Problem 1.1

a) $$\underline{W}_{rad} = \frac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta} \cdot \vec{a}_r = \frac{5^2 \vec{a}_r}{2 \cdot 120 \cdot \pi} = 0.03315 \vec{a}_r \ \ Watt/m^2$$

b) $$P_{rad} = \oint_S W_{rad} \cdot dS = \int_0^{2\pi} \int_0^{\pi} 0.03315 \cdot (r^2 \sin(\theta) \cdot d\theta \cdot d\varphi =$$

$$\int_0^{2\pi} \int_0^{\pi} 0.03315 \cdot (100^2 \sin(\theta) \cdot d\theta \cdot d\varphi =$$

$$2\pi \cdot 0.03315 \cdot 100^2 \cdot \int_0^{\pi} \sin(\theta) \cdot d\theta = 4165.75 \ \ Watt$$

Problem 1.2

a)

$$D_0 = \frac{4\pi \cdot U_{\max}}{P_{rad}} = \frac{4\pi(200 \cdot 10^{-3})}{0.9 \cdot (125.66 \cdot 10^{-3})} = 22.22 = 13.47 dB$$

$$G_0 = e_{cd} \cdot D_0 = 0.9 \cdot 22.22 = 20 = 13.01 dB$$

b)

$$D_0 = \frac{4\pi \cdot U_{\max}}{P_{rad}} = \frac{4\pi(200 \cdot 10^{-3})}{125.66 \cdot 10^{-3}} = 20 = 13.01 dB$$

$$G_0 = e_{cd} \cdot D_0 = 0.9 \cdot 20 = 18 = 12.55 dB$$

Problem 1.3

$$U(\theta,\phi)=\left\{\begin{array}{lll} 1 & 0^\circ \le \theta & \le 20^\circ \\ 0.342 & 20^\circ \le \theta & \le 60^\circ \\ 0 & 60^\circ \le \theta & \le 180^\circ \end{array}\right\} \qquad 0^\circ \le \phi \le 360^\circ$$

$$P_{rad}=\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\cdot d\theta\cdot d\phi = 2\pi[\int_0^{20^\circ}\sin(\theta)\cdot d\theta+\int_0^{20^\circ}0.342\cdot\csc(\theta)\cdot\sin(\theta)\cdot d\theta]$$

$$=2\pi\left\{-\cos(\theta)\Big|_0^{\pi/9}+0.342\cdot\theta\Big|_{\pi/9}^{\pi/3}\right\}=1.87912$$

$$D_0=\frac{4\pi U_{\max}}{P_{rad}}=\frac{4\pi}{1.87912}=6.68737=8.25dB$$

Problem 1.4

a)

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\cdot d\theta\cdot d\phi = \int_0^{2\pi}\sin^2(\phi)\cdot d\phi\cdot\int_0^{\frac{\pi}{2}}\cos^4(\theta)\cdot\sin(\theta)\cdot d\theta = \frac{\pi}{5}$$

$$U_{\max} = U(\theta = 0, \phi = \frac{\pi}{2}) = 1$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi}{\pi/5} = 20 = 13.0dB$$

b)

Elevation plane: theta varies, phi fixed. => choose $\phi = \frac{\pi}{2}$

$$U(\theta, \phi = \pi/2) = \cos^4(\theta) \qquad 0 \le \theta \le \pi/2$$

$$\cos^4\left[\frac{HPBW(elevation)}{2}\right] = \frac{1}{2}$$

$$HPBW(elevation) = 2\cdot\cos^{-1}(\sqrt{0.5}) = 65.5^\circ$$