Exercise Sheet 3

Literature:

G.F. Franklin, J.D. Powell and A. Emami-Naeini: *Feedback Control of Dynamic Systems*, 6th edition, pp. 484-490, pp. 496-503, pp. 505-509.

Exercise 1

Determine for each of the following systems, whether it is observable:

$$(1): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}, \quad (2): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}, \quad (3): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} x \end{cases}$$

$$(4): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{cases}, \quad (5): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 2 \end{pmatrix} x \end{cases}, \quad (6): \begin{cases} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{cases}$$

Exercise 2

In the previous exercise sheet, we considered the following controllable system:

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{aligned} \tag{1}$$

which can be shown to be also observable with the output above. The state feedback

$$u = Fx = \begin{pmatrix} -2 & 2 \end{pmatrix} x$$

was shown to achieve a pole placement corresponding to the characteristic polynomial $s^2 + 3s + 2$.

1. Design a full order observer for (1) having characteristic polynomial $s^2 + 9s + 20$, i.e. find an observer gain L such that the eigenvalues of $A + LC$ become $\lambda_1 = -4$ og $\lambda_2 = -5$
2. Draw a diagram for an observer based compensator using F and L.
3. Verify, by computing the eigenvalues of the system matrix for the closed loop system:

$$\dot{\begin{pmatrix} x \\ \hat{x} \end{pmatrix}} = \begin{pmatrix} A & BF \\ -LC & A + LC + BF \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

Exercise 3

The figure illustrates two bodies connected via a spring, moving on a surface with viscous friction.

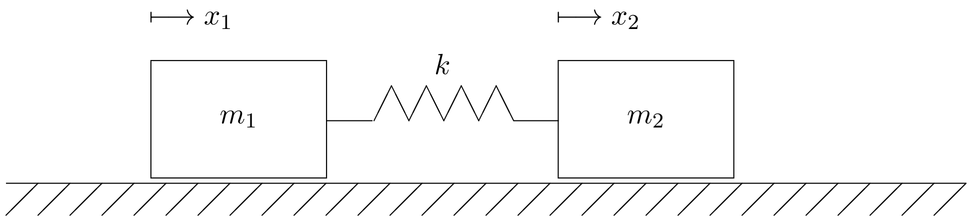

Figure 1: Coupled carts

It is assumed that the two bodies have masses m_1 resp. m_2, and that the frictional forces acting on these two bodies are equal. The system is equipped with sensors, making it possible to measure the positions of the two bodies x_1 and x_2 as well as their velocities $v_1 = \dot{x}_1$ and $v_2 = \dot{x}_2$.

1. Determine (without doing any algebra) which measurements or combinations of measurements render the system observable.

2. The system can be described by the following state space model:

$$\dot{\begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix}} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{c}{m_1} & 0 \\ \frac{k}{m_2} & -\frac{k}{m_2} & 0 & -\frac{c}{m_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix}$$

   Consider the correctness of this model, and verify the heuristic result from 1 by choosing a couple of arbitrary numerical values in MATLAB™.