Assignment 6

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1 Overview

In this assignment, we automatically combine multiple exposures into a single high dynamic range radiance map and then convert this radiance map to an image suitable for display through tone mapping.

2 Task 1 - Calculate the camera response function

2.1 Approach

In this section, we first calculate the Camera Response Function (CRF) with respect to the different channels. The Camera Response Function will map the log-exposure value to the intensity levels in the input images. According to the film reciprocity equation:

$$Z_{ij} = f(E_i \Delta t_j) \tag{1}$$

where Z_{ij} represents the pixel value at the spatial index i in the image for exposure index j. Here, E_i is the scene radiance at pixel i, indicating the actual light intensity from the scene falling on that pixel, and Δt_j is the exposure time for image j which represents the exposure of the image. Since f is monotonic and invertible, we can rewrite the equation as:

$$f^{-1}(Z_{ij}) = E_i \Delta t_j \Rightarrow \ln f^{-1}(Z_{ij}) = \ln E_i + \ln \Delta t_j \tag{2}$$

Let $g = \ln f^{-1}$, we have the equation:

$$g(Z_{ij}) = \ln E_i + \ln \Delta t_j \tag{3}$$

where g is the camera response function. In this equation, g and E_i are the unknown variables that we want to find, which can be recovered by applying an optimization method to minimize the error between the calculated and observed pixel values across multiple images with varying exposure levels. This optimization involves solving a system of equations derived from multiple exposures of the same scene, effectively allowing us to deduce both the camera response function g and the scene radiance E_i for each pixel. In our case, $Z_{min} = 0$ to $Z_{max} = 255$ since we are working with 8-bit images. The calculation of CRF involves three steps:

1. Select N random pixel coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_N, y_N)$ from the image stack, each with varying exposure levels, to retrieve their respective intensity values for the optimization process. Where N is defined as:

$$N = \lfloor (5 \times \frac{256}{(P-1)} \rfloor \tag{4}$$

, where P is the number of different exposures we have in the image stack.

2. We define the triangle function to emphasize the smoothness and fitting terms toward the middle of the curve. It assigns higher weights to pixel values closer to the midpoint of the dynamic range and lower weights to those near the extremes. The function is defined as follows:

$$w(z) = \begin{cases} z - Z_{min} & \textbf{if } z \le \frac{Z_{min}+Z_{max}}{2} \\ Z_{max} - z & \textbf{otherwise} \end{cases} \tag{5}$$

3. We optimize the objective function using the `gsolve` function to solve for $g(z)$ and E_i. Here we set $\lambda = 50$.

$$O = \sum_{i=1}^{N} \sum_{j=1}^{P} \left[w(Z_{ij}) \cdot (g(Z_{ij}) - \ln E_i - \ln \Delta t_j) \right]^2 + \lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} \left[w(z) \cdot g''(z) \right]^2 \tag{6}$$

As $g(z)$ represents the log exposure corresponding to the pixel value z, we can visualize the Camera Response Function (CRF) by plotting $g(z)$ on the x-axis against the pixel value z on the y-axis. The $g(z)$ values for each color channel are computed independently to accommodate variations in the response across different channels. We use this method and find the CRF for the Chapel and Office examples.

2.2 Results

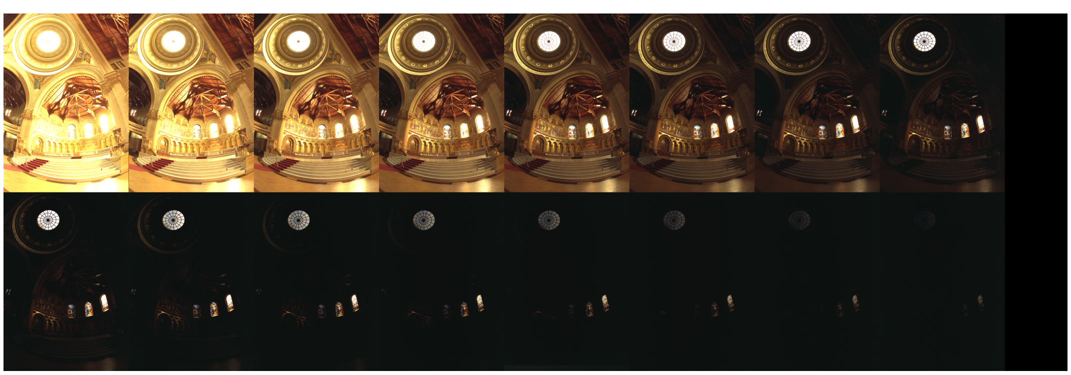

Figure 1: Example Chapel. The exposure ranges from 32 to $\frac{1}{1024}$.

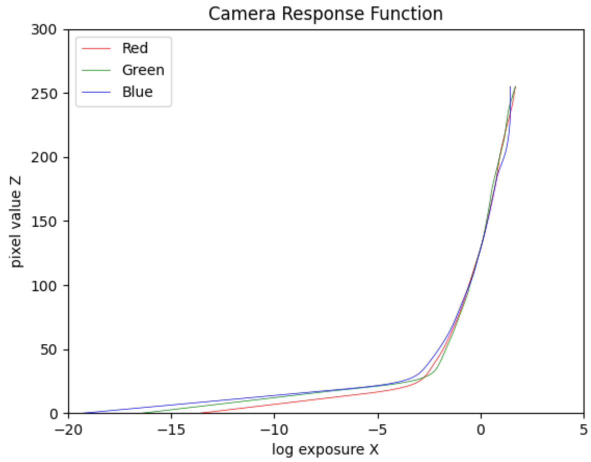

Figure 2: The camera response function (CRF) from the Chapel image set.

Figure 3: Example Office. The exposure ranges from $\frac{1}{6}$ to $\frac{1}{1600}$.

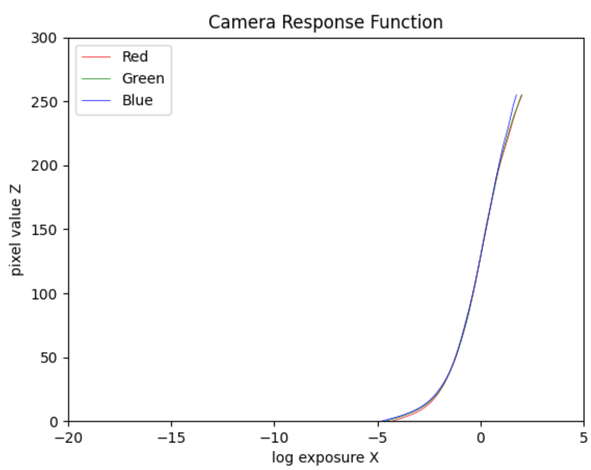

Figure 4: The camera response function (CRF) from the Office image set.

3 Task 2 - Reconstruct the image

3.1 Approach

After determining the Camera Response Function, we proceed with the reconstruction of the HDR image, which involves two major steps: calculating the scene's radiance and applying tone mapping to produce the final result.

Calculating the scene's radiance To recover high dynamic range radiance values, we need to use the CRF for each color channel we calculated in the previous section. For each pixel, we should use all the different exposure images in the image stack to compute its radiance. From equation 3, we can rewrite it such that:

$$\ln E_i = g(Z_{ij}) - \ln \Delta t_j \tag{7}$$

We also reuse the triangle function in equation 2 to give higher weight to exposures in which the pixel value is closer to the middle of the response function. Therefore, the combined equation will be:

$$\ln E_i = \frac{\sum_{j=1}^{P} w(Z_{ij}) \cdot (g(Z_{ij}) - \ln \Delta t_j)}{\sum_{j=1}^{P} w(Z_{ij})} \tag{8}$$

Since all the variables on the right side are known, we can recover the scene radiance E_i. Note that we calculate g independently for each color channel; similarly, we recover E_i for each channel independently and then stack them afterward.

Tone Mapping Here, we implement two methods of tone mapping: global tone mapping and local tone mapping.

- Global Tone Mapping: obtain a tone-mapped image using gamma compression as follows:

$$T = (\frac{E}{\max{(E)}})^{\gamma} \tag{9}$$

- Local Tone Mapping: The local tone mapping requires multiple steps:

  1. Your input E is linear RGB values of radiance.
  2. Compute the intensity (I) by averaging the color channels.
  3. Compute the chrominance channels: $(R/I, G/I, B/I)$
  4. Compute the log intensity: $L = \log_2(I)$
  5. Filter that with a Gaussian filter: $B = \text{filter}(L)$ with standard deviation σ
  6. Compute the detail layer: $D = L - B$
  7. Apply an offset and a scale to the base: $B' = (B - o) * s$, where $o = \max(B)$ and $s = \frac{dR}{\max(B) - \min(B)}$
  8. Reconstruct the log intensity: $O = 2^{(B'+D)}$
  9. Put back the colors: $R', G', B' = O * (R/I, G/I, B/I)$
  10. Apply gamma compression with parameter γ

Therefore, for global tone mapping, we have one parameter γ while for local tone mapping, we have three parameters (σ, dR, γ). In the results produced below, we set the parameters as follows:

- Global Tone Mapping: $\gamma = 0.1$.
- Local Tone Mapping: $(\sigma, dR, \gamma) = (0.5, 4, 0.5)$.

3.2 Results

As we can see from the results, images generated with global tone mapping tend to have a more uniformly illuminated appearance, while those generated with local tone mapping display greater contrast and color saturation. One reason might because global tone mapping applies the same transformation to all pixels without considering local variations in brightness. On the other hand, local tone mapping adjusts the luminance of each pixel based on its surroundings, allowing for a broader dynamic range that preserves details in both highlights and shadows. This approach can more effectively accentuate the local features of an image.

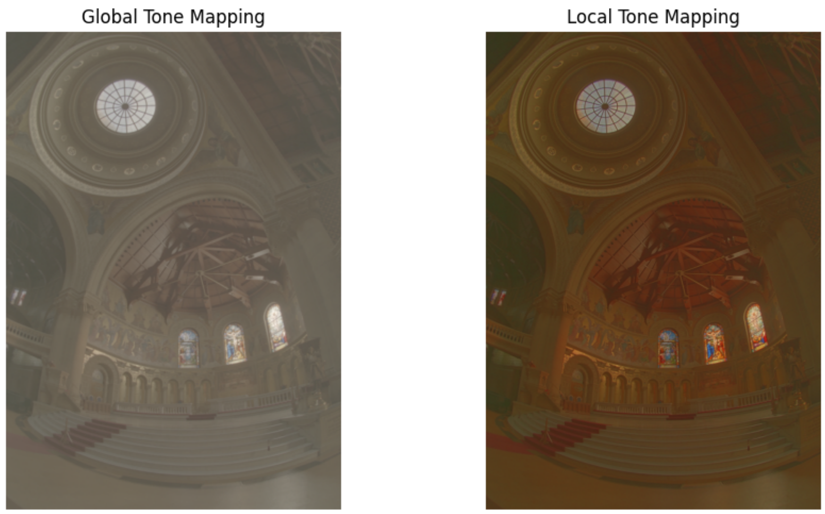

Figure 5: The 'Chapel' Results: The image on the left has been processed with global tone mapping with $\gamma = 0.1$, and the one on the right has undergone local tone mapping with parameters $(\sigma, dR, \gamma) = (0.5, 4, 0.5)$.

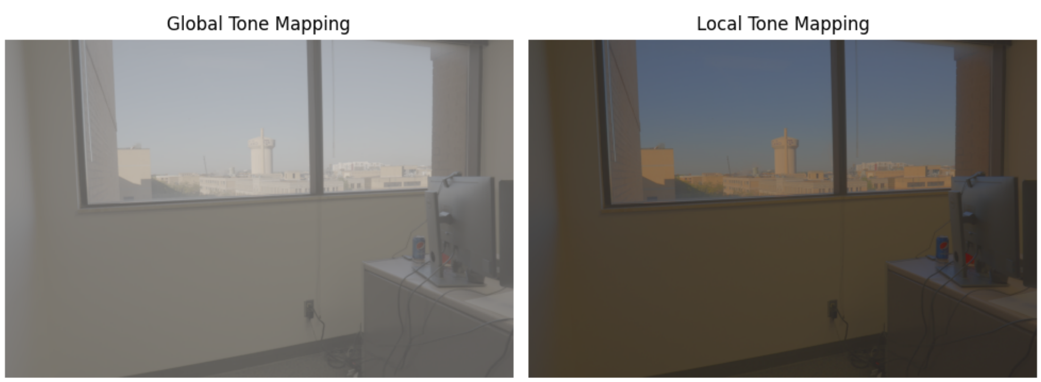

Figure 6: The 'Office' Results: The image on the left has been processed with global tone mapping with $\gamma = 0.1$, and the one on the right has undergone local tone mapping with parameters $(\sigma, dR, \gamma) = (0.5, 4, 0.5)$.