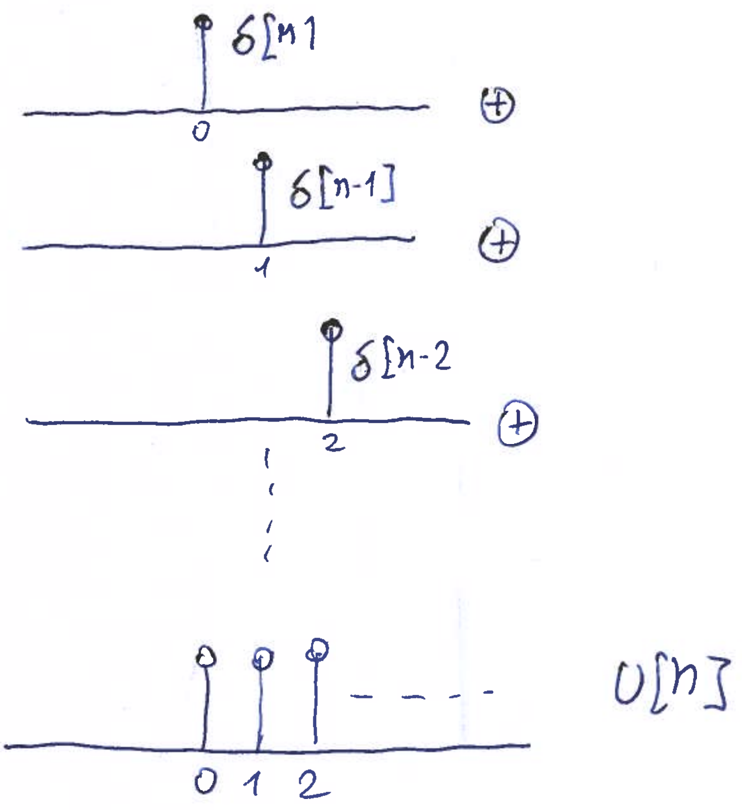
Module 1

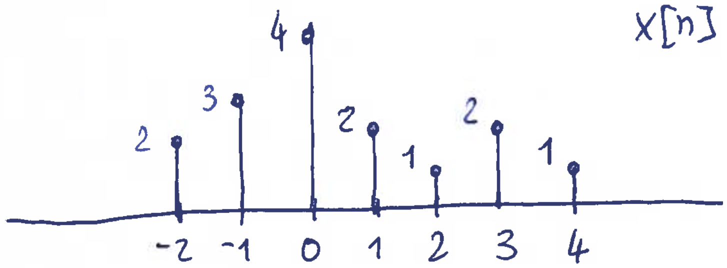
1) $$\sum_{k=0}^{\infty} \delta[n-k] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

$$\delta[n] = 1 \quad \text{if } n=0$$
$$\delta[n-1] = 1 \quad \text{if } n=1 \qquad \delta[n-k]=0 \text{ if } n \neq k$$
$$\delta[n-2] = 1 \quad \text{if } n=2$$
$$\vdots$$

Since $u[n]=1$ if $n \geq 0 \rightarrow \delta[n] + \delta[n-1] + \delta[n-2] + \dots$
$$= u[n]$$



c)



$$x[n] = 2\delta[n+2] + 3\delta[n+1] + 4\delta[n] + 2\delta[n-1] + \delta[n-2] +$$
$$+ 2\delta[n-3] + \delta[n-4].$$

- $x[n-2] = 2\delta[(n-2)+2] + 3\delta[(n-2)+1] + 4\delta[n-2] + 2\delta[(n-2)-1] +$

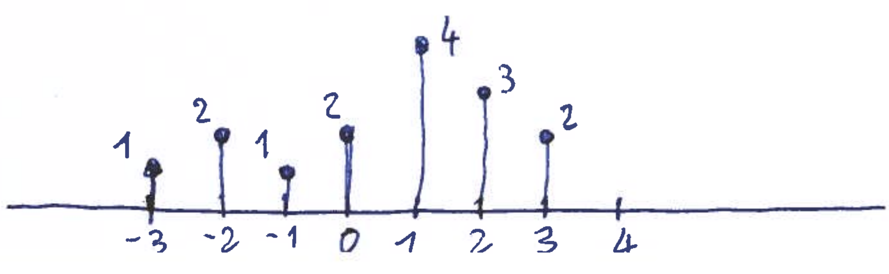
$$+ \delta[(n-2)-2] + 2\delta[(n-2)-3] + \cancel{4}\delta[(n-2)-4] =$$
$$= 2\delta[n] + 3\delta[n-1] + 4\delta[n-2] + 2\delta[n-3] + \delta[n-4] +$$
$$+ 2\delta[n-5] + \delta[n-6]$$

- $x[1-n] = 2\delta[(1-n)+2] + 3\delta[(1-n)+1] + 4\delta[1-n] + 2\delta[(1-n)-1] +$

$$+ \delta[(1-n)-2] + 2\delta[(1-n)-3] + \delta[(1-n)-4] =$$
$$= 2\delta[3-n] + 3\delta[2-n] + 4\delta[1-n] + 2\delta[-n] +$$
$$+ \delta[-n-1] + 2\delta[-n-2] + \delta[-n-3] =$$

$\delta[-n-x] = \delta[n+x]$

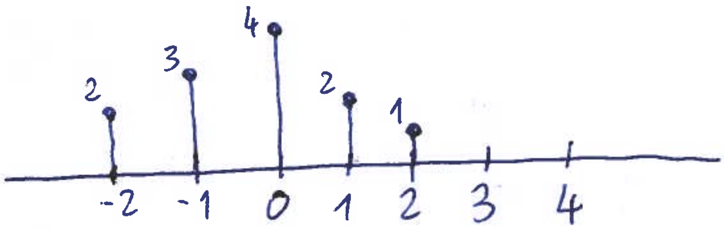
$$= 2\delta[n-3] + 3\delta[n-2] + 4\delta[n-1] + 2\delta[n] +$$
$$+ \delta[n+1] + 2\delta[n+2] + \delta[n+3]$$



- $x[n]\,u[2-n]$

$$u[2-n]=\begin{cases}1 & 2-n\geqslant 0 \rightarrow n\leq 2\\ 0 & 2-n<0 \rightarrow n>2\end{cases}$$

$$x[n]\,u[2-n]=2\delta[n+2]+3\delta[n+1]+4\delta[n]+2\delta[n-1]+$$
$$+\delta[n-2]$$

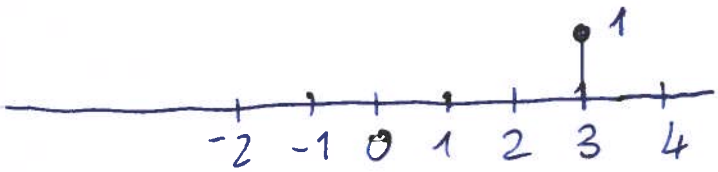


- $x[n-1]\,\delta[n-3]$

$$\delta[n-3]=\begin{cases}1 & n=3\\ 0 & n\neq 3\end{cases}$$

$$x[n-1]=2\delta[n+1]+3\delta[n]+4\delta[n-1]+2\delta[n-2]+$$
$$+\delta[n-3]+2\delta[n-4]+\delta[n-5]$$

$$x[n-1]\,\delta[n-3]=\delta[n-3]$$



Module 2

1) $x[n] = 2\delta[n+2] + 3\delta[n] + 2\delta[n-1] - \delta[n-4]$

$h[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

We can calculate the response to the individual samples of the input.

$k=-2 \quad x[-2] = 2$

$x[-2]\,h[n+2] = 2(\delta[n+2] + 3\delta[n+1] + 2\delta[n] + \delta[n-1]) =$

$= 2\delta[n+2] + 6\delta[n+1] + 4\delta[n] + 2\delta[n-1]$

$k=0 \quad x[0] = 3$

$x[0]\,h[n] = 3(\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]) =$

$= 3\delta[n] + 9\delta[n-1] + 6\delta[n-2] + 3\delta[n-3]$

$k=1 \quad x[1] = 2$

$x[1]\,h[n-1] = 2(\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]) =$

$= 2\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

$k=4 \quad x[4] = -1$

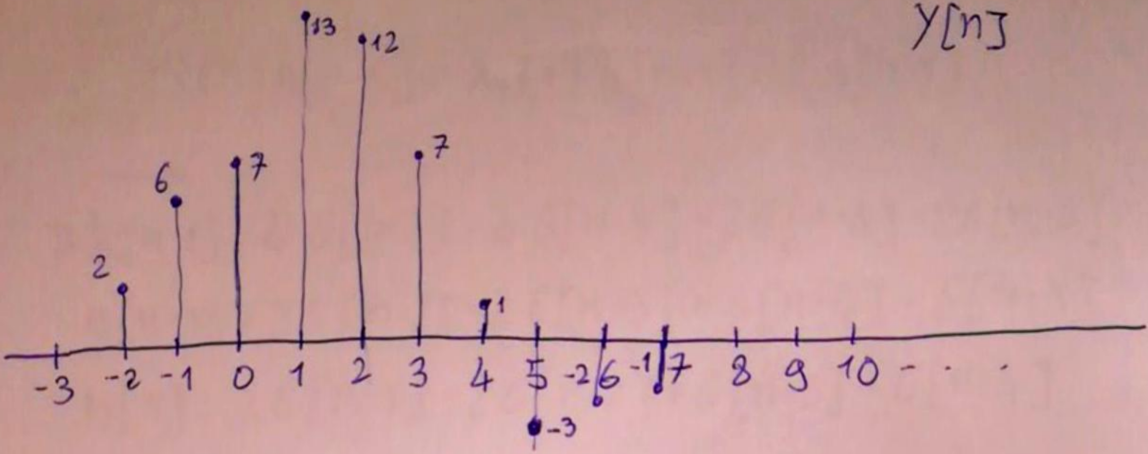
$x[4]\,h[n-4] = -(\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + \delta[n-7]) =$

$= -\delta[n-4] - 3\delta[n-5] - 2\delta[n-6] - \delta[n-7]$

$$y[n] = x[-2]\,h[n+2] + x[0]\,h[n] + x[1]\,h[n-1] + x[4]\,h[n-4] =$$

$$= 2\delta[n+2] + 6\delta[n+1] + 7\delta[n] + 13\delta[n-1] + 12\delta[n-2] +$$

$$+ 7\delta[n-3] + \delta[n-4] - 3\delta[n-5] - 2\delta[n-6] - \delta[n-7]$$



2) If h represents an LTI system, then:

$$y_1[n] = \sum_{k=-\infty}^{+\infty} x_1[k]\, h[n-k] = 4\delta[n-2] - 4\delta[n-3] + 2\delta[n-4] - 2\delta[n-5]$$

$$\sum_{k=-\infty}^{+\infty} x_1[n]\, h[n-k] = x_1[1]\, h[n-1] = 2h[n-1]$$

$\longrightarrow$

$$2h[n-1] = 4\delta[n-2] - 4\delta[n-3] + 2\delta[n-4] - 2\delta[n-5]$$

$$h[n-1] = 2\delta[n-2] - 2\delta[n-3] + \delta[n-4] - \delta[n-5]$$

$$h[n] = 2\delta[n-1] - 2\delta[n-2] + \delta[n-3] - \delta[n-4]$$

We need to verify that

$$y_2[n] = \sum_{k=-\infty}^{+\infty} x_2[n]\, h[n-k] = x_2[-3]\, h[n+3] + x_2[-1]\, h[n+1] =$$

$$= -4\delta[n+2] + 4\delta[n+1] + \delta[n-2] - \delta[n-3]$$

- $x_2[-3]\, h[n+3] = -2h[n+3] = -2\cdot(2\delta[n+2] - 2\delta[n+1] + \delta[n] - \delta[n-1]) = -4\delta[n+2] + 4\delta[n+1] - 2\delta[n] + 2\delta[n-1]$
- $x_2[-1]\, h[n+1] = h[n+1] = 2\delta[n] - 2\delta[n-1] + \delta[n-2] - \delta[n-3]$

$$y_2[n] = -4\delta[n+2] + 4\delta[n+1] + \delta[n-2] - \delta[n-3] \quad \checkmark$$

h can then be an LTI system!

Since $h[n]$ does not depend on future values of n, the system is causal.