Digital Signal Processing
ESD-5 & IV-5 (elektro), E24

6. Filters with Finite Impulse Response, cont.

Assoc. Prof. Peter Koch, AAU

In this lecture…

…we will initially provide a recap of the most essential points and conclusions that we made at our previous lecture, which addressed the fundamental ideas of Finite Impulse Response (FIR) Filters with Linear Phase.

Next, we will turn into a discussion on the implications we see in the time- and in the frequency domain when an infinite length sequence is truncated by a finite lenght window function.

Finally, we will introduce a window function, known as the Kaiser window, for which it is possible to control some essential filter design parameters…

Our prime purpose today is to enhance your awareness about the relations between the time- and frequency domain.

A short recap of FIR filters – theory and design

Systems with Linear Phase are characterized by the frequency response

1 $$H(e^{j\omega}) = A(e^{j\omega})e^{-j(\alpha\omega-\beta)}$$

Real Amplitude Function   Complex Phase Function

We now consider the frequency response for a general system

2 $$H(e^{j\omega}) = \sum_{n=\infty}^{\infty} h[n]\, e^{-j\omega n}$$

For 1 and 2 we next derive expressions for $\tan\{\angle H(e^{j\omega})\}$, and then equalizing the two, leading to

$$\sum_{n=-\infty}^{\infty} h[n]\sin(\beta + (n-\alpha)\omega) = 0$$ which should hold for $\forall\ \omega$.

This is a necessary condition for Linear Phase…

$$\sum_{n=-\infty}^{\infty} h[n]\sin(\beta + (n-\alpha)\omega) = 0 \qquad \text{which should hold for } \forall\ \omega.$$

Find values for α, β, and $h[n]$ such that this condition holds.

Without proof, the following values fulfills the condition;

- $h[n] = \begin{cases} h[M-n] \\ -h[M-n] \end{cases}$, i.e., Symmetric or Anti-symmetric Impulse Response
- $M = 2\alpha,\quad M$ is an integer, even or odd.
- $\beta = \begin{cases} 0 \\ \pi \end{cases}$ or $\beta = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases}$
- We require that the system is causal, $h[n] = 0$, for n outside the interval $[0; M]$

$h[n]$ \ M	Even	Odd
Symmetric $\beta = 0$ or π	I	II
Anti-symmetric $\beta = \pi/2$ or $3\pi/2$	III	IV

Four different types of FIR filters:

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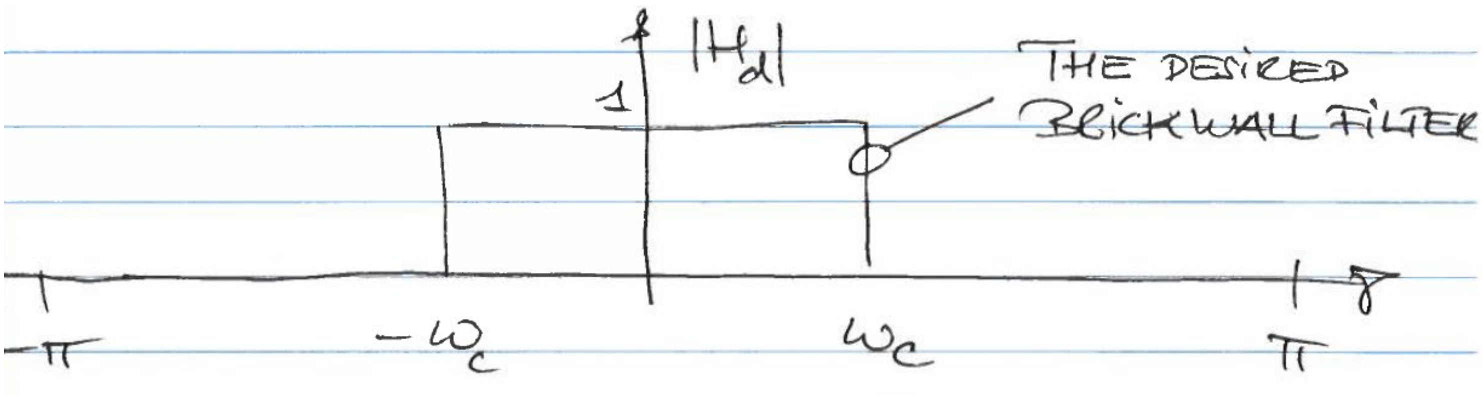
Designing an FIR Filter

- The desired Amplitude Response
- Information on Linear Phase Response, Type-I



The Desired Frequency Response, $H_d(e^{j\omega})$

$$H_d\left(e^{j\omega}\right) = \begin{cases} 1 \cdot e^{-j\omega^{M}/_{2}} & |\omega| \leq \omega_c \\ 0 & \omega_c < \omega \leq \pi \end{cases}$$

$\alpha = {}^{M}/_{2}$ and $\beta = 0$

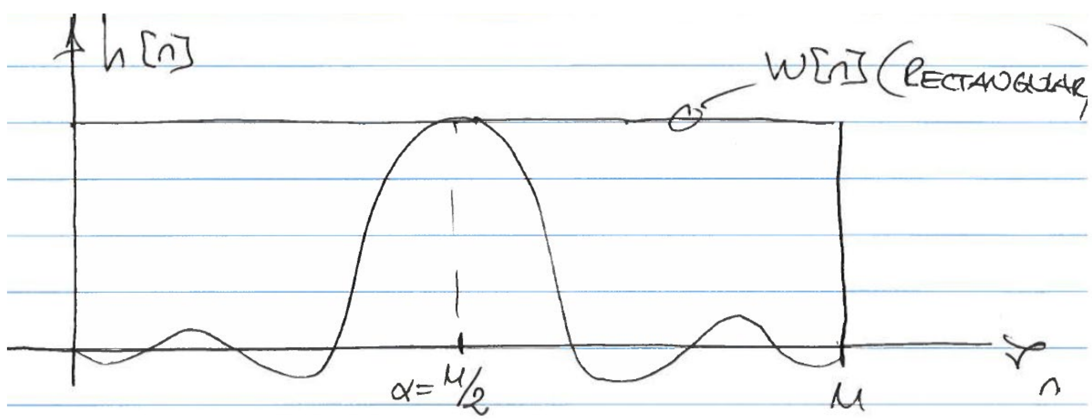
Using the Inverse DTFT to find the desired Impulse Response

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\, d\omega$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega\frac{M}{2}} e^{j\omega n}\, d\omega = \frac{\sin(\omega_c(n - {}^{M}/_{2}))}{\pi(n - {}^{M}/_{2})}$$

$h_d[n]$ is a symmetric sequence, but it is also Non-causal and Infinite. Also note that for $n = {}^{M}/_{2}$, both the numerator and the denominator are zero (0) – the cure is l'Hospital's theorem…

Now, truncate $h_d[n]$ with a window function $w[n]$; $h[n] = h_c[n] \cdot w[n]$
Since $w[n]$ is symmetric, $h[n]$ is also symmetric…!!



NOTE THAT α „MOVES" AS WE ADJUST THE WINDOW LENGTH.

How does the Aplitude Response looks like…???

We discussed previous, that the Frequency Response can be written as

$$H(e^{j\omega}) = \{h[{}^{M}/_{2}] + 2\sum_{k=1}^{M/2} h[{}^{M}/_{2} + k]\cos(\omega k)\} \cdot e^{-j\omega M/2}$$

See my slides, lecture 5

In O&S, a slightly different notation is used;

$$H(e^{j\omega}) = \{\sum_{k=0}^{M/2} a[k]\cos(\omega k)\} \cdot e^{-j\omega M/2}$$

Equ. 140a, p. 343

$a[k] = 2h[{}^{M}/_{2} - k]$ for $k = 1..{}^{M}/_{2}$

$a[0] = h[{}^{M}/_{2}]$

Using this expression, the amplitude response is;

$$|H(e^{j\omega})| = \left|\sum_{k=0}^{M/2} a[k]\cos(\omega k)\right|$$

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$$\left|H\left(e^{j\omega}\right)\right| = \left|\sum_{k=0}^{M/2} a[k]\cos(\omega k)\right|$$

The filter specification dictates that we should have a $0\ dB$ DC-gain, i.e.,

$$G \cdot \left|H\left(e^{j\omega}\right)\right| = G \cdot \left|\sum_{k=0}^{M/2} a[k]\cos(\omega k)\right| = 1 \quad \text{for } \omega = 0.$$

$$G \cdot \left|\sum_{k=0}^{M/2} a[k]\right| = 1 \quad \Rightarrow \quad G = \frac{1}{\left|\sum_{k=0}^{M/2} a[k]\right|}$$

For a given window function $w[n]$, our task now is to find the smallest possible value for the filter order M such that the amplitude response complies with the specifications, i.e., the pasband and the stopband requirements. Note that we cannot do anything about the cut-off frequency – it is given for the brick-wall filter.

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The challenge we are facing here…

$$\boxed{H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})}$$

Since in the discrete-time domain everything is 2π periodic, this expression indicates a periodic convolution.

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta}) W(e^{j(\omega-\theta)}) d\theta$$

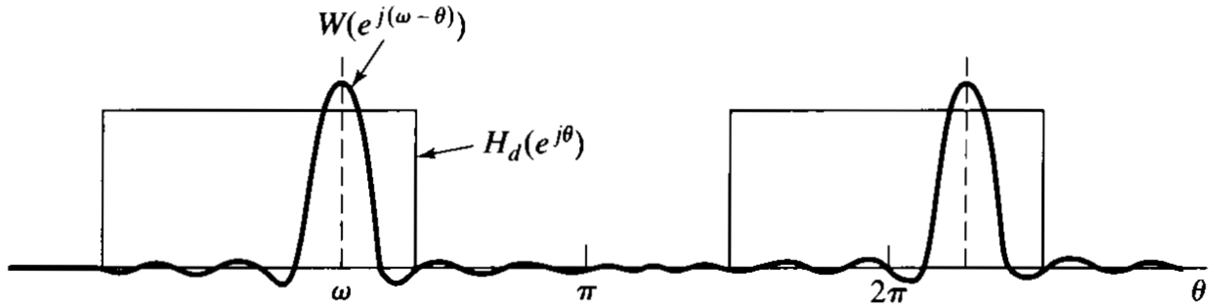
If we want $H(e^{j\omega}) = H_d(e^{j\omega})$, we have no other options than requiring

$$W(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

i.e., that the Fourier Transform of the window function should be an impulse train.

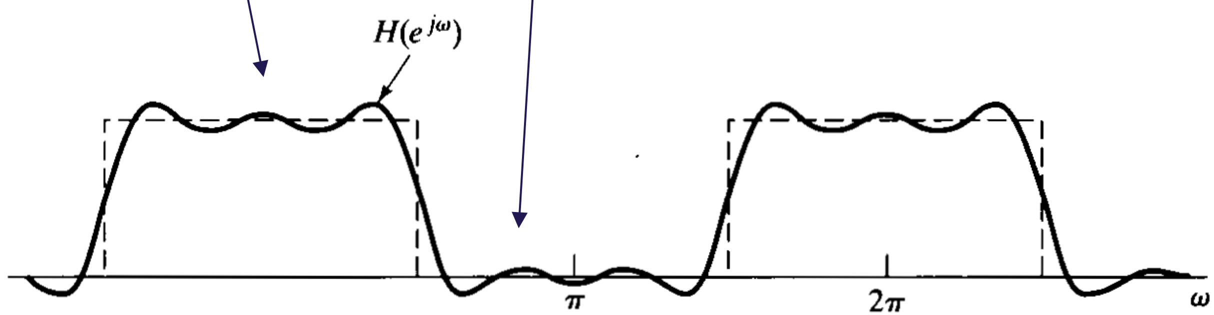
This implies, however, that the window function $w[n]$ in the time domain is also an infinite impulse train, and thus there is NO TRUNCATION of $h_d[n]$.

So, we are looking for a COMPROMISE…!!!



(a)

Ripples both in the pas- and the stop-band



(b)

The compromise we are looking for is a window function which has a Fourier transform with

- a narrow main lope ⇒ good approximation to the transitions
- small amplitude side lopes ⇒ reduced ripples in the pass- and the stop band

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

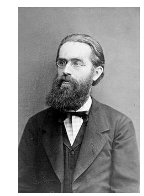
Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \;\; M \text{ even} \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Maurice Stevenson Bartlett, England

Hann *(Hanning)*

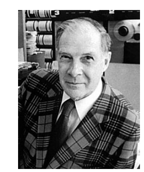
$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$



Julius Ferdinand von Hann, Austria

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$



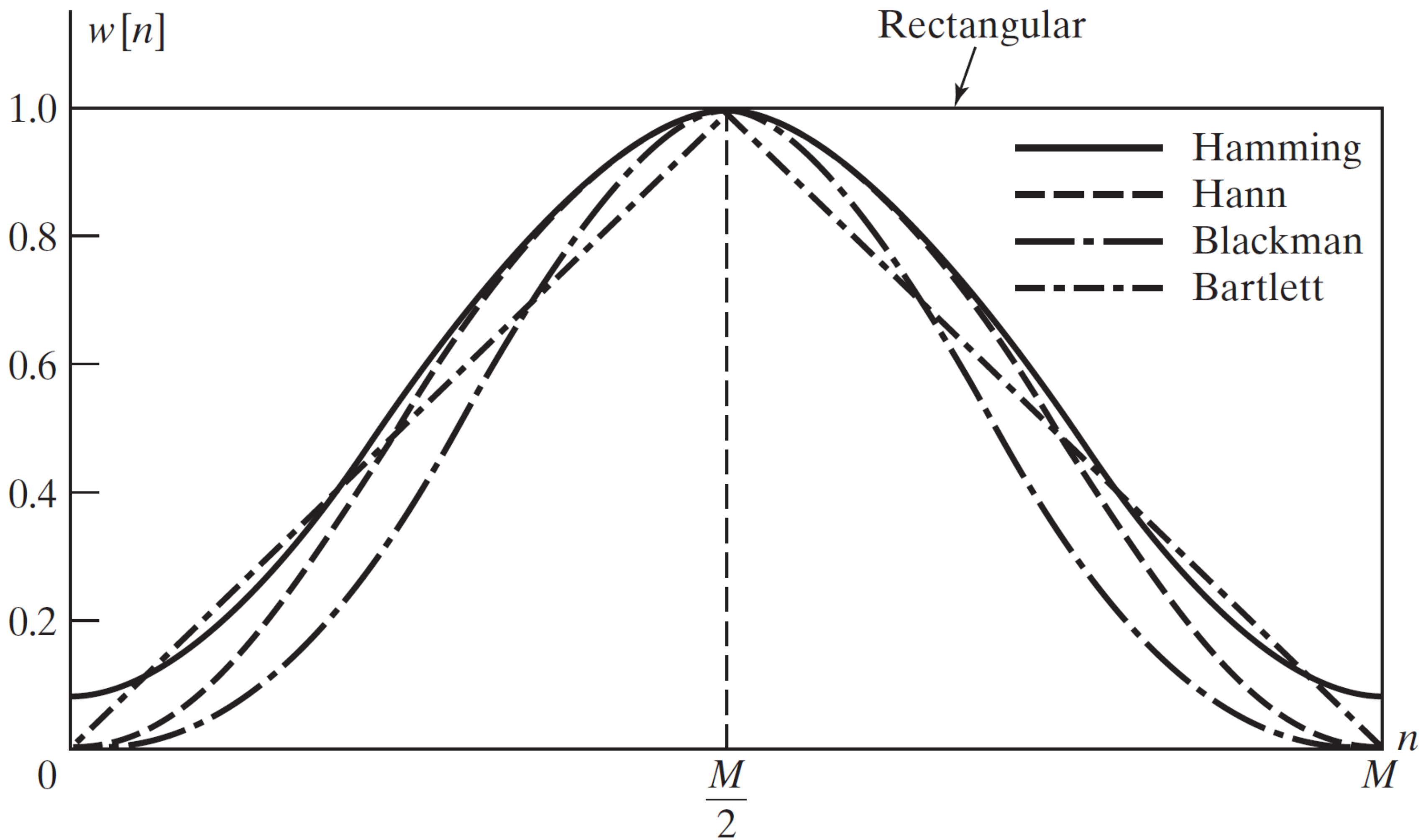
Richard Wesley Hamming, USA

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

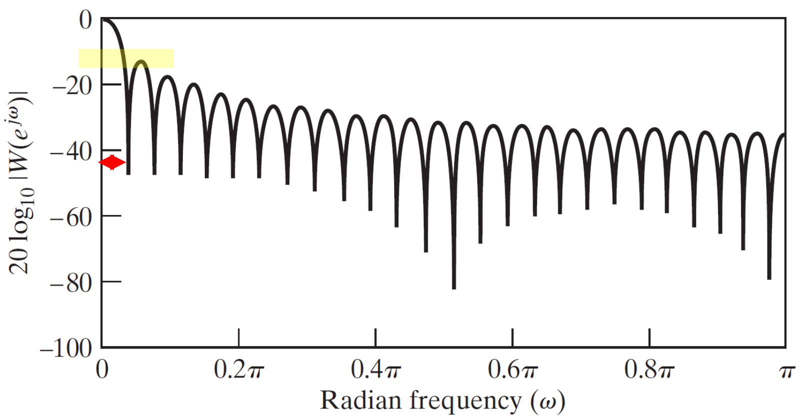
Ralph Beebe Blackman, USA

Time Domain Representation of Window Functions

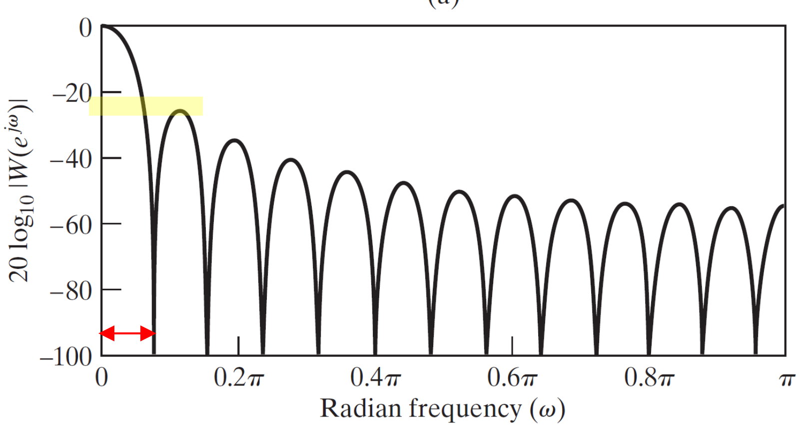


Frequency domain representation for window length $M = 50$

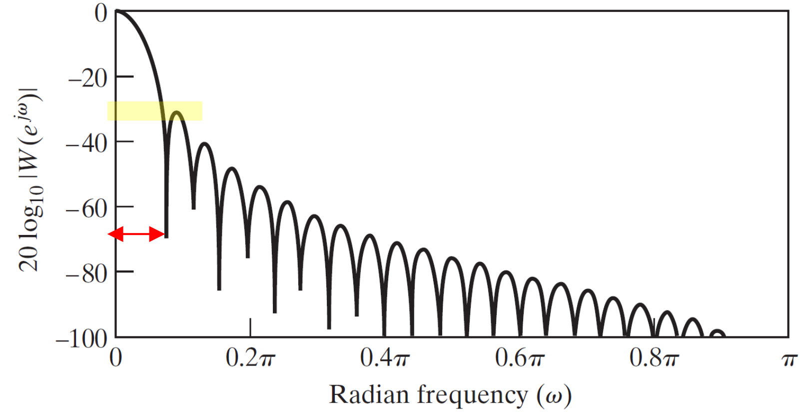
a) Rectangular
b) Bartlett
c) Hanning
d) Hamming
e) Blackman



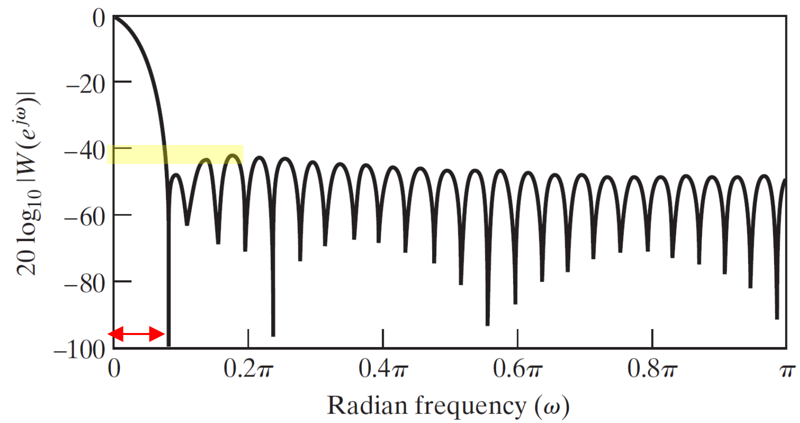
(a)



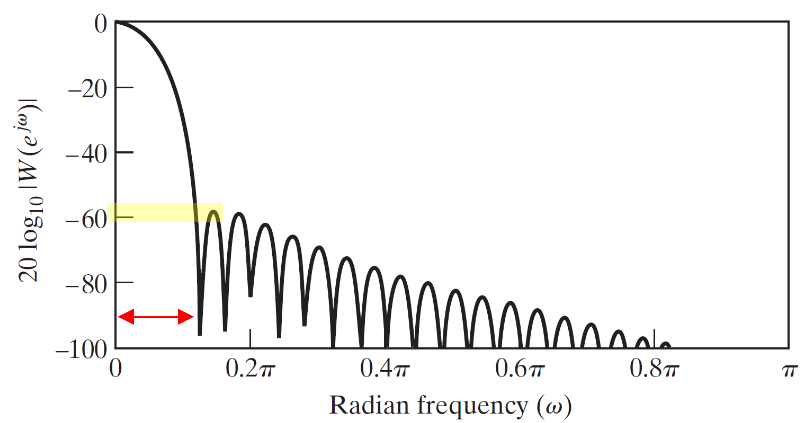
(b)



(c)



(d)



(e)

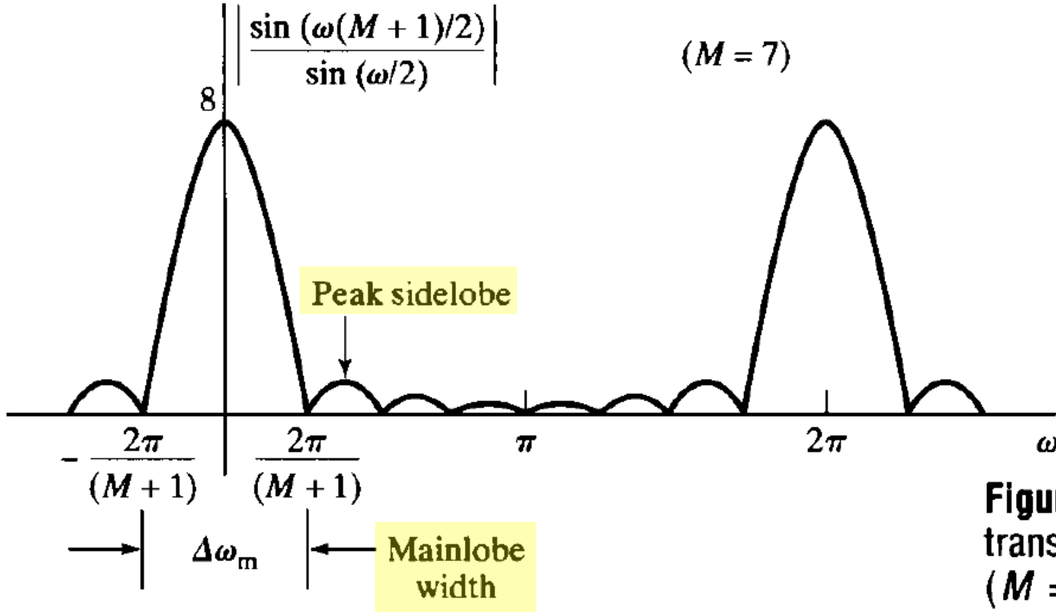


Figure 7.20 Magnitude of the Fourier transform of a rectangular window ($M = 7$).

TABLE 7.1 COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hanning	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

1) Width of the main-lope is inverse proportional to the filter order M. 2) Area under the lopes remains constant for varying M. ⇒ Relative peak of the side-lopes is constant.

So, in conclusion…

- No matter how large a filter order M we choose, we CANNOT (by using the same window function $w[n]$) reduce the amplitude of the pass- and the stop-band ripples.

- By increasing M, however, we can get a better approximation to the abrupt transition from the pass-band to the stop-band of $|H(e^{j\omega})|$, i.e., we can obtain a more narrow transition band.

- Modification (normally that means reduction) of the ripple amplitude can be done only by choosing a different window function $w[n]$.

- Different window functions, however, lead to different degrees of slope in the transition band due to variation in their main-lope width.

- So, using the Window Design Method, we have no other options than doing experimentation (trial and error), i.e., adjust M and $w[n]$, and then evaluate the amplitude response of the resulting filter. Linear phase is guaranteed…!

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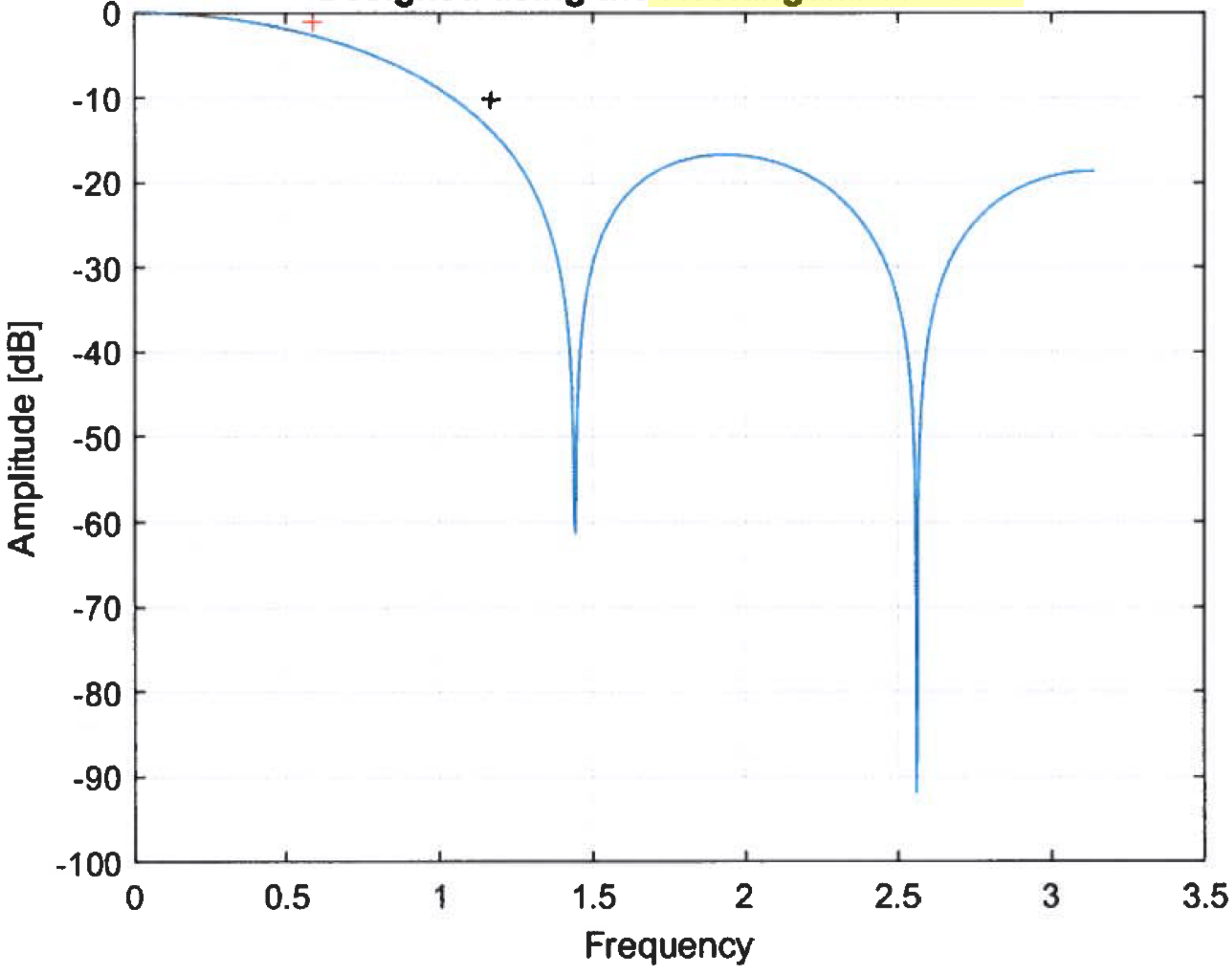
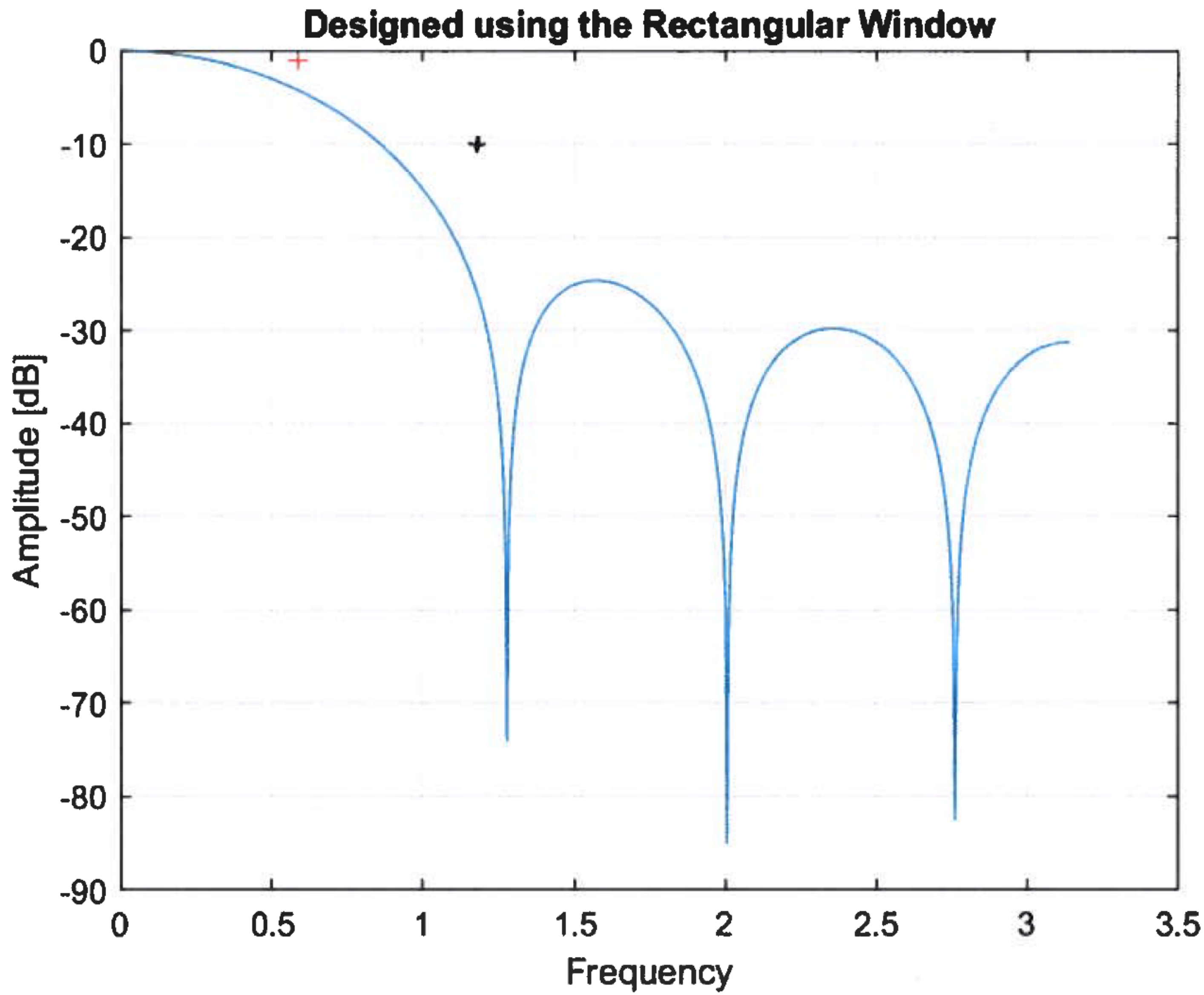
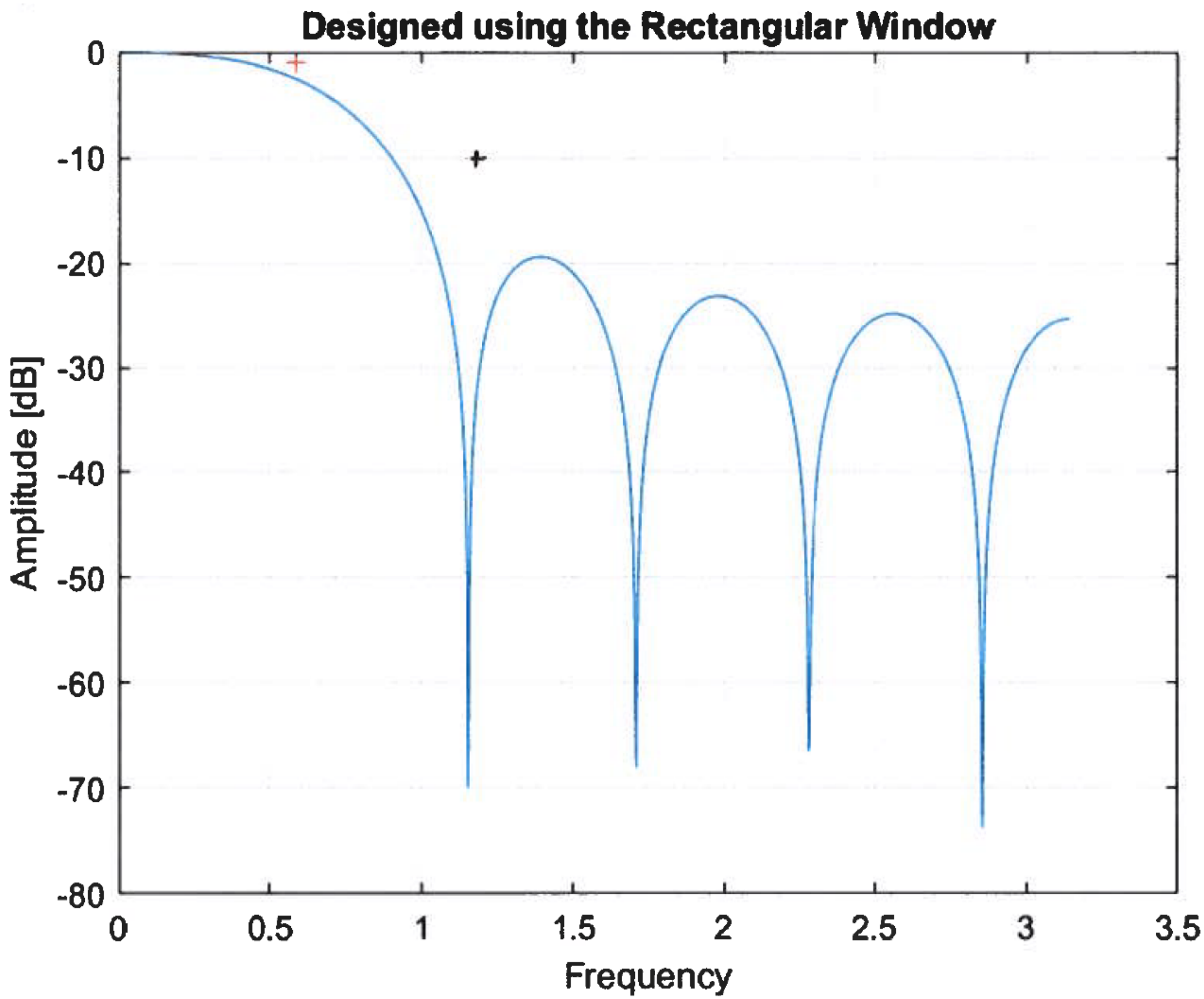
Designed using the Rectangular Window



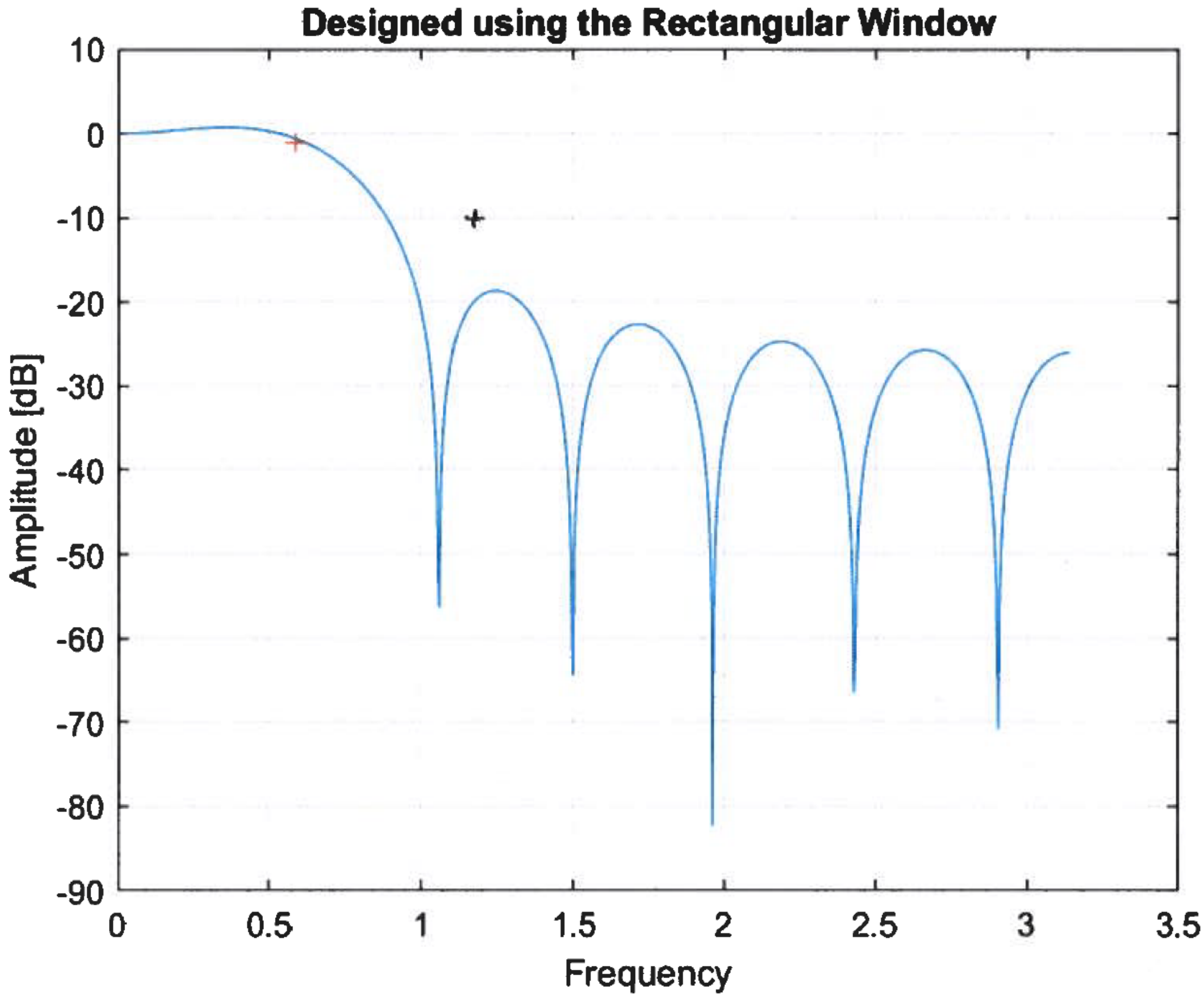
$M = 4$


Designed using the Rectangular Window
Amplitude [dB]
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
0
0.5
1
1.5
2
2.5
3
3.5
Frequency

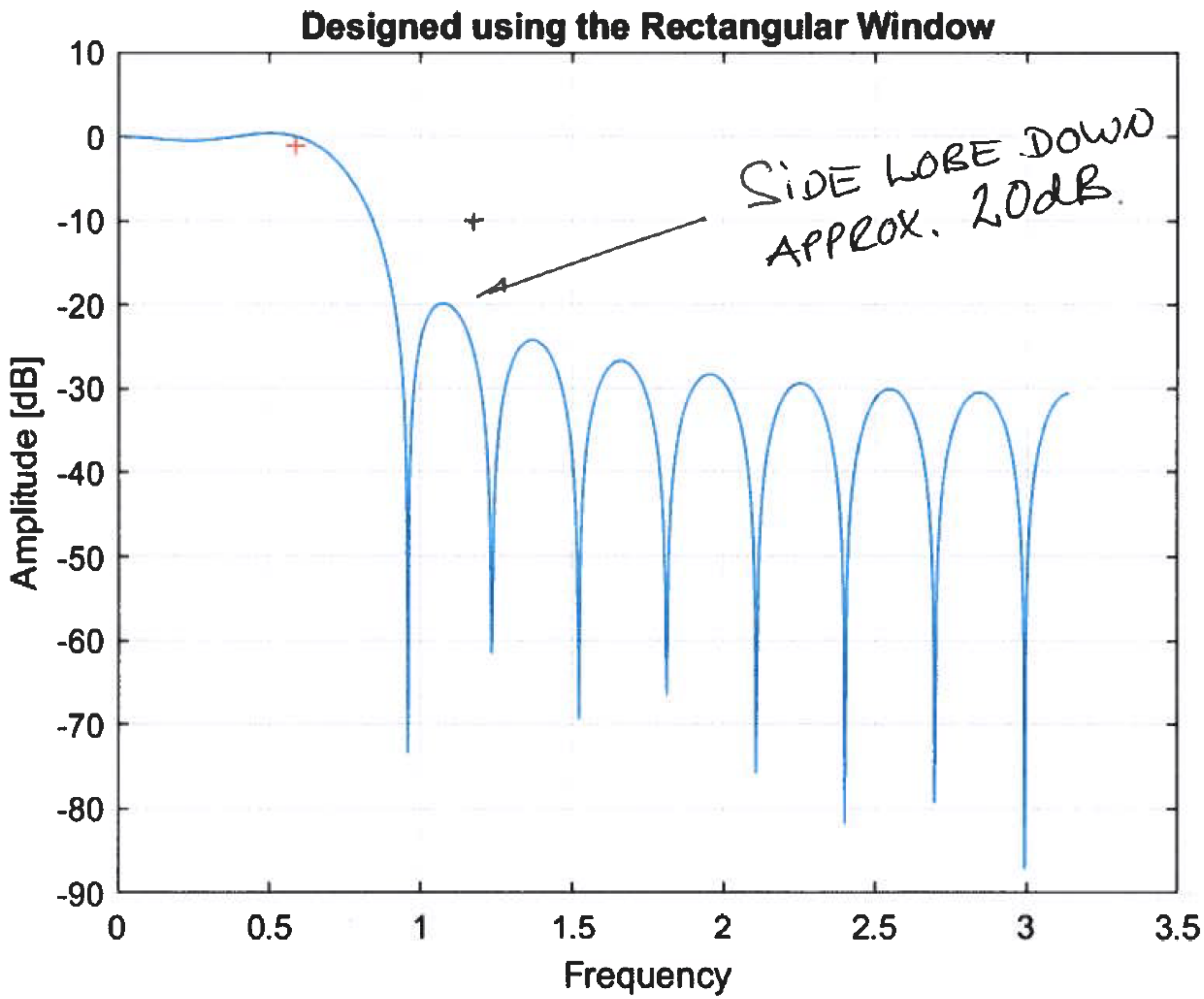
$M=8$


Designed using the Rectangular Window
0
-10
-20
-30
-40
-50
-60
-70
-80
Amplitude [dB]
0
0.5
1
1.5
2
2.5
3
3.5
Frequency

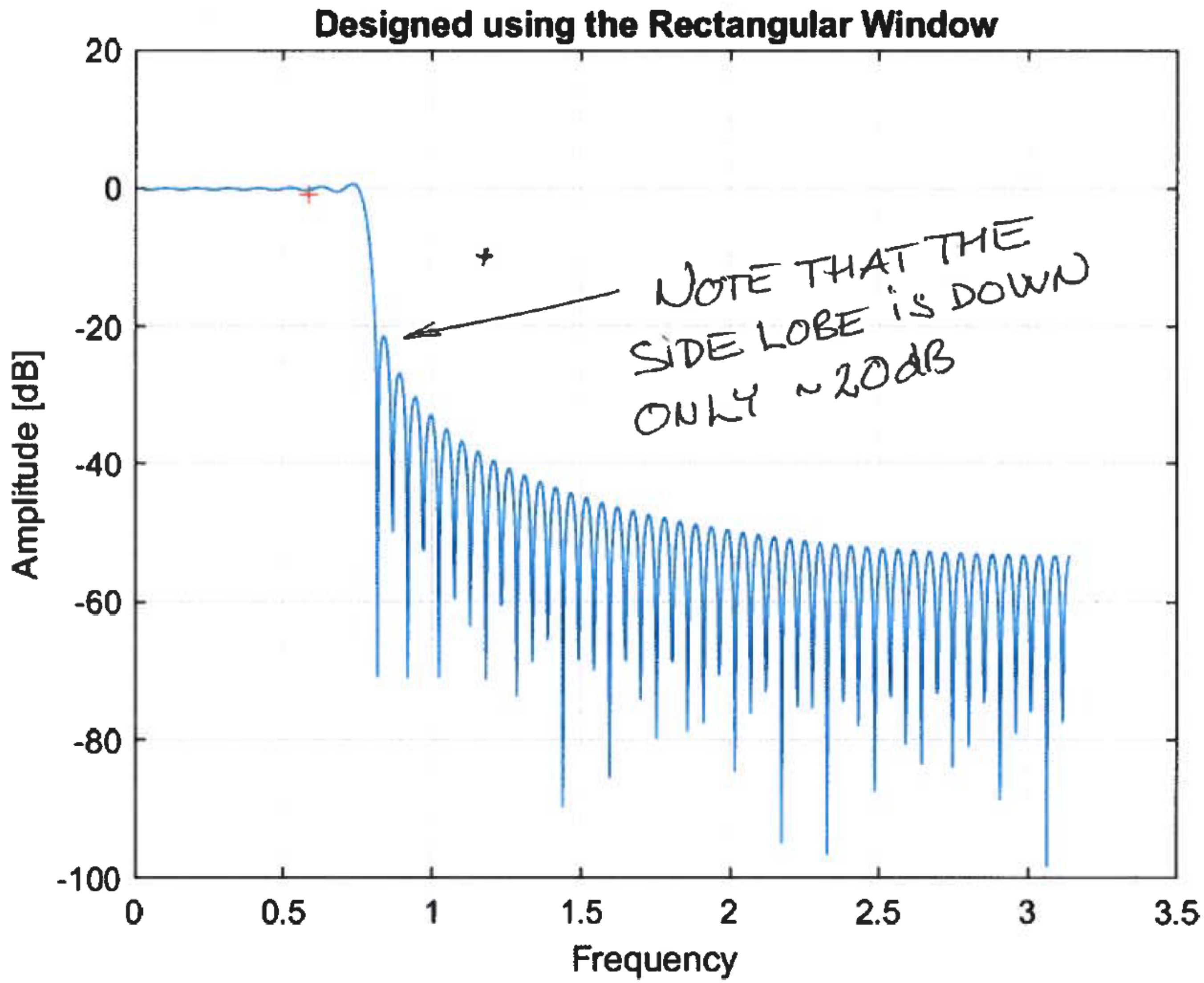
$M=10$


Designed using the Rectangular Window
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Amplitude [dB]
0
0.5
1
1.5
2
2.5
3
3.5
Frequency

$M=12$

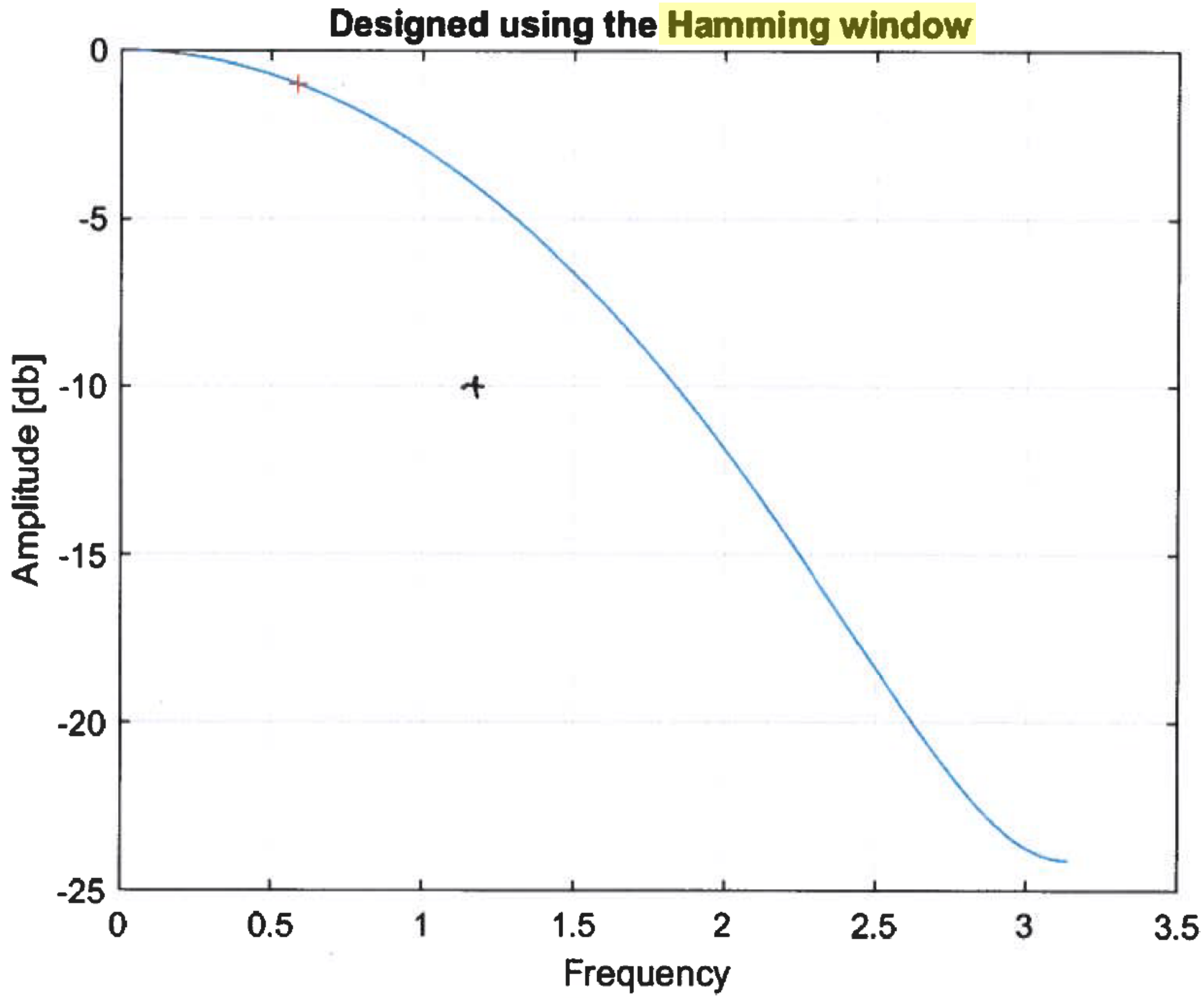

Designed using the Rectangular Window
Amplitude [dB]
Frequency
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
0
0.5
1
1.5
2
2.5
3
3.5
SIDE LOBE DOWN
APPROX. 20dB.

$M=20$

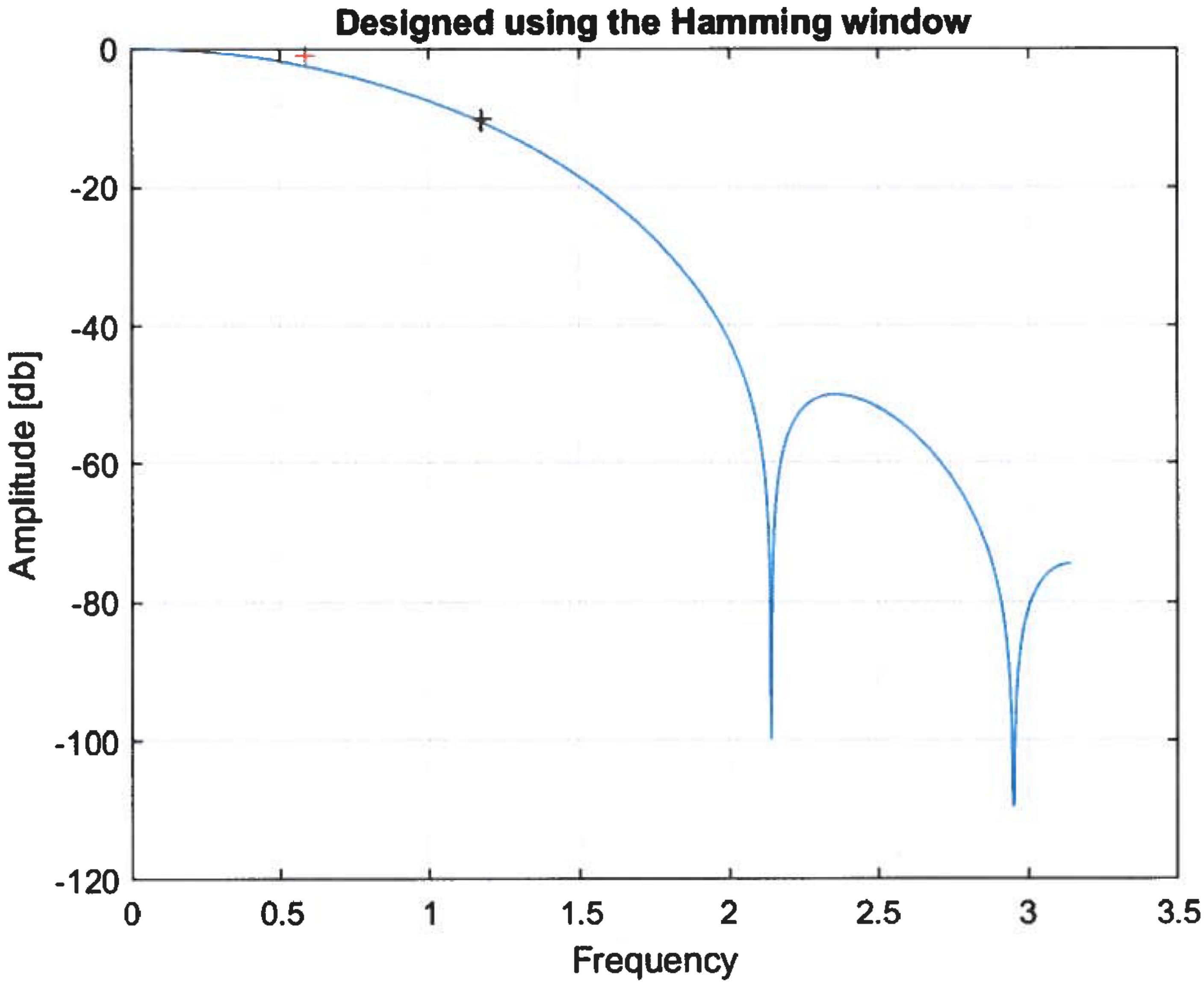


Designed using the Rectangular Window
20
0
-20
-40
-60
-80
-100
Amplitude [dB]
0
0.5
1
1.5
2
2.5
3
3.5
Frequency
NOTE THAT THE
SIDE LOBE IS DOWN
ONLY ~20dB

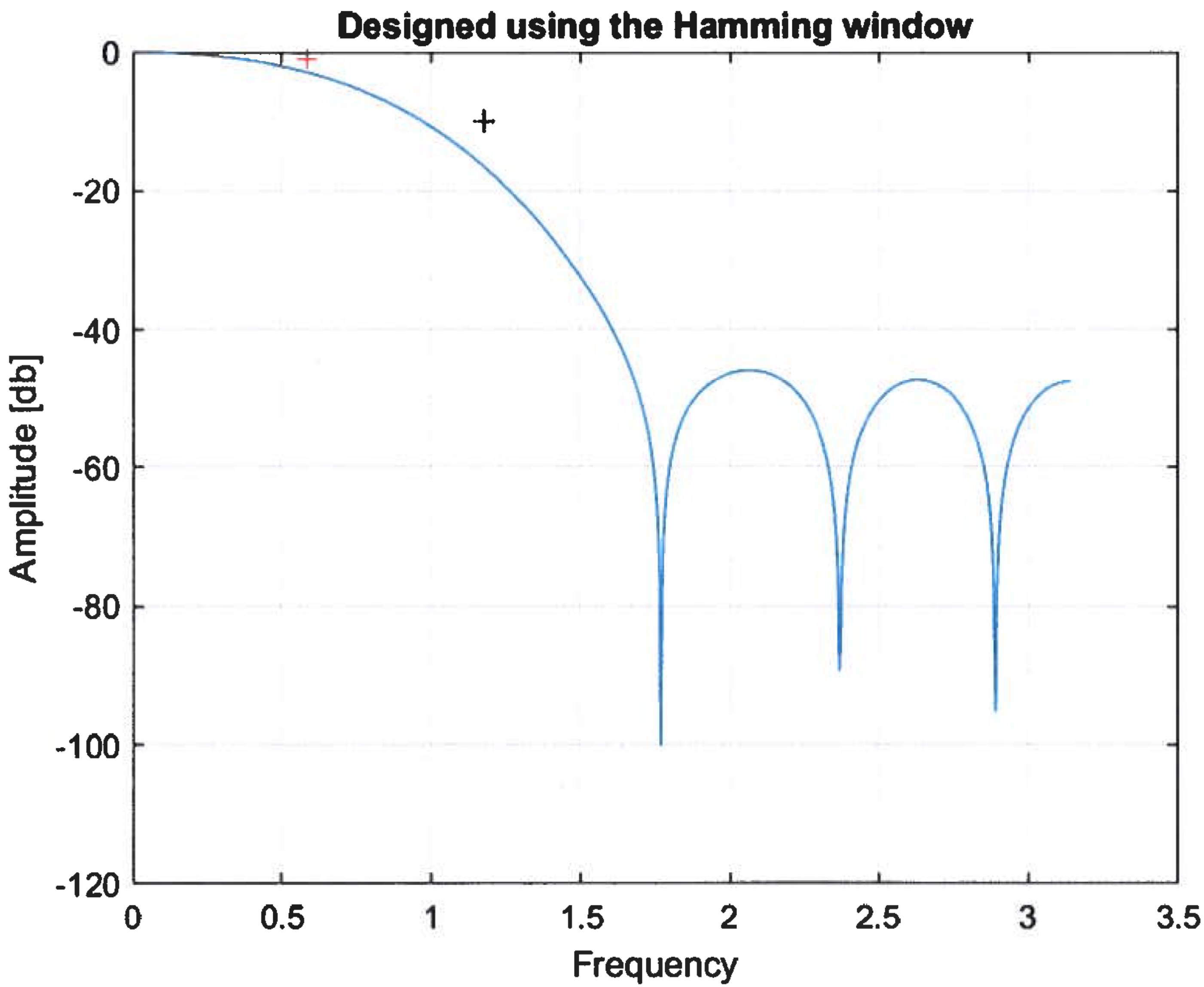
$M=120$


Designed using the Hamming window
Amplitude [db]
Frequency
0
-5
-10
-15
-20
-25
0
0.5
1
1.5
2
2.5
3
3.5

$u=4$

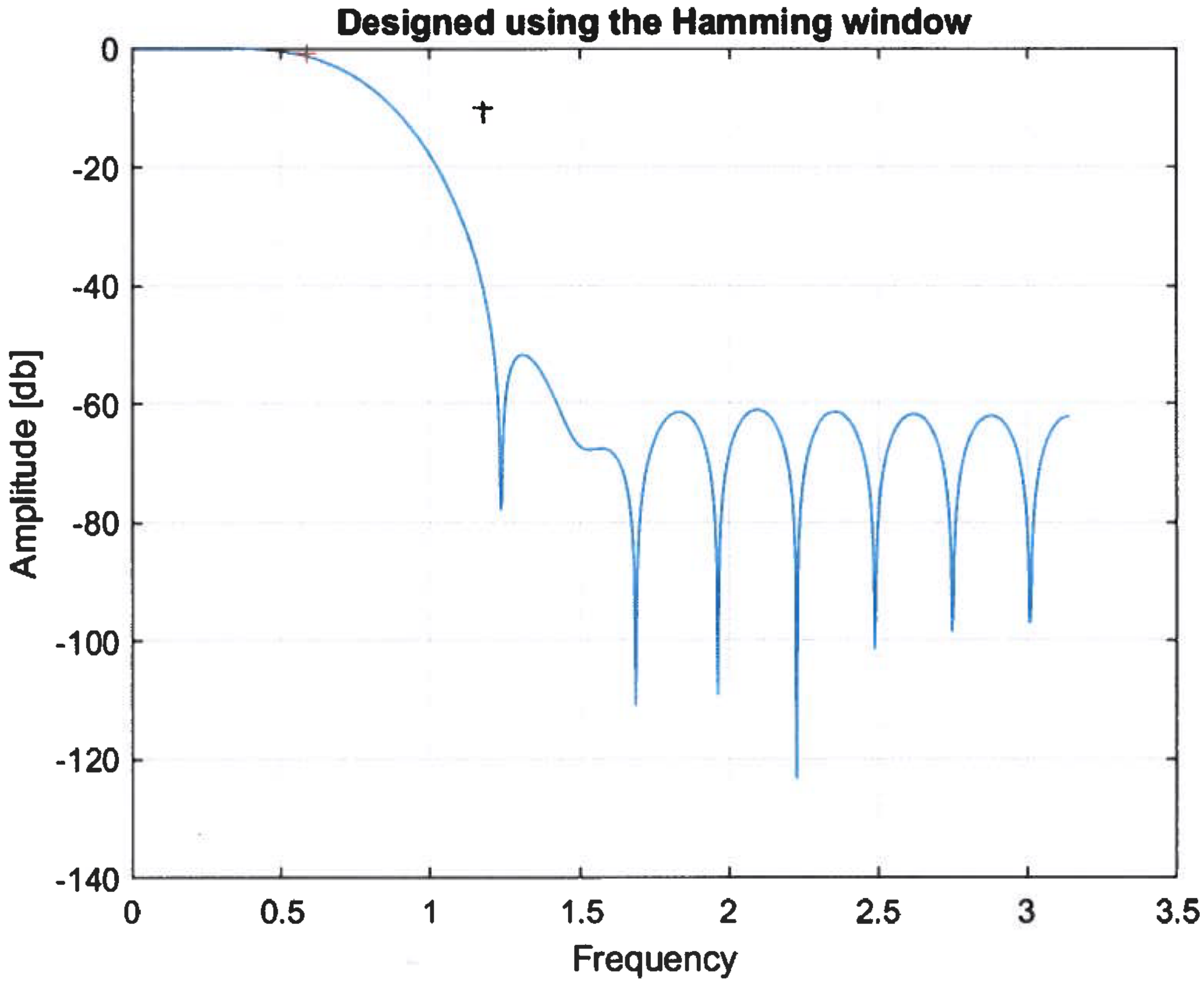

Designed using the Hamming window
Amplitude [db]
Frequency
0
-20
-40
-60
-80
-100
-120
0
0.5
1
1.5
2
2.5
3
3.5

$M=8$

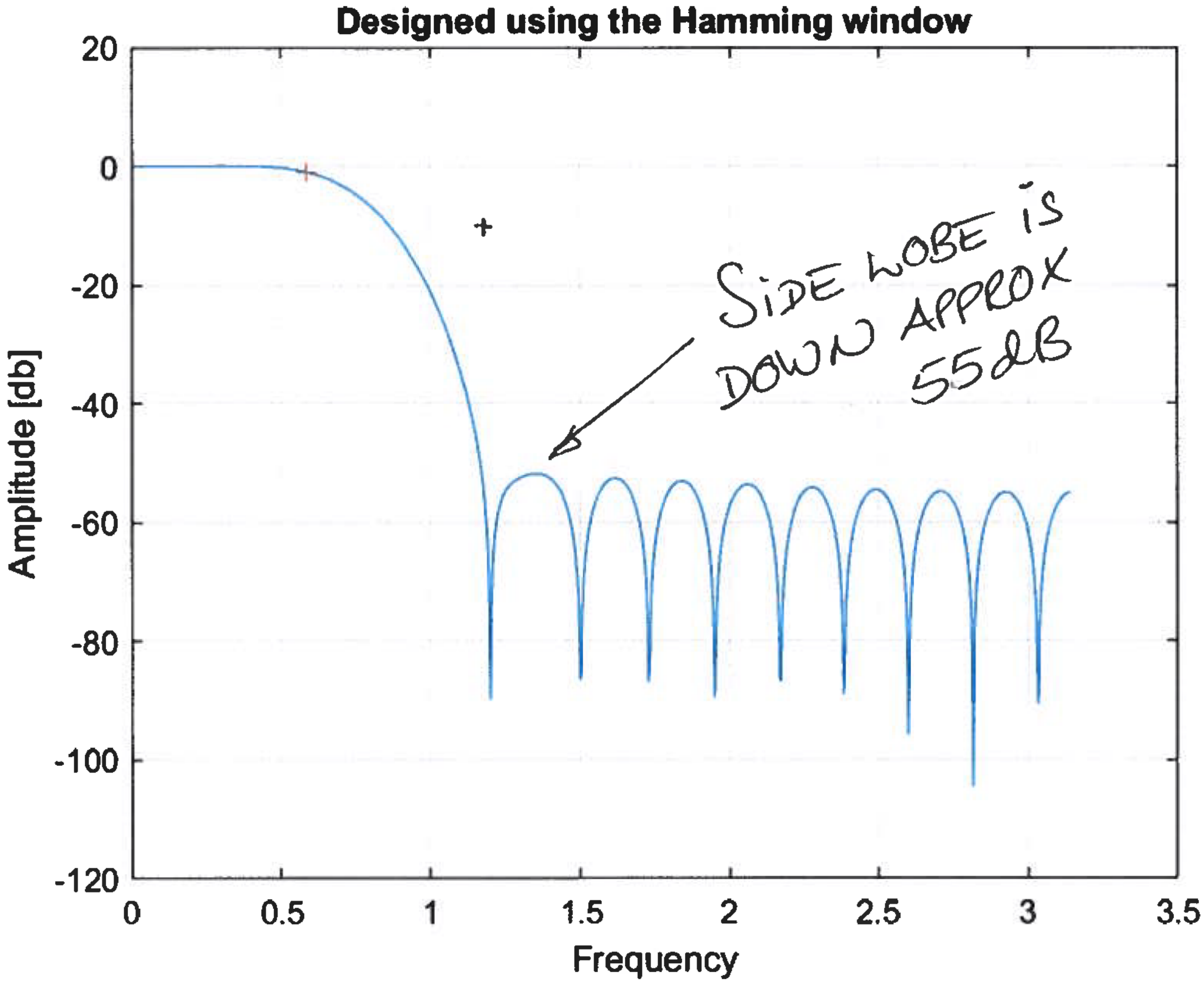


Designed using the Hamming window
Amplitude [db]
Frequency
0
-20
-40
-60
-80
-100
-120
0
0.5
1
1.5
2
2.5
3
3.5

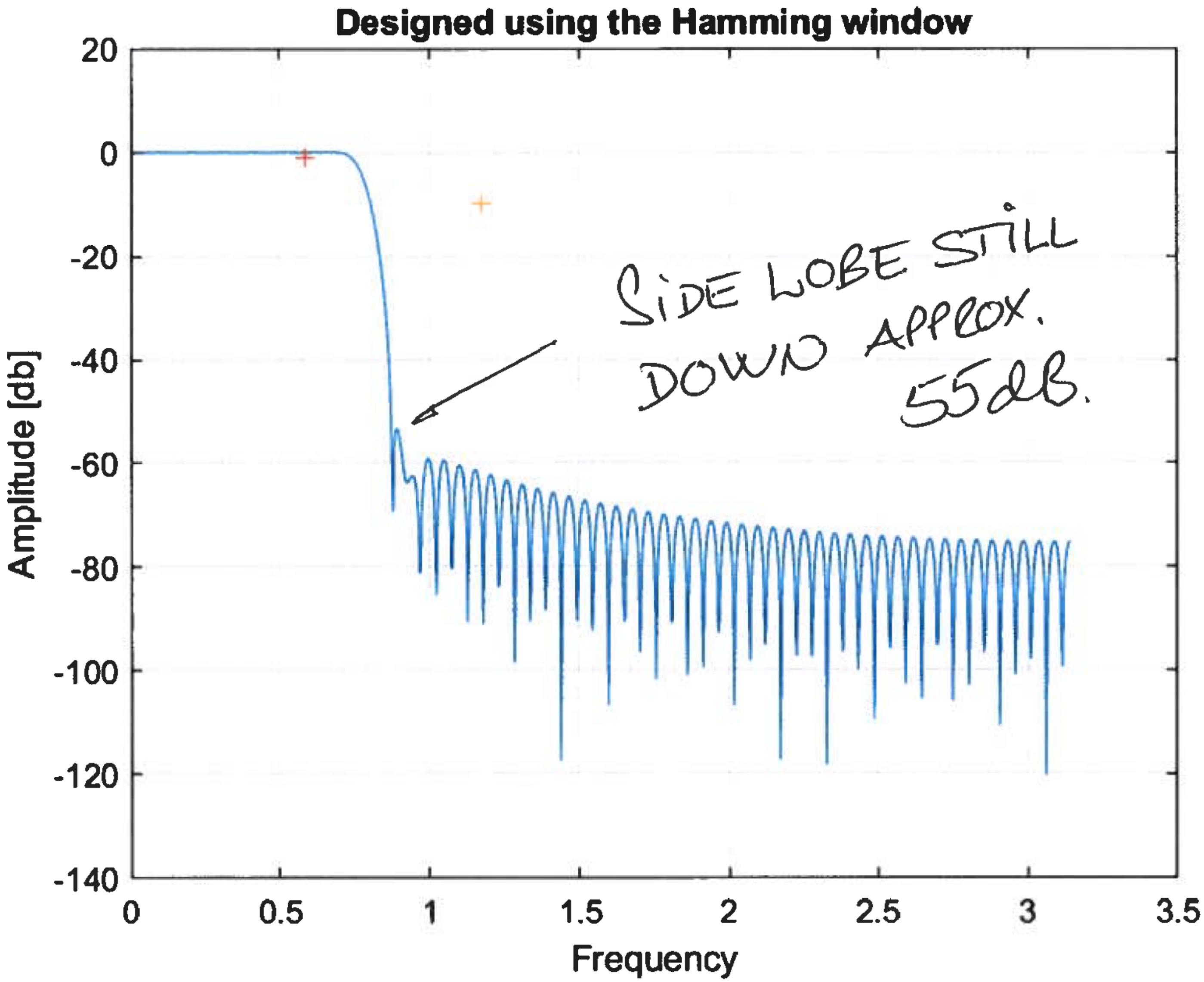
$M = 12$


Designed using the Hamming window
Amplitude [db]
Frequency
0
-20
-40
-60
-80
-100
-120
-140
0
0.5
1
1.5
2
2.5
3
3.5

$M=24$


Designed using the Hamming window
Amplitude [db]
20
0
-20
-40
-60
-80
-100
-120
0
0.5
1
1.5
2
2.5
3
3.5
Frequency
SIDE LOBE IS DOWN APPROX 55dB

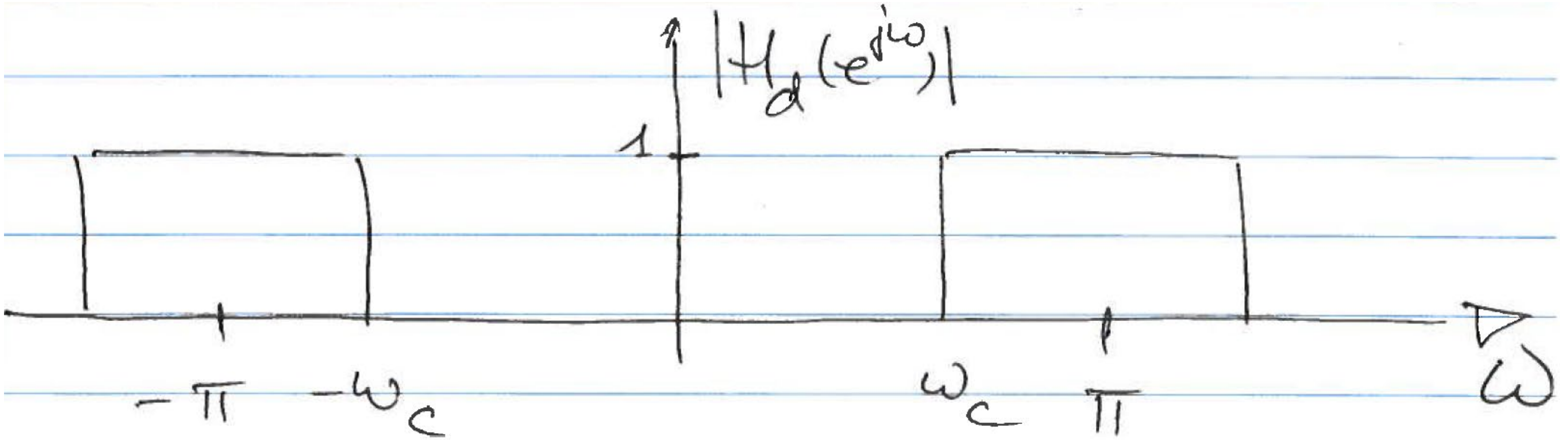
M = 28



Designed using the Hamming window
20
0
-20
-40
-60
-80
-100
-120
-140
Amplitude [db]
0
0.5
1
1.5
2
2.5
3
3.5
Frequency
SIDE LOBE STILL
DOWN APPROX.
55dB.

M=120

Other types of filters – FIR Type-I High Pass



$$H_d(e^{j\omega}) = \begin{cases} 0 & |\omega| \le \omega_c \\ e^{-j\omega M/2} & \omega_c \le |\omega| \le \pi \end{cases}$$

We can re-write the desired frequency response;

$$H_{d_{HP}}(e^{j\omega}) = e^{-j\omega M/2} - H_{d_{LP}}(e^{j\omega})$$

$$h_{d_{HP}}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{d_{HP}}(e^{j\omega})e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}(e^{-j\omega M/2} - H_{d_{LP}}(e^{j\omega}))e^{j\omega n}\,d\omega$$

$$h_{d_{HP}}[n] = \frac{\sin(\pi(n - M/2))}{\pi(n - M/2)} - \frac{\sin\left(\omega_c(n - M/2)\right)}{\pi(n - M/2)} \qquad -\infty < n < \infty$$

Equ. 80, p.571

$$h_{d_{HP}}[n] = \frac{\sin(\pi(n - M/2))}{\pi(n - M/2)} - \frac{\sin\left(\omega_c(n - M/2)\right)}{\pi(n - M/2)} \qquad -\infty < n < \infty$$

$$= \begin{cases} 1 & n = M/2 \\ 0 & otherwise \end{cases}$$

Identical to LP

So, the impulse response, and thus the filter coefficients, for the HP-filter can be expressed in terms of the LP-filter;

$$b_{i_{HP}} = -b_{i_{LP}} \qquad i = 0, \dots, M/2 - 1, M/2 + 1, \dots, M$$

$$b_{i_{HP}} = 1 - b_{i_{LP}} \quad i = M/2$$

The impulse response is still symmetric, and thus the HP-filter also has linear phase.

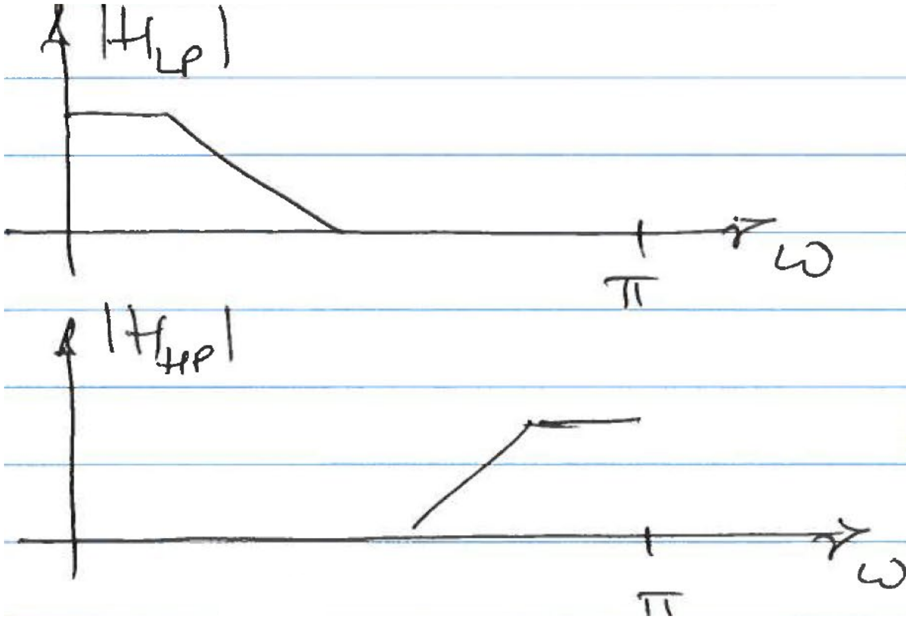
So, designing a High-pass filter can be done by first designing a ”corresponding” Low-pass filter, and then changing the sign of the filter coefficients, and $1 - b_{M/2_{LP}}$.

Pass-band gain for FIR Type-I HP-filter

$$H(e^{j\omega}) = \{\sum_{k=0}^{M/2} a[k]\cos(\omega k)\} \cdot e^{-j\omega M/2}$$

$$a[k] = 2h[M/2 - k] \quad \text{for} \quad k = 1..M/2$$

$$a[0] = h[M/2]$$



$$G \cdot |H_{LP}| = 1 \Big|_{\omega=0}$$

$$G \cdot |H_{HP}| = 1 \Big|_{\omega=\pi}$$

$$G \cdot \left|\sum_{k=0}^{M/2} a[k]\cos(\omega k)\right| = 1 \quad \omega = \pi \quad \Rightarrow \quad G = \frac{1}{|a[0] - a[1] + a[2] - a[3] + \cdots + a[M/2]|}$$

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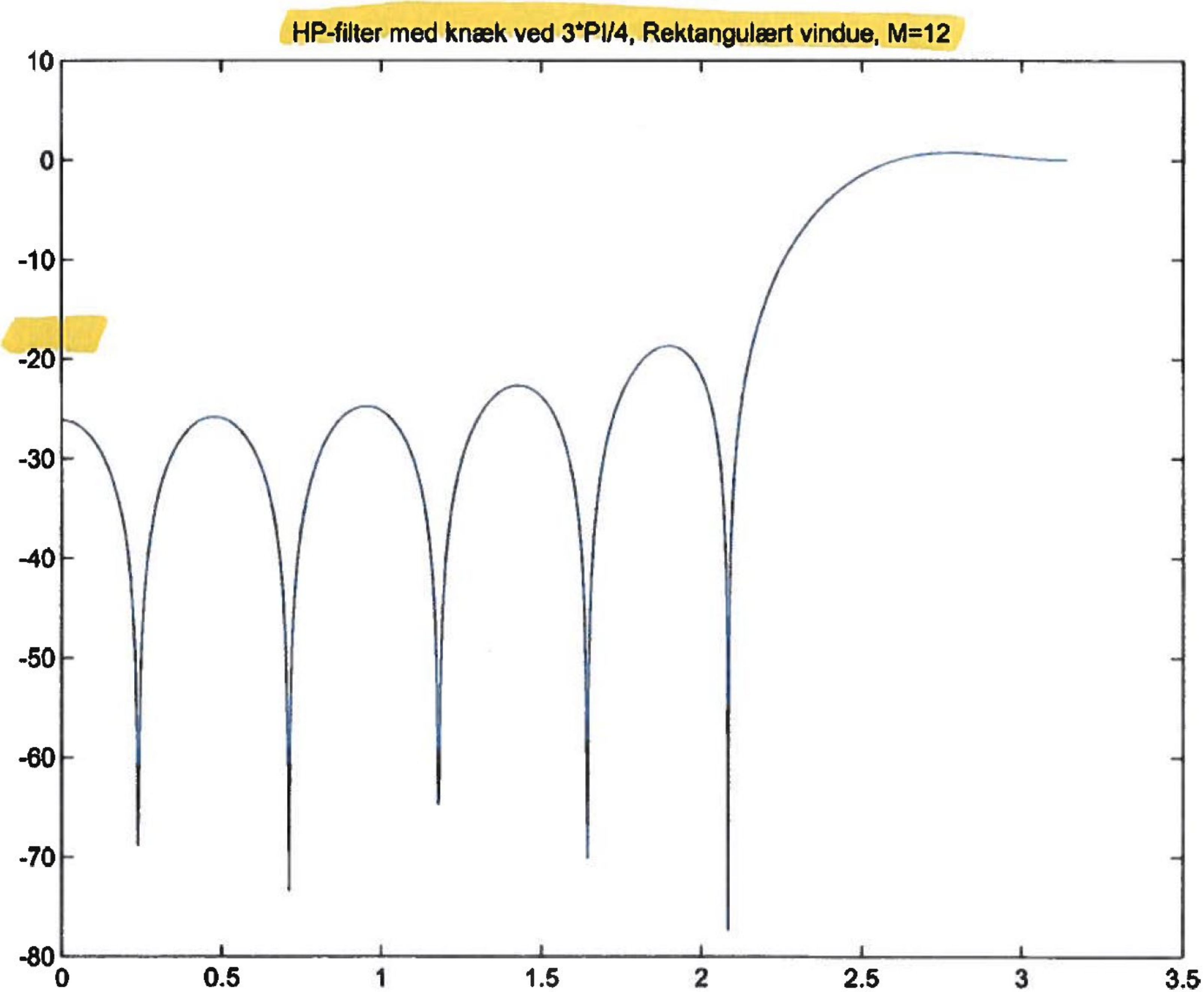
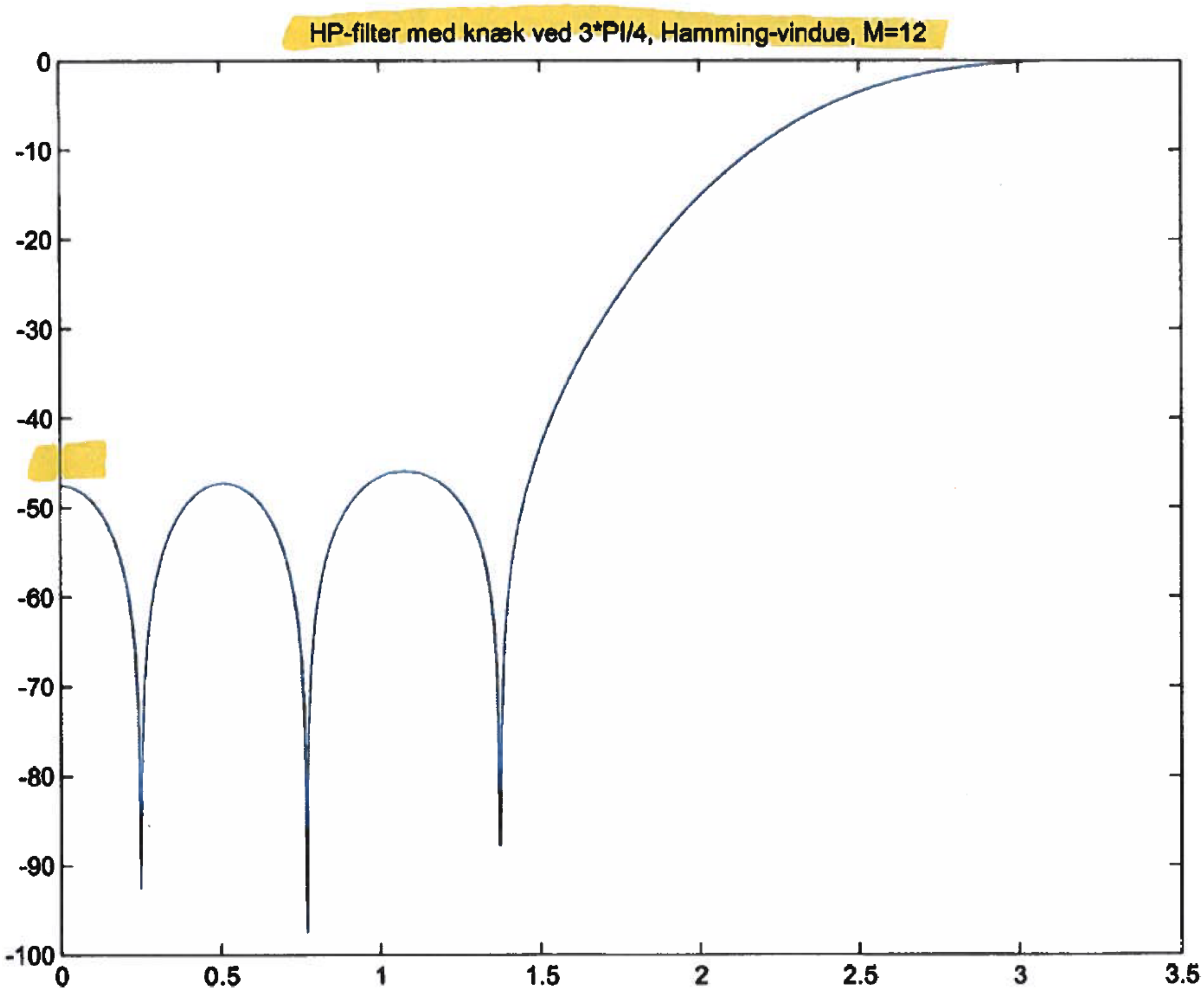
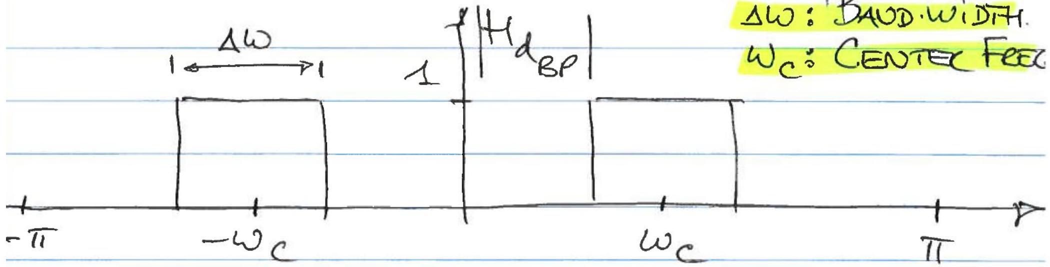

HP-filter med knæk ved 3*PI/4, Rektangulært vindue, M=12
10
0
-10
-20
-30
-40
-50
-60
-70
-80
0
0.5
1
1.5
2
2.5
3
3.5


HP-filter med knæk ved 3*PI/4, Hamming-vindue, M=12
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
0
0.5
1
1.5
2
2.5
3
3.5

Other types of filters – FIR Band Pass

BAND·PASS FILTERS

$\Delta\omega$: BAND·WIDTH.
ω_c : CENTER FREQ



$$H_{d_{BP}}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & \begin{cases} -\omega_c - \frac{\Delta\omega}{2} \le \omega \le -\omega_c + \frac{\Delta\omega}{2} \\ \omega_c - \frac{\Delta\omega}{2} \le \omega \le \omega_c + \frac{\Delta\omega}{2} \end{cases} \\ 0 & \text{OTHERWISE} \end{cases}$$

$\Downarrow$

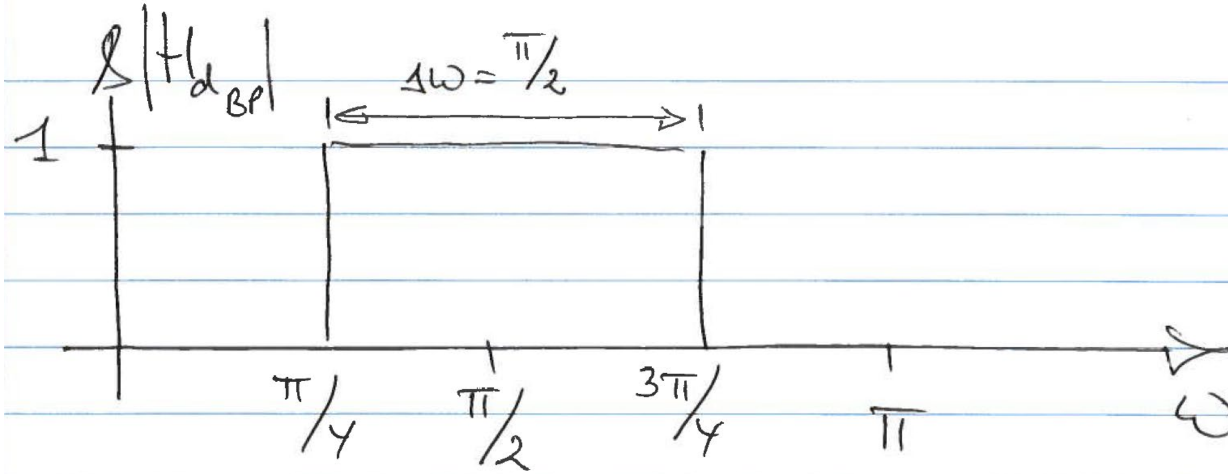
$$h_{d_{BP}}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{d_{BP}}(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$$

$\|$

$$h_{d_{BP}}[n] = \frac{1}{2\pi}\left\{\int_{-\omega_c - \frac{\Delta\omega}{2}}^{-\omega_c + \frac{\Delta\omega}{2}} e^{-j\omega M/2} \cdot e^{j\omega n}\, d\omega + \int_{\omega_c - \frac{\Delta\omega}{2}}^{\omega_c + \frac{\Delta\omega}{2}} e^{-j\omega M/2} \cdot e^{j\omega n}\, d\omega\right\}$$

$\|$

$$h_{d_{BP}}[n] = \frac{\sin\left(\left(\omega_c + \frac{\Delta\omega}{2}\right)\left(n - \frac{M}{2}\right)\right) - \sin\left(\left(\omega_c - \frac{\Delta\omega}{2}\right)\left(n - \frac{M}{2}\right)\right)}{\pi\left(n - \frac{M}{2}\right)}$$



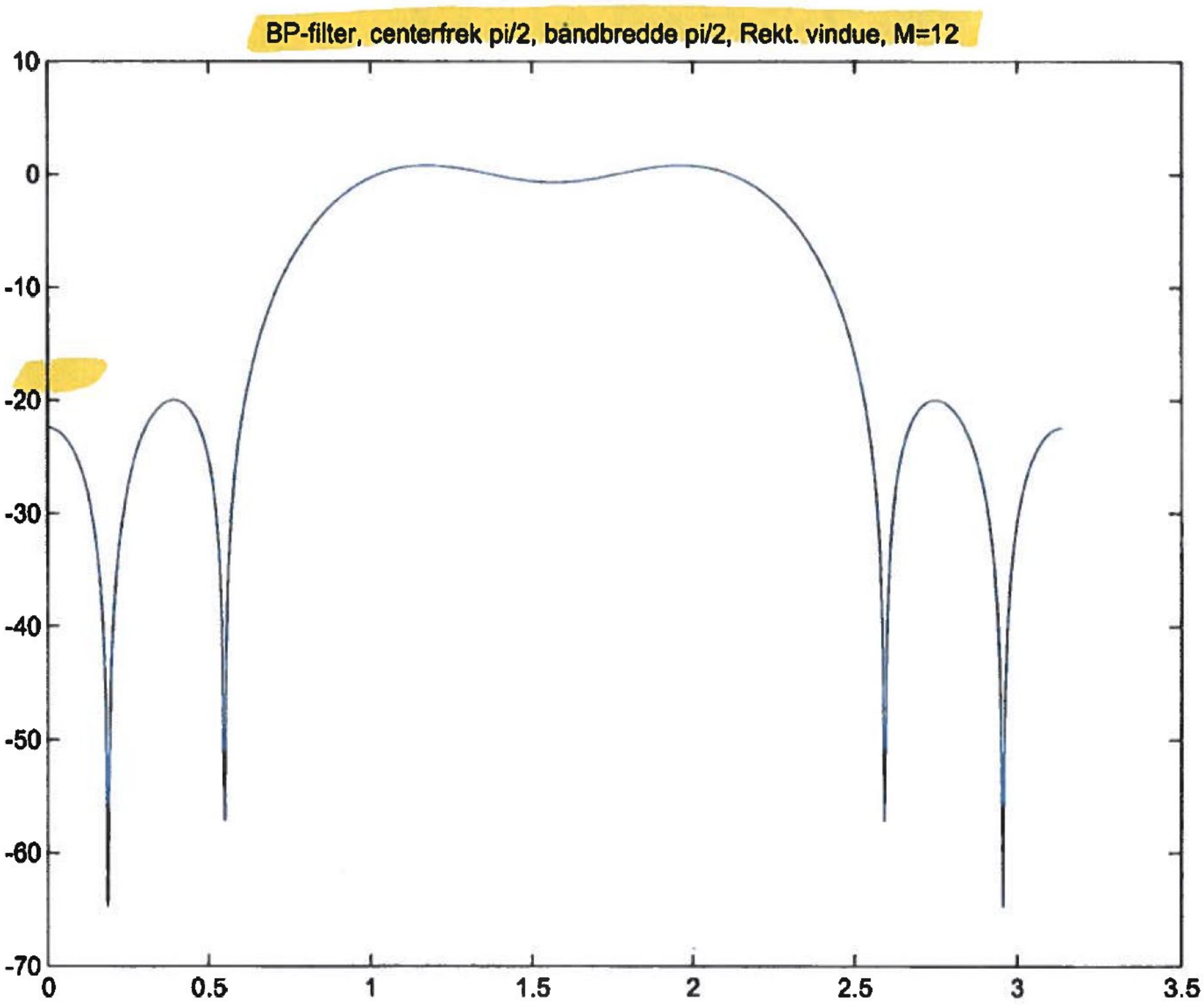
CENTER FREQUENCY : $\omega_c = \pi/2$

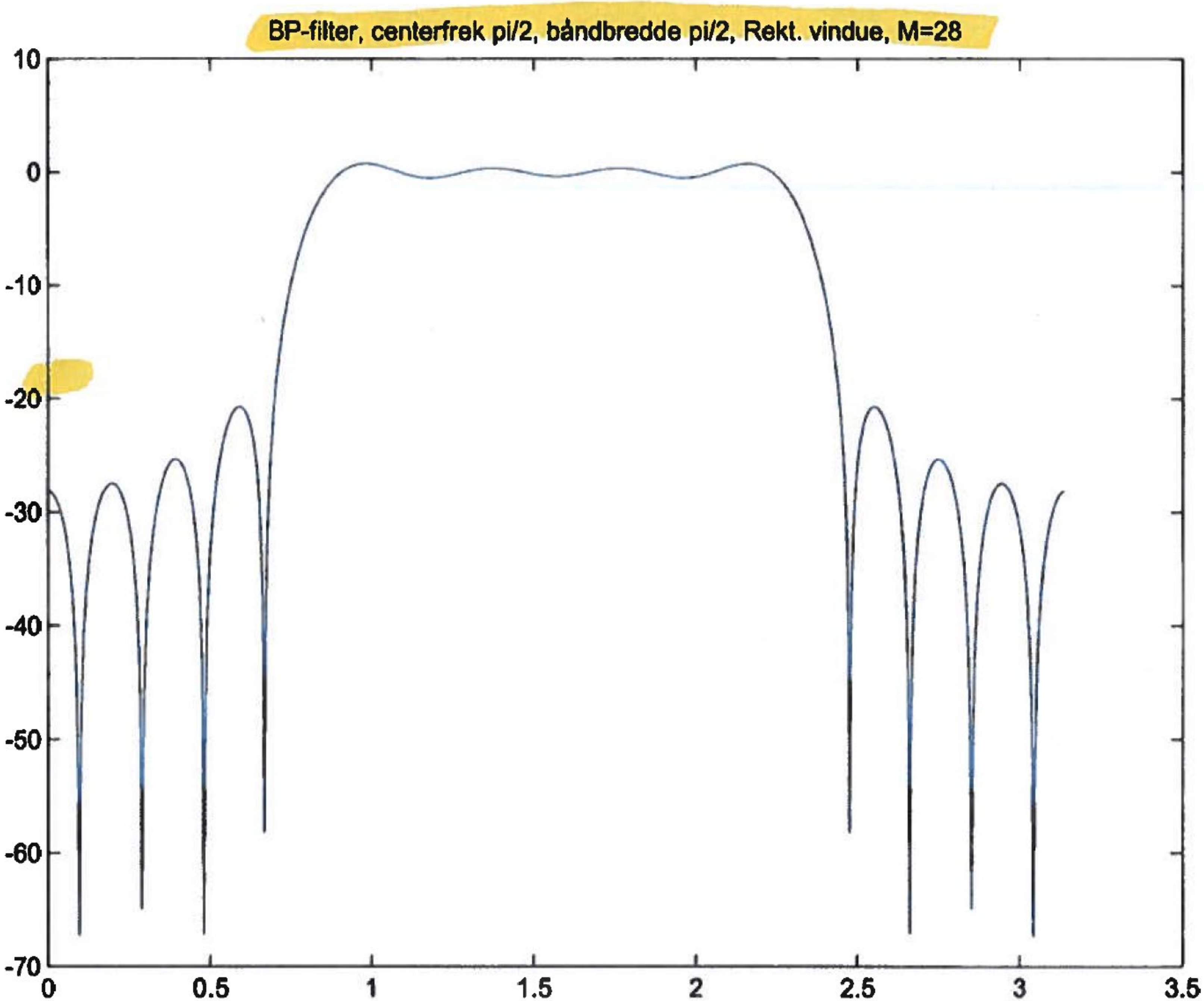
BANDWIDTH : $\Delta\omega = \pi/2$

$$h_{d_{BP}}[n] = \frac{\sin\left(\frac{3\pi}{4}(n - M/2)\right) - \sin\left(\frac{\pi}{4}(n - M/2)\right)}{\pi(n - M/2)}$$

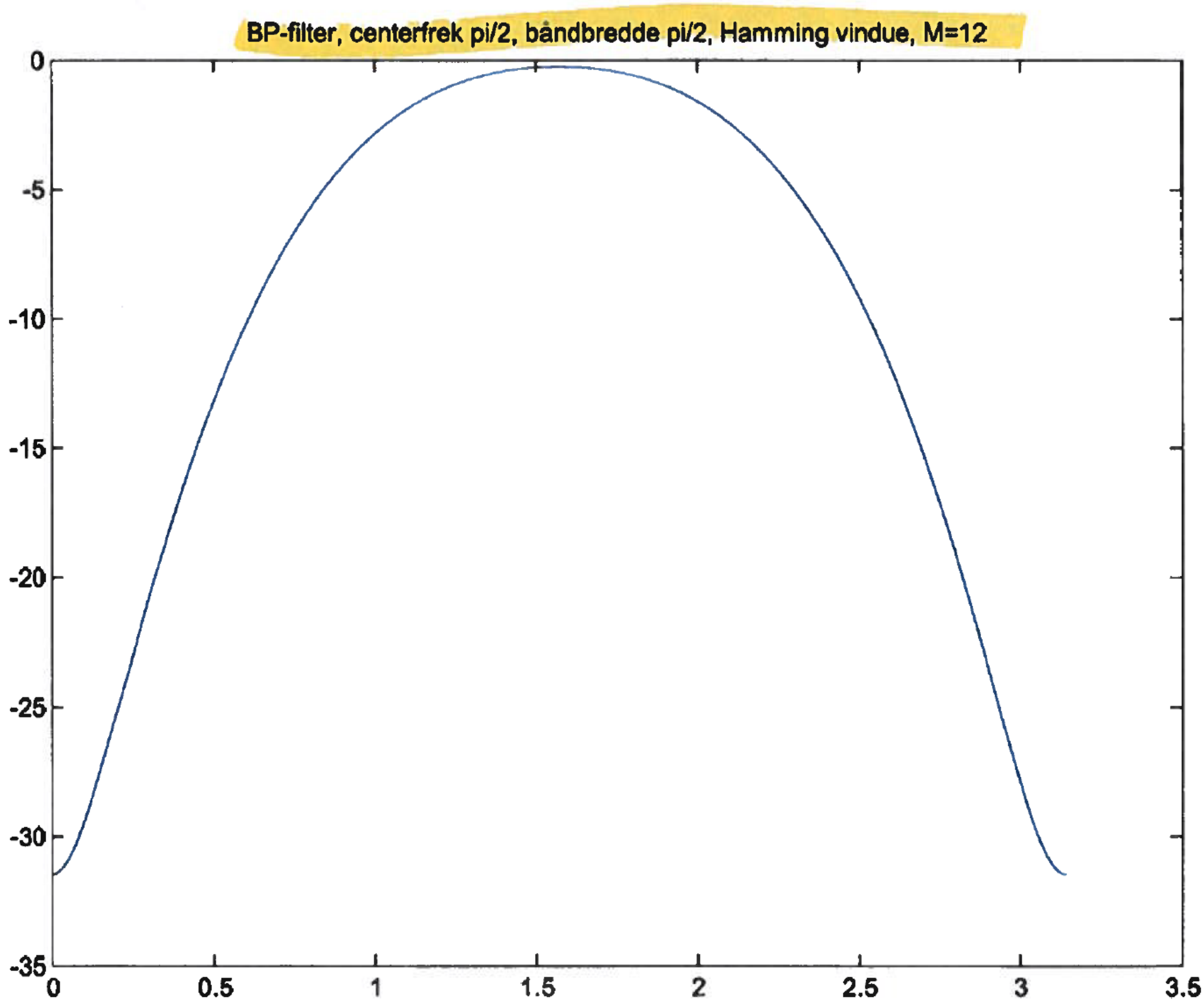
$$h[n] = h_{d_{BP}}[n] \cdot w[n]$$

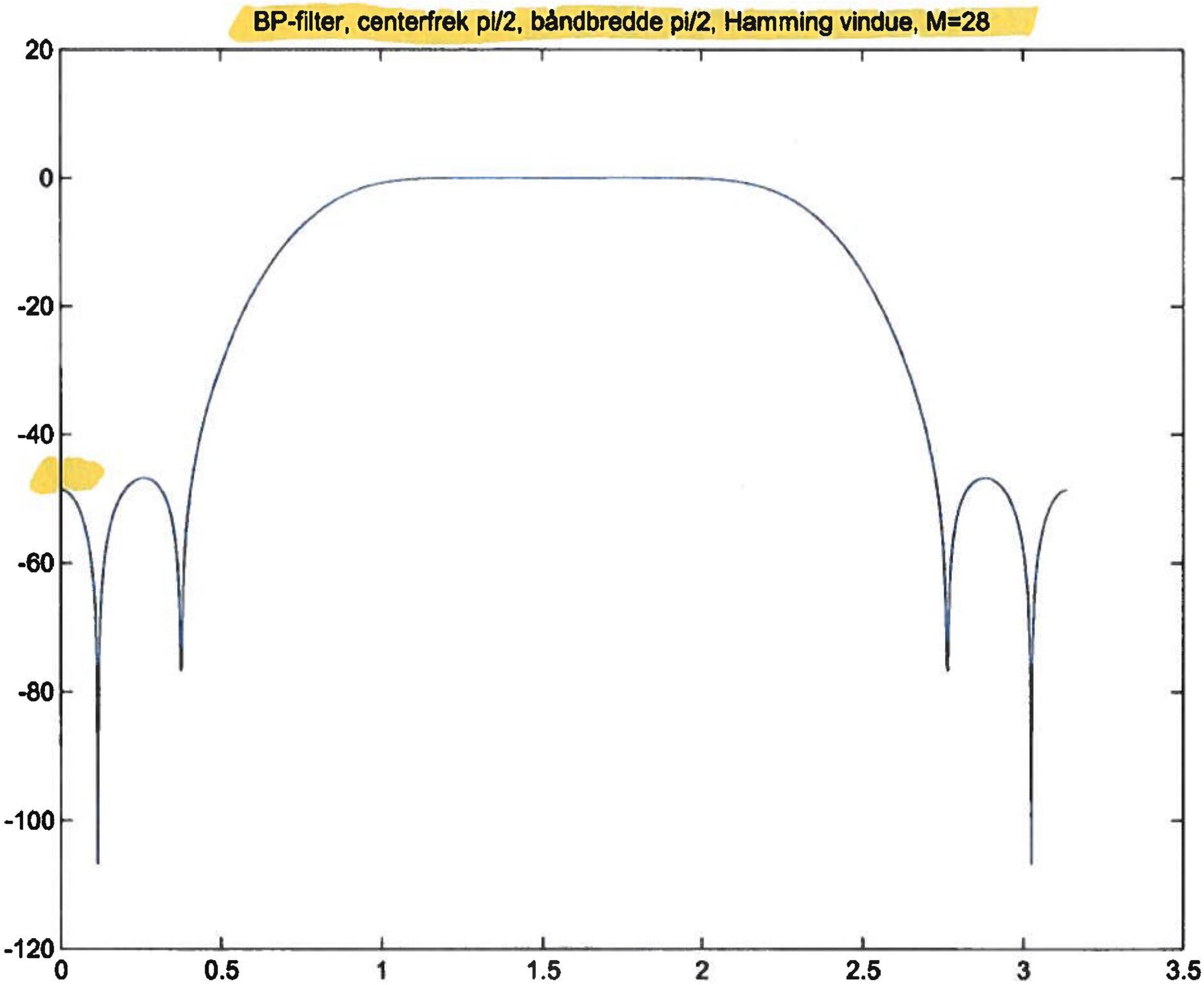
AGAIN, LET'S EXPERIMENT WITH M AND THE WINDOW FUNCTION.


BP-filter, centerfrek pi/2, båndbredde pi/2, Rekt. vindue, M=12
10
0
-10
-20
-30
-40
-50
-60
-70
0
0.5
1
1.5
2
2.5
3
3.5

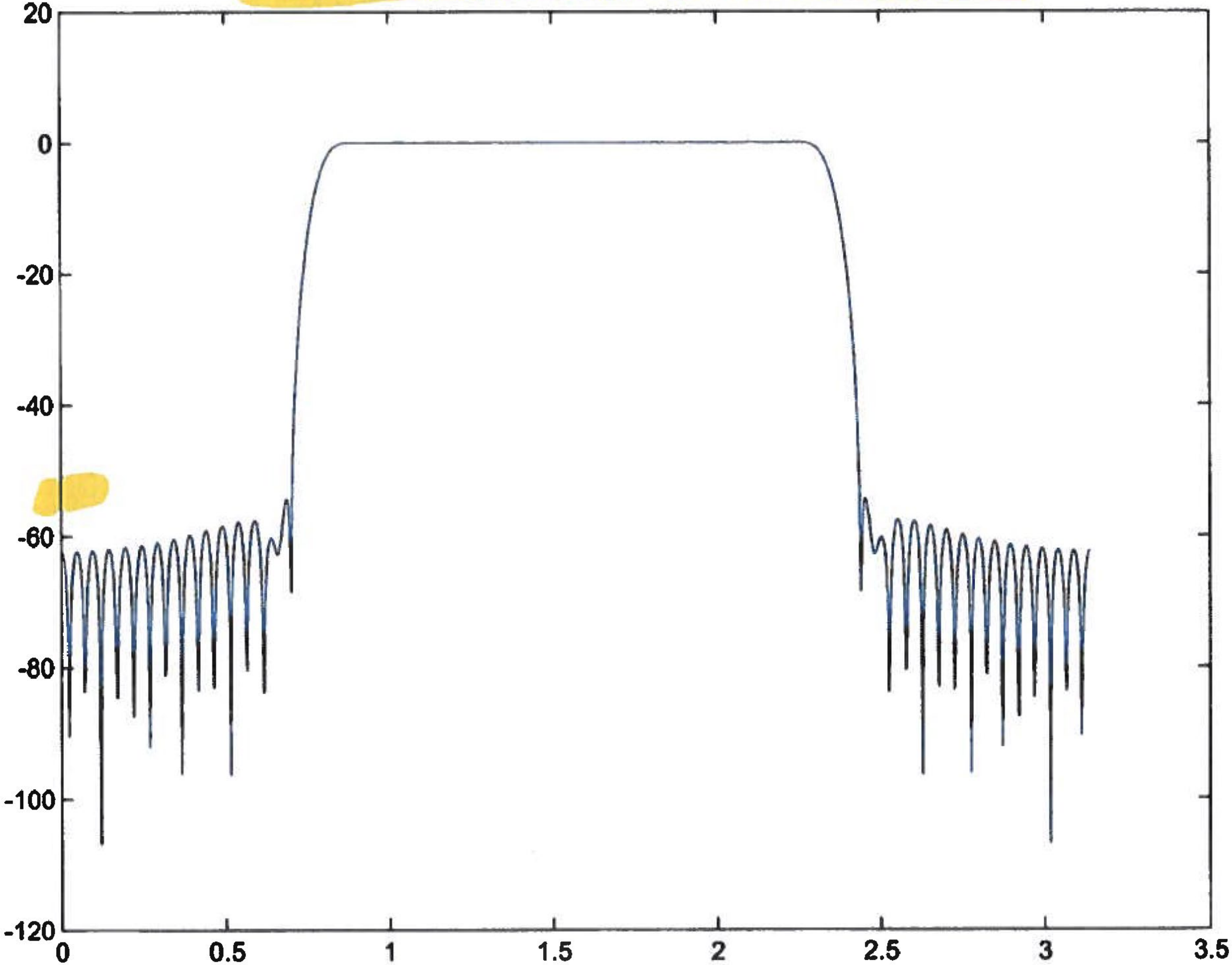


BP-filter, centerfrek pi/2, båndbredde pi/2, Rekt. vindue, M=28
10
0
-10
-20
-30
-40
-50
-60
-70
0
0.5
1
1.5
2
2.5
3
3.5


BP-filter, centerfrek pi/2, båndbredde pi/2, Hamming vindue, M=12
0
-5
-10
-15
-20
-25
-30
-35
0
0.5
1
1.5
2
2.5
3
3.5


BP-filter, centerfrek pi/2, båndbredde pi/2, Hamming vindue, M=28
20
0
-20
-40
-60
-80
-100
-120
0
0.5
1
1.5
2
2.5
3
3.5

BP-filter, centerfrek pi/2, båndbredde pi/2, Hamming vindue, M=128



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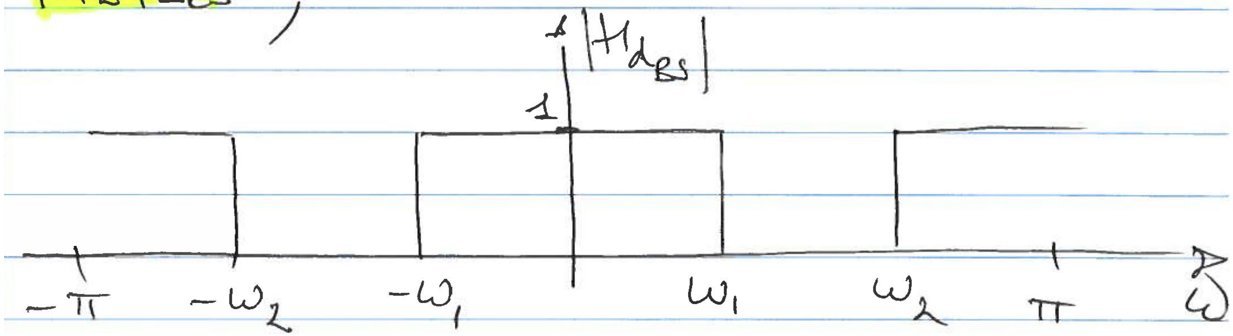
... AND, OF COURSE, SIMILARLY FOR BAND STOP FILTERS:



$$H_{d_{BS}}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & \begin{cases} -\pi \le \omega \le -\omega_2 \\ -\omega_1 \le \omega \le \omega_1 \\ \omega_2 \le \omega \le \pi \end{cases} \\ 0 & \text{OTHERWISE} \end{cases}$$

$\Downarrow$

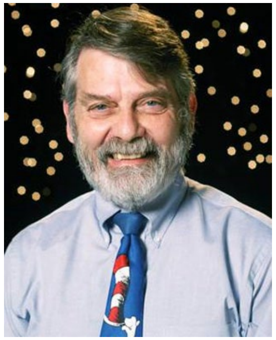
$$h_{d_{BS}}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{d_{BS}}(e^{j\omega}) \cdot e^{j\omega n} \, d\omega$$

etc ...

If you are curious and want to study more…

The most comprehensive study of Window Functions ever published is the work by Frederic J. Harris;

"On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform"
Proceedings of The IEEE, vol. 66, no. 1, January 1978, pp. 51-83.
Cited 10.365 times…(!), Oct. 18, 2024

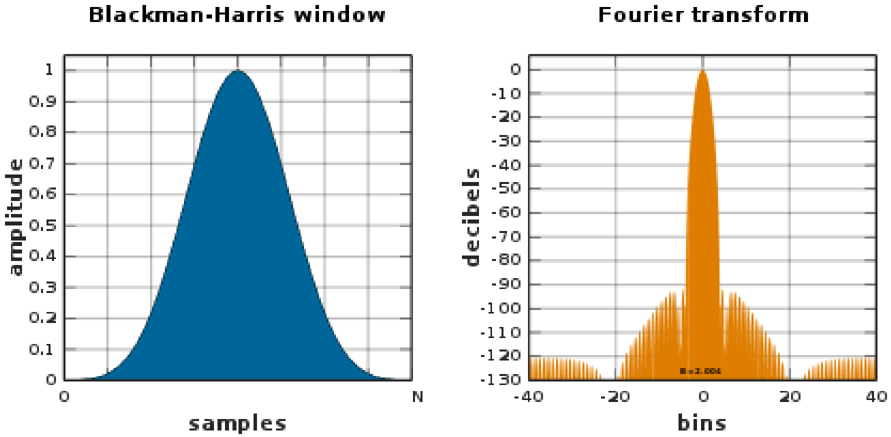


Harris achieved his PhD degree from Aalborg University, 2008

The Blackman–Harris window

$$w[n] = a_0 - a_1 \cos\left(\frac{2\pi n}{N}\right) + a_2 \cos\left(\frac{4\pi n}{N}\right) - a_3 \cos\left(\frac{6\pi n}{N}\right)$$

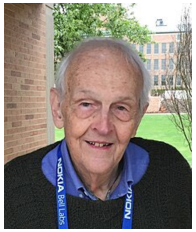
$$a_0 = 0.35875, a_2 = 0.48829, a_2 = 0.14128, a_3 = 0.01168$$



FIR Filter Design using the Kaiser Window

Designing an FIR filter by trial-and-error is not (necessarily) an efficient approach, so people have sought for an alternative one-step method…

Such a method was suggested by James F. Kaiser, USA (1929-2020)



His window uses a 0^{th} order modified Bessel function of the first kind (!)

The $M+1$ length window function is given as;

$$w[n] = \begin{cases} \dfrac{I_0\left\{\beta\left(1-\left[{}^{(n-\alpha)}/_{\alpha}\right]^2\right)^{1/2}\right\}}{I_0\{\beta\}}, & 0 \le n \le M, \alpha = {}^{M}/_{2}, \beta \text{ is a ”shape parameter”} \\ 0, & otherwise \end{cases}$$

where the Bessel function can be calculated as

$$I_0(x) = \sum_{k=0}^{\infty}\left[\frac{\left(\frac{x}{2}\right)^k}{k!}\right]^2$$

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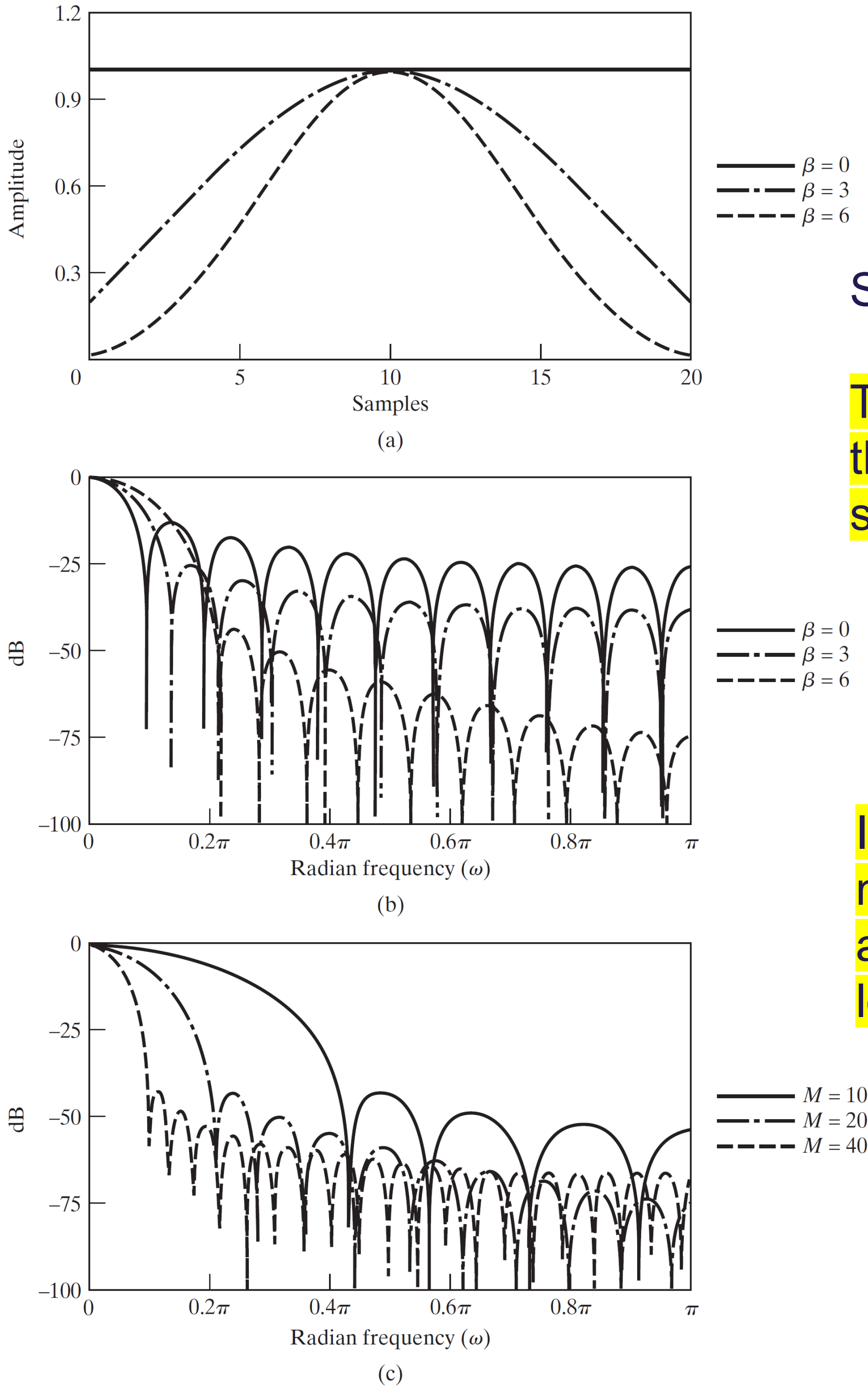
$M = 20$

$M = 20$

$\beta = 6$

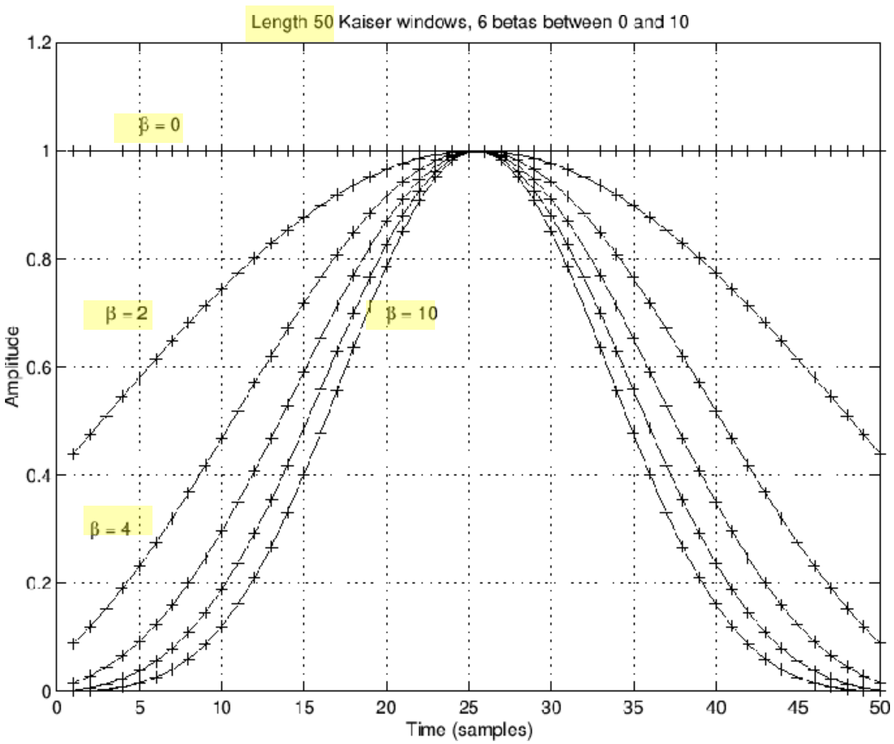


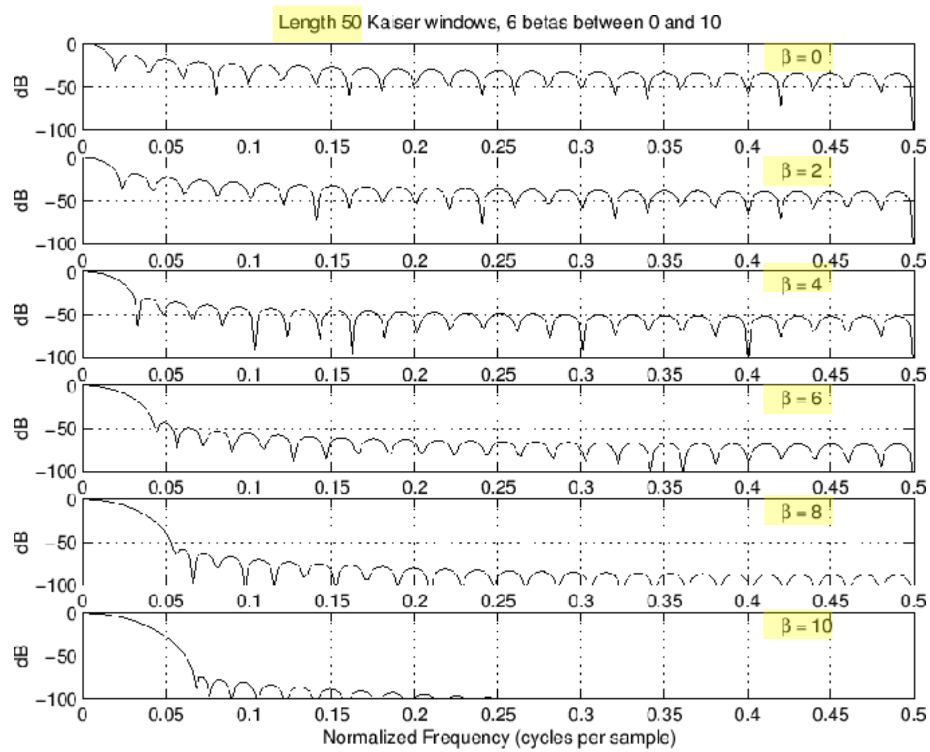
So, in conclusion (we see again);

The more $w[n]$ is tapered towards 0, the wider the main lope and the smaller the side lopes.

Increasing M for fixed β provides a more narrow main lope but does not affect the peak amplitude of the side lopes.

Here are some other plots which illustrate that larger β values give lower side lobe levels, but at the price of a wider main lobe, for maintained window length M.





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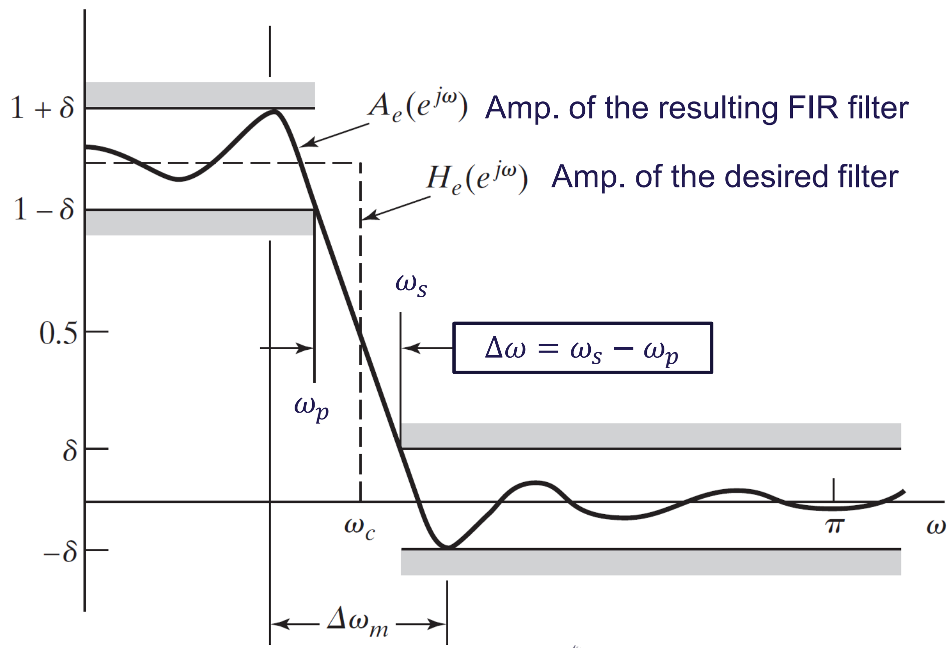
Kaiser has suggested (based on numerical experiments) a procedure which can used to predit the values of M and β needed to comply with given filter specifications.

The idea is that the ”peak approximation error” δ is related to the choice of β.

For a LP-filter with fixed δ

- the passband frequency ω_p is the highest frequency such that $|H(e^{j\omega})| \geq 1 - \delta$
- the stopband frequency ω_s is the lowest frequency such that $|H(e^{j\omega})| \leq \delta$
- these two frequencies define the width of the transition band, $\Delta\omega = \omega_s - \omega_p$



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The Kaiser FIR Filter Design Method

Determine the transition band width $\Delta\omega = \omega_s - \omega_p$

Based on the Peak Approximation Error δ, find $A = -20\log(\delta)$

Now, the value of β needed to achieve a specified value of A is given by;

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50, \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \leq A \leq 50, \\ 0.0, & A < 21. \end{cases}$$

where $A = 21$ ($\delta = 0.0891$) corresponds to the Rectangular Window (i.e., $\beta = 0$).

Finally, the window length, which is independent of the shape factor, can be calculated as

$$M = \frac{A - 8}{2.285 \cdot \Delta\omega}$$

With these formulas, there are (essentially) no need for trial-and-error, and thus the FIR filter can be designed in one iteration…