Developing a Multi-Microphone Impedancetube Focused on Determining Room Acoustic Material Properties at Low-Frequencies

Bachelor Project ESD6

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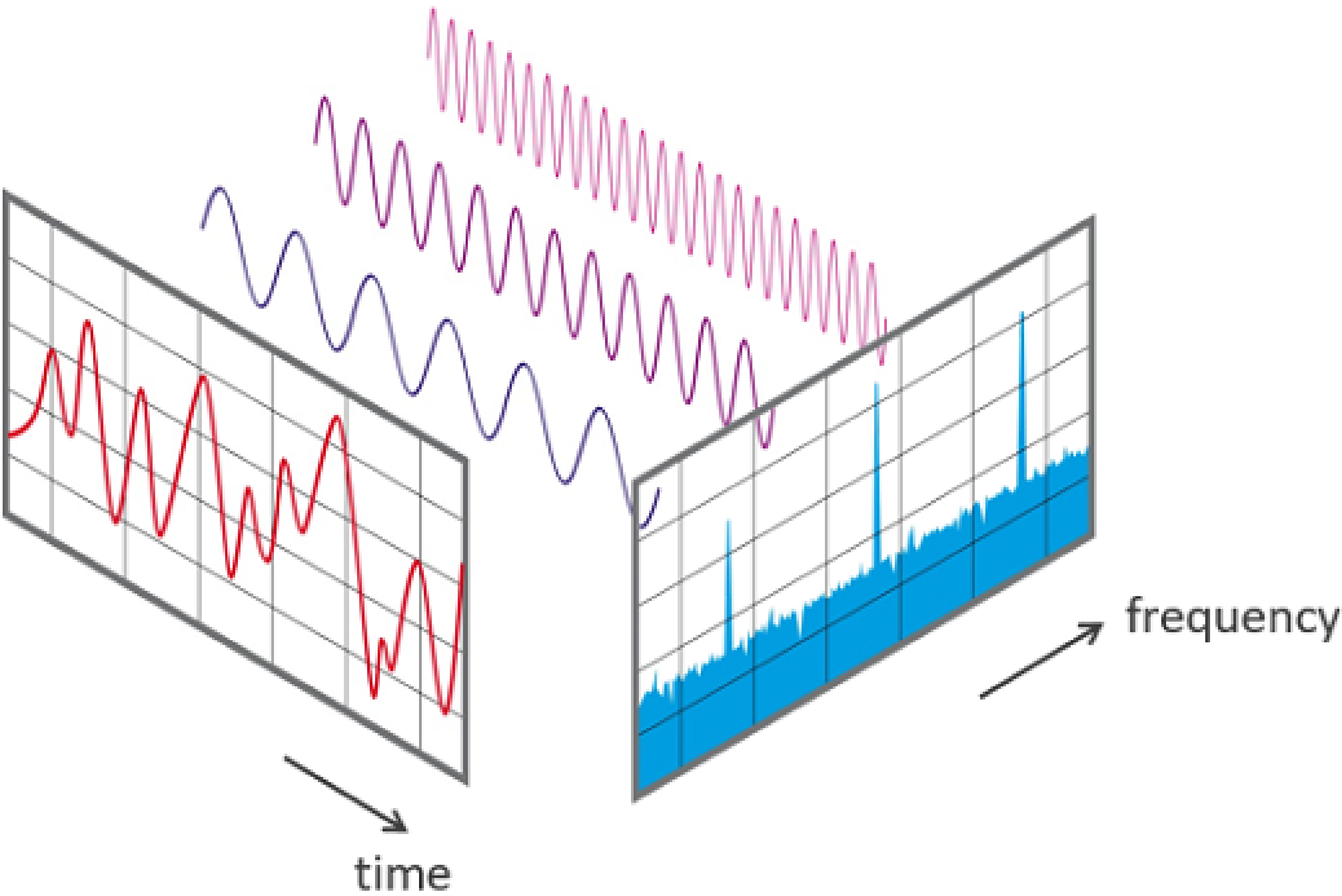

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Preface

This report searches the requirements to develop an impedance tube based upon DS/EN ISO 10534-2:2023. In appendix B on page 64 all formulas and variables will be noted, in order of appearance.

Aalborg University, March 31, 2025

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1 Introduction

Acoustics are present in every aspect of life, no matter how small or large, resulting in acoustics significantly impacting the very perception of life. Scientific acoustical studies began emerging as far back as the 6th century BC with Greek philosophers and later Roman architects/engineers embracing acoustic properties in construction, [1]. However, the contemporary understanding of acoustics has only existed for approximately 200 years, [1]. Within those 200 years, acoustics has changed from a phenomenon only understood by scholars to an aspect of life, that most can relate to or know of to some degree.

In modern construction, acoustical properties have traversed from being reserved for performance centers, to being integrated into almost anything. It has become crucial for the average homeowner to achieve "good acoustics", along with acoustics occupying an increasingly large facet of large-scale construction. To achieve "good acoustics" a multitude of methods are available, ranging from adding a thick carpet or heavy drapes to bass-traps, vibration minimizing, and similar sound-deadening techniques.

Unfortunately, creating an acoustically optimal room is expensive, and in many cases wouldn't resemble a habitable room in common sense. Therefore the next-best option is adding unnoticeable elements such as padded furniture, rugs, pillows, drapes, and acoustic art/images or incorporating constructional options such as floor dampening, carpets, acoustic ceilings, and other, more intrusive options, [2, 3].

To bridge the gap between the historical and practical perspectives on acoustics, it is essential to consider the challenges and trade-offs involved in achieving optimal acoustical environments. While the importance of acoustics in everyday life and construction is evident, the practical implementation of "good acoustics" is often constrained by cost, aesthetics, and feasibility. This raises a fundamental question that initializes this project:

"How can "good acoustics" be defined and quantified in a way that balances theoretical ideals with real-world applicability?"

2 | Problem Analysis

2.1 Room Acoustics

Determining good acoustics is as subjective as determining which music is pleasant. While subjective qualities are present, objective aspects should be considered as well, as the very shape of a person (ears, head, shoulders, etc.) influence the perception of sound as well, [4, 5, 6, 7, 8]. Furthermore, good acoustical properties are not as "simple" as attempting to achieve anechoic conditions, as the inherent psychoacoustical properties of an anechoic room are not considered pleasant. Fortunately, guidelines exist for what is, in general, considered good acoustics. The physical basis for acoustics is given from section 2.2 to 2.3, the architectural view is found from section 2.5 to 2.6, the acoustical metrics commonly used, are described in section 2.7, whereas and the measuring techniques for determining the acoustic properties are introduced in section 2.8. To describe the performance of concert halls and auditoriums, the metrics: room gain (G), reverberation time (RT60), early decay time (EDT), speech intelligibility (C50), definition (D50), musical clarity, and temporal distribution, were conceived.

2.2 Introduction to sound

To establish sound as an entity, a lot of equations will be instantiated, with most coming from [7]. As most of the notation and equations in [7] are equal or at the very least similar to other sources, it has been chosen as the main source for equations, meaning all formulas used in this section can be found in [7] as well, unless otherwise noted. Other sources used for mathematical formulas are [4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. For the time being, only ideal sound will be described, assuming no losses in propagation or obstacles. Furthermore, the medium is considered homogeneous and at rest, making the velocity of sound constant in relation to space and time. If the medium were air, the velocity is given by:

$$c = 331.4 + 0.6 * T \left[\frac{m}{s}\right] \tag{2.1}$$

With T being the temperature in degrees centigrade. The speed of sound c is set to be $343\frac{m}{s}$ for all following computations, based on an average temperature of 23.2°. The average is used as exact calculations would require infinitesimal precision and, in practice, would be impossible to compute. An example could be a sports hall, wherein the temperature around players and audience is higher than under the rafters, [4, 5, 7, 10].

Another fundamental equation is the wave equation. The wave equation describes how sound propagates over time and space relative to the speed of sound. It is given by:

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \left[\frac{Pa}{m^2} \right] \tag{2.2}$$

where the squared speed of sound is defined by:

$$c^2 = \kappa \frac{p_0}{\rho_0} \left[\frac{m^2}{s^2} \right] \tag{2.3}$$

in these equations, p is the sound pressure, rho_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$, p_0 is the static sound pressure, and κ is the adiabatic component (for air $\kappa = 1.4$), [7]. In cartesian coordinates, the laplacian operator Δ is given by:

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \tag{2.4}$$

To elaborate, the wave equation is derived from the conservation of momentum relation:

$$\nabla p = -\rho_0 * \frac{\partial v}{\partial t} \tag{2.5}$$

with its one-dimensional version being:

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \tag{2.6}$$

where: v is a vector representing particle velocity, and t is time. Furthermore, the mass conservation requirement leads to:

$$\rho_0 \; div \; v = -\frac{\partial \rho}{\partial t} \tag{2.7}$$

with ρ being the variable gas density. It is generally assumed that the variation in gas pressure and density is small compared to their static values. All of the above can be related by:

$$\frac{p}{p_0} = \kappa \frac{\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} * \frac{\delta * T}{T + 273} \tag{2.8}$$

By eliminating the particle velocity v and variable gas density ρ from equations 2.5-2.8 one would arrive at the wave equation given in equation 2.2. The wave equation holds true for sound waves in any lossless fluid and for the resulting pressure, density, and temperature variations.

2.2.1 Actual waves

In section "Plane Waves" and "Spherical Waves" ideal scenarios are described, and a more realistic approach is made in the following. Therefore directional sound radiation with angular dependence is now considered. It is best imagined by a singular speaker pointing into an ideal environment. As the speaker is pointing in a direction and is not omnidirectional, the resulting sound wave will not be uniformly spherical, [7, 8, 21]. This expands the spherical wave equation to now include the polar and azimuth angles as well:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} + \frac{1}{r^2} \left(\frac{1}{sin(\theta)} * \frac{\partial}{\partial * \theta} \left(sin(\theta) * \frac{\partial}{\partial * \theta} \right) + \frac{1}{sin^2(\theta)} * \frac{\partial^2}{\partial \phi^2} \right) = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{2.9}$$

To solve the complex spherical wave equation, the sound pressure can be expressed as a Fourier series separated into its radial and angular variables:

$$p(r,\theta,\phi) = A * \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Gamma_{nm} h_n(kr) Y_n^m(\theta,\phi) \tag{2.10}$$

Where: Γ_{nm} are the Fourier series coefficients of order n and degree m, $h_n(kr)$ is the Hankel function of the first kind with order n, $Y_n^m(\theta,\phi)$ are the spherical harmonics, forming an orthonormal basis, which is used in the Legendre polynomial:

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4*\pi} * \frac{(n-|m|)!}{(n+|m|)!}} * P_n^{|m|} * (cos(\theta) * \begin{cases} cos|m|\phi & if \;\; m \geq 0 \\ sin|m|\phi & if \;\; m < 0 \end{cases} \tag{2.11}$$

To actually use the above equations for anything, a spherical microphone array surrounding the speaker must be used to obtain empirical data. When such a measuring system is used, the coefficients of Γ_{nm} can be found with an inverse two-dimensional Fourier transform — often referred to as the inverse spherical harmonic transform.

It should be noted that degrees (n) and orders (m) of a spherical harmonic refer to the number of harmonics present along the radial and azimuthal angles. The degree n characterizes the overall angular frequency or spatial resolution of the wave pattern on the sphere, while the order m details the specific azimuthal variation, defining the symmetry of the sound field with respect to the polar axis.

2.2.2 Human Hearing and Perception

As 13 orders of magnitude differentiate the hearing threshold and the threshold for painful listening in the most sensitive part of human hearing (1-3kHz), it would not make sense to discuss the sound pressure, [7, 8, 12, 22, 23]. Therefore, sound pressure level SPL is used instead, given by:

$$SPL = 20 * log10\left(\frac{\tilde{p}}{\tilde{p_0}}\right) [dB] \tag{2.12}$$

Where $\tilde{p}$ denotes root mean square of the sound pressure, with $\sqrt{\bar{p^2}}$ and $\tilde{p_0} = 2 * 10^{-5}[\frac{N}{m^2}]$ as an internationally fixed value corresponding to the hearing threshold at 1kHz, [7]. The sound pressure level reveals the objective sound pressure but tells nothing about the subjective sound level. A 1kHz sinusoid at 120dB is deafening, while a 30kHz sinusoid at the same SPL is inaudible. To compensate for this subjective perception, the unit "phon" and the associated "sone" are used. Phons are in an idealized world equal to the SPL of a 1kHz sinusoid at any given dB, and can therefore be approximated as $10phons = 10dB@1kHz$. To yield a better understanding of phons and sones, figure 2.1 shows what is known as equal loudness curves (also known as Fletcher-Munson curves) from the DS/ISO standard 226:2023, [24]. Equal loudness curves show the subjective loudness perception of puretone signals at varying frequencies with known SPL, and can be used to find the exact perception of loudness from 20-12500Hz. The threshold of hearing is found at the dashed line in the bottom, signifying 0 phon, established in DS/ISO 389-7, [25]. The curves for 10 and 100 phon are dotted lines as very little experimental data exists for these levels, [24]. *NOTE: the equal loudness curves are only applicable for pure-tone signals, as complex sounds, i.e. a hammer hitting a nail, contain masking* *of some spectral quantities, and can therefore not be placed specifically on the curves. In such an instance dB(A) measurements should be used instead, [7, 24, 25].*

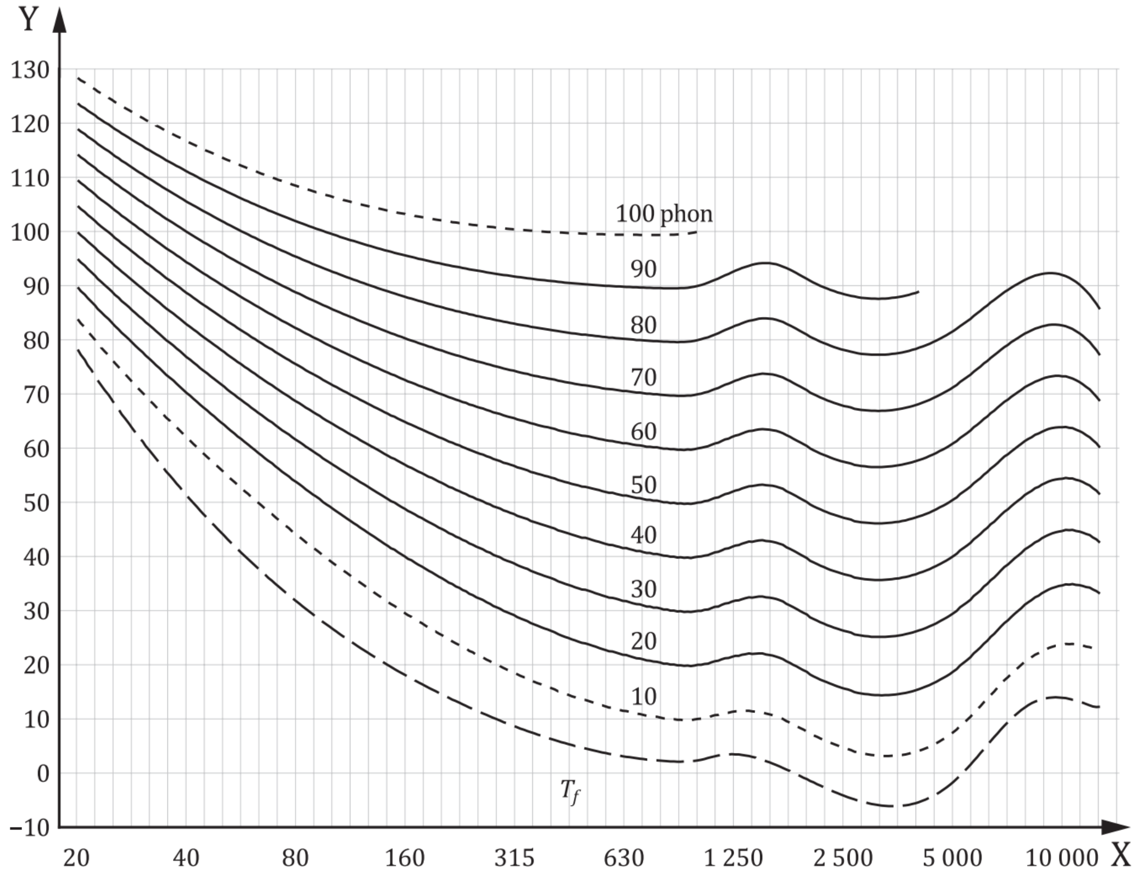


Figure 2.1: Equal loudness curves for pure tone listening in free field conditions, [24]. X is frequency, expressed in Hz, Y sound pressure level, expressed in dB, and T_f hearing threshold.

The metric "sone" describes when the subjective loudness is doubled, as doubling the number of phons does not equal a doubling in perceived loudness. The first sone equals 40 phons, and thereafter 1 sone equals 10 phons. The relation is best understood by viewing table 2.1, and is otherwise given by equations 2.13 and 2.14, with N being loudness in sones and L_N being loudness in phons.

$$N = \left(10^{\frac{L_N - 40}{10}}\right)^{0.30103} \approx 2^{\frac{L_N - 40}{10}} \tag{2.13}$$

$$L_N = 40 + log_2(N) \tag{2.14}$$

Phon	0	10	20	30	40	50	60	70	80	90	100
Sone	0 (Inaudible)	0.019	0.138	0.439	1	2	4	8	16	32	64

Table 2.1: Relationship between phons and sones, calculated using the Zwicker method in Matlab, as described in ISO 532-1, [26, 27].

The remaining aspects related to human perception of sounds are mostly oriented towards psychoacoustics and binaural perception, such as head-related transfer functions (HRTF) and binaural transfer functions,[7, 28, 29]. While the HRTF is applicable in the virtual auralization of rooms, it will not be expanded upon in this project, and neither will psychoacoustical properties in depth.

2.3 Sound in Front of a Wall

Instead of ideal free-field propagation, actual constraints such as a wall will now be introduced. A wall will have some inherent properties, such as its reflection, diffusion, transmission, resonance, and absorption properties, that together determine the actual acoustical properties of each specific wall. In table 2.2 each property and its typical values are presented, with relevant equations following.

Property	Definition	Typical Values
Reflection Coefficient (R)	Fraction of sound energy reflected	0.8 - 0.98 (Concrete), 0.1 - 0.5 (Foam)
Diffusion Coefficient (D)	Measure of even sound scattering	0.2 (Flat wall), 0.6 - 0.9 (Diffuser)
Transmission Loss (TL)	Reduction of sound through a barrier (dB)	25 dB (Drywall), 50+ dB (Concrete)
Resonance Frequency (f_r)	Frequency at which the wall vibrates	50 - 500 Hz (Depending on material)
Absorption Coefficient (α)	Fraction of energy absorbed	0.02 (Concrete), 0.8+ (Acoustic panels)
Wall Impedance (Z)	Opposition to sound transmission, depends on density and stiffness.	1.5×10^6 kg·m^{-2}·s^{-1} (Concrete), 5×10^5 kg·m^{-2}·s^{-1} (Brick), 10^4 kg·m^{-2}·s^{-1} (Foam)
Wall Admittance (ζ)	Measure of a wall's ability to accept sound energy, reciprocal of impedance.	0.001 (Concrete), 0.003 (Brick), 0.1 (Foam)

Table 2.2: Acoustic properties of walls and their typical values, [30, 16, 15].

- **Reflection Coefficient:**

$$R = \frac{I_r}{I_i} \tag{2.15}$$

where: I_r is reflected intensity and I_i is incident intensity.

- **Diffusion Coefficient:**

$$D = 1 - \frac{\sum S(\theta)}{NS_{\text{ideal}}} \tag{2.16}$$

where: $S(\theta)$ is the scattered energy in direction θ, and S_{ideal} is the ideally diffused scattered energy.

- **Transmission Loss (TL):**

$$TL = 10 \log_{10}\left(\frac{I_i}{I_t}\right) \tag{2.17}$$

where: I_t is the transmitted intensity.

- **Resonance Frequency (Panel):**

$$f_r = \frac{60}{d}\sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{2.18}$$

where: d = panel thickness, E = Young's modulus, ρ = density, and ν = Poisson's ratio.

- **Absorption Coefficient:**

$$\alpha = 1 - |R|^2 \left[\frac{sabin}{m^2}\right] \tag{2.19}$$

- **Wall Impedance:**

$$Z = \left(\frac{p}{v_n}\right) \tag{2.20}$$

where: v_n denotes the particle component normal to the wall.

- **Wall Admittance:**

$$\zeta = \frac{Z}{\rho_0 * c} \tag{2.21}$$

All walls are assumed to be plane, unbounded, and smooth. Moreover, the sound source will be placed at a distance allowing the spherical waves to be considered planar.

2.3.1 Reflection at Normal Incidence

When referring to normal incidence, a reflection perpendicular to the wall should be imagined. To comply with previous formulas, the axis of propagation remains along the x-axis, resulting in a wall being aligned with the z- and y-axis. Furthermore, the wall is set to $x = 0$ and the wave will arrive from the negative direction, yielding the following sound pressure:

$$p_i(x,t) = \hat{p}_0 * e^{i*(\omega*t-k*x)} \tag{2.22}$$

And particle velocity:

$$v_i(x,t) = \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t-k*x)} \tag{2.23}$$

Now that the sound wave has hit the wall, the reflected wave in completion is given by the wall's characteristics. For now, only the reflection coefficient R is used, as that can help describe both the amplitude attenuation and the phase shift, which also results in k flipping signs. Following equation C.5, the sign should be flipped for the reflected particle velocity as well. By using this, the reflected waves' sound pressure and particle velocity are given by:

$$p_r(x,t) = R * \hat{p}_0 * e^{i*(\omega*t+k*x)} \tag{2.24}$$

$$v_r(x,t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t+k*x)} \tag{2.25}$$

To find the impedance of the wall, x should be set equal to the coordinates of the wall (0), yielding equations 2.26 and 2.27. It should be noted in those equations that they are a product of both the incident and reflected sound wave, yielding the $1 + R$ and $1 - R$ relations:

$$p(0,t) = (1+R) * \hat{p}_0 * e^{i*(\omega*t)} \tag{2.26}$$

$$v(0,t) = (1-R) * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t)} \tag{2.27}$$

With the sound wave evaluated at the wall, it is possible to use equation 2.20 to derive the exact wall impedance:

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \rho_0 * c * \frac{1+R}{1-R} \tag{2.28}$$

Which can be reformulated for the wall admittance, given in equation 2.21:

$$\zeta = \frac{Z}{\rho_0 * c} \rightarrow \zeta = \frac{\rho_0 * c * \left(\frac{1+R}{1-R}\right)}{\rho_0 * c} \rightarrow \zeta = \frac{1+R}{1-R} \tag{2.29}$$

With this, the reflection coefficient can also be described as:

$$R = \frac{\zeta - 1}{\zeta + 1} \tag{2.30}$$

And the absorption coefficient can be found as:

$$\alpha = \frac{4 * Re(\zeta)}{|\zeta|^2 + 2 * Re(\zeta) + 1} \left[\frac{sabin}{m^2}\right] \tag{2.31}$$

Graphically the wall impedance can be seen in figure 2.2. As α increases, the circles draw nearer to $\zeta = 1$ corresponding to impedance matching of the medium. A completely rigid wall will have infinite impedance, as $R = 1$, while a soft wall with $R = -1$ will yield no impedance.

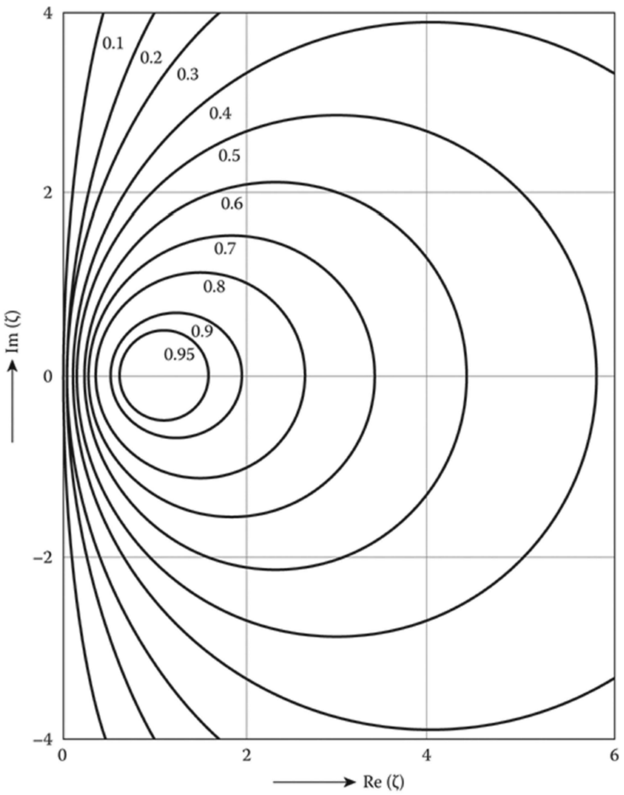


Figure 2.2: Circles of constant absorption coefficient in the complex wall impedance plane for normal sound incidence, with the absorption coefficient values indicated by the numbers next to the circles, [8].

2.3.2 Reflection at Oblique Incidence

In reality, some reflections happen at normal incidence, but due to the spherical propagation of sound, a significantly larger amount will have its incidence at oblique angles. Therefore, θ will be added to all previous calculations, representing the angle between the incident wave and the normal to the surface. By doing so, the coordinate system is transformed with $x \to x'$ defining a new propagation axis, which is given by:

$$x' = x * cos(\theta) + y * sin(\theta) \tag{2.32}$$

With x' the coordinate system expands from only regarding the x-axis to being two-dimensional, including the y-axis. Modifying the calculations from normal incidence to oblique incidence is done simply by replacing x with x'. This yields modified expressions for the incident and reflected wave properties. The key equations are:

- **Incident sound pressure:** Equation (2.33)

$$p_i(x',t) = \hat{p}_0 * e^{i*(\omega*t-k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.33}$$

- **Incident particle velocity:** Equation (2.34)

$$v_i(x',t) = \frac{\hat{p}_0}{\rho_0 * c} * cos(\theta) * e^{i*(\omega*t-k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.34}$$

- **Reflected sound pressure:** Equation (2.35)

$$p_r(x',t) = R * \hat{p}_0 * e^{i*(\omega*t+k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.35}$$

- **Reflected particle velocity:** Equation (2.36)

$$v_r(x',t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t+k*(x*cos(\theta)+y*sin(\theta)))} \tag{2.36}$$

- **Wall impedance:** Equation (2.37)

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \frac{\rho_0 * c}{cos(\theta)} * \frac{1+R}{1-R} \tag{2.37}$$

- **Reflection coefficient:** Equation (2.38)

$$R = \frac{Z * cos(\theta) - \rho_0 * c}{Z * cos(\theta) + \rho_0 * c} = \frac{\zeta * cos(\theta) - 1}{\zeta * cos(\theta) + 1} \tag{2.38}$$

- **Absorption coefficient:** Equation (2.39)

$$\alpha(\theta) = \frac{4 * Re(\zeta) * cos(\theta)}{(|\zeta| * cos(\theta))^2 + 2 * Re(\zeta) * cos(\theta) + 1} \tag{2.39}$$

- **Sound pressure at the wall:** Equation (2.40)

$$p(x,y) = \hat{p}[1 + |R|^2 + 2 * |R| * cos(2 * k * x * cos(\theta) + \chi]^{1/2} * e^{-i*k*y*(sin\theta)} \tag{2.40}$$

- **Particle velocity at the wall:** Equation (2.41)

$$c_y = \frac{\omega}{k_y} = \frac{\omega}{k * sin(\theta)} = \frac{c}{sin(\theta)} \tag{2.41}$$

As with the reflections at normal incidence, the reflected wave will have a phase shift and attenuation, which leads to signs being flipped when looking at the reflected wave. Furthermore, the impedance, reflection coefficient, and absorption coefficients are evaluated at $x = 0$, corresponding to the location of the wall.

2.4 Closed Space Sound Field

With reflections for normal and oblique incidents described, sound propagation in closed spaces can be described. In closed spaces, the propagation is notably different from free-field due to the reflections interacting with the sound source, other reflections, and boundaries anew, creating even more reflections, [7, 8, 31, 10, 11, 5, 18]. This continuous interaction between reflected waves and sound sources creates modal interactions, reverberation, and standing waves, which all are determining factors in a room's acoustics. To create a representation of the associated wave theory, the wave equation 2.2 is modified to resemble the Helmholtz equation:

$$\Delta p + k^2 p = 0 \tag{2.42}$$

The Helmholtz equation describes wave behavior and sound propagation in closed spaces, by the spatial variation of sound pressure fields with harmonic time dependencies. Therefore it can also only be used to describe waves under the assumption of time harmonic waves with an angular frequency ω. This is given by:

$$\frac{\partial^2}{\partial t^2}\left(P(x)e^{i\omega t}\right) = -\omega^2 P(x)e^{i\omega t} \rightarrow -\omega^2 P(x)e^{i\omega t} = c^2\nabla^2 P(x)e^{i\omega t} \rightarrow \Delta p + k^2 p = 0 \tag{2.43}$$

Where the second time derivative is taken and substituted into the wave equation. Thereafter $e^{i*\omega*t}$ is canceled as it is on both sides of the equation, minimizing the expression to equal 2.42. The wave number k is an eigenvalue, as the wave equation can only yield non-zero solutions for equations 2.44 and 2.45 for particular discrete values, as k is still given by $k = \frac{\omega}{c}$. For k values at specific nodes, equation 2.46 is used.

$$Z * \frac{\partial p}{\partial n} + i * \omega * \rho_0 * p = 0 \tag{2.44}$$

$$\zeta * \frac{\partial p}{\partial n} + i * k * p = 0 \tag{2.45}$$

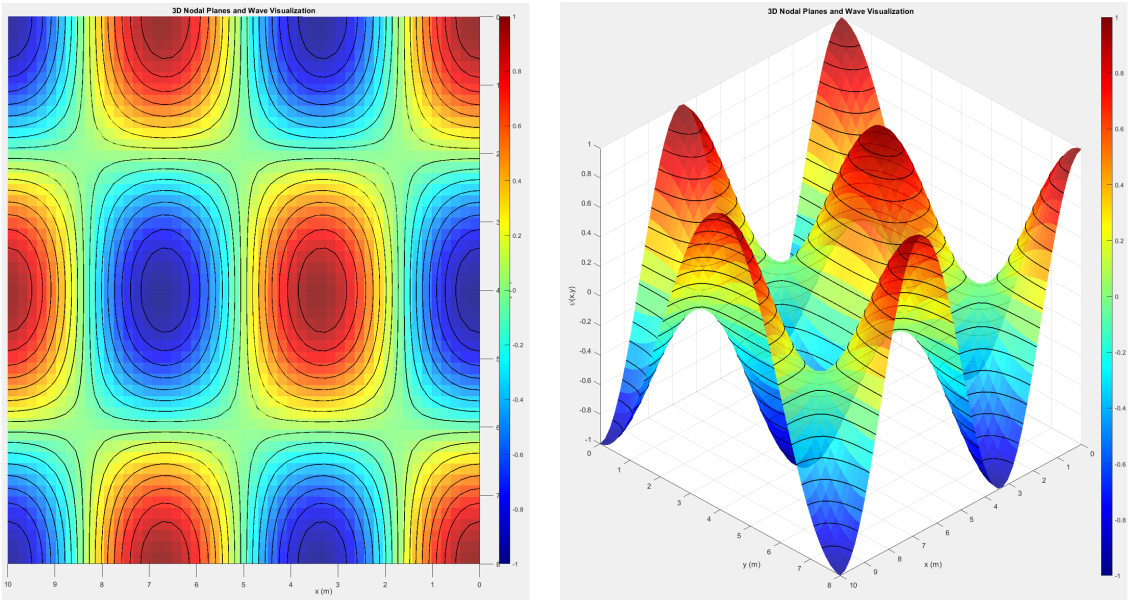
$$k_{n_x n_y n_z} = \pi * \left(\left(\frac{n_x}{L_x}\right)^2 \left(\frac{n_y}{L_y}\right)^2 \left(\frac{n_z}{L_z}\right)^2\right)^{\frac{1}{2}} \tag{2.46}$$

In a rectangular room of dimensions, $L_x \times L_y \times L_z$, the general solution for the pressure distribution considering rigid boundaries is given by:

$$p(x,y,z,t) = P_0 \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) e^{i\omega t}, \tag{2.47}$$

Where n_x, n_y, n_z are index numbers along the respective axes signifying which nodal plane is being referred to. Equation 2.47 describes a three-dimensional standing wave, wherein the sound pressure is zero for all times, at any point where the cosines equal to zero. All values of x (and y or z) which are odd integers of $\frac{L_x}{n_x}$ result in cosines equal to zero, [7, 8]. The intersection between the x, y, and z-axis equidistant planes are referred to as nodal planes, which are mutually orthogonal. It should be noted that nodal surfaces differ from nodal planes, as nodal surfaces are used to describe non-orthogonal surfaces, which may not even be planar. The intersections between nodal planes will therefore also always have a sound pressure of zero, [7, 8]. In figure 2.3 a sound pressure distribution can be seen in an 8x10 meter room subjected to 40 and

50Hz. In other words, this particular room has 3 room nodes in the x direction and 2 room nodes in the y direction, at z=0.



(a) Nodal planes seen from the top.

(b) Nodal planes as seen from a corner.

Figure 2.3: Nodal plane illustrations in an 8x10 meter room subjected to 40 and 50Hz.

By using equations 2.47 eigenvalues can be found, and with those in place, the eigenfrequencies of a room can be found with equation 2.48, where $k_{n_x n_y n_z}$ os given by equation 2.46. In table 2.3 the first 20 eigenfrequencies of a room with dimensions 4.7 × 4.1 × 3.1 m can be seen, using 340 m/s as c.

$$f_{n_c n_y n_z} = \frac{c}{2 * \pi} * k_{n_x n_y n_z} \tag{2.48}$$

fn	nx	ny	nz	fn	nx	ny	nz
36.17	1	0	0	90.47	1	2	0
41.46	0	1	0	90.78	2	0	1
54.84	0	0	1	99.42	0	2	1
55.02	1	1	0	99.80	2	1	1
65.69	1	0	1	105.79	1	2	1
68.75	0	1	1	108.51	3	0	0
72.34	2	0	0	109.68	0	0	2
77.68	1	1	1	110.05	2	2	0
82.93	0	2	0	115.49	1	0	2
83.38	2	1	0	116.16	3	1	0

Table 2.3: Eigenfrequencies of a rectangular room with dimensions 4.7 × 4.1 × 3.1 m (in Hz), [7].

To find the number of room modes in a room, every combination of indexes must be calculated for a given frequency range. This is done by equations 2.49 and 2.50 and is realistically done by looping a code such as ***HER SKAL INDSÆTTES KODELINK***, as the amount of computations necessary is unfathomably large. Using the same room as in [7], a frequency threshold of 200 Hz has 78 modes and 71 unique eigenfrequencies, whereas a threshold of 2 kHz has a threshold of 52.126 modes and 9.850 eigenfrequencies. If the hearing spectrum up to 20 kHz is used, it leads to a staggering 49.855.567 modes and 143.378 unique eigenfrequencies.

$$\left(\frac{2 * f_{max}}{c}\right)^2 = \left(\frac{2 * 20000}{c}\right)^2 = 116.619^2 \tag{2.49}$$

$$\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2 \leq 116.619^2 \tag{2.50}$$

2.5 Room Geometry in Relation to Acoustics

Until now, only rectangular rooms have been discussed, as they form a more intuitive understanding. When more flamboyant room geometry is in use, all sound propagation is thought of as rays instead of spheres. By using sound rays to describe propagation instead of spheres, simplifies the understanding of a room's reflections by reducing the computational complexity and increasing the graphical understanding, [7, 8, 10, 11, 15, 16, 31]. It is possible by the limiting case of very high frequencies, where any frequency above 1 kHz is considered high, as the wavelength (34.3cm) is considered small in comparison with typical room dimensions.

2.5.1 Image Sources

Image sources are an intuitive (yet conceptual) way to describe reflections using sound rays. Figure 2.4 shows a basic image source at A'. Image sources are best described as an identical sound source placed mirrored according to a wall. The original sound source A has in figure 2.4 3 sound rays hitting a wall. As the wall is considered smooth, the reflection will have an identical angle of departure as the angle of incidence, [7, 8, 10, 11, 15, 16, 31]. This is furthermore underlined as identical sound rays propagate from A'.

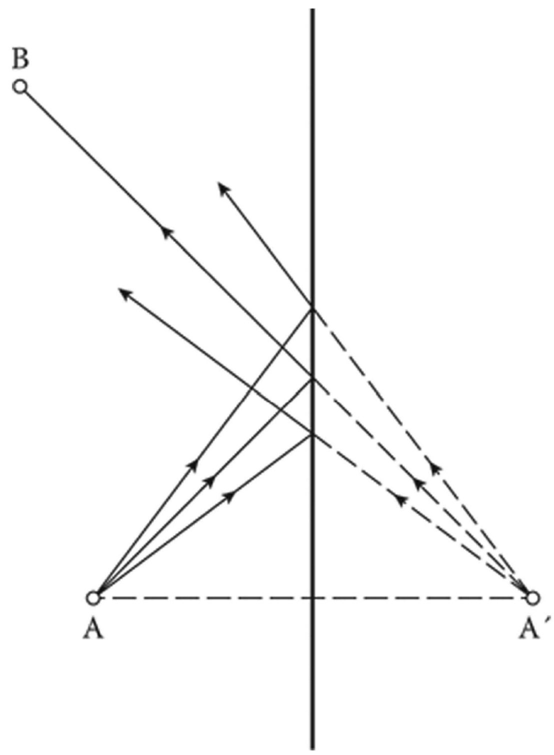
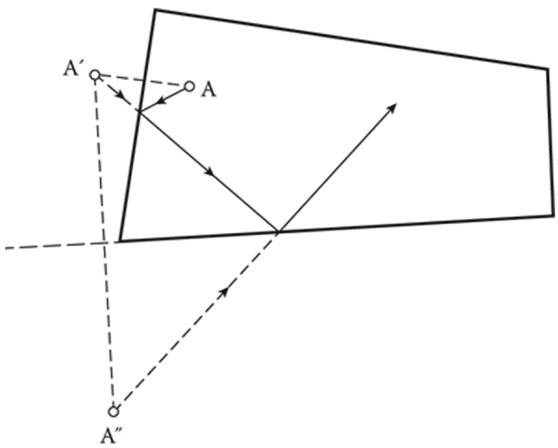


Figure 2.4: Original sound source A and image source A', [7]. Note the angle of incidence and departure for sound rays.

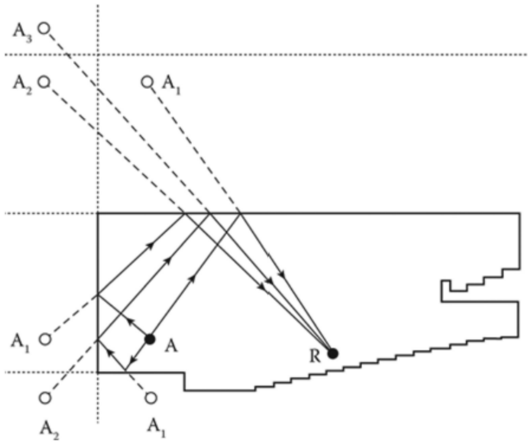
In this exact example, the image source is meant to determine where on the wall A's reflection will coincide with B, which is also why the direct sound ray is neglected. The image source finds that the middle sound ray is the only matching reflection, using sound rays at least. It should be remembered that sound sources still propagate omnidirectionally unless otherwise specified. It should also be noted that for now, reflection/absorption coefficients are currently not used to describe ray propagation. If those coefficients were applied, the sound rays would decay accordingly, [7, 8, 10, 11, 15, 16, 31].

Figure 2.5 is a more interesting example, as a non-rectangular room is used. The first image source A' is placed mirrored to the original source, but the second image source A'' is not mirrored to the original source. Instead, the image source A'' is the image source to A' if the mirror plane is placed at the next wall, which the reflection should hit. This can be done continuously unless reflection/absorption coefficients are used.

In general, image sources and their corresponding reflection diagrams are used with 3-5 orders for initial investigation, and by using programs such as ODEON, higher-order diagrams can be made.



(a) First and second order reflections in an oddly shaped room, [8].



(b) Image sources in an auditorium, found by mirroring along the walls being hit, [8].

Figure 2.5: In an oddly shaped room, image sources are best found by mirroring the source along the wall, which the reflection hits next. A', A_1, etc denotes the order of image sources, [8].

A significant detail regarding image sources is that they are infinite in the same way as orthogonal mirrors create an infinite space. As most rooms have at least four walls, a floor, and a ceiling, image sources will multiply incredibly fast. For a room with N plane walls, the total number of image sources for a given order i is $N * (N-1)^{i-1}$, for $i \geq 1$. The total number of images is given by equation 2.51, which for $N = 4$ and $i = 5$ yields a total of 160 image sources:

$$N(i_0) = N * \frac{(N-1)^{i_0} - 1}{N-2} \tag{2.51}$$

Moreover, as ceilings and floors has to be taken into consideration as well, an amount of mirrored rooms will also be present above and below the original room, each with similar amounts of image sources for their respective plane. For a simple rectangular room, this could look as shown in figure 2.6, where the image should be imagined as one plane, with a mirrored one on top and below for order i.

Image sources/image rooms can be used to determine the rate of reflections, yielding a temporal distribution, [7, 8, 10, 11, 15, 16, 31]. The temporal distribution describes the intensity and time dependency of reflections. Using equation 2.52, the temporal distribution can be calculated for each time t, if absorption coefficients of all walls were frequency independent. However, as all materials are frequency dependent, equation 2.53 is used instead, as it describes the response of a single pulse, with A_n being the strength and t_n the time of arrival for reflection n. The temporal distribution is partitioned into: direct sound, early reflections, and reverberation. The time delay and

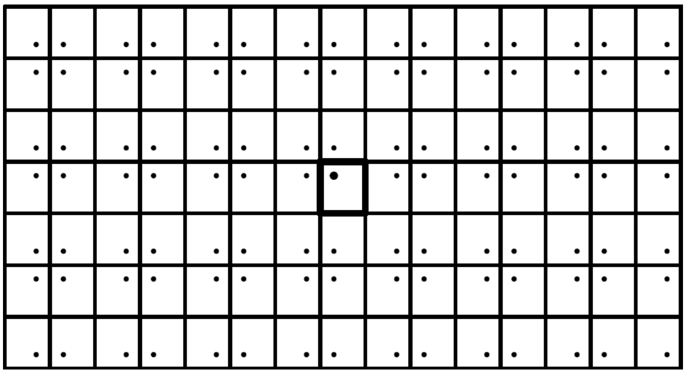


Figure 2.6: Image sources as they appear in adjacent theoretical rooms, with the original room in the middle. This plane of image sources is multiplied by the order i of image planes.

intensity of the early reflections as well as the reverberation are detrimental for the reverberation of a room, and in general for a room's acoustical properties, as described in section 2.7.2 on page 21. In figure 2.7, a temporal distribution can be seen using an echogram. The echogram shows the pulses and their intensity along the time axis, with the first pulse being the direct sound at $t = 0$. All subsequent reflections carry less and less density, but will also increase exponentially in density, as an increasing amount of continously weaker reflections appear.

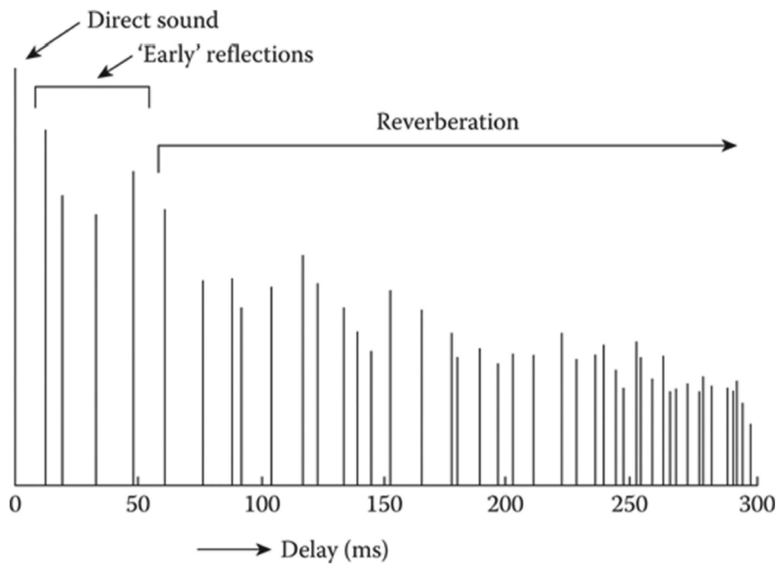


Figure 2.7: Temporal distribution showing the impulse response of a room, [8].

$$s'(t) = \sum_n A_n * s(t - t_n) \tag{2.52}$$

$$g(t) = \sum_n A_n * \delta(t - t_n) \tag{2.53}$$

2.6 Sound Absorbers

Sound absorbers are present in all acoustical measurements, no matter the material. By absorbers, an attenuation of the original sound pulse is meant. That attenuation can either be achieved by conversion into heat, transmission to a different medium, or simply by propagation in free space, which essentially is a conversion into heat as well. In air, the attenuation coefficient m is found by dividing the absorption coefficient by the distance in meters:

$$m = \frac{\alpha}{distance} \tag{2.54}$$

Or from a known intensity it can be found by:

$$I(x) = I_0 * e^{-m*x} \tag{2.55}$$

In table 2.4 the attenuation relative to humidity and frequency can be seen. Note that with increase in frequency, the attenuation increases as well. This is due to the non-ideality of the real world, as the mechanical energy irreversibly converts into heat, as sound waves propagate air. Furthermore, soundwaves create an oscillating rarefication and compression of air molecules, which mechanically will create friction between them, thus creating heat.

	Frequency (kHz)						
Relative Humidity (%)	0.5	1	2	3	4	6	8
40	0.60	1.07	2.58	5.03	8.40	17.71	30.00
50	0.63	1.08	2.28	4.20	6.84	14.26	24.29
60	0.64	1.11	2.14	3.72	5.91	12.08	20.52
70	0.64	1.15	2.08	3.45	5.32	10.62	17.91

Table 2.4: Attenuation constant m of air at 20°C and normal atmospheric pressure, in 10^{-3} m^{-1}, [7, 8].

Apart from heat, sound can also be "absorbed" by transmission, such as with open windows and doors. A more complex scenario is the transmission of sound through a wall or other structural material. A perfectly rigid and reflecting wall does not exist, therefore we assume that some value different from zero will always be absorbed or transmitted through any wall. If the sound pressures of an incident sound wave is denoted p_1, p_2, and p_3 for the incident, reflected, and transmitted components, the pressure acting on a wall can be described by $p_1 + p_2 - p_3$. The inertial force of the wall balances this by $i * \omega * M * v$, where M is the mass per unit area and v is the velocity of motion of the wall. This is equal to the particle velocity of the wave radiating from the back of the wall, given as $p_3 = \rho_0 * c * v$. From this, the impedance of a wall can be given as:

$$Z = \frac{p_1 + p_2}{v} = i * \omega * M * v + \rho_0 * c \tag{2.56}$$

Based upon equation 2.20. Implementing the wall admittance given by equation 2.21 as well, yields the following absorption coefficient, also known as the mass law:

$$\alpha = \left(\frac{2 * \rho_0 * c}{\omega * M}\right)^2 \tag{2.57}$$

The above calculations are only available for walls whose mass reactance is large in comparison with the characteristic impedance of air. As frequency decreases, the absorption coefficient drops as well, and poses the fact that lower frequencies will transmit from one room to the next, easier than high frequencies, due to the impedance mismatch between air and the wall. Typically, this is dealt with by impedance matching or by increasing the mass of the wall per unit area, if lower transmission is desired, [7, 8, 10, 11, 15, 16, 31, 32, 33, 34, 35]. If higher absorption is desired, lighter materials such as foam or similar porous materials should be applied. A result of lower absorption coefficients for lower frequencies is that a room can sound more "crisp", as the sound wave is neither absorbed nor reflected, but rather transmitted by the wall, whereas higher frequency sound waves yield a larger absorption and reflection coefficient.

2.6.1 Absorbers in Construction

In construction, absorbers are present in many forms, such as perforated walls, resonance absorbers, porous materials, heavy walls, and acoustically optimized windows. Perforated walls are commonly found in acoustic panels or as stand-alone walls placed some distance from a rigid backwall. The perforated plates act as both a reflecting medium and a transmitting medium, as each hole can be considered a short tube with length b equal to the thickness of the material, , [7, 8, 10, 11, 15, 16, 31, 5, 17]. The air within each hole is given by $\rho_0 * b$. A perforated panel has an "equivalent mass", which is given by equation 2.58, where σ is given by the perforation ratio $\frac{S_1}{S_2}$, wherein S_1 is the area of the whole and S_2 is the panel area per hole. The remaining variable b' is given by the effective length $b' = b + 2 * \delta b$, wherein δb is the end correction of a tube, which considers the fact pressure waves cannot contract and expand abruptly when entering or leaving an aperture. For a circular hole with radius a, the end correction is given by $\delta b = \frac{\pi}{4} * a$. For a rectangular hole, the end correction is approximated by $\delta b = \frac{\pi}{4} * \frac{a+b}{2}$. With these variables in place, it is possible to calculate the absorption coefficient of the perforated panel, as shown in equation 2.59, where M' is the equivalent mass and M'_0 is the specific mass for all solid parts of the panel, [7, 8].

$$M' = \frac{\rho_0 * b'}{\sigma} \tag{2.58}$$

$$M = \left(\frac{1}{M'} + \frac{1}{M'_0} \right)^{-1} \tag{2.59}$$

By calculating all of the above, the absorption coefficient can be found, by updating variables with their respective values and calculating equation 2.57 with 2.59 as M.

Another practical absorption method that can be implemented in the construction phase, is the integration of resonance absorbers. The general idea is to place a membrane in front of a cavity, where the membrane will be excited by a sound wave hitting it, thus creating vibrations controlled by the specific mass M and the cavity behind it. These vibrations represent an absorption and can be represented with vibrational losses, represented by the loss resistance r_s. Using equation 2.31 and 2.60, where d is the distance between the rigid wall and the membrane and M' is the specific mass of the membrane, the absorption coefficient can be calculated for various ratios of $\frac{r_s}{\rho_0 * c}$ under the assumption that $M' * \omega = 10 * \rho_0 * c$.

$$Z = r_s + i \left(\omega * M' - \frac{\rho_0 * c^2}{\omega * d} \right) \tag{2.60}$$

As r_s gets closer to $\rho_0 * c$, the absorption coefficient increases, until $\alpha = 1$ is achieved by $r_s = \rho_0 * c$. For all values far from r_s, the ratio yields broadening curves, as can be seen in figure 2.8.

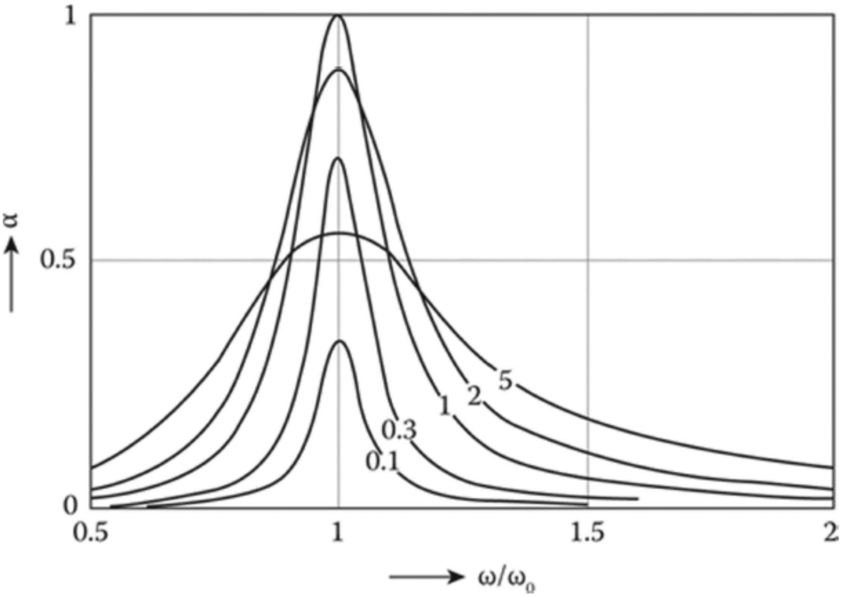


Figure 2.8: Calculated absorption coefficients for resonance absorbers at normal sound incidence, with determining parameter $r_s = \rho_0 * c$,[8].

When using resonance absorbers in construction, the membrane often consists of plywood, plasterboard, metal plates, or similar materials. Depending on the resonating medium, different hurdles can arise, such as intrinsic impedance, elastic deformation, or other properties inherent to the material. Furthermore, the material must be attached to the rigid backwall, requiring more substantial fastening for some materials than others. A method for adapting the materials' characteristics and thereby r_s, is filling a percentage of the cavity with porous material, such as rockwool or foam. For normal sound incidence, the practical resonance frequency of an absorber is given by equation 2.61:

$$f_0 = \frac{600}{\sqrt{M' * d}}[Hz] \tag{2.61}$$

Where M' is in $kg\ m^{-2}$ and d in cm. If the cavity is filled with porous material, the value "600" should be replaced by "500", to account for it, [7]. As this equation is a practical formula rather than a scientifically based formula, it can only tell the ballpark for the resonant frequency, as the mounting method will have a significant impact on the actual resonant frequency. The only way to get better results would be to test the construction in a reverberation chamber, examining the absorption coefficient.

Porous materials such as rockwool, Troldtekt, Rockfon, and similar acoustic altering material work by employing the principle of viscous and thermal processes at the boundary layer. The boundary layer is between 0.01 and 0.2mm depending on frequency. For smooth surfaces the absorption is rather small, but as the surface gets rougher, the absorption increases. Therefore, as a porous material can be described as an almost infinitely rough surface, the absorption increases dramatically, with the best effect achieved by channels/openings to the outside air. As infinitely many surfaces appear, an equal amount of reflections will appear as well. Depending on the frequency, these reflections can interfere destructively or constructively. However, because of the infinite number of waves present within the porous layer of decreasing strength, the attenuation should increase in relation to the depth of the medium.

The pressure fluctuations within the porous material/along the edges of each surface will result in a considerable amount of mechanical energy being withdrawn from the impinging soundfield, thus converting it into heat. Due to the inherently rough surface of some construction materials, such as bricks, mortar, and plaster (*not plasterboard*) will show similar characteristics as the porous materials. Unfortunately, as the rough layer is not nearly deep enough before becoming solid, the acoustical benefits of these materials are not comparable to products designed for absorption.

The characteristics of porous materials can be described mathematically by i.e. the Rayleigh model. However, mathematically describing such materials is too comprehensive and time-consuming, compared with measuring them in a reverberation room, [7, 8]. Therefore, the mathematics behind it is left out of this project.

2.6.2 Absorbers in Furnishment

Furnishment is the go-to solution for altering the acoustic properties of a room if it cannot be done structurally. Some typical examples of furniture/decor that will alter the acoustical properties are:

- **Carpets and Rugs:**
  Thick soft carpets, large rugs, or similar material that can shield off some of the hard flooring, would be able to reduce the reflection coefficient of the floor significantly. Due to the large area and hard backing, low-frequency attenuation is achievable with the correct composition.

- **Upholstered furniture:**
  Lounge chairs, sofas, regular chairs, pillows, and other furniture can beneficially be upholstered instead of leathered/wooden, to create a porous surface with foam, felt, or similar backing. With correct placement in a room, the furniture can help control reflections.

- **Curtains:** Curtains can lower the reverberation time significantly if made of sufficiently thick or heavy material. Not to say that thin drapes will not have an effect, as such could act as resonance absorbers. Furthermore, louvers can be fitted to direct reflections in specific directions.

- **Acoustic images, art, etc.:** The latest craze is adding acoustic images to a room, to alter the reverberation time. These are often composed of printed fabric wrapped around a porous material. Another option is art sculpted into the porous material, enabling effective attenuation in specific frequency bands to be placed on walls, ceilings, or other flat surfaces.

While all of the above is applicable in most rooms, other situations require more substantial incorporation. One such situation is improving the acoustics of a concert hall, lecture room, auditorium, sports hall, cinema, or similar large-scale hall. As such rooms are meant to be used by a large number of people, the acoustic properties must be determined by including the number of people in the equation. The absorption coefficient of a guest in any room is highly reliant on the clothing, as different fabrics, styles, and seasonal clothing alter the final measurement. Apart from clothing, the arrangement of an audience is highly influential as well, as the density is determined by the number of people per square meter. In table 2.5 the absorption coefficient of a person can be viewed, obtained by placing them in a reverberation room. Unfortunately, this method does not take into account what would happen if a large amount of people were placed closely together, as reverberation rooms large enough to handle several hundred people do not exist. An alternative is measuring the change of reverberation in finished halls, but since most halls vary in size and shape, the results are rarely applicable for other halls than the measured. It is however still possible to get useful measurements in a reverberation room, which can be seen in table 2.6, where the absorption coefficients of different chairs with an audience are shown.

Kind of Person	125 Hz	250 Hz	500 Hz	1,000 Hz	2,000 Hz	4,000 Hz
Male standing in heavy coat	0.17	0.41	0.91	1.30	1.43	1.47
Male standing without coat	0.12	0.24	0.59	0.98	1.13	1.12
Musician seated, with instrument	0.60	0.95	1.06	1.08	1.08	1.08

Table 2.5: Equivalent absorption area of persons, in m^2, [8].

Type of Seats	125 Hz	250 Hz	500 Hz	1,000 Hz	2,000 Hz	4,000 Hz	6,000 Hz
Audience seated on wooden chairs, two persons per m^2	0.24	0.40	0.78	0.98	0.96	0.87	0.80
Audience seated on wooden chairs, one person per m^2	0.16	0.24	0.56	0.69	0.81	0.78	0.75
Audience seated on moderately upholstered chairs, 0.85 m × 0.63 m	0.72	0.82	0.91	0.93	0.94	0.87	0.77
Audience seated on moderately upholstered chairs, 0.90 m × 0.55 m	0.55	0.86	0.83	0.87	0.90	0.87	0.80
Moderately upholstered chairs, unoccupied, 0.90 m × 0.55 m	0.44	0.56	0.67	0.74	0.83	0.87	

Table 2.6: Absorption coefficients of audience and chairs (reverberation chamber), [8].

As can be seen from table 2.6, the seat type has a pronounced effect on absorption, especially at low frequencies, where upholstered seats more than triple the absorption coefficient, compared with wooden chairs. Unfortunately, as seating is divided into blocks divided by pathways rather than uniformly across a hall, exact absorption is best found by measuring the reverberation time before and after the installation of seats in a hall.

An effect that has been measured in finished concert halls is known as "the seat dip effect". This effect is the attenuation present across empty rows of seats, with the highest attenuation from 80-250 Hz. While the effect has been replicable for occupied seats as well, it is not as common as for unoccupied seats, [7, 8, 10, 11].

The attenuation of direct sound is furthermore increased with an increased amount of rows in front of the listening position. One aspect is the natural attenuation in air, but the most prominent factor is diffraction of sound, as it hits seats and occupants of the rows in front. A typical countermeasure to this phenomenon is the vineyard topology of many concert halls. That is, the placement of seats on a slope, maximizes the area that is subjected to direct sound while minimizing grazing incidence.

2.7 Metrics for Room Acoustics

2.7.1 Room Gain - G:

The gain of a room is very literally the amplification or attenuation that a room provides. It can be measured using a calibrated omnidirectional speaker (such as a dodecahedral speaker), where an impulse is first measured in a free field (an anechoic room), and then in the room to be examined. The measurement in the free field is done by measuring at a distance of 10 m from the sound source and is repeated for at least 5 measurements at various locations to create an average, [22, 9]. As in free field, the room to measure should have an equal amount of measurements made. The gain of a room can be described with the following equations from DS 3382-1, [9]:

$$G = 10 * log10\left(\frac{\int_0^\infty p^2(t)\,dt}{\int_0^\infty p_{10}^2(t)\,dt}\right) = L_{pE} - L_{pE,10}[dB] \tag{2.62}$$

$$L_{pE} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p^2(t)\,dt}{p_0^2}\right][dB] \tag{2.63}$$

$$L_{pE,10} = 10 * log10\left[\frac{1}{T_0} * \int_0^\infty \frac{p_{10}^2(t)\,dt}{p_0^2}\right][dB] \tag{2.64}$$

Where: $p(t)$ is the instantaneous sound pressure of the impulse response measured at a point in the room measured in Pascal, where $1\ Pascal = 1\frac{kg}{m*s^2}$, $p_{10}(t)$ is the instantaneous sound pressure of the impulse response measured at 10m in free field, p_0 is $20\mu Pa$, T_0 is 1 second, L_{pE} is the sound pressure exposure level of $p(t)$, and $L_{pE,10}$ is the sound pressure exposure level of $p_{10}(t)$.

If any of the rooms are not large enough to complete the measurements at 10 meters, they can instead be done at 3 meters and modified with:

$$L_{pE,10} = L_{pE,d} + 20 * log10(d/10)[dB] \tag{2.65}$$

Or even simpler by using, [22, 9]:

$$L_p = L_{p,10}[dB] \tag{2.66}$$

Where: d is distance in meters, L_p is the sound pressure level averaged across every measurement point, and $L_{p,10}$ is the sound pressure level measured at 10 m in the free field.

Alternatively, if a sound source with a known sound power level is available, the gain can be obtained by the following, [22, 9]:

$$G = L_p - L_W + 31[dB] \tag{2.67}$$

Where: L_p is the sound pressure level averaged across every measurement point, and L_W is the sound power level of the sound source, and should be measured according to DS 3741.

2.7.2 Reverberation Time 60 - RT60:

Reverberation **T**ime 60 describes the time it takes for sound a level to decay 60 dB. In many practical applications, background noise makes it difficult to measure a full 60 dB decay, and therefore, T20 or T30 is used instead. T20 and T30 differ from RT60 by measuring either 20 or 30 dB decay and following the description set in DS ISO 3382-2, the decay time must be measured from -5 to -25/30 dB, and not from 0 to -20/30 dB, [36, 22, 37, 38]. In figure 2.9a, an example of measuring T30 can be seen, where it should be noted that the decay is first measured from -5 dB. When using T20 or T30, the time measured should be multiplied by 3 or 2 to harmonize with RT60. It should furthermore be noted that T20 is most frequently used, as DS 3382-2 states that *"the subjective evaluation of reverberation is related to the early part of the decay"*, and that *"the signal-to-noise ratio is often a problem in field measurements, and it is often difficult or impossible to get an evaluation range of more than 20 dB."*, [36].

Sabine's equation for a diffuse sound field can be used to estimate a room's reverberation time. It takes the input V for the volume of the room in cubic meters, α for the absorption coefficient in $\left[\frac{sabin}{m^2}\right]$, and S for the surface area of the room in square meters, and is used as:

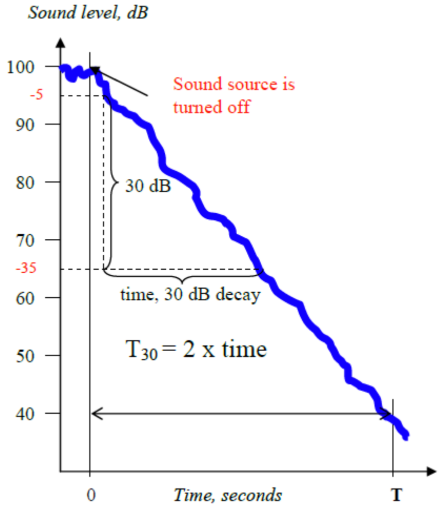
$$RT60 = \frac{0.161 * V}{S_i * \alpha_i} \leftrightarrow RT60 = \frac{0.161 * V}{A}[s] \tag{2.68}$$

Where:

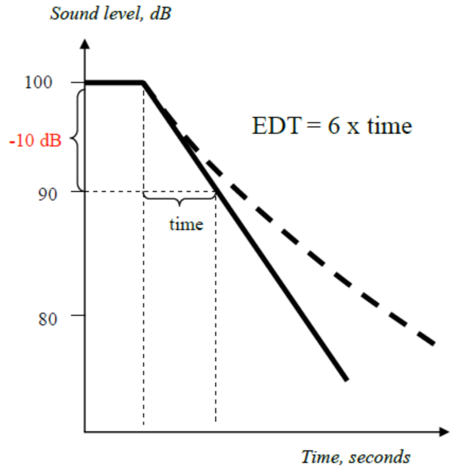
$$A = \sum_{i=o}^{i=\infty} S_i * \alpha_i \tag{2.69}$$

The reasoning behind equation 2.69/the lower fraction of 2.68 is to compensate for different materials with different absorption coefficients in a room. It should, however be denoted that since Sabine's formula assumes a diffuse sound field, it is best used concerning EDT rather than T30 or T20 due to the corresponding decay curves. This can be seen in figure 2.9b, where the solid straight line matches a continued approximation derived from the EDT. The dashed line is a more realistic decay curve of a room with absorbent materials, such as acoustic ceilings, [22]. In general, some optimal RT ranges are, [10, 11, 12, 22, 23]:

- Speech-oriented spaces, such as classrooms, meeting rooms, lecture halls, etc., require a short reverberation time of 0.3-1 second, to ensure speech intelligibility.
- Musical performance rooms, such as opera houses, concert halls, clubs, etc., would typically benefit from longer reverberation times, ranging from 1.5-3 seconds.
- Multi-use rooms such as dining halls, auditoriums, etc. benefit from a compromise, with reverberation times of 1.2-2 seconds.
- Sacred rooms/rooms of worship, such as churches, chapels, monasteries, etc., have much wider ranges, as they (dependent on religious purposes) have a reverberation time between 1-10 seconds, with gothic churches aiming at 6-13 seconds, [10].



(a) Definition of 30 dB decay (T30). Notice the timer starts at -5dB, [22].



(b) Illustration of EDT, [22].

Figure 2.9: T30 and EDT illustrations, [22].

2.7.3 Early Decay Time - EDT/T10:

Early decay time describes the first 10 dB decay within a room and is sometimes denoted T10, similarly to T20 and T30. The main difference from T20 and T30 is that EDT does not include the same -5 dB buffer but instead measures the actual first 10 dB decay. According to [22], [39], and DS 3382-1, [9], the EDT should be the subjectively most important reverberation characteristic when determining the acoustic properties of a room. EDT is determined by the first 10 dB of attenuation multiplied by 6 to reach a number comparable with RT60. The EDT equation is most beneficial when it comes to the psychoacoustic perception of a room and for determining "ideal" decay time, [39]. This is caused by the minimum of summed reflections present within the first 10 dB decay.

2.7.4 Clarity - C50/C80:

To determine the clarity relation of a room as an acoustical parameter, C50 or C80 is used. C50 estimates the reflections and their energy present after 50 ms, while C80 estimates them after 80 ms. The idea is that reflections present after the critical time limit muddles perceived speech and reduce clarity. The critical time limit varies from person to person, with age, and is furthermore dependent on what exactly is being listened to, [22, 10, 11, 23, 9]. C50 is most often used for speech intelligibility and C80 is used for music clarity. The C50 and C80 relation is calculated by:

$$C50 = 10 * log10\left(\frac{Energy(0-50ms)}{Energy(51ms-end)}\right)[dB] \tag{2.70}$$

$$C80 = 10 * log10\left(\frac{Energy(0-80ms)}{Energy(81ms-end)}\right)[dB] \tag{2.71}$$

Where the "Energy" values are derived from a broadband impulse response in the room. The impulse response is recorded and divided into integrated squared energy values before and after the critical time limit, [22, 10, 11, 23, 9]. Both can also be found generally by:

$$C_{t_e} = 10 * log10 \left(\frac{\int_0^{t_e} p^2(t)\, dt}{\int_{t_e}^{\infty} p^2(t)\, dt} \right) [dB] \tag{2.72}$$

Where: C_{t_e} is the early to late index. t_e is the early time limit (e.g. 50 or 80 ms) and $p(t)$ is the measured sound pressure of the impulse response, [9].

For speech clarity, values above 0 dB are considered good, while for music clarity it depends on music type and preference, but in most cases, a value between -2 dB and +4 dB is considered acceptable, [23].

2.7.5 Definition - D50:

Definition/Deutlichkeit after 50 ms is a percentage describing how many percent of the total energy is present within the first 50 ms. D50 is strongly correlated to C50, as D50 (and vice-versa C50) can be found by, [9]:

$$C50 = 10 * log10 \left(\frac{D50}{1 - D50} \right) [dB] \tag{2.73}$$

Or by itself using:

$$D50 = \frac{\int_0^{0.050} p^2(t)\, dt}{\int_0^{\infty} p^2(t)\, dt} [\cdot] \tag{2.74}$$

2.8 Measuring Techniques

As mentioned many times throughout this report already, the best way to determine the acoustical parameters of materials or rooms is by measuring them. The method for measuring acoustical properties has been developed considerably over the last two decades, with the introduction of increasingly powerful computers, algorithms, and electroacoustical instruments.

2.8.1 Impulse Response of a Room

The impulse response of a room reveals everything there is to know about a signal's propagation from one point to another in a room if it is assumed to be time-invariant. The transfer function created from the Fourier transform of an impulse response yields the linear transmission system. Spatial and directional information requires a binaural impulse response. A regular impulse response, on the other hand, uses only one omnidirectional microphone and speaker. In contrast, a binaural impulse response requires a dummy head or a person with microphones inserted into each ear canal.

To be considered, the impulse response must be measured using distortionless components, and the room should be in the same state as when in use. The impulse itself is a short burst, often in the low millisecond range, excited with intensity high enough for decaying sound fields to be measured. The impulse can be created by various methods, such as pistol shots, wooden clappers, or an omnidirectional loudspeaker playing a known signal. By using the latter method, exact impulses can be designed and used. Another benefit of using the loudspeaker with a known impulse response is that the convolved signal can be broken into fragments, containing the impulse response of the speaker and the actual room impulse response. Mathematically, it can be written in the time domain as:

$$g'(t) = g(t) * g_L(t) \tag{2.75}$$

Where $g(t)$ is the room's impulse response, and $g_L(t)$ is the response of the loudspeaker. The deconvolution is best done in the frequency domain by Fourier transforming the response, yielding:

$$G'(f) = G(f) * G_L(f) \tag{2.76}$$

The signal exciting the speaker is given as $s't$ in the time domain and as $S'(f)$ in the frequency domain:

$$s'(t) = \int_{-\infty}^{\infty} s(\tau) * g(t-\tau) d\tau \tag{2.77}$$

$$S'(f) = S(f) * G(f) \tag{2.78}$$

From the above equations, the impulse response of the room ($g(t)$) can be found from the inverse Fourier of either equation 2.76 or 2.78. This method is known as dual channel analysis, as both the original signal and the speaker's impulse response is known, isolating $g(t)$ as the only unknown.

To remove uncertainties, measurements should be done several times to achieve mean values. With each increase in the number of measurements N, the energy of the impulse responses in total grows by N^2, while background noise and measuring noise only grow by N. By doing so, the SNR is increased by $10 * log_{10}(N)$.

2.8.2 Acoustical Properties of Materials

While the impulse response of a room is the most accurate way of determining the characteristics of a room, it relies on the room being built beforehand. To mitigate unwanted acoustical properties in the sketching phase, the room can be simulated, using software such as Odeon, COMSOL Multiphysics, etc., if the materials and schematics of the room have been finalized. Therefore, it is highly beneficial to know the acoustical properties of materials used in the room beforehand, such as sound absorption, reflection, transmission, and diffusion. Said properties can be determined by measuring impulse responses of the materials when placed in known environments, such as reverberation rooms or impedance tubes.

The room can be designed to meet acoustic benchmarks by acquiring these properties for all materials used. Unfortunately, measuring such materials is prone to faulty or unreplicable results, as quite a few aspects can alter them. For reverberation rooms, the main fault for material testing is the lack of strictly standardized sizes and forms. Some standards do exist though, as specified in DS/ISO 354:2003, such as a minimum volume V of $150m^3$, a recommended volume of $200m^3$, and a maximum volume of $500m^3$ to reduce the possibility of attenuation by air and a maximum length in a straight line anywhere in the room of $l_{max} = 1.9 * V^{\frac{1}{3}}$, [40]. Furthermore, diffusers should be placed in the room until the absorption coefficient reaches a maximum and remains there no matter how many diffusers are added - a rule of thumb is that the area equals 15-25% of the rooms total surface area, [40]. More rules exist for the reverberation room, but as can be seen by these general rules, a lot of variation is permissible, yielding in non-equal measurements of the same material, placed in different reverberation rooms.

Regarding the impedance tube, two methods currently exist: 1. the standing wave method. 2. The multimicrophone method. The standing wave method has been the industry standard for many years (since 1902, [21]), but has recently been replaced by the multi-microphone method (2023, [14]). Unfortunately, both methods have their flaws: Common for both methods is mounting of test specimens, as specimens are easily subjected to deformation and/or being cut slightly in the wrong size due to the circular form, resulting in air gaps. Another issue present mostly for the standing wave method is the human error present when reading measurements off of analog instruments for each individual frequency. The multimicrophone method is as prone to user error, as it is essential that microphones are calibrated correctly before each test.

Depending on the purpose of the material test, the impedancetubes are limited to the frequencies for which they can test. The Brüel & Kjær 4002 standing wave apparatus has working frequencies $\approx$90-6500Hz, and the type 4206 multimicrophone impedance tube has $\approx$50-6400Hz. However, these working frequencies are dependent on the attached tube, resulting in two different tubes for use in different frequency bands. While both impedance tube types cover the frequency band most influential on human hearing, they still leave a rather large frequency band untested.

2.8.3 Problematic Frequencies

As can be extrapolated from the report so far, many acoustical models exist for behavior from 100Hz to 6kHz, but outside of this band, models and results become increasingly experimental. This leaves room for improvement on the remaining spectrums, especially as background noise often has components in these regions. Low-frequency noise has been an increasing health focus worldwide and would benefit from being attenuated, both at the source and within rooms. While most low-frequency noise is vibrational, large amounts are still audible. The infrasound spectrum (0-20Hz) is especially interesting, as most levels are not audible, but felt instead. Despite being largely inaudible, infrasound can still have psychological and physiological effects on individuals, potentially causing discomfort or even health problems, especially at higher amplitudes. This makes the study and mitigation of infrasound an important area of research. Therefore, extending research and testing to cover these problematic frequencies, particularly in the infrasound range, would help create more comprehensive acoustical models that account for the full spectrum of human hearing and environmental noise. Addressing this gap could lead to better control of noise pollution, especially in environments where noise exposure is a concern for health and well-being.

2.9 Problem Statement

Based on the problem analysis/dive into room acoustics, several statements can be made when reviewing the initial problem statement:

"How can "good acoustics" be defined and quantified in a way that balances theoretical ideals with real-world applicability?"

The analysis shed light on the main subject of quantizing "good" acoustics, before embarking in an effort to describe acoustical phenomena by math and intuitive relations. The immediate characteristics for obtaining good acoustics were D50, C50, and most importantly RT60 and EDT. However, these properties were inherent by a room, and could be hard to quantify, if material properties were unknown. The most influential properties were found to be: Reflection, diffusion, and absorption coefficients, wall impedance/admittance, transmission loss and resonance frequency, as shown in table 2.2.
With common acoustical measures and wall properties in place, room acoustics in relation to modal planes and image sources was investigated, leading to the possibility of describing soundfields using ray-tracing, rather than spherical waves. This idealization enables easier computation of modeled reverberation times, while also making it possible to predict orientation of reverberated sound rays.

However, a prevailing issue began appearing while analyzing room acoustics as a whole. When it came to absorbers and impulse responses of rooms, it became clear that many models rely on data for 100-6000Hz rather than the full audible spectrum, or atleast the spectrum used for equal loudness curves (20-12500Hz). With increasing amounts of research pointing towards psychological and physiological concers regarding especially low frequency sounds. It would pose an interesting task to determine acoustical properties below 100Hz, and preferably entering the infrasound band for optimal modeling.

The low frequency predicament is found in impedance tubes and reverberation rooms as well. This adds to the inherent problems of absorption coefficient measurements of materials in impedance tubes and especially reverberation rooms mentioned in 2.8.2. Due to the un-replicable nature related to reverberation rooms, impedance tubes would be a better starting point for developing a system capable of determining acoustical properties. The main issue of impedance tubes is the circular form, as samples are hard to create without risking deformation and/or air gaps. Therefore, this projects problem statement is:

"How can a square impedancetube capable of measuring very low frequency properties of construction materials be developed?"

3 | Technical Analysis

In an effort to analyze the technical needs of the project, common methods for measuring acoustic properties are examined in depth. Furthermore, signal processing procedures for easing acoustical measurements will be examined as well, to gain insight into designing a complete system capable of measuring wideband frequency properties, along with the focus on low frequencies given by the problem statement.

When measuring the acoustic properties, the R_{10} series of the international "preferred numbers" should be used, based on the standardized Renard numbers, [32]. The frequencies used closely match the center frequencies of $\frac{1}{3}$ octave bands. Octave bands are described in appendix E on page 74, and the complete table of preferred Renard numbers applied in acoustics can be found in appendix F on page 77.

3.1 Reverberation Room Method

The reverberation room is a common method used to test large samples, such as entire rows of seats from a concert hall, resonating panels, entire rockwool batts, furniture, large-scale absorbers, edge absorption, etc. The reverberation room functions by following the dimensions specified in DS/ISO 354:2003, and placing "hard" walls with little to no absorption at all edges, [40]. To prolong the reverberation time, diffusers are placed within the room, minimizing the amount of parallel surfaces and scattering sound energy in all directions.

However, as pointed out in several publications by different authors, the reverberation room is not a trustworthy source of absorption/reflection coefficients, in spite of semi-standardized room configurations, [32, 34, 33, 35, 31, 7, 8, 41, 42, 17, 5].

3.1.1 Measurement of Sound in a Reverberation Room - DS 354:2003

As briefly mentioned in "Acoustical Properties of Materials", reverberation rooms are standardized through DS/ISO 354:2003. The room itself should as a minimum have a volume V of $150m^3$, a recommended volume of $200m^3$, and a maximum volume of $500m^3$ to reduce the possibility of attenuation by air and a maximum length in a straight line anywhere in the room of $l_{max} = 1.9 * V^{\frac{1}{3}}$, [40]. The number of diffusers present within the reverberation chamber is given by Annex A in DS/ISO 354:2003. The diffusers should be sheets of different sizes between 0.8-$3m^2$ (single side) with a mass of $5\frac{kg}{m^2}$, [40]. When finding the threshold of diffusivity, a sample of 5-10cm thick homogeneous, porous material with an absorption coefficient over 0.9 in the frequency range of 0.5-4kHz is placed in the room. The room is then tested without diffusers, and thereafter with increases of $5m^2$ until the absorption coefficient reaches a constant value in spite of an increasing number of diffusers mounted. For rectangular rooms, a diffuser area (both sides) matching 15-25% of the total surface area of the room is sufficient, [40].

When testing plane absorbers in a reverberation room, they should be 10-12m^2 for rooms with a volume of 200m^3. For larger rooms, the test specimen area should increase with a factor of $(\frac{V}{200})^{\frac{2}{3}}$. Furthermore, the plane absorbers should be mounted according to Annex B in DS/ISO 354:2003. Discrete objects such as furniture, free-standing objects, or similar, should be installed in the reverberation rooms in a similar fashion, as when installed in practice. When testing discrete objects, several samples should be placed at a distance of 2m between them. For both plane absorbers and discrete objects, the distance to any boundaries in the chamber should be 1 meter or more, unless otherwise specified by the installation manual.

To ensure the reverberation room complies with the standard, table 3.1 is used. The equivalent sound absorption area is a descriptor of "*hypothetical area of a totally absorbing surface without diffraction effects which, if it were the only absorbing element in the room, would give the same reverberation time as the room under consideration*", and is given by equation 3.1 where V is the volume of the empty reverberation room, c is the propagation speed of sound in air, T_1 is the reverberation time of the empty reverberation room, and m_1 is the power attenuation coefficient calculated according to ISO 9613-1, [40, 43]. The values given for equivalent sound absorption area are the maximum permissible value for the reverberation room to be accepted as per the standard.

$$A_1 = \frac{55.3 * V}{c * T_1} - 4 * V * m_1 \tag{3.1}$$

Frequency, Hz	100	125	160	200	250	315	400	500	630
Equivalent sound absorption area, m^2	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
Frequency, Hz	800	1,000	1,250	1,600	2,000	2,500	3,150	4,000	5,000
Equivalent sound absorption area, m^2	6.5	7.0	7.5	8.0	9.5	10.5	12.0	13.0	14.0

Table 3.1: Maximum equivalent sound absorption areas for room volume $V = 200m^3$ for $\frac{1}{3}$ octave bands with given centerfrequencies, [40].

3.1.2 Subtleties of Using a Reverberation Room

While widely accepted as the standard measurement method for acoustic properties of large specimens, the reverberation room method poses several subtleties that should be considered. The main consideration should be that no reverberation rooms are identical, leaving measurements as a value only obtained in that specific room. Effectively, this renders measurements more or less useless, as they are not replicable in different rooms, and possibly not even in the same room.

Moreover, if samples are mounted slightly differently in testing, they might not produce a similar result when installed at the job site. Orientation of samples, along with distances between samples, will also yield different results. A factor largely contributing to this is the increased absorption at the edges of porous materials. In general, the reverberation room is great for achieving a vague idea of a sample's acoustic properties, but values are rarely applicable outside of that exact test environment, [32, 34, 33, 35, 31, 7, 8, 41, 42, 17, 5].

3.2 Standing Wave Method

The standing wave method has been the go-to method of obtaining consistent and accurate acoustical properties for decades. It is considered the most reliant method for assessing reflection and absorption coefficients of materials. However, the standing wave method has significant drawbacks in the time needed to perform measures for large frequency bands, while also being incapable of achieving correct measurements of resonance absorbers in most cases.

To comprehend the standing wave method, the DS 10534-1 2001, Brüel & Kjær Application Manuals, and transmission line theory will be utilized, [13, 32, 33, 34, 35]. But first, a graphical example/analogy will be introduced:

3.2.1 Graphical Analogy

Impedance tubes utilizing the standing wave method are often portrayed as seen in figure 3.1 or similarly, most resembling SWR patterns as seen in transmission line theory. However, it can be quite difficult to get a constructive idea of what is happening physically from such an image. Therefore, figure 3.3 is used instead. This figure describes the incident sound wave by equation 3.2, the reflected sound wave by equation 3.3, and the summed waveform by equation 3.4. Now, the way that a drawing such as 3.1 should be interpreted is, that each "hoop" is the upper part of both waveforms. This can be seen in figure 3.2 where the upper half of the waves can be seen, with x_{min1}, x_{min2}, and x_{max} marked as well. This interpretation can be made, as the reflected wave is phase shifted 180°, meaning that its upper maximum is equivalent in position to the lower maximum of the incident wave. This also means that the intersection between the waves matches minimums.

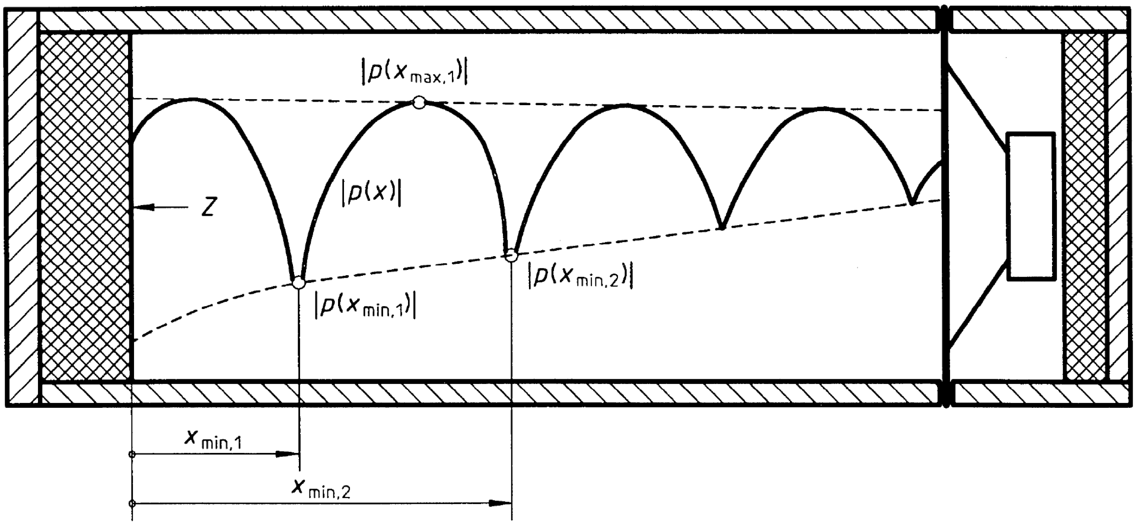


Figure 3.1: Typical graphic representation of the standing wave pattern in an impedance tube, [13].

$$p_i(x) = A * cos(kx - \omega T) \tag{3.2}$$

$$p_r(x) = r * A * cos(-kx - \omega T + \pi) \tag{3.3}$$

$$p_t(x) = A * cos(kx - \omega T) + r * A * cos(-kx - \omega T + \pi) \tag{3.4}$$

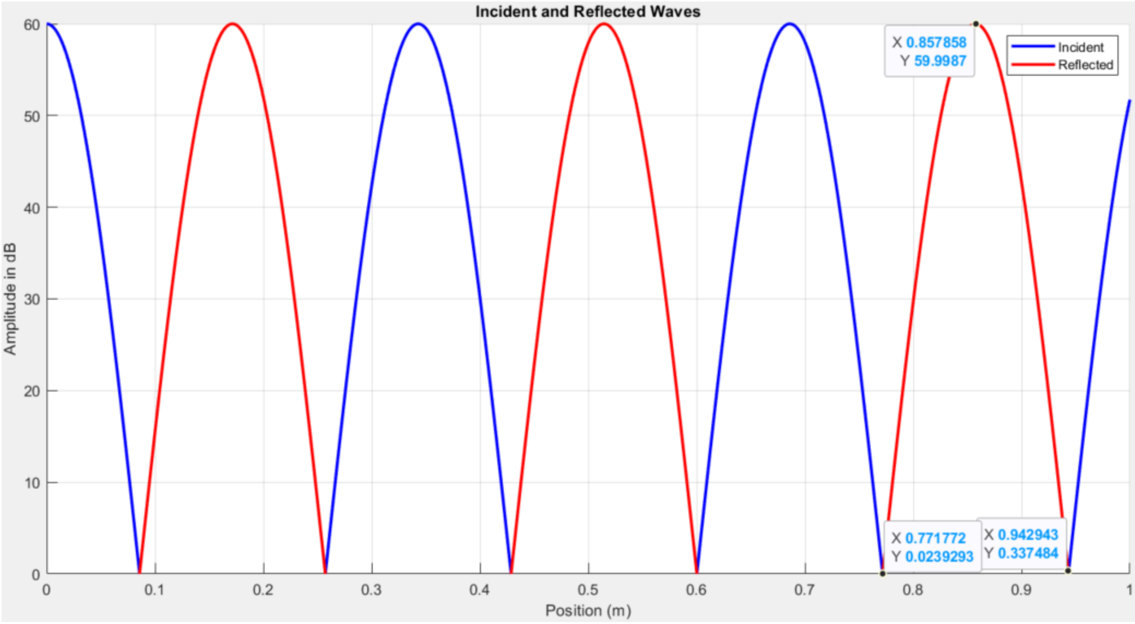
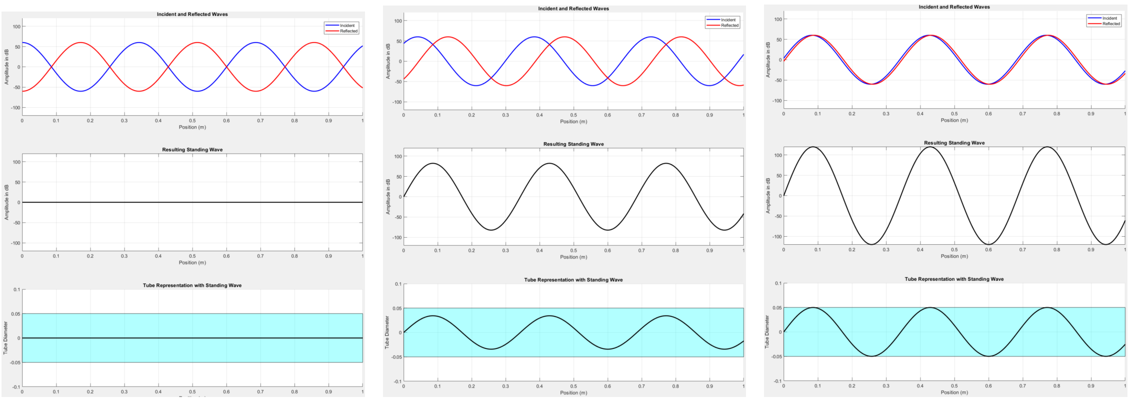


Figure 3.2: Interpretation of standing waves in an impedance tube, with x_{min1}, x_{min2}, and x_{max} marked.



(a) **(b)** **(c)**

Figure 3.3: Standing waves visualized as incident, reflected, and summed waveform as well as the summed waveform in an impedance tube. The blue waveform is the incident wave, and red is the reflected wave. All figures have the tube-end to the right on the x-axis in meters, and the two top illustrations have amplitude in dB on the y-axis, with the lower blue illustration having tube diameter in meters on the y-axis.

3.2.2 Standing Wave Method Standard - DS 10534-1-2001

To obtain qualitative measurements using an impedance tube, DS 10534-1-2001 should be used as a guideline. The standard describes all related variables and how they should be used, along with defining the physical aspects that an impedance tube should meet. While the standard introduces significant formulas, the related math will be explained in section 3.2.3 starting page 32. Therefore, this section will mostly relate to the physical characteristics of an impedance tube, such as diameter, length, material, etc. This also means that the main specifications for the use of an impedance tube are best found in the standard itself, and will be left out of this section.

As the name suggests, an impedance tube should be tubular, but is not restricted in shape, and could therefore be circular or rectangular. The first parameter to acquire is the length of a tube, given by equation 3.5, with λ being the desired frequency wavelength to test. It should however be noted that this equation yields a length far greater than the one used in the Brüel & Kjær Type 4002, which will be explained in greater detail in section 3.2.3 on page 32. When defining the diameter, equation 3.6 is used to find the frequency diameter boundaries when length is known, and equation 3.7 is used to find the upper boundary for rectangular tubes, and 3.9 for circular tubes. In equations 3.6, 3.7, and 3.9 the variables f and λ are interchanged with a given value for a specific frequency. Equations 3.8 and 3.10 are used to establish the upper frequency threshold.

The equations can only be used by specifying variables, such as the desired frequency boundaries or physical sizes, to determine the corresponding frequency boundaries. By investigating the equations, it also becomes clear that for rectangular tubes, a square form is desired in most cases, as it is the longest side which determines the upper frequency boundary.

$$l \geq \frac{3 * \lambda}{4} \tag{3.5}$$

$$l = \frac{250}{f} + 3 * d \rightarrow d = \frac{l}{3} - \frac{250}{f} \tag{3.6}$$

$$d \leq 0.5 * \lambda \tag{3.7}$$

$$f_U * d \leq 170 \tag{3.8}$$

$$d \leq 0.58 * \lambda \tag{3.9}$$

$$f_U * d \leq 200 \tag{3.10}$$

The tube itself should be made of a rigid and smooth material that will not be excited by vibrations. Furthermore, the tube must be straight within 0.2%. The material which the tube is made from is effectively not as important as the smoothness and rigidity, however, metal or concrete tubes will be easier to create uniform surfaces with. Rectangular tubes will especially have extra emphasis on the rigidity of the structure and smoothness in corners, as the form itself is not as rigid as circular tubes.

The sample holder can be constructed as either an integrated terminating end with room for a sample in front, or by making the end removable. With the terminating end, a section of the tube must be removable, to insert a sample. The removable section should be long enough to enable the possibility of testing cavities behind a material sample. By using a removable end, the end section must still conform to the other specifications of the tube, while also enabling the possibility of inserting a sample into the end. When mounting samples, seals should be made airtight by elastic gaskets, such as vaseline.

The electrical equipment necessary to facilitate use of an impedance tube is:

- A moveable microphone
- Signal processing equipment
- Loudspeaker
- Signal generator
- Thermometer

It should go without saying that each of these tools should be of sufficient quality to make measurements viable. Specific relations between the equipment are best found by reading the standard, [13].

3.2.3 Math Behind the Standing Wave Method

This section outlines key equations used for the standing wave method. Depending on which source material is used, some formulas differ but essentially calculate the same, exemplified by the equations for SWR in 3.11 and absorption coefficient in 3.12, [13, 32, 33, 34, 35].

$$n = \frac{(A+B)}{(A-B)} = \frac{p_{max}}{p_{min}} = \frac{1+|r|}{1-|r|} = \frac{N}{\rho_0 * c} \tag{3.11}$$

$$\alpha = 1 - \frac{B^2}{A^2} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{4}{n + \frac{1}{n} + 2} = 1 - |r|^2 = \frac{4}{\frac{(n+1)}{(n+2)}} = \frac{4n}{n^2 + 2n + 1} \tag{3.12}$$

Where:

- n is the standing wave ratio
- A is the measured amplitude of the incident sound wave
- B is the measured amplitude of the reflected sound wave
- p_{max} is the maximum sound pressure given by equation 3.19
- p_{min} is the minimum sound pressure given by equation 3.20
- r is the reflection coefficient of a sample given by equation 3.14
- N is the "true impedance" given by equation 3.13
- ρ_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$ for air)
- c is the speed of sound (set to $343[\frac{m}{s}]$

The SWR n is used to determine how much of the wave is reflected from a material by measuring the difference between minima and maxima of the standing wave, which can be found by equations 3.20 and 3.19. It is not a percentage but a dimensionless variable used to determine the reflection coefficient r given in equation 2.15 and/or the absorption coefficient α.

$$n = \frac{N}{\rho_0 * c} \leftrightarrow N = n * \rho_0 * c \tag{3.13}$$

$$|r| = \frac{n-1}{n+1} \tag{3.14}$$

With the reflection coefficient known, it is possible to calculate the reflected wave as shown in equation 3.3. Another possibility is measuring the reflected wave at a given point as in 3.16. The incident sound wave is given by equation 3.15.

$$p_i = A * cos(2 * \pi * f * t) \tag{3.15}$$

$$p_r = B * cos(2 * \pi * f * \left(t - \frac{2y}{c}\right) \tag{3.16}$$

where p_i is sound pressure of the incident sound wave in Pa, p_r is sound pressure of the reflected sound wave in Pa, f is frequency of excitation in Hz, y is distance of observed point from the surface of the sample in m, c is velocity of sound within the tube in m.s-1, t is time in seconds.

By adjoining these equations, the total pressure at point y can be found by equation 3.17. Unto which the addition theorem shown in equation 3.18 can be used to yield the minimum and maximum pressures in equations 3.20 and 3.19, which then ultimately can be used to determine the reflection/absorption of a material.

$$p_y = p_i + p_r = A * cos(2 * \pi * f * t) + B * cos(2 * \pi * f * \left(t - \frac{2y}{c}\right) \tag{3.17}$$

$$cos(\theta - \phi) = cos(\theta) * cos(\phi) + sin(\theta) * \sin(\phi) \tag{3.18}$$

$$p_{y_{max}} = (A + B) * cos(2 * \pi * f * t) \tag{3.19}$$

$$p_{y_{min}} = (A - B) * cos(2 * \pi * f * t) \tag{3.20}$$

In section 3.2.2 it was mentioned that the dimensions of the tube could be determined differently, explaining how the standing wave apparatus type 4002 can rely on a tubelength of only a meter, instead of 2.8583 meters, as is calculated by using equation 3.5. In the instruction manual for the 4002 impedance tube revised in 1979, [34], it is stated that length of the tube must be at least $\frac{\lambda}{4}$ for low frequencies and the diameter $0.586 * \lambda$ for higher frequencies, virtually identical to 3.9. By using $\frac{\lambda}{4}$ to determine the lower boundary, it is seen that the type 4002 should be capable of measuring frequencies down to 85.75 Hz with a length of 1 meter. This matches the specification plus some overhead for the type 4002, as it specified to measure down to 90 Hz. When the frequency no longer shows isolated maximums, the pressure in front of the sample should be used.

3.2.4 Transmission Line Theory - Smith Chart

In transmission line theory, impedance, admittance, and Smith charts are fundamental for analysis of wave propagation in networks, especially at increasingly high frequencies. The standing wave method in acoustics has many similarities to the standing wave ratio in transmission line theory. In transmission line theory, SWR is used to impedance-match antennas and transmission lines, minimizing lost power. In acoustics, something similar is happening, as acoustical impedance can reveal where in a material reflections are happening, thus making it possible to tune materials to absorb/reflect exact frequencies. The normal acoustic impedance of a material is given as Z_n and describes the sound pressure at the surface and its associated particle velocity, [34].

$$Z_n = \frac{p}{v} \tag{3.21}$$

Which for the standing wave tube becomes:

$$Z_n = \frac{p_i + p_r}{v_i + v_r} \tag{3.22}$$

As the characteristic impedance of air ρ_0 influences both p_i and p_r, the relations become:

$$p_i = \rho_0 * c * v_i \text{ and } p_r = \rho_0 * c * (-v_r) \tag{3.23}$$

$$Z_n = \left(\frac{p_i + p_r}{p_i - p_r}\right) * \rho_0 * c \tag{3.24}$$

The equation for p_r can be rewritten into:

$$p_r = |r| * p_i * e^{j*\Delta} \tag{3.25}$$

Where Δ is the phase angle between incident and reflected sound pressures. This rewrite can be inserted into equation 3.24 yielding equation 3.26 and further derived equation 3.27:

$$Z_n = \left(\frac{1 + |r| * p_i * e^{j*\Delta}}{1 - |r| * p_i * e^{j*\Delta}}\right) * \rho_0 * c \tag{3.26}$$

$$Z_n = (Re(Z_n) + jIm(Z_n)) * \rho_0 * c \tag{3.27}$$

The real and imaginary parts of 3.27 are given by, [34]:

$$Re(Z_n) = \frac{1 - r^2}{1 + r^2 - 2 * r * cos(\Delta)} \tag{3.28}$$

$$Im(Z_n) = \frac{2 * r * sin(\Delta)}{1 + r^2 - 2 * r * cos(\Delta)} \tag{3.29}$$

As the only variables are the reflection coefficient and the phase angle, normal acoustic impedance can be calculated by equations 3.28 and 3.29. The phase angle Δ is given by equation 3.30, where y_1 and y_2 are the distances from the sample material to minima 1 and 2, as can be seen in figure 3.4. It is important to remember the distance y_0 as well, as both equations in 3.30 otherwise will return 0.

$$\Delta = \left(\frac{4 * y_1}{\lambda} - 1\right) * \pi \leftrightarrow \Delta = \left(\frac{2 * y_1}{y_2 - y_1} - 1\right) * \pi \tag{3.30}$$

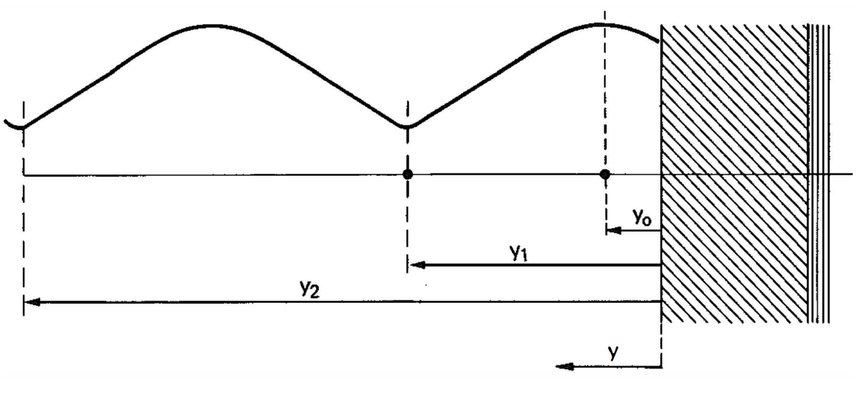


Figure 3.4: Diagram of where to find y_1 and y_2, [34].

When Δ has been calculated, its value must be converted from radians to degrees to make use of the Smith chart seen in figure 3.5. To use the Smith chart, a circle is first drawn from the center with radius equal to the reflection coefficient value r. When that is done, the degree notation around the periphery of the Smith chart reveals which circle should be followed. The Smith chart in use can be seen in figure 3.6, where the green circle matches the value for a reflection coefficient of 0.3, and the phase difference is 120 degrees, shown by the red fractional circle. The blue and orange cross signifies where Re and Im components should be read off of. The values are read off as $0.88 + j0.58$ and $1.52 + j0.58$. Equations 3.31 and 3.32 are used to double check the interpretation, and determine which value is correct.

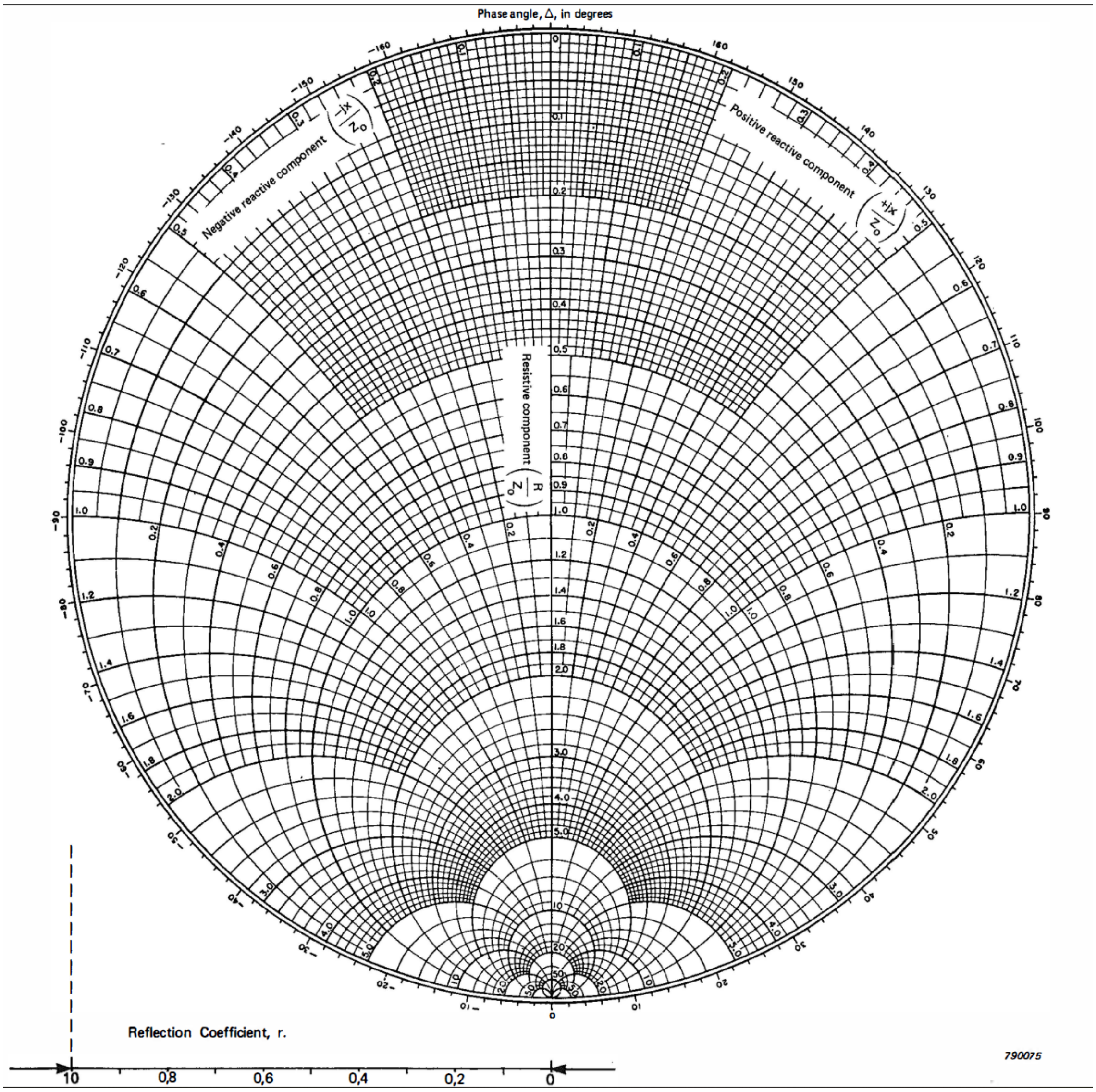


Figure 3.5: Acoustic smith chart, [34].

$$Re(Z_n) = \frac{1 - 0.3^2}{1 + 0.3^2 - 2 * 0.3 * cos(120)} = 1.5129 \tag{3.31}$$

$$Im(Z_n) = \frac{2 * 0.3 * sin(120)}{1 + 0.3^2 - 2 * 0.3 * cos(120)} = 0.57917 \tag{3.32}$$

As can be seen from the equations, the orange cross marks the correct spot within a small error margin. Graphically, the choice cannot be explained, as vanishingly little information can be found regarding use of a Smith chart for acoustic impedance matching, with most coming from Per Brüels own literature, [32, 34, 33]. Nevertheless, Z_n can now be given as:

$$Z_n = (1.5129 + j0.57917) * \rho_0 * c \tag{3.33}$$

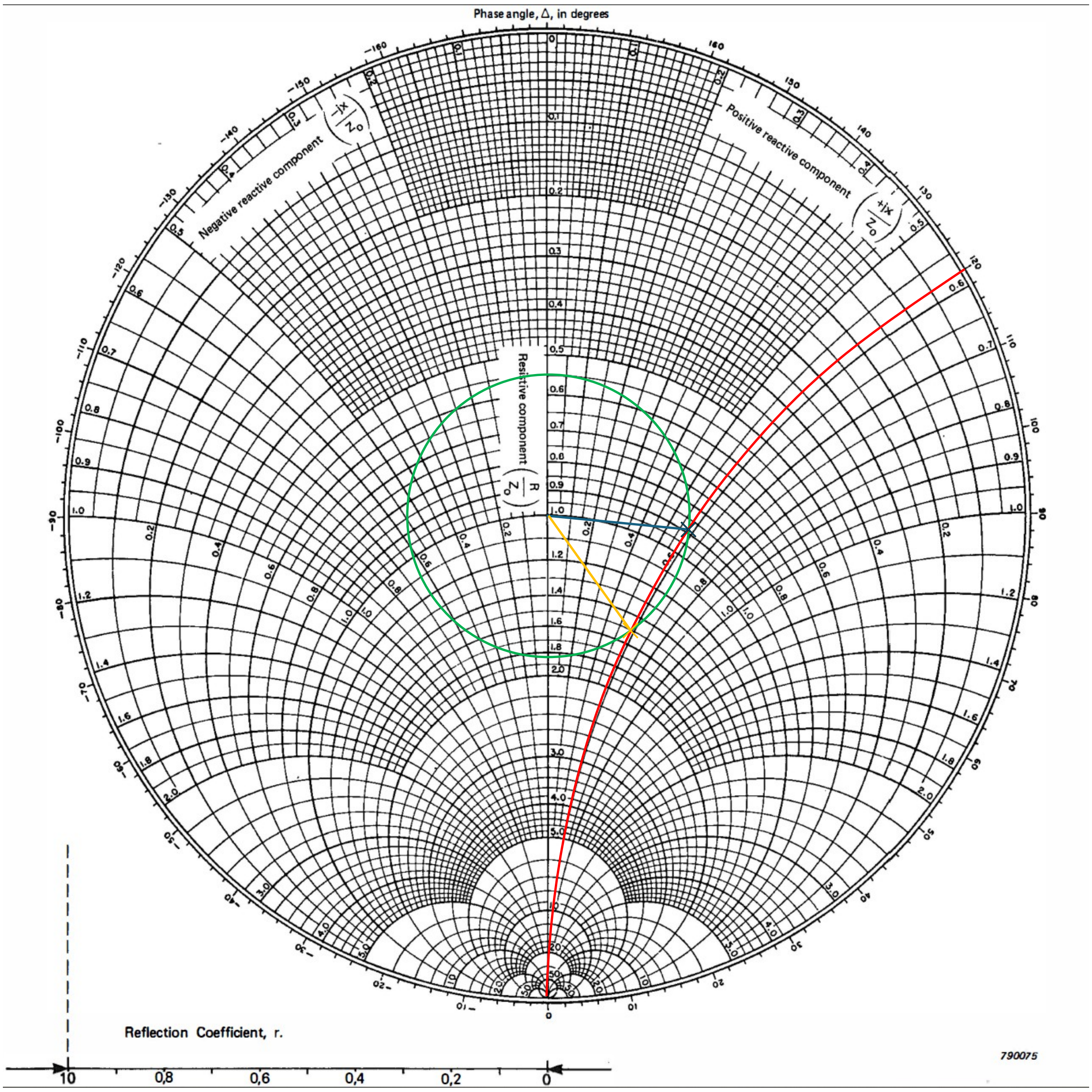


Figure 3.6: Acoustic smith chart in use.

By finding the impedance of a material, it is possible to find the reflection from equation 3.24 and the admittance of a wall by equation 2.21. More importantly, the impedance qualities can reveal where in a material reflection happens for specific frequencies and how sound travels within the material. Depending on the mismatch between air's characteristic impedance given by $\rho_0 * c$, the reflection will happen at different depths, [34, 44, 30]. The mismatch is given as a ratio of $\frac{Z_n}{\rho_0 * c}$, effectively revealing the admittance, as in equation 2.21, [7, 8, 34, 44, 30]. A larger impedance mismatch results in reflection happening closer to the surface, just as in transmission line theory. Therefore, if an impedance match can be made, the material will be completely admittant, which should not be confused with absorption. This can be used if specific frequencies should be attenuated, and others should be reflected as is. By inspecting the reactive j component, it can be determined if the material acts as a mass or a spring, with negative reactance being spring-like behavior, and positive being mass-like behavior. A perforated panel absorber with an air gap behind it will typically have a negative reactance at certain frequencies, meaning that the sound pressure builds up inside the cavity. A thin metal sheet or a dense vibrating membrane can have a positive reactance at certain frequencies, meaning it behaves like a mass, slowing down and reflecting sound waves at those frequencies.

3.3 Multi Microphone Method

In short, the multimicrophone method (MMM) is very similar to the standing wave method, as it also employs a homogenous rigid tube, with a speaker at one end and a sample at the other. The main difference is the lack of probing microphones and the possible addition of a secondary tube for transmission measurements. The MMM has several benefits in comparison with the standing wave method. The first and foremost is the independence of having $\frac{1}{4}\lambda$ as the minimum length of a tube, which enables measurements at frequencies lower than possible in a standing wave impedance tube. The digital signal processing capabilities inherent to the MMM also enables the possibility of obtaining values for a frequency spectrum a lot faster, than with the stand wave method.

The MMM utilizes the transfer matrix method, which is known from the optical domain as well. The transfer matrix method enables the determination of transfer functions of different materials when placed in a known environment. From these transfer functions, impedances, absorption, and reflection coefficients can be found.

3.3.1 Two Microphone Technique for Normal Sound Absorption Coefficient and Normal Surface Impedance Standard - DS 10534-2-2023

As for the standing wave method, international standards ensure equal measurements can be made. For the MMM, the current related standards are DS 10534-2-2023 and ASTM E2611-24. Unfortunately, only the current DS standard has been available for this project, along with a dated version of the ASTM standard. However, from the dated ASTM standard, it can be derived, that the requirements set by it are nearly identical to those set by DS, [14, 45, 46].

As in section "Standing Wave Method Standard - DS 10534-1-2001", the following section will be mostly related to the physical characteristics of an impedance tube utilizing the MMM. The mathematical background for deriving coefficients from the MMM is given in section 3.3.2.

Impedance tubes using the MMM can be made circular and rectangular, just as when using the SWT. Furthermore, several physical equations are similar to those of the SWT. When using the MMM, the tube length is dictated by the diameter, which in turn is still dictated by the upper frequency to measure. To determine the length of the impedance tube, it is necessary to know the distance between microphones, the distance from the speaker to the first microphone, and the distance from the second microphone to the sample, which is determined by the diameter. In equation 3.34 the diameter is given, as a function of the shortest wavelength. Equation 3.35 reveals the maximum distance between microphones s in meters, whereas equation 3.36 yields the recommended minimum distance between microphones, given by 1.5% of the lowest frequency wavelength,

[14]. In equation 3.37, the complete range of distances is given as S, wherein all values can be chosen. It should however be noted, that larger distances between microphones increase accuracy for low frequencies, whereas small distances increase accuracy for high frequencies. Therefore, a higher bandwidth with high accuracy can be obtained by multiple microphone positions available.

$$rec = d \leq 0.5 * \lambda_U \text{ , } circ = d \leq 0.58 * \lambda_U \text{ } [m] \tag{3.34}$$

$$f_U * s \leq 0.45 * c \text{ } [m] \tag{3.35}$$

$$0.015 * f_L \text{ } [m] \tag{3.36}$$

$$S = 0.015 * f_L \leq f_U * s \leq 0.45 * c \text{ } [m] \tag{3.37}$$

When determining the length of the impedance tube, the first microphone should be placed at least $\frac{1}{d}$ away, but should ideally be $2 * d$ from the sample. Additionally, it is recommended that the microphone farthest from the sample be at most 250 mm away. In equation 3.38 the possible placements of the microphone nearest to the sample can be seen. The microphone closest to the speaker must be at least 3 diameters away from the speaker, which is calculated in equation 3.39. Assuming that the recommended distance of 250mm from the sample to the microphone farthest away is followed by placing the first microphone 250mm from the sample, the minimum length of an impedance tube is given by equation 3.40. However, the sample size still has to be added to the equation as well, yielding the actual minimum length to be given by equation 3.41. However, neither the DS nor the dated ASTM standard notes any maximum lengths of the impedance tube, meaning that the length can be adapted to fit the standing wave method as well, as a possible variation measured by multiple microphones instead of a moveable microphone.

$$2 * d_{highest} \leq 0.25 - S_{highest} \tag{3.38}$$

$$min_D = 3 * d_{highest} \tag{3.39}$$

$$0.25 + min_D \tag{3.40}$$

$$0.25 + sample_{size} + min_D \tag{3.41}$$

Many of the same physical constraints for the material choice is similar as to the SWT, such as straightness within 0.2%, rigid construction, terminating end, etc. The main difference in setup is found in the electrical components, as the MMM utilizes microphones placed at sidewalls and some sort of DSP, to process the measurements. Otherwise, equipment such as the signal generator, loudspeaker, and thermometer are still necessary.

When placing the microphones, they should be flush with the interior surface of the tube, or slightly recessed. The placement of the microphone center must be known within 0.2mm. To ensure complete air-tightness, vaseline should be used in the threads for the microphone.

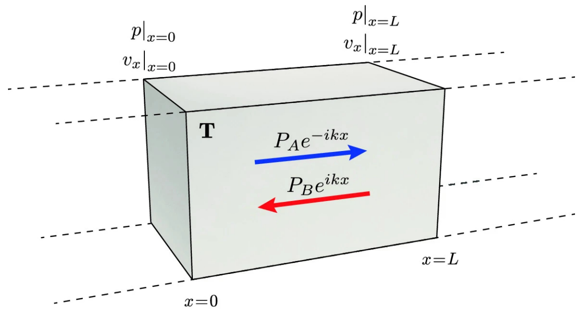


Figure 3.7: Enter Caption

As with the SWT, the sample holder should be large enough to contain the desired samples, and otherwise follow the requirements of the tube itself. When inserting samples, sealants such as vaseline should be used on all joining surfaces.

The signal processing system necessary will be described in section 3.3.4 on page 40. The specific relations between equipment, algorithms, materials, etc. is best found by reading the standards, [14, 45].

3.3.2 Math Behind the Multi Microphone Method

based on [47]

$$p(x) = A * e^{-i*k*x} + B * e^{i*k*x} \tag{3.42}$$

$$v_x(x) = \frac{A}{Z_0} * e^{-i*k*x} - \frac{B}{Z_0} * e^{i*k*x} \tag{3.43}$$

Where $Z_0 = \rho_0 * c$, $k = \frac{\omega}{c}$, $\omega = 2 * \pi * f$ at x=0

$$p(x)|_{x=0} = A + B \tag{3.44}$$

$$Zv_x(x)|_{x=0} = A - B \tag{3.45}$$

While at x=L

$$p(x)|_{x=L} = (A + B) * cos(k * L) - i * (A - B) * cos(k * L) \tag{3.46}$$

$$Zv_x(x)|_{x=L} = \frac{A-B}{Z_0} * cos(k * L) - i * \frac{A-B}{Z_0} * sin(k * L) - \tag{3.47}$$

De første formler sættes sammen med de næste, så man får:

$$p(x)|_{x=L} = cos(k * L) * p(x)|_{x=0} - i * Z * sin(k * L) * v_x(x)|_{x=0} \tag{3.48}$$

$$v_x(x)|_{x=L} = cos(k * L) * v_x(x)|_{x=0} - i * \frac{1}{Z} * sin(k * L) * p(x)|_{x=0} \tag{3.49}$$

Which can be set into matrix form as:

$$\begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=L} = \begin{bmatrix} cos(k * L) & -i * Z * sin(k * L) \\ cos(k * L) & -i * \frac{1}{Z} * sin(k * L) \end{bmatrix} * \begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=0} \tag{3.50}$$

$$\begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=0} = \begin{bmatrix} cos(k * L) & i * Z * sin(k * L) \\ i * \frac{1}{Z} * sin(k * L) & cos(k * L) \end{bmatrix} * \begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=L} \tag{3.51}$$

$$\begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=0} = T * \begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=L} \tag{3.52}$$

$$\begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} * \begin{bmatrix} p \\ v_x \end{bmatrix} |_{x=L} \tag{3.53}$$

$$T = T_1 * T_2 * T_3 \tag{3.54}$$

$$T = \Pi_{n=1}^{N} T_n \tag{3.55}$$

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} cos(k_{eff} * L) & i * Z * sin(k_{eff} * L) \\ i * \frac{1}{Z} * sin(k_{eff} * L) & cos(k_{eff} * L) \end{bmatrix}, \tag{3.56}$$

3.3.3 Comparison to the Standing Wave Method

3.3.4 Algorithm

[14]
standard
matematik
fordele/ulemper
Algoritme/dsp

3.4 Signal Processing

To gain any knowledge about what is happening within the impedance tube, some level of signal processing is necessary. To create a full perspective of significant aspects, the signal processing will include knowledge of the signal generation, as this enables further optimization of the complete system. The idea is to use fixed-point arithmetic for all computations, maximum length sequence (and/or chirps) for signal generation, and FFTs to analyze the acoustic field within the tube.

3.4.1 Fixed Point Arithmetic

To increase the computational efficiency of the SPA, fixed-point arithmetic (FPA) is used. The benefit of using FPA is that inherently fewer operations are necessary to conduct common operations, such as $+, -, *, /$. When binary FPA is used for adding and subtracting, it can be done in a single clock cycle. Furthermore, when dividing or multiplying by 2, bitshifting enables such operands to happen in a single cycle as well. Compared with floating point arithmetic, FPA represents all numbers as scalable integers. When representing numbers in FPA, the bit value is divided into a fractional and an integer part. The major flaw of FPA is acquiring the optimal ratio of which bits should represent the fractional part, and how many bits should represent the integer. By determining the optimal ratio, it is meant to determine how many bits are used to describe each value.

If a 16-bit number is taken as an example, it represents $2^{16} = 65536$ different values in total. However, equally dividing it into 8 bits for the integer and 8 bits for the fraction leaves 256 different values for each, which, depending on the purpose, can be appropriate or flawed. By taking 1 bit from the integer part and adding it to the fraction, the ratio shifts to 128 and 512 values. In this example, the integer is now too small to function in some audio applications, as sound easily can surpass 128dB, BUT, if the application regards assessing background noise at high precision, it might still be valid. In general, a fixed-point number's bit ratio can be described as:

$$2^b . \frac{1}{2^{B-b}} \tag{3.57}$$

Where b is the number of bits used for the integer, and B is the total number of bits available. The $\frac{1}{2^{B-b}}$ is meant to show the resolution achievable with the remaining bits. By inserting the desired maximum value or minimum resolution, it is easy to conclude whether or not the current bit-ratio is correct.

However, all is not perfect in the land of FPA, as the first bit (MSB) often signifies a sign, thus reducing the amount of bits. Furthermore, when choosing bit sizes, an overhead should be added to the requirements as well, to minimize the possibility of bit overflow, rendering data flawed. As mentioned previously, the available amount of bits can also lead to insufficient precision in the fractional part, as the number is a fraction and therefore discrete, rather than continuous.

Fortunately, FPA provides many benefits as well, such as predictable execution time, reduced hardware complexity, increased usability for real-time applications, and, when used correct, complete control of how data is handled across both software and hardware, creating a sturdier system.

Within audio and acoustic signal processing, the use of FPA is highly beneficial, as FFTs, IIR/FIR filters, etc., all utilize a series of additions and multiplications. To see how FPA is used and the code behind it, it is better to visit section 5 on page 50 or the accompanying GitHub with code.

3.4.2 Maximum Length Sequence - MLS

Maximum length sequences (MLS) are a type of pseudorandom binary sequence, which can be generated by a linear feedback shift register (LFSR), such as the one shown in figure 3.8. The main benefit of MLS is that, though the sequence mimics random noise, it is deterministically produced. The name comes from the sequence using all nonzero binary values in an LFSR, thus creating the maximum length sequence. The largest possible length is given by $2^n - 1$, where n is the number of stages in the register, and the -1 removes the possibility of hitting an all-zero stage.

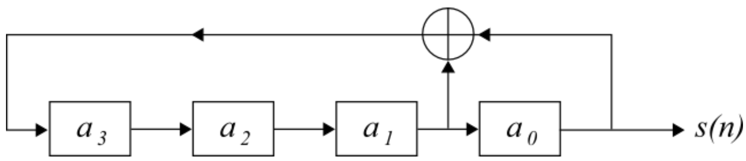


Figure 3.8: Linear feedback shift register of length 4, [48].

MLS are generated by the shift register from a given seed to ensure that sequences are reproducible. As the shift register cycles through all possible values, the register seed can be any value except for the all-zero state. The recursive function for figure 3.8 is described in equation 3.58, where it can be seen to be periodic.

$$\begin{cases} a_3[n \oplus 1] = a_0 \oplus a_1[n] \\ a_2[n \oplus 1] = a_3[n] \\ a_1[n \oplus 1] = a_2[n] \\ a_0[n \oplus 1] = a_1[n] \end{cases} \tag{3.58}$$

Where n is the time index and $\oplus$ is the modulo-2 addition, equaling the XOR operation. When implemented, the shift register can create sequences based on its length N, by 2^N. Therefore, an LFSR with 16 flip-flops can create a sequence of $65536 - 1$ samples, with the -1 sample being all-zero.

When examining a period, the bitstream can be divided into runs. A run is defined by a subsequence of identical symbols within a period, with the length of this subsequence being the length of the run, [49]. For all periods/sequences, the following properties should be met:

- The number of ones equals the number of zeros plus 1
- Each period must contain a run of length N
- One half of the runs are of length 1
- One quarter of the runs are of length 2
- One eighth of the runs are of length 3
- One-sixteenth of the runs are of length 4
- One-sixtyfourth of the runs are of length 5

When creating the LFSR, XOR gates are put at specific "taps" to yield the MLS. The taps are best found by inspecting a table, such as G.1 found in the appendix on page 78, as manually finding them can be tedious. However, they can be found as coefficients to a primitive polynomial on GF(2)[1], which can be calculated using Eulers totient function, [48, 50]. The tap's main function is to specify which flip-flop outputs should be XOR'ed together. In equation 3.59, a 4-bit feedback polynomial can be seen, with coefficients at x^4 and x^3. This means that the taps are at the third and fourth bit positions, and they should be XOR'ed together to yield the correct feedback. When XOR'ing the bits together, a maximum length sequence for 4 bits is given in table 3.2, where the "output bit" column yields the MLS with this specific seed. If a different set of flip-flops is used to generate the sequence, it is not certain that it will be an MLS, [51, 48, 49, 52].

$$x^4 + x^3 + 1 \tag{3.59}$$

Iteration	Start Value	XOR Result	New Value	Output Bit
1	1001	1	0011	1
2	1100	0	0110	0
3	0110	1	1100	0
4	1011	1	0101	1
5	0101	1	1010	1
6	1010	1	1101	0
7	1101	1	1110	1
8	1110	0	1111	0
9	1111	0	0111	1
10	0111	0	0011	1
11	0011	0	0001	1
12	0001	1	1000	1
13	1000	0	0100	0
14	0100	0	0010	0
15	0010	1	1001	0
16 (1)	1001	1	1100	1

Table 3.2: Table of XOR operations with taps at the third and fourth bit. The output bits are the values in the MLS, meaning that the corresponding MLS for a 4-bit LFSR with this seed (1001) will cycle these 15 values indefinitely, [51].

In table 3.2, it can be seen that the LFSR is periodic within 15 cycles, matching the $2^N - 1$ specification. Furthermore, with the taps correctly placed, it will never yield an all-zero value, which can also be seen, as it is the only combination left out in the table. The output MLS is "100110101111000", which can be divided into the following runs: 1 | 00 | 11 | 0 | 1 | 0 | 1111 | 000. This matches the properties mentioned earlier, as the number of zeros is 1 less than ones, half the runs are of length 1, a quarter is of length 2, an eighth is of length 3, and the first flaw is found, as more than one sixteenth is of length 4. However, this meets the criteria of a run being the same length as N.

As the amount of bits N expands, the period expands by twos complement, which can also be seen in the period column of table G.1 on page 78. The properties given by a larger number of bits yield a significantly larger period, which on the surface will seem

[1]Galois finite field: a finite set of numbers with defined addition, subtraction, multiplication, and division operations, where every nonzero element has a multiplicative inverse.

even more random than the 4-bit example, while still being 100% deterministic.

Autocorrelation Properties of MLS

One of the many useful features of MLS is the autocorrelation property. For any binary sequence $s(t)$ of period $2^N - 1$, the autocorrelation function at a time is defined as:

$$R(\tau) = \sum_{t=0}^{2^N-1-1} (-1)^{s(t) \oplus s(t+\tau)} \tag{3.60}$$

wheres $s(t)$ is the bit at time t and $\oplus$ is modulo-2 addition (XOR). The sequence is often portrayed with $0 = -1$ and $1 = +1$ for the respective bit values, when doing correlation analysis. At zero lag ($\tau = 0$), the sequence isn't shifted, resulting in bits lining up perfectly, and a perfect correlation $R(0) = 2^N - 1$. However, for all other shifts/$\tau \neq 0$ the correlation will be -1, making it seemingly random. The goal of autocorrelation is to check whether or not the shifted sequence resembles the non-shifted sequence. The constant mismatch results in a correlation of -1, matching total randomness.

The correlation itself is an important measure as it in practice can act as an identifier when measuring an impulse response in a noisy environment. As the correlation is 1 at only one spot, the impulse response can be swept through until found, and thereafter the original signal can be omitted from the response.

Creating White Noise from MLS

The MLS is highly beneficial to use in acoustic system identification, as the spectral composition of an MLS is nearly identical to the randomness of white noise, [53, 54, 50, 55]. When playing an MLS through a speaker, it can be seen as a linear time-invariant (LTI) system. The bipolar signal of MLS (-1 to +1) corresponds to the minimum and maximum positions of the speaker cone, and the constant switching creates a vaguely sinusoidal signal of different wavelengths. While the speaker is not playing exact white noise, the MLS is near enough for human hearing to consider it white noise. Furthermore, as the Fourier transform of an MLS approximates a flat spectrum, all frequencies are played equally, making the MLS signal indistinguishable from white noise. Mathematically, the LTI system can be described by equation 3.61, and the Fourier transform is given by equation 3.62, [53, 55].

$$y[n] = x[n] * h[n] = \sum_{k=0}^{N-1} x[k] * h[n-k] \tag{3.61}$$

$$X(f) = \sum_{n=0}^{N-1} x[n] * e^{-j*\frac{2*\pi*f*n}{N}} \tag{3.62}$$

Where $x[n]$ is the MLS signal and $h[n]$ is the system response. If the example used in [53] is used instead, the MLS used in regards to room acoustics can be given by equation 3.63, and the system as a whole can be seen in figure 3.9. The general idea is to play the MLS on an LTI system (speaker), creating an impulse response of a room. Thereafter, a fast Hadamard transform can be used to derive the room's impulse response. From that impulse response, a band filter can reveal the decay curve for specific octave bands[2], and an FFT will reveal the spectrum. The main benefit from utilizing this method is

[2]For information about octaves, see appendix E on page 74.

that the SNR can be increased by 3 dB for each average obtained, effectively enabling a noise-free impulse response with enough averages available, [53, 55].

$$s(t) = m(t) * h(t) \leftrightarrow s[n] = m[n] * h[n] \tag{3.63}$$

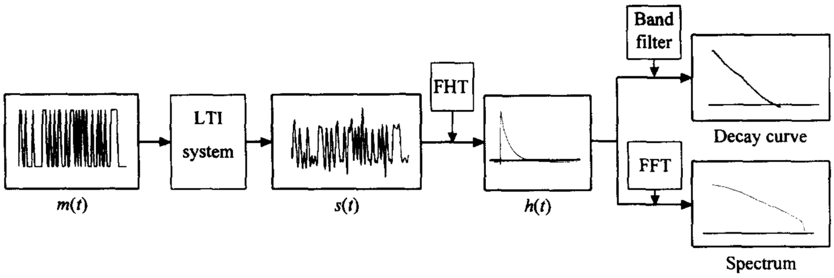


Figure 3.9: Signal flow schematic of MLS measurement, [53].

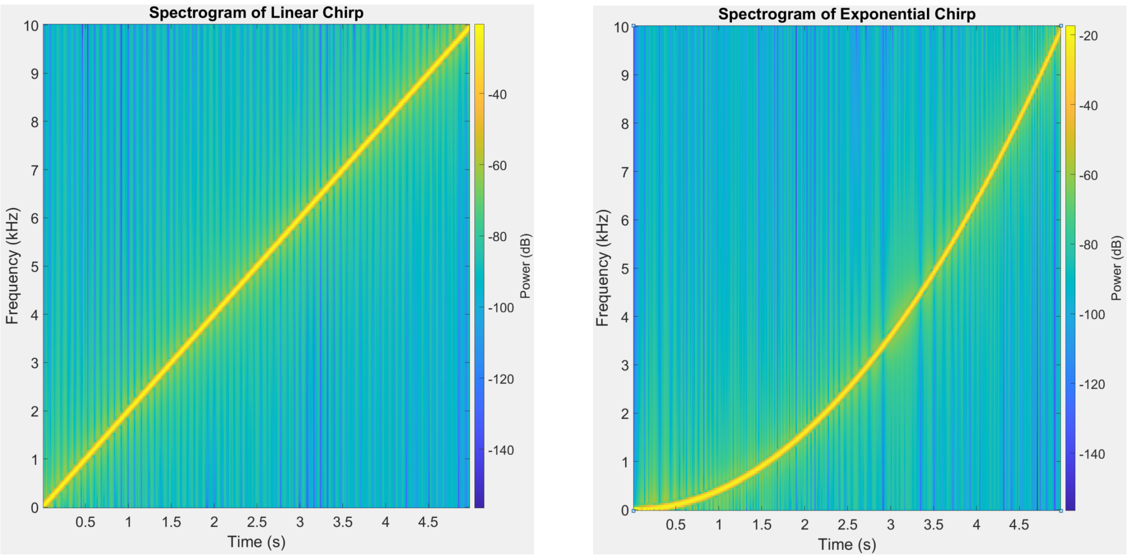
3.4.3 Chirps

Chirps are a common method of gaining impulse responses over a broad range of frequencies, [56, 57, 58, 59]. A chirp is a form of frequency sweep that will either increase in frequency (upsweep) or decrease (downsweep). Several types of chirp exist, but only the linear and exponential types will be described in this section. Mathematically, the linear chirp can be described by equation 3.64 and the exponential chirp by equation 3.65. Spectrograms of both types can be seen in figure 3.10, [56, 57, 58, 59].

$$x(t) = A * cos(2 * \pi(f_0 t + \frac{k}{2}t^2) + \phi_0) \tag{3.64}$$

$$x(t) = A * cos(2 * \pi * f_0 \left(\frac{b^t - 1}{ln(b)}\right) + \phi_0) \tag{3.65}$$

Where A is the amplitude, f_0 is the initial frequency, $k = \frac{f_1 - f_0}{T}$ is the "chirp rate", f_1 is the final frequency at time T, ϕ_0 is the initial phase, and b is a scaling factor for the exponential growth of the frequency given by $b = \frac{f_1}{f_0}^{\frac{1}{T}}$.



(a) Spectrogram of a linear chirp, generated with MATLAB's chirp function.

(b) Spectrogram of an exponential chirp, generated with MATLAB's chirp function.

Figure 3.10

In general, chirps are useful in system identification, as each chirp can be modified to fit broad frequency bands or smaller octave bands, fitting the application at hand. Compared with white noise and MLS, a chirp provides equal energy across the frequency spectrum, but the main advantage is a higher SNR. For MLS and white noise, the response has to be averaged across multiple tests, whereas with chirps, matched filters allow higher SNR, [56, 57, 58, 59].

4 | Demand Specification

4.1 High Level Specification

This section describes a high-level overview of what the impedance tube should be capable of. The functionality is written as seen by an acoustical engineer interested in a new impedance tube focused on low frequencies. Detailed specifications can be found in section 4.2 starting page 48.

As an acoustical engineer with a focus on low frequencies, I want:

- *An impedance tube which can measure at infrasound.*
- *An impedance tube capable of fitting square samples.*
- *An impedance tube capable of fitting large samples.*
- *An impedance tube wherein perforated materials can be tested.*
- *An impedance tube outputting .CSV data for data handling.*
- *An impedance tube with an intuitive interface.*
- *An impedance tube that can reveal absorption coefficients for an $\frac{1}{3}$ octave at a time.*
- *An impedance tube that can reveal reflection coefficients for an $\frac{1}{3}$ octave at a time.*
- *An impedance tube where I can inspect whether my sample is placed correctly in the tube.*
- *An impedance tube where I can easily switch speaker output.*
- *An impedance tube wherein deep samples can be placed to test low frequency absorption.*
- *An impedance tube that auto-calibrates microphones.*
- *An impedance tube which calculates the impedance and admittance of a material.*
- *An impedance tube that complies with DS/ISO 10534-2-2023.*
- *An impedance tube that complies with ASTM 2611-24.*

4.2 Functional Specification

This section describes the functional criteria of the product. The criteria are seen from an end-user perspective and made as user stories, where acceptance criteria (AC1 & AC2, e.g.) must be fulfilled. A test of the functional specifications is made in section 7.

4.2.1 Measuring Infrasound

As an acoustical engineer, I want an impedance tube that can measure at infrasound. **Accept Criteria: AC1:** The impedance tube must be able to measure below 20 Hz. **AC2:** The impedance tube should be able to measure below 10 Hz. **AC3:** The impedance tube should be able to measure down to 1 Hz.

4.2.2 Fitting Square Samples

As an acoustical engineer, I want an impedance tube capable of fitting square samples. **Accept Criteria: AC1:** The tube style must be square.

4.2.3 Fitting Large Samples

As an acoustical engineer, I want an impedance tube capable of fitting large samples. **Accept Criteria: AC1:** The tube should, at a minimum, be capable of fitting 10x10cm samples. **AC2:** Adapters should enable the tube to fit samples up to 30x30cm.

4.2.4 Data Output in CSV Format

As an acoustical engineer, I want an impedance tube outputting .CSV data for data handling. **Accept Criteria: AC1:** Data must be saved or transmitted to local storage in .CSV format.

4.2.5 Intuitive Interface

As an acoustical engineer, I want an impedance tube with an intuitive interface. **Accept Criteria: AC1:** The interface should be accessible at the impedance tube. **AC2:** Menus, data, and interpretations must be easily read. **AC3:** Results must be easy to extrapolate. **AC4:** Results, measurements, etc. should be saveable to external storage.

4.2.6 Absorption Coefficients for 1/3 Octave

As an acoustical engineer, I want an impedance tube that can reveal absorption coefficients for a $\frac{1}{3}$ octave at a time. **Accept Criteria: AC1:** The absorption coefficient should be calculated and easily read from measurements. **AC2:** The impedance tube must be capable of measuring $\frac{1}{3}$ octaves at a time.

4.2.7 Reflection Coefficients for 1/3 Octave

As an acoustical engineer, I want an impedance tube that can reveal reflection coefficients for a $\frac{1}{3}$ octave at a time. **Accept Criteria: AC1:** The reflection coefficient should be calculated and easily read from measurements. **AC2:** The impedance tube must be capable of measuring $\frac{1}{3}$ octaves at a time.

4.2.8 Correct Sample Placement Inspection

As an acoustical engineer, I want an impedance tube where I can inspect whether my sample is placed correctly in the tube. **Accept Criteria: AC1:** The tube should be see-through. **AC2:** Samples can be placed in a jig out of the tube, which can be placed within the tube.

4.2.9 Easy Speaker Output Switching

As an acoustical engineer, I want an impedance tube where I can easily switch speaker output. **Accept Criteria: AC1:** The speaker must be capable of playing an MLS signal. **AC2:** The speaker must be capable of playing white noise. **AC3:** The speaker must be capable of playing a chirp or similar frequency sweep. **AC4:** The speaker must be capable of playing a pure sinusoid.

4.2.10 Testing Deep Samples for Low Frequency Absorption

As an acoustical engineer, I want an impedance tube wherein deep samples can be placed to test low-frequency absorption. **Accept Criteria: AC1:** The sample holder must be adjustable from 2-10 cm. **AC2:** An extra sample holder adjustable from 10-50 cm must be available. **AC3:** An extra sample holder adjustable from 50-100 cm must be available.

4.2.11 Auto-Calibration of Microphones

As an acoustical engineer, I want an impedance tube that auto-calibrates microphones. **Accept Criteria: AC1:** Instead of switching microphone positions for calibration, the algorithm must calculate corrected transfer functions based on microphone position, temperature, humidity, and adequate testing signals. **AC2:** The calibration must adhere to DS/ISO and ASTM standards.

4.2.12 Impedance and Admittance Calculation

As an acoustical engineer, I want an impedance tube which calculates the impedance and admittance of a material. **Accept Criteria: AC1:** The calculated impedance must match the Smith chart method. **AC2:** The calculated admittance must match the Smith chart method. **AC3:** The real and imaginary values must be outputted as well.

4.2.13 Compliance with DS/ISO 10534-2-2023

As an acoustical engineer, I want an impedance tube that complies with DS/ISO 10534-2-2023. **Accept Criteria: AC1:** Must comply with the relevant criteria set by DS/ISO 10534-2-2023.

4.2.14 Compliance with ASTM 2611-24

As an acoustical engineer, I want an impedance tube that complies with ASTM 2611-24. **Accept Criteria: AC1:** Must comply with the relevant criteria set by ASTM 2611-24.

5 System Design

Oversigt over systemdesign i helhed

5.1 Physical Setup

Oversigt, rå skitse

5.1.1 Determining Desired Physical Dimensions

5.1.2 Choosing Adequate Materials

5.1.3 Creating a Sample Holder

5.1.4 Speaker Chamber

5.1.5 Microphones and Their Placement

5.1.6 Choosing a Microcontroller

5.2 Creating Sound

5.2.1 Inputting A Signal

5.3 Measuring Sound

5.3.1 Creating a Complete System

5.4 Establishing an Algorithm For Data Handling

Specifikationer så som:

- Maximum Length Sequence
- Fixed point

Forklaring af algoritme - formål
Gennemgang af matematik
Gennemgang af kode
valg af enhed til at køre algoritmen - fordele, ulemper og constraints
Test af algoritmen på kendte signaler

6 Integration

7 | Acceptance test

General acceptance test

Sammenligningstest med Bruel og Kjær 4002 Standbølge Apparat af forskellige materialer.

8 | Discussion

9 Conclusion

"?"

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Glossary

C50 Clarity 50 ms. 2, 22, 23

C80 Clarity 80 ms. 22

D50 Definition 50 ms. 2, 23

EDT Early Decay Time. 2, 21, 22

MSB Most significant byte. 41

RT60 Reverberation Time 60 dB. 2, 21, 22

SPL Sound pressure level. 4

T10 Reverberation Time 10 dB. 22

T20 Reverberation Time 20 dB. 21, 22

T30 Reverberation Time 30 dB. 21, 22

A | Appendix

B Mathematical Notations

B.1 Variables

$p(t)$ is the instantaneous sound pressure of the impulse response measured at a point in the room.
$p_{10}(t)$ is the instantaneous sound pressure of the impulse response measured at 10m in free field.
p_0 is $20\mu Pa$.
T_0 is 1 second.
L_{pE} is the sound pressure exposure level of $p(t)$.
$L_{pE,10}$ is the sound pressure exposure level of $p_{10}(t)$.
L_p is the sound pressure level averaged across every measurement point.
$L_{p,10}$ is the sound pressure level measured at 10 m in free field.
L_p is the sound pressure level averaged across every measurement point.
L_W is the sound power level of the sound source, and should be measured according to DS 3741.
$A = \sum S_i * \alpha_i$.
C_{t_e} is the early to late index.
t_e is the early time limit (e.g. 50 or 80 ms).
$p(t)$ is the measured sound pressure of the impulse response.
$c^2 = \kappa \frac{p_0}{\rho_0}$.
p is the sound pressure.
ρ_0 is the static gas density value ($\approx 1.2[\frac{kg}{m^3}]$ for air).
p_0 is the static sound pressure (measured in Pascal, where 1 $Pascal = 1\frac{kg}{m*s^2}$.
κ is the adiabatic component.
v is a vector representing particle velocity.
t is time.
ρ being the variable gas density.
$\hat{p}$ is amplitude.
k is the propagation constant $k = \frac{\omega}{c}$.
ω is the angular frequency, which can be used to obtain the temporal period $T = \frac{2*\pi}{\omega}$.
$f = \frac{\omega}{2*\pi} * \frac{1}{T}$ with the unit Hz.
$\rho_0 * c$ found by table values for ρ_0 and c in the correct medium.
θ and ϕ angles,
c^2 on both sides,
Q, being the rate in $\frac{m^3}{s}$ of "liquid" being expelled by the sound source located at $r = 0$.
The dot above Q denotes differentation w.r.t. time (t).
r and frequency $\omega = k * c$.
$\frac{p}{v}$ tends asymptotically to $\rho_0 c$.
Γ_{nm} are the Fourier series coefficients of order n and degree m,

$h_n(kr)$ is the Hankel function of the first kind with order n,
$Y_n^m(\theta, \phi)$ are the spherical harmonics,
$\tilde{p}$ denotes root mean square of the sound pressure, i.e
$\sqrt{\bar{p^2}}$ and $\tilde{p_0} = 2 * 10^{-5}[\frac{N}{m^2}]$ is an internationally fixed value corresponding to the hearing threshold at 1000Hz,
N being loudness in sones and
L_N being loudness in phons
$x = 0$ and the wave will arrive from the negative direction,

B.2 Formulas

$$G = 10 * log10 \left(\frac{\int_0^\infty p^2(t)\, dt}{\int_0^\infty p_{10}^2(t)\, dt} \right) = L_{pE} - L_{pE,10}[dB] \tag{B.1}$$

$$L_{pE} = 10 * log10 \left[\frac{1}{T_0} * \int_0^\infty \frac{p^2(t)\, dt}{p_0^2} \right] [dB] \tag{B.2}$$

$$L_{pE,10} = 10 * log10 \left[\frac{1}{T_0} * \int_0^\infty \frac{p_{10}^2(t)\, dt}{p_0^2} \right] [dB] \tag{B.3}$$

$$L_{pE,10} = L_{pE,d} + 20 * log10(d/10)[dB] \tag{B.4}$$

$$L_p = L_{p,10}[dB] \tag{B.5}$$

$$G = L_p - L_W + 31dB \tag{B.6}$$

$$RT60 = \frac{0.161 * V}{S_i * \alpha_i} \leftrightarrow RT = \frac{0.161 * V}{A} \tag{B.7}$$

$$A = \sum S_i * \alpha_i \tag{B.8}$$

$$C50 = 10 * log10 \left(\frac{Energy(0 - 50ms)}{Energy(51ms - end)} \right) [dB] \tag{B.9}$$

$$C80 = 10 * log10 \left(\frac{Energy(0 - 80ms)}{Energy(81ms - end)} \right) [dB] \tag{B.10}$$

$$C_{t_e} = 10 * log10 \left(\frac{\int_0^{t_e} p^2(t)\, dt}{\int_{t_e}^\infty p^2(t)\, dt} \right) [dB] \tag{B.11}$$

$$C50 = 10 * log10 \left(\frac{D50}{1 - D50} \right) [dB] \tag{B.12}$$

$$D50 = \frac{\int_0^{0.050} p^2(t)\, dt}{\int_0^\infty p^2(t)\, dt} \tag{B.13}$$

$$c = 331.4 + 0.6 * T[\frac{m}{s}] \tag{B.14}$$

$$c^2 \Delta p = \frac{\partial^2 p}{\partial t^2} \tag{B.15}$$

$$c^2 = \kappa \frac{p_0}{\rho_0} \tag{B.16}$$

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \tag{B.17}$$

$$grad\ p = -\rho_0 * \frac{\partial v}{\partial t} \tag{B.18}$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \tag{B.19}$$

$$\rho_0\ div\ v = -\frac{\partial \rho}{\partial t} \tag{B.20}$$

$$\frac{p}{p_0} = \kappa \frac{\rho}{\rho_0} = \frac{\kappa}{\kappa - 1} * \frac{\delta * T}{T + 273} \tag{B.21}$$

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{B.22}$$

$$p(x,t) = F(c * t - x) + G(c * t + x) \tag{B.23}$$

$$p(x,t) = \hat{p} * e^{i*k*(c*t-x)} = \hat{p} * e^{i*(\omega*t-k*x)} \tag{B.24}$$

$$\lambda = \frac{2 * \pi}{k} \leftrightarrow \frac{2 * \pi}{\frac{\omega}{c}} \leftrightarrow \frac{2 * \pi * c}{\omega} \leftrightarrow \frac{c}{\frac{\omega}{2*\pi}} \leftrightarrow \frac{c}{f} \tag{B.25}$$

$$v(x,t) = \frac{1}{\rho_0 * c} [F(c * t - x) - G(c * t + x)] \tag{B.26}$$

$$\frac{p}{v} = \rho_0 * c \tag{B.27}$$

$$\rho_{0_{air}} * c_{air} = 1.2 \left[\frac{kg}{m^3}\right] * 343 \left[\frac{m}{s}\right] = 414 \left[\frac{kg}{m^2 s}\right] \tag{B.28}$$

$$I = \bar{p}\bar{v} = \frac{\bar{p}^2}{\rho_0 * c} \tag{B.29}$$

$$w = \frac{I}{c} = \frac{\bar{p}^2}{\rho_0 * c^2} \tag{B.30}$$

$$\Delta p = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial p}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 p}{\partial \phi^2}. \tag{B.31}$$

$$\Delta p = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) \tag{B.32}$$

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r} \frac{\partial p}{\partial r} \tag{B.33}$$

$$c^2\left(\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}\frac{\partial p}{\partial r}\right)=\frac{\partial^2 p}{\partial t^2} \tag{B.34}$$

$$\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}*\frac{\partial p}{\partial r}=\frac{1}{c^2}*\frac{\partial^2 p}{\partial t^2} \tag{B.35}$$

$$p(r,t)=\frac{\rho_0}{4*\pi*r}*\dot{Q}\left(t-\frac{r}{c}\right) \tag{B.36}$$

$$v_r=\frac{1}{4*\pi*r^2}\left[Q\left(t-\frac{r}{c}\right)+\frac{r}{c}*\dot{Q}\left(t-\frac{r}{c}\right)\right] \tag{B.37}$$

$$v_r=\frac{p}{\rho_0*c}*\left(1+\frac{1}{i*k*r}\right) \tag{B.38}$$

$$\frac{\partial^2 p}{\partial r^2}+\frac{2}{r}*\frac{\partial p}{\partial r}+\frac{1}{r^2}\left(\frac{1}{sin(\theta)}*\frac{\partial}{\partial*\theta}\left(sin(\theta)*\frac{\partial}{\partial*\theta}\right)+\frac{1}{sin^2(\theta)}*\frac{\partial^2}{\partial\phi^2}\right)=\frac{1}{c^2}*\frac{\partial^2 p}{\partial t^2} \tag{B.39}$$

$$p(r,\theta,\phi)=A*\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Gamma_{nm}h_n(kr)Y_n^m(\theta,\phi) \tag{B.40}$$

$$Y_n^m(\theta,\phi)=\sqrt{\frac{(2n+1)}{4*\pi}*\frac{(n-|m|)!}{(n+|m|)!}}*P_n^{|m|}*(cos(\theta)*\begin{cases}cos|m|\phi & if\ \ m\geq 0\\ sin|m|\phi & if\ \ m<0\end{cases} \tag{B.41}$$

$$SPL=20*log10(\frac{\tilde{p}}{\tilde{p_0}}[dB] \tag{B.42}$$

$$N=\left(10^{\frac{L_N-40}{10}}\right)^{0.30103}\approx 2^{\frac{L_N-40}{10}} \tag{B.43}$$

$$L_N=40+log_2(N) \tag{B.44}$$

Phon	0	10	20	30	40	50	60	70	80	90	100
Sone	0 (Inaudible)	0.0625	0.125	0.25	1	2	4	8	16	32	64

Table B.1: Relationship between phons and sones.

- **Reflection Coefficient:**

$$R=\frac{I_r}{I_i} \tag{B.45}$$

where I_r is reflected intensity and I_i is incident intensity.

- **Diffusion Coefficient:**

$$D=1-\frac{\sum S(\theta)}{NS_{\text{ideal}}} \tag{B.46}$$

where $S(\theta)$ is the scattered energy in direction θ, and S_{ideal} is the ideally scattered energy.

- **Transmission Loss (TL):**

$$TL=10\log_{10}\left(\frac{I_i}{I_t}\right) \tag{B.47}$$

where I_t is the transmitted intensity.

Property	Definition	Typical Values
Reflection Coefficient (R)	Fraction of sound energy reflected	0.8 - 0.98 (Concrete), 0.1 - 0.5 (Foam)
Diffusion Coefficient (D)	Measure of even sound scattering	0.2 (Flat wall), 0.6 - 0.9 (Diffuser)
Transmission Loss (TL)	Reduction of sound through a barrier (dB)	25 dB (Drywall), 50+ dB (Concrete)
Resonance Frequency (f_r)	Frequency at which the wall vibrates	50 - 500 Hz (Depending on material)
Absorption Coefficient (α)	Fraction of energy absorbed	0.02 (Concrete), 0.8+ (Acoustic panels)
Wall Impedance (Z)	Opposition to sound transmission, depends on density and stiffness.	1.5×10^6 kg·m^{-2}·s^{-1} (Concrete), 5×10^5 kg·m^{-2}·s^{-1} (Brick), 10^4 kg·m^{-2}·s^{-1} (Foam)
Wall Admittance (ζ)	Measure of a wall's ability to accept sound energy, reciprocal of impedance.	0.001 (Concrete), 0.003 (Brick), 0.1 (Foam)

Table B.2: Acoustic properties of walls and their typical values.

- **Resonance Frequency (Panel):**

$$f_r = \frac{60}{d}\sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{B.48}$$

where:d = panel thickness, E = Young's modulus, ρ = density, and ν = Poisson's ratio.

- **Absorption Coefficient:**

$$\alpha = 1 - |R|^2 \tag{B.49}$$

- **Wall Impedance:**

$$Z = \left(\frac{p}{v_n}\right) \tag{B.50}$$

where: v_n denotes the particle component normal to the wall.

- **Wall Admittance:**

$$\zeta = \frac{Z}{\rho_0 * c} \tag{B.51}$$

$$p_i(x,t) = \hat{p}_0 * e^{i*(\omega*t-k*x)} \tag{B.52}$$

$$v_i(x,t) = \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t-k*x)} \tag{B.53}$$

$$p_r(x,t) = R * \hat{p}_0 * e^{i*(\omega*t+k*x)} \tag{B.54}$$

$$v_r(x,t) = -R * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t+k*x)} \tag{B.55}$$

$$p(0,t) = (1+R) * \hat{p}_0 * e^{i*(\omega*t)} \tag{B.56}$$

$$v(0,t) = (1-R) * \frac{\hat{p}_0}{\rho_0 * c} * e^{i*(\omega*t)} \tag{B.57}$$

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \rho_0 * c * \frac{1+R}{1-R} \tag{B.58}$$

$$\zeta = \frac{Z}{\rho_0 * c} \rightarrow \zeta = \frac{\rho_0 * c * \left(\frac{1+R}{1-R}\right)}{\rho_0 * c} \rightarrow \zeta = \frac{1+R}{1-R} \tag{B.59}$$

$$R = \frac{\zeta - 1}{\zeta + 1} \tag{B.60}$$

$$\alpha = \frac{4 * Re(\zeta)}{|\zeta|^2 + 2 * Re(\zeta) + 1} \tag{B.61}$$

$$x' = x * cos(\theta) + y * sin(\theta) \tag{B.62}$$

$$p_i(x', t) = \hat{p}_0 * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.63}$$

$$v_i(x', t) = \frac{\hat{p_0}}{\rho_0 * c} * cos(\theta) * e^{i*(\omega*t - k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.64}$$

$$p_r(x', t) = R * \hat{p}_0 * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.65}$$

$$v_r(x', t) = -R * \frac{\hat{p_0}}{\rho_0 * c} * e^{i*(\omega*t + k*(x*cos(\theta)+y*sin(\theta)))} \tag{B.66}$$

$$Z = \left(\frac{p}{v_n}\right) \rightarrow Z = \frac{\rho_0 * c}{cos(\theta)} * \frac{1+R}{1-R} \tag{B.67}$$

$$R = \frac{Z * cos(\theta) - \rho_0 * c}{Z * cos(\theta) + \rho_0 * c} = \frac{\zeta * cos(\theta) - 1}{\zeta * cos(\theta) + 1} \tag{B.68}$$

$$\alpha(\theta) = \frac{4 * Re(\zeta) * cos(\theta)}{(|\zeta| * cos(\theta))^2 + 2 * Re(\zeta) * cos(\theta) + 1} \tag{B.69}$$

$$p(x, y) = \hat{p}[1 + |R|^2 + 2 * |R| * cos(2 * k * x * cos(\theta) + \chi]^{1/2} * e^{-i*k*y*(sin\theta)} \tag{B.70}$$

$$c_y = \frac{\omega}{k_y} = \frac{\omega}{k * sin(\theta)} = \frac{c}{sin(\theta)} \tag{B.71}$$

C | Plane Waves

As most of the notation and equations in [7] are equal or at the very least similar to other sources, it has been chosen as the main source for equations, meaning all formulas used in this section can be found in [7] as well, unless otherwise noted. Other sources used for mathematical formulas are [4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. A plane wave simply put is a fraction of a spherical wave small enough that it can be assumed to have no curvature, [36, 9, 13, 14, 4, 5, 7, 8]. Another physical assumption that can be made, is that if a sound wave is present within a circular tube, with a diameter or a rectangular tube with a height/width significantly smaller than the wavelength of said wave, it can be assumed to be planar, see section 3.2 on page 29 for more. It is effectively an orthogonal plane to a vector in a cartesian coordinate system. If that vector is equal to the x-axis, a plane wave can be described by the wave equation (2.2) with:

$$c^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} \tag{C.1}$$

With a corresponding general solution given by:

$$p(x,t) = F(c * t - x) + G(c * t + x) \tag{C.2}$$

In equation C.2 the first term represents a plane wave propagating positively along the x-axis and the second term a negative direction. From equations C.1 and C.2 it can also be seen that a constant pressure level is present at each wavefront. By specifying F and G as exponential functions including the imaginary components, the propagation is given as:

$$p(x,t) = \hat{p} * e^{i*k*(c*t-x)} = \hat{p} * e^{i*(\omega*t-k*x)} \tag{C.3}$$

Where $\hat{p}$ is amplitude, k is the propagation constant $k = \frac{\omega}{c}$, ω is the angular frequency, which can be used to obtain the temporal period $T = \frac{2*\pi}{\omega}$. This all connects to several interpretations of the wavelength formula:

$$\lambda = \frac{2*\pi}{k} \leftrightarrow \frac{2*\pi}{\frac{\omega}{c}} \leftrightarrow \frac{2*\pi*c}{\omega} \leftrightarrow \frac{c}{\frac{\omega}{2*\pi}} \leftrightarrow \frac{c}{f} \tag{C.4}$$

Now that amplitude, wavelength, and direction are known, the only missing parameters for expressing the plane wave are frequency and intensity. The frequency is given by $f = \frac{\omega}{2*\pi} * \frac{1}{T}$ with the unit Hz. If equation 2.5 is applied to equation C.2, the only non-vanishing point of the wave is parallel to the x-axis, meaning that sound behaves as longitudinal waves in fluids, and the particle velocity can be found by:

$$v(x,t) = \frac{1}{\rho_0 * c}[F(c * t - x) - G(c * t + x)] \tag{C.5}$$

To obtain the ratio between sound pressure and particle velocity, also known as the characteristic impedance of a medium, the following is used:

$$\frac{p}{v} = \rho_0 * c \tag{C.6}$$

With $\rho_0 * c$ found by table values for ρ_0 and c in the correct medium. For air, the characteristic impedance is:

$$\rho_{0_{air}} * c_{air} = 1.2 \left[\frac{kg}{m^3}\right] * 343 \left[\frac{m}{s}\right] = 414 \left[\frac{kg}{m^2 s}\right] \tag{C.7}$$

Understanding the above relation makes it possible to determine the intensity of each wave. The plane imagined to be orthogonal to the x-axis will have energy present across its entire surface, and the average (denoted by the bar notation) product of the pressure and particle velocity on the surface yields the intensity:

$$I = \bar{pv} = \frac{\bar{p^2}}{\rho_0 * c} \tag{C.8}$$

To find the energy density of a wave, equation C.9 is used:

$$w = \frac{I}{c} = \frac{\bar{p^2}}{\rho_0 * c^2} \tag{C.9}$$

And with that, sufficient information should be available to describe any plane wave.

D | Spherical Waves

As most of the notation and equations in [7] are equal or at the very least similar to other sources, it has been chosen as the main source for equations, meaning all formulas used in this section can be found in [7] as well, unless otherwise noted. Other sources used for mathematical formulas are [4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The sound wave will now propagate with no boundaries whatsoever, leading to ideal spherical waves. Graphically this is best understood as an inflating ball stretching equally in all directions from an infinitesimally small point source. Due to the nature of spherical waves, a spherical coordinate system will be used, using polar coordinates. A benefit from the spherical symmetry is that pressure is represented as r and is independent of the θ and ϕ angles. To convert the wave equation from cartesian to spherical coordinates, the laplacian of a function $p(r, \theta, \phi)$has to expressed in spherical coordinates as well:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 p}{\partial \phi^2}. \tag{D.1}$$

As the acoustical properties of a spherical wave are equal across all θ and ϕ angles, it's description can be made with only r and t, simplifying the laplacian to:

$$\Delta p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) \tag{D.2}$$

The simplified laplacian should now be expanded to:

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} \tag{D.3}$$

And is now ready to be inputted into the wave equation:

$$c^2\left(\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}\right) = \frac{\partial^2 p}{\partial t^2} \tag{D.4}$$

By dividing with c^2 on both sides, the spherical wave equation is found:

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r} * \frac{\partial p}{\partial r} = \frac{1}{c^2} * \frac{\partial^2 p}{\partial t^2} \tag{D.5}$$

A simple solution to the spherical wave equation is given in equation D.6, representing a spherical wave with the volume velocity Q, being the rate in $\frac{m^3}{s}$ of "liquid" being expelled by the sound source located at $r = 0$. The dot above Q denotes differentation w.r.t. time (t).

$$p(r,t) = \frac{\rho_0}{4 * \pi * r} * \dot{Q}\left(t - \frac{r}{c}\right) \tag{D.6}$$

As for plane waves, spherical waves have a non-vanishing component of their particle velocity. Instead of being along the x-axis, the non-vanishing point for a spherical wave is radial, and can be calculated by applying equation 2.5 to D.6, yielding:

$$v_r = \frac{1}{4 * \pi * r^2} \left[Q \left(t - \frac{r}{c} \right) + \frac{r}{c} * \dot{Q} \left(t - \frac{r}{c} \right) \right] \tag{D.7}$$

To find the particle velocity of a spherical wave, equation D.8 is used. It shows that the sound pressure and velocity in a spherical wave is inversely proportional to the size r and frequency $\omega = k * c$. For distances large in comparison with the wavelength, the ratio $\frac{p}{v}$ tends asymptotically to $\rho_0 c$.

$$v_r = \frac{p}{\rho_0 * c} * \left(1 + \frac{1}{i * k * r} \right) \tag{D.8}$$

E | Octave Bands

When examining sound and acoustic properties, it is often done in octave bands or fractions thereof, [60, 61, 62, 63]. Octaves emerge from the world of music, given at the interval between two musical pitches, double or half the frequency. Octave values loosely follows 2s complement, with some rounding error, [61]. In figure E.1 the most common centerfrequencies can be seen to broadly follow 2s complement. The audible spectrum from 20Hz-20kHz is often divided into $\tilde{1}1$ octave bands, with the 7th band being set to 1kHz. Mathematically the octave bands are given by equation E.1, where f_c is set to be 1kHz and n is the octave band number. The upper and lower frequency band cutoff can be found by equation E.2, where f_c is set to the relevant centerfrequency.

$$Lower : f_c(n-1) = \frac{f_c(n)}{2} \qquad Upper : f_c(n+1) = 2 * f_c(n) \tag{E.1}$$

$$f_c = \sqrt{2} * f_{min} = \frac{f_{max}}{\sqrt{2}} \tag{E.2}$$

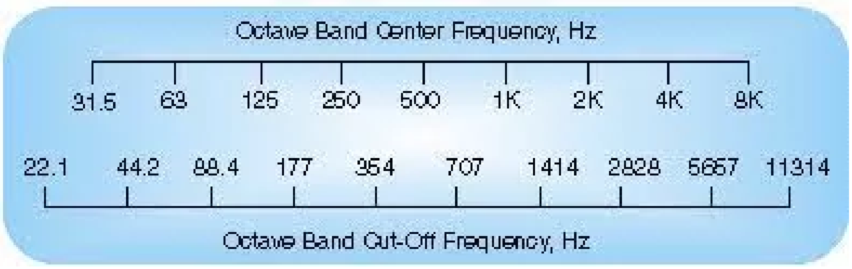

Figure E.1: 1/1 octave bands, with centerfrequencies at the top, and cutoff-frequencies at the bottom, [62].

To gain greater resolution most measurements within room acoustics are done in fractional octave bands, often $\frac{1}{3}$. When calculating the $\frac{1}{3}$ octave bands, equation E.1 is modified to:

$$Lower : f_c(n-1) = \frac{f_c(n)}{2^{\frac{1}{3}}} \qquad Upper : f_c(n+1) = 2^{\frac{1}{3}} * f_c(n) \tag{E.3}$$

And instead of $\tilde{1}1$ octave bands, $\tilde{3}1$ is used instead, making 1khz the 19th octave bands centerfrequency. As the octave bands have gotten singificantly smaller, the upper and lower cutoff is adjusted accordingly to:

$$f_{c\frac{1}{3}} = \frac{1}{3} * \sqrt{2} * f_{min\frac{1}{3}} = \frac{1}{3} * \frac{f_{max\frac{1}{3}}}{\sqrt{2}} \tag{E.4}$$

In table E.1 on page 75 the hearing spectrum is mapped into $\frac{1}{1}$ and $\frac{1}{3}$ octave bands.

Octave Bands			1/3 Octave Bands		
Lower Band Limit (Hz)	**Center Frequency (Hz)**	**Upper Band Limit (Hz)**	**Lower Band Limit (Hz)**	**Center Frequency (Hz)**	**Upper Band Limit (Hz)**
			14.1	16	17.8
11	16	22	17.8	20	22.4
			22.4	25	28.2
			28.2	31.5	35.5
22	31.5	44	35.5	40	44.7
			44.7	50	56.2
			56.2	63	70.8
44	63	88	70.8	80	89.1
			89.1	100	112
			112	125	141
88	125	177	141	160	178
			178	200	224
			224	250	282
177	250	355	282	315	355
			355	400	447
			447	500	562
355	500	710	562	630	708
			708	800	891
			891	1000	1122
710	1000	1420	1122	1250	1413
			1413	1600	1778
			1778	2000	2239
1420	2000	2840	2239	2500	2818
			2818	3150	3548
			3548	4000	4467
2840	4000	5680	4467	5000	5623
			5623	6300	7079
			7079	8000	8913
5680	8000	11360	8913	10000	11220
			11220	12500	14130
			14130	16000	17780
11360	16000	22720	17780	20000	22390

Table E.1: Octave Bands and 1/3 Octave Bands, [61].

F Standardized Renard Numbers

Standard frequencies (c/s)	American 1/3-octave band number
100	20
125	21
160	22
200	23
250	24
315	25
400	26
500	27
630	28
800	29
1000	30
1250	31
1600	32
2000	33
2500	34
3150	35
4000	36
5000	37
6300	38

40-series	20-series	10-series	5-series	Exact Value	Mantissa
1.00	1.00	1.00	1.00	10000	000
1.06				10593	025
1.12	1.12			11220	050
1.18				11885	075
1.25	1.25	1.25		12589	100
1.32				13335	125
1.40	1.40			14125	150
1.50				14962	175
1.60	1.60	1.60	1.60	15849	200
1.70				16788	225
1.80	1.80			17783	250
1.90				18836	275
2.00	2.00	2.00		19953	300
2.12				21135	325
2.24	2.24			22387	350
2.36				23714	375
2.50	2.50	2.50	2.50	25119	400
2.65				26607	425
2.80	2.80			28184	450
3.00				29854	475
3.15	3.15	3.15		31623	500
3.35				33497	525
3.55	3.55			35481	550
3.75				37594	575
4.00	4.00	4.00	4.00	39811	600
4.25				42170	625
4.50	4.50			44668	650
4.75				47315	675
5.00	5.00	5.00		50119	700
5.30				53088	725
5.60	5.60			56234	750
6.00				59566	775
6.30	6.30	6.30	6.30	63096	800
6.70				66834	825
7.10	7.10			70795	850
7.50				74989	875
8.00	8.00	8.00		79433	900
8.50				84140	925
9.00	9.00			89125	950
9.50				94406	975

Table F.1: Standard frequencies which should be chosen when taking measurements in the apparatus. The frequencies are based on the R_{10} series of the international "Preferred Numbers". A complete table of these so-called "standardized Renard numbers" dividing a decade into 10 equal logarithmic steps is shown at the right, [32].

G | Maximum Length LFSR Lookup Table

Bits (n)	Feedback Polynomial	Taps	Taps (hex)	Period
2	x^2+x+1	11	0x3	3
3	x^3+x^2+1	110	0x6	7
4	x^4+x^3+1	1100	0xC	15
5	x^5+x^3+1	10100	0x14	31
6	x^6+x^5+1	110000	0x30	63
7	x^7+x^6+1	1100000	0x60	127
8	$x^8+x^6+x^5+x^4+1$	10111000	0xB8	255
9	x^9+x^5+1	100010000	0x110	511
10	$x^{10}+x^7+1$	1001000000	0x240	1,023
11	$x^{11}+x^9+1$	10100000000	0x500	2,047
12	$x^{12}+x^{11}+x^{10}+x^4+1$	111000001000	0xE08	4,095
13	$x^{13}+x^{12}+x^{11}+x^8+1$	1110010000000	0x1C80	8,191
14	$x^{14}+x^{13}+x^{12}+x^2+1$	11100000000010	0x3802	16,383
15	$x^{15}+x^{14}+1$	110000000000000	0x6000	32,767
16	$x^{16}+x^{15}+x^{13}+x^4+1$	1101000000001000	0xD008	65,535
17	$x^{17}+x^{14}+1$	10010000000000000	0x12000	131,071
18	$x^{18}+x^{11}+1$	100000010000000000	0x20400	262,143
19	$x^{19}+x^{18}+x^{17}+x^{14}+1$	1110010000000000000	0x72000	524,287
20	$x^{20}+x^{17}+1$	10010000000000000000	0x90000	1,048,575
21	$x^{21}+x^{19}+1$	101000000000000000000	0x140000	2,097,151
22	$x^{22}+x^{21}+1$	1100000000000000000000	0x300000	4,194,303
23	$x^{23}+x^{18}+1$	10000100000000000000000	0x420000	8,388,607
24	$x^{24}+x^{23}+x^{22}+x^{17}+1$	111000010000000000000000	0xE10000	16,777,215

Table G.1: Feedback Polynomials, Taps, and Periods for Various Bit-Lengths, [48].