Control Engineering

Topics:

- Cascade control

Exercises:

1. The closed loop gives

$$\frac{\theta(s)}{R(s)} = \frac{KK_p/\tau}{s^2 + \frac{1+KK_v}{\tau}s + \frac{K_pK}{\tau}}$$

   Here the bandwidth ω_n is given by

$$\omega_n = \sqrt{\frac{K_pK}{\tau}}$$

   only depends on K_p. (The system parameters are assumed constant) The damping ζ is found by

$$2\zeta\omega_n = \frac{1+KK_v}{\tau}$$

   Keeping K_p at a constant value, change in K_v will change the damping ζ.

2. - The first two bullets are still missing.
   - Determine for the series controlled system the value of K_s giving an overshoot of 16 % (ζ=0.5).
     The closed loop transfer function is

$$Y(s) = \frac{K_s\frac{1}{(s+1)(s+1)}}{1+K_s\frac{1}{(s+1)(s+1)}}R(s) = \frac{K_s}{s^2+2s+1+K_s}R(s)$$

     Comparing with the standard 2 order system

$$Y(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

gives

$$\omega_n = \sqrt{1+K_s} \quad \wedge \quad 2\zeta\omega_n = 2 \Rightarrow \sqrt{1+K_s} = \frac{1}{\zeta} \Rightarrow K_s = \frac{1}{\zeta^2} - 1$$

$\zeta = 0.5$ gives $K_s = 3$.

- On figure 2 a cascade controller with proportional gain of 4 is inserted. Determine K_c giving a overshoot of 16 % (ζ=0.5).
  The closed loop transfer function is

$$Y(s) = \frac{K_c \frac{4}{(s+5)(s+1)}}{1 + K_c \frac{4}{(s+5)(s+1)}} R(s) = \frac{4K_c}{s^2 + 6s + 5 + 4K_c} R(s)$$

  Comparing with the standard 2 order system

$$Y(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

  gives

$$\omega_n = \sqrt{5+4K_c} \quad \wedge \quad 2\zeta\omega_n = 6 \Rightarrow \sqrt{5+4K_c} = \frac{3}{\zeta} \Rightarrow K_c = \frac{\frac{9}{\zeta^2} - 5}{4}$$

  $\zeta = 0.5$ gives $K_c = 7.75$.

- The two controller have the same overshoot, is the rise times the same? (Calculate the rise times for the two controlled systems).
  For a second order system the rise time T_r and the natural undamped frequency ω_n is given by

$$T_r \approx \frac{1.8}{\omega_n}$$

  This gives for the series controlled system

$$\omega_n = \sqrt{1+K_s} = 2 \Rightarrow T_r \approx \frac{1.8}{2} = 0.9 \;\; [sec]$$

  For the cascade controlled system

$$\omega_n = \sqrt{5+4K_c} = 6 \Rightarrow T_r \approx \frac{1.8}{6} = 0.3 \;\; [sec]$$

- The two controller have the same overshoot, is the steady state errors the same? (Calculate the steady state error for the two controlled systems)

  Both systems are special case type 0. For the series controlled system the dc-gain in the open loop is

  $$Kp = \lim_{s\to\infty} \frac{K_s}{(s+1)(s+1)} = K_s = 3.$$

  giving the steady state error

  $$e_{ss} = \frac{1}{1+K_p} = 0.25$$

  For the cascade controlled system the dc-gain is

  $$Kp = \lim_{s\to\infty} \frac{4K_c}{(s+5)(s+1)} = \frac{4\cdot 7.75}{5} = 6.2$$

  giving the steady state error

  $$e_{ss} = \frac{1}{1+K_p} = 0.14$$

- Determine the transfer function from $W(s)$ to $Y(s)$ in the two cases. Using Matlab, plot bode-plots of the two transfer function and discuss the two.

  The transfer function is the series controlled system

  $$Y(s) = \frac{\frac{1}{s+1}}{1+\frac{K_s}{(s+1)(s+1)}}W(s) = \frac{s+1}{s^2+2s+1+Ks}W(s) = \frac{s+1}{s^2+2s+4}W(s)$$

  The transfer function is the cascade controlled system

  $$\begin{aligned} Y(s) &= \frac{s+1}{s+5}\frac{\frac{1}{s+1}}{1+\frac{4K_c}{(s+5)(s+1)}}W(s) \\ &= \frac{s+1}{s+5}\frac{s+5}{s^2+6s+5+4K_c}W(s) = \frac{s+1}{s^2+6s+5+4K_c}W(s) \\ &= \frac{s+1}{s^2+6s+36}W(s) \end{aligned}$$

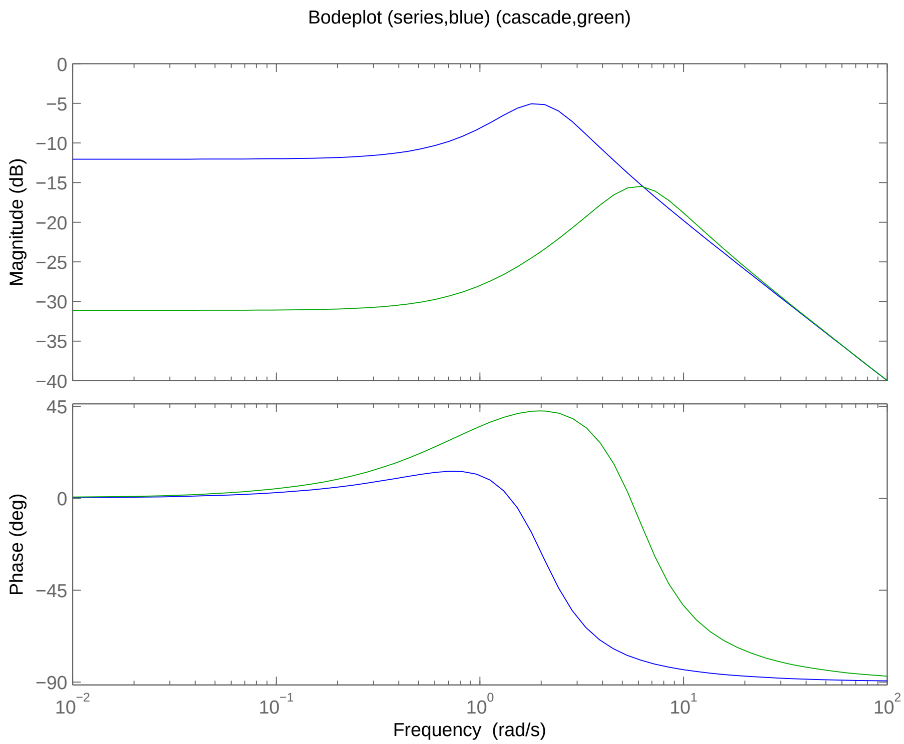

Figure 1: *Bode plots of series and cascade controlled system from disturbance*

- Find $G(s)$ and $F(s)$ so that figure 3 gives the same closed loop transfer functions as figure 2.

  The two transfer functions are given by

$$F(s) = \frac{s+1}{s+5}$$

$$G(s) = \frac{4}{s+5}$$