# Pattern Recognition Experiment 1

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1.

a) Generate a data set $X_1$ of $N = 1{,}000$ 2-D vectors that stem from three equiprobable classes modeled by normal distribution with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [8,12]^T$, $\boldsymbol{\mu}_3 = [1,16]^T$ and covariance matrices $\Sigma_1 = \Sigma_2 = \Sigma_3 = 4I$, where $I$ is the $2 \times 2$ identity matrix.

b) Apply the Bayesian, the Euclidean, and the Mahalanobis classifiers on $X_1$.

c) Compute the classification error for each classifier and draw your conclusions.

Conclusion: In this experiment, the classification error of three classifiers are all the same. Below is the result of the experiment repeating 5 times. This refers to the case 1 with equiprobable classes with the same covariance matrix, maximizing $g_i(x)$ = of each class = minimizing the Euclidean distance $||x - \mu_i||^2$, thus x is assigned to the class of the nearest mean.

| | | |
|---|---|---|
| X1_b_err: 0.06<br>X1_m_err: 0.06<br>X1_e_err: 0.06 | X1_b_err: 0.058<br>X1_m_err: 0.058<br>X1_e_err: 0.058 | X1_b_err: 0.051<br>X1_m_err: 0.051<br>X1_e_err: 0.051 |
| X1_b_err: 0.046<br>X1_m_err: 0.046<br>X1_e_err: 0.046 | X1_b_err: 0.063<br>X1_m_err: 0.063<br>X1_e_err: 0.063 | |

2.

a) Generate a data set $X_2$ of $N = 1{,}000$ 2-D vectors that stem from three equiprobable classes modeled by normal distribution with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [7,14]^T$, $\boldsymbol{\mu}_3 = [1,16]^T$ and covariance matrices $\Sigma_1 = \Sigma_2 = \Sigma_3 = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$.

b) Apply the Bayesian, the Euclidean, and the Mahalanobis classifiers on $X_2$

c) Compute the classification error for each classifier and draw your conclusions.

Conclusion: In this experiment, the classification error of Bayesian classifier and Mahalanobis classifier is lower than Euclidean classifier. This refers to case 3 on the slides with equiprobable prior and the covariance matrices for all classes are the same but arbitrary. In this case, maximizing $g_i(x)$ =

minimizing the Mahalanobis distance$((x-\mu_i)^T \sum^{-1} (x-\mu_i))^{\frac{1}{2}}$ from x to $\mu_i$. Since the Euclidean distance only deals with the mean of each class and no covariance, it results in having higher classification error.

| | | |
|---|---|---|
| X2_b_err: 0.008<br>X2_m_err: 0.008<br>X2_e_err: 0.015 | X2_b_err: 0.005<br>X2_m_err: 0.005<br>X2_e_err: 0.009 | X2_b_err: 0.006<br>X2_m_err: 0.006<br>X2_e_err: 0.012 |
| X2_b_err: 0.005<br>X2_m_err: 0.005<br>X2_e_err: 0.015 | X2_b_err: 0.005<br>X2_m_err: 0.005<br>X2_e_err: 0.017 | |

3.

a) Generate a data set $X_3$ of $N = 1{,}000$ 2-D vectors that stem from three equiprobable classes modeled by normal distribution with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [6,8]^T$, $\boldsymbol{\mu}_3 = [1,13]^T$ and covariance matrices $\Sigma_1 = \Sigma_2 = \Sigma_3 = 6I$, where $I$ is the $2 \times 2$ identity matrix.

b) Apply the Bayesian, the Euclidean, and the Mahalanobis classifiers on $X_3$

c) Compute the classification error for each classifier and draw your conclusions.

Conclusion: This experiment is similar to the first one. It is also equiprobable for each class and the covariance matrices are all 6*I*. Therefore, the classification error of three classifiers are also the same. Below is the classification error derived from 5 repeat experiments.

| | | |
|---|---|---|
| X3_b_err: 0.082<br>X3_m_err: 0.082<br>X3_e_err: 0.082 | X3_b_err: 0.069<br>X3_m_err: 0.069<br>X3_e_err: 0.069 | X3_b_err: 0.066<br>X3_m_err: 0.066<br>X3_e_err: 0.066 |
| X3_b_err: 0.072<br>X3_m_err: 0.072<br>X3_e_err: 0.072 | X3_b_err: 0.074<br>X3_m_err: 0.074<br>X3_e_err: 0.074 | |

4.

a) Generate a data set $X_4$ of $N = 1{,}000$ 2-D vectors that stem from three equiprobable classes modeled by normal distribution with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [5,10]^T$, $\boldsymbol{\mu}_3 = [1,11]^T$ and covariance matrices $\Sigma_1 = \Sigma_2 = \Sigma_3 = \begin{bmatrix} 5 & 4 \\ 4 & 7 \end{bmatrix}$.

b) Apply the Bayesian, the Euclidean, and the Mahalanobis classifiers on $X_4$

c) Compute the classification error for each classifier and draw your conclusions.

Conclusion: This experiment is similar to the second experiment, where three classes are equiprobable, and the covariance matrix are the same via classes but arbitrary instead of constant times identity matrix. Therefore, Bayesian and Mahalanobis classifier has lower error rate than Euclidean classifier.

| | | |
|---|---|---|
| X4_b_err: 0.07<br>X4_m_err: 0.07<br>X4_e_err: 0.108 | X4_b_err: 0.079<br>X4_m_err: 0.079<br>X4_e_err: 0.11 | X4_b_err: 0.086<br>X4_m_err: 0.086<br>X4_e_err: 0.119 |
| X4_b_err: 0.071<br>X4_m_err: 0.071<br>X4_e_err: 0.125 | X4_b_err: 0.085<br>X4_m_err: 0.085<br>X4_e_err: 0.128 | |

5.

a) Generate two data sets $X_5$ and $X_5'$ of $N = 1{,}000$ 2-D vectors each that stem from three classes modeled by normal distributions with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [4,4]^T$, $\boldsymbol{\mu}_3 = [1,8]^T$ and covariance matrices $\Sigma_1 = \Sigma_2 = \Sigma_3 = 2I$. In the generation of $X_5$, the classes are assumed to be equiprobable, while in the generation of $X_5'$, the prior probabilities of the classes are given by $P_1 = 0.7, P_2 = 0.1, P_3 = 0.2$.

b) Apply the Bayesian and the Euclidean, classifiers on both $X_5$ and $X_5'$.

c) Compute the classification error for each classifier for both datasets and draw your conclusions.

Conclusion: This experiment generates two datasets X5 and X5' that has same means and covariance matrices but with different prior probability for each class. Below is the table of classification of each dataset. For the equiprobable dataset X5, the classification error of Bayesian and Euclidean is the same, it is similar to the first and third experiment. Now, for dataset X5', the distribution of each class is not equiprobable anymore, and the Euclidean only take means into account, but Bayesian classifier takes prior into account is the discriminant function also, so it has lower error than Euclidean classifier, for the error rate of two datasets, I think there is no significant difference.

| | | |
|---|---|---|
| X5_b_err: 0.064<br>X5_e_err: 0.064 | X5_b_err: 0.073<br>X5_e_err: 0.073 | X5_b_err: 0.061<br>X5_e_err: 0.061 |

| | | |
|---|---|---|
| X5_b_err: 0.07<br>X5_e_err: 0.07 | X5_b_err: 0.08<br>X5_e_err: 0.08 | |
| X5_prime_b_err: 0.039<br>X5_prime_e_err: 0.063 | X5_prime_b_err: 0.044<br>X5_prime_e_err: 0.076 | X5_prime_b_err: 0.039<br>X5_prime_e_err: 0.067 |
| X5_prime_b_err: 0.049<br>X5_prime_e_err: 0.069 | X5_prime_b_err: 0.045<br>X5_prime_e_err: 0.06 | |

6.

a) Generate two data sets $X_6$ and $X_6'$ of $N = 1{,}000$ 2-D vectors each that stem from three classes modeled by normal distributions with mean vectors $\boldsymbol{\mu}_1 = [1,1]^T$, $\boldsymbol{\mu}_2 = [7,7]^T$, $\boldsymbol{\mu}_3 = [1,15]^T$ and covariance matrices $\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 12 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 2 & 3 \\ 3 & 8 \end{bmatrix}, \Sigma_3 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. In the generation of $X_6$, the classes are assumed to be equiprobable, while in the generation of $X_6'$, the prior probabilities of the classes are given by $P_1 = 0.6, P_2 = 0.3, P_3 = 0.1$..

b) Apply the Bayesian and the Euclidean, classifiers on both $X_6$ and $X_6'$.

c) Compute the classification error for each classifier for both datasets and draw your conclusions.

Conclusion: Last experiment are two datasets with same/different prior and the covariance matrices of three classes are now different. This refers to case 4 on the slides and the discriminant function is not guaranteed to be linear classifier anymore. The behavior of the experiment is similar to experiment 5, but since of covariance matrix three matrix is different now, the error rate of classifier is different, also. For X6' dataset(especially X6'), Bayesian classifier takes prior into account so it has lower error than Euclidean classifier.

| | | |
|---|---|---|
| X6_b_err: 0.007<br>X6_e_err: 0.043 | X6_b_err: 0.003<br>X6_e_err: 0.043 | X6_b_err: 0.004<br>X6_e_err: 0.045 |
| X6_b_err: 0.004<br>X6_e_err: 0.032 | X6_b_err: 0.005<br>X6_e_err: 0.035 | |
| X6_prime_b_err: 0.005<br>X6_prime_e_err: 0.05 | X6_prime_b_err: 0.007<br>X6_prime_e_err: 0.052 | X6_prime_b_err: 0.006<br>X6_prime_e_err: 0.057 |
| X6_prime_b_err: 0.003<br>X6_prime_e_err: 0.041 | X6_prime_b_err: 0.005<br>X6_prime_e_err: 0.045 | |