CSCE 448/748 – Computational Photography

Homographies and Mosaics

Nima Kalantari

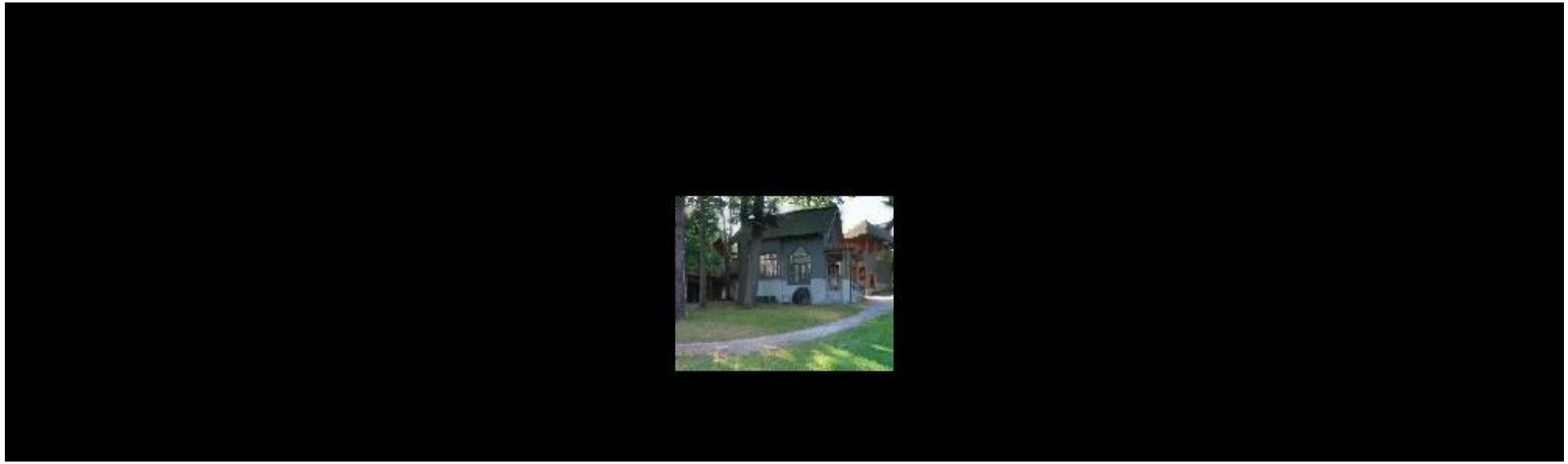
Many slides from Alexei A. Efros, Richard Szeliski, and Steve Seitz

Mosaics: stitching images together

virtual wide-angle camera

Why Mosaic?

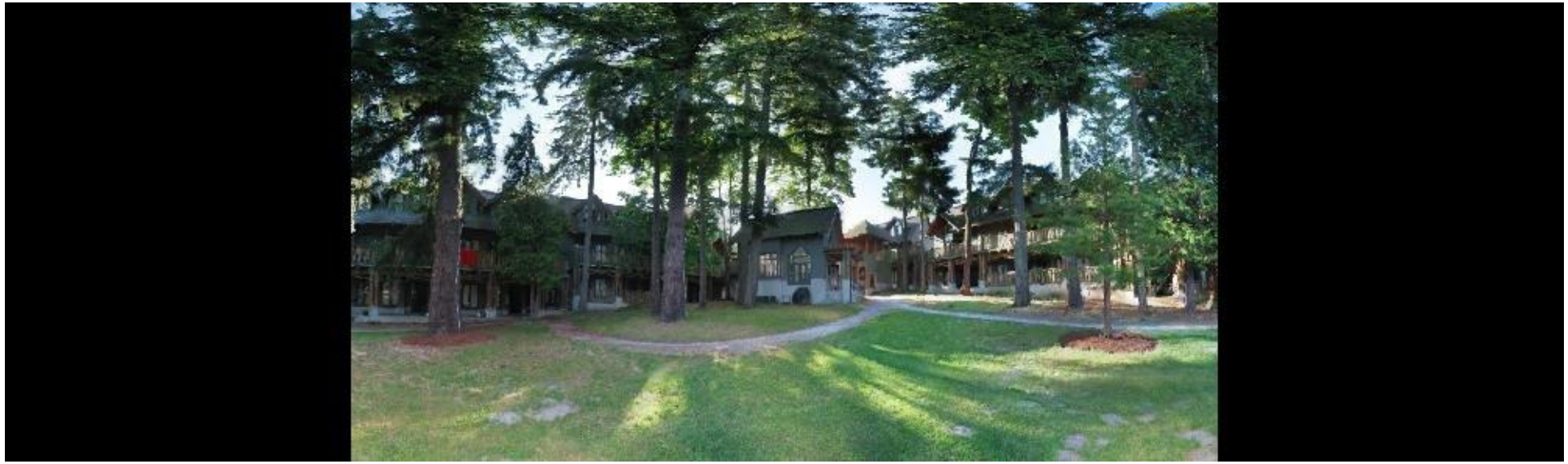
- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°



Slide from Brown & Lowe

Why Mosaic?

- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°
  - Human FOV = 200 x 135°



Slide from Brown & Lowe

Why Mosaic?

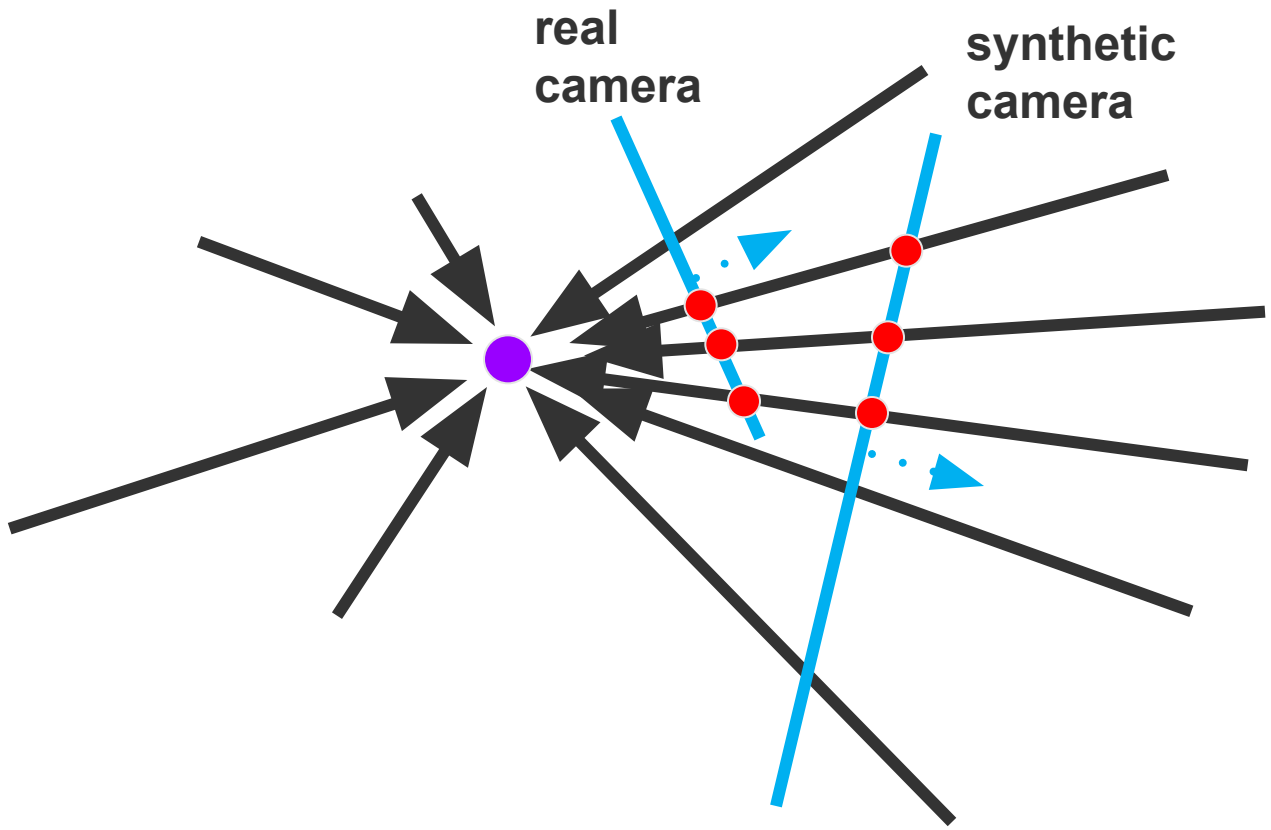
- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°
  - Human FOV = 200 x 135°
  - Panoramic Mosaic = 360 x 180°

Slide from Brown & Lowe

How to do it?

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation between second image and first
- Transform the second image to overlap with the first
- Blend the two together to create a mosaic
- If there are more images, repeat

A pencil of rays contains all views



Can generate any synthetic camera view
as long as it has the same center of projection!

Image reprojection

- The mosaic has a natural interpretation in 3D
  - The images are reprojected onto a common plane
  - The mosaic is formed on this plane
  - Mosaic is a *synthetic wide-angle camera*

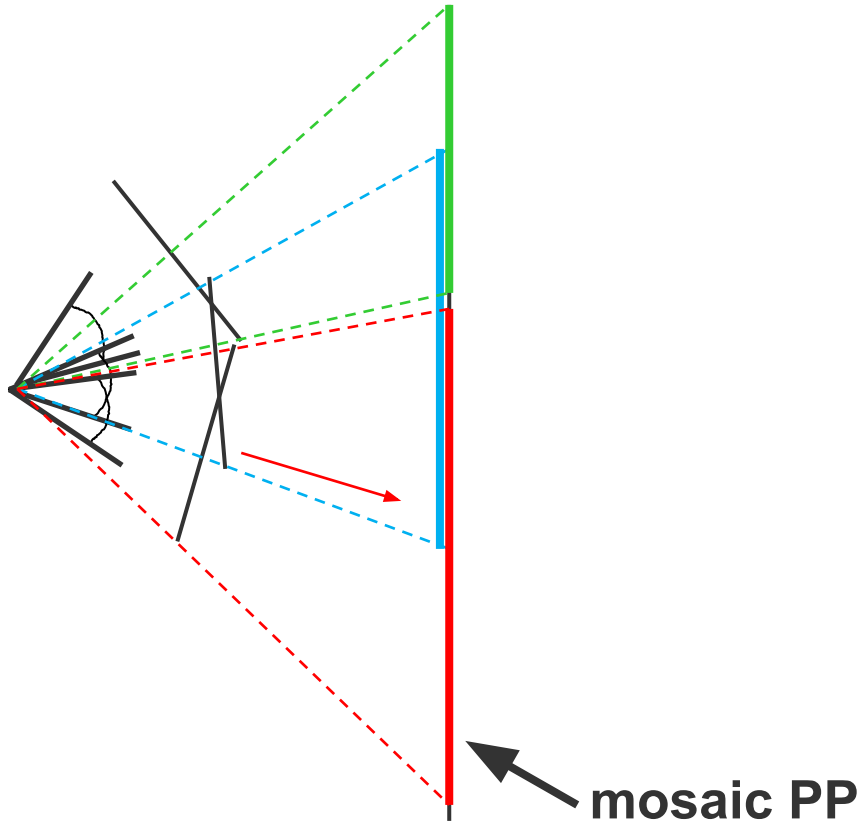


Image reprojection

Basic question

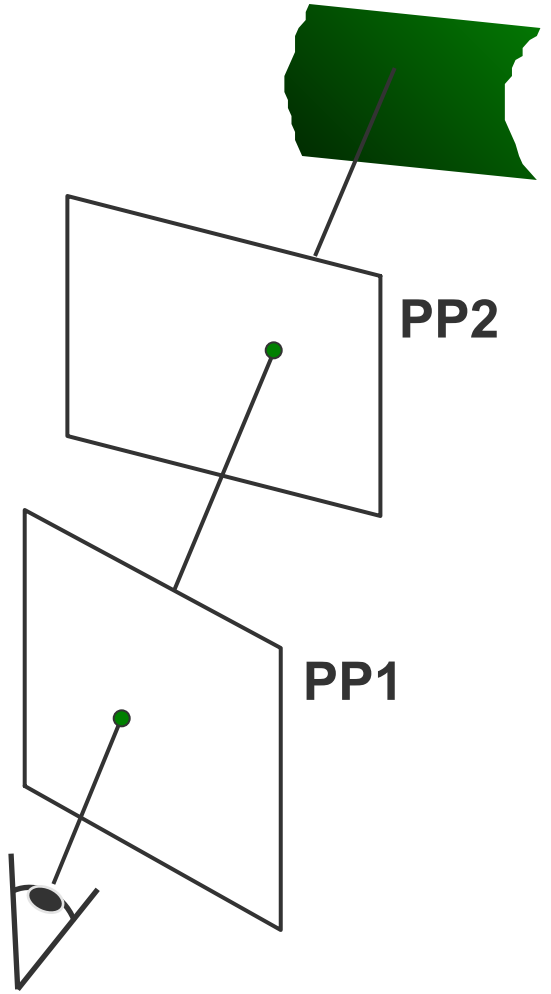
- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2

Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

But don't we need to know the geometry of the two planes with respect to the eye?

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another



Aligning images

left on top

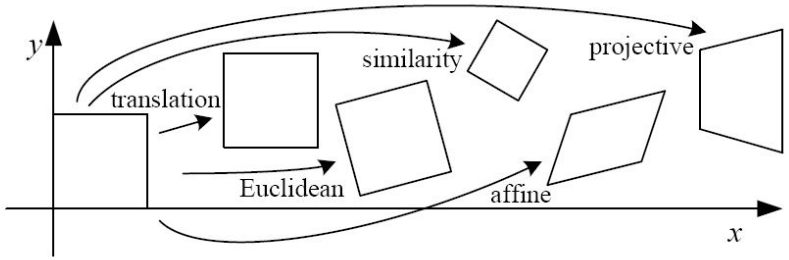
right on top

Translations are not enough to align the images

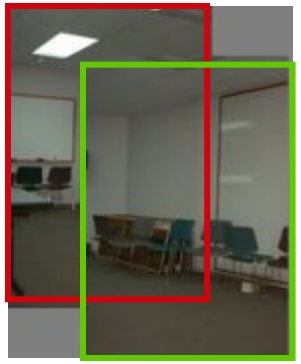
Back to Image Warping

Which t-form is the right one for warping PP1 into PP2?

e.g. translation, Euclidean, affine, projective

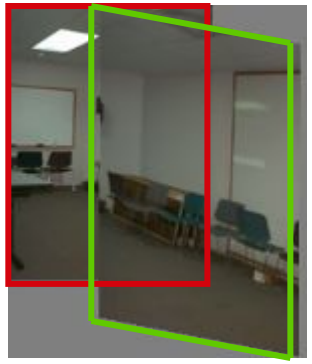


Translation



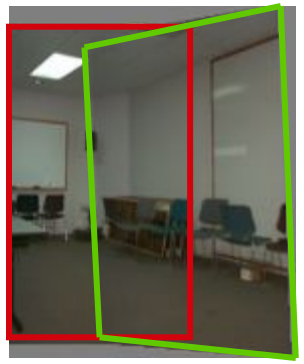
2 unknowns

Affine



6 unknowns

Perspective



8 unknowns

Homography

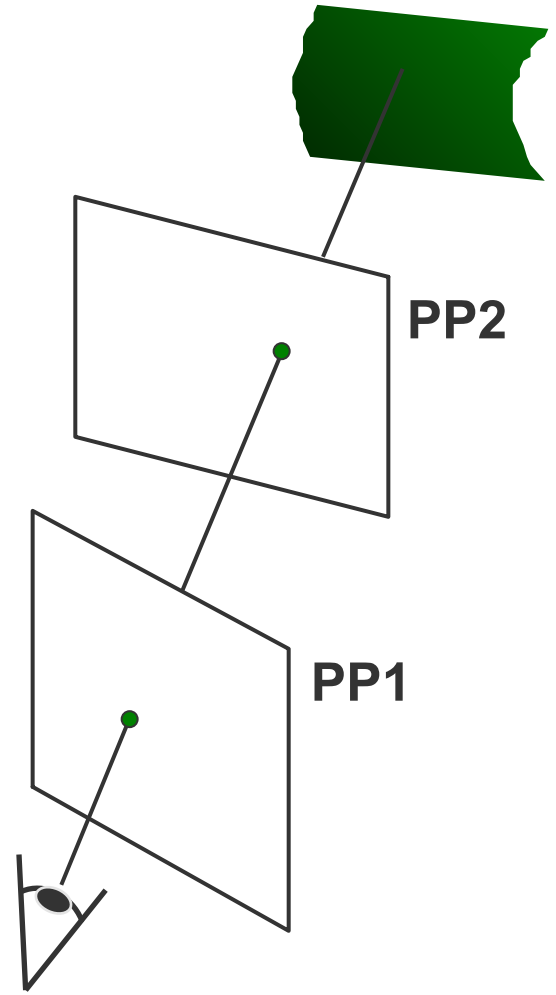
- Projective – mapping between any two PPs with the same center of projection
  - rectangle should map to arbitrary quadrilateral
  - parallel lines aren't
  - but must preserve straight lines
- called Homography

$$\begin{bmatrix} x'' \\ y'' \\ w'' \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

P'' H p

To apply a homography H

- Compute p'' = Hp (regular matrix multiply)
- Convert p'' from homogeneous to image coordinates



Mosaics: main steps

- Collect correspondences (manually)
- Solve for homography matrix H

- Warp content from one image frame to the other to combine: say im1 into im2 reference frame

- Overlay im2 content onto the warped im1 content.

Mosaics: main steps

- **Collect correspondences (manually)**
- Solve for homography matrix H
  - Least squares solution
- Warp content from one image frame to the other to combine: say im1 into im2 reference frame
  - Determine bounds of the new combined image
    - Where will the corners of im1 fall in im2's coordinate frame?
    - We will attempt to lookup colors for any of these positions we can get from im1.
  - Compute coordinates in im1's reference frame (via homography) for all points in that range
  - Lookup all colors for all these positions from im1
    - Inverse warp
- Overlay im2 content onto the warped im1 content.
  - Careful about new bounds of the output image: minx, miny

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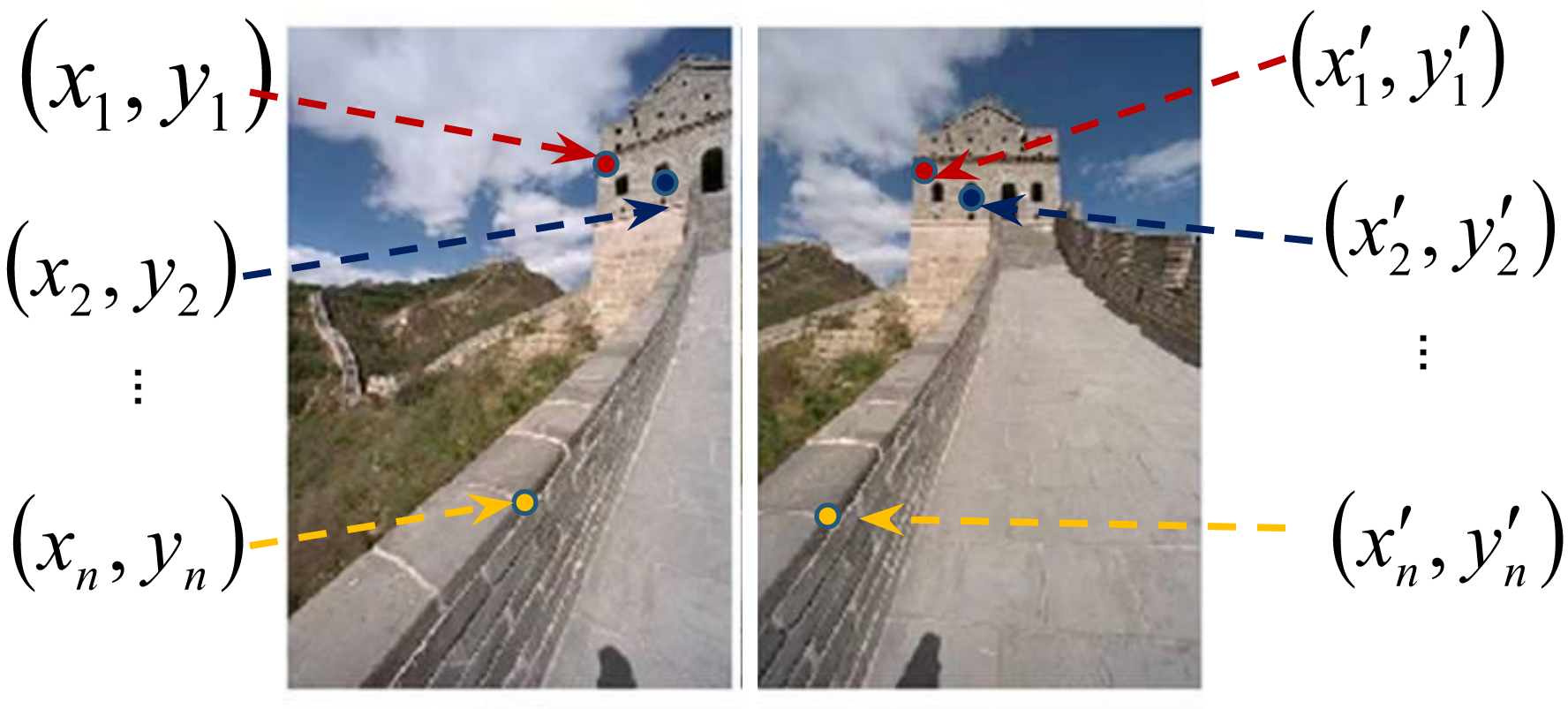
Homography



To compute the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of H are the unknowns…

Solving for homographies

$$P'' = Hp$$

$$\begin{bmatrix} x'' \\ y'' \\ w'' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Can set scale factor *i*=1. So, there are 8 unkowns.
- Set up a system of linear equations:

$$Ah = b$$

- where vector of unknowns h = [a,b,c,d,e,f,g,h]^T
- Need at least 8 eqs, but the more the better…
- Solve for h. If overconstrained, solve using least-squares:

$$\min \|Ah - b\|^2$$

Example system for finding homography

destination coordinates of the four corners of a quadrilateral. Let the correspondence map $(u_k, v_k)^t$ to $(x_k, y_k)^T$ for vertices numbered cyclically $k = 0, 1, 2, 3$. All coordinates are assumed to be real (finite). To compute the forward mapping matrix $\mathbf{M}_{sd}$, assuming that $i = 1$, we have eight equations in the eight unknowns a-h:

$$x_k = \frac{au_k + bv_k + c}{gu_k + hv_k + 1} \quad \Rightarrow \quad u_k a + v_k b + c - u_k x_k g - v_k x_k h = x_k$$

$$y_k = \frac{du_k + ev_k + f}{gu_k + hv_k + 1} \quad \Rightarrow \quad u_k d + v_k e + f - u_k y_k g - v_k y_k h = y_k$$

for $k = 0, 1, 2, 3$. This can be rewritten as an 8×8 system:

$$\begin{pmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & -u_0x_0 & -v_0x_0 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1x_1 & -v_1x_1 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -u_2x_2 & -v_2x_2 \\ u_3 & v_3 & 1 & 0 & 0 & 0 & -u_3x_3 & -v_3x_3 \\ 0 & 0 & 0 & u_0 & v_0 & 1 & -u_0y_0 & -v_0y_0 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1y_1 & -v_1y_1 \\ 0 & 0 & 0 & u_2 & v_2 & 1 & -u_2y_2 & -v_2y_2 \\ 0 & 0 & 0 & u_3 & v_3 & 1 & -u_3y_3 & -v_3y_3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{pmatrix} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Source: Paul Heckbert, ***Fundamentals of Texture Mapping and Image Warping***

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    - We will attempt to lookup colors for any of these positions we can get from im1. : `meshgrid`
  - Compute coordinates in im1's reference frame (via homography) for all points in that range: H^{-1}
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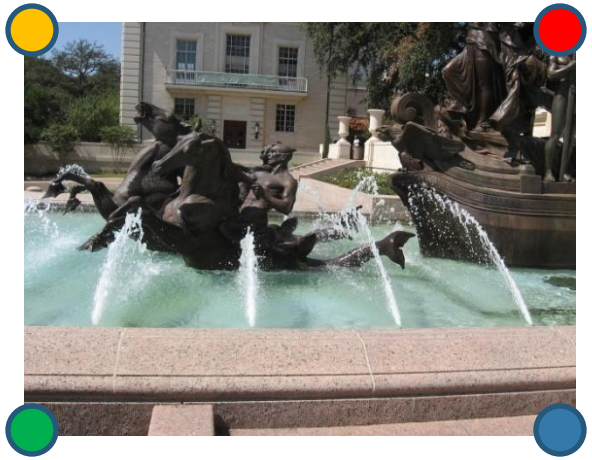

im1

im2

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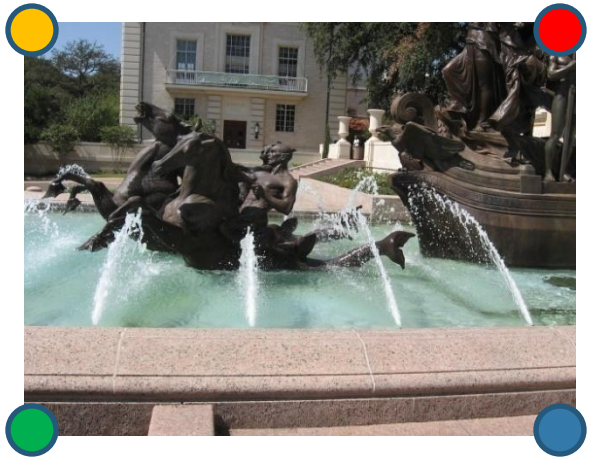

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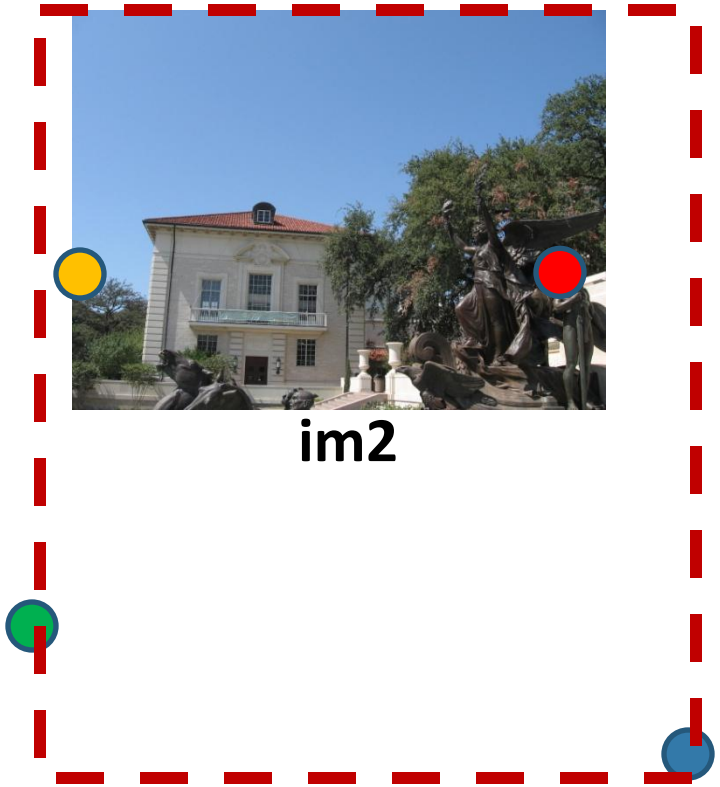

im2

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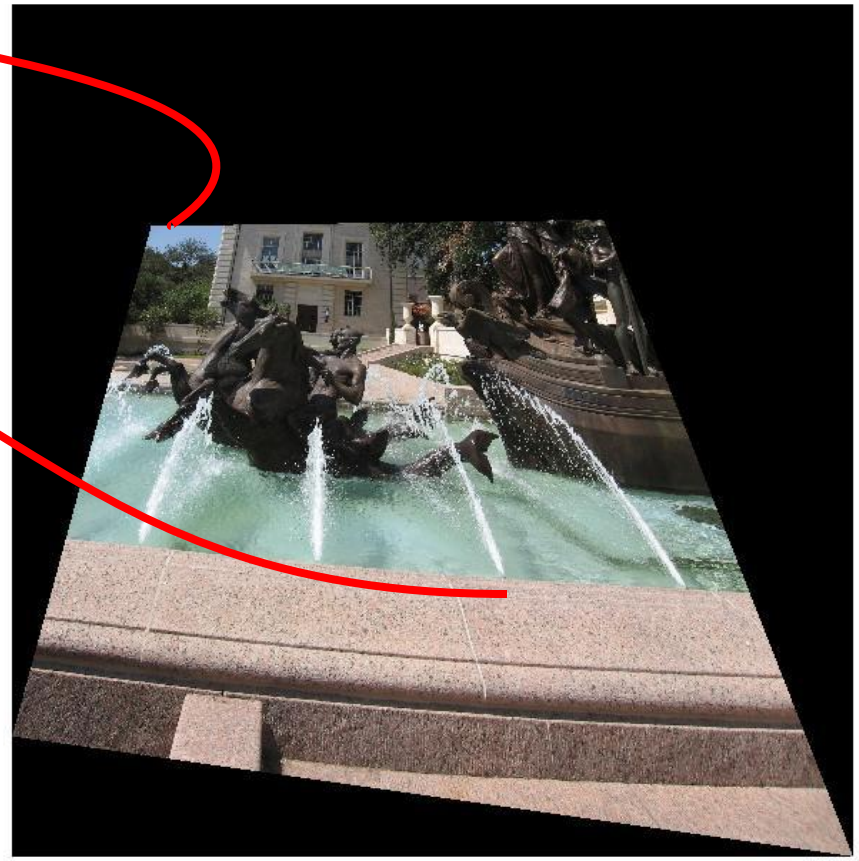

im1 warped into reference frame of im2.

Use interp2 to ask for the colors (possibly interpolated) from im1 at all the positions needed in im2's reference frame.

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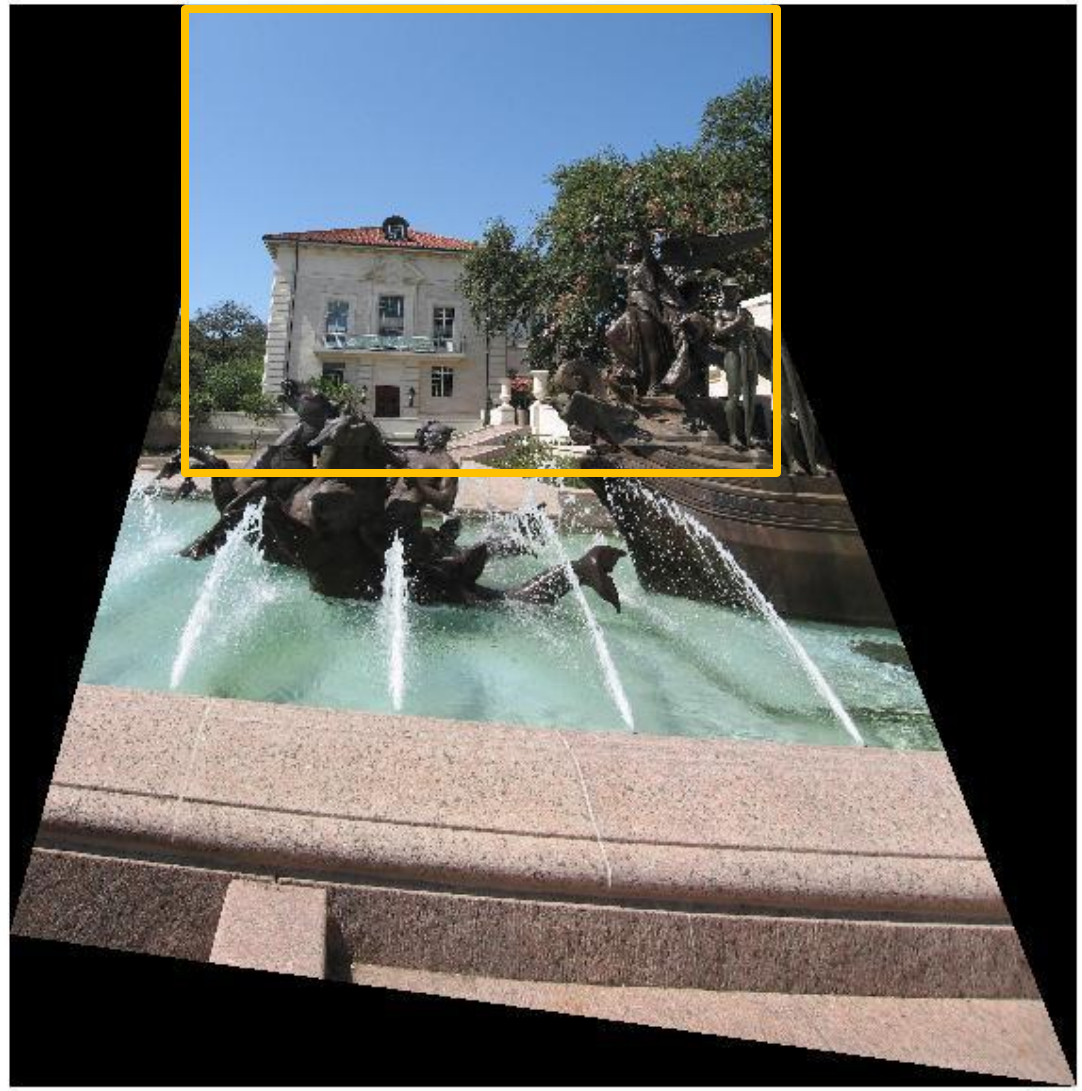

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