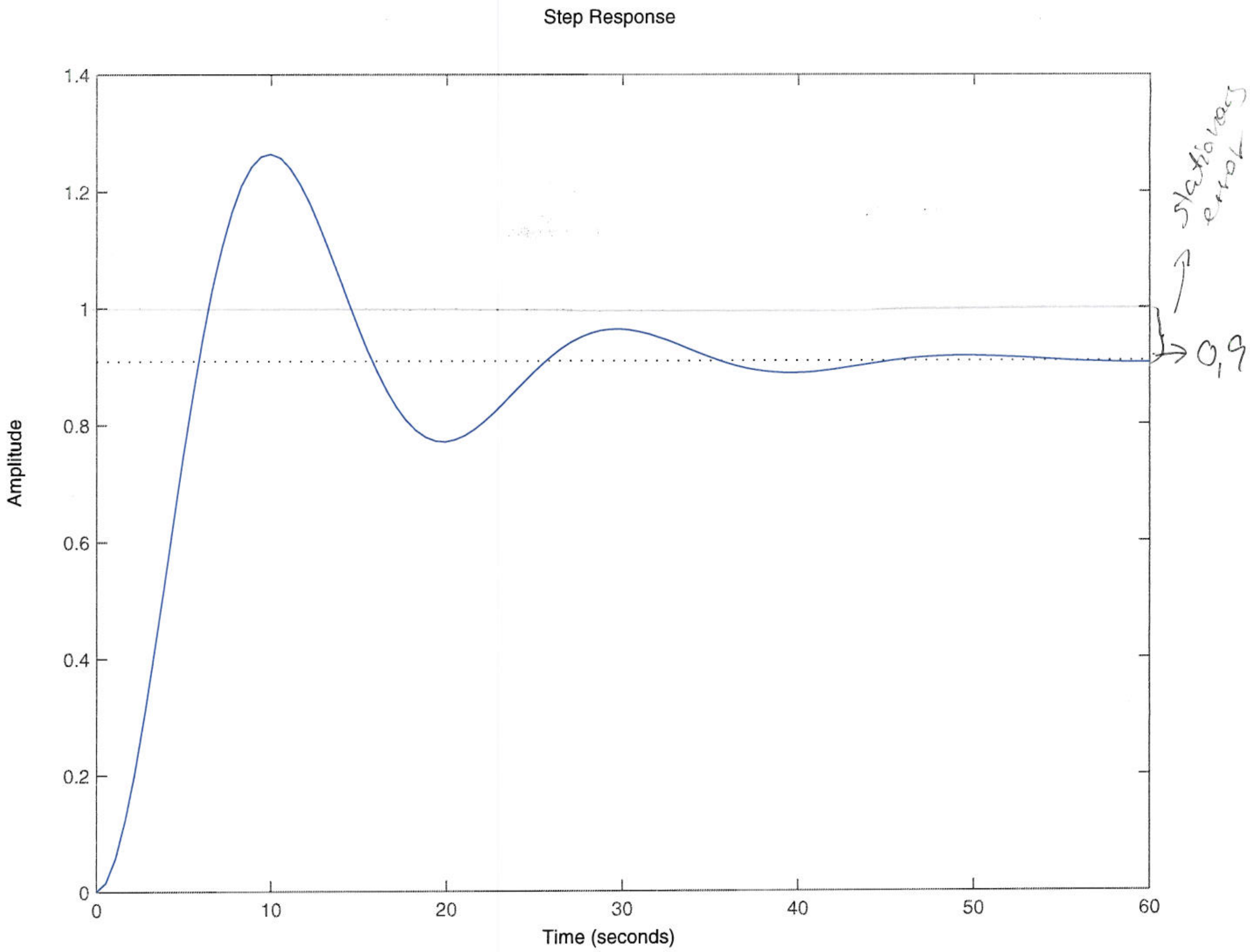
P - control: stationary error on step; the noise are reduced not eliminated

PI - control: no stationary error on step; the noise are eliminated

(9)

Closed loop step response $\frac{T(s)}{R(s)}$

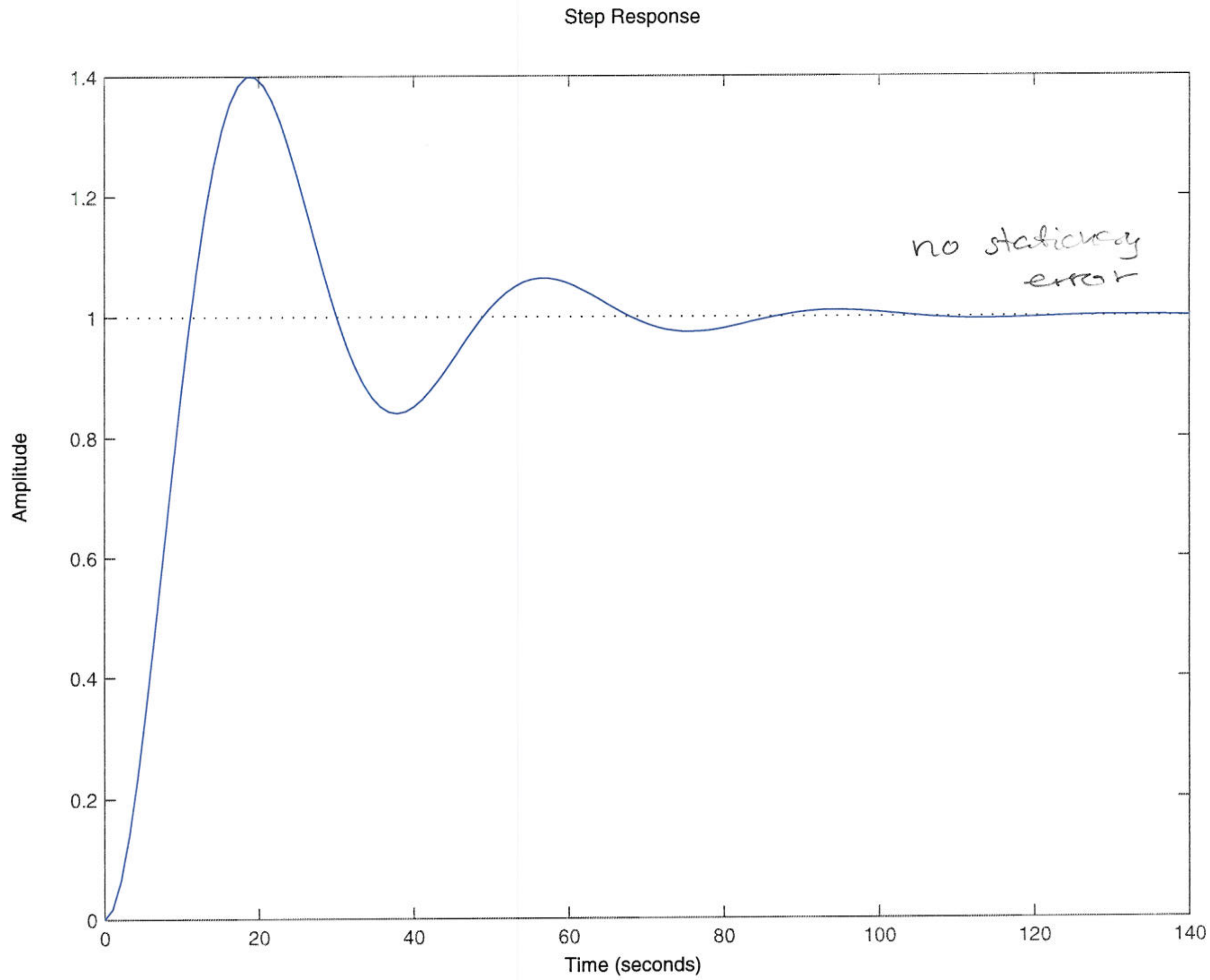
$D(s) = 10$




stationary error = 0.1

(10) closed loop stepresponse $\frac{T(s)}{R(s)}$

$$D(s) = 3\,\frac{s+0.1}{s}$$

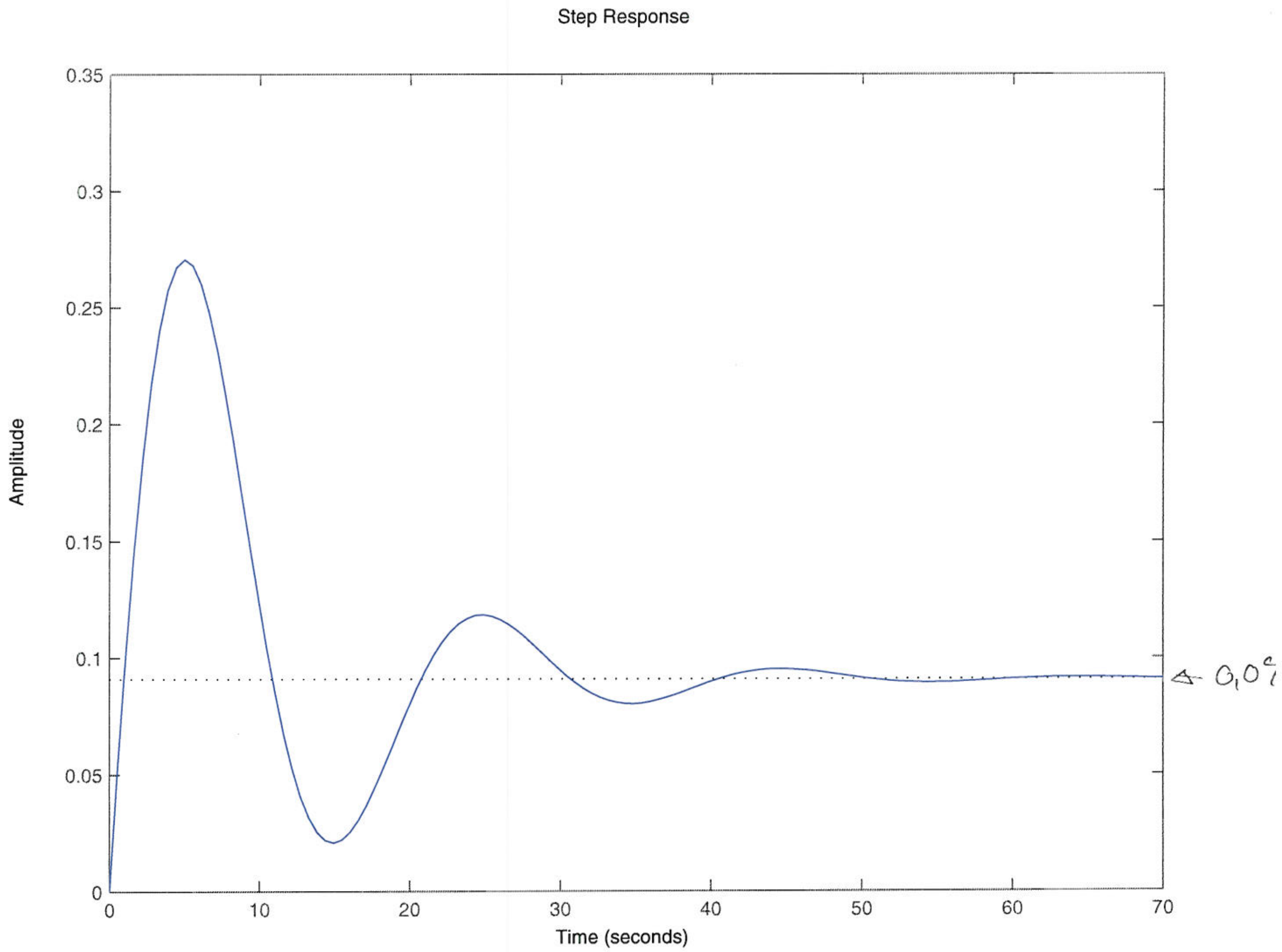



no stationary error on step

(11)

closed loop $\frac{Y(s)}{W(s)}$

$D(s) = 10$

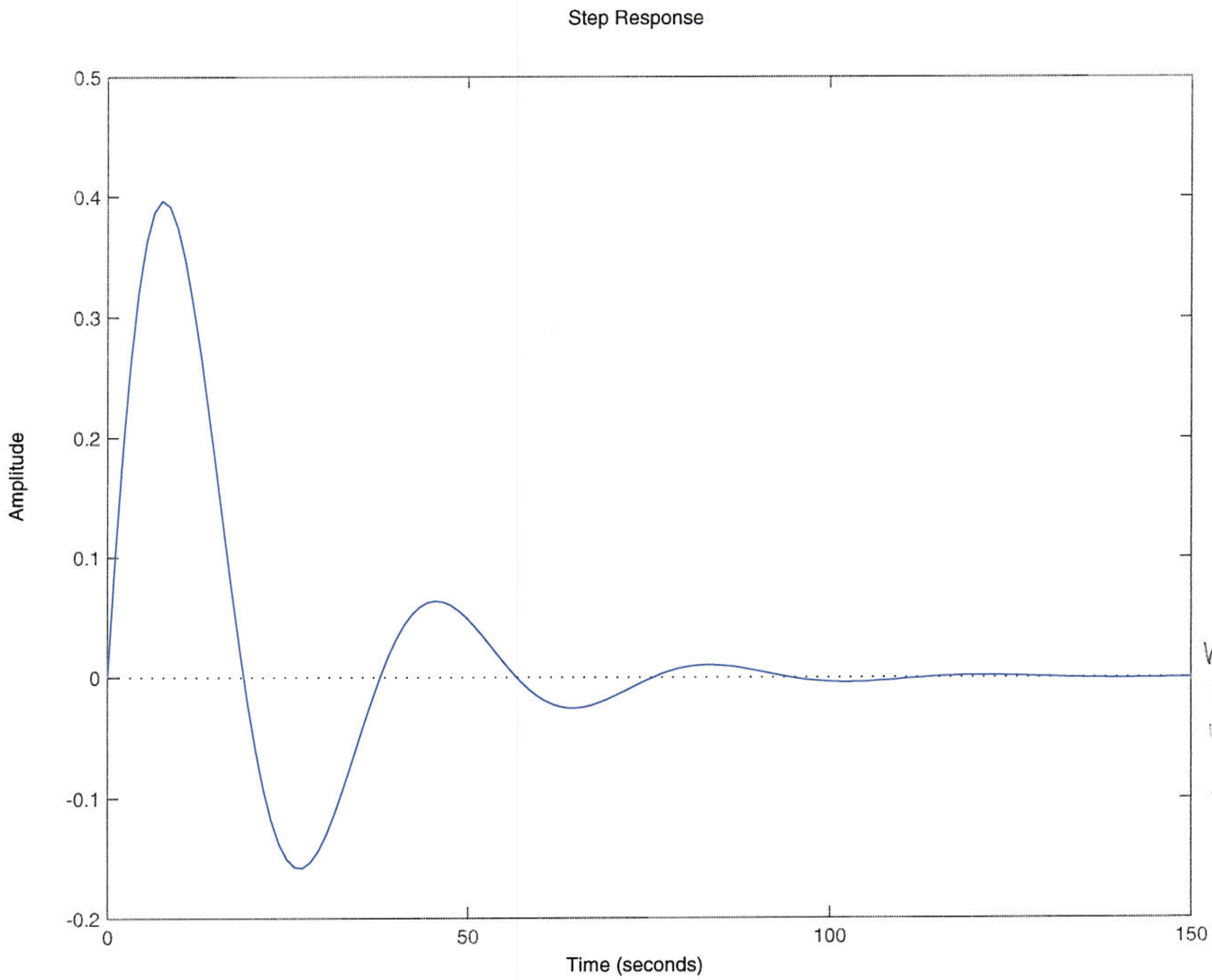



the controller can not eliminate
a step on the noise $W(s)$

(12)

closed loop $\frac{Y(s)}{W(s)}$

$$D(s) = 3\,\frac{s+0.1}{s}$$




no static value error

The PI controller eliminates the error