1. Givet $H(s) = \dfrac{2}{s^2 + 4s + 3}$

A) Overføringsfunktionen har to poler — $s_1 = -1$ og $s_2 = -3$. Partialbrøks-opspaltning

$$H(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3}$$

$$A_1 = (s+1)H(s)\Big|_{s=-1} = 1$$

$$A_2 = (s+3)H(s)\Big|_{s=-3} = -1$$

$\Downarrow$

$$H(s) = \frac{1}{s+1} - \frac{1}{s+3}$$

Givet at H(s) nu er udtrykt som en sum af to 1. orden led, kan vi bestemme H(z)

$$H(z) = \sum_{k=1}^{2} \frac{T \cdot A_k}{1 - e^{s_k T} z^{-1}}$$

$$= \frac{T}{1 - e^{-T} z^{-1}} - \frac{T}{1 - e^{-3T} z^{-1}}$$

$$H(z) = T \cdot \frac{(1-e^{-3T}z^{-1}) - (1-e^{-T}z^{-1})}{(1-e^{-T}z^{-1})\cdot(1-e^{-3T}z^{-1})}$$

$$= T \cdot \frac{(e^{-T}-e^{-3T})z^{-1}}{1-e^{-3T}z^{-1}-e^{-T}z^{-1}+e^{-4T}z^{-2}}$$

$$= T \cdot \frac{(e^{-T}-e^{-3T})z^{-1}}{1-(e^{-T}+e^{-3T})z^{-1}+e^{-4T}z^{-2}}$$

B) $f_s = 1\,Hz \Rightarrow T = \frac{1}{1} = 1$

$$H(z) = \frac{(0.3679-0.0498)\,z^{-1}}{1-(0.3679+0.0498)z^{-1}+0.0183z^{-2}}$$

$$= \frac{0.3181z^{-1}}{1-0.4177z^{-1}+0.0183z^{-2}}$$

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{0.3181z}{z^2-0.4177z+0.0183}\Bigg|_{z=e^{j\omega}}$$

$$= \frac{0.3181e^{j\omega}}{e^{2j\omega}-0.4177e^{j\omega}+0.0183}$$

$$= \frac{0.3181(\cos\omega + j\sin\omega)}{\cos 2\omega + j\sin 2\omega - 0.4177\cos\omega - j0.4177\sin\omega + 0.0183}$$

$$= \frac{0.3181(\cos\omega + j\sin\omega)}{(\cos 2\omega - 0.4177\cos\omega + 0.0183) + j(\sin 2\omega - 0.4177\sin\omega)}$$

$\Downarrow$

$$|H(e^{j\omega})| = 0.3181 \cdot \frac{\sqrt{\cos^2\omega + \sin^2\omega}}{\sqrt{(\cos 2\omega - 0.4177\cos + 0.0183)^2 + (\sin 2\omega - 0.4177\sin\omega)^2}}$$

Skriv et MATLAB program, som ~~udregne~~ plotter $|H(e^{j\omega})|$ i intervallet $[0;\pi[$.

c) 3dB frekvensen er den frekvens ved hvilken dæmpningen er faldet med en faktor $\frac{1}{\sqrt{2}}$ ift. DC

— Se Matlab program

$\omega_{3dB} = 1.0336$ rad

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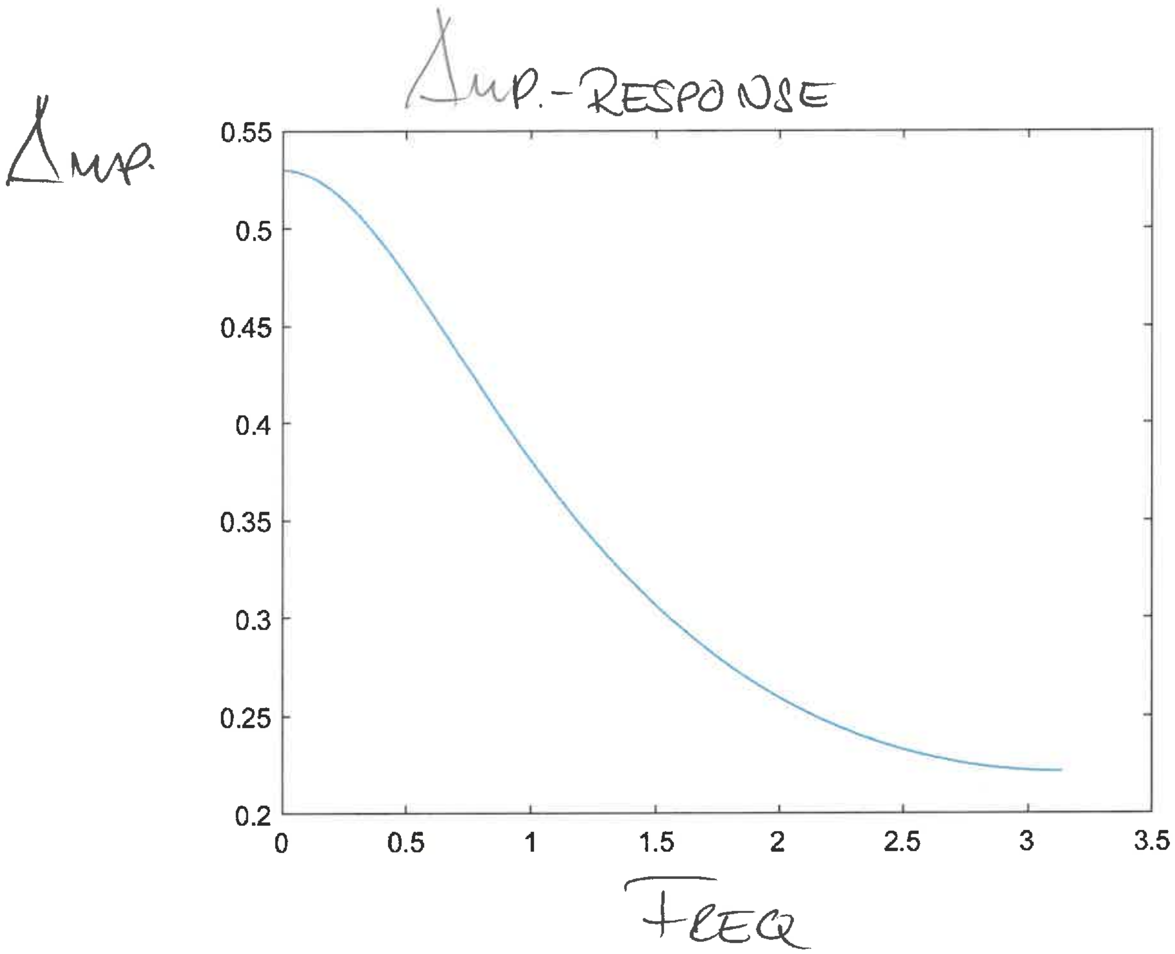
```
% Make a frequency axis with 1000 points in the interval 0 to pi
for i=1:1000
    f(i)=((i-1)/1000)*pi;
end

% Calculate the amp. response
for i=1:1000
    numerator(i)=sqrt((cos(f(i))^2 + (sin(f(i))^2)));
    denominator(i)=sqrt( (cos(2*f(i)) - 0.4177*cos(f(i)) + 0.0183)^2 + (sin(2*f(i)) -↙
0.4177*sin(f(i)))^2 );
    freq_resp(i)=0.3181*(numerator(i)/denominator(i));
end

% Plot the amplitude response
plot(f,freq_resp);



% We can now use the amp. response to find the 3dB-frequency.
% The 3dB-frequency is the frequency at which the attenuation is down with
% 3dB, i.e., 1/sqrt(2), as compared to the DC attenuation.
att_3db=freq_resp(1)/(sqrt(2));
% Now search through the frequency response to find the first value which
% is less than att_3db
i=1;
while (freq_resp(i)-att_3db)>0
    j=i;
    i=i+1;
end

% The 3dB-frequncy is
freq_resp(j)
% ...and it is located at
(j/1000)*pi
```


Amp.-Response
Amp.
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0
0.5
1
1.5
2
2.5
3
3.5
Freq

D) Difference ligning:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.3181z^{-1}}{1-0.4177z^{-1}+0.0183z^{-2}}$$

$$\Downarrow$$

$$Y(z)\left(1-0.4177z^{-1}+0.0183z^{-2}\right) = X(z)\cdot 0.3181z^{-1}$$

$$\Downarrow$$

$$Y(z) = 0.3181X(z)z^{-1} + 0.4177Y(z)z^{-1} - 0.0183Y(z)z^{-2}$$

$$\mathcal{Z}^{-1} \Downarrow$$

$$y[n] = 0.3181x[n-1] + 0.4177y[n-1] - 0.0183y[n-2]$$

· OPG 2 $\quad |H_c(j\Omega)|^2 = \dfrac{1}{1+\left(\Omega/\Omega_c\right)^{2N}}$

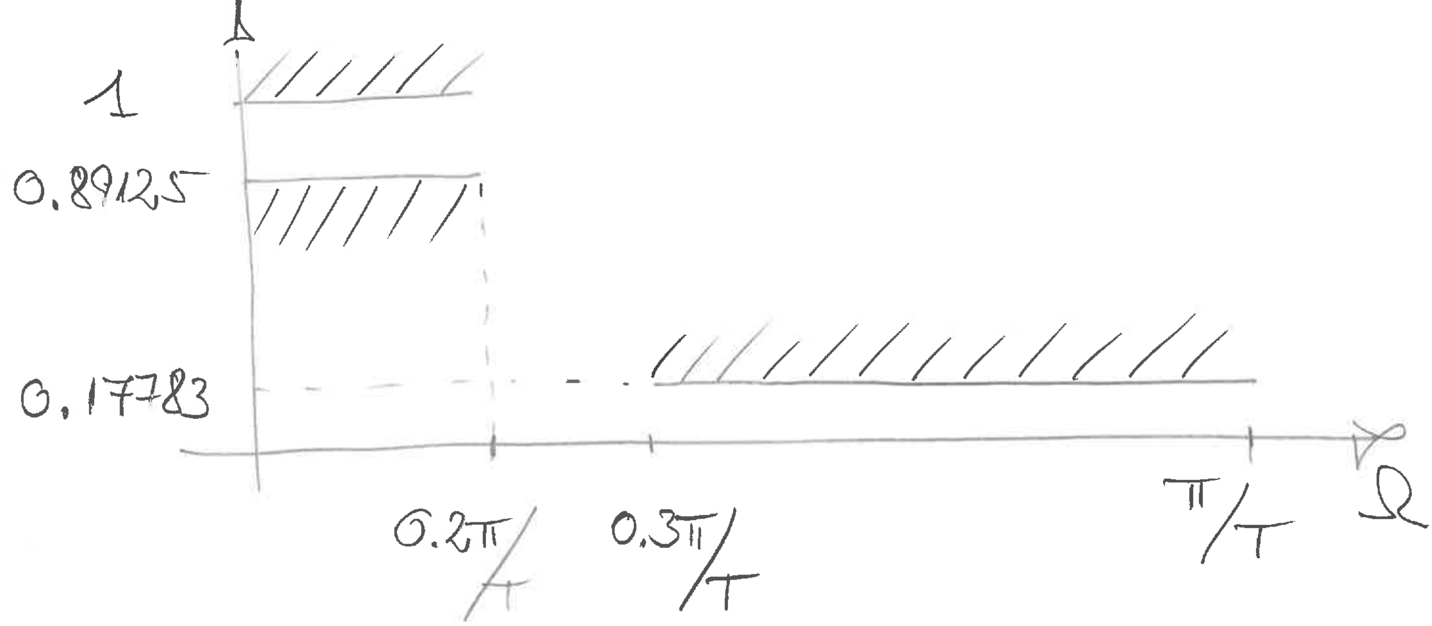
Spec's for the discrete-time filter is the same as for Example 2 (p. 524);

$$0.89125 \le |H(e^{j\omega})| \le 1 \qquad 0 \le |\omega| \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.17783 \qquad 0.3\pi \le |\omega| \le \pi$$

a) Tolerance bounds of the freq.-response of $|H_c(j\Omega)|$. $H_c(s)$ is Butterworth.

Remember that $\omega = \Omega \cdot T \Rightarrow \Omega = \omega/T$ where T is the sample period.



b) Determine the integer order N and $T \cdot \Omega_c$ such that the continuous-time Butterworth filter exactly meets the specs from a) at the passband edge.

Since we don't know neither N nor Ω_c we need two equations:

- $$|H_c(j\,0{,}2\pi/T)|^2 = \frac{1}{1+\left(\frac{0{,}2\pi/T}{\Omega_c}\right)^{2N}}$$

$$= \frac{1}{1+\left(\frac{0{,}2\pi}{\Omega_c \cdot T}\right)^{2N}} = (0.89125)^2$$

- $$|H_c(j\,0{,}3\pi/T)|^2 = \frac{1}{1+\left(\frac{0.3\pi/T}{\Omega_c}\right)^{2N}}$$

$$= \frac{1}{1+\left(\frac{0{,}3\pi}{\Omega_c \cdot T}\right)^{2N}} = (0.17783)^2$$

Note: We don't know whether the passband or the stopband specification is the „strongest" requirement.

Two equations with two unknown
(N and $\Omega_c T$)

$$\begin{cases} \dfrac{1}{1+\left(\dfrac{0.2\pi}{\Omega_c T}\right)^{2N}} = 0.79433 \\[2ex] \dfrac{1}{1+\left(\dfrac{0.3\pi}{\Omega_c T}\right)^{2N}} = 0{,}03162 \end{cases}$$

$\Downarrow$

$$\begin{cases} \dfrac{1}{0.79433} - 1 = \left(\dfrac{0.2\pi}{\Omega_c \cdot T}\right)^{2N} \\[2ex] \dfrac{1}{0{,}03162} - 1 = \left(\dfrac{0.3\pi}{\Omega_c T}\right)^{2N} \end{cases}$$

$$\begin{cases} \log(0{,}25893) = 2N \cdot \log\left(\dfrac{0.2\pi}{\Omega_c \cdot T}\right) = -0.58682 \\[2ex] \log(30{,}6220) = 2N \log\left(\dfrac{0.3\pi}{\Omega_c T}\right) = 1.48603 \end{cases}$$

$$\begin{cases} N = \dfrac{-0.58682}{2\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\ \cancel{2}\cdot\left(\dfrac{-0.58682}{\cancel{2}\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right)}\right)\cdot\log\left(\dfrac{0.3\pi}{\Omega_c T}\right) = 1.48603 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\ \log\left(\dfrac{0.3\pi}{\Omega_c T}\right) = -2.53249\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right) \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\ \dfrac{0.3\pi}{\Omega_c T} = 10^{-2.53249\cdot\log\left(\frac{0.2\pi}{\Omega_c T}\right)} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2\cdot\log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\ \dfrac{0.3\pi}{\Omega_c T} = \left(10^{\log\left(\frac{0.2\pi}{\Omega_c T}\right)}\right)^{-2.53249} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2 \cdot \log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\[2ex] \dfrac{0.3\pi}{\Omega_c T} = \left(\dfrac{0.2\pi}{\Omega_c T}\right)^{-2.53249} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2 \cdot \log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\[2ex] \dfrac{0.3\pi}{\Omega_c T} = \dfrac{(0.2\pi)^{-2.53249}}{(\Omega_c T)^{-2.53249}} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2 \cdot \log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\[2ex] \dfrac{0.3\pi}{(0.2\pi)^{-2.53249}} = \dfrac{\Omega_c T}{(\Omega_c T)^{-2.53249}} = (\Omega_c T)^{1+2.53249} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = \dfrac{-0.58682}{2 \cdot \log\left(\dfrac{0.2\pi}{\Omega_c T}\right)} \\[2ex] \Omega_c T = \sqrt[3.53249]{\dfrac{0.3\pi}{(0.2\pi)^{-2.53249}}} = 0.70474 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} N = 5.8859 \\ \Omega_c T = 0.70474 \end{cases}$$

Now, N must be an integer, and thus N=6.

If N=6 and we have to meet the specs, i.e, the passband end should match.

$$|H_c(j\,{}^{0.2\pi}/_T)|^2 = \frac{1}{1+\left(\frac{0.2\pi}{\Omega_c T}\right)^{2N}} = (0.89125)^2$$

$\Downarrow$

$$\frac{1}{1+\left(\frac{0.2\pi}{\Omega_c T}\right)^{12}} = 0.79433$$

$\Downarrow$

$$\left(\frac{0.2\pi}{\Omega_c T}\right)^{12} = \frac{1}{0.79433} - 1$$

$\Downarrow$

$$\frac{0.2\pi}{\Omega_c T} = \sqrt[12]{0.25893} = 0.89351$$

$\Downarrow$

$$\Omega_c T = \frac{0.2\pi}{0.89351} = 0.7032$$