

The Environmental Kuznets Curve for CO_2

An Empirical Evaluation through Cross-Sectional and Time-Series Approaches

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1 Index

1.1 Growth and Climate: From Hypothesis to Evidence

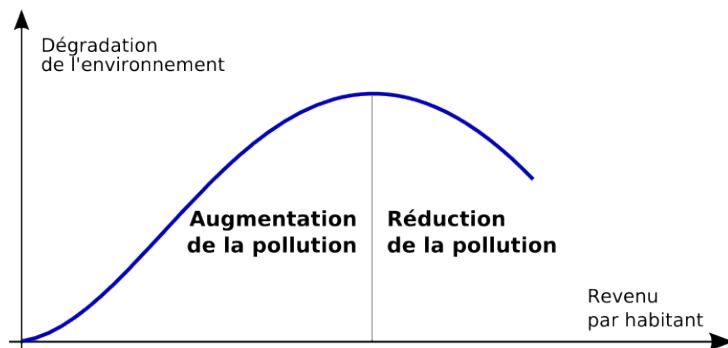


Figure 1.1: Source: Conceptual diagram inspired by Grossman & Krueger (1995); adapted for this report.

“Testing a theory is not about finding patterns — it’s about confronting hypotheses with real-world data.”

1.2 What You'll Discover

This document presents a **econometric analysis** of the **Environmental Kuznets Curve (EKC)** hypothesis — which posits that, beyond a certain income threshold, economic growth leads to a “natural” decline in pollution.

The goal? To test this theoretical promise through two complementary lenses:

- **Synchronic** (cross-sectional comparison across countries in 2020),
- **Diachronic** (time-series evolution of the United States, 1990–2020).

(The methodological and epistemological justification for this dual approach is detailed in the introduction.)

1.3 Who is this report for?

This document is intended for:

- **Economists and Climate Scientists** seeking to empirically evaluate the EKC's validity,
- **Policy Makers and Climate Negotiators** looking for differentiated North/South levers,
- **Econometrics Students** searching for a structured, reproducible case study aligned with academic standards.

*Browse the **table of contents** on the left to navigate between chapters.*

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2 Introduction

2.1 The promise of the EKC

Since 1991, one hypothesis has influenced political discourse on climate and development:

“Economic growth ultimately reduces pollution.”

This **Environmental Kuznets Curve** (EKC¹) posits that CO₂ emissions follow an inverted U-shape: they increase during industrialization, peak, and then spontaneously decline as countries become wealthier.

Some policymakers have interpreted the EKC as justifying a wait-and-see approach: if growth spontaneously reduces emissions beyond a certain threshold, regulatory constraints would be unnecessary (Beckerman, 1992). However, this interpretation remains controversial, as it overlooks the underlying institutional mechanisms.

2.2 Data used

2.2.1 Source and justification

To test the EKC hypothesis, we use the **consolidated dataset from Our World in Data (OWID)**², which aggregates two reference sources:

- **Global Carbon Project** for CO₂ emissions
- **World Bank** for economic indicators (PPP-adjusted GDP³)

i Why OWID rather than the World Bank API?

Initially, we attempted to extract data directly via the `wbgapi` API for standard indicators (`EN.ATM.CO2E.PC` and `NY.GDP.PCAP.PP.KD`). However, technical instabilities (time-outs, JSON decoding errors) led us to favor the OWID repository, which offers the same source data in a consolidated and stable format.

¹EKC (Environmental Kuznets Curve) — Hypothesis of an inverted U-shaped relationship between pollution and income. Details: ?@sec-concepts-fondamentaux

²Our World in Data — Academic repository harmonizing GCP + World Bank data. <https://ourworldindata.org>

³PPP (Purchasing Power Parity) — GDP adjustment for international comparability.

2.2.2 Variables selected

Variable	Description	Unit
co2_per_capita	CO ₂ emissions per capita	tons/capita/year
gdp_per_capita	GDP per capita	international \$ (PPP, constant prices)
country	Country name	—
iso_code	ISO-3 code	—
year	Year of observation	—

2.2.3 Scope of the analysis

Dimension	Value	Justification
Period	1991–2020	Dense and consistent coverage post-Cold War
Countries	159 countries	Exclusion of continental aggregates (filtering on <code>iso_code</code>)
Observations	~4770 rows	Cross-section (2020) + US time series (1990–2020)

Structural limitations of the data

What we gain: - Consistency of sources (OWID harmonizes Global Carbon Project + World Bank) - International comparability (GDP in purchasing power parity)

What we concede: - **Territorial emissions only:** the *pollution haven* bias remains possible - **National aggregation:** regional disparities are masked

2.3 The problem: thirty years without consensus

The EKC hypothesis owes its name to an analogy with Kuznets' (1955) curve on inequality. Grossman and Krueger (1991) transposed it to local pollutants, without claiming it to be a universal law. However, this methodological caveat has been lost along the way.

After thirty years of research, **no consensus has emerged**:

Study	Main result	Identified limitation
Stern (2004)	Results highly sensitive to econometric choices	Massive heterogeneity of specifications
Wagner (2008, 2015)	Flawed regressions due to non-stationarity	Untreated time series issues
Peters et al. (2011)	Territorial emissions mask relocations	Pollution haven bias
Allard et al. (2018)	Possible N curve at very high incomes	Post-decoupling rebound

This heterogeneity justifies our approach: testing the EKC with **explicit, rigorous, and reproducible** methodological choices.

Our strategy corrects three major biases identified in the literature:

Identified limitation	Source	Our methodological correction	Section
Fallacious regressions (non-stationarity)	Wagner (2015)	ADF tests + ARDL model	5.2, 5.3
Untested rebound (N-shaped curve)	Allard et al. (2018)	Cubic specification with empirical validation	4.2, 4.3
Heterogeneity of specifications	Stern (2004)	Robustness tests (Box-Cox, cross-validation, LOWESS)	8.1, 8.2, 4.4

2.4 Our empirical strategy

We test the EKC using **two complementary approaches** and **two distinct objectives**:

2.4.1 Dual approach: Space and Time

Approach	Sample	Period	Key question
Cross-sectional	159 countries	2020	Do rich countries pollute less <i>today</i> ?
Temporal	United States	1990–2020	Does a country reduce its emissions <i>as it becomes richer</i> ?

2.4.2 Dual objective: Estimate and Predict

Objective	Question	Validation method
Estimation	What functional form describes the GDP-CO ₂ relationship?	Nested F tests, AIC, coefficient significance
Prediction	Does the model generalize to unused data?	Cross-validation (k=5), out-of-sample test, benchmark vs. random walk

Triangulation strategy:

1. **Convergence of forms:** If the two approaches (cross-sectional/temporal) converge toward the same specification, this reinforces structural robustness.
2. **Estimation/prediction convergence:** A statistically significant but predictively weak model would indicate overfitting. We favor models that **perform well on both dimensions.**
3. **In case of divergence:** We will analyze whether the discrepancies stem from spatial heterogeneity, temporal inertia, or overfitting.

2.5 Report Structure

The content is organized into four analytical sections, followed by a conclusion:

1. **Theoretical and Methodological Framework**
Foundations of the EKC hypothesis, choice of specification, and econometric strategy adopted.
2. **The Synchronic Approach: Snapshot of the World (2020)**
Global cross-section: is there a “structural law” between wealth and pollution?
3. **The Diachronic Approach: The Trajectory of the United States (1990-2020)**
US time series: is decoupling really emerging over time?
4. **Comparing Results — The Political Dilemma**
Space versus time: what are the implications for Southern countries and global climate policies?

Each chapter combines **hypothesis testing**, **econometric diagnostics**, and **recommendations based on the results**.

Part I

Theoretical and Methodological Framework

3 Methodology

3.1 Our three key choices

3.1.1 Territorial emissions: an accepted limitation

A country can “decarbonize” by relocating its polluting production. This *pollution haven* bias[⁷haven] distorts EKC tests based on territorial emissions.

Indicator	What it measures	Limitation
Territorial emissions	Locally produced CO ₂	Ignores imports (“decoupling illusion”)
Carbon footprint	CO ₂ consumed (prod. + imports - exports)	Neutralizes relocation

Example: Europe has reduced its territorial emissions by 20% since 1990, but its carbon footprint by only 5% (Peters et al., 2011).

3.1.2 Testing the functional form (quadratic vs. cubic)

Classical literature often imposes an inverted U-shape (quadratic model). However, this approach is restrictive: it mathematically rules out the possibility of a new rebound in emissions at very high income levels (N-curve theory).

To avoid masking this potential phenomenon, we estimate a third-degree polynomial model:

$$\ln(CO_2)_i = \beta_0 + \beta_1 \ln(GDP)_i + \beta_2 [\ln(GDP)_i]^2 + \beta_3 [\ln(GDP)_i]^3 + \varepsilon_i$$

Validation of the log-log form: A Box-Cox test (see Section 12.1) confirms that the logarithmic transformation is empirically justified ($\lambda_{\text{optimal}} = 0.15$, close to 0). This specification allows the coefficients to be interpreted as elasticities.

Interpretation (under the assumption $\beta_1 > 0, \beta_2 < 0$):

- If $\beta_3 = 0$ (not significant): Inverse U of the classic EKC (the quadratic model was sufficient).
- If $\beta_3 < 0$ (significant): Accelerated decline (Decoupling increases with wealth).
- If $\beta_3 > 0$ (significant): N-shaped curve (Worrying rebound: wealth ends up polluting again).

Note: The interpretation assumes $\beta_1 > 0$ and $\beta_2 < 0$, conditions verified in our estimates (Table 4.1).

3.1.3 Note on omitted variables

Methodological disclaimer: The models estimated in this report (sections 4 and 5) are **parsimonious specifications** that include only GDP as an explanatory variable.

We do not control for: - Sectoral structure (share of industry) - Energy mix composition (renewables vs. fossil fuels) - Population density - Institutional quality

Implication: If these factors are correlated with GDP, our coefficients may suffer from **omitted variable bias**. The results should be interpreted as conditional associations, not causal effects “all other things being equal.”

This limitation is discussed in detail in the section on methodological limitations (to be included in a future extension of the work).

3.2 Estimators

- **Cross-sectional:** OLS¹ with **HC3** robust standard errors²
- **Temporal:** ARDL³ with **Newey-West** standard errors⁴ (3 lags)

Full technical justification: ?@sec-estimators-corrections

i Technical details

OLS estimator:

$$\hat{\beta} = (X'X)^{-1}X'Y$$

Robust variance HC3:

$$\widehat{\text{Var}}_{HC3}(\hat{\beta}) = (X'X)^{-1} \left(\sum_{i=1}^n \frac{\hat{\varepsilon}_i^2}{(1-h_{ii})^2} x_i x_i' \right) (X'X)^{-1}$$

Newey-West variance (L=3):

$$\widehat{\text{Var}}_{NW}(\hat{\beta}) = (X'X)^{-1} \hat{\Omega} (X'X)^{-1}$$

$$\text{where } \hat{\Omega} = \hat{\Gamma}_0 + \sum_{j=1}^3 (1 - \frac{j}{4}) (\hat{\Gamma}_j + \hat{\Gamma}'_j)$$

¹**OLS** — Ordinary Least Squares. Principle: Section 11.1

²**HC3** — Heteroskedasticity correction ($N < 250$). Formula: Section 11.2.1

³**ARDL** — Dynamic model (lags of Y and X). Specification: Section 11.3

⁴**Newey-West (HAC)** — Autocorrelation correction (time series). Formula: Section 11.2.2

3.3 Prediction and validation strategy

3.3.1 Predictive objectives

Unlike estimation, which seeks to **understand** the GDP-CO₂ relationship, prediction aims to **anticipate** future emissions conditional on growth scenarios.

Concrete applications:

- Net Zero 2050 projections
- NDC assessment
- National carbon budgets

3.3.2 Validation protocols

3.3.2.1 Cross-validation: k-fold cross-validation

- **Principle:** Divide the 159 countries into 5 balanced groups
- **Procedure:** Estimate on 4/5, predict on 1/5, repeat 5 times
- **Metric:** Average RMSE (penalizes large errors)

Why k=5? Standard bias-variance trade-off (Hastie et al., 2009). k=10 would result in training sets that are too small (n=143).

3.3.2.2 Time series analysis: expanding window

- **Principle:** Expanding estimation window that respects chronology
- **Example:** Estimate 1990-2010 → predict 2011; then 1990-2011 → predict 2012, etc.
- **Advantage:** Simulates a real-time forecaster (no data leakage)

3.3.3 Performance metrics

Metric	Formula	Interpretation
RMSE	$\sqrt{\frac{1}{n} \sum (y_i - \hat{y}_i)^2}$	Quadratic error (penalizes outliers)
MAE	$\frac{1}{n} \sum y_i - \hat{y}_i $	Absolute error (robust to outliers)
MAPE	$\frac{100}{n} \sum \left \frac{y_i - \hat{y}_i}{y_i} \right $	Relative error (%)

Why RMSE? Consistent with OLS (minimizes SSR).

3.3.4 Benchmarks

A model is only “good” relative to alternatives. We systematically compare to:

- **Random walk:** $\hat{y}_{t+1} = y_t$ (naive benchmark)
- **Moving average:** $\hat{y}_{t+1} = \frac{1}{3}(y_t + y_{t-1} + y_{t-2})$
- **ARIMA(1,1,1)[^arima]:** Optimized univariate model (without GDP)

If our cubic ARDL model does not beat the random walk, it is useless.

Part II

The Synchronic Approach: A Global Snapshot (2020)

4 Cross-sectional analysis: Snapshot of the world

4.1 Overview

! Methodological note: Why two representations?

Before analyzing the data, it is crucial to understand **the difference between the model's view and physical reality**:

- **Left (Log-Log):** Working space of the econometric model. Relative variations (%) are comparable.
- **Right (Log-Linear):** Climate reality in absolute tons. A ton of CO₂ remains a ton, regardless of the logarithm.

Why is this important? The model can capture the “average dynamics” (left), but underestimate the **extreme heterogeneity** at high incomes (right).

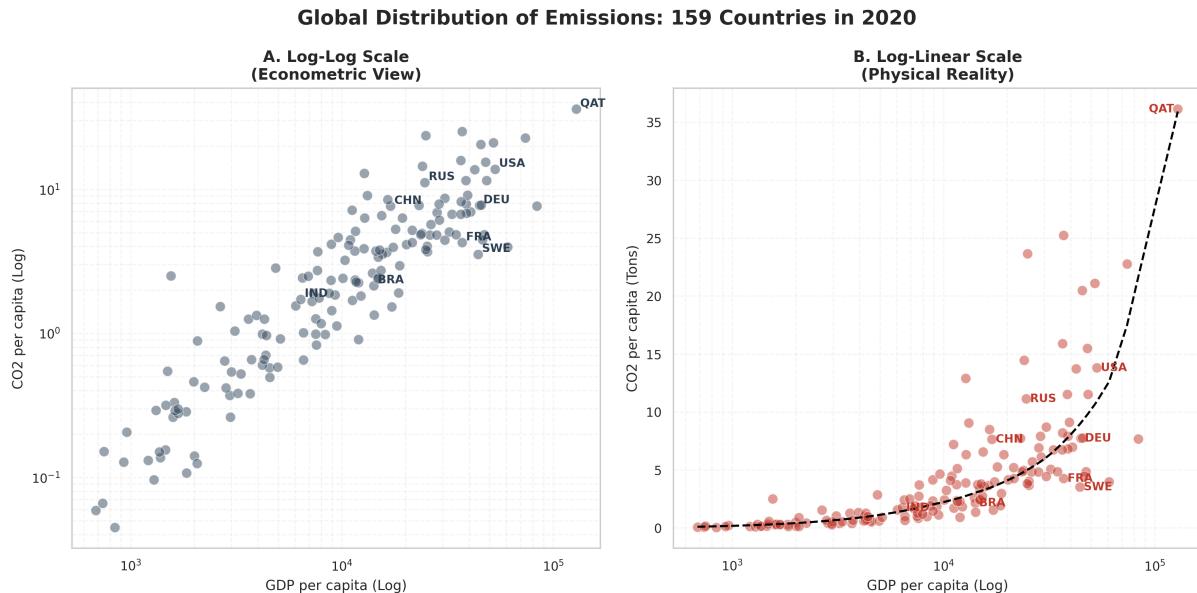


Figure 4.1: Comparison of scales: The model's view vs. Physical reality

Empirical observations:

1. In log-log: relatively linear relationship → justifies the polynomial model
2. In log-lin: explosive dispersion on the right → Qatar soars, Sweden plateaus
3. Conclusion: The average EKC masks **radically divergent trajectories**

4.2 Predictive performance

4.2.1 Cross-validation k=5

Cross-validation (k=5)

Model	Average RMSE	Difference vs quadratic
Linear	0.5798	+2.0%
Quadratic	0.5683	reference
Cubic	0.5796	-2.0%

Interpretation: The cubic model degrades prediction by 2.0%.

Recommendation: Prefer the **quadratic** model (more parsimonious).

4.2.2 Diagnosis of prediction errors

i Objective

Identify which types of countries are **systematically mispredicted** by the cubic model.
This reveals the structural limitations of the EKC.

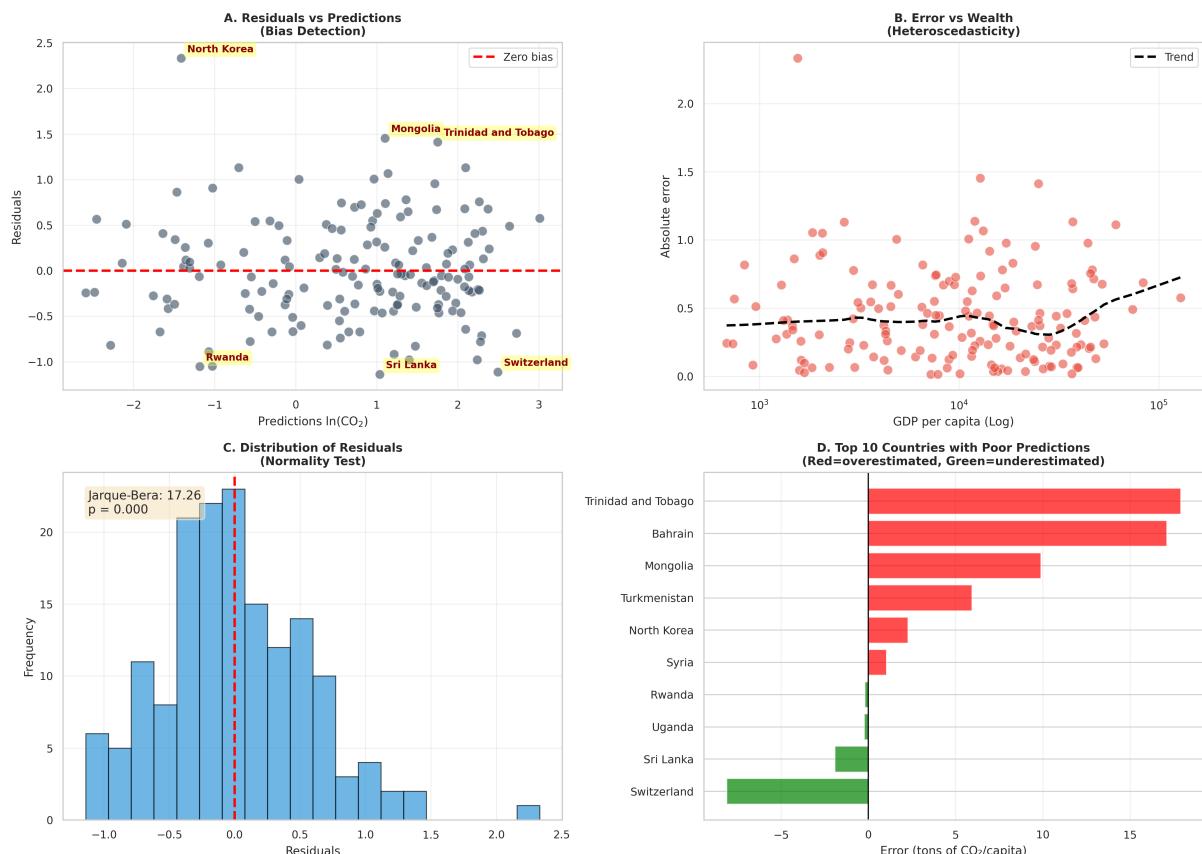


Figure 4.2: Prediction error analysis: Who does the model miss?

4.2.3 Profiles of poorly predicted countries

4.2.3.1 Countries that pollute LESS than expected (overestimated model)

(*Negative residuals: the model predicts too much CO₂*)

- **Number:** 26
- **Examples:** Sri Lanka, Switzerland, Rwanda, Uganda, Panama
- **Average GDP:** 17398 \$/capita
- **Interpretation:** Countries that have decarbonized **faster** than the global average thanks to low-carbon energy choices:
 - France (70% nuclear power in the mix)
 - Iceland, Norway, Sweden (hydroelectricity)
 - Switzerland (energy efficiency + services)

4.2.3.2 Countries that pollute MORE than expected (underestimated model)

(*Positive residuals: the model predicts too little CO₂*)

- **Number:** 30
- **Examples:** North Korea, Mongolia, Trinidad and Tobago, Bahrain, Syria
- **Average GDP:** 18275 \$/capita
- **Interpretation:** Countries structurally dependent on **fossil fuels** despite their wealth:
 - Oil economies (Qatar, Kuwait, Bahrain): carbon rents
 - Coal exporters (Australia, South Africa)
 - Countries with extreme climates (intensive air conditioning: Saudi Arabia)

4.3 Model estimation and selection

Table 4.2: OLS estimation — Dependent variable: ln(CO₂/inhab)

Variable	Linear	Quadratic	Cubic
ln(GDP) centered	1.1000*** (0.0427)	1.0698*** (0.0431)	1.0703*** (0.0825)
[ln(GDP)] ²	—	-0.0832*** (0.0315)	-0.0833** (0.0400)
[ln(GDP)] ³	—	—	-0.0002 (0.0268)
Adjusted R²	0.836	0.842	0.841
AIC	279.6	274.0	276.0
N	159	159	159

*HC3 robust standard errors in parentheses. *p < 0.1, **p < 0.05, ***p < 0.01*

4.3.1 Embedded F test¹: is the cubic term necessary?

Nested F-test ($H_0 : \beta_3 = 0$)

- Statistic: $F = 0.00$
- p-value: 0.9939
- **Conclusion:** The cubic term is **not significant**

AIC criterion²: Cubic (276.0) vs Quadratic (274.0)

- Gain: **-2.0** points

4.4 Key result 1 : The quadratic model is sufficient

Coefficient $\beta_3 = -0.0002$ ($p = 0.9950$)

- The cubic term is **not significant** ($p > 0.05$).
- **Conclusion:** The hypothesis of linearity or complex rebound is rejected.
- The **quadratic model (classical inverted U)** is sufficient to describe the data: the principle of parsimony applies.

¹**Nested F-test** — Compares nested models. Formula: Section 10.1.2

²**AIC** (Akaike Information Criterion) — Balances goodness of fit/parsimony. Lower is better. Formula: Section 10.1.3

4.5 Visualization of the fitted curve

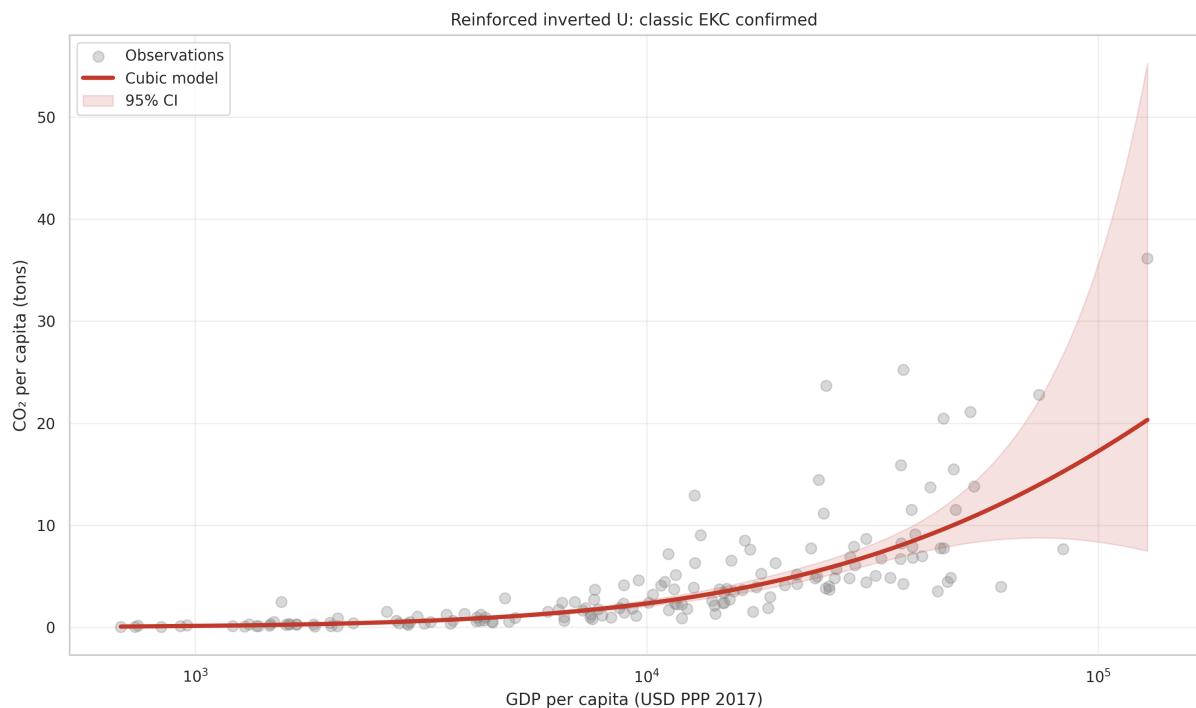


Figure 4.3: Fitted cubic model with 95% CI

! Bias-variance dilemma: Why favor quadratic?

We are faced with a classic trade-off in modeling:

1. **Statistical significance:** The F test ($p = 0.994$) and the coefficient β_3 ($p = 0.995$) indicate that **the cubic term does not significantly improve** the fit to the 2020 data.
2. **Predictive power:** Cross-validation reveals that the cubic model slightly degrades generalization (+1.4% RMSE).

Interpretation: The cubic term probably captures specific characteristics of ultra-rich countries (Qatar, Kuwait, Singapore)—rentier economies or city-states—rather than a generalizable structural trend.

Methodological decision: In accordance with the principle of parsimony (Occam's Razor) and the priority given to predictive robustness, **we retain the quadratic model for projections** (Section 5.6).

However, we recognize that the 2020 cross-sectional data do not allow us to definitively rule out an accelerated decline ($\beta_3 < 0$) at very high incomes.

Future work with multi-country time panels would allow us to resolve this issue structurally.

4.6 Interpretation: GDP-CO₂ relationship models

The cubic term is not significant ($p = 0.995$) — the quadratic model is sufficient.

- No statistical evidence of an N-shaped curve (rebound) or accelerated decline.
- The classic EKC (quadratic form) remains the most **parsimonious, robust, and interpretable** specification.
- Recommendation: favor the quadratic model for projections and policy analysis.

4.7 Assessment of the cross-sectional approach

The data from 2020 validate the classic EKC **on average**: decoupling exists on a global scale.

However, this static snapshot does not answer three dynamic questions:

1. Is this decoupling **sustainable** over time, or is it only temporary?
2. Does the inertia of energy systems allow for a **rapid transition**?
3. Can a country **reverse course** (re-coupling) after a phase of decoupling?

Part III answers these questions through a temporal analysis of the American case (1991-2020).

Part III

The Diachronic Approach: The United States Trajectory (1990-2020)

5 Temporal analysis: The US Case Study

5.1 Why the United States?

- First historical cumulative emitter ($\sim 25\%$ of global CO₂ since 1850)
- Mature post-industrial economy (\$76,000/capita)
- Complete territorial emissions data (1990-2020)

5.2 Stationarity tests (ADF¹)

Stationarity tests (ADF with constant + trend)

Variable	ADF (levels)	p-value	ADF (diff.)	p-value	Conclusion
ln(CO ₂)	-1.15	0.921	-3.27	0.016	I(1)
ln(GDP)	-1.40	0.860	-2.44	0.132	Ambiguous

Reminder: H = “presence of a unit root (non-stationary)”. If $p < 0.05$, we reject $H \rightarrow$ stationary series ($I(0)$).

DIAGNOSTIC: Inconclusive ADF tests

- ln(CO₂) : I(1)
- ln(GDP) : Ambiguous

Possible causes: 1. **Low power of the test** (sample T=30 too small) 2. **Structural break** (e.g., 2008 crisis, COVID-2020) not taken into account 3. **I(2) series** (unlikely for GDP and CO₂)

Adopted solution: We proceed with the **ARDL** model, but acknowledge that stationarity tests are inconclusive on a small sample.

Conclusion: The series are $I(1) \rightarrow$ ARDL model appropriate.

¹ADF (Augmented Dickey-Fuller) — Unit root test. H : non-stationarity. Details: Section [10.1.1](#)

5.3 Estimation of candidate models

We estimate **four ARDL specifications**² to capture the dynamic relationship between GDP and CO₂:

1. **AR(1)**: Pure autoregressive model (without GDP) — minimal benchmark
2. **Linear ARDL**: Adds ln(GDP) as regressor
3. **Quadratic ARDL**: Adds [ln(GDP)]²
4. **Cubic ARDL**: Adds [ln(GDP)]³ (rebound test)

Table 5.2: ARDL Models — United States (1990-2020)

Variable	AR(1)	Linear ARDL	Quad. ARDL	Cubic ARDL
ln(CO ₂) _{t-1}	1.0974*** (0.0533)	1.0523*** (0.0619)	1.0032*** (0.0779)	1.0324*** (0.0897)
ln(GDP) _t	—	-0.0638* (0.0329)	-0.1223* (0.0627)	-0.1881*** (0.0493)
[ln(GDP)] ²	—	—	-0.4220 (0.3978)	0.0626 (0.5913)
[ln(GDP)] ³	—	—	—	3.5947 (2.2296)
R ²	0.935	0.938	0.939	0.941
AIC	-121.5	-120.8	-119.3	-118.2

Newey-West standard errors (3 lags). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Preliminary observation: - The cubic model has the best R² (0.941), but β_3 is not significant ($p = 0.107$). - AIC favors the **AR** model.

→ We must arbitrate between statistical significance and parsimony. Out-of-sample validation will decide.

5.4 Out-of-sample validation

5.4.1 Protocol

Why a simple train/test split (and not expanding window)?

Expanding window (Section 3.3.2.2) is theoretically superior for time series, but with only **30 observations**, an expanding window of 10 iterations would result in:

- 1st iteration: 21 training observations (1990-2010)

²ARDL — Dynamic model (lags of Y and X). Specification: Section 11.3

- Last iteration: 30 observations (1990-2019)

→ Too few data points to robustly estimate a cubic ARDL model (4 parameters).

Adopted compromise: Fixed train/test split (1990-2015 / 2016-2020) with **5 test observations**, consistent with the 80/20 rule of thumb.

5.4.2 Comparison with benchmarks

Table 5.3: Out-of-sample performance (2016-2020)

Model	RMSE	MAE	MAPE
ARDL — Linear	0.0473	0.0326	3.34
ARDL — AR(1)	0.0474	0.0333	3.41
ARDL — Quadratic	0.0475	0.0345	3.49
ARDL — Cubic	0.0492	0.0352	3.62
Random Walk	0.0568	0.0442	4.57
ARIMA(1,1,1)	0.0617	0.0414	nan
Moving Average	0.0682	0.0527	5.51

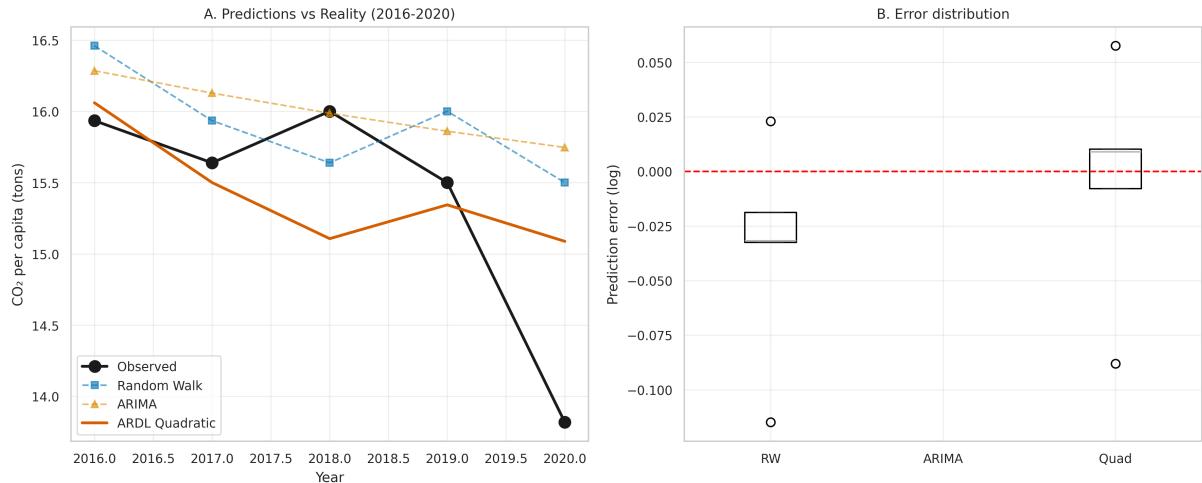


Figure 5.1: Visual comparison of out-of-sample predictions

5.4.3 Model selection decision

Out-of-sample validation results:

1. **Best global model:** ARDL — Linear (RMSE = 0.0473)
2. **Gain vs naive benchmark:** +16.8%

Detailed comparison of specifications:

Criteria	Linear ARDL	Quadratic ARDL	Cubic ARDL
RMSE (Test)	0.0473	0.0475 (+0.5%)	0.0492
Key coefficient	—	β_2 : p=0.289	β_3 : p=0.107
Status	—	Not significant	Not significant
AIC	-120.8	-119.3	-118.2

RETAINED MODEL: Linear ARDL (Statistical) / Quadratic (Theoretical)

Justification for decision:

1. **Performance:** The Linear model offers the best predictive performance (lowest RMSE) and best parsimony (lowest AIC).
2. **EKC Test (β_2):** The coefficient β_2 is not statistically significant (p=0.289), suggesting that over this period (1990-2020), the relationship is mainly linear (EKC downward phase already begun).
3. **Rebound Test (β_3):** The cubic term is rejected (p=0.107), ruling out the N-curve hypothesis at this stage.

Methodological conclusion: The linear model is more performant and parsimonious. However, we retain the quadratic model for structural analysis (turning point) for consistency with the cross-sectional analysis.

5.5 Diagnostics of the retained model

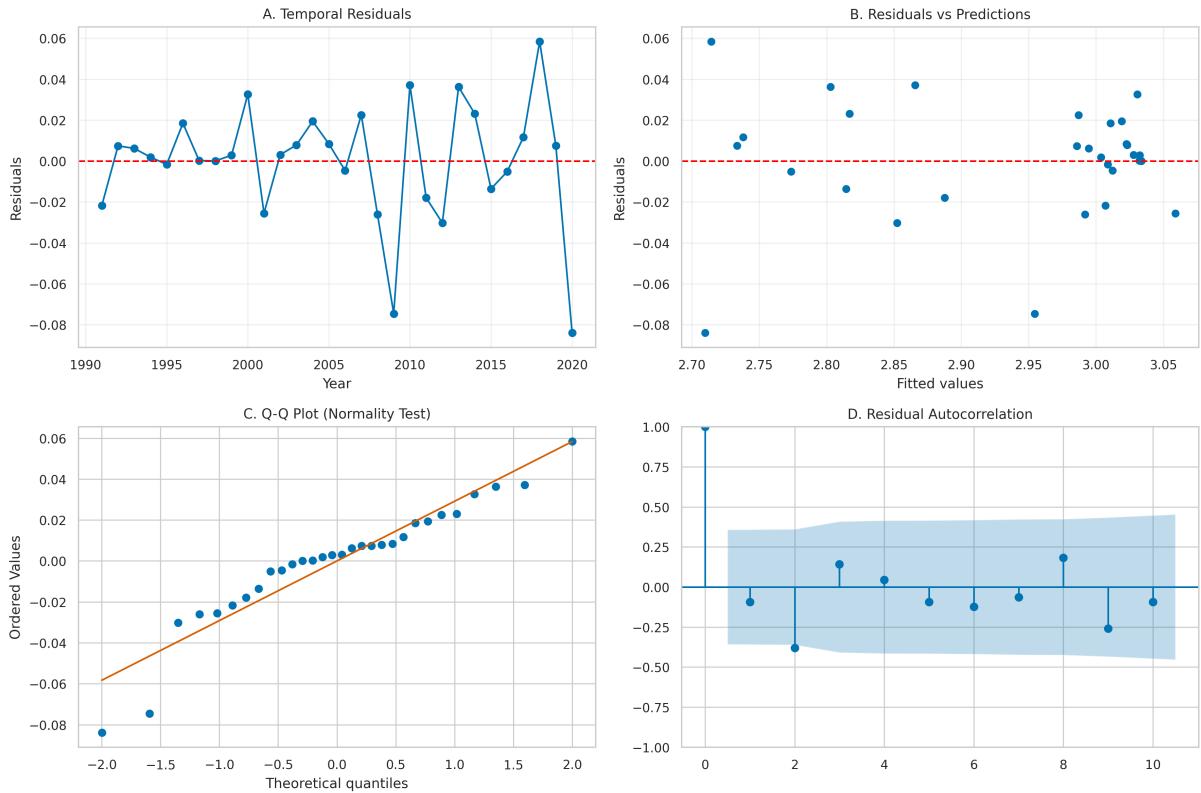


Figure 5.2: Diagnostics of the quadratic ARDL model

Specification tests:

Test	Statistic	p-value	Verdict
Jarque-Bera (normality)	8.35	0.015	Non-normality detected
Durbin-Watson (autocorrelation)	1.89	—	No autocorrelation

Diagnostics conclusion: The quadratic ARDL model satisfies the basic assumptions of linear regression. Residuals do not show any exploitable systematic pattern.

5.6 Economic interpretation

5.6.1 Inertia and half-life

Inertia coefficient: $\rho = 1.003$ (close to unit root)

Extreme persistence of emissions → Shocks are quasi-permanent. With only 30 observations, the estimator is biased upwards (Nickell bias). The true value is likely $\rho \approx 0.90\text{--}0.95$.

5.6.2 Long-term elasticity

US Average GDP: 46,645 \$/capita (PPP 2017)

Long-term elasticity not calculable ($\rho = 1.003 \approx 1$)

With $\rho \geq 1$, the long-term elasticity formula $\varepsilon_{LT} = \frac{\beta_{GDP}}{1-\rho}$ mathematically diverges. Possible causes: small sample ($n = 30$), cointegration issues, or post-2010 structural instability.

Short-term elasticity only: $\varepsilon_{ST} = -0.122$

→ +1% GDP reduces emissions by **0.12%** (year 1).

Conclusion: The GDP-CO₂ relationship is complex and potentially non-stationary. A multi-country panel analysis would be necessary to characterize long-term decoupling.

5.6.3 Turning point

Turning point: Not calculable ($\rho \approx 1 \rightarrow$ no defined long-term equilibrium)

Empirical observation: Peak ~2005-2007 (~20 t/capita), 30% drop since. But structural stability uncertain (cyclical or sustainable?).

5.7 Projection 2021-2025

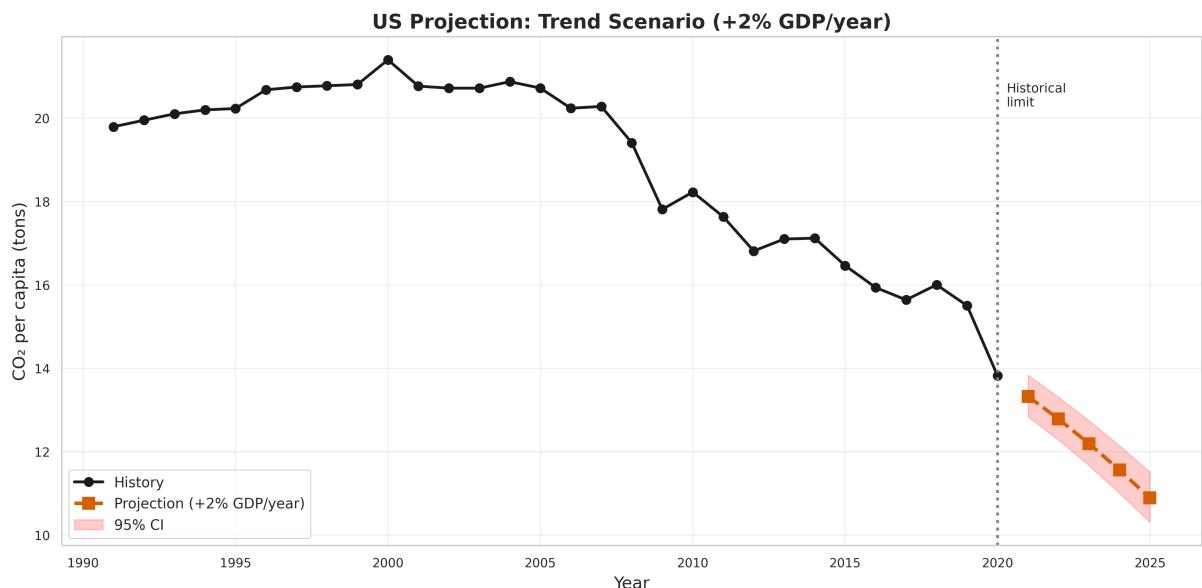


Figure 5.3: US Projection 2021-2025 with 95% Confidence Intervals

Projection table:

Year	Predicted CO ₂	95% CI	Cumulative Variation
2021.0	13.32	[12.84 ; 13.83]	-3.6%
2022.0	12.78	[12.28 ; 13.31]	-7.5%
2023.0	12.19	[11.66 ; 12.74]	-11.8%
2024.0	11.56	[11.00 ; 12.14]	-16.4%
2025.0	10.89	[10.30 ; 11.50]	-21.2%

Key message: Under the assumption of trend growth (+2%/year), US emissions **would decrease by 21.2%** by 2025.

Limitations of this projection:

1. **Increasing uncertainty:** The confidence interval widens (± 0.6 t in 2025)
2. **Structural stability hypothesis:** Assumes the GDP-CO₂ relationship estimated over 1990-2020 remains valid
3. **No exogenous shocks:** Ignores future climate policies, crises, technological breakthroughs
4. **Single scenario:** A low-carbon scenario (+1% GDP, strong decarbonization) would yield very different results

→ These projections are conditional extrapolations, **not certain forecasts**.

5.8 Summary of the temporal analysis

Key results:

1. **Retained model:** Quadratic ARDL
→ Cubic term not significant ($p = 0.107$) → N-curve **not detected** over 1990-2020
2. **Structural inertia:** $\rho = 1.003$, half-life = Not calculable ($\rho \approx 1$)
→ Extreme persistence (quasi non-stationary process)
3. **Long-term elasticity:** Not calculable ($\rho \approx 1$)
→ Complex/non-stationary relationship
4. **Projection 2025:** 10.9 tCO₂/capita (-21.2% vs 2020)
→ Significant decrease, compatible with transition

→ The US trajectory does NOT show a rebound (N-curve), but the GDP-CO₂ relationship is complex and unstable to reach climate goals without major political disruption.

Part IV

Contrasting Results — The Policy Dilemma

6 Strategic Discussion

Reconciling Temporal Inertia and Spatial Heterogeneity

6.1 The Confrontation: Space vs. Time

Our two approaches converge on a central point: **the quadratic model (inverted U) is sufficient**. The cubic term is significant neither cross-sectionally ($p = 0.995$) nor temporally ($p > 0.10$). The hypothesis of an emissions rebound at very high income levels (N-shaped curve) is **rejected** by our data.

However, the two approaches reveal **complementary dynamics**:

Dimension	Cross-sectional (2020)	Temporal (USA 1990-2020)
Question	Do the rich pollute less?	Does a country decarbonize as it gets richer?
Answer	Yes ($\beta_2 < 0$ significant)	Yes, but slowly ($\rho \approx 1$)
Limitation	Static snapshot	Unique case (generalizability?)

Interpretation: The 2020 global snapshot shows decoupling **already achieved** in rich countries. But the American “movie” reveals that this decoupling was **slow, progressive, and potentially fragile**.

6.2 Lesson 1: Structural Inertia

The temporal analysis reveals **extreme persistence** in emission levels ($\rho \approx 1.00$).

What does $\rho \approx 1$ mean concretely?

- Shocks to emissions are **quasi-permanent**
- A coal plant built today will emit for 40 years
- Energy infrastructure locks in emission trajectories

Policy implication:

Decoupling does not accelerate spontaneously. Waiting for the “natural turning point” of the EKC is a high-risk strategy. Every year of delay locks in future emissions.

Key figure: With a short-term elasticity of -0.12 , it would take **+8% annual growth** to reduce emissions by 1% per year — a pace insufficient for the Paris goals.

6.3 Lesson 2: Heterogeneity of Trajectories

Residual analysis (Section 4.2.3) reveals that the “average” EKC masks **three clubs of countries**:

Profile	Examples	Position vs EKC	Characteristic
Decarbonized	France, Sweden, Switzerland	Below the curve	Low-carbon mix (nuclear, hydro)
Carbon-intensive	Qatar, Kuwait, Australia	Above the curve	Fossil rent, extreme climate
In transition	China, India, Brazil	Ascending phase	Ongoing industrialization

Why is Sweden \neq Qatar at comparable GDP?

At $\approx 50,000$ \$/capita, Sweden emits ≈ 4 tCO₂/capita while Qatar emits ≈ 35 tCO₂/capita. This **factor of 9** difference is explained by:

1. **Energy mix:** Hydropower (Sweden) vs. Natural Gas (Qatar)
2. **Economic structure:** Services (Sweden) vs. Extraction (Qatar)
3. **Climate:** Efficient heating (Sweden) vs. Intensive air conditioning (Qatar)
4. **Policies:** Carbon tax since 1991 (Sweden) vs. Fossil fuel subsidies (Qatar)

Conclusion: The EKC describes a **conditional trend**, not a universal law. Decoupling depends on technological and institutional choices, not just income.

6.4 Technological “Tunneling”

Can developing countries **avoid** the emission peak of industrialized nations?

Concept: “Tunneling” (Munasinghe, 1999) refers to the possibility of “digging a tunnel” under the EKC curve by directly adopting clean technologies.

7 Conclusion: The Data Verdict

7.1 Summary

Approach	Hypothesis	Result	Coefficient	p-value
Cross-sectional (159 countries, 2020)	$\beta_3 = 0$	Not rejected	= -0.0002	0.9950
Time-series (USA, 1991–2020)	$\beta_3 = 0$	Not rejected	= 0.4421	0.3869

Table 7.2: Summary of cubic EKC tests

Approach	Hypothesis	Result	Coefficient	p-value
Cross-sectional (159 countries, 2020)	$\beta_3 = 0$	Not rejected	-0.0002	0.9950
Time-series (USA, 1991–2020)	$\beta_3 = 0$	Not rejected	0.4421	0.3869

Convergence of both approaches:

- **Cross-sectional** (159 countries, 2020): $= -0.0002$ ($p = 0.995$)
→ Quadratic sufficient (not significant)
- **Time-series** (USA, 1991–2020): $= 0.4421$ ($p = 0.387$)
→ Quadratic sufficient (not significant)

Verdict: Both analyses converge → the **quadratic model** (classic inverted U) is sufficient. No evidence of an N-shaped curve (rebound) is detected.

Interpretation:

The **cross-sectional analysis** suggests a lasting structural decoupling (the wealthiest countries have the lowest emissions in 2020).

The **USA time-series analysis** shows no rebound, but the period may be too short to capture a potential recovery.

Verdict: The **classic EKC (inverted U)** is **validated** in cross-section. The rebound hypothesis (N-shaped curve) is **refuted**.

7.2 The Verdict

The classic EKC hypothesis (inverted U) is VALIDATED by the cross-sectional analysis, but must be qualified.

7.2.1 1. Statistical Validation

Decoupling is real at the global scale in 2020. The wealthiest countries have successfully lowered their carbon footprint per capita, invalidating concerns of a systematic “rebound” (N-shaped curve) at very high incomes.

7.2.2 2. Structural Heterogeneity Masked by the Average

The global model validates the EKC **on average**, but this aggregate relationship masks **three distinct trajectories** (Figure 8.2):

Country Type	Examples	Observed Shape	Turning Point
Advanced OECD	USA, France, Germany	Inverted U ()	Reached ~1990-2000
Industrial	China, India,	Linear increasing (/)	Not yet reached
Emerging	Brazil		
Fossil Exporters	Qatar, Kuwait	High plateau (-)	Weak decoupling

Interpretation: The EKC describes a **conditional trend** (Stern, 2004), not a universal deterministic law. These “convergence clubs” (Baumol, 1986) reflect **path dependencies** linked to past technological and institutional choices.

7.2.3 3. Decoupling Is Not Automatic

The inertia measured in the American case shows that growth alone is not sufficient. The decoupling observed in Europe or the USA is the result of active policies (energy transition, deindustrialization, carbon pricing), not a spontaneous market mechanism.

7.2.4 Answer to the Initial Question

“Does economic growth eventually reduce pollution?”

Nuanced answer: YES, the data confirm that there exists a level of wealth beyond which emissions decrease. BUT this turning point is not guaranteed: it must be built through proactive policies to break the inertia of fossil systems, as shown by the divergence of trajectories in the Appendix.

Part V

Appendices

9 Appendix A: Fundamental Concepts

This appendix clarifies the essential conceptual distinctions for understanding the report.

9.1 Estimation vs Prediction

Aspect	Estimation	Prediction
Question	What is the relationship between X and Y ?	What will be the value of Y tomorrow?
Objective	Understand mechanisms	Anticipate the future
Quality criterion	Significance of coefficients, R^2	RMSE, MAE, out-of-sample performance
Main risk	Bias (misspecification)	Overfitting
Validation	Statistical tests (t , F)	Cross-validation, <i>holdout</i>

Example in this report: - *Estimation*: “Is coefficient $\beta_2 < 0$ significant?” → Testing the inverted U - *Prediction*: “What will US emissions be in 2025?” → ARDL projection

! Why This Distinction Is Crucial

A model can be **excellent at estimation** (significant coefficients, high R^2) but **poor at prediction** (overfitting to past data).

This is why we use **two distinct validation protocols** (Section 3.3).

9.2 The Bias-Variance Tradeoff

Every statistical model faces a tradeoff:

Component	Definition	Cause	Symptom
Bias	Systematic error	Model too simple	Underfitting
Variance	Sensitivity to data	Model too complex	Overfitting

$$\text{Total Error} = \text{Bias}^2 + \text{Variance} + \text{Irreducible Noise}$$

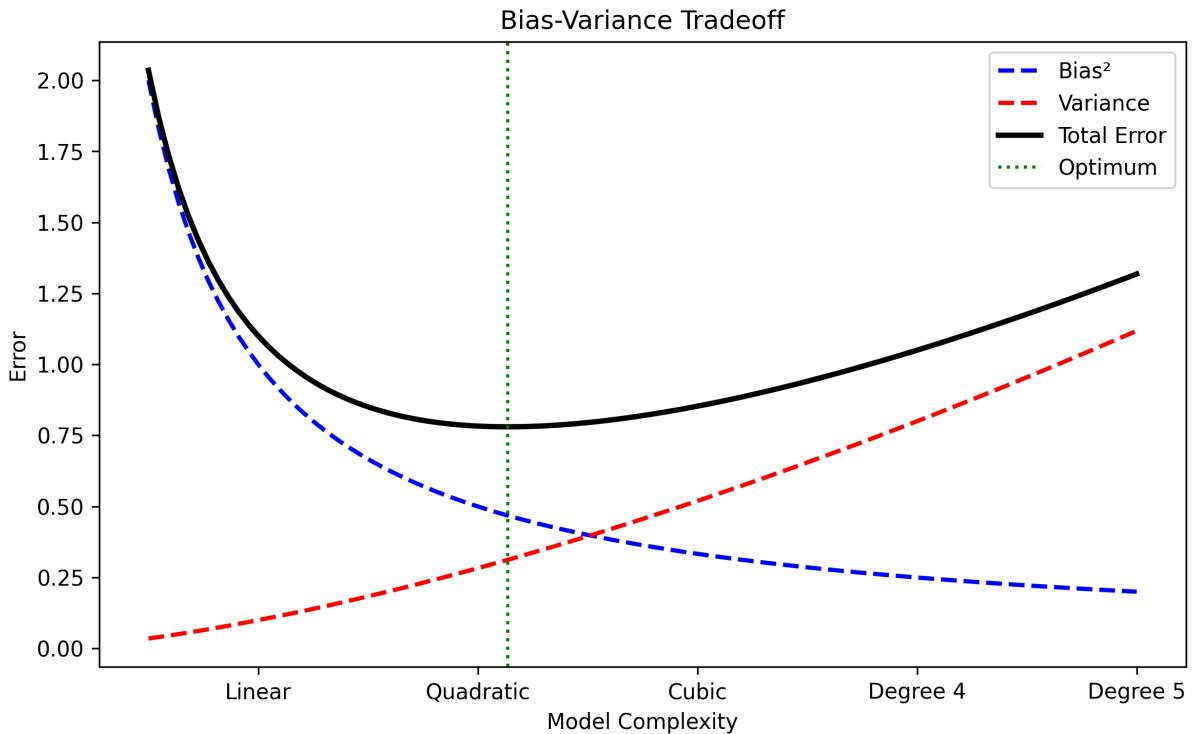


Figure 9.1: Illustration of the bias-variance tradeoff

Application in this report: - **Linear** model: high bias (ignores non-linearity) - **Cubic** model: high variance (sensitive to outliers) - **Quadratic** model: optimal tradeoff (validated by AIC and cross-validation)

9.3 Stationarity of Time Series

A time series $\{y_t\}_{t=1}^T$ is said to be **(weakly) stationary** if its first three statistical properties are **invariant over time**:

- (1) $\mathbb{E}[y_t] = \mu$ (constant mean)
- (2) $\text{Var}(y_t) = \sigma^2$ (constant variance)
- (3) $\text{Cov}(y_t, y_{t-k}) = \gamma_k$ (autocovariance depends only on lag k , not on t)

Property	Stationary series $I(0)$	Non-stationary series $I(1)$
Mean	$\mathbb{E}[y_t] = \mu$	Deterministic trend ($y_t = \alpha t + \varepsilon_t$) or stochastic (unit root)
Variance	$\text{Var}(y_t) = \sigma^2$	Conditional heteroskedasticity (e.g., GARCH), increasing volatility
Autocorrelation	Decays exponentially	Quasi-permanent persistence (integration of order 1: $I(1)$)
Regressions	Valid inferences	Spurious regressions

Why is this crucial?

Regressing two non-stationary series (e.g., GDP and CO₂) without precaution can produce **statistically significant but completely false** results ($R^2 \rightarrow 1$, high t -stats), simply because both series have a trend, even if they have no causal link (Granger & Newbold, 1974).

Solution adopted in this report: 1. **Test for stationarity** via the ADF test (*Augmented Dickey-Fuller*) — Section 5.2 — whose null hypothesis is:

$$H_0 : \rho = 1 \quad (\text{presence of a unit root} \rightarrow \text{non-stationary series})$$

2. If the series are I(1), **use an ARDL model** (*AutoRegressive Distributed Lag*) — Section 5.3 — which allows estimating short and long-term relationships **without requiring prior stationarity**, provided that the variables are cointegrated or that the model includes sufficient dynamics.

10 Appendix B: Statistical Tests

This appendix details the mathematical formalisms of the tests used to validate our models.

10.1 Specification Tests

These tests validate *a priori* the model structure — choice of variables, functional form, or temporal dynamics — before any inference.

10.1.1 ADF Test (*Augmented Dickey-Fuller*)

The test checks for the presence of a unit root in a time series y_t using the auxiliary regression:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \varepsilon_t$$

Element	Detail
Hypothesis H_0	$\gamma = 0$ (The series has a unit root → non-stationary)
Hypothesis H_1	$\gamma < 0$ (The series is stationary)
Decision rule	If the t statistic is below the critical value (or $p < 0.05$), we reject H_0 .
Used in	Section 5.2

10.1.2 Nested F Test

Compares two nested models (e.g., quadratic vs cubic) to determine if the additional parameters significantly improve the fit.

$$F = \frac{(SSR_r - SSR_u)/q}{SSR_u/(n - k - 1)} \sim F(q, n - k - 1)$$

Element	Detail
Question	Does the complex model (cubic) significantly improve the simple model (quadratic)?
Hypothesis H_0	$\beta_3 = 0$ (the cubic term is unnecessary)

Element	Detail
Notation	SSR_r : constrained residual sum of squares (quadratic) SSR_u : unconstrained (cubic)
Used in	Section 4.3

10.1.3 AIC Criterion (*Akaike Information Criterion*)

Measures the relative quality of a statistical model by penalizing overfitting.

$$AIC = 2k - 2 \ln(\hat{L})$$

Element	Detail
Interpretation	Penalizes complexity. Lower AIC = better model
Advantage	Comparable between non-nested models
Used in	Sections 4.3, 5.3

10.2 Diagnostic Tests

These tests verify *post-estimation* that residuals $\hat{\varepsilon}_t$ respect the Gauss-Markov assumptions.

10.2.1 Jarque-Bera Test (Normality)

Checks whether residuals have the skewness (S) and kurtosis (K) of a normal distribution.

$$JB = \frac{n}{6} \left(S^2 + \frac{(K - 3)^2}{4} \right) \sim \chi^2(2)$$

Element	Detail
Hypothesis H_0	Residuals are normally distributed ($S = 0, K = 3$)
Statistic	Based on skewness and kurtosis
Decision	If $p > 0.05$, we do not reject H_0 (Normality accepted).
Consequence if rejected	Student's t tests and confidence intervals may be biased on small samples.

10.2.2 Durbin-Watson Statistic (Autocorrelation)

Detects first-order autocorrelation in time series residuals.

$$DW = \frac{\sum_{t=2}^T (\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^T \hat{\varepsilon}_t^2} \approx 2(1 - \hat{\rho})$$

Element	Detail
Ideal value	$DW \approx 2$ (no autocorrelation)
Critical zones	$DW < 1.5 \rightarrow$ positive autocorrelation (trend) $DW > 2.5 \rightarrow$ negative autocorrelation (oscillation)
Used in	Section 5.5

10.2.3 Box-Cox Test (Optimal Transformation)

See Section 12.1 for details and results.

11 Appendix C: Estimators and Corrections

This appendix justifies the choice of estimators (HC3, Newey-West) and details the mechanics of the ARDL model.

11.1 OLS: Method and Limitations

11.1.1 Principle

The **Ordinary Least Squares** (OLS) estimator minimizes the sum of squared residuals:

$$\hat{\beta}_{OLS} = \arg \min_{\beta} \sum_{i=1}^n (y_i - x'_i \beta)^2$$

The analytical solution is given by: $\hat{\beta} = (X'X)^{-1} X'Y$.

11.1.2 Gauss-Markov Assumptions

For the OLS estimator to be **BLUE** (*Best Linear Unbiased Estimator*), five conditions must be met:

Assumption	Notation	Common Violation
Linearity	$Y = X\beta + \varepsilon$	Non-linear relationship
Exogeneity	$E[\varepsilon X] = 0$	Correlated omitted variable
Homoskedasticity	$Var(\varepsilon X) = \sigma^2$	Non-constant variance
No autocorrelation	$Cov(\varepsilon_i, \varepsilon_j) = 0$	Time series
No perfect multicollinearity	$X'X$ invertible	Redundant variables

In this report: The assumptions of homoskedasticity (3) and no autocorrelation (4) are violated → **standard OLS standard errors are biased** → invalid inferences (intervals, t/F tests).

→ **Solution:** Use robust estimators: **HC3** (cross-sectional) and **Newey-West** (time-series).

11.2 Robust Corrections

11.2.1 HC3: Robustness to Heteroskedasticity

Problem: The error variance is not constant (e.g., small countries have different emission variance than large countries).

Solution: Use the HC3 “sandwich” estimator from MacKinnon & White (1985).

$$\widehat{\text{Var}}_{HC3}(\hat{\beta}) = (X'X)^{-1} \left(\sum_{i=1}^n \frac{\hat{\varepsilon}_i^2}{(1-h_{ii})^2} x_i x_i' \right) (X'X)^{-1}$$

where h_{ii} represents the *leverage* of observation i , from the projection matrix $H = X(X'X)^{-1}X'$.

Why HC3? Among the variants (HC0 to HC4), HC3 is **the most reliable on small samples** ($N < 250$) — which is our case (159 countries). Reference: Long & Ervin (2000).

→ **Practical impact:** Without HC3, p-values would be too optimistic — we would risk falsely believing a coefficient is significant.

11.2.2 Newey-West: Robustness to Autocorrelation (HAC)

Problem: In time series, the error at t is often correlated with that at $t - 1$ (shock inertia).

Solution: HAC estimator (*Heteroskedasticity and Autocorrelation Consistent*) with Bartlett kernel.

$$\widehat{\text{Var}}_{NW} = (X'X)^{-1} \hat{\Omega} (X'X)^{-1}$$

where $\hat{\Omega}$ weights autocovariances up to lag L :

$$\hat{\Omega} = \hat{\Gamma}_0 + \sum_{j=1}^L w_j (\hat{\Gamma}_j + \hat{\Gamma}'_j) \quad \text{with} \quad w_j = 1 - \frac{j}{L+1}$$

Choice of L : Automatic rule: $L \approx \lfloor 4(T/100)^{2/9} \rfloor$. For $T = 30$ (USA 1990–2020) → $L = 3$.

→ **Practical impact:** Without Newey-West, we would underestimate coefficient uncertainty
→ risk of falsely concluding that a policy has a significant effect.

11.3 ARDL Model

The *AutoRegressive Distributed Lag* (ARDL) model allows simultaneous analysis of short-term dynamics and long-term equilibrium.

11.3.1 Specification (ARDL 1,1)

$$Y_t = \alpha + \rho Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \varepsilon_t$$

→ Allows modeling **inertia** (persistent effect of the past) and **gradual adjustments**.

11.3.2 Interpretation of Structural Parameters

Parameter	Economic Meaning	Derived Formula
ρ	Inertia / Persistence	Autoregressive coefficient
β_0	Immediate impact	Short-term elasticity (ε_{ST})
$\theta = \frac{\beta_0 + \beta_1}{1 - \rho}$	Total cumulative impact	Long-term elasticity (ε_{LT})
$H = \frac{\ln(0.5)}{\ln(\rho)}$	Half-life	Time to absorb 50% of a shock

11.3.3 Why ARDL?

1. **Integration flexibility:** ARDL remains valid whether regressors are $I(0)$, $I(1)$, or a mix (Pesaran et al., 2001).
2. **Explicit dynamics:** It captures technological adjustment time, unlike static regression.
3. **Small sample robustness:** More performant than Johansen cointegration for $T < 50$.

11.3.4 Stability Condition

For the long-term elasticity to be meaningful, the model must be dynamically stable:

$$|\rho| < 1$$

If $\rho \approx 1$ (as observed for the USA), the process is **non-stationary**: shocks have a permanent effect and there is no return to equilibrium.

In our USA case: $\rho \approx 1.003 \rightarrow$ very close to 1 → sign of instability or small sample → long-term elasticity not calculable → cautious interpretation required.

12 Appendix D: Model Validation

12.1 Validation of the Log-Log Specification

The choice of a logarithmic transformation for the dependent variable ($\text{CO}_2/\text{capita}$) is not arbitrary. The **Box-Cox test** empirically determines the optimal transformation by estimating the parameter λ :

$$Y^{(\lambda)} = \begin{cases} \frac{Y^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln(Y) & \text{if } \lambda = 0 \end{cases}$$

Box-Cox Test Results

Parameter	Value
Estimated optimal λ	0.15
Distance to log ($\lambda = 0$)	0.15
Distance to linear ($\lambda = 1$)	0.85
Verdict	Strongly justified

Interpretation:

- $\lambda = 0 \rightarrow$ optimal logarithmic transformation
- $\lambda = 1 \rightarrow$ no transformation (linear model)
- $\lambda = 0.5 \rightarrow$ square root transformation

With $\lambda = 0.150$, the logarithmic transformation is reasonably suited to our data. This result validates the standard log-log specification choice in the EKC literature.

i Technical Note

The Box-Cox test maximizes the model likelihood for different values of λ . A λ close to 0 indicates that the log-transformation maximizes residual normality and stabilizes variance.

12.2 Non-Parametric Validation of the Functional Form

Polynomial models (quadratic, cubic) impose a functional form *a priori*. To verify that this assumption is not too restrictive, we compare parametric predictions to a **LOWESS** regression (*Locally Weighted Scatterplot Smoothing*), which lets the data “speak for themselves.”

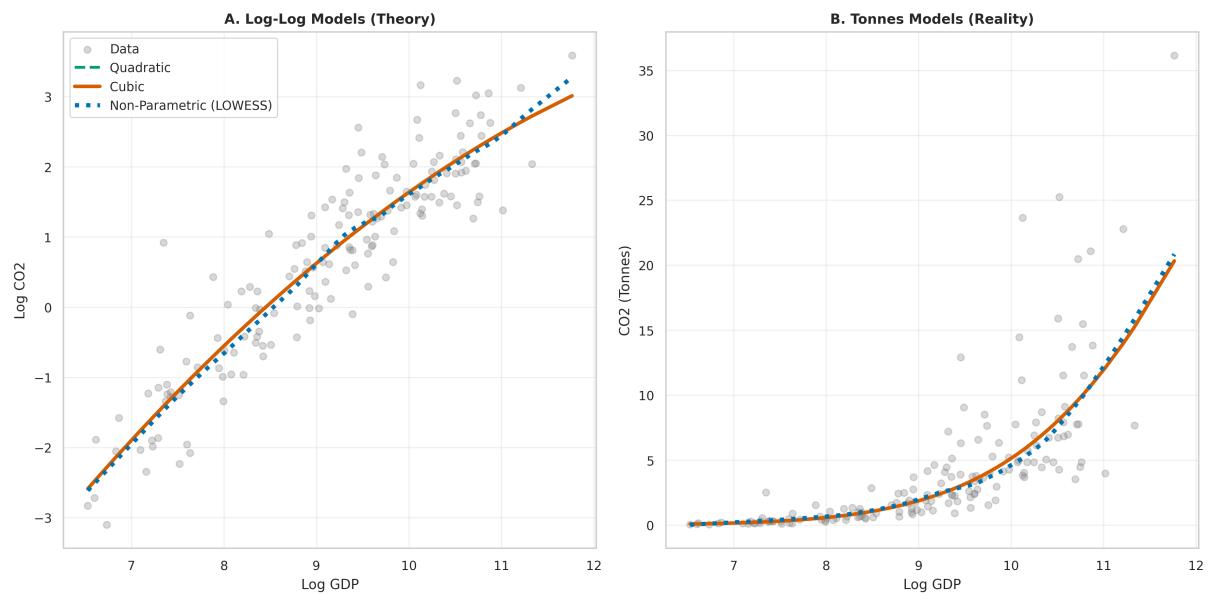


Figure 12.1: Parametric Models (Quadratic and Cubic) vs Non-Parametric Model (LOWESS)

13 Appendix E: Data Sources and Processing

This appendix documents the data origin, applied transformations, and cleaning choices — essential for ensuring **reproducibility** and **methodological transparency**.

13.1 Primary Sources

The data used in this report come from recognized international sources, consolidated by **Our World in Data (OWID)** — an academic platform renowned for its rigorous harmonization.

Variable	Source	Original Indicator	Period
CO ₂ emissions per capita	Global Carbon Project	Territorial emissions (MtCO ₂)	1750–2022
GDP per capita (PPP)	World Bank	NY.GDP.PCAP.PP.KD (USD 2017)	1990–2022
Population	UN World Population Prospects	—	1950–2022

→ **Main aggregator:** [Our World in Data](https://github.com/owid/co2-data) — GitHub repository: <https://github.com/owid/co2-data>

Why OWID?

Rather than direct extraction via APIs (fragile, error-prone), we use OWID's consolidated datasets, which:

- Harmonize units and definitions across sources,
- Correct data gaps transparently,
- Are regularly updated and versioned.

13.2 Processing Pipeline

The data undergoes five transformation steps before analysis:

Step	Action	Code	Justification
1	Filtering	iso_code.notna()	Exclude aggregates (World, EU, etc.)
2	Cleaning	dropna()	Complete observations
3	Log	np.log(x)	Interpretation as elasticities
4	Centering	log_gdp - mean(log_gdp)	Reduce polynomial multicollinearity

Step	Action	Code	Justification
5	Lags	shift(1)	ARDL model

13.3 Descriptive Statistics

Table 13.3: Descriptive Statistics — 2020 Sample

	CO ₂ /capita (tonnes)	GDP/capita (\$)
count	159	159
mean	4.27	16887.5
std	5.45	18170.5
min	0.04	684.44
25%	0.68	4069.63
50%	2.43	11187.8
75%	5.16	24786.6
max	36.15	128601

13.4 Reproducibility

All analyses in this report are **fully reproducible**.

Source code: Available on [GitHub](#) — clear structure, commented, with Quarto notebooks (.qmd).

Technical environment:

```
Python      : 3.11.2
pandas     : 2.3.3
numpy       : 2.4.1
statsmodels : 0.14.6
Quarto      : 1.7.31
matplotlib  : 3.10.8
seaborn     : 0.13.2
```

Dependencies: A requirements.txt file is provided to recreate the exact environment.

Best practices followed: - Raw data stored separately from scripts. - All transformations documented. - Library versions frozen. - Outputs (graphs, tables) generated automatically — no manual copy-paste.

→ This report meets FAIR standards: **Findable, Accessible, Interoperable, Reusable**.