Exercise Sheet 4

Literature:

G.F. Franklin, J.D. Powell and A. Emami-Naeini: *Feedback Control of Dynamic Systems*, 6th edition, pp. 490-493, pp. 504-505, pp. 520-537.

Exercise 1 (Reduced order control)

In the previous exercise sheet, we considered the following controllable and observable system:

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{aligned} \tag{1}$$

The state feedback

$$u = Fx = \begin{pmatrix} -2 & 2 \end{pmatrix} x$$

was shown to achieve a pole placement corresponding to the char. polynomial $s^2 + 3s + 2 = (s+1)(s+2)$.

1. Compute $\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = T^{-1}AT$, $\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = T^{-1}B$, and $\begin{pmatrix} F_1 & F_2 \end{pmatrix} = FT$, where T is a transform matrix, $\det T \neq 0$, such that $CT = \begin{pmatrix} 1 & 0 \end{pmatrix}$. One possible choice for T is:

$$T = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

2. Design a reduced order observer for (1) having characteristic polynomial $s + 4$, i.e. find an observer gain L such that the eigenvalue of $A_{22} + LA_{12}$ becomes $\lambda = -4$.

3. Compute in Matlab™ the transfer function of the reduced order observer based controller with the above observer and feedback gains. The following state space formulae can be used:

$$K(s) = C_r \left(sI - A_r\right)^{-1} B_r + D_r$$

where

$$\begin{aligned} A_r &= A_{22} + LA_{12} + (B_2 + LB_1)F_2 \\ B_r &= A_{21} + LA_{11} + (B_2 + LB_1)F_1 - A_r L \\ C_r &= F_2 \\ D_r &= F_1 - F_2 L \end{aligned}$$

(it is not difficult to derive these formulae directly from the block diagram of a reduced order observer based controller).

4. Close the loop e.g. with the command `feedback`, and verify that the closed loop poles are as expected.

Exercise 2 (integral control)

Once again, we consider the system:

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 7 & -9 \\ 6 & -8 \end{pmatrix} x + \begin{pmatrix} 4 \\ 3 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 1 \end{pmatrix} x \end{aligned}$$

In the previous exercise sheet, we found the following observer gain:

$$L = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

which assigned poles of $A + LC$ in $\{-4, -5\}$.

1. Construct the extended state space matrices $A_e = \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix}$ and $B_e = \begin{pmatrix} B \\ 0 \end{pmatrix}$ for this system.
2. Find an extended state feedback F_e which assigns poles of $A_e + B_e F_e$ in $\{-1, -2, -3\}$. Hint, use the MATLAB™ command `Fe = -place(Ae,Be,-[1 2 3])` .
3. Extract the feedback gain F and the integral gain F_I from F_e.
4. Compute the closed loop system:

$$\begin{aligned} \dot{x}_{\text{cl}} &= A_{\text{cl}} x_{\text{cl}} + B_{\text{cl}} r \\ y &= C_{\text{cl}} x_{\text{cl}} + D_{\text{cl}} r \end{aligned}$$

   resulting from this controlller. The matrices can be computed as:

$$\begin{aligned} A_{\text{cl}} &= \begin{pmatrix} A & BF & BF_I \\ -LC & A + BF + LC & BF_I \\ C & 0 & 0 \end{pmatrix} & B_{\text{cl}} &= \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \\ C_{\text{cl}} &= \begin{pmatrix} C & 0 & 0 \end{pmatrix} & D_{\text{cl}} &= 0 \end{aligned}$$

5. Compute the unit step response, e.g. by the MATLAB™ commands `ss` and `step` as in `step(ss(Acl,Bcl,Ccl,Dcl))` and verify that the output converges to 1 as guaranteed by the integrator.