

# Lecture 4

## Linked List

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# Our Roadmap

前面 (AA)

linked list (DS)

## ◆ Linked List Definition

linked list, map, Vector [封装好的容器]  
↳ array 类似

## ◆ Linked List Operators

array:  
① item[i]  
② very compact

## ◆ Illustration Example

# Representing a Sequence of Data

- ◆ An ordered collection of items (position matters)
  - ◆ Array, lists, stacks, and queues
- ◆ What did you study before? Array!
- ◆ Advantages of using an array
  - ◆ Easy and efficient access to any item in the sequence
  - ◆ `item[i]`: return the i-th element in array item
  - ◆ Every item can be accessed in constant time
  - ◆ This feature of arrays is known as “random access”
  - ◆ Very compact (in terms of memory)
- ◆ Disadvantages of using an array ?

# Disadvantages of an Array

- ◆ Have to specify an initial array size
- ◆ Resize an array is possible, but not so easy
- ◆ Difficult to insert/delete elements at arbitrary positions
  - ◆ Delete 10 in array A, time complexity?

/vector 尽管你声明  
但构造新数组帮你  
补充说明。

任意

$O(n)$

<b>A</b>	5	8	10	13	16	19	27	46	51	86
----------	---	---	----	----	----	----	----	----	----	----

<b>A</b>	5	8	✓	13	16	19	27	46	51	86
----------	---	---	---	----	----	----	----	----	----	----

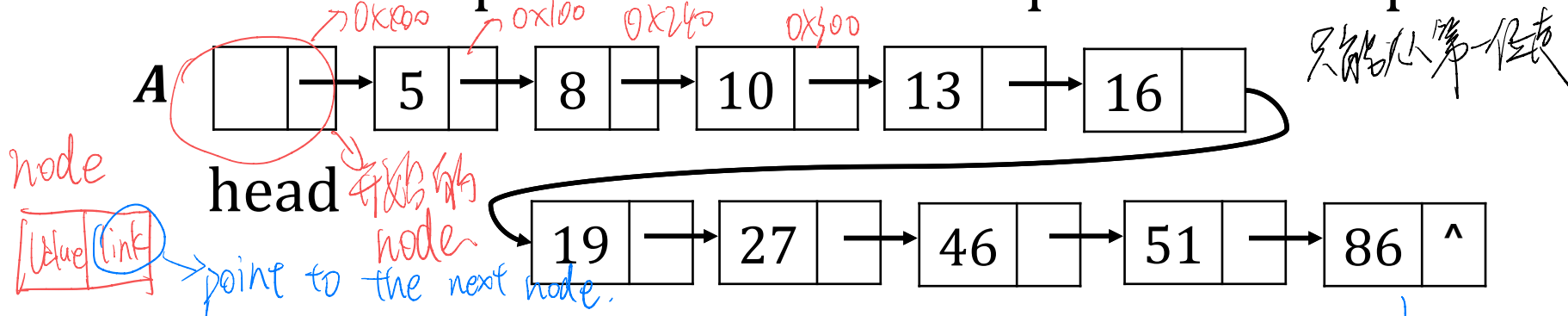
not used memory

<b>A</b>	5	8	13	16	19	27	46	51	86	
----------	---	---	----	----	----	----	----	----	----	--

# A Linked List

访问16, 要访问13  
一步步往前走, 无法跳过  
只能从第一个开始

## Alternative Representation of a sequence. Example:



- ◆ A linked list stores a sequence of elements in separate nodes
- ◆ Each node contains: a single item, a “link” to the node containing the next item: 

13	→
----	---
- ◆ The last node in the linked list has a link value of “NULL”: 

86	^
----	---
- ◆ The linked list as a whole is represented by a variable that hold a reference to the first node (e.g., **A**)

# Array vs. Linked List in Memory

- ◆ In an array, the elements occupy consecutive memory locations:

5	8	10	13	16	19	27	46	51	86
0x100	0x104	0x108	0x112	0x116	0x120	0x124	0x128	0x132	0x136

**A**

0x320

0x320

- ◆ In linked list, each node is a distinct object. The nodes do NOT have to be next to each other in memory. That's why we need the links to get from one node to the next.

0x100

8

0x480

...

...

0x240

13

0x640

...

...

5

0x100

...

...

0x480

10

0x240

...

...

0x640

16

0x800

.....

...

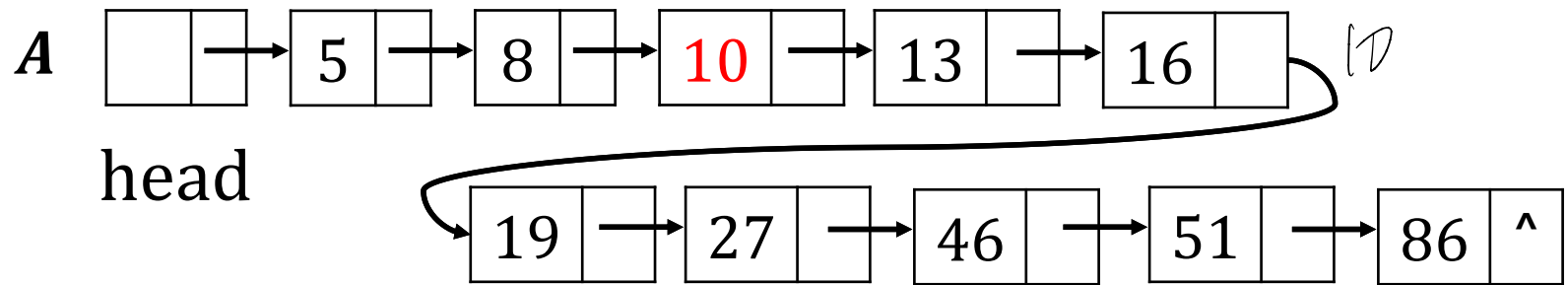
# Features of Linked List

- ◆ It can grow without limit (not fixed length)

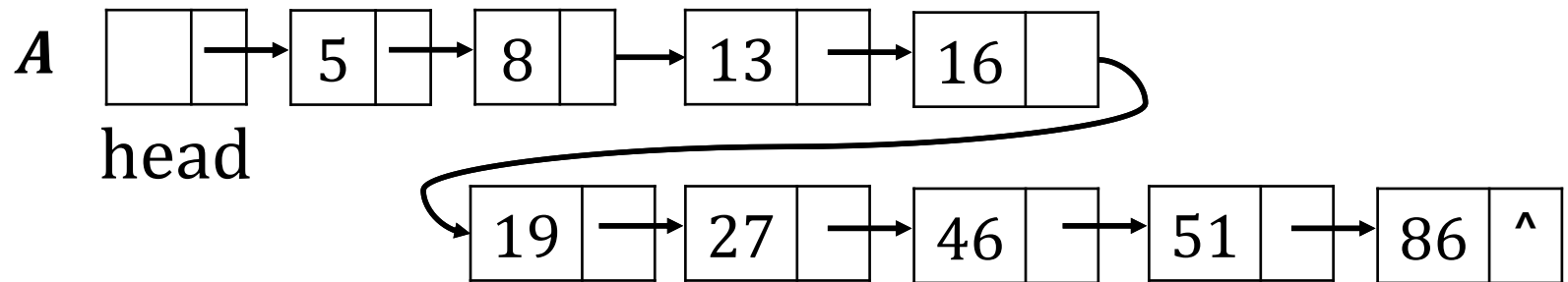
- ◆ Easy to insert/delete an element

- ◆ Delete 10 in Linked List A, before:

找到10, 将10前指10的, (让8指向13), 删除10

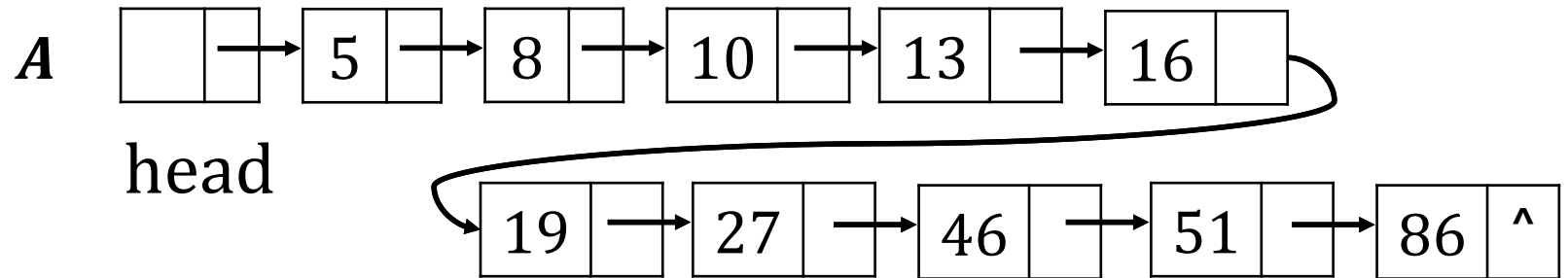


- ◆ After:

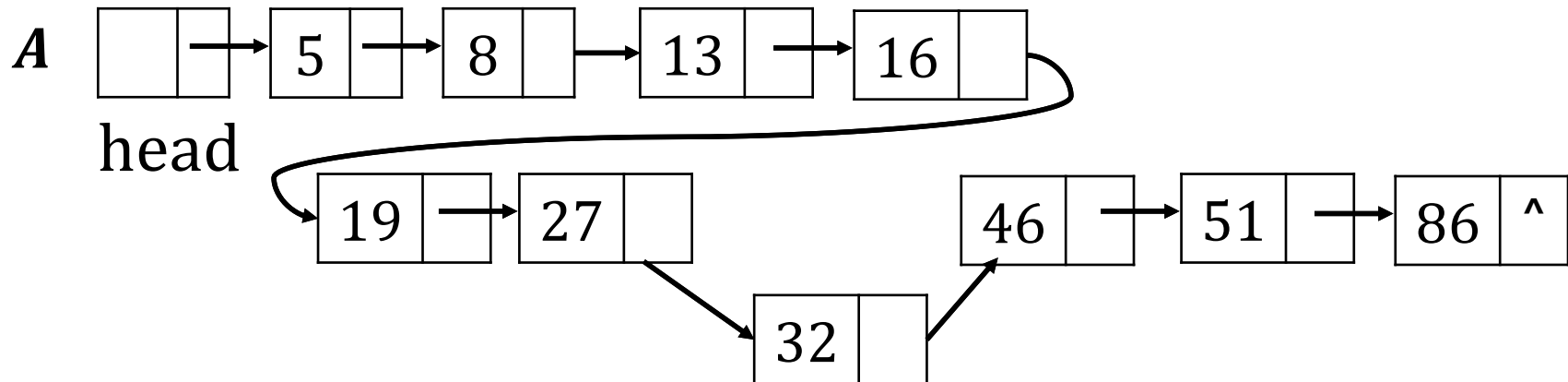


# Features of Linked List

- ◆ Insert 32 in Linked List A, before:



- ◆ After:



- ◆ Time Complexity?



# Features of Linked List

## ◆ Disadvantages of Linked List

- ◆ They do not provide random access
- ◆ Need to “walk down” the list to access an item
- ◆ The links take up additional memory
- ◆ Not compact (in terms of Memory)

## ◆ Linked List vs. Array

- ◆ Space complexity
- ◆ Time Complexity: Insert, Delete, Find

basic operators

$P, q \in P$

Memory

LL non-continuous

Arr continuous

node size

value pointer

value

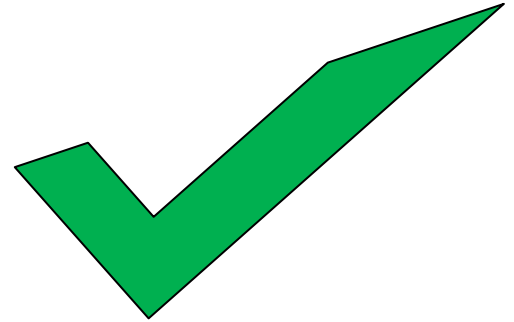
Access

walk down

Random

# Our Roadmap

- ◆ Linked List Definition
- ◆ Linked List Operators
- ◆ Illustration Example

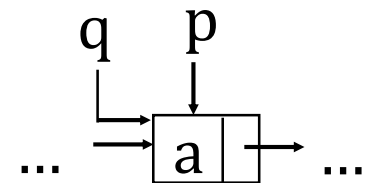
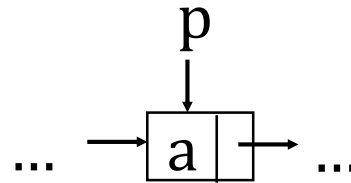


# Basic Operators of Linked List

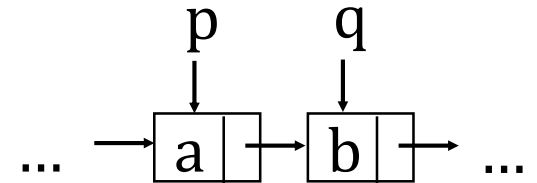
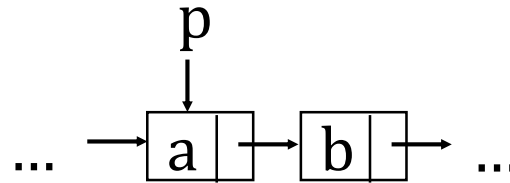
Before

After

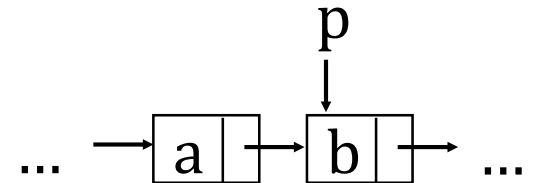
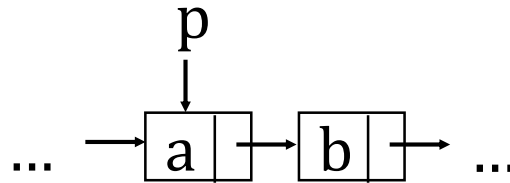
①  $q \leftarrow p$



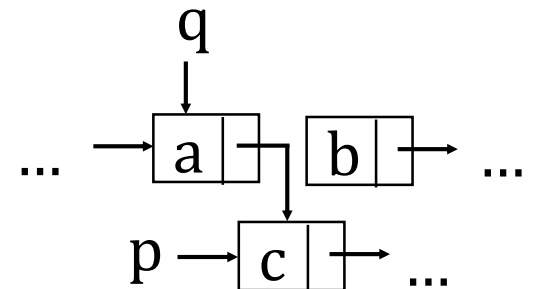
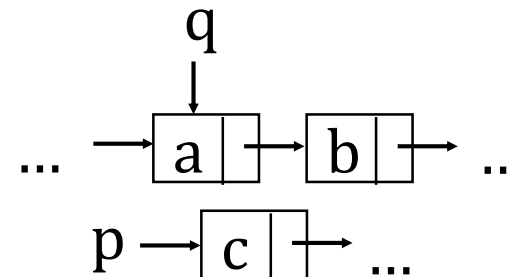
②  $q \leftarrow \text{next of } p$



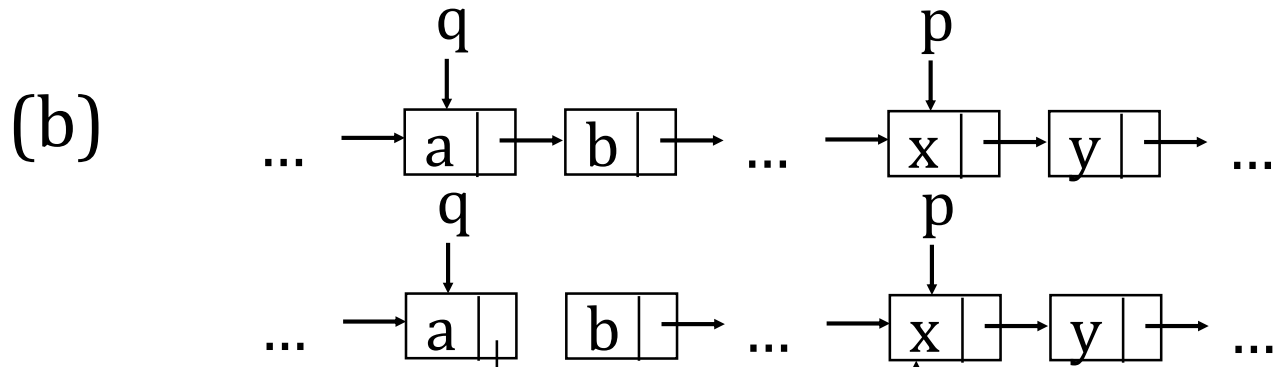
③  $p \leftarrow \text{next of } p$



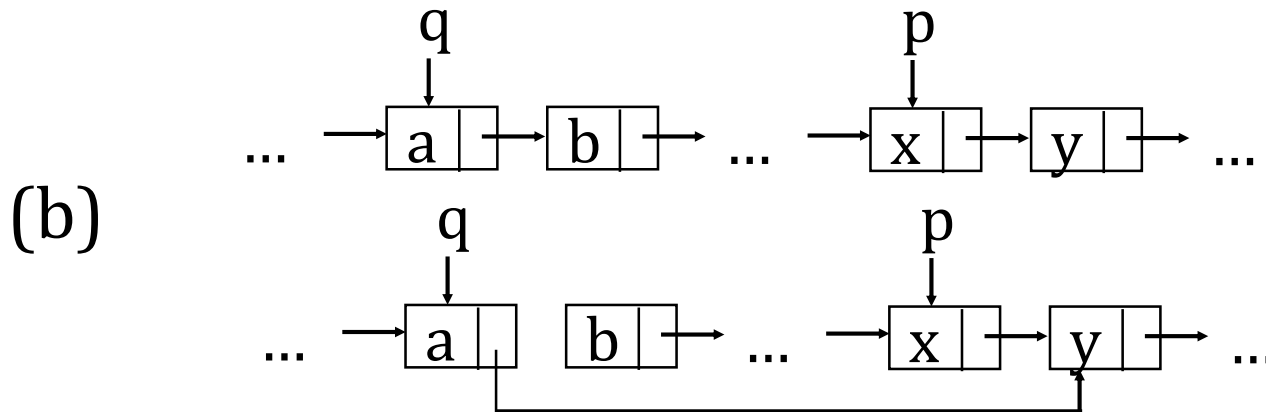
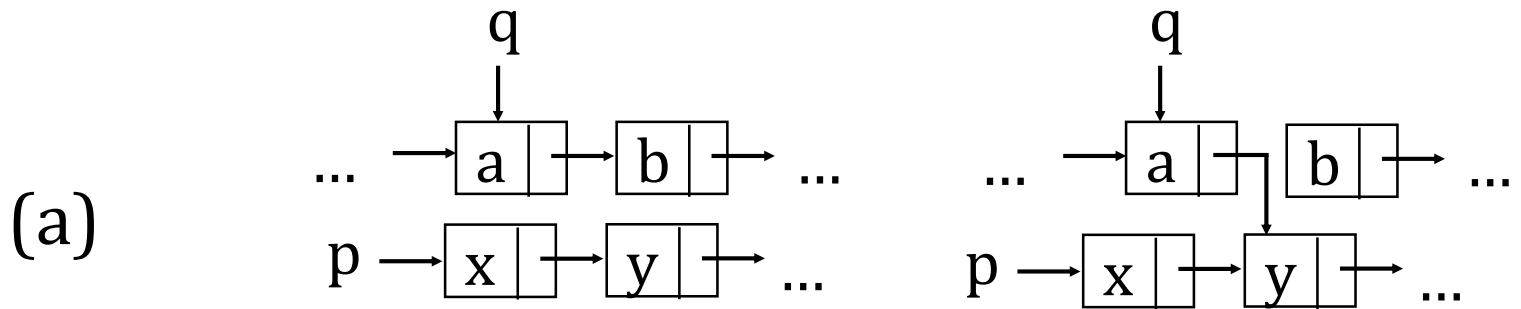
④ next of  $q \leftarrow p$   
(a)



# Basic Operators of Linked List



⑤ next of q  $\leftarrow$  next of p



# Traverse a Linked List

- Many tasks require us to traverse or “walk down” a linked list

- Recursion Pseudocode

- Algorithm:** traverse(A):

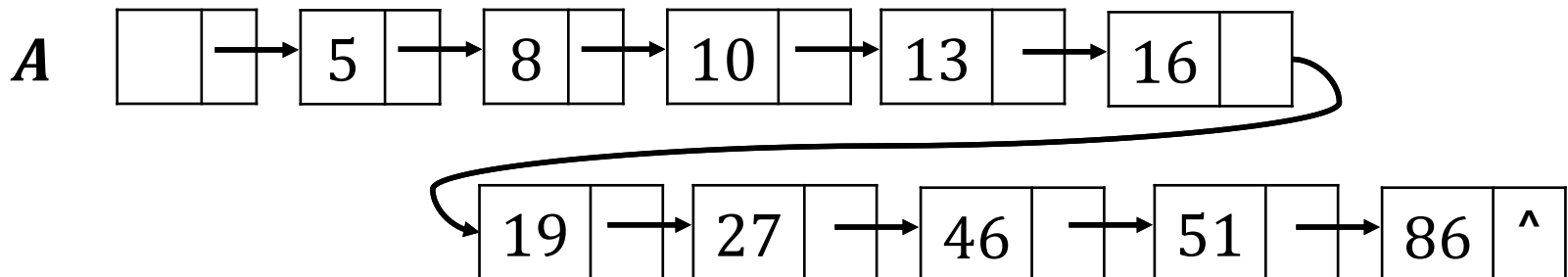
```
1. if (A=NULL)
2.     return
3. else
4.     print A.value
5.     traverse(A.next)
```

5. temp ← p.next

6. p.next ← q

7. q.next ← temp

8. return

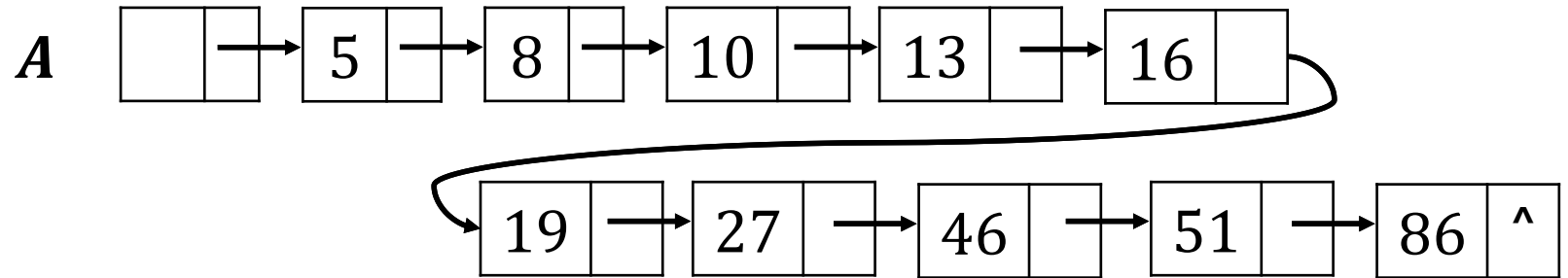


# Traverse a Linked List

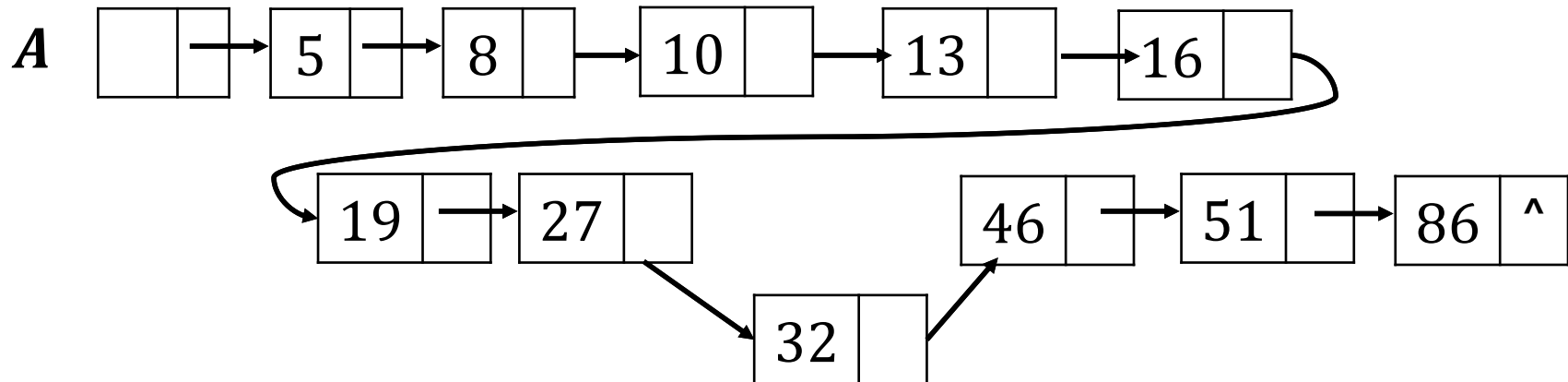
- ◆ It can also be done using iteration (for loops, while loops, etc.)
- ◆ Iteration Pseudocode
- ◆ **Algorithm:** `traverseIteration(A)`:
  1. `node trav ← A`
  2. `While (trav != NULL)`
  3.     `print trav.value`
  4.     `trav ← trav.next`
- ◆ We use iteration in the following operators, but you can try to use recursion to implement these operators.

# Inserting an Item at Position i

- ◆ Insert 32 in Linked List A at position 8, before:



- ◆ After:



- ◆ How to do that?

# Inserting an Item at Position i

◆ **Problem:** insert node q in Linked List A at Position i

◆ **Algorithm:** insertNode(A, node q, i):

1.  $a \leftarrow 0$ , node  $p \leftarrow A$ ,

2. while ( $i-1 > a$ )

3.  $p \leftarrow p.next$

4.  $a \leftarrow a + 1$

5.  $tmp \leftarrow p.next$

6.  $p.next \leftarrow q$

7.  $q.next \leftarrow tmp$

8. return A

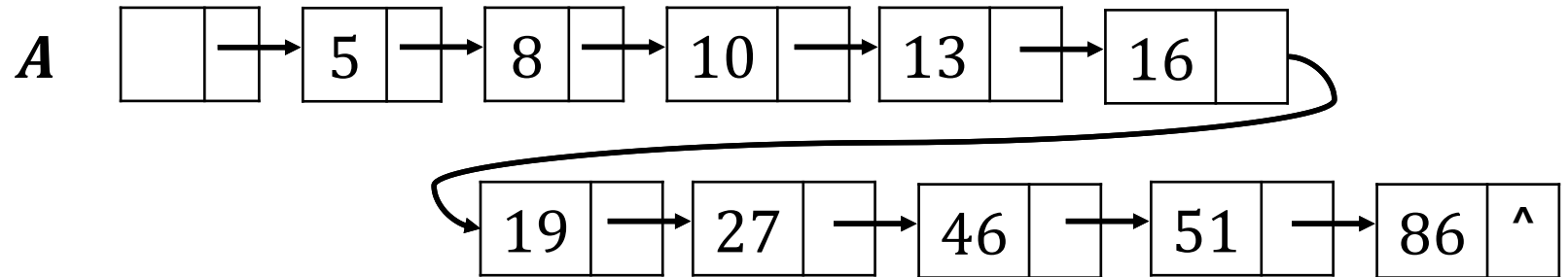
◆ Time Complexity:  $O(n)$

◆ Space Complexity:  $O(1)$

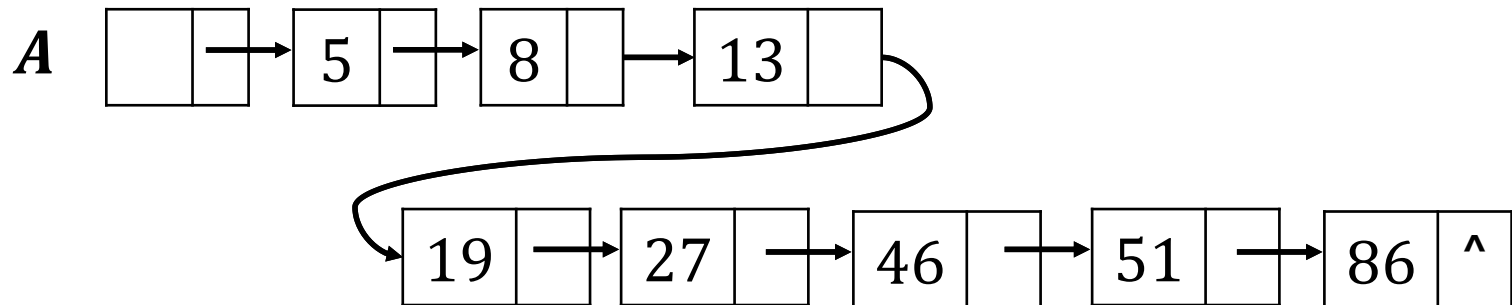


# Deleting an Item at Position i

- ◆ Delete position 5 in Linked List A, before:



- ◆ After:

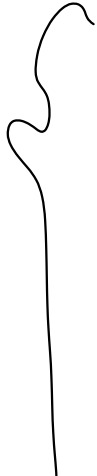


- ◆ How to do that?

# Deleting an Item at Position i

◆ **Problem:** delete node in Linked List A at Position i

◆ **Algorithm:** deleteNode(A, i):



1.  $a \leftarrow 0$ , node  $p \leftarrow A$ ,
2. **while** ( $i-1 > a$ )
3.            $p \leftarrow p.next$
4.            $a \leftarrow a + 1$
5.  $p.next \leftarrow p.next.next$
6. **return** A

◆ Time Complexity:  **$O(n)$**

◆ Space Complexity:  **$O(1)$**

# Finding an Item at Position i

◆ **Problem:** Find value x in Linked List A

◆ **Algorithm:** findValue(A, x):

1.  $a \leftarrow 0$ , node  $p \leftarrow A$ ,
2. **while** ( $p \neq \text{NULL}$ )
4.       **if** ( $x = p.\text{value}$ )
5.               **return** p
6.        $p \leftarrow p.\text{next}$
7. **return** -1

◆ **Time Complexity:**  $O(n)$

◆ **Space Complexity:**  $O(1)$


DeleteNode(node A, i)

1.  $a \leftarrow 0$ , node  $p \leftarrow A$
2. **while** ( $i \neq a$ )
3.        $p \leftarrow p.\text{next}$
4.        $a \leftarrow a + 1$
5.        $p.\text{next} \leftarrow p.\text{next}.\text{next}$   
(没删干净)(干净的:  $\text{temp} \leftarrow p.\text{next};$   
free temp).
- 6.

# Updating an Item at Position i

- ◆ **Problem:** Update nodes with value x to y in Linked List A

- ◆ **Algorithm:** updateNodes(A, x):



1.  $a \leftarrow \emptyset$ , node  $p \leftarrow A$ ,
2. **while** ( $p \neq \text{NULL}$ )
4.       **if** ( $x = p.\text{value}$ )
5.              $p.\text{value} \leftarrow y$
6.        $p \leftarrow p.\text{next}$
7. **return** A

$$\frac{d f(x)}{dx} = 2 + 8x + 6x^2 + 6x^3$$

X

- ◆ Time Complexity:  $O(n)$
- ◆ Space Complexity:  $O(1)$

# Our Roadmap

- ◆ Linked List Definition
- ◆ Linked List Operators
- ◆ Illustration Example



# Operators on polynomials

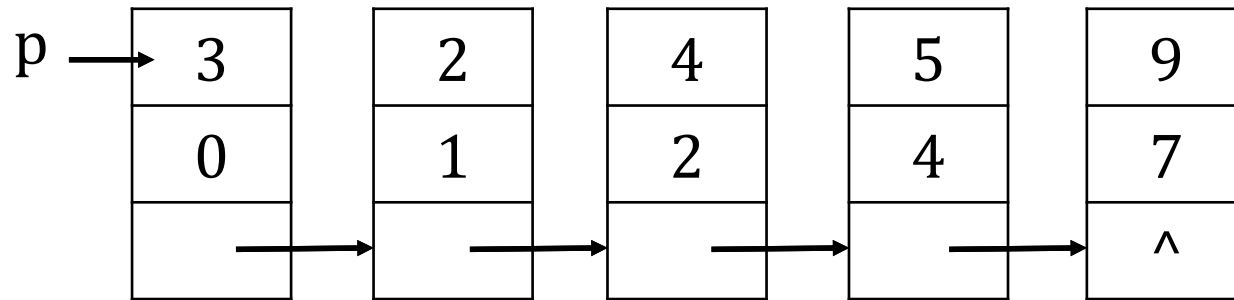
- ◆ **Polynomials:**  $p(x) = p_0 + p_1x + p_2x^2 + \dots + p_nx^n$
- ◆ a set of ordered pairs of  $\langle p_i, i \rangle$  where  $p_i$  is the coefficient and  $i$  is the exponent.
- ◆ We use linked list store the  $\langle p_i, i \rangle$  pairs of  $p(x)$
- ◆ Without loss of generality, we skip all nodes w/  $p_i = 0$
- ◆ Node representation:

```
node polyItem{  
    float coef    // record  $p_i$   
    int  expo    // record exponent  
    node next    // reference to next polyItem  
}
```

- ◆ **Question:** how about use array?

# Finding degree of a Polynomials

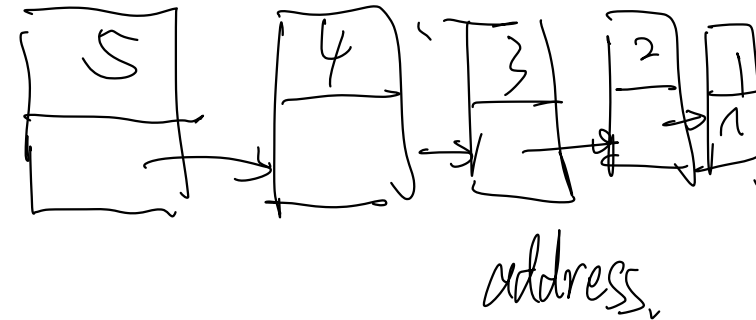
◆ **Polynomials:**  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$



◆ Degree of  $p(x)$ : 7

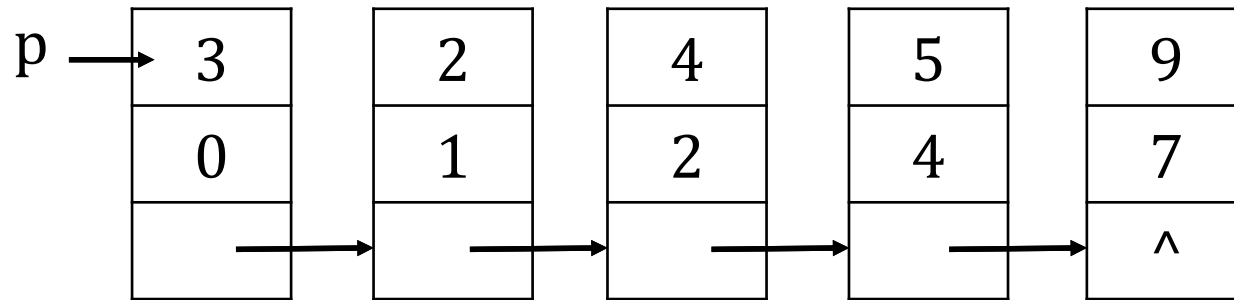
◆ **Algorithm:** findDegree(p):

1. node tmp  $\leftarrow$  p
2. **While** (tmp.next  $\neq$  NULL)
3.        tmp  $\leftarrow$  tmp.next
4. **return** tmp.expo

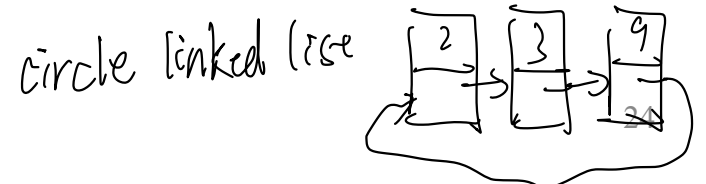
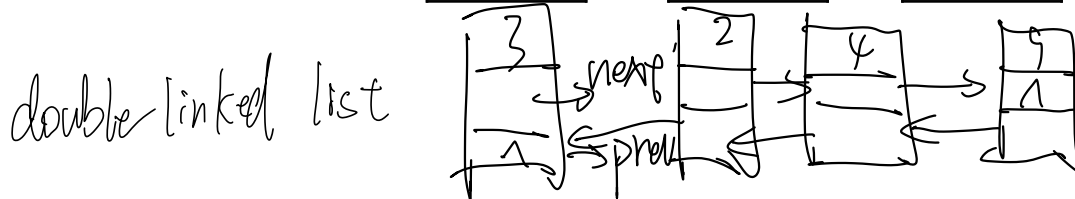
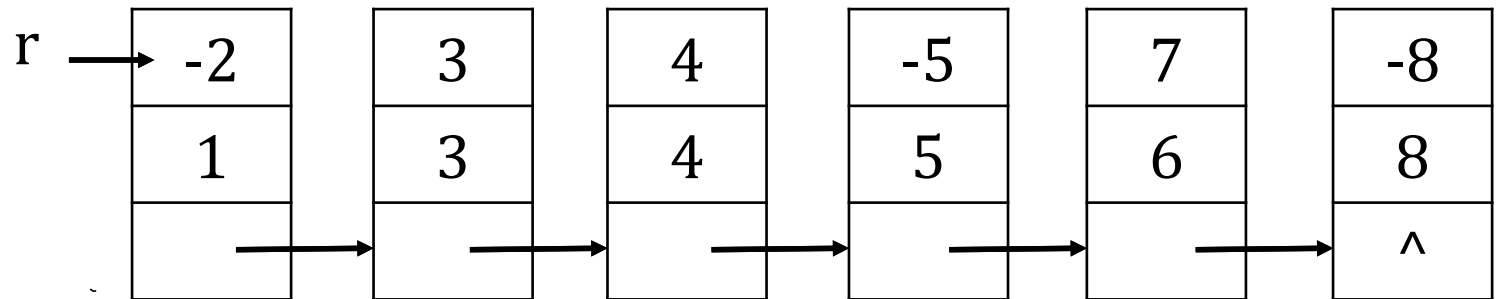


# Adding two polynomials

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$



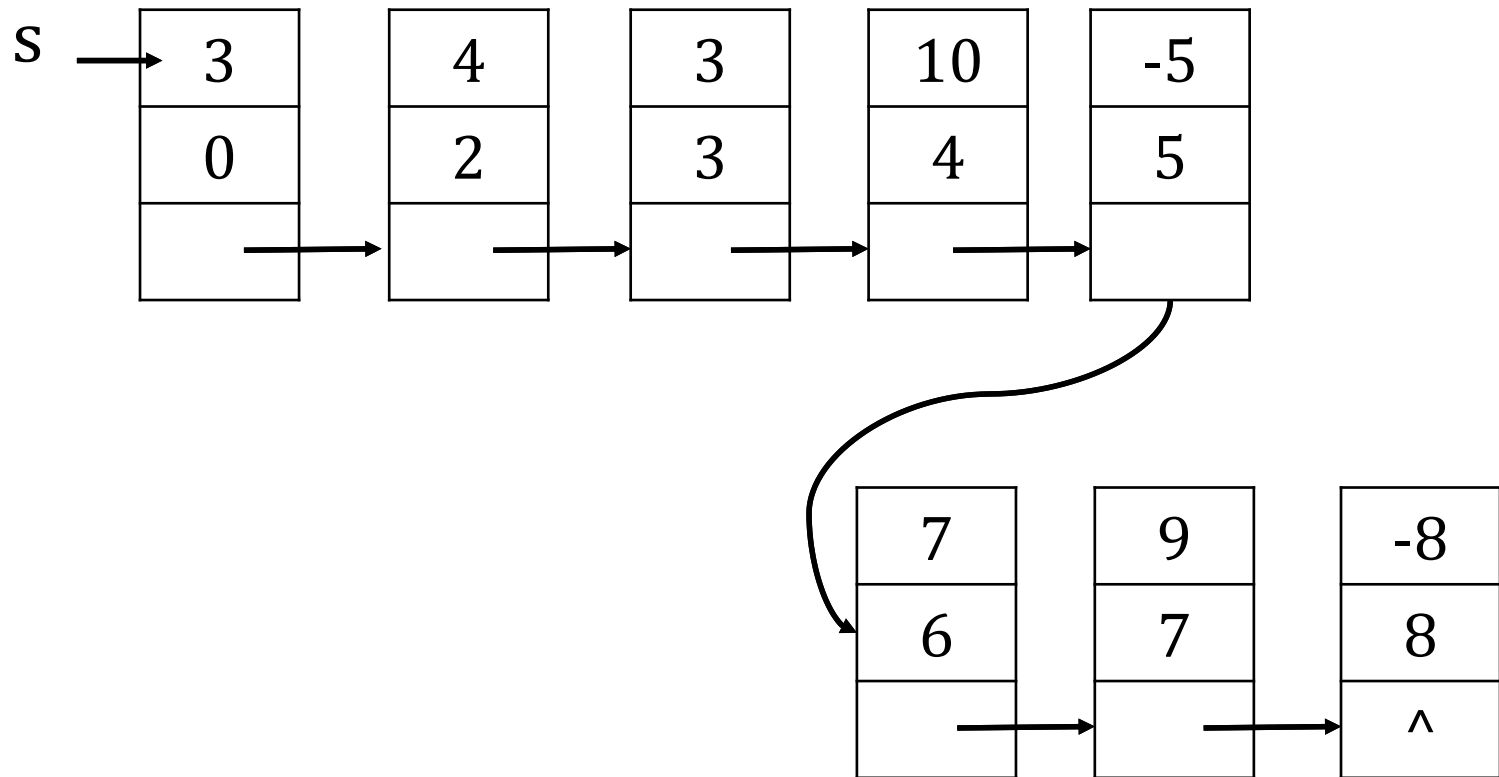
◆  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$





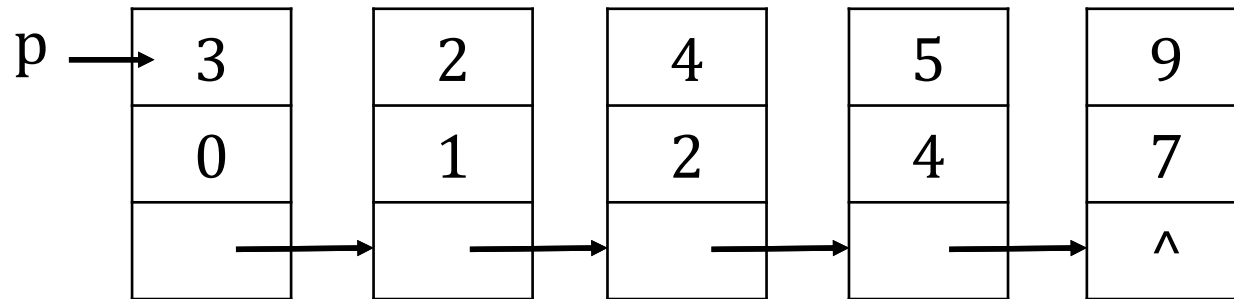
# Adding two polynomials

◆  $s(x) = p(x) + r(x)$   
 $= 3 + 4x^2 + 3x^3 + 10x^4 - 5x^5 + 7x^6 + 9x^7 - 8x^8$

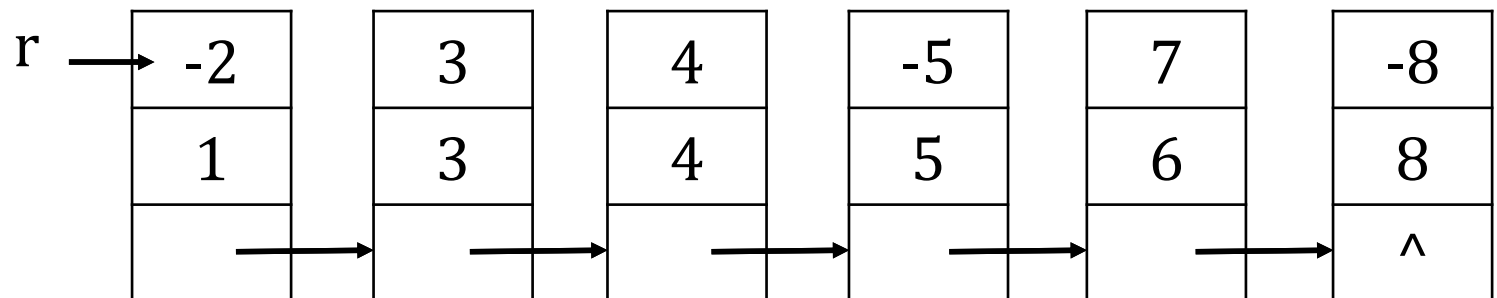


# Subtracting two polynomials

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$

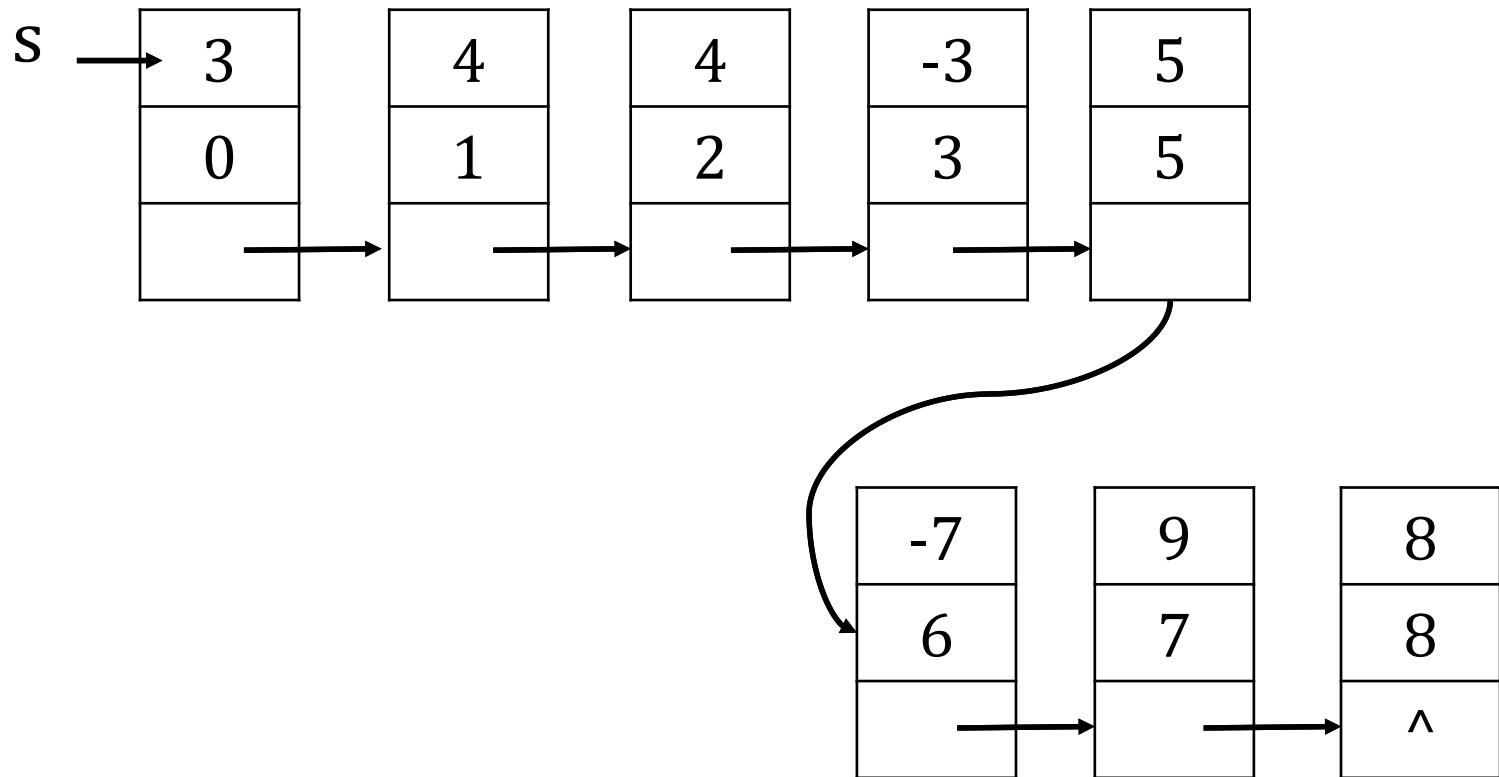


◆  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$



# Subtracting two polynomials

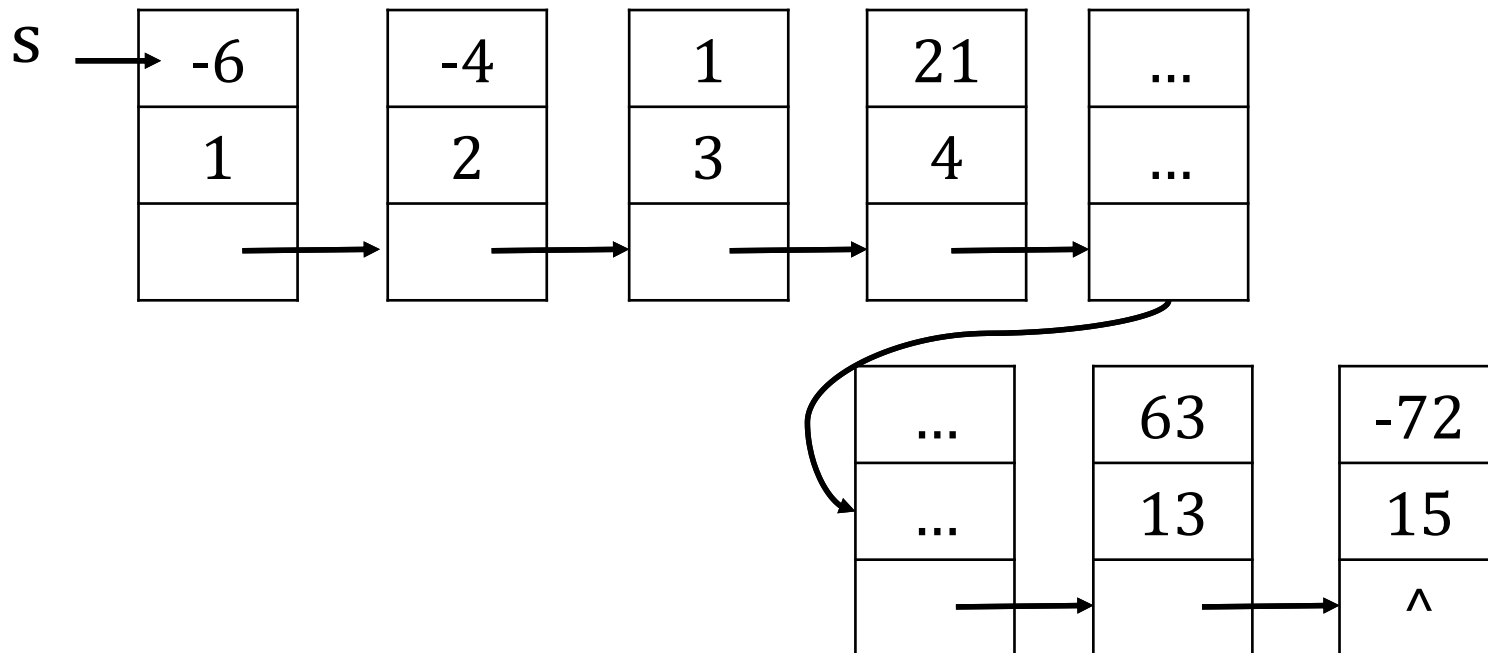
◆  $s(x) = p(x) - r(x)$   
 $= 3 + 4x + 4x^2 - 3x^3 + 5x^5 - 7x^6 + 9x^7 + 8x^8$



# Multiplying two polynomials

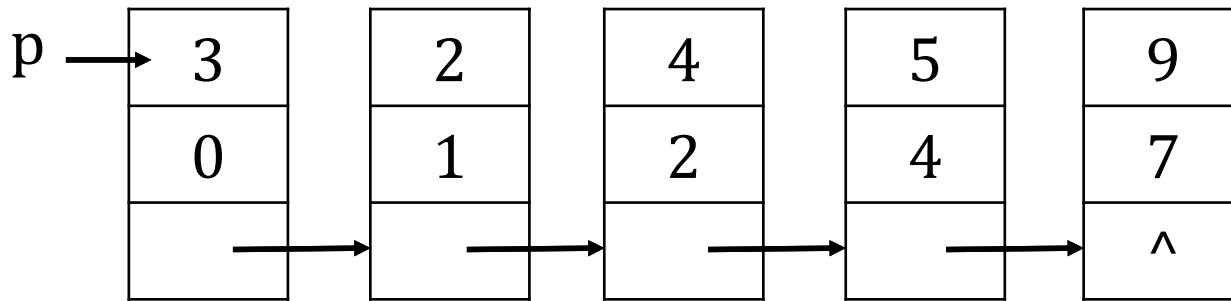
- ◇  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$
- ◇  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$
- ◇  $s(x) = p(x) * r(x)$   

$$= -6x - 4x^2 + x^3 + 21x^4 - 3x^5 + 31x^6 + 9x^7 + 11x^8 - 41x^9 + 30x^{10} + 45x^{11} - 85x^{12} + 63x^{13} - 72x^{15}$$

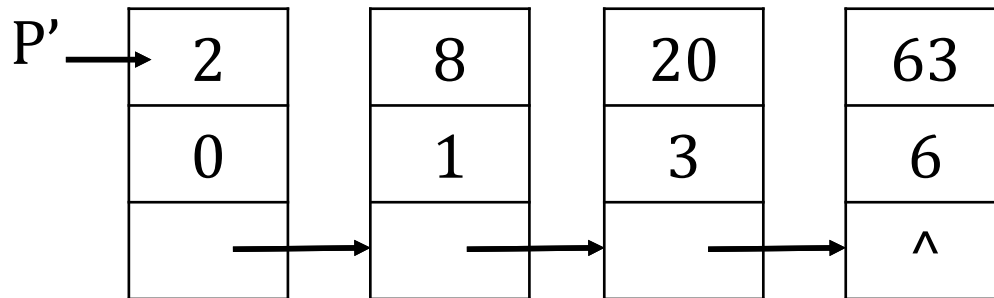


# Differentiating of a polynomial

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$

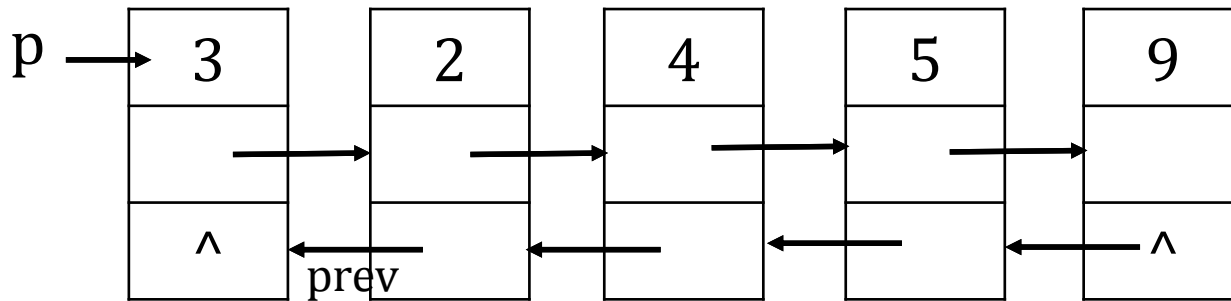


◆  $p'(x) = 2 + 8x + 20x^3 + 63x^6$



# Other variants of Lined List

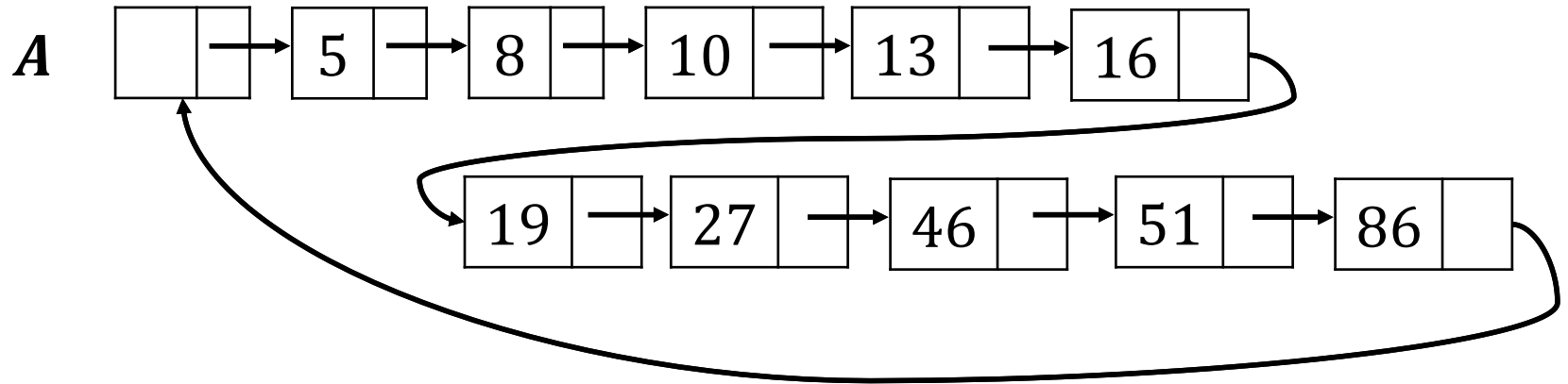
## ◆ Double linked list



- ◆ add a prev reference to each node: refers to the previous node
- ◆ allow us to “back up” from a given node

# Other variants of Lined List

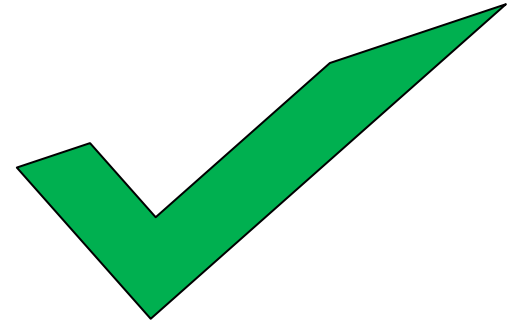
## ◆ Circular linked list



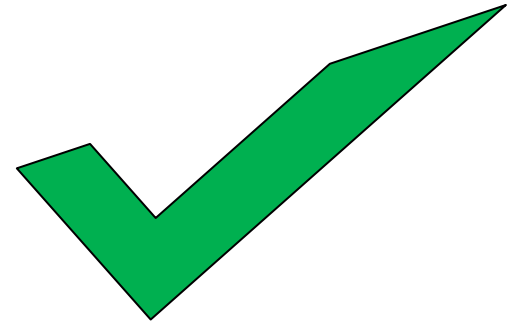
- ◆ Is it a empty list? `head.next = head` ?
- ◆ Is it the end of list? `tmp.next = head`?

# Our Roadmap

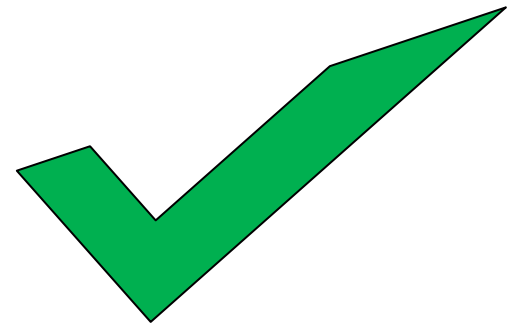
- ◆ Linked List Definition



- ◆ Linked List Operators



- ◆ Illustration Example





Thank You!