ESD5 – Fall 2024
Lecture Notes – Lecture 5

Department of Electronic Systems
Aalborg University

September 16, 2024

Example 1 – Bit and Packet Error Rate

Consider a scenario of point-to-point communication, where the sender, say, Alice, attempts to transmit her n bits messages to the receiver, say, Bob, through the communication channel. As we introduced in the lecture, bit errors can happen when information is transmitted over an unreliable wireless medium. This example will give you an intuitive interpretation of the *Bit Error Rate* (BER). Fig. 1 shows Alice transmitting the 8-bit message “11001001” through the channel, and Bob receiving “01011011”. In this case, we can see that 3 bits (1st, 4th, and 7th) out of 8 bits are flipped on the receiver side. In this case, we can observe the bit error rate is $\frac{3}{8}$.

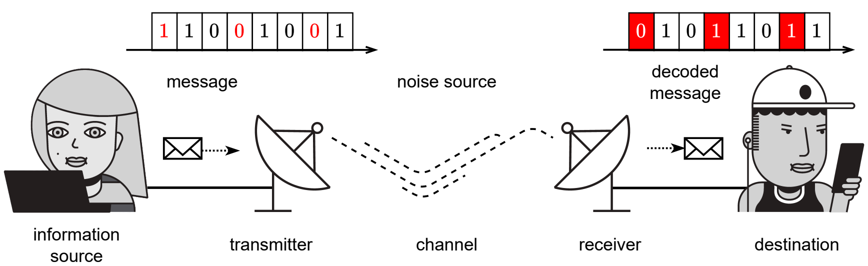

Figure 1: The information source (Alice) sends a packet to the destination (Bob) comprised by 8 bits and 3 bits are erroneously received at the destination.

In the actual communication, the data are transmitted in the form of packets. In most cases, the designer is interested in how reliable the transmitted packets are delivered to the destination without error. To depict this, let us consider a scenario where Alice transmits the 2-bit information (x_1, x_2). Then, we define the packet error as the probability of the event where at least one transmitted bit is flipped at the receiver. To simplify the assumption, here we consider the binary symmetric channel with the flipping probability p. Then, we can describe the possible combination of received signal and corresponding probability as shown below. From this Table, you can see the packet error probability.

Transmitted packet	Potentially received packet	Probability
(x_1, x_2)	(x_1, x_2)	$(1-p)^2$
	$(x_1, \bar{x}_2)$	$(1-p)p$
	$(\bar{x}_1, x_2)$	$p(1-p)$
	$(\bar{x}_1, \bar{x}_2)$	p^2

Example 2 – Throughput

In the context of communication protocol design, we often evaluate the system-level performance using the metric called throughput. The *throughput* is defined as the number of messages successfully delivered to the receiver per time unit, usually represented in bit/s. It accounts for the network overhead due to, for instance, the exchange of control information and retransmission of erroneous packets. Fig. 2 shows the example of traffic among 10 s, where Alice sends three packets whose size is 8 bits. In this example, due to transmission errors, only a single packet is successfully received by Bob. Then, the throughput can be calculated as $\frac{1\times 8}{10}$ bit/s.

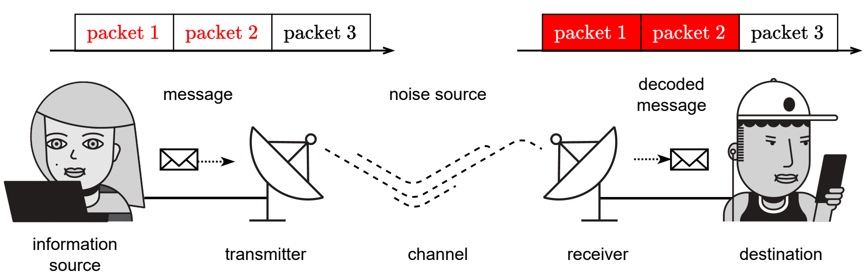

Figure 2: The information source (Alice) sends 3 packets to the destination (Bob) over 10 s and only a single packet is successfully received at the destination.

Example 3 – Automatic Repeat Request Protocol

Let us consider the scenario where Alice wants to send a single packet to Bob using the *Automatic Repeat reQuest* (ARQ) protocol. In ARQ protocol, if the sender detects the failure of data transmission through the absence of ACK, it retransmits the same packet. Let p_{PER} be the packet error probability. Then, let us derive the expected delay caused by the data transmission. Here, we define the delay as the time required to succeed in data transmission (receiving the ACK) since the sender attempts the data transmission. We denote the time required for the data transmission and that for ACK as T_{Data} and T_{ACK}, respectively. Then, when there is no limit on the number of retransmissions, the expected delay for data transmission is defined by:

$$E[T] = \sum_{i=1}^{\infty} i(T_{\mathrm{Data}} + T_{\mathrm{ACK}}) p_{\mathrm{PER}}^{i-1}(1 - p_{\mathrm{PER}}) = \frac{T_{\mathrm{Data}} + T_{\mathrm{ACK}}}{1 - p_{\mathrm{PER}}}. \tag{1}$$