Note: No electronics (calculators, cellphones, etc.) are allowed on this exam. You are allowed to have one sheet of notes (front and back).

1. [42 points] Short answers

a) (6 points) Explain orthographic projection and one application of such a projection?

Orthographic projection projects point far away onto an image plane. Assume we have point (x, y, z), we want to project on image plane at $-d$ of z $\Rightarrow$ $(x', y') = \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$

-4

$\Rightarrow$ This helps us get rid of the z axis and shows the 3D object in a 2D plane. (No scaling issues)

b) (6 points) Why camera lenses are necessary (i.e., what's the problem with the pinhole camera)?

The lenses are important because it helps increasing the aperture (since lenses can help focusing the light.) ✓

For pinhole camera, the aperture must be small to have clearer image but small aperture leads to lower intensity & possibility diffraction.

c) (4 point) Mention one way we can change the focal plane of a camera (the distance at which the objects would appear sharp on the sensor).

We can change the focal length or the image distance

✓ as $\frac{1}{d'} + \frac{1}{d} = \frac{1}{f}$, where d' is the distance from the lens to the focal plane, d is the image distance & f is the focal length.

d) (6 points) Thin lens formula describes the relationship between what quantities?

Thin lens formula describes the relationship between diameter D of the aperture, focal length and the f-number, where

-6

$D = \frac{f}{N}$, The area of aperture is $A = \pi \left(\frac{D}{2}\right)^2 \propto \frac{1}{N^2}$

X

e) (4 points) Mention the two factors that control field of view?

Field of view is the angle through which camera can see the world. We can change the focal length or sensor size (h) to affect the FoV. as $FoV = 2 \arctan\left(\frac{h}{2f}\right)$.

f) (6 point) How can we achieve weak perspective when capturing an image?

To achieve weak perspective ⇒ we want to increase the focal length.

g) (6 points) What is the purpose of the demosaicing process?

-6 × Mosaics is where we find correspondences to calculate the homography H of transforming im1 onto im2. After obtaining the H, we can compute H^{-1} and query the color at coordinate (x', y') in the reprojected position in im1 (Demosaicing)

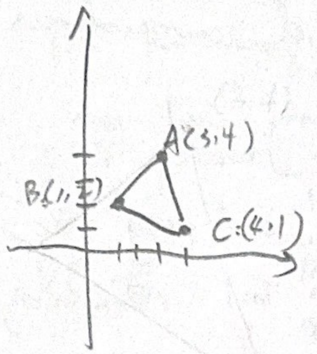
h) (4 points) What are two ways to remove aliasing?

1. We can increase the sampling rate (however most time is not feasible) ✓

2. We can apply filters (such as Gaussian filter) to filter out the high frequency part. ✓

2. [15 points] Let's say you have a triangle with three vertices A = (3, 4), B = (1, 2), C = (4, 1). What are the vertices of the triangle after being transformed using the following matrix? (Note that the coordinates are provided in (x, y) order)

$$\begin{bmatrix}2 & 3 & 1\\4 & 1 & 2\\1 & 1 & 1\end{bmatrix}$$



for A:

$$\begin{bmatrix}2 & 3 & 1\\4 & 1 & 2\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}3\\4\\1\end{bmatrix} = \begin{bmatrix}6+12+1\\12+4+2\\3+4+1\end{bmatrix} = \begin{bmatrix}19\\18\\8\end{bmatrix} \Rightarrow \begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\frac{19}{8}\\\frac{18}{8}\end{bmatrix}$$ ✓

for B:

$$\begin{bmatrix}2 & 3 & 1\\4 & 1 & 2\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix} = \begin{bmatrix}2+6+1\\4+2+2\\1+2+1\end{bmatrix} = \begin{bmatrix}9\\8\\4\end{bmatrix} \Rightarrow \begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\frac{9}{4}\\\frac{8}{4}\end{bmatrix}$$ ✓

for C:

$$\begin{bmatrix}2 & 3 & 1\\4 & 1 & 2\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}4\\1\\1\end{bmatrix} = \begin{bmatrix}8+3\\16+1\\4+1\end{bmatrix} = \begin{bmatrix}11\\17\\5\end{bmatrix} \Rightarrow \begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\frac{11}{5}\\\frac{17}{5}\end{bmatrix}$$

Small mistake

✓

3. [20 points] For the following target, source, and mask, compute the 3x4 blended image using Poisson image blending.

Target

0.5	0.3	0.8
0.4	0.2	0.6
0.1	0.5	0.2
0.5	0.9	0.4

Source

0.7	0.6	0.8
0.1	0.4	0.1
0.5	0.6	0.4
0.5	0.8	0.2

Mask

0	0	0
0	1	0
0	1	0
0	0	0

$$\begin{matrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \\ f_{10} & f_{11} & f_{12} \end{matrix}$$

$$\underset{A}{\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&-1&0&-1&4&-1&0&-1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&-1&0&-1&4&-1&0&-1&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}} \underset{f}{\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \end{bmatrix}} = \underset{v}{\begin{bmatrix} 0.5 \\ 0.3 \\ 0.8 \\ 0.4 \\ \bigcirc \\ 0.6 \\ 0.1 \\ \bigcirc \\ 0.2 \\ 0.5 \\ 0.9 \\ 0.4 \end{bmatrix}}$$

$0.4 \times 4 - 0.1 - 0.1 - 0.6 - 0.6 = 0.2$

$0.6 \times 4 - 0.4 - 0.4 - 0.5 - 0.8 = 0.3$

$$\begin{cases} 4f_5 - f_2 - f_4 - f_6 - f_8 = 0.2 \\ 4f_8 - f_5 - f_7 - f_9 - f_{11} = 0.3 \end{cases}$$

$$\begin{cases} 4f_5 - 0.3 - 0.4 - 0.6 - f_8 = 0.2 \\ 4f_8 - f_5 - 0.1 - 0.2 - 0.9 = 0.3 \end{cases}$$

$$\begin{cases} 4f_5 - f_8 = 1.5 \\ 4f_8 - f_5 = 1.5 \end{cases} \Rightarrow \begin{cases} 4f_5 - f_8 = 1.5 \\ 16f_8 - 4f_5 = 6 \end{cases}$$

$15f_8 = 7.5$

$f_8 = 0.5$

$f_5 = 0.5$

$$\text{Ans:} \begin{bmatrix} 0.5 & 0.3 & 0.8 \\ 0.4 & 0.5 & 0.6 \\ 0.1 & 0.5 & 0.2 \\ 0.5 & 0.9 & 0.4 \end{bmatrix}$$

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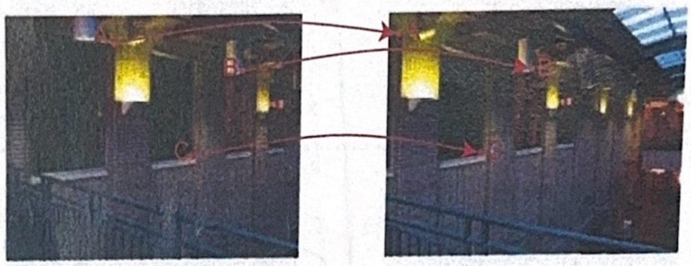
4. [23 points] For the two images below, three pairs of corresponding points are given.

$(x, y) \rightarrow (x', y')$

A = (0, 0) → A' = (1, 7)
B = (4, 0) → B' = (5, 7)
C = (5, 3) → C' = (3, 1)

Note that the coordinates are in (x, y) order. Recover the affine transformation given these three pairs of points, i.e., calculate the 6 parameters of the following 3x3 matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 1 \\ 5 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 5 \\ 7 \\ 3 \\ 1 \end{bmatrix}$$

$$x' = \frac{ax+by+c}{1} = ax+by+c$$

$$y' = dx+ey+f$$

$$\begin{cases} c = 1 \\ f = 7 \\ 4a + c = 5 \\ 4d + f = 7 \\ 5a + 3b + c = 3 \\ 5d + 3e + f = 1 \end{cases} \Rightarrow \begin{cases} c = 1 \\ f = 7 \\ a = 1 \\ d = 0 \\ b = -1 \\ e = -2 \end{cases}$$

$$\Rightarrow A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

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