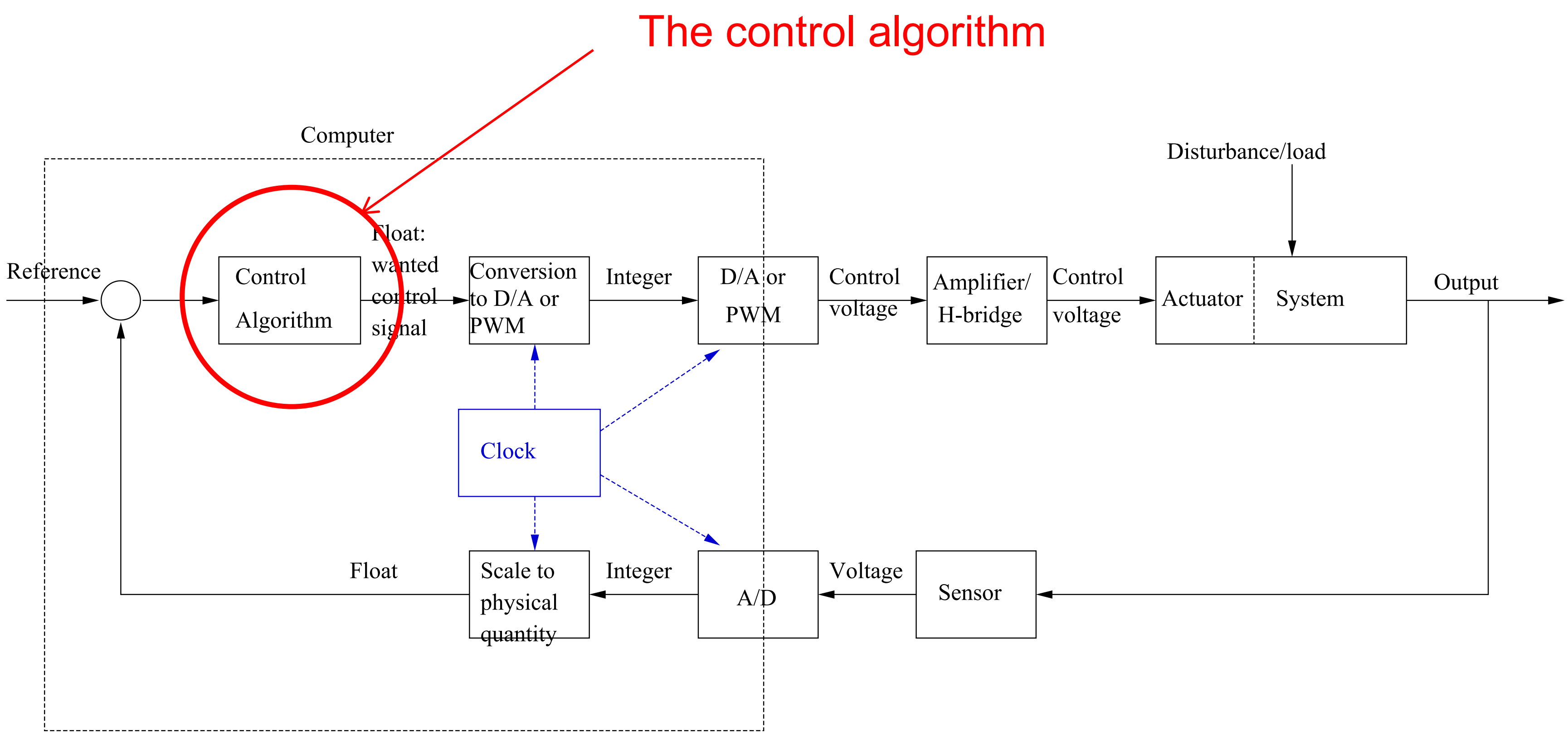
CONTROL

Frequency domain specifications

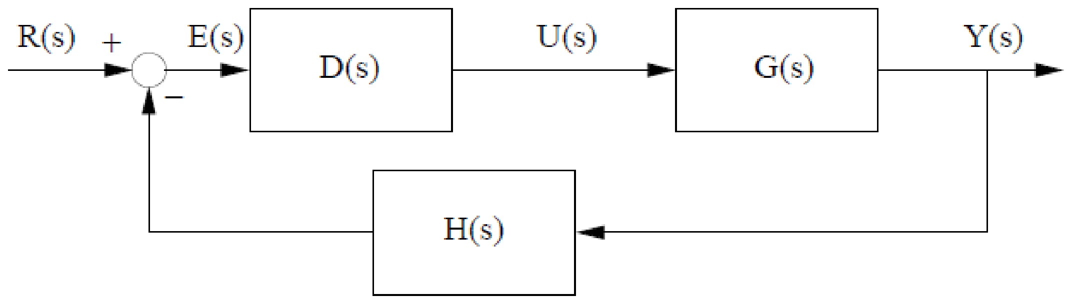
outline

- Repetition (time domain control)
- Bode plot design
- Control to follow a reference
  - stability
  - dynamics
  - bandwith
- Control for noise reduction

Standard control system



Standard set-up



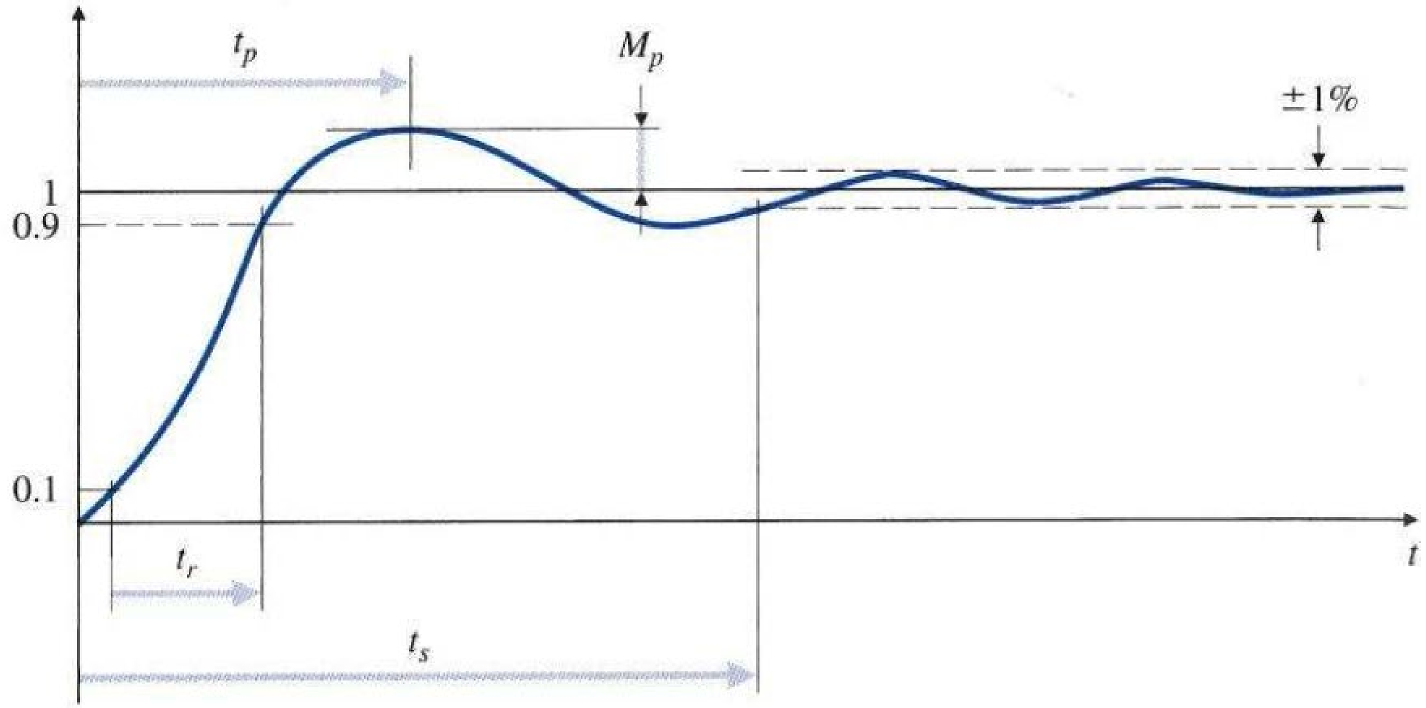
$$Closed \quad loop: \qquad T(s) = \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)}$$

$$Open \quad loop: \qquad L(s) = D(s)G(s)H(s)$$

$$Direct \quad term: \qquad D(s)G(s)$$

$$closed \quad loop = \frac{direct \quad term}{1 + open \quad loop}$$

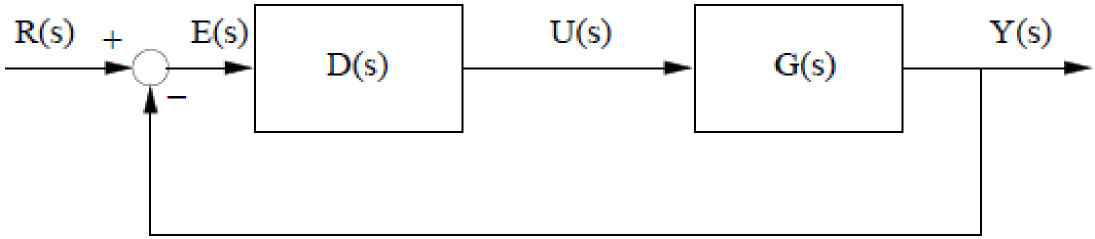
Time domain specifications (closed loop)



Rise time $t_r \sim$ stigetid
Settling time $t_s \sim$ indsvingningstid

Overshoot $M_p \sim$ oversving

Special case H(s)=1



System type=number of poles in 0 in $D(s)G(s)$.

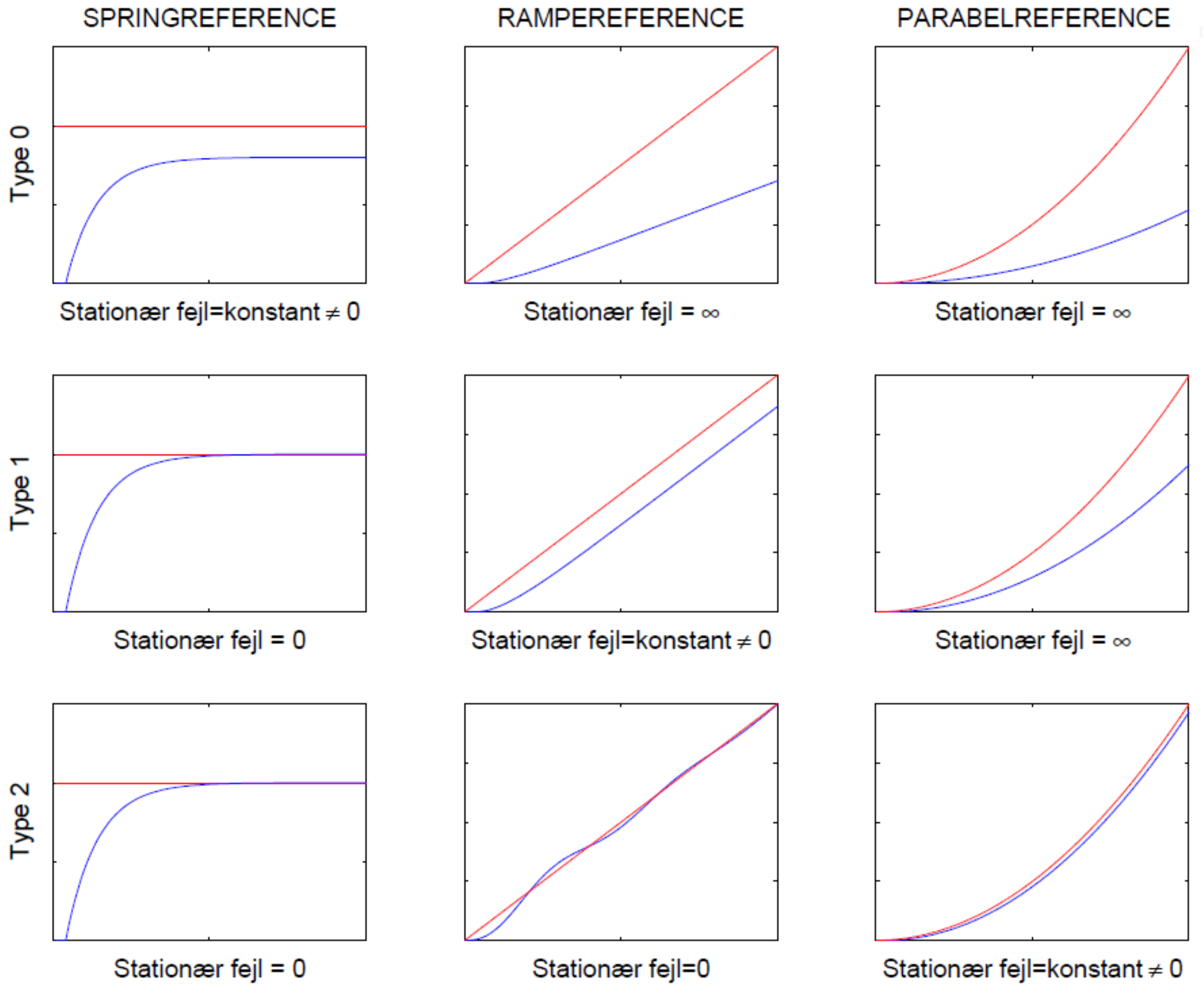
Example:

if $D(s)G(s) = \frac{10}{s(s+5)}$ one pole i 0 (and a pole in -5) => type 1

If $D(s)G(s) = \frac{10}{(s+10)(s+5)}$ poles in -10 and -5 system type = 0.

Notice that $D(s)G(s) = L(s)$ OPEN LOOP

Steady state errors system type

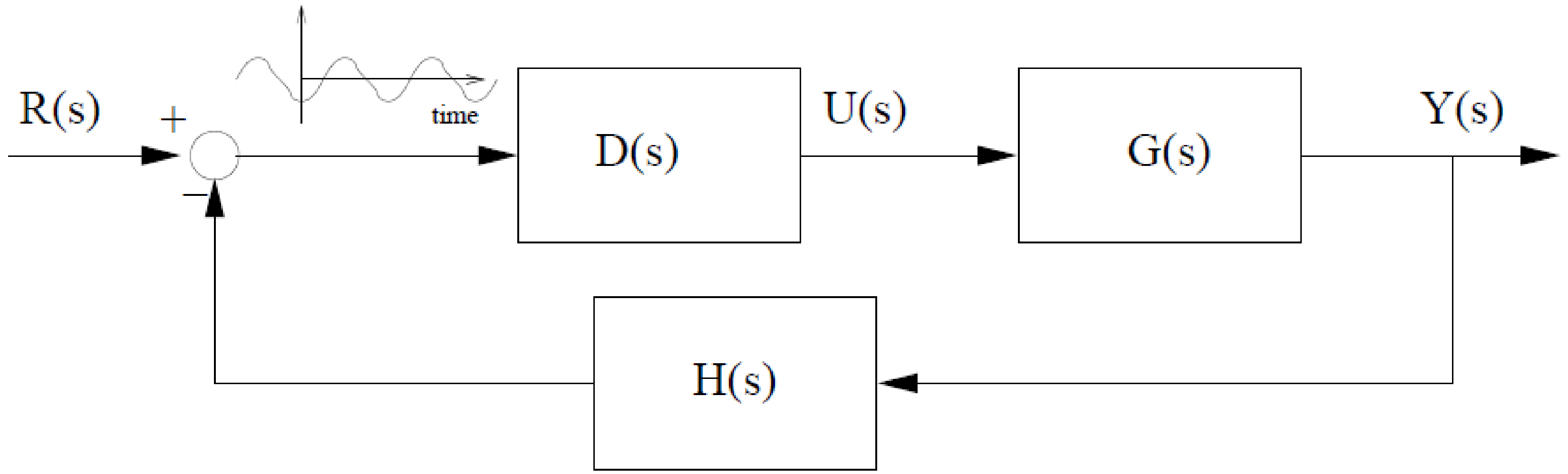


Frequency domain analysis

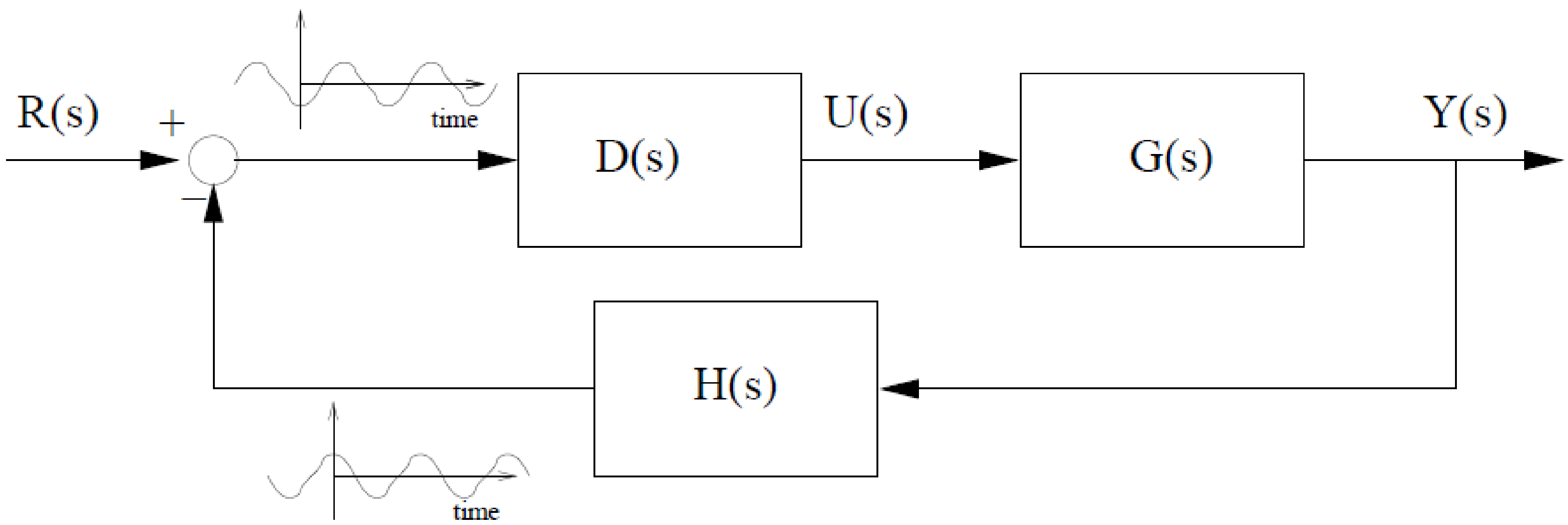
- Assumption: signals are a sum of sines an cosines
- Frequency domain response specify what happens to sine signals when they are passed through a linear system – changes in phase and magnitude

A change in phase and amplitude can be illustrated in a Bode plot

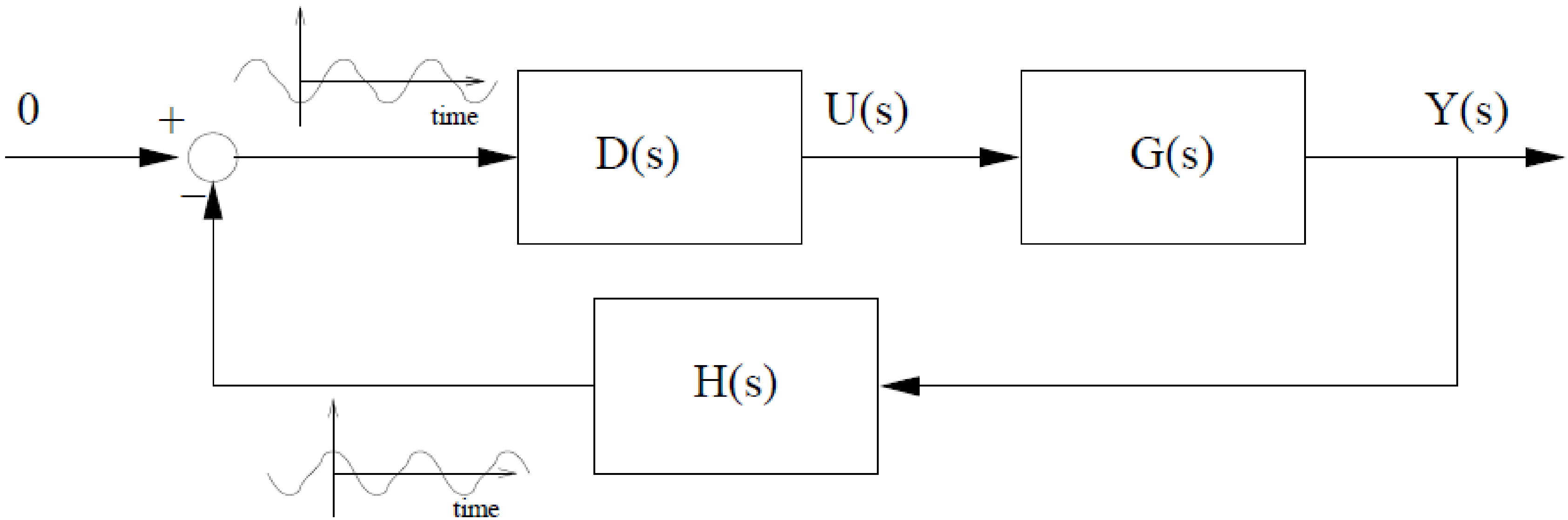
Frequency domain specifications



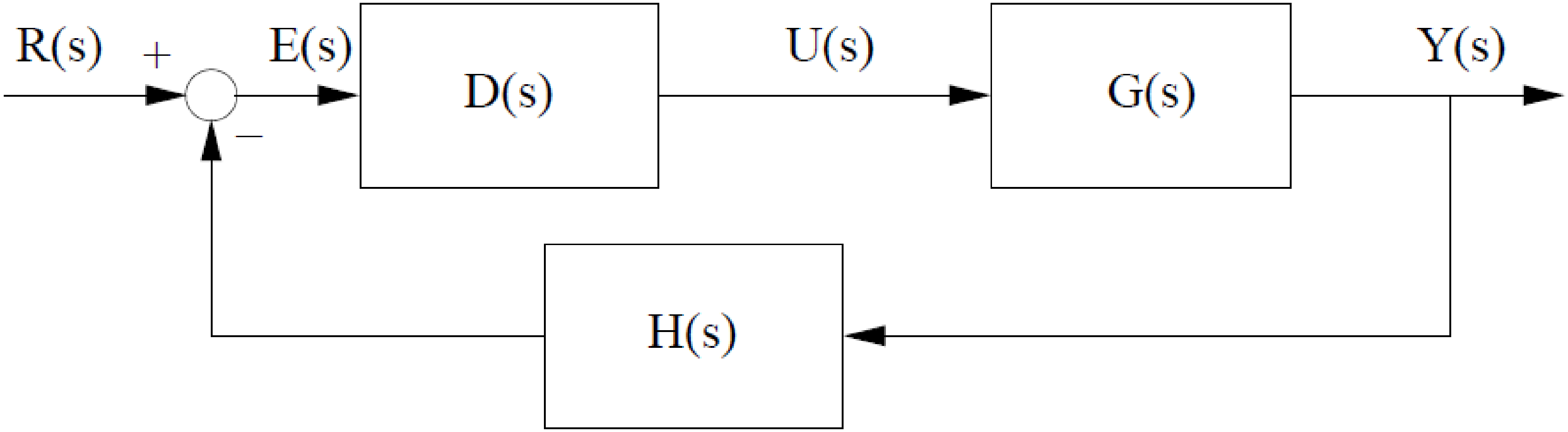
Frequency domain specifications



Frequency domain specifications



Frequency domain specifications

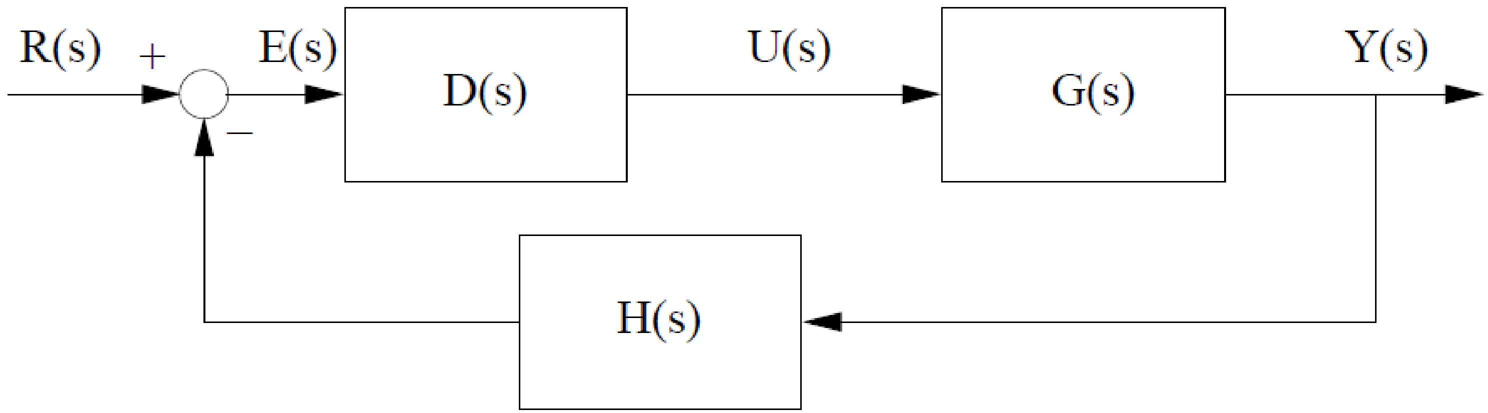


Investigate the open loop D(s)G(s)H(s)

The controller react on the error signal E(s)

Frequency domain specifications

Open loop



Oscillating:

$$D(s)G(s)H(s) = -1 = \ 1 \ \angle -180$$

Unstable:

$$D(s)G(s)H(s) = \ > 1 \ \angle -180$$

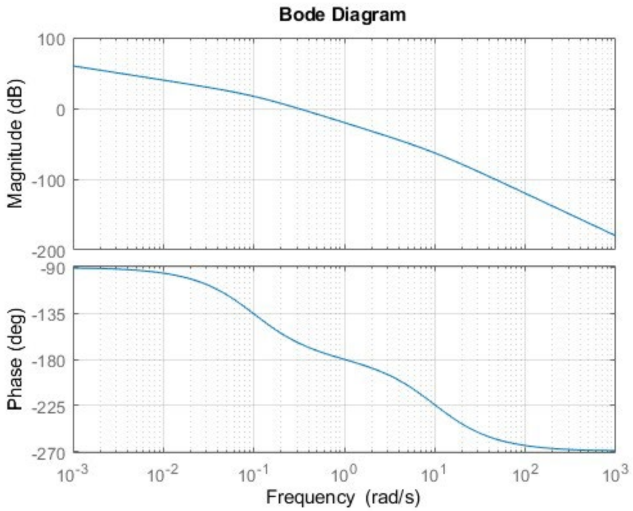
If there is a frequency ω_1 where the phase $\angle D(\omega_1)G(\omega_1)H(\omega_1)$ is -180 then the gain $|D(\omega_1)G(\omega_1)H(\omega_1)|$ must be smaller than 1 (0 dB) for stability

Frequency domain specifications

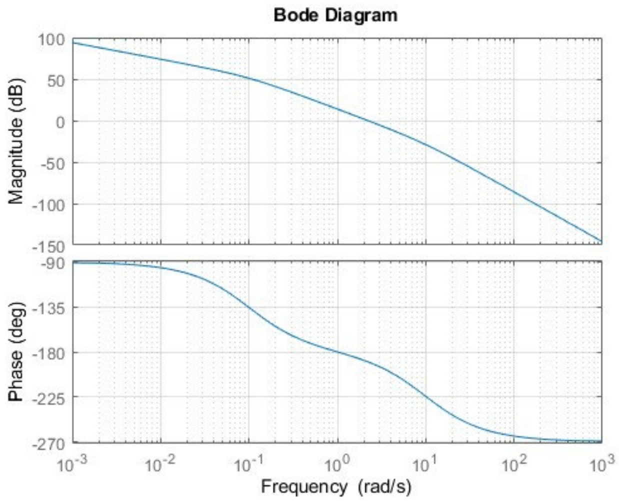
Bode plot og open loop system

$$OL_1(s) = \frac{1}{s(s+0.1)(s+10)}$$

$$OL_2(s) = \frac{50}{s(s+0.1)(s+10)}$$

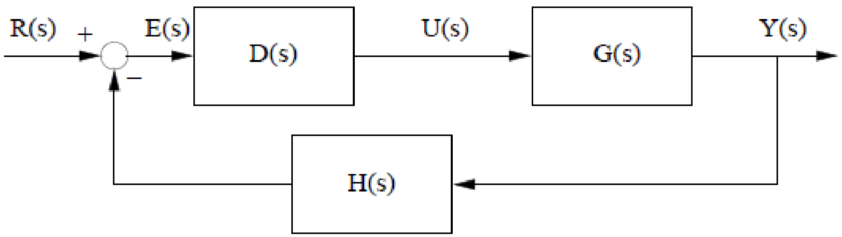


Stable system



Unstable system

Frequency domain specifications



Oscillating:

$$D(s)G(s)H(s) = -1 = \ 1 \ \angle - 180$$

Stability margins:

Gain margin (GM) is the number of dB missing before the gain is 0dB at the frequency where the phase is -180 degrees. Gain and phase is calculated on the open loop function!

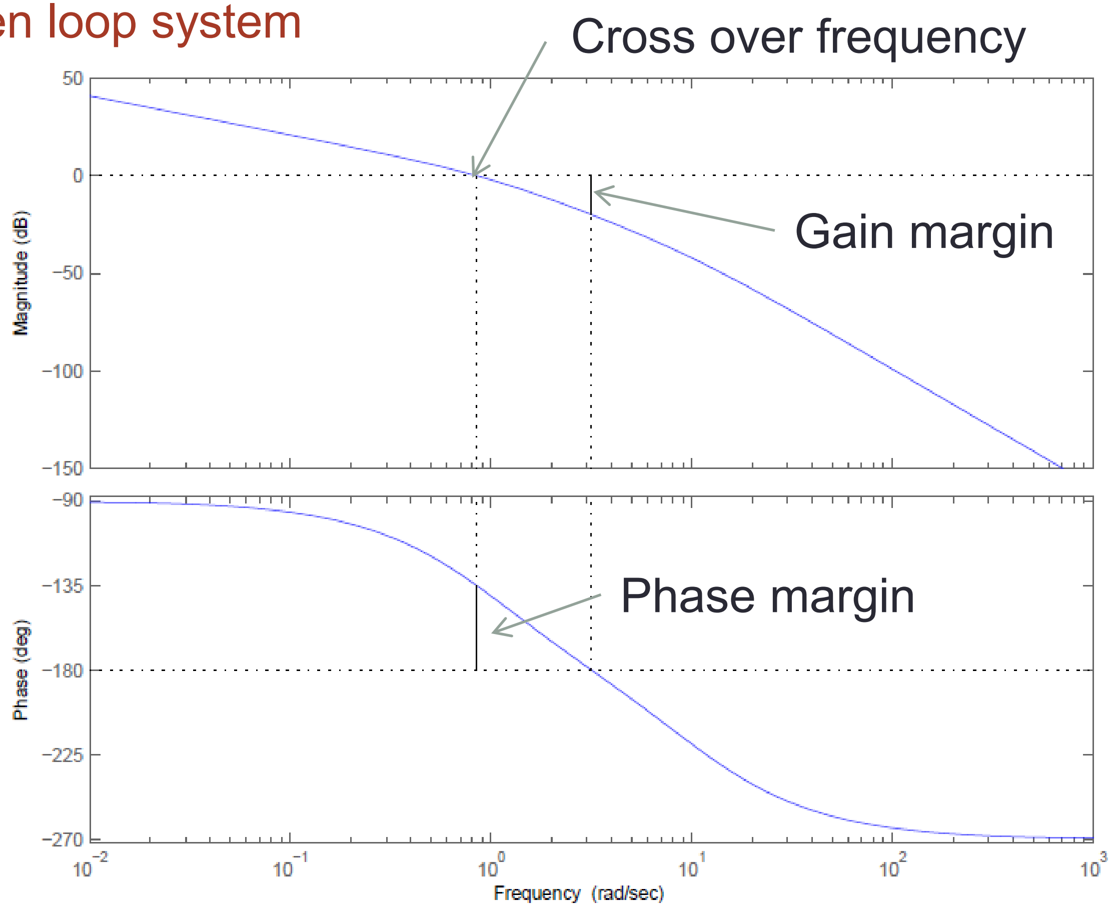
Phase margin, (PM) is the number of degrees missing before the phase is -180 at the frequency where the gain is 0 dB. Gain and phase is calculated on the open loop function!

Cross over frequency ω_c is the frequency where the gain is 0 dB.

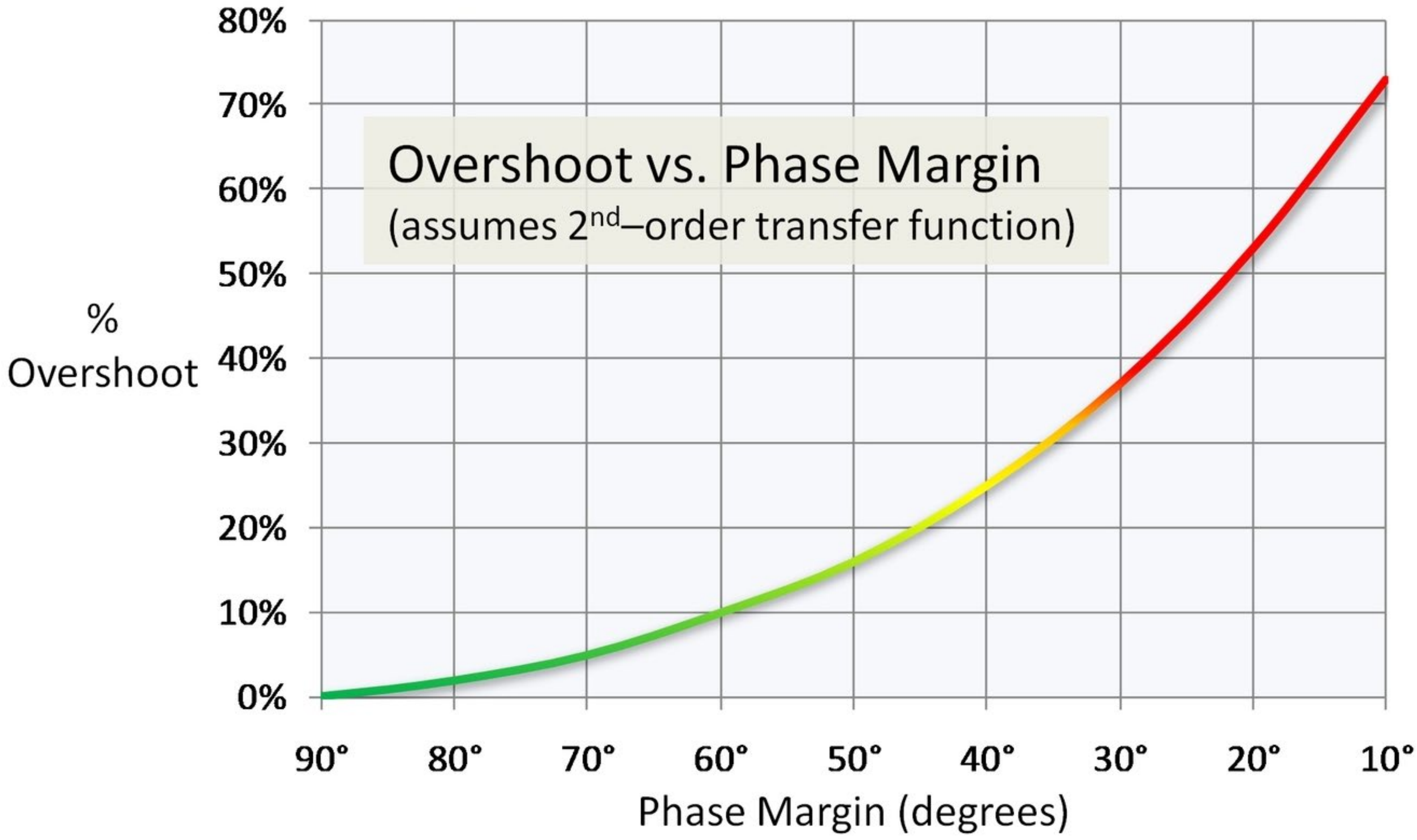
−p

Frequency domain specifications

Bode plot og open loop system



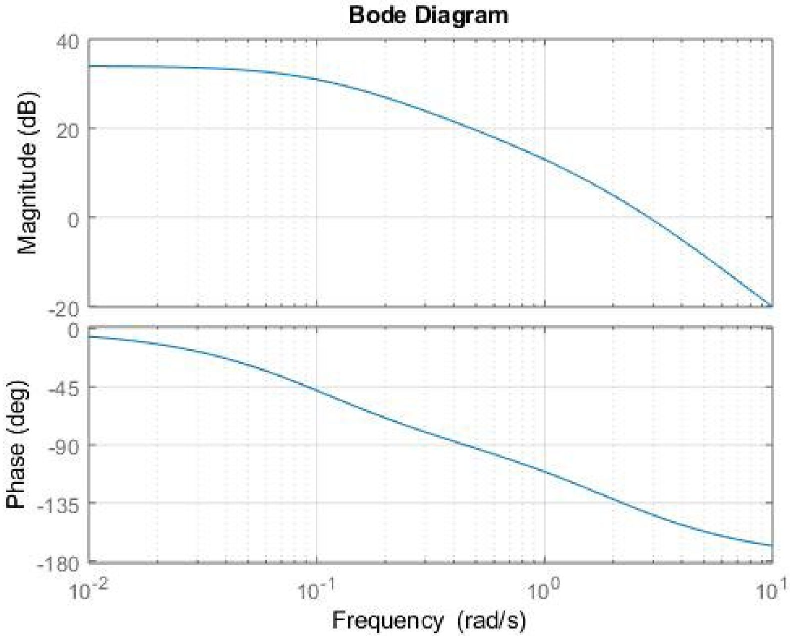
Relations between margins and dynamics



Find the phase and gain margin

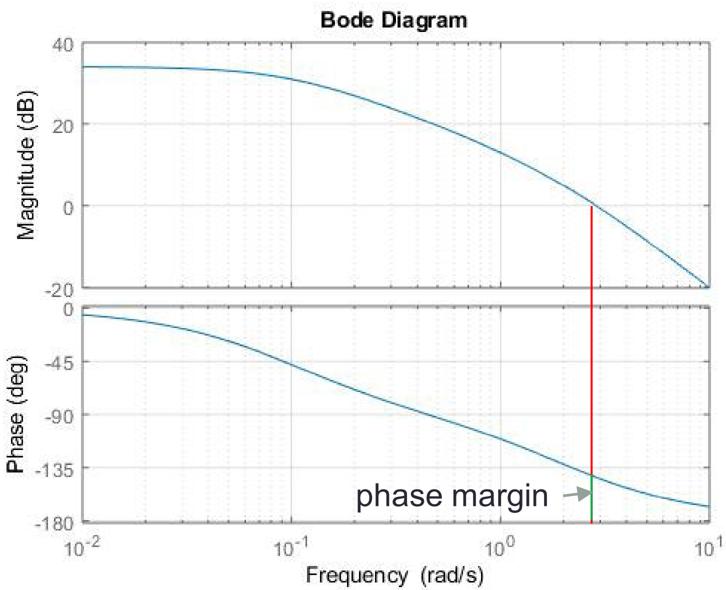
Is the systems stable

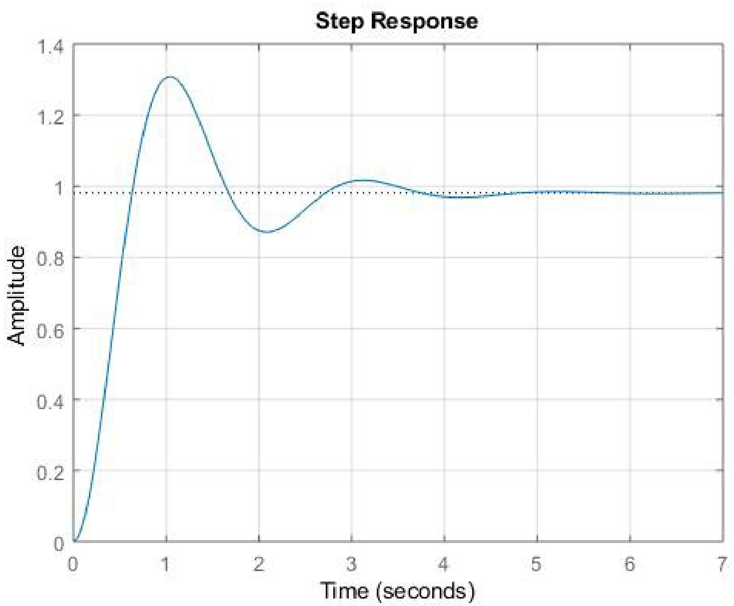
What is the overshoot



Phase margin and overshoot

A large phase margin => a small overshoot





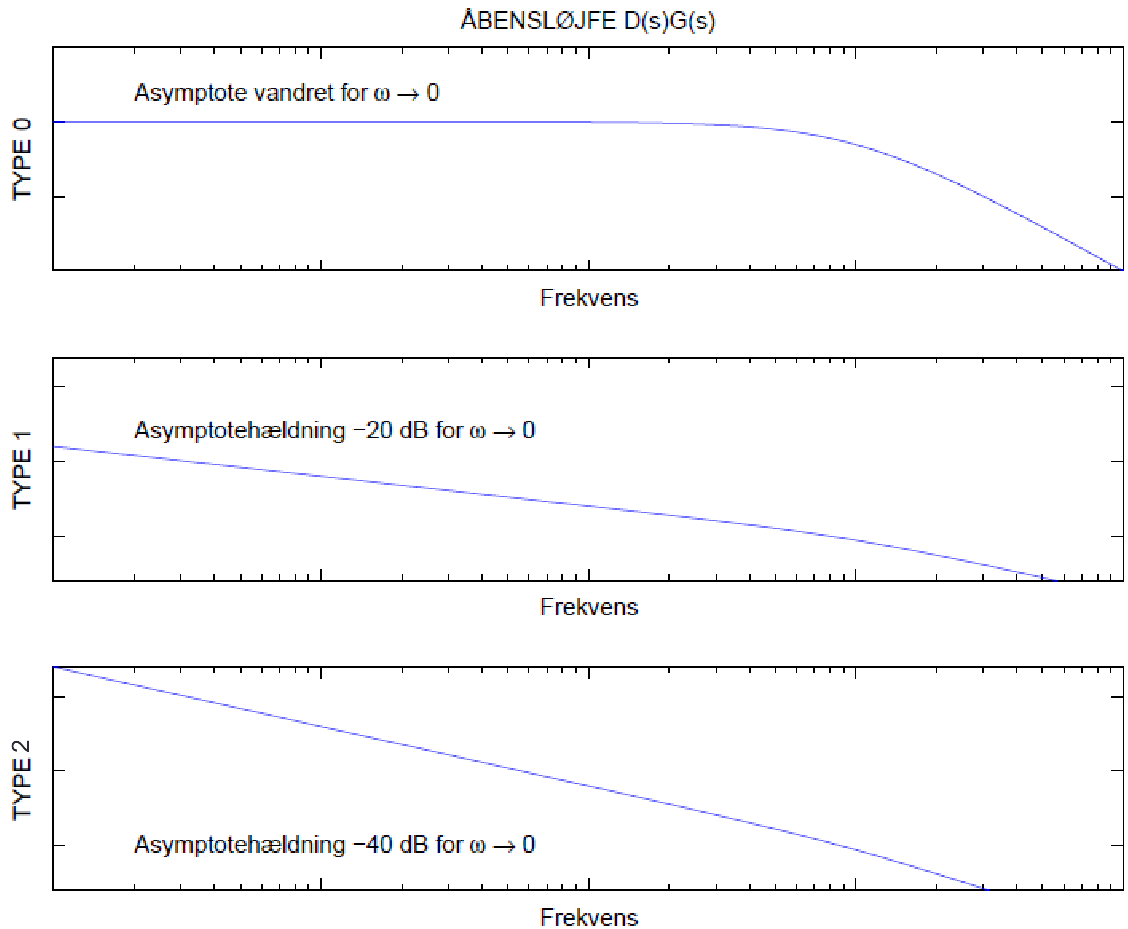
Special case : H(s)=1
System type   bode plot

Open Loop

No poles in zero

One pole in zero

Two poles in zero



Controller specifications

Specifications:

Time domaine spec's	*Frequency domaine spec's*		*Type of spec.*
Closed loop:	Closed loop:	Open loop:	
Overshoot M_p	Resonant peak M_r	Phase margin PM	Stability
Rise time t_r	Bandwidth ω_{BW}	Crossover frequency ω_c	Dynamics
Settling time t_s			Dynamics
Peak time t_p	Resonant frequency ω_r		Dynamics
		Gain margin GM	Stability
Steady state error e_{ss}	Steady state error e_{ss}	Asymptote $\omega \rightarrow 0$	Steady state

Controller specifications

For the standard 2. order cloosed loop system $T(s) = \frac{A}{s^2+2\zeta\omega_n s+\omega_n^2}$

Specifications	*Equations*
Overshoot,Resonant peak,Phase margin	$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\quad M_r \approx \frac{1}{2sin\frac{PM}{2}}$ $\quad \zeta \approx \frac{PM}{100}$
Rise time,Bandwidth, Cross over frekvens	$t_r \approx \frac{1.8}{\omega_n}$ $\quad \omega_{BW} \approx 1.4 \cdot \omega_n$ $\quad \omega_c \approx 0.5 \cdot \omega_{BW}$
Settling time	$t_s = \frac{-ln(x)}{\zeta\omega_n}$ $\quad x = baand$
Peak time, Resonant frequency	$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad \omega_r = \omega_n\sqrt{1-2\zeta^2}$

Controller design

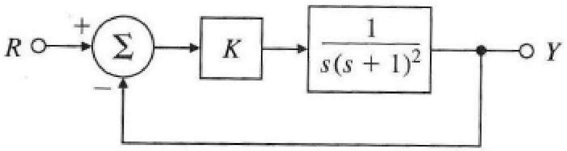
How can we change D(s) if we want to obtain

- better stability
- better stationary conditions
- faster dynamics

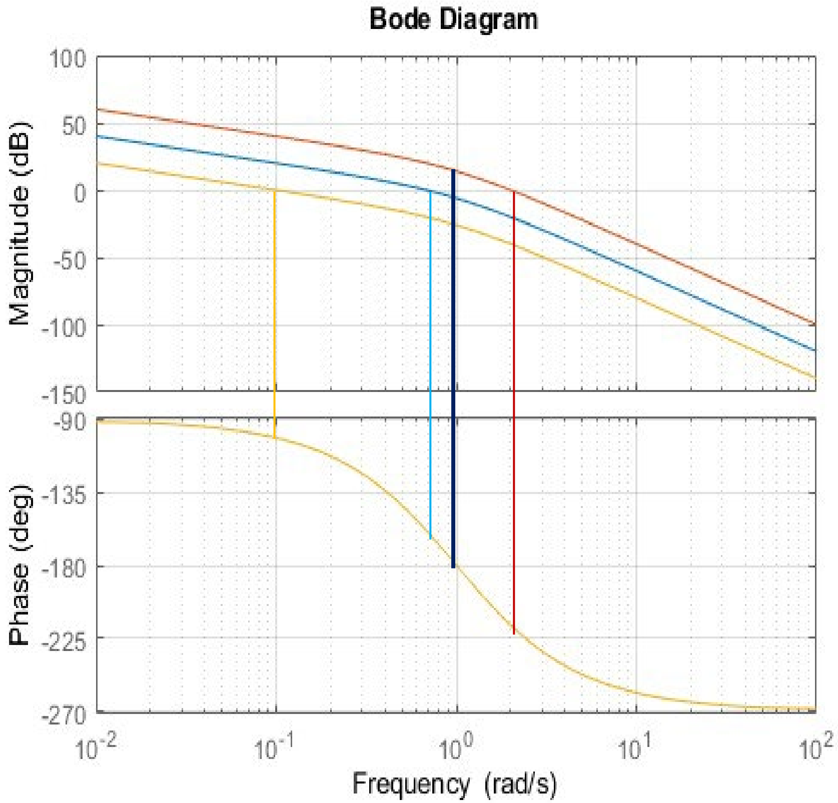
Frequency domain specifications

Most simple controller : a constant K
K in relation to stability

Example



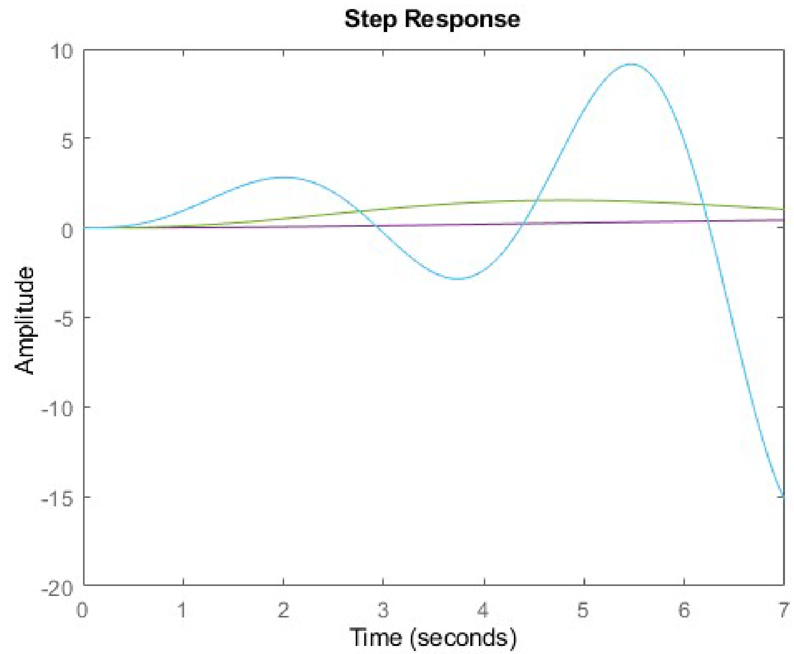
K = 0.1, 1, 10

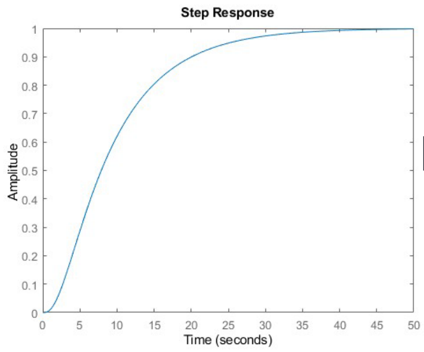


OPEN LOOP

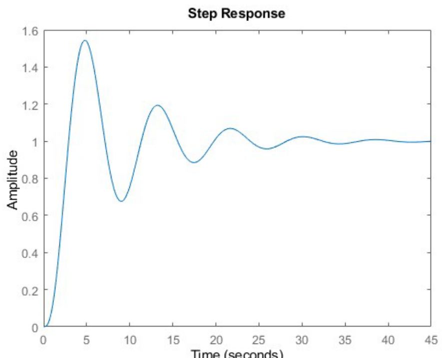
- Gain margin
- Phase margin
- Cross over frequency

Closed loop stepresponse for K=0.1, K=1, K=10

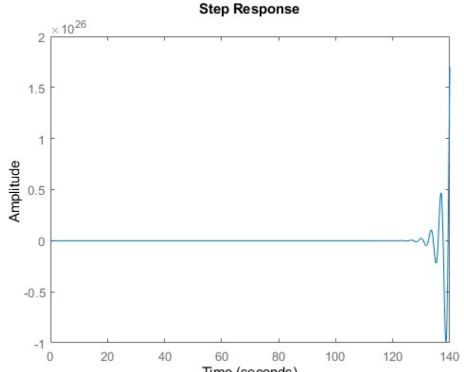




K=0.1



K=1

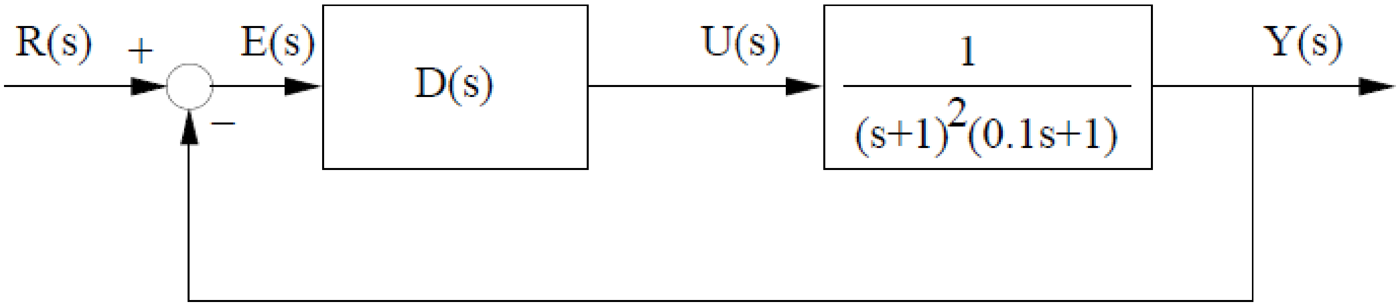


K=10

BREAK

Frequency domain design (on black board)

Design example

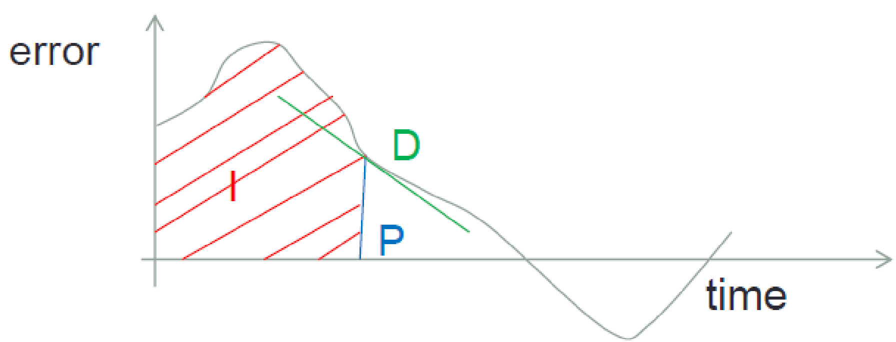


Phase margin 45^o

PID controller

A standard controller is:

$$
\begin{aligned}
D(s) &= K_p(1 + \frac{1}{T_i \cdot s} + T_d \cdot s) \\
&= Proportional(1 + Integral + Differential) \\
&= PID - controller
\end{aligned}
$$



Used in combinations

P control K

I control K/s

PI control $K(T_i s+1)/T_d s$

PD control $K(T_d s+1)$

PID control $K(T_i T_d s^2+T_i s+1)/T_i s$

Bode plot – effect of gain, poles and zeroes

- Poles :
- Gain: – 20 dB/decade
- Phase: - 90°

- Zeros
- Gain: 20 dB/decade
- Phase 90°

- Gain:
- $20\log_{10}$
- Phase 0 for positive gain, -180 for negative gain

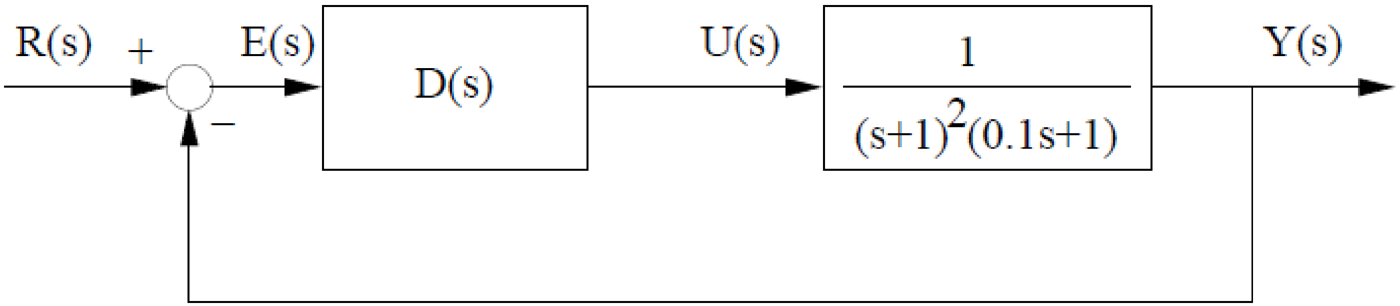
Used in combinations

P control	K
I control	K/s
PI control	$K(T_i s+1)/T_d s$
PD control	$K(T_d s+1)$
PID control	$K(T_i T_d s^2+T_i s+1)/T_i s$

Bodeplot for the controllers on the blackboard

Frequency domain design

Design example



Specifications:

- Steady state error for step = 0
- Phase margin ≅ 45 degrees
- Cross over frequency > 0.5 rad/sek

Determine $D(s)$

Frequency domain design

Open loop with out controller

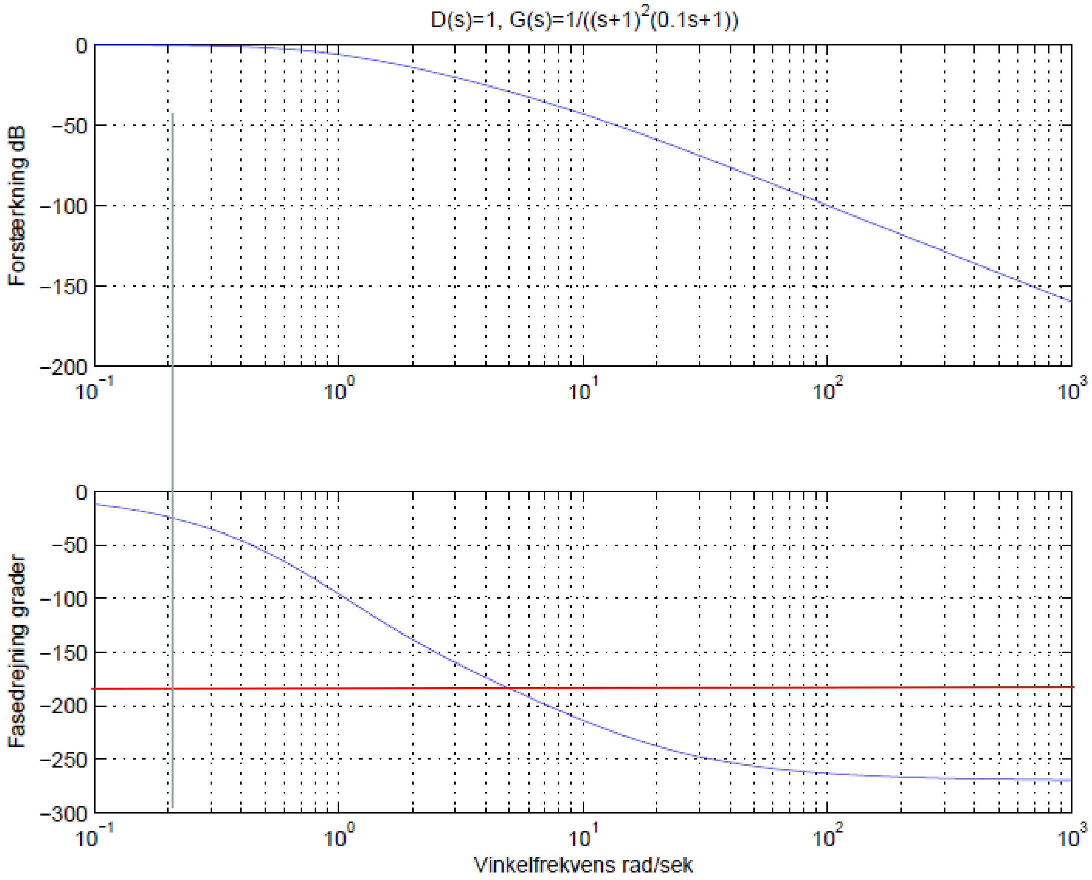
G(s) = $\frac{1}{(S+1)^2(0.1s+1)}$

Gain < 1 for all ω means that the phase margin are undefined,
The gain margin are ok

Type 0 system=> there will be a stationary error

P control (P>1) will give a phase and gain margin + a cross over frequency

The stationary error will go down



$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{(s+1)^2(0.1s+1)}}{1+\frac{K}{(s+1)^2(0.1s+1)}} = \frac{K}{(s+1)^2(0.1s+1)+K} \rightarrow \frac{K}{1+K}\ for\ \omega \rightarrow 0$$

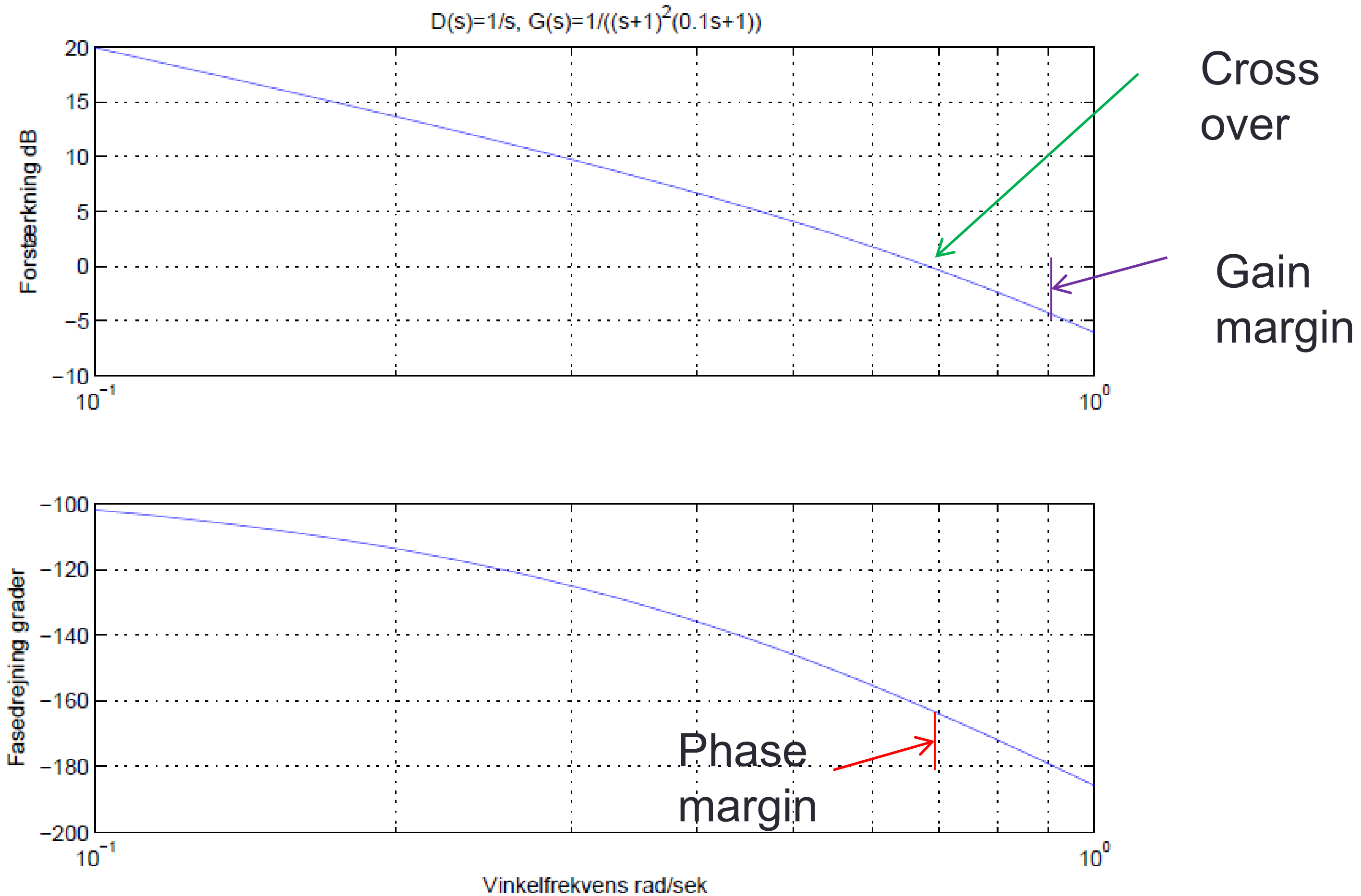
Frequency domain design

Open loop

$$D(s) = \frac{1}{s}$$

Steady state error =0

Phase margin =18



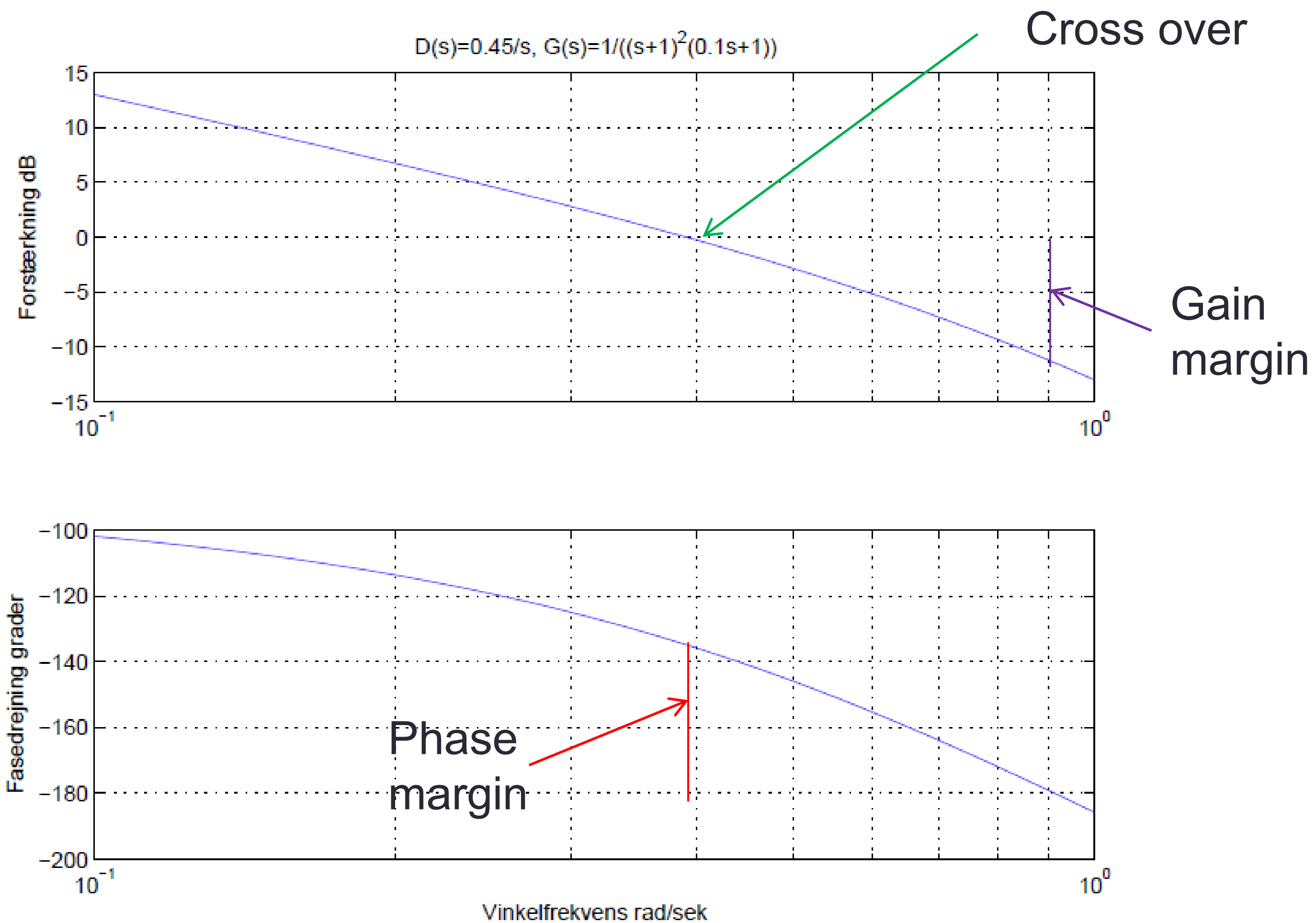
Frequency domain design

Open loop

$$D(s) = \frac{0.45}{s}$$

Cross over= 0.4

Phase margin = 45



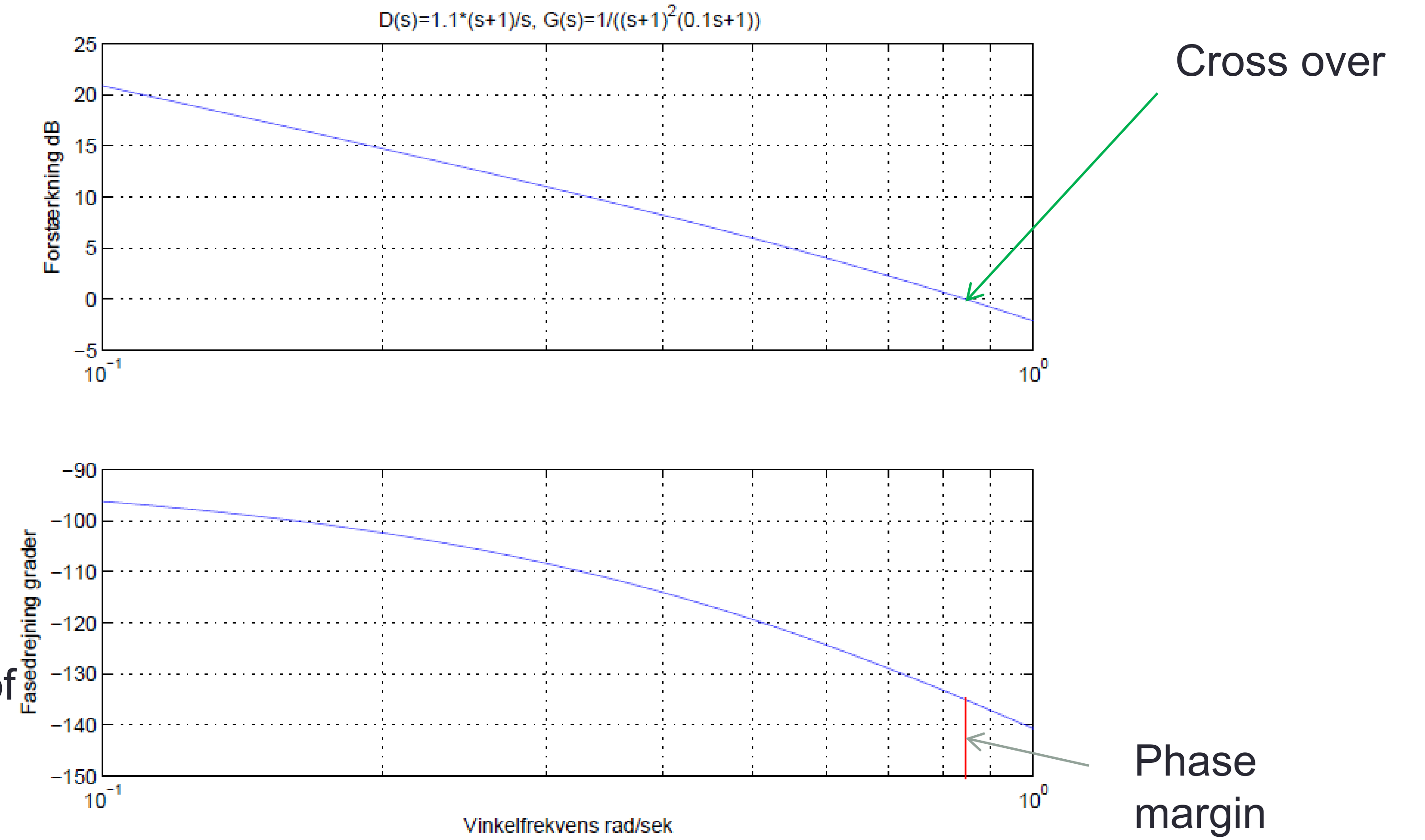
Frequency domain design

We insert a zero and adjust the gain

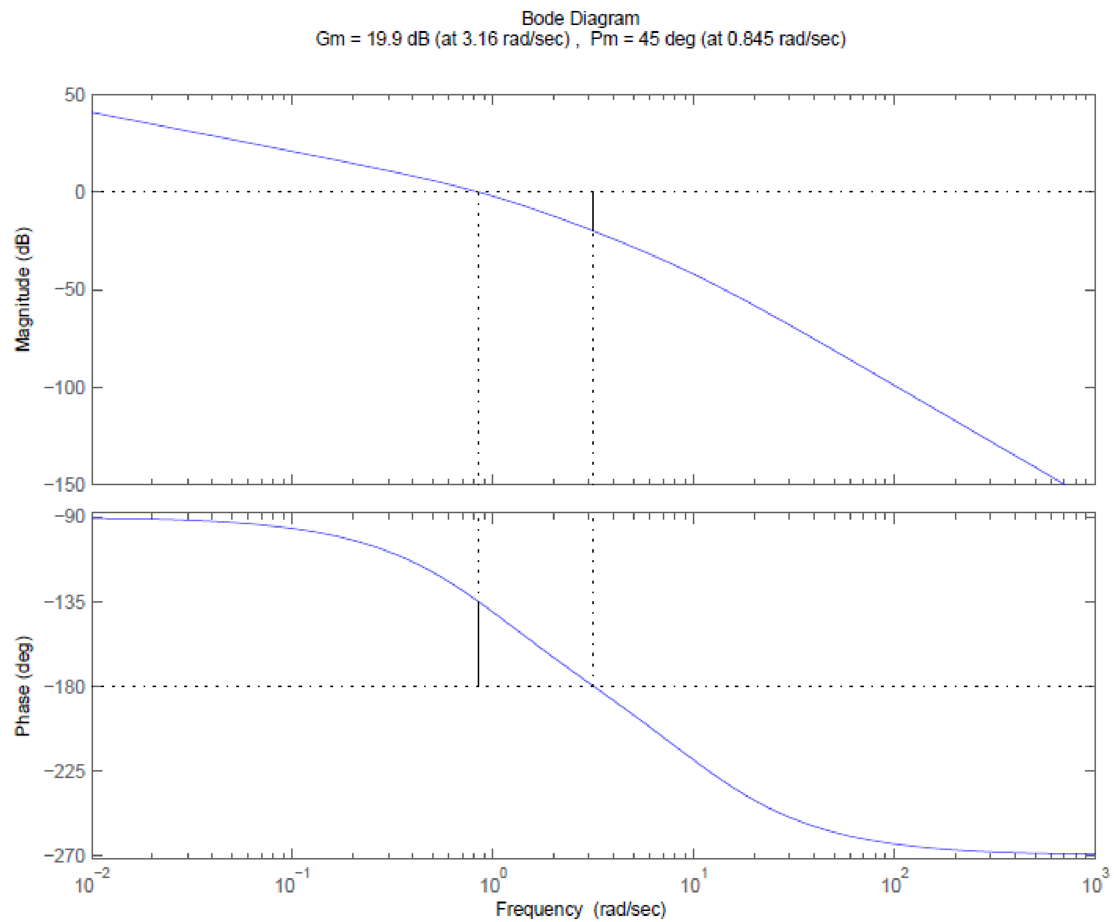
Open loop

$$D(s) = \frac{1.1(s+1)}{s}$$

The zero cancels one of the poles



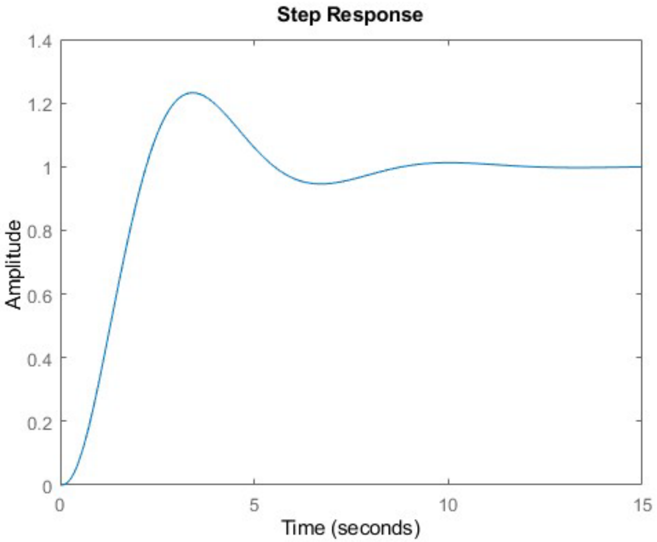
Frequency domain design



Controller type

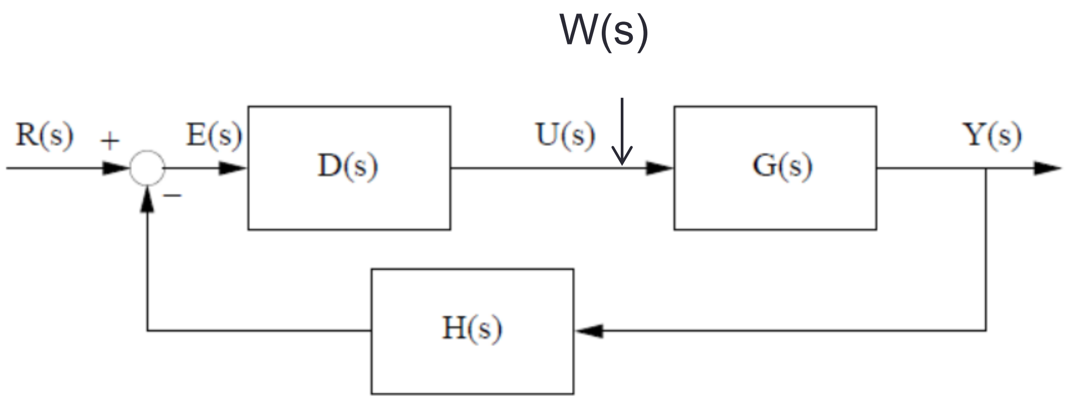
Controller

$$
\begin{aligned}
D(s) &= 1.11\frac{s+1}{s} = 1.11(1+\frac{1}{s}) \\
&= Proportional(1 + Integral) = PI - controller
\end{aligned}
$$



No stationary error

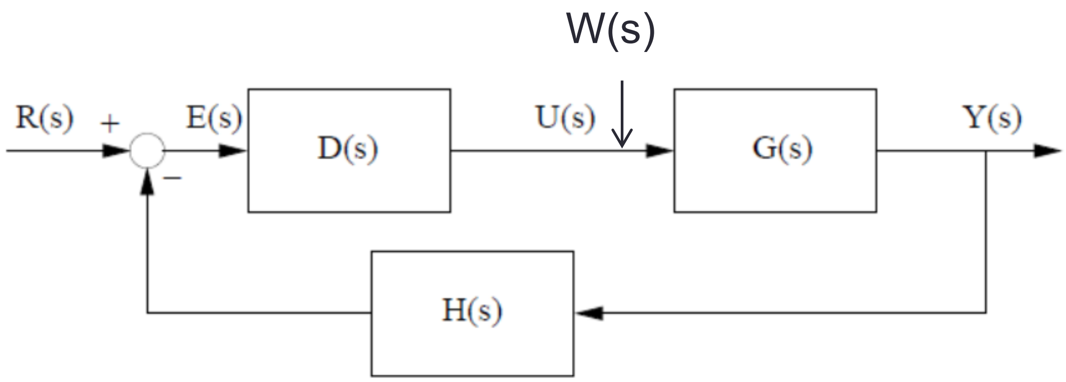
Noise reduction

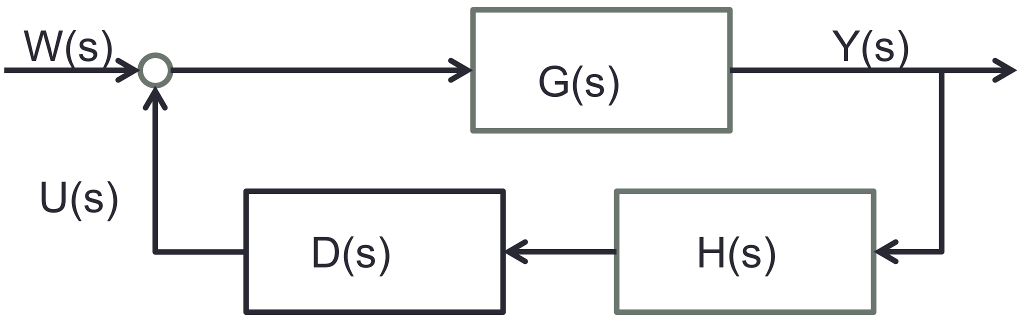


Calculate the transfer function:

$$\frac{Y(s)}{W(s)}$$

Noise reduction



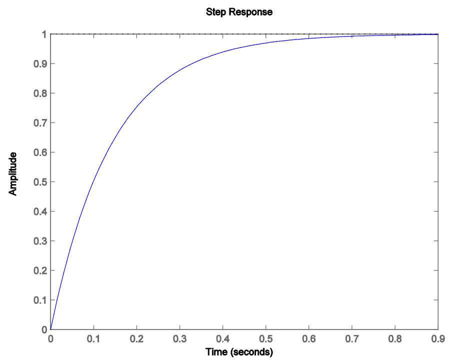


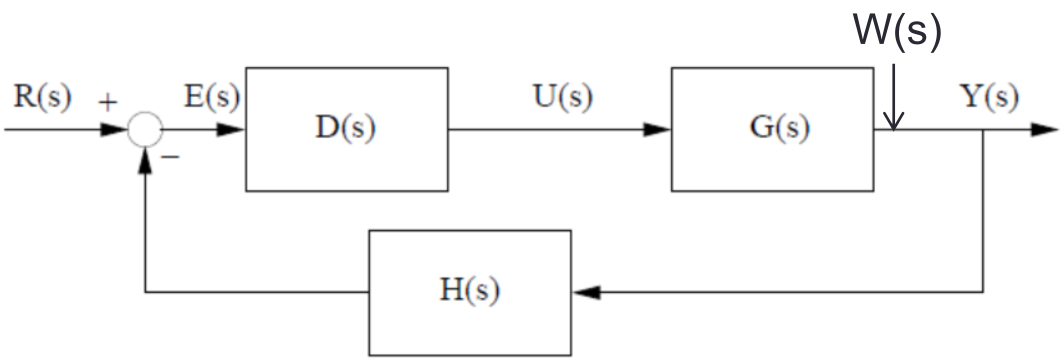
$$\frac{Y(s)}{W(s)} = \frac{G(s)}{1 + G(s)H(s)D(s)}$$

Reference following

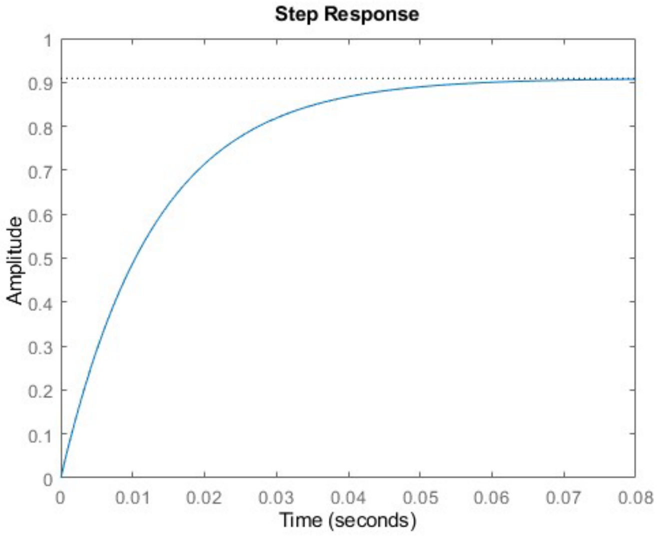
$$G(s) = \frac{7}{s+7}$$

$$K = 10, \qquad H = 1$$





Unitstep for R(s)



$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + G(s)H(s)D(s)}$$

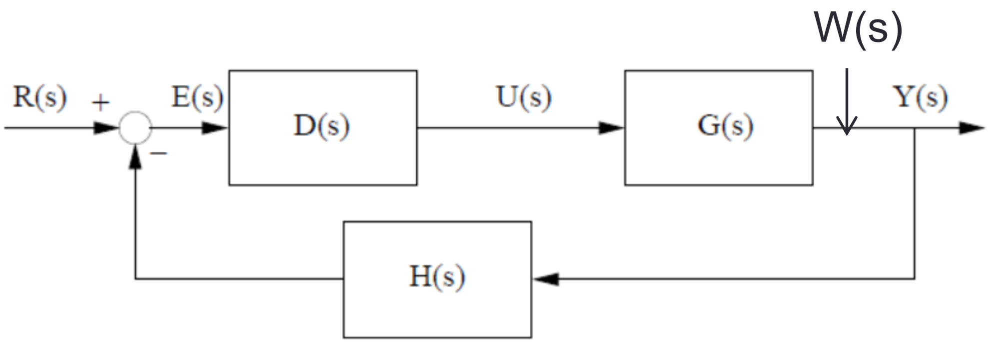
$$= \frac{10 * \frac{7}{s+7}}{1 + \frac{7}{s+7} * 10 * 1} = \frac{70}{s+77}$$

DC gain (s=0) = $\frac{10}{11}$

Noise reduction

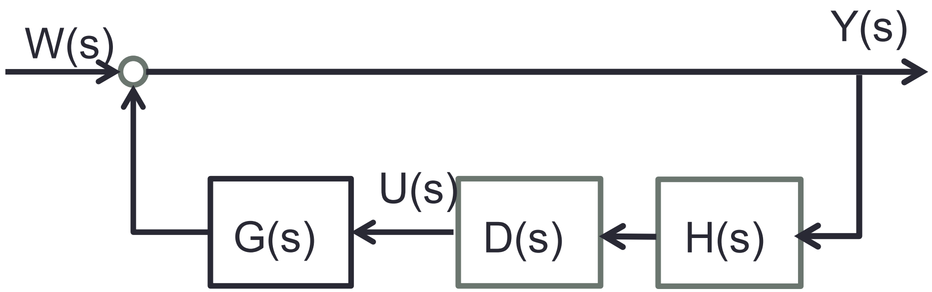
$$G(s) = \frac{7}{s+7}$$

$$\mathrm{K} = 10, \qquad \mathrm{H} = 1$$



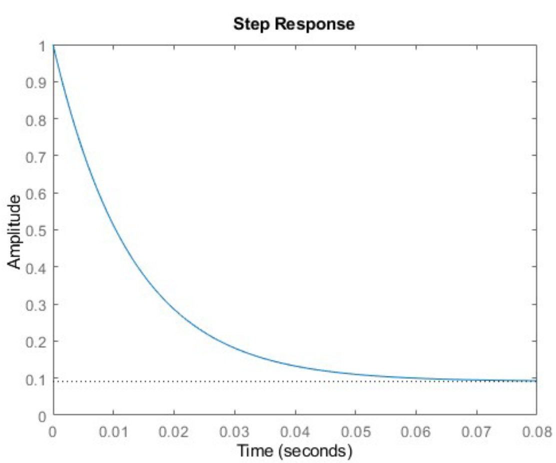
$$\frac{Y(s)}{W(s)} = \frac{1}{1 + G(s)H(s)D(s)}$$

$$= \frac{1}{1 + \frac{7}{s+7} * 10 * 1} = \frac{s+7}{s+77}$$



DC gain (s=0) = $\frac{1}{11}$

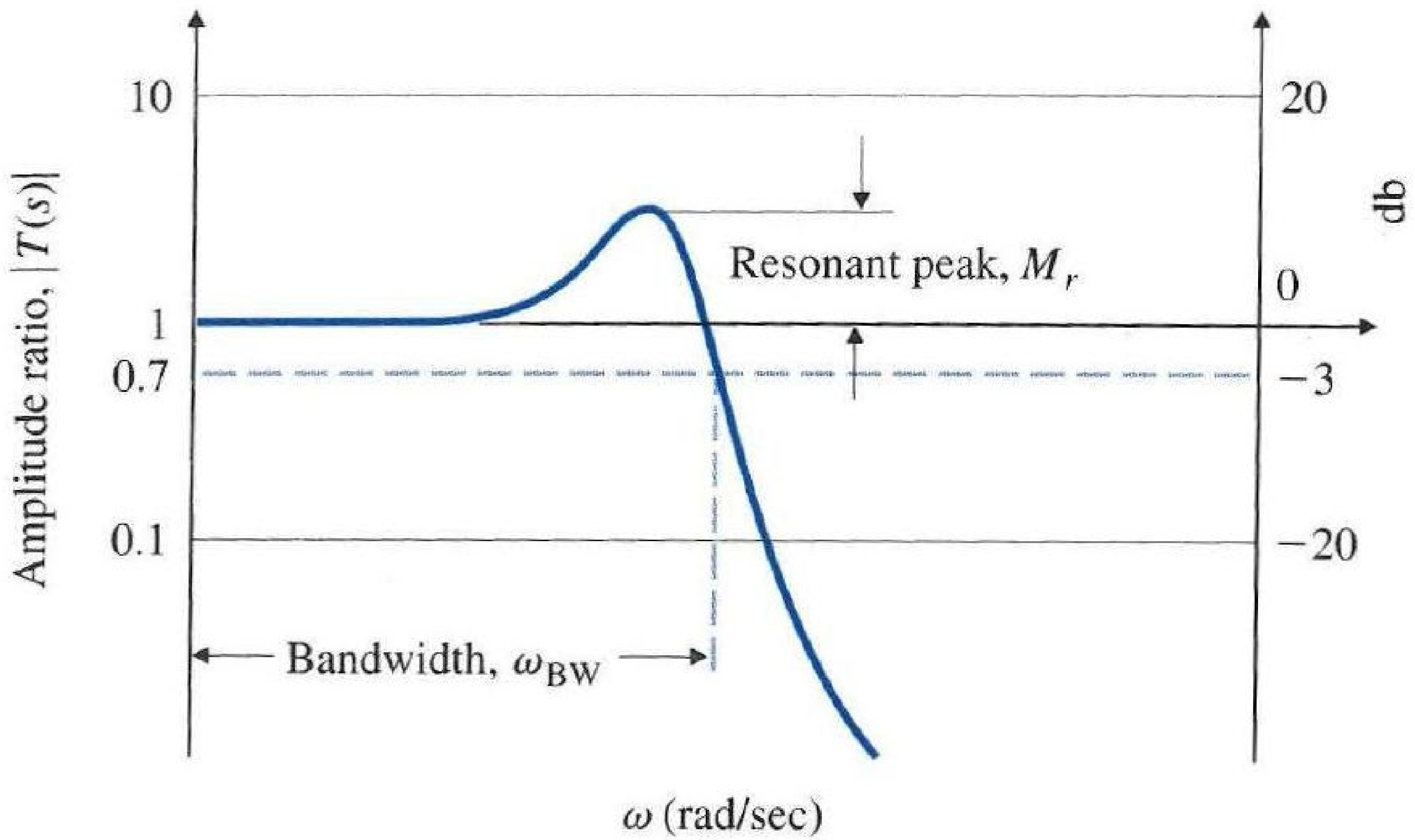
Unitstep for w(s)



Frequency domain specifications

Bandwith

CLOSED LOOP



Closed loop Bodeplot for the controlled system - finding the bandwith

```
s=tf('s')
D=1.11*(s+1)/s;
G=0.1/(s+1)*(s+1)*(0.1*s+1);
H=1;
T=feedback(D*G,H);
bode (T)
```

Bandwith