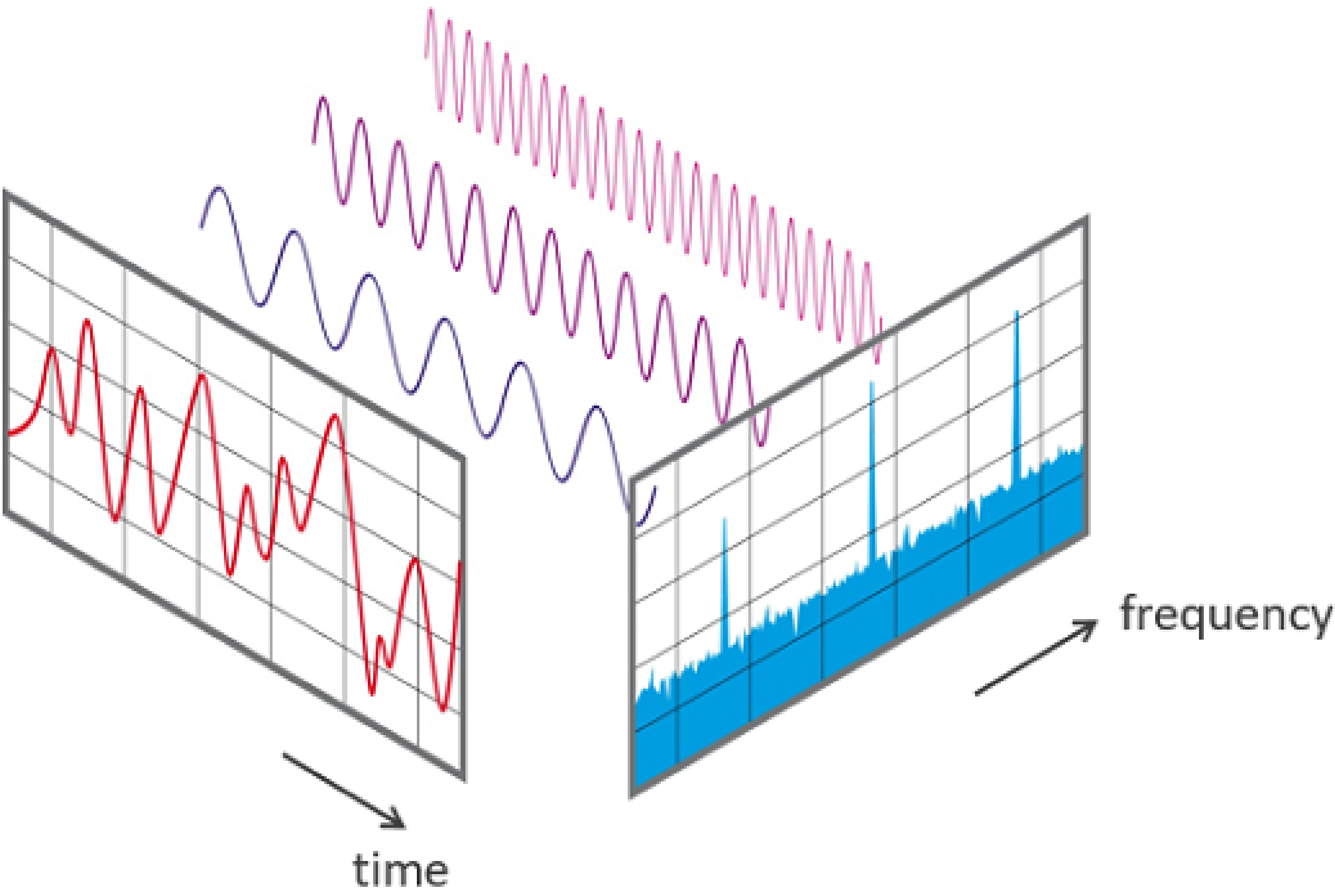
Room Acoustic Material Property Determination Using a Multi-Microphone Impedancetube

Bachelor Project ESD6

Christian Lykke Jørgensen



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Lorem Ipsum.

Contents

Preface

This report searches the requirements to develop an impedance tube based upon DS/EN ISO 10534-2:2023. In appendix B on page 40 all formulas and variables will be noted, in order of appearance.

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1 Introduction

Acoustics are present in every aspect of life, no matter how small or large, resulting in acoustics significantly impacting the very perception of life. Scientific acoustical studies began emerging as far back as the 6th century BC with Greek philosophers and later Roman architects/engineers embracing acoustic properties in construction, [1]. However, the contemporary understanding of acoustics has only existed for approximately 200 years, [1]. Within those 200 years, acoustics has changed from a phenomenon only understood by scholars to an aspect of life, that most can relate to or know of to some degree.

In modern construction, acoustical properties have traversed from being reserved for performance centres, to being integrated into almost anything. It has become crucial for the average homeowner to achieve "good acoustics", along with acoustics occupying an increasingly large facet of large-scale construction. To achieve "good acoustics" a multitude of methods are available, ranging from adding a thick carpet or heavy drapes to bass-traps, vibration minimizing, and similar sound-deadening techniques.

Unfortunately, creating an acoustically perfect room is expensive, and in many cases wouldn't resemble a habitable room in common sense. Therefore the next-best option is adding unnoticeable elements such as padded furniture, rugs, pillows, drapes, and acoustic art/images or incorporating constructional options such as floor dampening, carpets, acoustic ceilings, and other, more intrusive options, [2, 3].

To bridge the gap between the historical and practical perspectives on acoustics, it is essential to consider the challenges and trade-offs involved in achieving optimal acoustical environments. While the importance of acoustics in everyday life and construction is evident, the practical implementation of "good acoustics" is often constrained by cost, aesthetics, and feasibility. This raises a fundamental question that initializes this project:

"How can "good acoustics" be defined and quantified in a way that balances theoretical ideals with real-world applicability?"

2 | Problem Analysis

2.1 Room Acoustics

Determining good acoustics is as subjective as determining which music is pleasant. While subjective qualities are present, objective aspects should be considered as well, as the very shape of a person (ears, head, shoulders, etc.) influence the perception of sound as well, [4, 5, 6, 7, 8]. Furthermore, good acoustical properties are not as "simple" as attempting to achieve anechoic conditions, as the inherent psychoacoustical properties of an anechoic room are not considered pleasant. Fortunately, guidelines exist for what is, in general, considered good acoustics. The metrics are room gain (G), reverberation time (RT60), early decay time (EDT), speech intelligibility (C50), definition (D50), musical clarity, and temporal distribution. The first four can be estimated by using the following:

2.1.1 G:

The gain of a room is very literally the amplification or attenuation that a room provides. It can be measured using a calibrated omnidirectional speaker (such as a dodecahedral speaker), where an impulse is first measured in free field (an anechoic room), and then in the room to be examined. The measurement in free field is done by measuring at a distance of 10 m from the sound source, and is repeated for at least 5 measurements at various locations to create an average, [9, 10]. As in free field, the room to measure should have an equal amount of measurements made. The gain of a room can be described with the following equations from DS 3382-1, [10]:

$$G = 10 * log10\left(\frac{\int_0^\infty p^2(t)\,dt}{\int_0^\infty p_{10}^2(t)\,dt}\right) = L_{pE} - L_{pE,10}[dB] \tag{2.1}$$

$$L_{pE} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p^2(t)\,dt}{p_0^2}\right][dB] \tag{2.2}$$

$$L_{pE,10} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p_{10}^2(t)\,dt}{p_0^2}\right][dB] \tag{2.3}$$

Where: $p(t)$ is the instantaneous sound pressure of the impulse response measured at a point in the room, $p_{10}(t)$ is the instantaneous sound pressure of the impulse response measured at 10m in free field, p_0 is $20\mu Pa$, T_0 is 1 second, L_{pE} is the sound pressure exposure level of $p(t)$, and $L_{pE,10}$ is the sound pressure exposure level of $p_{10}(t)$.

If any of the rooms are not large enough to complete the measurements at 10 meters, they can instead be done at 3 meters and modified with:

$$L_{pE,10} = L_{pE,d} + 20 * log10(d/10)[dB] \tag{2.4}$$

Or even simpler by using, [9, 10]:

$$L_p = L_{p,10}[dB] \tag{2.5}$$

Where: L_p is the sound pressure level averaged across every measurement point and $L_{p,10}$ is the sound pressure level measured at 10 m in free field.

Alternatively, if a sound source with known sound power level is available, the gain can be obtained by the following, [9, 10]:

$$G = L_p - L_W + 31dB \tag{2.6}$$

Where: L_p is the sound pressure level averaged across every measurement point and L_W is the sound power level of the sound source, and should be measured according to DS 3741.

2.1.2 RT60:

Reverberation **T**ime 60 describes the time it takes for sound a level to decay 60 dB. In many practical applications, background noise makes it difficult to measure a full 60 dB decay, and therefore, T20 or T30 is used instead. T20 and T30 differ from RT60 by measuring either 20 or 30 dB decay and following the description set in DS ISO 3382-2, the decay time must be measured from -5 to -2(3)5 dB, and not from 0 to -2(3)0 dB, [11, 9, 12, 13]. In figure B.1a, an example of measuring T30 can be seen, where it should be noted that the decay is first measured from -5 dB. When using T20 or T30, the time measured should be multiplied by 3 or 2 to harmonize with RT60. It should furthermore be noted that T20 is most frequently used, as DS 3382-2 states that *"the subjective evaluation of reverberation is related to the early part of the decay"*, and that *"the signal-to-noise ratio is often a problem in field measurements, and it is often difficult or impossible to get a evaluation range of more than 20 dB."*, [11].

Sabine's equation for a diffuse sound field can be used to estimate a room's reverberation time. It takes the input V for the volume of the room in cubic meters, α for the absorption coefficient, and S for the surface area of the room in meters, and is used as:

$$RT60 = \frac{0.161 * V}{S_i * \alpha_i} \leftrightarrow RT = \frac{0.161 * V}{A} \tag{2.7}$$

Where:

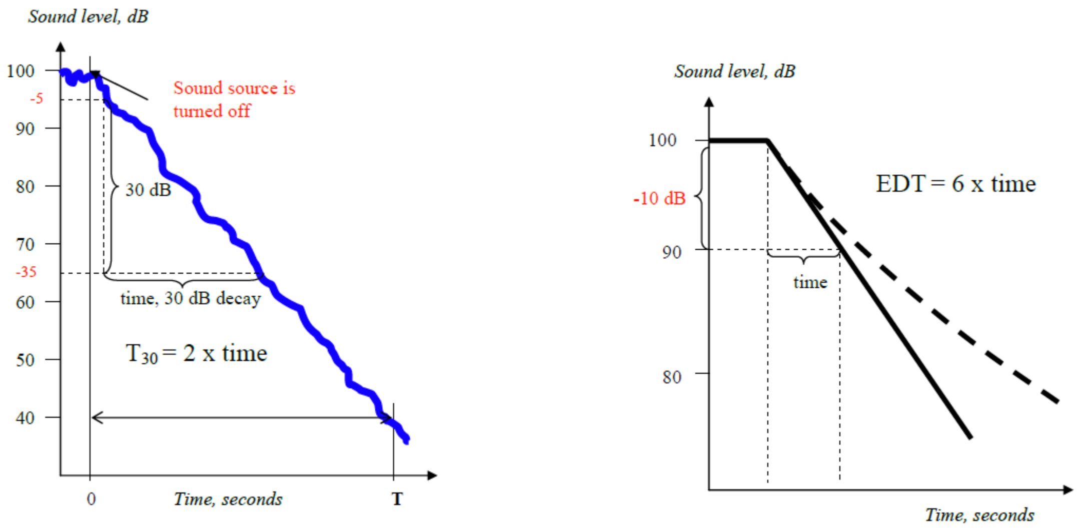
$$A = \sum S_i * \alpha_i \tag{2.8}$$

The reasoning behind equation B.79/the lower fraction of B.78 is to compensate for different materials with different absorption coefficients in a room. It should, however be denoted that since Sabine's formula assumes a diffuse sound field, it is best used concerning EDT rather than T30 or T20 due to the corresponding decay curves. This can be seen in figure B.1b, where the solid straight line matches a continued approximation derived from the EDT. The dashed line is a more realistic decay curve of a room with absorbent materials, such as acoustic ceilings, [9].

In general, some optimal RT ranges are, [14, 15, 16, 9, 17]:

- Speech-oriented spaces, such as classrooms, meeting rooms, lecture halls, etc. require a short reverberation time of 0.3-1 second, to ensure speech intelligibility.

- Musical performance rooms, such as opera houses, concert halls, clubs, etc. would typically benefit from longer reverberation times, ranging from 1.5-3 seconds.
- Multi-use rooms such as dining halls, auditoriums, etc. benefit from a compromise, with reverberation times of 1.2-2 seconds.
- Sacred rooms/rooms of worship, such as churches, chapels, monasteries, etc. have much wider ranges, as they (dependent on religious purposes) have a reverberation time between 3-10 seconds.



(a) Definition of 30 dB decay (T30). Notice the timer starts at -5dB, [9].

(b) Illustration of EDT, [9].

Figure 2.1: T30 and EDT illustrations, [9].

2.1.3 EDT/T10:

Early decay time describes the first 10 dB decay within a room and is sometimes denoted T10, similarly to T20 and T30. The main difference from T20 and T30 is that EDT does not include the same -5 dB buffer but instead measures the actual first 10 dB decay. According to [9], [18] and DS 3382-1, [10], the EDT should be the subjectively most important reverberation characteristic when determining the acoustic properties of a room. EDT is determined by the first 10 dB of attenuation multiplied by 6 to reach a number comparative with RT60. The EDT equation is most beneficial when it comes to the psychoacoustic perception of a room and for determining "ideal" decay time, [18]. This is caused by the minimum of summed reflections present within the first 10 dB decay.

2.1.4 C50/C80:

To determine the clarity relation of a room as an acoustical parameter, C50 or C80 is used. C50 estimates the reflections and their energy present after 50 ms, while C80 estimates them after 80 ms. The idea is that reflections present after the critical time limit muddles perceived speech and reduce clarity. The critical time limit varies from person to person, with age, and is furthermore dependent on what exactly is being listened to, [9, 14, 15, 17, 10]. C50 is most often used for speech intelligibility and C80 is used for music clarity. The C50 and C80 relation is calculated by:

$$C50 = 10 * log10\left(\frac{Energy(0-50ms)}{Energy(51ms-end)}\right)[dB] \tag{2.9}$$

$$C80 = 10 * log10\left(\frac{Energy(0-80ms)}{Energy(81ms-end)}\right)[dB] \tag{2.10}$$

Where the "Energy" values are derived from a broadband impulse response in the room. The impulse response is recorded and divided into integrated squared energy values before and after the critical time limit, [9, 14, 15, 17, 10]. Both can also be found generally by:

$$C_{t_e} = 10 * log10\left(\frac{\int_0^{t_e} p^2(t)\,dt}{\int_{t_e}^{\infty} p^2(t)\,dt}\right)[dB] \tag{2.11}$$

Where: C_{t_e} is the early to late index. t_e is the early time limit (e.g. 50 or 80 ms) and $p(t)$ is the measured sound pressure of the impulse response, [10].

For speech clarity, values above 0 dB are considered good, while for music clarity it depends on music type and preference, but in most cases, a value between -2 dB and +4 dB is considered acceptable, [17].

2.1.5 D50:

Definition/Deutlichkeit after 50 ms is a percentage describing how many percent of the total energy is present within the first 50 ms. D50 is strongly correlated to C50, as D50 (and vice-versa C50) can be found by, [10]:

$$C50 = 10 * log10\left(\frac{D50}{1-D50}\right)[dB] \tag{2.12}$$

Or by itself using:

$$D50 = \frac{\int_0^{0.050} p^2(t)\,dt}{\int_0^{\infty} p^2(t)\,dt} \tag{2.13}$$

2.1.6 Introduction to sound

Now that the basic metrics for evaluating a room's acoustic properties have been established, sound as an entity can be determined. To do so, a lot of equations will be instantiated, with most coming from [7]. As most of the notation and equations in [7] are equal or at the very least similar to other sources, it has been chosen as the main source for equations, meaning all formulas used in this can be found in [7] as well, unless otherwise noted. Other sources used for mathematical formulas are [4, 5, 6, 8, 10, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26]. For the time being, only ideal sound will be described, assuming no losses in propagation or obstacles. Furthermore, the medium is considered homogeneous and at rest, making the velocity of sound constant in relation to space and time. If the medium were air, the velocity is given by:

$$c = 331.4 + 0.6 * T\left[\frac{m}{s}\right] \tag{2.14}$$

With T being the temperature in degrees centigrade. C is set to be 343m/s for all following computations, based on an average temperature of 23.2 degrees. The average is used as exact calculations would require infinitesimal precision and, in practice, would be impossible to compute. An example could be a sports hall, wherein the temperature around players and audience is higher than under the rafters, [4, 5, 7, 14].

Another fundamental equation is the wave equation. The wave equation describes how sound propagates over time and space relative to the speed of sound. It is given by:

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \tag{2.15}$$

where the squared speed of sound is defined by:

$$c^2 = \kappa \frac{p_0}{\rho_0} \tag{2.16}$$

in these equations, p is the sound pressure, rho_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$, p_0 is the static sound pressure (measured in Pascal, where $1\ Pascal = 1 \frac{kg}{m*s^2}$, and κ is the adiabatic component (for air $\kappa = 1.4$), [7]. In cartesian coordinates, the laplacian operator Δ is given by:

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \tag{2.17}$$

To elaborate, the wave equation is derived from the conservation of momentum relation:

$$grad\ p = -\rho_0 * \frac{\partial v}{\partial t} \tag{2.18}$$

with its one-dimensional version being:

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \tag{2.19}$$

where: v is a vector representing particle velocity, and t is time. Furthermore, the mass conservation requirement leads to:

$$\rho_0\ div\ v = -\frac{\partial \rho}{\partial t} \tag{2.20}$$

with ρ being the variable gas density. It is generally assumed that the variation in gas pressure and density is small compared to their static values. All of the above can be related by:

$$\frac{p}{p_0} = \kappa \frac{\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} * \frac{\delta * T}{T + 273} \tag{2.21}$$

By eliminating the particle velocity v and variable gas density ρ from equations B.89-B.92 one would arrive at the wave equation given in equation B.86. The wave equation holds true for sound waves in any lossless fluid and for the resulting pressure, density, and temperature variations.

Plane Waves

A plane wave simply put is a fraction of a spherical wave small enough that it can be assumed to have no curvature, [11, 10, 19, 20, 4, 5, 7, 8]. Another physical assumption that can be made, is that if a sound wave is present within a tube, with a diameter/height/width significantly smaller than the wavelength of said wave, it can be assumed to be planar, see section 3.2 on page 24 for more. It is effectively an orthogonal plane to a vector in a cartesian coordinate system. If that vector is equal to the x-axis, a plane wave can be described by the wave equation (B.86) with:

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{2.22}$$

With a corresponding general solution given by:

$$p(x,t) = F(c * t - x) + G(c * t + x) \tag{2.23}$$

In equation B.94 the first term represents a plane wave propagating positively along the x-axis and the second term a negative direction. From equations B.93 and B.94 it can also be seen that a constant pressure level is present at each wavefront. By specifying F and G as exponential functions including the imaginary components, the propagation is given as:

$$p(x,t) = \hat{p} * e^{i*k*(c*t-x)} = \hat{p} * e^{i*(\omega*t-k*x)} \tag{2.24}$$

Where $\hat{p}$ is amplitude, k is the propagation constant $k = \frac{\omega}{c}$, ω is the angular frequency, which can be used to obtain the temporal period $T = \frac{2*\pi}{\omega}$. This all connects to several interpretations of the wavelength formula:

$$\lambda = \frac{2 * \pi}{k} \leftrightarrow \frac{2 * \pi}{\frac{\omega}{c}} \leftrightarrow \frac{2 * \pi * c}{\omega} \leftrightarrow \frac{c}{\frac{\omega}{2*\pi}} \leftrightarrow \frac{c}{f} \tag{2.25}$$

Now that amplitude, wavelength, and direction are known, the only missing parameters for expressing the plane wave are frequency and intensity. The frequency is given by $f = \frac{\omega}{2*\pi} * \frac{1}{T}$ with the unit Hz. If equation B.89 is applied to equation B.94, the only non-vanishing point of the wave is parallel to the x-axis, meaning that sound behaves as longitudinal waves in fluids, and the particle velocity can be found by:

$$v(x,t) = \frac{1}{\rho_0 * c}[F(c * t - x) - G(c * t + x)] \tag{2.26}$$

To obtain the ratio between sound pressure and particle velocity, also known as the characteristic impedance of a medium, the following is used:

$$\frac{p}{v} = \rho_0 * c \tag{2.27}$$

With $\rho_0 * c$ found by table values for ρ_0 and c in the correct medium. For air, the characteristic impedance is:

$$\rho_{0_{air}} * c_{air} = 1.2 \left[\frac{kg}{m^3}\right] * 343 \left[\frac{m}{s}\right] = 414 \left[\frac{kg}{m^2 s}\right] \tag{2.28}$$

Understanding the above relation makes it possible to determine the intensity of each wave. The plane imagined to be orthogonal to the x-axis will have energy present across its entire surface, and the average (denoted by the bar notation) product of the pressure and particle velocity on the surface yields the intensity:

$$I = \bar{pv} = \frac{\bar{p^2}}{\rho_0 * c} \tag{2.29}$$

To find the energy density of a wave, equation B.101 is used:

$$w = \frac{I}{c} = \frac{\bar{p^2}}{\rho_0 * c^2} \tag{2.30}$$

And with that, sufficient information should be available to describe any plane wave.

Spherical Waves

The sound wave will now propagate with no boundaries whatsoever, leading to ideal spherical waves. Graphically this is best understood as an inflating ball stretching equally in all directions from an infinitesmally small point source. Due to the nature of spherical waves, a spherical coordinate system will be used, using polar coordinates. A benefit from the spherical symmetry is that pressure is represented as r and is independent of the θ and ϕ angles. To convert the wave equation from cartesian to spherical coordinates, the laplacian of a function $p(r, \theta, \phi)$has to expressed in spherical coordinates as well:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p}{\partial \phi^2}. \tag{2.31}$$

As the acoustical properties of a spherical wave are equal across all θ and ϕ angles, it's description can be made with only r and t, simplifying the laplacian to:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) \tag{2.32}$$

The simplified laplacian should now be expanded to:

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} \tag{2.33}$$

And is now ready to be inputtet into the wave equation:

$$c^2\left(\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}\right) = \frac{\partial^2 p}{\partial t^2} \tag{2.34}$$

By dividing with c^2 on both sides, the spherical wave equation is found:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{2.35}$$

A simple solution to the spherical wave equation is given in equation B.107, representing a spherical wave with the volume velocity Q, being the rate in $\frac{m^3}{s}$ of "liquid" being expelled by the sound source located at $r = 0$. The dot above Q denotes differentation w.r.t. time (t).

$$p(r,t) = \frac{\rho_0}{4 * \pi * r} * \dot{Q}\left(t - \frac{r}{c}\right) \tag{2.36}$$

As for plane waves, spherical waves have a non-vanishing component of its particle velocity. Instead of being along the x-axis, the non-vanishing point for a spherical wave is radial, and can be calculated by applying equation B.89 to B.107, yielding:

$$v_r = \frac{1}{4 * \pi * r^2}\left[Q\left(t - \frac{r}{c}\right) + \frac{r}{c} * \dot{Q}\left(t - \frac{r}{c}\right)\right] \tag{2.37}$$

To find the particle velocity of a spherical wave, equation B.109 is used. It show that the sound pressure and velocity in a spherical wave is inversely proportional to the size

r and frequency $\omega = k * c$. For distances large in comparison with the wavelength, the ratio $\frac{p}{v}$ tends asymptotically to $\rho_0 c$.

$$v_r = \frac{p}{\rho_0 * c} * \left(1 + \frac{1}{i * k * r}\right) \tag{2.38}$$

Actual waves

Now that the ideal scenarios are described, a more realistic approach should also be made. Therefore directional sound radiation with angular dependence is considered in the following. It is best imagined by a singular speaker pointing into an ideal environment. As the speaker is pointing in a direction and is not omnidirectional, the resulting sound wave will not be uniformly spherical. This expands the spherical wave equation to now include the polar and azimuth angles as well:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} + \frac{1}{r^2}\left(\frac{1}{sin(\theta)} * \frac{\partial}{\partial * \theta}\left(sin(\theta) * \frac{\partial}{\partial * \theta}\right) + \frac{1}{sin^2(\theta)} * \frac{\partial^2}{\partial \phi^2}\right) = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{2.39}$$

To solve the complex spherical wave equation, the sound pressure can be expressed as a Fourier series separated into its radial and angular variables:

$$p(r, \theta, \phi) = A * \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Gamma_{nm} h_n(kr) Y_n^m(\theta, \phi) \tag{2.40}$$

Where: Γ_{nm} are the Fourier series coefficients of order n and degree m, $h_n(kr)$ is the Hankel function of the first kind with order n, $Y_n^m(\theta, \phi)$ are the spherical harmonics, forming an orthonormal basis, which are used in the Legendre polynomial:

$$Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)}{4 * \pi} * \frac{(n - |m|)!}{(n + |m|)!}} * P_n^{|m|} * (cos(\theta) * \begin{cases} cos|m|\phi & if \;\; m \geq 0 \\ sin|m|\phi & if \;\; m < 0 \end{cases} \tag{2.41}$$

To actually use the above equations for anything, a spherical microphone array surrounding the speaker must be used to obtain empirical data. When such a measuring system is used, the coefficients of Γ_{nm} can be found with an inverse two-dimensional Fourier transform — often referred to as the inverse spherical harmonic transform.

It should be noted that degrees (n) and orders (m) of a spherical harmonic refer to the number of harmonics present along the radial and azimuthal angles. The degree n characterizes the overall angular frequency or spatial resolution of the wave pattern on the sphere, while the order m details the specific azimuthal variation, defining the symmetry of the sound field with respect to the polar axis.

Human Hearing and Perception

As 13 orders of magnitude differentiate the hearing threshold and the threshold for painful listening in the most sensitive part of human hearing (1000-3000Hz), it would not make sense to discuss the sound pressure, [7]. Therefore, sound pressure level SPL is used instead, given by:

$$SPL = 20 * log10(\frac{\tilde{p}}{\tilde{p_0}}[dB] \tag{2.42}$$

Where $\tilde{p}$ denotes root mean square of the sound pressure, i.e. $\sqrt{\bar{p^2}}$ and $\tilde{p}_0 = 2 * 10^{-5}[\frac{N}{m^2}]$ is an internationally fixed value corresponding to the hearing threshold at 1000Hz, [7]. The sound pressure level reveals the objective sound pressure, but tells nothing about the subjective sound level. A 1000Hz sinusoid at 120dB is deafening, while a 30kHz sinusoid at the same SPL is inaudible. To compensate for this subjective perception, the unit "phon" and the associated "sone" is used. Phons are in an idealised world equal to the SPL of a 1kHz sinusoid at any given dB, and can therefore be approximated as $10phons = 10dB@1kHz$. To yield a better understanding of phons and sones, figure B.2 shows what is known as equal loudness curves (also known as Fletcher-Munson curves) from the DS/ISO standard 226:2023, [27]. Equal loudness curves show the subjective loudness perception of puretone signals at varying frequencies with known SPL, and can be used to find the exact perception of loudness from 20-12500Hz. The threshold of hearing is found at the dashed line in the bottom, signifying 0 phon, established in DS/ISO 389-7, [28]. The curves for 10 and 100 phon are dotted lines as very little experimental data exists for these levels, [27]. *NOTE: the equal loudness curves are only applicable for puretone signals, as complex sounds, i.e. a hammer hitting a nail, contain masking of some spectral quantities, and can therefore not be placed specifically on the curves. In such an instance dB(A) measurements should be used instead.*

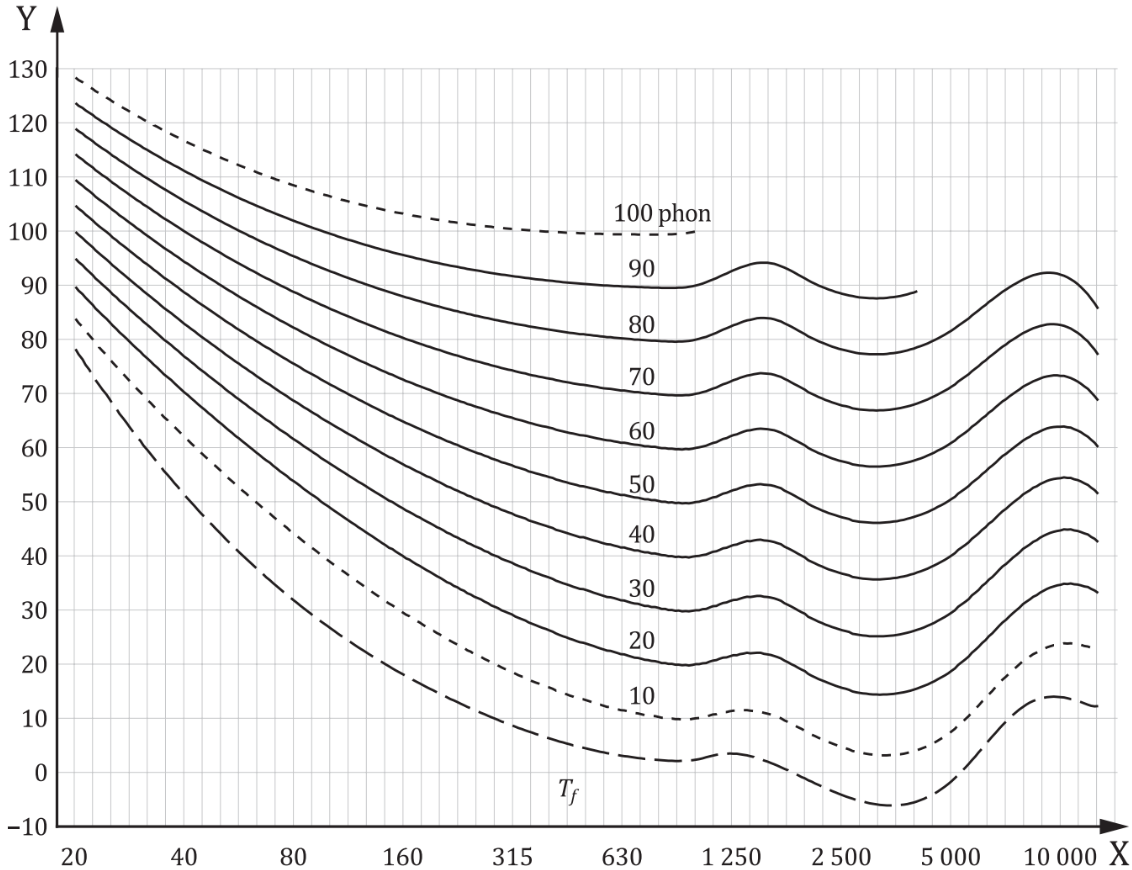


Figure 2.2: Equal loudness curves for pure tone listening in free field conditions, [27]. X is frequency, expressed in Hz, Y sound pressure level, expressed in dB, and T_f hearing threshold.

The metric "sone" describes when the subjective loudness is doubled, as doubling the number of phons does not equal a doubling in perceived loudness. The first sone equals 40 phons, and thereafter 1 sone equals 10 phons. The relation is best understood by viewing table B.3, and is otherwise given by equations B.114 and B.115, with N being loudness in sones and L_N being loudness in phons.

$$N = \left(10^{\frac{L_N-40}{10}}\right)^{0.30103} \approx 2^{\frac{L_N-40}{10}} \tag{2.43}$$

$$L_N = 40 + log_2(N) \tag{2.44}$$

Phon	0	10	20	30	40	50	60	70	80	90	100
Sone	0 (Inaudible)	0.0625	0.125	0.25	1	2	4	8	16	32	64

Table 2.1: Relationship between phons and sones.

The remaining aspects related to human perception of sounds are mostly oriented towards psychoacoustics and binaural perception, such as head-related transfer functions (HRTF) and binaural transfer functions. While the HRTF is applicable in virtual auralization of rooms, it will not be expanded upon in this project, and neither will psychoacoustical properties in depth.

2.1.7 Sound in Front of a Wall

Instead of ideal free-field propagation, actual constraints such as a wall will now be introduced. A wall will have some inherent properties, such as its reflection, diffusion, transmission, resonance, and absorption properties, that together determine the actual acoustical properties of each specific wall. In table B.4 each property and its typical values are presented, with relevant equations following.

Property	**Definition**	**Typical Values**
Reflection Coefficient (R)	Fraction of sound energy reflected	0.8 - 0.98 (Concrete), 0.1 - 0.5 (Foam)
Diffusion Coefficient (D)	Measure of even sound scattering	0.2 (Flat wall), 0.6 - 0.9 (Diffuser)
Transmission Loss (TL)	Reduction of sound through a barrier (dB)	25 dB (Drywall), 50+ dB (Concrete)
Resonance Frequency (f_r)	Frequency at which the wall vibrates	50 - 500 Hz (Depending on material)
Absorption Coefficient (α)	Fraction of energy absorbed	0.02 (Concrete), 0.8+ (Acoustic panels)
Wall Impedance (Z)	Opposition to sound transmission, depends on density and stiffness.	1.5×10^6 kg·m^{-2}·s^{-1} (Concrete), 5×10^5 kg·m^{-2}·s^{-1} (Brick), 10^4 kg·m^{-2}·s^{-1} (Foam)
Wall Admittance (ζ)	Measure of a wall's ability to accept sound energy, reciprocal of impedance.	0.001 (Concrete), 0.003 (Brick), 0.1 (Foam)

Table 2.2: Acoustic properties of walls and their typical values.

- **Reflection Coefficient:**
$$R = \frac{I_r}{I_i} \tag{2.45}$$
where I_r is reflected intensity and I_i is incident intensity.

- **Diffusion Coefficient:**
$$D = 1 - \frac{\sum S(\theta)}{NS_{\text{ideal}}} \tag{2.46}$$
where $S(\theta)$ is the scattered energy in direction θ, and S_{ideal} is the ideally scattered energy.

- **Transmission Loss (TL):**
$$TL = 10 \log_{10} \left(\frac{I_i}{I_t} \right) \tag{2.47}$$
where I_t is the transmitted intensity.

- **Resonance Frequency (Panel):**
$$f_r = \frac{60}{d} \sqrt{\frac{E}{\rho(1 - \nu^2)}} \tag{2.48}$$
where:d = panel thickness, E = Young's modulus, ρ = density, and ν = Poisson's ratio.

- **Absorption Coefficient:**
$$\alpha = 1 - |R|^2 \tag{2.49}$$

- **Wall Impedance:**
$$Z = \left(\frac{p}{v_n} \right) \tag{2.50}$$
where: v_n denotes the particle component normal to the wall.

- **Wall Admittance:**
$$\zeta = \frac{Z}{\rho_0 * c} \tag{2.51}$$

All walls will be assumed to be plane, unbounded, and smooth. Moreover, the sound source will be placed at a distance allowing the spherical waves to be considered planar.

Reflection at Normal Incidence

When referring to normal incidence, a reflection perpendicular to the wall should be imagined. To comply with previous formulas, the axis of propagation remains along the x-axis, resulting in a wall being aligned with the z- and y-axis. Furthermore, the wall is set to $x = 0$ and the wave will arrive from the negative direction, yielding the following sound pressure:

$$p_i(x, t) = \hat{p}_0 * e^{i*(\omega*t - k*x)} \tag{2.52}$$

And particle velocity:

$$v_i(x, t) = \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t - k*x)} \tag{2.53}$$

Now that the sound wave has hit the wall, the reflected wave in completion is given by the wall's characteristics. For now, only the reflection coefficient R is used, as that can help describe both the amplitude attenuation and the phase shift, which also results in k flipping signs. Following equation B.97, the sign should be flipped for the reflected particle velocity as well. By using this, the reflected waves' sound pressure and particle velocity are given by:

$$p_r(x, t) = R * \hat{p}_0 * e^{i*(\omega*t + k*x)} \tag{2.54}$$

$$v_r(x, t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t + k*x)} \tag{2.55}$$

To find the impedance of the wall, x should be set equal to the coordinates of the wall (0), yielding equations B.127 and B.128. It should be noted in those equations that they are a product of both the incident and reflected sound wave, yielding the 1+R and 1-R relations:

$$p(0,t) = (1+R) * \hat{p}_0 * e^{i*(\omega*t)} \tag{2.56}$$

$$v(0,t) = (1-R) * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t)} \tag{2.57}$$

With the sound wave evaluated at the wall, it is possible to use equation B.121 to derive the exact wall impedance:

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \rho_0 * c * \frac{1+R}{1-R} \tag{2.58}$$

Which can be reformulated for the wall admittance, given in equation B.122:

$$\zeta = \frac{Z}{\rho_0 * c} \rightarrow \zeta = \frac{\rho_0 * c * \left(\frac{1+R}{1-R}\right)}{\rho_0 * c} \rightarrow \zeta = \frac{1+R}{1-R} \tag{2.59}$$

With this, the reflection coefficient can also be described as:

$$R = \frac{\zeta - 1}{\zeta + 1} \tag{2.60}$$

And the absorption coefficient can be found as:

$$\alpha = \frac{4 * Re(\zeta)}{|\zeta|^2 + 2 * Re(\zeta) + 1} \tag{2.61}$$

Graphically the wall impedance can be seen in figure B.3. As α increases, the circles draw nearer to $\zeta = 1$ corresponding to impedance matching of the medium. A completely rigid wall will have infinite impedance, as R=1, while a soft wall with R=-1 will yield no impedance.

Reflection at Oblique Incidence

In reality, some reflections happen at normal incidence, but due to the spherical propagation of sound, a significantly larger amount will have its incidence at oblique angles. Therefore, θ will be added to all previous calculations, representing the angle between the incident wave and the normal to the surface. By doing so, the coordinate system is transformed with $x \rightarrow x'$ defining a new propagation axis, which is given by:

$$x' = x * cos(\theta) + y * sin(\theta) \tag{2.62}$$

With x′ the coordinate system expands from only regarding the x-axis to being two-dimensional, including the y-axis. Modifying the calculations from normal incidence to oblique incidence is done simply by replacing x with x′. This yields modified expressions for the incident and reflected wave properties. The key equations are:

- **Incident sound pressure:** Equation (B.134)

$$p_i(x',t) = \hat{p}_0 * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.63}$$

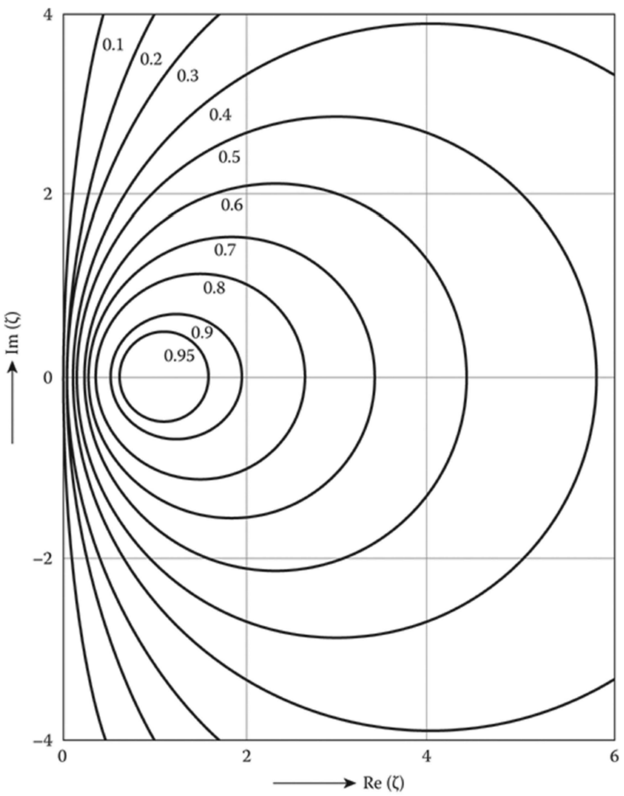


Figure 2.3: Circles of constant absorption coefficient in the complex wall impedance plane for normal sound incidence, with the absorption coefficient values indicated by the numbers next to the circles, [8].

- **Incident particle velocity:** Equation (B.135)

$$v_i(x', t) = \frac{\hat{p_0}}{\rho_0 * c} * cos(\theta) * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.64}$$

- **Reflected sound pressure:** Equation (B.136)

$$p_r(x', t) = R * \hat{p}_0 * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.65}$$

- **Reflected particle velocity:** Equation (B.137)

$$v_r(x', t) = -R * \frac{\hat{p_0}}{\rho_0 * c} * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.66}$$

- **Wall impedance:** Equation (B.138)

$$Z = \left(\frac{p}{v_n}\right) \to Z = \frac{\rho_0 * c}{cos(\theta)} * \frac{1+R}{1-R} \tag{2.67}$$

- **Reflection coefficient:** Equation (B.139)

$$R = \frac{Z * cos(\theta) - \rho_0 * c}{Z * cos(\theta) + \rho_0 * c} = \frac{\zeta * cos(\theta) - 1}{\zeta * cos(\theta) + 1} \tag{2.68}$$

- **Absorption coefficient:** Equation (B.140)

$$\alpha(\theta) = \frac{4 * Re(\zeta) * cos(\theta)}{(|\zeta| * cos(\theta))^2 + 2 * Re(\zeta) * cos(\theta) + 1} \tag{2.69}$$

- **Sound pressure at the wall:** Equation (B.141)

$$p(x,y) = \hat{p}[1 + |R|^2 + 2 * |R| * cos(2 * k * x * cos(\theta) + \chi]^{1/2} * e^{-i*k*y*(sin\theta)} \quad (2.70)$$

- **Particle velocity at the wall:** Equation (B.142)

$$c_y = \frac{\omega}{k_y} = \frac{\omega}{k * sin(\theta)} = \frac{c}{sin(\theta)} \quad (2.71)$$

As with the reflections at normal incidence, the reflected wave will have a phase shift and attenuation, which leads to signs being flipped when looking at the reflected wave. Furthermore, the impedance, reflection coefficient, and absorption coefficients are evaluated at x=0, corresponding to the walls location.

2.1.8 Closed Space Sound Field

With reflections for normal and oblique incidents described, sound propagation in closed spaces can be described. In closed spaces, the propagation is notably different from free-field due to the reflections interacting with the sound source, other reflections, and boundaries anew, creating even more reflections. This continuous interaction between reflected waves and sound sources creates modal interactions, reverberation, and standing waves, which all are determining factors in a room's acoustics. To create a representation of the associated wave theory, the wave equation B.86 is modified to resemble the Helmholtz equation:

$$\Delta p + k^2 p = 0 \quad (2.72)$$

The Helmholtz equation describes wave behavior and sound propagation in closed spaces, by the spatial variation of sound pressure fields with harmonic time dependencies. Therefore it can also only be used to describe waves under the assumption of time harmonic waves with an angular frequency ω. This is given by:

$$\frac{\partial^2}{\partial t^2}\left(P(x)e^{i\omega t}\right) = -\omega^2 P(x)e^{i\omega t} \rightarrow -\omega^2 P(x)e^{i\omega t} = c^2\nabla^2 P(x)e^{i\omega t} \rightarrow \Delta p + k^2 p = 0 \quad (2.73)$$

Where the second time derivative is taken and substituted into the wave equation. Thereafter $e^{i*\omega*t}$ is cancelled as it is on both sides of the equation, minimizing the expression to equal B.143. The wave number k is an eigenvalue, as the wave equation can only yield non-zero solutions for equations B.145 and B.146 for particular discrete values, as k is still given by $k = \frac{\omega}{c}$. For k values at specific nodes, equation B.147 is used.

$$Z * \frac{\partial p}{\partial n} + i * \omega * \rho_0 * p = 0 \quad (2.74)$$

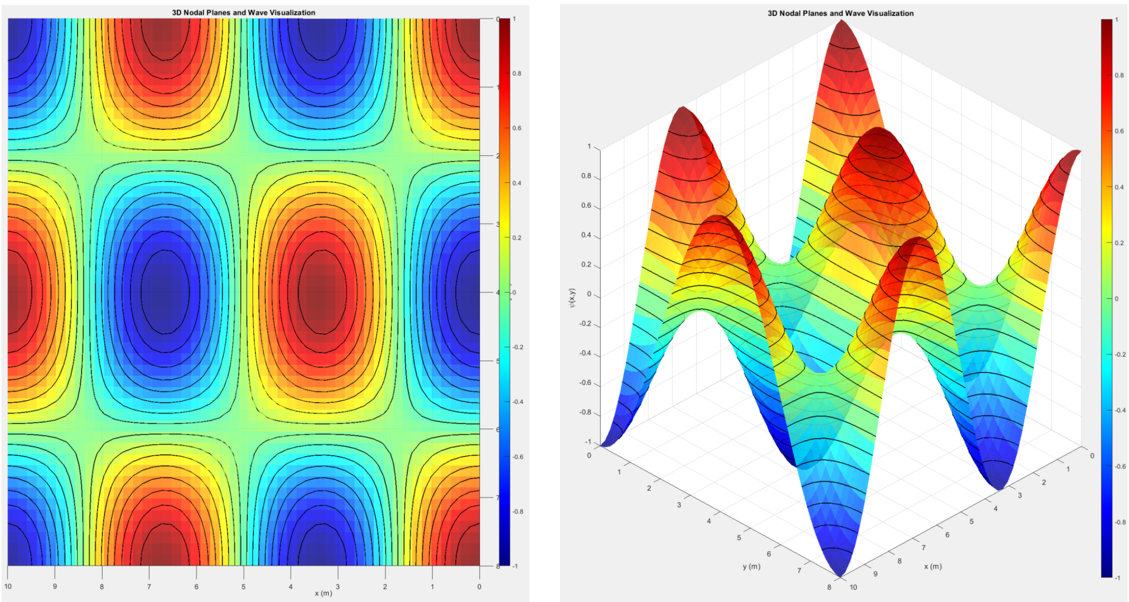
$$\zeta * \frac{\partial p}{\partial n} + i * k * p = 0 \quad (2.75)$$

$$k_{n_x n_y n_z} = \pi * \left(\left(\frac{n_x}{L_x}\right)^2 \left(\frac{n_y}{L_y}\right)^2 \left(\frac{n_z}{L_z}\right)^2\right)^{\frac{1}{2}} \quad (2.76)$$

In a rectangular room of dimensions $L_x \times L_y \times L_z$, the general solution for the pressure distribution considering rigid boundaries is given by:

$$p(x, y, z, t) = P_0 \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) e^{i\omega t}, \tag{2.77}$$

Where n_x, n_y, n_z are index numbers along the respective axes signifying which nodal plane is being referred to. Equation B.148 describes a three-dimensional standing wave, wherein the sound pressure is zero for all times, at any point where the cosines equal to zero. All values of x (and y or z) which are odd integers of $\frac{L_x}{n_x}$ result in cosines equal to zero. The intersection between the x, y, and z axis equidistant planes are referred to as nodal planes, which are mutually orthogonal. It should be noted that nodal surfaces differ from nodal planes, as nodal surfaces are used to describe non-orthogonal surfaces, which may not even be planar. The intersections between nodal planes will therefore also always have a sound pressure of zero. In figure B.4 a sound pressure distribution can be seen in an 8x10 meter room subjected to 40 and 50Hz. In other words, this particular room has 3 room nodes in the x direction and 2 room nodes in the y direction, at z=0.



(a) Nodal planes seen from the top

(b) Nodal planes as seen from a corner.

Figure 2.4: Nodal plane illustrations in an 8x10 meter room subjected to 40 and 50Hz.

By using equations B.148 eigenvalues can be found, and with those in place, the eigenfrequencies of a room can be found with equation B.149, where $k_{n_x n_y n_z}$ os given by equation B.147. In table B.5 the first 20 eigenfrequencies of a room with dimensions 4.7 × 4.1 × 3.1 m can be seen, using 340 m/s as c.

$$f_{n_c n_y n_z} = \frac{c}{2 * \pi} * k_{n_x n_y n_z} \tag{2.78}$$

To find the number of room modes in a room, every combination of indexes must be calculated for a given frequency range. This is done by equations B.150 and B.151 and is realistically done by looping a code such as ***HER SKAL INDSÆTTES KODELINK***, as the amount of computations necessary is unfathomably large. Using the same room as in [7], a frequency threshold of 200 Hz has 78 modes and 71 unique eigenfrequencies, whereas a threshold of 2000 Hz has a threshold of 52.126 modes and 9850 eigenfrequencies. If the hearing spectrum up to 20 kHz is used, it leads to a staggering 49.855.567 modes and 49.808.708 eigenfrequencies.

fn	nx	ny	nz	fn	nx	ny	nz
36.17	1	0	0	90.47	1	2	0
41.46	0	1	0	90.78	2	0	1
54.84	0	0	1	99.42	0	2	1
55.02	1	1	0	99.80	2	1	1
65.69	1	0	1	105.79	1	2	1
68.75	0	1	1	108.51	3	0	0
72.34	2	0	0	109.68	0	0	2
77.68	1	1	1	110.05	2	2	0
82.93	0	2	0	115.49	1	0	2
83.38	2	1	0	116.16	3	1	0

Table 2.3: Eigenfrequencies of a rectangular room with dimensions 4.7 × 4.1 × 3.1 m (in Hz), [7].

$$\left(\frac{2 * f_{max}}{c}\right)^2 = \left(\frac{2 * 20000}{c}\right)^2 = 116.619^2 \tag{2.79}$$

$$\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2 \leq 116.619^2 \tag{2.80}$$

2.1.9 Room Geometry in Relation to Acoustics

Until now, only rectangular rooms have been discussed, as they form a more intuitive understanding. When more flamboyant room geometry is in use, all sound propagation is thought of as rays instead of spheres. By using sound rays to describe propagation instead of spheres, it simplifies the understanding of a room's reflections by reducing the computational complexity and increasing the graphical understanding. It is possible by the limiting case of very high frequencies, where any frequency above 1 kHz is considered high, as the wavelength (34.3cm) is considered small in comparison with typical room dimensions.

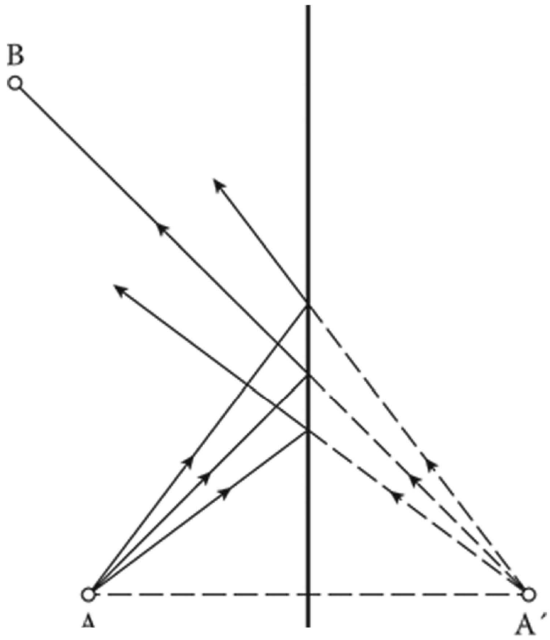
Image Sources

Image sources are an intuitive way to describe reflections using sound rays. Figure B.5 shows a basic image source at A'. Image sources are best described as an identical sound source placed exactly mirrored according to a wall. The original sound source A has in figure B.5 3 sound rays hitting a wall. As the wall is considered smooth, the reflection will have an identical angle of departure as the angle of incidence. This is furthermore underlined as identical sound rays propagate from A'.

In this exact example, the image source is meant to determine where on the wall A's reflection will coincide with B, which is also why the direct sound ray is neglected. The image source finds that the middle sound ray is the only matching reflection, using sound rays at least. It should still be remembered that sound sources still propagate omnidirectionally unless otherwise specified. It should also be noted that for now, reflection/absorption coefficients are currently not used to describe ray propagation. If those coefficients were applied, the sound rays would decay accordingly.

Figure B.6 is a more interesting example, as a non-rectangular room is used. The first image source A′ is placed mirrored to the original source, but the second image source A″ is not mirrored to the original source. Instead, the image source A″ is the image source to A′ if the mirrorplane is placed at the next wall, which the reflection should hit. This can be done continuously, unless reflection/absorption coefficients are used.

In general, image sources and their corresponding reflection diagrams are used with 3-5 orders for initial investigation, and by using programs such as ODEON, higher order diagrams can be made.



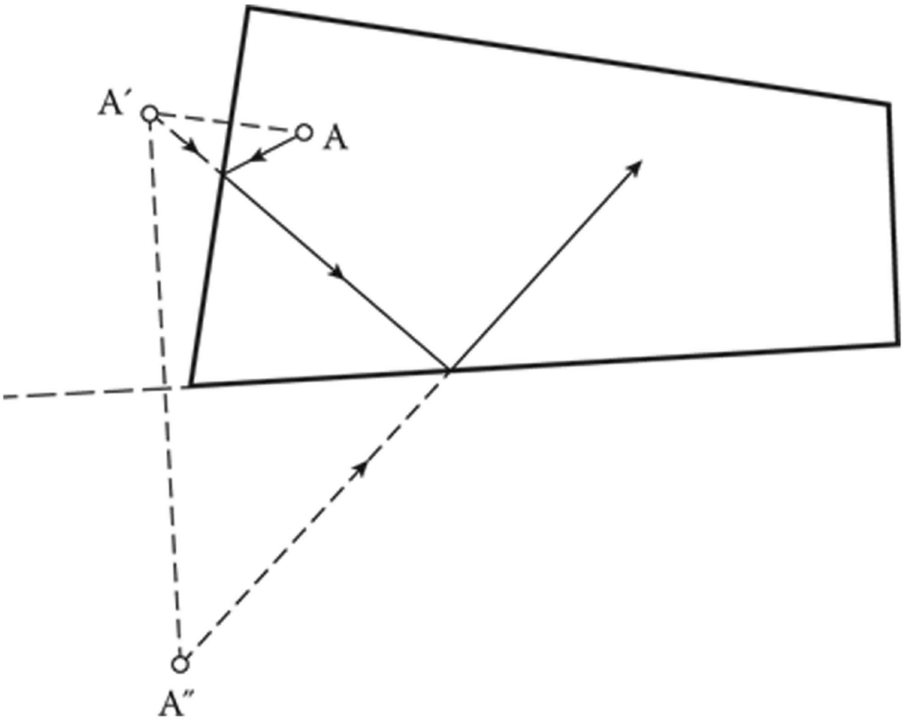


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Figure 2.6: Enter Caption

A significant detail regarding image sources is that they are infinite in the same way as orthogonal mirrors create an infinite space. As most rooms have at least four walls, a floor, and a ceiling, image sources will multiply incredibly fast. For a room with N-plane walls, the total number of image sources for a given order i is $N * (N-1)^{i-1}$, for $i \geq 1$. The total number of images is given by equation B.152, which for $N = 4$ and $i = 5$ yields a total of 160 image sources:

$$N(i_0) = N * \frac{(N-1)^{i_0} - 1}{N-2} \tag{2.81}$$

Moreover, as ceilings and floors has to be taken into consideration as well, an amount of mirrored rooms will also be present above and below the original room, each with similar amounts of image sources for their respective plane. For a simple rectangular room, this could look as shown in figure 2.7, where the image should be imagined as one plane, with a mirrored one on top and below for order i.

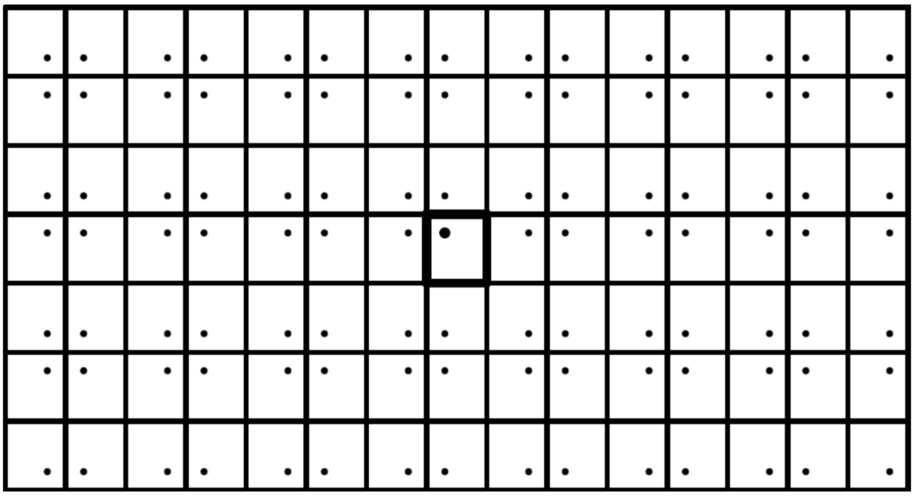

Figure 2.7: Image sources as they appear in adjacent theoretical rooms, with the original room in the middle. This plane of image sources is multiplied by the order i of image planes.

Image sources/image rooms can be used to determine the rate of reflections, yielding a temporal distribution. The temporal distribution describes the intensity and time dependency of reflections. Using equation 2.82, the temporal distribution can be calculated for each time t, if absorption coefficients of all walls were frequency independent. However, as all materials are frequency dependent, equation 2.83 is used instead, as it describes the response of a single pulse, with A_n being the strength and t_n the time of arrival for reflection n. The temporal distribution is partitioned into: direct sound, early reflections, and reverberation. The time delay and intensity of the early reflections as well as the reverberation are detrimental for the reverberation of a room, and in general for a room's acoustical properties, as described in section B.3 on page 47. In figure 2.8, a temporal distribution can be seen using an echogram. The echogram shows the pulses and their intensity along the time axis, with the first pulse being the direct sound at $t = 0$. All subsequent reflections carry less and less density, but will also exponentially multiply in density, as an increasing amount of continously weaker reflections appear.

$$s'(t) = \sum_n A_n * s(t - t_n) \tag{2.82}$$

$$g(t) = \sum_n A_n * \delta(t - t_n) \tag{2.83}$$

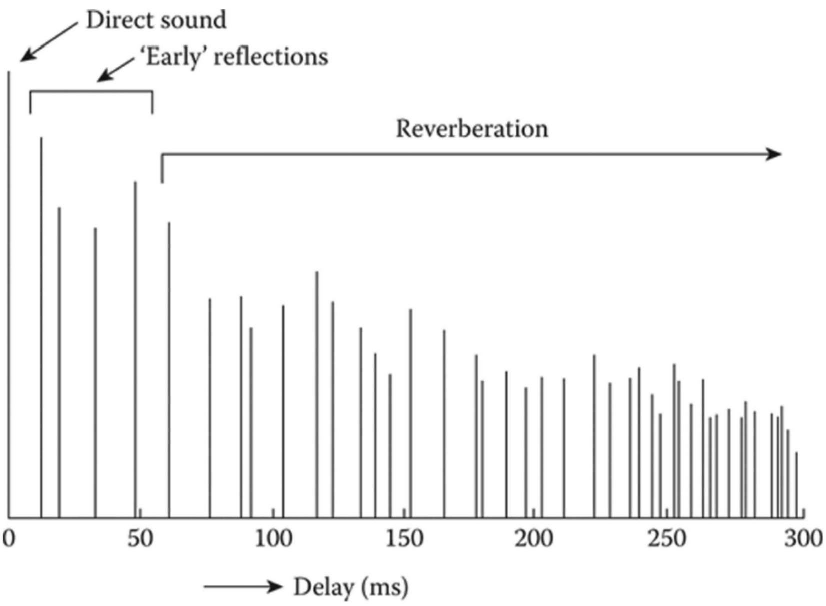

Figure 2.8: Temporal distribution showing the impulse response of a room, [8].

2.1.10 Sound Absorbers

Absorbers in Construction

Absorbers in Furnishment

2.1.11 Measuring Techniques

Impulse Response of a Room

Acoustical Properties of Materials

skal være segway til impulsrør og teknisk analyse

2.2 Existing Solutions For Determining Acoustics

2.2.1 Impulse Response of a Room

2.2.2 Impedance Tube

2.3 Problem Statement

Noget a la:

"How can a square impedancetube capable of measuring very low frequency properties of construction materials be developed?"

3 Technical Analysis

In an effort to analyze the technical needs of the project, the demands set for a prototype have been revisited. This identified the following areas of interest:

3.1 Reverberation Room Method

3.2 Standing Wave Method

[19]

3.3 Multi Microphone Method

[20]

3.4 Signal Processing Algorithm

4 | Demand Specification

4.1 High Level Specification

4.2 Demands Set by AAU

4.3 Functional Specification

5 System Design

5.1 Establishing an Algorithm

Specifikationer så som:

- Maximum Length Sequence
- Fixed point

Forklaring af algoritme - formål
Gennemgang af matematik
Gennemgang af kode
valg af enhed til at køre algoritmen - fordele, ulemper og constraints
Test af algoritmen på kendte signaler

5.2 Physical Setup

Størrelse på rør i forhold til sample sizes - eventuelt gennemgang af begrænsninger

Udforsk betydning af firkantet rør kontra rundt - fiberline rør måske? https://fiberline.com/da/produ
x-100-x-6-mm-060280

Afstande på rør

Elektrisk opsætning/valg af speaker, mic, amp osv

6 | Integration

7 Acceptance test

General acceptance test

Sammenligningstest med Bruel og Kjær 4002 Standbølge Apparat af forskellige materialer.

8 Discussion

9 Conclusion

"?"

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Glossary

C50 Clarity 50 ms. 2, 4, 5, 46, 48, 49

C80 Clarity 80 ms. 4, 48

D50 Definition 50 ms. 2, 5, 46, 49

EDT Early Decay Time. 2–4, 46–48

RT60 Reverberation Time 60 dB. 2–4, 46–48

SPL Sound pressure level. 9, 10, 53

T10 Reverberation Time 10 dB. 4, 48

T20 Reverberation Time 20 dB. 3, 4, 47, 48

T30 Reverberation Time 30 dB. 3, 4, 47, 48

A | Appendix

B Mathematical Notations

B.1 Variables

$p(t)$ is the instantaneous sound pressure of the impulse response measured at a point in the room.
$p_{10}(t)$ is the instantaneous sound pressure of the impulse response measured at 10m in free field.
p_0 is $20\mu Pa$.
T_0 is 1 second.
L_{pE} is the sound pressure exposure level of $p(t)$.
$L_{pE,10}$ is the sound pressure exposure level of $p_{10}(t)$.
L_p is the sound pressure level averaged across every measurement point.
$L_{p,10}$ is the sound pressure level measured at 10 m in free field.
L_p is the sound pressure level averaged across every measurement point.
L_W is the sound power level of the sound source, and should be measured according to DS 3741.
$A = \sum S_i * \alpha_i$.
C_{t_e} is the early to late index.
t_e is the early time limit (e.g. 50 or 80 ms).
$p(t)$ is the measured sound pressure of the impulse response.
$c^2 = \kappa \frac{p_0}{\rho_0}$.
p is the sound pressure.
ρ_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$ for air).
p_0 is the static sound pressure (measured in Pascal, where 1 $Pascal = 1\frac{kg}{m*s^2}$.
κ is the adiabatic component.
v is a vector representing particle velocity.
t is time.
ρ being the variable gas density.
$\hat{p}$ is amplitude.
k is the propagation constant $k = \frac{\omega}{c}$.
ω is the angular frequency, which can be used to obtain the temporal period $T = \frac{2*\pi}{\omega}$.
$f = \frac{\omega}{2*\pi} * \frac{1}{T}$ with the unit Hz.
$\rho_0 * c$ found by table values for ρ_0 and c in the correct medium.
θ and ϕ angles,
c^2 on both sides,
Q, being the rate in $\frac{m^3}{s}$ of "liquid" being expelled by the sound source located at $r = 0$.
The dot above Q denotes differentation w.r.t. time (t).
r and frequency $\omega = k * c$.
$\frac{p}{v}$ tends asymptotically to $\rho_0 c$.
Γ_{nm} are the Fourier series coefficients of order n and degree m,

$h_n(kr)$ is the Hankel function of the first kind with order n,
$Y_n^m(\theta, \phi)$ are the spherical harmonics,
$\tilde{p}$ denotes root mean square of the sound pressure, i.e
$\sqrt{\bar{p^2}}$ and $\tilde{p_0} = 2 * 10^{-5}[\frac{N}{m^2}]$ is an internationally fixed value corresponding to the hearing threshold at 1000Hz,
N being loudness in sones and
L_N being loudness in phons
$x = 0$ and the wave will arrive from the negative direction,

B.2 Formulas

$$G = 10 * log10\left(\frac{\int_0^\infty p^2(t)\,dt}{\int_0^\infty p_{10}^2(t)\,dt}\right) = L_{pE} - L_{pE,10}[dB] \tag{B.1}$$

$$L_{pE} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p^2(t)\,dt}{p_0^2}\right][dB] \tag{B.2}$$

$$L_{pE,10} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p_{10}^2(t)\,dt}{p_0^2}\right][dB] \tag{B.3}$$

$$L_{pE,10} = L_{pE,d} + 20 * log10(d/10)[dB] \tag{B.4}$$

$$L_p = L_{p,10}[dB] \tag{B.5}$$

$$G = L_p - L_W + 31dB \tag{B.6}$$

$$RT60 = \frac{0.161 * V}{S_i * \alpha_i} \leftrightarrow RT = \frac{0.161 * V}{A} \tag{B.7}$$

$$A = \sum S_i * \alpha_i \tag{B.8}$$

$$C50 = 10 * log10\left(\frac{Energy(0 - 50ms)}{Energy(51ms - end)}\right)[dB] \tag{B.9}$$

$$C80 = 10 * log10\left(\frac{Energy(0 - 80ms)}{Energy(81ms - end)}\right)[dB] \tag{B.10}$$

$$C_{t_e} = 10 * log10\left(\frac{\int_0^{t_e} p^2(t)\,dt}{\int_{t_e}^\infty p^2(t)\,dt}\right)[dB] \tag{B.11}$$

$$C50 = 10 * log10\left(\frac{D50}{1 - D50}\right)[dB] \tag{B.12}$$

$$D50 = \frac{\int_0^{0.050} p^2(t)\,dt}{\int_0^\infty p^2(t)\,dt} \tag{B.13}$$

$$c = 331.4 + 0.6 * T[\frac{m}{s}] \tag{B.14}$$

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \tag{B.15}$$

$$c^2 = \kappa \frac{p_0}{\rho_0} \tag{B.16}$$

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \tag{B.17}$$

$$grad\ p = -\rho_0 * \frac{\partial v}{\partial t} \tag{B.18}$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \tag{B.19}$$

$$\rho_0\ div\ v = -\frac{\partial \rho}{\partial t} \tag{B.20}$$

$$\frac{p}{p_0} = \kappa \frac{\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} * \frac{\delta * T}{T + 273} \tag{B.21}$$

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{B.22}$$

$$p(x,t) = F(c * t - x) + G(c * t + x) \tag{B.23}$$

$$p(x,t) = \hat{p} * e^{i*k*(c*t-x)} = \hat{p} * e^{i*(\omega*t-k*x)} \tag{B.24}$$

$$\lambda = \frac{2 * \pi}{k} \leftrightarrow \frac{2 * \pi}{\frac{\omega}{c}} \leftrightarrow \frac{2 * \pi * c}{\omega} \leftrightarrow \frac{c}{\frac{\omega}{2*\pi}} \leftrightarrow \frac{c}{f} \tag{B.25}$$

$$v(x,t) = \frac{1}{\rho_0 * c}[F(c * t - x) - G(c * t + x)] \tag{B.26}$$

$$\frac{p}{v} = \rho_0 * c \tag{B.27}$$

$$\rho_{0_{air}} * c_{air} = 1.2 \left[\frac{kg}{m^3}\right] * 343 \left[\frac{m}{s}\right] = 414 \left[\frac{kg}{m^2 s}\right] \tag{B.28}$$

$$I = \bar{p}\bar{v} = \frac{\bar{p^2}}{\rho_0 * c} \tag{B.29}$$

$$w = \frac{I}{c} = \frac{\bar{p^2}}{\rho_0 * c^2} \tag{B.30}$$

$$\Delta p = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial p}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 p}{\partial \phi^2}. \tag{B.31}$$

$$\Delta p = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial p}{\partial r}\right) \tag{B.32}$$

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r} \frac{\partial p}{\partial r} \tag{B.33}$$

$$c^2\left(\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}\frac{\partial p}{\partial r}\right)=\frac{\partial^2 p}{\partial t^2} \tag{B.34}$$

$$\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}*\frac{\partial p}{\partial r}=\frac{1}{c^2}*\frac{\partial^2 p}{\partial t^2} \tag{B.35}$$

$$p(r,t)=\frac{\rho_0}{4*\pi*r}*\dot{Q}\left(t-\frac{r}{c}\right) \tag{B.36}$$

$$v_r=\frac{1}{4*\pi*r^2}\left[Q\left(t-\frac{r}{c}\right)+\frac{r}{c}*\dot{Q}\left(t-\frac{r}{c}\right)\right] \tag{B.37}$$

$$v_r=\frac{p}{\rho_0*c}*\left(1+\frac{1}{i*k*r}\right) \tag{B.38}$$

$$\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}*\frac{\partial p}{\partial r}+\frac{1}{r^2}\left(\frac{1}{sin(\theta)}*\frac{\partial}{\partial*\theta}\left(sin(\theta)*\frac{\partial}{\partial*\theta}\right)+\frac{1}{sin^2(\theta)}*\frac{\partial^2}{\partial\phi^2}\right)=\frac{1}{c^2}*\frac{\partial^2 p}{\partial t^2} \tag{B.39}$$

$$p(r,\theta,\phi)=A*\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Gamma_{nm}h_n(kr)Y_n^m(\theta,\phi) \tag{B.40}$$

$$Y_n^m(\theta,\phi)=\sqrt{\frac{(2n+1)}{4*\pi}*\frac{(n-|m|)!}{(n+|m|)!}}*P_n^{|m|}*(cos(\theta)*\begin{cases}cos|m|\phi & if\ \ m\geq 0\\ sin|m|\phi & if\ \ m<0\end{cases} \tag{B.41}$$

$$SPL=20*log10(\frac{\tilde{p}}{\tilde{p_0}}[dB] \tag{B.42}$$

$$N=\left(10^{\frac{L_N-40}{10}}\right)^{0.30103}\approx 2^{\frac{L_N-40}{10}} \tag{B.43}$$

$$L_N=40+log_2(N) \tag{B.44}$$

Phon	0	10	20	30	40	50	60	70	80	90	100
Sone	0 (Inaudible)	0.0625	0.125	0.25	1	2	4	8	16	32	64

Table B.1: Relationship between phons and sones.

- **Reflection Coefficient:**

$$R=\frac{I_r}{I_i} \tag{B.45}$$

where I_r is reflected intensity and I_i is incident intensity.

- **Diffusion Coefficient:**

$$D=1-\frac{\sum S(\theta)}{NS_{\text{ideal}}} \tag{B.46}$$

where $S(\theta)$ is the scattered energy in direction θ, and S_{ideal} is the ideally scattered energy.

- **Transmission Loss (TL):**

$$TL=10\log_{10}\left(\frac{I_i}{I_t}\right) \tag{B.47}$$

where I_t is the transmitted intensity.

Property	Definition	Typical Values
Reflection Coefficient (R)	Fraction of sound energy reflected	0.8 - 0.98 (Concrete), 0.1 - 0.5 (Foam)
Diffusion Coefficient (D)	Measure of even sound scattering	0.2 (Flat wall), 0.6 - 0.9 (Diffuser)
Transmission Loss (TL)	Reduction of sound through a barrier (dB)	25 dB (Drywall), 50+ dB (Concrete)
Resonance Frequency (f_r)	Frequency at which the wall vibrates	50 - 500 Hz (Depending on material)
Absorption Coefficient (α)	Fraction of energy absorbed	0.02 (Concrete), 0.8+ (Acoustic panels)
Wall Impedance (Z)	Opposition to sound transmission, depends on density and stiffness.	1.5×10^6 kg·m^{-2}·s^{-1} (Concrete), 5×10^5 kg·m^{-2}·s^{-1} (Brick), 10^4 kg·m^{-2}·s^{-1} (Foam)
Wall Admittance (ζ)	Measure of a wall's ability to accept sound energy, reciprocal of impedance.	0.001 (Concrete), 0.003 (Brick), 0.1 (Foam)

Table B.2: Acoustic properties of walls and their typical values.

- **Resonance Frequency (Panel):**

$$f_r = \frac{60}{d}\sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{B.48}$$

where:d = panel thickness, E = Young's modulus, ρ = density, and ν = Poisson's ratio.

- **Absorption Coefficient:**

$$\alpha = 1 - |R|^2 \tag{B.49}$$

- **Wall Impedance:**

$$Z = \left(\frac{p}{v_n}\right) \tag{B.50}$$

where: v_n denotes the particle component normal to the wall.

- **Wall Admittance:**

$$\zeta = \frac{Z}{\rho_0 * c} \tag{B.51}$$

$$p_i(x,t) = \hat{p}_0 * e^{i*(\omega*t-k*x)} \tag{B.52}$$

$$v_i(x,t) = \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t-k*x)} \tag{B.53}$$

$$p_r(x,t) = R * \hat{p}_0 * e^{i*(\omega*t+k*x)} \tag{B.54}$$

$$v_r(x,t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t+k*x)} \tag{B.55}$$

$$p(0,t) = (1+R) * \hat{p}_0 * e^{i*(\omega*t)} \tag{B.56}$$

$$v(0,t) = (1-R) * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t)} \tag{B.57}$$

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \rho_0 * c * \frac{1+R}{1-R} \tag{B.58}$$

$$\zeta = \frac{Z}{\rho_0 * c} \rightarrow \zeta = \frac{\rho_0 * c * \left(\frac{1+R}{1-R}\right)}{\rho_0 * c} \rightarrow \zeta = \frac{1+R}{1-R} \tag{B.59}$$

$$R = \frac{\zeta - 1}{\zeta + 1} \tag{B.60}$$

$$\alpha = \frac{4 * Re(\zeta)}{|\zeta|^2 + 2 * Re(\zeta) + 1} \tag{B.61}$$

$$x' = x * cos(\theta) + y * sin(\theta) \tag{B.62}$$

$$p_i(x', t) = \hat{p}_0 * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.63}$$

$$v_i(x', t) = \frac{\hat{p_0}}{\rho_0 * c} * cos(\theta) * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.64}$$

$$p_r(x', t) = R * \hat{p}_0 * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.65}$$

$$v_r(x', t) = -R * \frac{\hat{p_0}}{\rho_0 * c} * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.66}$$

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \frac{\rho_0 * c}{cos(\theta)} * \frac{1+R}{1-R} \tag{B.67}$$

$$R = \frac{Z * cos(\theta) - \rho_0 * c}{Z * cos(\theta) + \rho_0 * c} = \frac{\zeta * cos(\theta) - 1}{\zeta * cos(\theta) + 1} \tag{B.68}$$

$$\alpha(\theta) = \frac{4 * Re(\zeta) * cos(\theta)}{(|\zeta| * cos(\theta))^2 + 2 * Re(\zeta) * cos(\theta) + 1} \tag{B.69}$$

$$p(x, y) = \hat{p}[1 + |R|^2 + 2 * |R| * cos(2 * k * x * cos(\theta) + \chi]^{1/2} * e^{-i*k*y*(sin\theta)} \tag{B.70}$$

$$c_y = \frac{\omega}{k_y} = \frac{\omega}{k * sin(\theta)} = \frac{c}{sin(\theta)} \tag{B.71}$$

B.3 Room Acoustics

Determining good acoustics is as subjective as determining which music is pleasant. While subjective qualities are present, objective aspects should be considered as well, as the very shape of a person (ears, head, shoulders, etc.) influence the perception of sound as well, [4, 5, 6, 7, 8]. Furthermore, good acoustical properties are not as "simple" as attempting to achieve anechoic conditions, as the inherent psychoacoustical properties of an anechoic room are not considered pleasant. Fortunately, guidelines exist for what is, in general, considered good acoustics. The metrics are room gain (G), reverberation time (RT60), early decay time (EDT), speech intelligibility (C50), definition (D50), musical clarity, and temporal distribution. The first four can be estimated by using the following:

G:

The gain of a room is very literally the amplification or attenuation that a room provides. It can be measured using a calibrated omnidirectional speaker (such as a dodecahedral speaker), where an impulse is first measured in free field (an anechoic room), and then in the room to be examined. The measurement in free field is done by measuring at a distance of 10 m from the sound source, and is repeated for at least 5 measurements at various locations to create an average, [9, 10]. As in free field, the room to measure should have an equal amount of measurements made. The gain of a room can be described with the following equations from DS 3382-1, [10]:

$$G = 10 * log10\left(\frac{\int_0^\infty p^2(t)\,dt}{\int_0^\infty p_{10}^2(t)\,dt}\right) = L_{pE} - L_{pE,10}[dB] \tag{B.72}$$

$$L_{pE} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p^2(t)\,dt}{p_0^2}\right][dB] \tag{B.73}$$

$$L_{pE,10} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p_{10}^2(t)\,dt}{p_0^2}\right][dB] \tag{B.74}$$

Where: $p(t)$ is the instantaneous sound pressure of the impulse response measured at a point in the room, $p_{10}(t)$ is the instantaneous sound pressure of the impulse response measured at 10m in free field, p_0 is $20\mu Pa$, T_0 is 1 second, L_{pE} is the sound pressure exposure level of $p(t)$, and $L_{pE,10}$ is the sound pressure exposure level of $p_{10}(t)$.

If any of the rooms are not large enough to complete the measurements at 10 meters, they can instead be done at 3 meters and modified with:

$$L_{pE,10} = L_{pE,d} + 20 * log10(d/10)[dB] \tag{B.75}$$

Or even simpler by using, [9, 10]:

$$L_p = L_{p,10}[dB] \tag{B.76}$$

Where: L_p is the sound pressure level averaged across every measurement point and $L_{p,10}$ is the sound pressure level measured at 10 m in free field.

Alternatively, if a sound source with known sound power level is available, the gain can be obtained by the following, [9, 10]:

$$G = L_p - L_W + 31dB \tag{B.77}$$

Where: L_p is the sound pressure level averaged across every measurement point and L_W is the sound power level of the sound source, and should be measured according to DS 3741.

RT60:
Reverberation **T**ime 60 describes the time it takes for sound a level to decay 60 dB. In many practical applications, background noise makes it difficult to measure a full 60 dB decay, and therefore, T20 or T30 is used instead. T20 and T30 differ from RT60 by measuring either 20 or 30 dB decay and following the description set in DS ISO 3382-2, the decay time must be measured from -5 to -2(3)5 dB, and not from 0 to -2(3)0 dB, [11, 9, 12, 13]. In figure B.1a, an example of measuring T30 can be seen, where it should be noted that the decay is first measured from -5 dB. When using T20 or T30, the time measured should be multiplied by 3 or 2 to harmonize with RT60. It should furthermore be noted that T20 is most frequently used, as DS 3382-2 states that *"the subjective evaluation of reverberation is related to the early part of the decay"*, and that *"the signal-to-noise ratio is often a problem in field measurements, and it is often difficult or impossible to get a evaluation range of more than 20 dB."*, [11].

Sabine's equation for a diffuse sound field can be used to estimate a room's reverberation time. It takes the input V for the volume of the room in cubic meters, α for the absorption coefficient, and S for the surface area of the room in meters, and is used as:

$$RT60 = \frac{0.161 * V}{S_i * \alpha_i} \leftrightarrow RT = \frac{0.161 * V}{A} \tag{B.78}$$

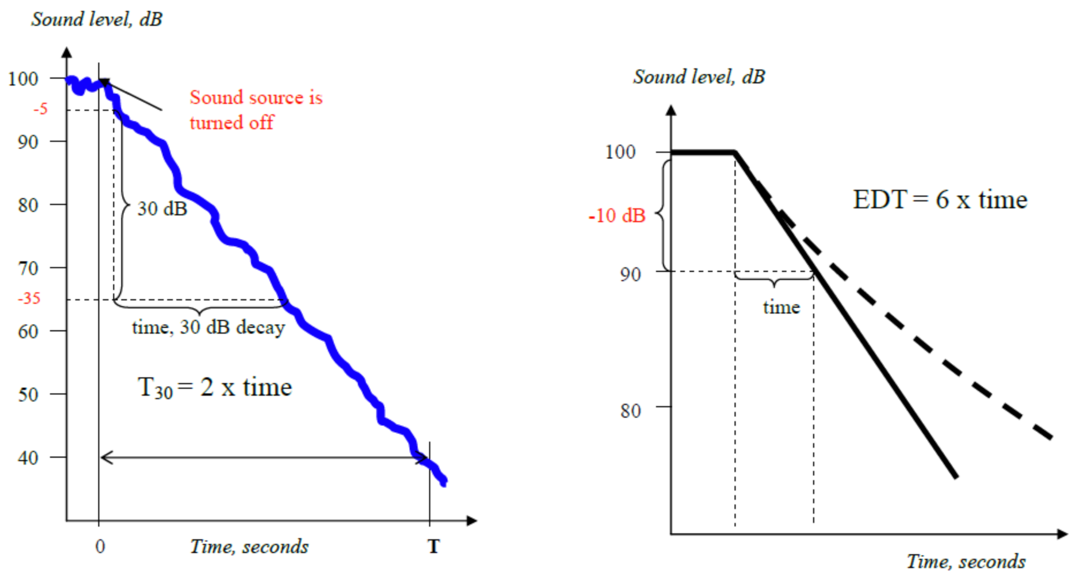
Where:

$$A = \sum S_i * \alpha_i \tag{B.79}$$

The reasoning behind equation B.79/the lower fraction of B.78 is to compensate for different materials with different absorption coefficients in a room. It should, however be denoted that since Sabine's formula assumes a diffuse sound field, it is best used concerning EDT rather than T30 or T20 due to the corresponding decay curves. This can be seen in figure B.1b, where the solid straight line matches a continued approximation derived from the EDT. The dashed line is a more realistic decay curve of a room with absorbent materials, such as acoustic ceilings, [9].

In general, some optimal RT ranges are, [14, 15, 16, 9, 17]:

- Speech-oriented spaces, such as classrooms, meeting rooms, lecture halls, etc. require a short reverberation time of 0.3-1 second, to ensure speech intelligibility.
- Musical performance rooms, such as opera houses, concert halls, clubs, etc. would typically benefit from longer reverberation times, ranging from 1.5-3 seconds.
- Multi-use rooms such as dining halls, auditoriums, etc. benefit from a compromise, with reverberation times of 1.2-2 seconds.
- Sacred rooms/rooms of worship, such as churches, chapels, monasteries, etc. have much wider ranges, as they (dependent on religious purposes) have a reverberation time between 3-10 seconds.



(a) Definition of 30 dB decay (T30). Notice the timer starts at -5dB, [9].

(b) Illustration of EDT, [9].

Figure B.1: T30 and EDT illustrations, [9].

EDT/T10:

Early decay time describes the first 10 dB decay within a room and is sometimes denoted T10, similarly to T20 and T30. The main difference from T20 and T30 is that EDT does not include the same -5 dB buffer but instead measures the actual first 10 dB decay. According to [9], [18] and DS 3382-1, [10], the EDT should be the subjectively most important reverberation characteristic when determining the acoustic properties of a room. EDT is determined by the first 10 dB of attenuation multiplied by 6 to reach a number comparative with RT60. The EDT equation is most beneficial when it comes to the psychoacoustic perception of a room and for determining "ideal" decay time, [18]. This is caused by the minimum of summed reflections present within the first 10 dB decay.

C50/C80:

To determine the clarity relation of a room as an acoustical parameter, C50 or C80 is used. C50 estimates the reflections and their energy present after 50 ms, while C80 estimates them after 80 ms. The idea is that reflections present after the critical time limit muddles perceived speech and reduce clarity. The critical time limit varies from person to person, with age, and is furthermore dependent on what exactly is being listened to, [9, 14, 15, 17, 10]. C50 is most often used for speech intelligibility and C80 is used for music clarity. The C50 and C80 relation is calculated by:

$$C50 = 10 * log10\left(\frac{Energy(0-50ms)}{Energy(51ms-end)}\right)[dB] \tag{B.80}$$

$$C80 = 10 * log10\left(\frac{Energy(0-80ms)}{Energy(81ms-end)}\right)[dB] \tag{B.81}$$

Where the "Energy" values are derived from a broadband impulse response in the room. The impulse response is recorded and divided into integrated squared energy values before and after the critical time limit, [9, 14, 15, 17, 10]. Both can also be found generally by:

$$C_{t_e} = 10 * log10\left(\frac{\int_0^{t_e} p^2(t)\,dt}{\int_{t_e}^{\infty} p^2(t)\,dt}\right)[dB] \tag{B.82}$$

Where: C_{t_e} is the early to late index. t_e is the early time limit (e.g. 50 or 80 ms) and $p(t)$ is the measured sound pressure of the impulse response, [10].

For speech clarity, values above 0 dB are considered good, while for music clarity it depends on music type and preference, but in most cases, a value between -2 dB and +4 dB is considered acceptable, [17].

D50:newline Definition/Deutlichkeit after 50 ms is a percentage describing how many percent of the total energy is present within the first 50 ms. D50 is strongly correlated to C50, as D50 (and vice-versa C50) can be found by, [10]:

$$C50 = 10 * log10\left(\frac{D50}{1 - D50}\right)[dB] \tag{B.83}$$

Or by itself using:

$$D50 = \frac{\int_0^{0.050} p^2(t)\,dt}{\int_0^{\infty} p^2(t)\,dt} \tag{B.84}$$

B.3.1 Introduction to sound

Now that the basic metrics for evaluating a room's acoustic properties have been established, sound as an entity can be determined. To do so, a lot of equations will be instantiated, with most coming from [7]. As most of the notation and equations in [7] are equal or at the very least similar to other sources, it has been chosen as the main source for equations, meaning all formulas used in this can be found in [7] as well, unless otherwise noted. Other sources used for mathematical formulas are [4, 5, 6, 8, 10, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26]. For the time being, only ideal sound will be described, assuming no losses in propagation or obstacles. Furthermore, the medium is considered homogeneous and at rest, making the velocity of sound constant in relation to space and time. If the medium were air, the velocity is given by:

$$c = 331.4 + 0.6 * T\left[\frac{m}{s}\right] \tag{B.85}$$

With T being the temperature in degrees centigrade. C is set to be 343m/s for all following computations, based on an average temperature of 23.2 degrees. The average is used as exact calculations would require infinitesimal precision and, in practice, would be impossible to compute. An example could be a sports hall, wherein the temperature around players and audience is higher than under the rafters, [4, 5, 7, 14].

Another fundamental equation is the wave equation. The wave equation describes how sound propagates over time and space relative to the speed of sound. It is given by:

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \tag{B.86}$$

where the squared speed of sound is defined by:

$$c^2 = \kappa \frac{p_0}{\rho_0} \tag{B.87}$$

in these equations, p is the sound pressure, rho_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$, p_0 is the static sound pressure (measured in Pascal, where 1 $Pascal = 1\frac{kg}{m*s^2}$, and κ is the adiabatic component (for air $\kappa = 1.4$), [7]. In cartesian coordinates, the laplacian operator Δ is given by:

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \tag{B.88}$$

To elaborate, the wave equation is derived from the conservation of momentum relation:

$$grad\ p = -\rho_0 * \frac{\partial v}{\partial t} \tag{B.89}$$

with its one-dimensional version being:

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \tag{B.90}$$

where: v is a vector representing particle velocity, and t is time. Furthermore, the mass conservation requirement leads to:

$$\rho_0\ div\ v = -\frac{\partial \rho}{\partial t} \tag{B.91}$$

with ρ being the variable gas density. It is generally assumed that the variation in gas pressure and density is small compared to their static values. All of the above can be related by:

$$\frac{p}{p_0} = \kappa \frac{\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} * \frac{\delta * T}{T + 273} \tag{B.92}$$

By eliminating the particle velocity v and variable gas density ρ from equations B.89-B.92 one would arrive at the wave equation given in equation B.86. The wave equation holds true for sound waves in any lossless fluid and for the resulting pressure, density, and temperature variations

Plane Waves

A plane wave simply put is a fraction of a spherical wave small enough that it can be assumed to have no curvature, [11, 10, 19, 20, 4, 5, 7, 8]. Another physical assumption that can be made, is that if a sound wave is present within a tube, with a diameter/height/width significantly smaller than the wavelength of said wave, it can be assumed to be planar, see section 3.2 on page 24 for more. It is effectively an orthogonal plane to a vector in a cartesian coordinate system. If that vector is equal to the x-axis, a plane wave can be described by the wave equation (B.86) with:

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{B.93}$$

With a corresponding general solution given by:

$$p(x,t) = F(c * t - x) + G(c * t + x) \tag{B.94}$$

In equation B.94 the first term represents a plane wave propagating positively along the x-axis and the second term a negative direction. From equations B.93 and B.94 it can also be seen that a constant pressure level is present at each wavefront. By specifying F and G as exponential functions including the imaginary components, the propagation is given as:

$$p(x,t) = \hat{p} * e^{i*k*(c*t-x)} = \hat{p} * e^{i*(\omega*t-k*x)} \tag{B.95}$$

Where $\hat{p}$ is amplitude, k is the propagation constant $k = \frac{\omega}{c}$, ω is the angular frequency, which can be used to obtain the temporal period $T = \frac{2*\pi}{\omega}$. This all connects to several interpretations of the wavelength formula:

$$\lambda = \frac{2*\pi}{k} \leftrightarrow \frac{2*\pi}{\frac{\omega}{c}} \leftrightarrow \frac{2*\pi*c}{\omega} \leftrightarrow \frac{c}{\frac{\omega}{2*\pi}} \leftrightarrow \frac{c}{f} \tag{B.96}$$

Now that amplitude, wavelength, and direction are known, the only missing parameters for expressing the plane wave are frequency and intensity. The frequency is given by $f = \frac{\omega}{2*\pi} * \frac{1}{T}$ with the unit Hz. If equation B.89 is applied to equation B.94, the only non-vanishing point of the wave is parallel to the x-axis, meaning that sound behaves as longitudinal waves in fluids, and the particle velocity can be found by:

$$v(x,t) = \frac{1}{\rho_0 * c}[F(c*t-x) - G(c*t+x)] \tag{B.97}$$

To obtain the ratio between sound pressure and particle velocity, also known as the characteristic impedance of a medium, the following is used:

$$\frac{p}{v} = \rho_0 * c \tag{B.98}$$

With $\rho_0 * c$ found by table values for ρ_0 and c in the correct medium. For air, the characteristic impedance is:

$$\rho_{0_{air}} * c_{air} = 1.2\left[\frac{kg}{m^3}\right] * 343\left[\frac{m}{s}\right] = 414\left[\frac{kg}{m^2 s}\right] \tag{B.99}$$

Understanding the above relation makes it possible to determine the intensity of each wave. The plane imagined to be orthogonal to the x-axis will have energy present across its entire surface, and the average (denoted by the bar notation) product of the pressure and particle velocity on the surface yields the intensity:

$$I = \bar{pv} = \frac{\bar{p}^2}{\rho_0 * c} \tag{B.100}$$

To find the energy density of a wave, equation B.101 is used:

$$w = \frac{I}{c} = \frac{\bar{p}^2}{\rho_0 * c^2} \tag{B.101}$$

And with that, sufficient information should be available to describe any plane wave.

Spherical Waves

The sound wave will now propagate with no boundaries whatsoever, leading to ideal spherical waves. Graphically this is best understood as an inflating ball stretching equally in all directions from an infinitesmally small point source. Due to the nature of spherical waves, a spherical coordinate system will be used, using polar coordinates. A benefit from the spherical symmetry is that pressure is represented as r and is independent of the θ and ϕ angles. To convert the wave equation from cartesian to spherical coordinates, the laplacian of a function $p(r, \theta, \phi)$has to expressed in spherical coordinates as well:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial p}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 p}{\partial \phi^2}. \tag{B.102}$$

As the acoustical properties of a spherical wave are equal across all θ and ϕ angles, it's description can be made with only r and t, simplifying the laplacian to:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial p}{\partial r}\right) \tag{B.103}$$

The simplified laplacian should now be expanded to:

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} \tag{B.104}$$

And is now ready to be inputtet into the wave equation:

$$c^2 \left(\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}\right) = \frac{\partial^2 p}{\partial t^2} \tag{B.105}$$

By dividing with c^2 on both sides, the spherical wave equation is found:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{B.106}$$

A simple solution to the spherical wave equation is given in equation B.107, representing a spherical wave with the volume velocity Q, being the rate in $\frac{m^3}{s}$ of "liquid" being expelled by the sound source located at $r = 0$. The dot above Q denotes differentation w.r.t. time (t).

$$p(r,t) = \frac{\rho_0}{4 * \pi * r} * \dot{Q}\left(t - \frac{r}{c}\right) \tag{B.107}$$

As for plane waves, spherical waves have a non-vanishing component of its particle velocity. Instead of being along the x-axis, the non-vanishing point for a spherical wave is radial, and can be calculated by applying equation B.89 to B.107, yielding:

$$v_r = \frac{1}{4 * \pi * r^2}\left[Q\left(t - \frac{r}{c}\right) + \frac{r}{c} * \dot{Q}\left(t - \frac{r}{c}\right)\right] \tag{B.108}$$

To find the particle velocity of a spherical wave, equation B.109 is used. It show that the sound pressure and velocity in a spherical wave is inversely proportional to the size r and frequency $\omega = k * c$. For distances large in comparison with the wavelength, the ratio $\frac{p}{v}$ tends asymptotically to $\rho_0 c$.

$$v_r = \frac{p}{\rho_0 * c} * \left(1 + \frac{1}{i * k * r}\right) \tag{B.109}$$

Actual waves

Now that the ideal scenarios are described, a more realistic approach should also be made. Therefore directional sound radiation with angular dependence is considered in the following. It is best imagined by a singular speaker pointing into an ideal environment. As the speaker is pointing in a direction and is not omnidirectional, the resulting sound wave will not be uniformly spherical. This expands the spherical wave equation to now include the polar and azimuth angles as well:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} + \frac{1}{r^2}\left(\frac{1}{sin(\theta)} * \frac{\partial}{\partial * \theta}\left(sin(\theta) * \frac{\partial}{\partial * \theta}\right) + \frac{1}{sin^2(\theta)} * \frac{\partial^2}{\partial \phi^2}\right) = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{B.110}$$

To solve the complex spherical wave equation, the sound pressure can be expressed as a Fourier series separated into its radial and angular variables:

$$p(r,\theta,\phi) = A * \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Gamma_{nm} h_n(kr) Y_n^m(\theta,\phi) \tag{B.111}$$

Where: Γ_{nm} are the Fourier series coefficients of order n and degree m, $h_n(kr)$ is the Hankel function of the first kind with order n, $Y_n^m(\theta,\phi)$ are the spherical harmonics, forming an orthonormal basis, which are used in the Legendre polynomial:

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4*\pi} * \frac{(n-|m|)!}{(n+|m|)!}} * P_n^{|m|} * (cos(\theta) * \begin{cases} cos|m|\phi & if \;\; m \geq 0 \\ sin|m|\phi & if \;\; m < 0 \end{cases} \tag{B.112}$$

To actually use the above equations for anything, a spherical microphone array surrounding the speaker must be used to obtain empirical data. When such a measuring system is used, the coefficients of Γ_{nm} can be found with an inverse two-dimensional Fourier transform — often referred to as the inverse spherical harmonic transform.

It should be noted that degrees (n) and orders (m) of a spherical harmonic refer to the number of harmonics present along the radial and azimuthal angles. The degree n characterizes the overall angular frequency or spatial resolution of the wave pattern on the sphere, while the order m details the specific azimuthal variation, defining the symmetry of the sound field with respect to the polar axis.

Human Hearing and Perception

As 13 orders of magnitude differentiate the hearing threshold and the threshold for painful listening in the most sensitive part of human hearing (1000-3000Hz), it would not make sense to discuss the sound pressure, [7]. Therefore, sound pressure level SPL is used instead, given by:

$$SPL = 20 * log10(\frac{\tilde{p}}{\tilde{p_0}}[dB] \tag{B.113}$$

Where $\tilde{p}$ denotes root mean square of the sound pressure, i.e. $\sqrt{\bar{p^2}}$ and $\tilde{p_0} = 2 * 10^{-5}[\frac{N}{m^2}]$ is an internationally fixed value corresponding to the hearing threshold at 1000Hz, [7]. The sound pressure level reveals the objective sound pressure, but tells nothing about the subjective sound level. A 1000Hz sinusoid at 120dB is deafening, while a 30kHz sinusoid at the same SPL is inaudible. To compensate for this subjective perception, the unit "phon" and the associated "sone" is used. Phons are in an idealised world equal to the SPL of a 1kHz sinusoid at any given dB, and can therefore be approximated as $10phons = 10dB@1kHz$. To yield a better understanding of phons and sones, figure B.2 shows what is known as equal loudness curves (also known as Fletcher-Munson curves) from the DS/ISO standard 226:2023, [27]. Equal loudness curves show the subjective loudness perception of puretone signals at varying frequencies with known SPL, and can be used to find the exact perception of loudness from

20-12500Hz. The threshold of hearing is found at the dashed line in the bottom, signifying 0 phon, established in DS/ISO 389-7, [28]. The curves for 10 and 100 phon are dotted lines as very little experimental data exists for these levels, [27]. *NOTE: the equal loudness curves are only applicable for puretone signals, as complex sounds, i.e. a hammer hitting a nail, contain masking of some spectral quantities, and can therefore not be placed specifically on the curves. In such an instance dB(A) measurements should be used instead.*

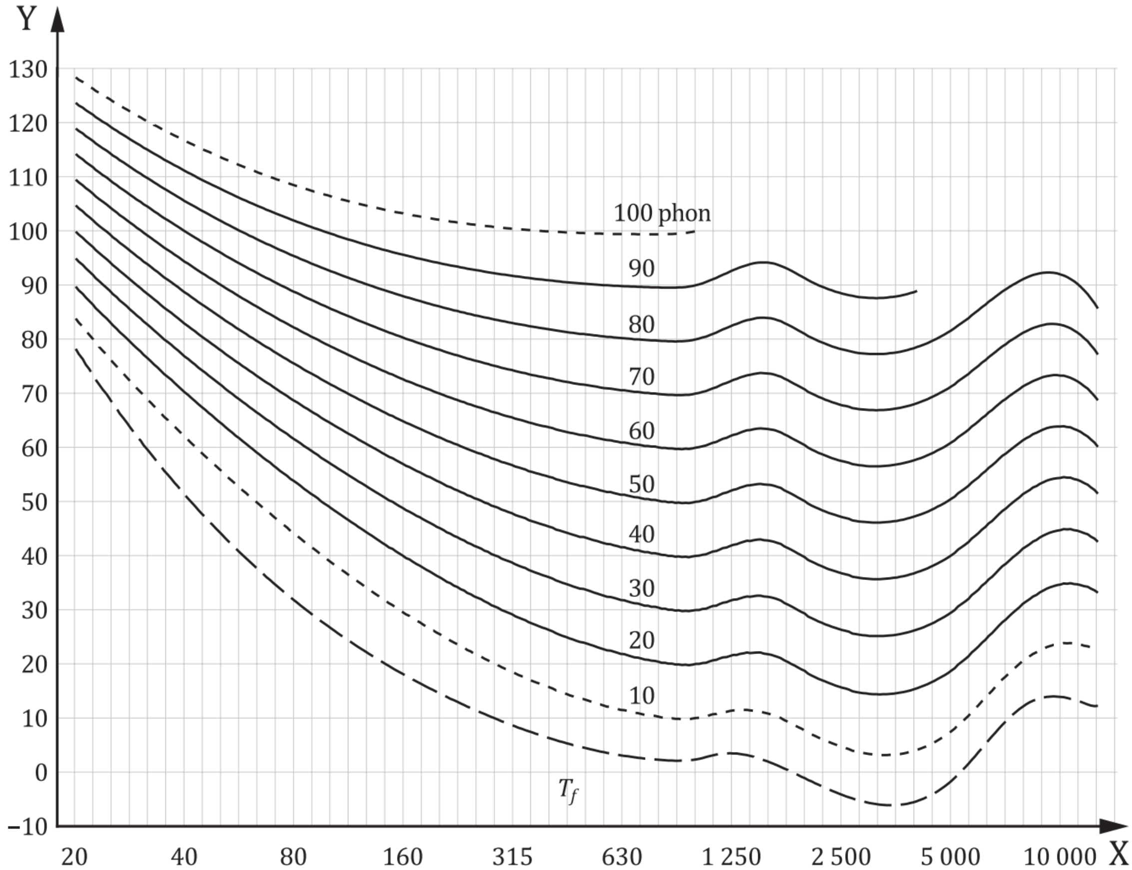


Figure B.2: Equal loudness curves for pure tone listening in free field conditions, [27]. X is frequency, expressed in Hz, Y sound pressure level, expressed in dB, and T_f hearing threshold.

The metric "sone" describes when the subjective loudness is doubled, as doubling the number of phons does not equal a doubling in perceived loudness. The first sone equals 40 phons, and thereafter 1 sone equals 10 phons. The relation is best understood by viewing table B.3, and is otherwise given by equations B.114 and B.115, with N being loudness in sones and L_N being loudness in phons.

$$N = \left(10^{\frac{L_N-40}{10}}\right)^{0.30103} \approx 2^{\frac{L_N-40}{10}} \tag{B.114}$$

$$L_N = 40 + log_2(N) \tag{B.115}$$

Phon	0	10	20	30	40	50	60	70	80	90	100
Sone	0 (Inaudible)	0.0625	0.125	0.25	1	2	4	8	16	32	64

Table B.3: Relationship between phons and sones.

The remaining aspects related to human perception of sounds are mostly oriented towards psychoacoustics and binaural perception, such as head-related transfer functions (HRTF) and binaural transfer functions. While the HRTF is applicable in virtual auralization of rooms, it will not be expanded upon in this project, and neither will psychoacoustical properties in depth.

B.3.2 Sound in Front of a Wall

Instead of ideal free-field propagation, actual constraints such as a wall will now be introduced. A wall will have some inherent properties, such as its reflection, diffusion, transmission, resonance, and absorption properties, that together determine the actual acoustical properties of each specific wall. In table B.4 each property and its typical values are presented, with relevant equations following.

Property	Definition	Typical Values
Reflection Coefficient (R)	Fraction of sound energy reflected	0.8 - 0.98 (Concrete), 0.1 - 0.5 (Foam)
Diffusion Coefficient (D)	Measure of even sound scattering	0.2 (Flat wall), 0.6 - 0.9 (Diffuser)
Transmission Loss (TL)	Reduction of sound through a barrier (dB)	25 dB (Drywall), 50+ dB (Concrete)
Resonance Frequency (f_r)	Frequency at which the wall vibrates	50 - 500 Hz (Depending on material)
Absorption Coefficient (α)	Fraction of energy absorbed	0.02 (Concrete), 0.8+ (Acoustic panels)
Wall Impedance (Z)	Opposition to sound transmission, depends on density and stiffness.	1.5×10^6 kg·m^{-2}·s^{-1} (Concrete), 5×10^5 kg·m^{-2}·s^{-1} (Brick), 10^4 kg·m^{-2}·s^{-1} (Foam)
Wall Admittance (ζ)	Measure of a wall's ability to accept sound energy, reciprocal of impedance.	0.001 (Concrete), 0.003 (Brick), 0.1 (Foam)

Table B.4: Acoustic properties of walls and their typical values.

- **Reflection Coefficient:**

$$R = \frac{I_r}{I_i} \tag{B.116}$$

where I_r is reflected intensity and I_i is incident intensity.

- **Diffusion Coefficient:**

$$D = 1 - \frac{\sum S(\theta)}{N S_{\text{ideal}}} \tag{B.117}$$

where $S(\theta)$ is the scattered energy in direction θ, and S_{ideal} is the ideally scattered energy.

- **Transmission Loss (TL):**

$$TL = 10 \log_{10} \left(\frac{I_i}{I_t} \right) \tag{B.118}$$

where I_t is the transmitted intensity.

- **Resonance Frequency (Panel):**

$$f_r = \frac{60}{d} \sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{B.119}$$

where:d = panel thickness, E = Young's modulus, ρ = density, and ν = Poisson's ratio.

- **Absorption Coefficient:**

$$\alpha = 1 - |R|^2 \tag{B.120}$$

- **Wall Impedance:**

$$Z = \left(\frac{p}{v_n}\right) \tag{B.121}$$

where: v_n denotes the particle component normal to the wall.

- **Wall Admittance:**

$$\zeta = \frac{Z}{\rho_0 * c} \tag{B.122}$$

All walls will be assumed to be plane, unbounded, and smooth. Moreover, the sound source will be placed at a distance allowing the spherical waves to be considered planar.

Reflection at Normal Incidence

When referring to normal incidence, a reflection perpendicular to the wall should be imagined. To comply with previous formulas, the axis of propagation remains along the x-axis, resulting in a wall being aligned with the z- and y-axis. Furthermore, the wall is set to $x = 0$ and the wave will arrive from the negative direction, yielding the following sound pressure:

$$p_i(x,t) = \hat{p}_0 * e^{i*(\omega*t-k*x)} \tag{B.123}$$

And particle velocity:

$$v_i(x,t) = \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t-k*x)} \tag{B.124}$$

Now that the sound wave has hit the wall, the reflected wave in completion is given by the wall's characteristics. For now, only the reflection coefficient R is used, as that can help describe both the amplitude attenuation and the phase shift, which also results in k flipping signs. Following equation B.97, the sign should be flipped for the reflected particle velocity as well. By using this, the reflected waves' sound pressure and particle velocity are given by:

$$p_r(x,t) = R * \hat{p}_0 * e^{i*(\omega*t+k*x)} \tag{B.125}$$

$$v_r(x,t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t+k*x)} \tag{B.126}$$

To find the impedance of the wall, x should be set equal to the coordinates of the wall (0), yielding equations B.127 and B.128. It should be noted in those equations that they are a product of both the incident and reflected sound wave, yielding the 1+R and 1-R relations:

$$p(0,t) = (1+R) * \hat{p}_0 * e^{i*(\omega*t)} \tag{B.127}$$

$$v(0,t) = (1-R) * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t)} \tag{B.128}$$

With the sound wave evaluated at the wall, it is possible to use equation B.121 to derive the exact wall impedance:

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \rho_0 * c * \frac{1+R}{1-R} \tag{B.129}$$

Which can be reformulated for the wall admittance, given in equation B.122:

$$\zeta = \frac{Z}{\rho_0 * c} \rightarrow \zeta = \frac{\rho_0 * c * \left(\frac{1+R}{1-R}\right)}{\rho_0 * c} \rightarrow \zeta = \frac{1+R}{1-R} \tag{B.130}$$

With this, the reflection coefficient can also be described as:

$$R = \frac{\zeta - 1}{\zeta + 1} \tag{B.131}$$

And the absorption coefficient can be found as:

$$\alpha = \frac{4 * Re(\zeta)}{|\zeta|^2 + 2 * Re(\zeta) + 1} \tag{B.132}$$

Graphically the wall impedance can be seen in figure B.3. As α increases, the circles draw nearer to $\zeta = 1$ corresponding to impedance matching of the medium. A completely rigid wall will have infinite impedance, as R=1, while a soft wall with R=-1 will yield no impedance.

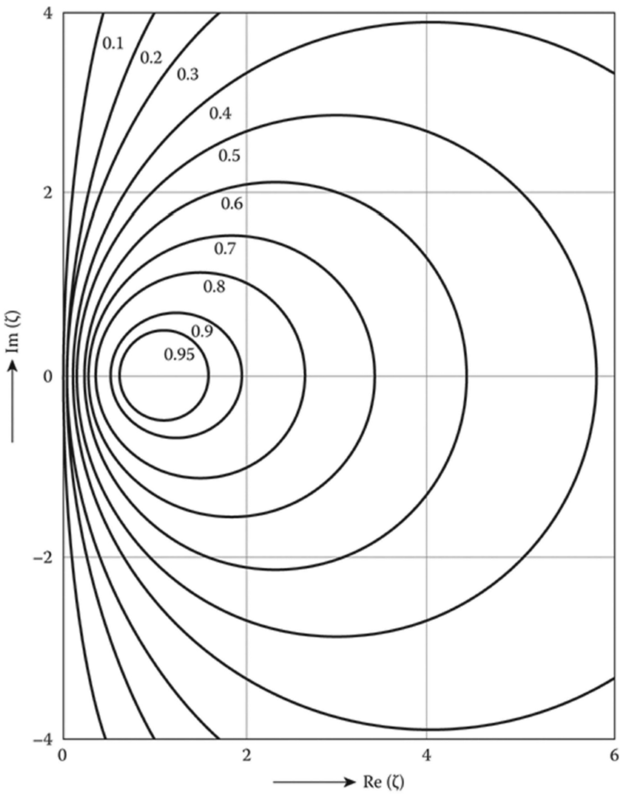


Figure B.3: Circles of constant absorption coefficient in the complex wall impedance plane for normal sound incidence, with the absorption coefficient values indicated by the numbers next to the circles, [8].

Reflection at Oblique Incidence

In reality, some reflections happen at normal incidence, but due to the spherical propagation of sound, a significantly larger amount will have its incidence at oblique angles. Therefore, θ will be added to all previous calculations, representing the angle between the incident wave and the normal to the surface. By doing so, the coordinate system is transformed with $x \rightarrow x'$ defining a new propagation axis, which is given by:

$$x' = x * cos(\theta) + y * sin(\theta) \tag{B.133}$$

With x′ the coordinate system expands from only regarding the x-axis to being two-dimensional, including the y-axis. Modifying the calculations from normal incidence to oblique incidence is done simply by replacing x with x′. This yields modified expressions for the incident and reflected wave properties. The key equations are:

- **Incident sound pressure:** Equation (B.134)

$$p_i(x', t) = \hat{p}_0 * e^{i*(\omega*t-k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.134}$$

- **Incident particle velocity:** Equation (B.135)

$$v_i(x', t) = \frac{\hat{p_0}}{\rho_0 * c} * cos(\theta) * e^{i*(\omega*t-k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.135}$$

- **Reflected sound pressure:** Equation (B.136)

$$p_r(x', t) = R * \hat{p}_0 * e^{i*(\omega*t+k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.136}$$

- **Reflected particle velocity:** Equation (B.137)

$$v_r(x', t) = -R * \frac{\hat{p_0}}{\rho_0 * c} * e^{i*(\omega*t+k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.137}$$

- **Wall impedance:** Equation (B.138)

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \frac{\rho_0 * c}{cos(\theta)} * \frac{1+R}{1-R} \tag{B.138}$$

- **Reflection coefficient:** Equation (B.139)

$$R = \frac{Z * cos(\theta) - \rho_0 * c}{Z * cos(\theta) + \rho_0 * c} = \frac{\zeta * cos(\theta) - 1}{\zeta * cos(\theta) + 1} \tag{B.139}$$

- **Absorption coefficient:** Equation (B.140)

$$\alpha(\theta) = \frac{4 * Re(\zeta) * cos(\theta)}{(|\zeta| * cos(\theta))^2 + 2 * Re(\zeta) * cos(\theta) + 1} \tag{B.140}$$

- **Sound pressure at the wall:** Equation (B.141)

$$p(x, y) = \hat{p}[1 + |R|^2 + 2 * |R| * cos(2 * k * x * cos(\theta) + \chi]^{1/2} * e^{-i*k*y*(sin\theta)} \tag{B.141}$$

- **Particle velocity at the wall:** Equation (B.142)

$$c_y = \frac{\omega}{k_y} = \frac{\omega}{k * sin(\theta)} = \frac{c}{sin(\theta)} \tag{B.142}$$

As with the reflections at normal incidence, the reflected wave will have a phase shift and attenuation, which leads to signs being flipped when looking at the reflected wave. Furthermore, the impedance, reflection coefficient, and absorption coefficients are evaluated at x=0, corresponding to the walls location.

B.3.3 Closed Space Sound Field

With reflections for normal and oblique incidents described, sound propagation in closed spaces can be described. In closed spaces, the propagation is notably different from free-field due to the reflections interacting with the sound source, other reflections, and boundaries anew, creating even more reflections. This continuous interaction between reflected waves and sound sources creates modal interactions, reverberation, and standing waves, which all are determining factors in a room's acoustics. To create a representation of the associated wave theory, the wave equation B.86 is modified to resemble the Helmholtz equation:

$$\Delta p + k^2 p = 0 \tag{B.143}$$

The Helmholtz equation describes wave behavior and sound propagation in closed spaces, by the spatial variation of sound pressure fields with harmonic time dependencies. Therefore it can also only be used to describe waves under the assumption of time harmonic waves with an angular frequency ω. This is given by:

$$\frac{\partial^2}{\partial t^2}\left(P(x)e^{i\omega t}\right) = -\omega^2 P(x)e^{i\omega t} \rightarrow -\omega^2 P(x)e^{i\omega t} = c^2\nabla^2 P(x)e^{i\omega t} \rightarrow \Delta p + k^2 p = 0 \tag{B.144}$$

Where the second time derivative is taken and substituted into the wave equation. Thereafter $e^{i*\omega*t}$ is cancelled as it is on both sides of the equation, minimizing the expression to equal B.143. The wave number k is an eigenvalue, as the wave equation can only yield non-zero solutions for equations B.145 and B.146 for particular discrete values, as k is still given by $k = \frac{\omega}{c}$. For k values at specific nodes, equation B.147 is used.

$$Z * \frac{\partial p}{\partial n} + i * \omega * \rho_0 * p = 0 \tag{B.145}$$

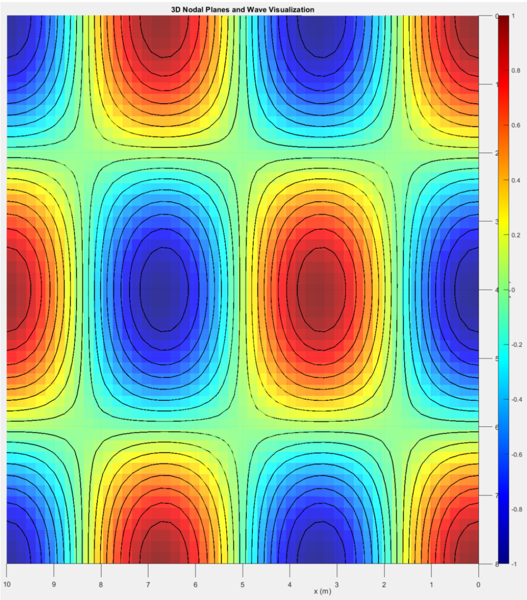
$$\zeta * \frac{\partial p}{\partial n} + i * k * p = 0 \tag{B.146}$$

$$k_{n_x n_y n_z} = \pi * \left(\left(\frac{n_x}{L_x}\right)^2 \left(\frac{n_y}{L_y}\right)^2 \left(\frac{n_z}{L_z}\right)^2\right)^{\frac{1}{2}} \tag{B.147}$$

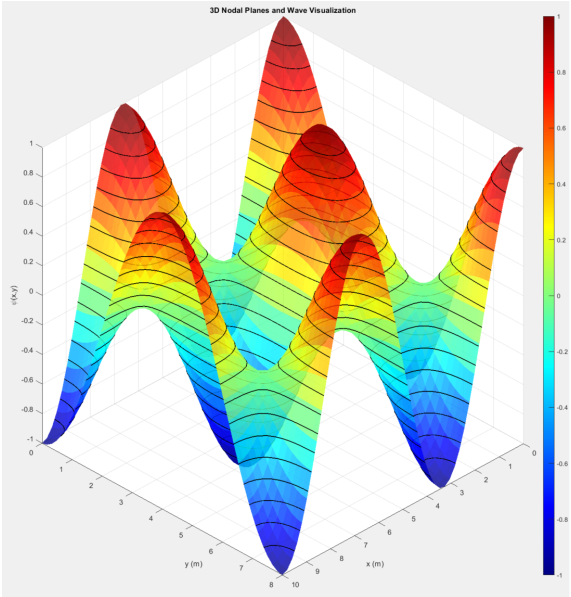
In a rectangular room of dimensions $L_x \times L_y \times L_z$, the general solution for the pressure distribution considering rigid boundaries is given by:

$$p(x, y, z, t) = P_0 \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) e^{i\omega t}, \tag{B.148}$$

Where n_x, n_y, n_z are index numbers along the respective axes signifying which nodal plane is being referred to. Equation B.148 describes a three-dimensional standing wave, wherein the sound pressure is zero for all times, at any point where the cosines equal to zero. All values of x (and y or z) which are odd integers of $\frac{L_x}{n_x}$ result in cosines equal to zero. The intersection between the x, y, and z axis equidistant planes are referred to as nodal planes, which are mutually orthogonal. It should be noted that nodal surfaces differ from nodal planes, as nodal surfaces are used to describe non-orthogonal surfaces, which may not even be planar. The intersections between nodal planes will therefore also always have a sound pressure of zero. In figure B.4 a sound pressure distribution can be seen in an 8x10 meter room subjected to 40 and 50Hz. In other words, this particular room has 3 room nodes in the x direction and 2 room nodes in the y direction, at z=0.



(a) Nodal planes seen from the top



(b) Nodal planes as seen from a corner.

Figure B.4: Nodal plane illustrations in an 8x10 meter room subjected to 40 and 50Hz.

By using equations B.148 eigenvalues can be found, and with those in place, the eigenfrequencies of a room can be found with equation B.149, where $k_{n_x n_y n_z}$ os given by equation B.147. In table B.5 the first 20 eigenfrequencies of a room with dimensions 4.7 × 4.1 × 3.1 m can be seen, using 340 m/s as c.

$$f_{n_c n_y n_z} = \frac{c}{2 * \pi} * k_{n_x n_y n_z} \tag{B.149}$$

fn	nx	ny	nz	fn	nx	ny	nz
36.17	1	0	0	90.47	1	2	0
41.46	0	1	0	90.78	2	0	1
54.84	0	0	1	99.42	0	2	1
55.02	1	1	0	99.80	2	1	1
65.69	1	0	1	105.79	1	2	1
68.75	0	1	1	108.51	3	0	0
72.34	2	0	0	109.68	0	0	2
77.68	1	1	1	110.05	2	2	0
82.93	0	2	0	115.49	1	0	2
83.38	2	1	0	116.16	3	1	0

Table B.5: Eigenfrequencies of a rectangular room with dimensions 4.7 × 4.1 × 3.1 m (in Hz), [7].

To find the number of room modes in a room, every combination of indexes must be calculated for a given frequency range. This is done by equations B.150 and B.151 and is realistically done by looping a code such as ***HER SKAL INDSÆTTES KODELINK***, as the amount of computations necessary is unfathomably large. Using the same room as in [7], a frequency threshold of 200 Hz has 78 modes and 71 unique eigenfrequencies, whereas a threshold of 2000 Hz has a threshold of 52.126 modes and 9850 eigenfrequencies. If the hearing spectrum up to 20 kHz is used, it leads to a staggering 49.855.567 modes and 49.808.708 eigenfrequencies.

$$\left(\frac{2 * f_{max}}{c}\right)^2 = \left(\frac{2 * 20000}{c}\right)^2 = 116.619^2 \tag{B.150}$$

$$\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2 \leq 116.619^2 \tag{B.151}$$

B.3.4 Room Geometry in Relation to Acoustics

Until now, only rectangular rooms have been discussed, as they form a more intuitive understanding. When more flamboyant room geometry is in use, all sound propagation is thought of as rays instead of spheres. By using sound rays to describe propagation instead of spheres, it simplifies the understanding of a room's reflections by reducing the computational complexity and increasing the graphical understanding. It is possible by the limiting case of very high frequencies, where any frequency above 1 kHz is considered high, as the wavelength (34.3cm) is considered small in comparison with typical room dimensions.

Image Sources

Image sources are an intuitive way to describe reflections using sound rays. Figure B.5 shows a basic image source at A'. Image sources are best described as an identical sound source placed exactly mirrored according to a wall. The original sound source A has in figure B.5 3 sound rays hitting a wall. As the wall is considered smooth, the reflection will have an identical angle of departure as the angle of incidence. This is furthermore underlined as identical sound rays propagate from A'.

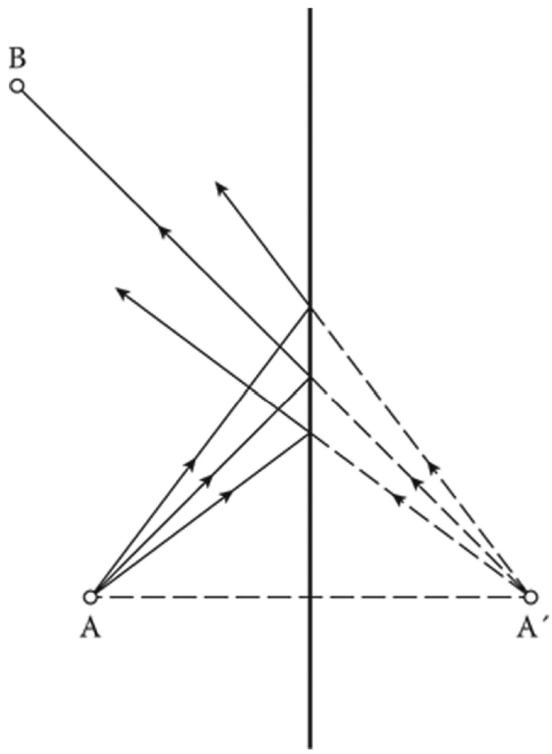


Figure B.5: Original sound source A and image source A', [7]. Note the angle of incidence and departure for sound rays.

In this exact example, the image source is meant to determine where on the wall A's reflection will coincide with B, which is also why the direct sound ray is neglected. The image source finds that the middle sound ray is the only matching reflection, using sound rays at least. It should still be remembered that sound sources still propagate omnidirectionally unless otherwise specified. It should also be noted that for now, reflection/absorption coefficients are currently not used to describe ray propagation. If those coefficients were applied, the sound rays would decay accordingly.

Figure B.6 is a more interesting example, as a non-rectangular room is used. The first image source A' is placed mirrored to the original source, but the second image source A" is not mirrored to the original source. Instead, the image source A" is the image source to A' if the mirrorplane is placed at the next wall, which the reflection should hit. This can be done continuously, unless reflection/absorption coefficients are used.

In general, image sources and their corresponding reflection diagrams are used with 3-5 orders for initial investigation, and by using programs such as ODEON, higher order diagrams can be made.

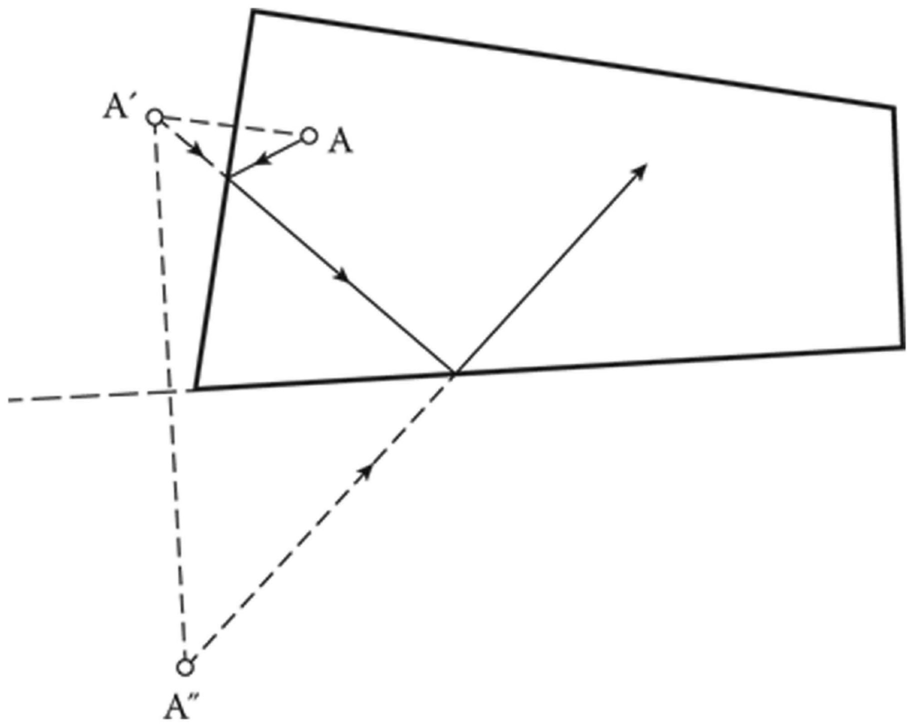


Figure B.6: Enter Caption

A significant detail regarding image sources is that they are infinite in the same way as orthogonal mirrors create an infinite space. As most rooms have at least four walls, a floor, and a ceiling, image sources will multiply incredibly fast. For a room with N-plane walls, the total number of image sources for a given order i is $N * (N-1)^{i-1}$, for $i \geq 1$. The total number of images is given by equation B.152, which for $N = 4$ and $i = 5$ yields a total of 160 image sources:

$$N(i_0) = N * \frac{(N-1)^{i_0} - 1}{N-2} \tag{B.152}$$

Moreover, as ceilings and floors has to be taken into consideration as well, an amount of mirrored rooms will also be present above and below the original room, each with similar amounts of image sources for their respective plane. For a simple rectangular room, this could look as shown in figure **??**, where the image should be imagined as one plane, with a mirrored one on top and below for order i.

With image sources found, two things can be done: 1. the total sound pressure at any place can be found. 2. the temporal distribution can be described.

The temporal distribution describes the intensity and time dependency of reflections.

Using equation **??**, the temporal distribution calculated for each time t. The temporal distribution is partitioned into: direct sound, first reflections, early reflections, and late reflections. The time delay and intensity of all three reflection types are detrimental for the reverberation of a room, and in general for a room's acoustical properties, as described in section B.3 on page 47. In figure **??**, a temporal distribution can be seen.ref

B.3.5 Sound Absorbers

Absorbers in Construction

Absorbers in Furnishment

B.3.6 Measuring Techniques

Impulse Response of a Room

Acoustical Properties of Materials

skal være segway til impulsrør og teknisk analyse