D5 − E5 Feedback 2

2. mm. Solution

1. a) The closed loop is given by:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{K_p}{s^2 + s + K_p}$$

$\zeta \approx 0.45$
The standard 2. order system is given by:

$$F(s) = \frac{A}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K_p}$$

$$2\zeta\omega_n = 2\zeta\sqrt{K_p} = 1 \Rightarrow \sqrt{K_p} = \frac{1}{2\zeta} = 1.11 \Rightarrow K_p = 1.23$$

b) Equation 3.60:

$$t_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{K_p}} = 1.6[sek]$$

Equation for settlingtime(2%):

$$t_s = \frac{-ln(0.02)}{\zeta\omega_n} = \frac{3.9}{0.45\sqrt{1.23}} = 7.8[sek]$$

c)Using the 'special case' the system type is qual to 1
In table 4.1 page 199:
Step: e_{ss}=0
Ramp:

$$e_{ss} = \frac{1}{K_v}$$

where (equation 4.33)

$$K_v = \lim_{s\to 0} sG_0(s) \Rightarrow K_v = \lim_{s\to 0} s\frac{K_p}{s+1}\frac{1}{s} = K_p = 1.23$$

giving:

$$e_{ss} = \frac{1}{K_v} = 0.81$$

Parabola: $e_{ss} = \infty$

2. a) The transfer function for the inner loop:

$$T_i(s) = \frac{K_2 \frac{1}{s+1}}{1 + K_2 \frac{1}{s+1}} = \frac{K_2}{s + 1 + K_2}$$

The closed loop:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{K_1 K_2 \frac{1}{s+1+K_2} \frac{1}{s}}{1 + K_1 K_2 \frac{1}{s+1+K_2} \frac{1}{s}} = \frac{K_1 K_2}{s^2 + (1 + K_2)s + K_1 K_2}$$

From the over shoot specification:

$$M_p = 20\% \Rightarrow \zeta \approx 0.45$$

From the settling time spec.:

$$t_s = \frac{3.9}{0.45\omega_n} = 1 \Rightarrow \omega_n = 8.7[sek]$$

K_1 and K_2 :

$$2\zeta\omega_n = 2 \cdot 0.45 \cdot 8.7 = 1 + K_2 \Rightarrow K_2 = 6.8$$

$$\omega_n^2 = K_1 K_2 = 8.7^2 \Rightarrow K_1 = 11.1$$

b)

$$t_r \approx \frac{1.8}{\omega_n} = 0.2[sek]$$

c) System type 1
Step: $e_{ss} = 0$
Ramp:$e_{ss} = 0.1$
Parabola:$e_{ss} = \infty$

3. a) The closed loop transfer function:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{1}{(s+1)s}}{1 + K(1 + T_d s)\frac{1}{(s+1)s}} = \frac{1}{s^2 + (1 + KT_d)s + K}$$

with $\zeta = 0.45$ and $\omega_n = 8.7$ [rad/sek] we find $K = \omega_n^2 = 75.7$ and $1 + KT_d = 2\zeta\omega_n = 7.83$ giving T_d=0.09 [sek].
b)

$$t_r \approx \frac{1.8}{\omega_n} = 0.2[sek]$$

c) As we do not have unit feedback we may not use the 'speciel case'. To find the system type equation we use the equation 4.37 page 200:

$$e_{ss} = \lim_{s\to 0} \frac{1 - T(s)}{s^k}$$

where the system type k is the value of k that will result in a non-zero constant steady state error e_{ss} . T(s) is the closed loop transfer function.

$$e_{ss} = \lim_{s\to 0} \frac{\frac{s^2+(1+KT_d)s+K-1}{s^2+(1+KT_d)s+K}}{s^k} = \lim_{s\to 0} \frac{1}{s^k}\frac{K-1}{K}$$

giving a non-zero value for k=0, meaning a system type 0.
Becauce the system type is 0 the steady state errors for ramp and parabola are ∞. The steady state error for a step is given by the equation 4.36 page 200

$$e_{ss} = \lim_{s\to 0} s(1 - T(s))R(s) = \lim_{s\to 0}(1 - T(s))$$

$$= \lim_{s\to 0} \frac{s^2 + (1 + KT_d)s + K - 1}{s^2 + (1 + KT_d)s + K} = \frac{K-1}{K} = 0.9868$$

This means that if an unitstep is used as reference the output will be y_{ss}=1-0.9868=0.0132.
The calculations follow the text book, but it is easy to see that the DC-gain of the system is $\frac{1}{K}$. The steady state error is then $1 - \frac{1}{K}$=0.9868

4. The closed loop transfer function $T(s)$ is calculated:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{K_pK}{\tau s^2 + s + K_pK} \Rightarrow$$

$$T(s) = \frac{K_pK}{\tau} \cdot \frac{1}{s^2 + \frac{1}{\tau}s + \frac{K_pK}{\tau}}$$

Compared to the 2. order standard system we find:

$$2\zeta\omega_n = \frac{1}{\tau} = 2\zeta\sqrt{\frac{K_pK}{\tau}} \Rightarrow$$

$$\frac{1}{4\zeta^2\tau} = K_pK \Rightarrow$$

$$K_p = \frac{1}{4\zeta^2\tau K}$$

The overshoot must be 10% meaning that $\zeta \approx 0.6$ and gives

$$K_p = \frac{0.69}{K\tau}$$