

Problem 23

In triangle  $ABC$ ,  $AB = 13$ ,  $BC = 14$ , and  $CA = 15$ . distinct points  $D$ ,  $E$ , and  $F$ , lie on segments  $BC$ ,  $CA$ , and  $\tilde{DE}$ , respectively, such that  $AD \perp \overline{BC}$ ,  $\overline{DE} \perp AC$ , and  $AF \perp \overline{BF}$ . The length of segment  $\overline{DF}$  can be written as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. What is  $m + n$ ?

(A) 18

(B) 21

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(D) 27

(E) 30

14a

Problem 16

In rectangle  $ABCD$ ,  $AB = 1$ ,  $BC = 2$  and points  $E$ ,  $F$ , and  $G$  are midpoints of  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{AD}$ , respectively. Point  $H$  is the midpoint of  $\overline{GE}$ . What is the area of the shaded region?

(A)  $\frac{1}{12}$

(B)  $\frac{\sqrt{3}}{18}$

©  $\frac{\sqrt{2}}{12}$

(D)  $\frac{\sqrt{3}}{12}$

(E)  $\frac{1}{6}$

Problem 18

A square in the coordinate plane has vertices whose  $y$ -coordinates are 0,1,4, and 5 . What is the area of the square?

(A) 16

(B) 17

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(D) 26

(E) 27

Problem 22

In rectangle  $ABCD$ ,  $AB = 20$  and  $BC = 10$ . Let  $E$  be a point on  $\overline{CD}$  such that  $\angle CBE = 15^\circ$ . What is  $\overline{AE}$ ?

(A)  $\frac{20\sqrt{3}}{3}$

(B)  $10\sqrt{3}$

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(D)  $11\sqrt{3}$

(E) 20