CSCE 448/748 – Computational Photography

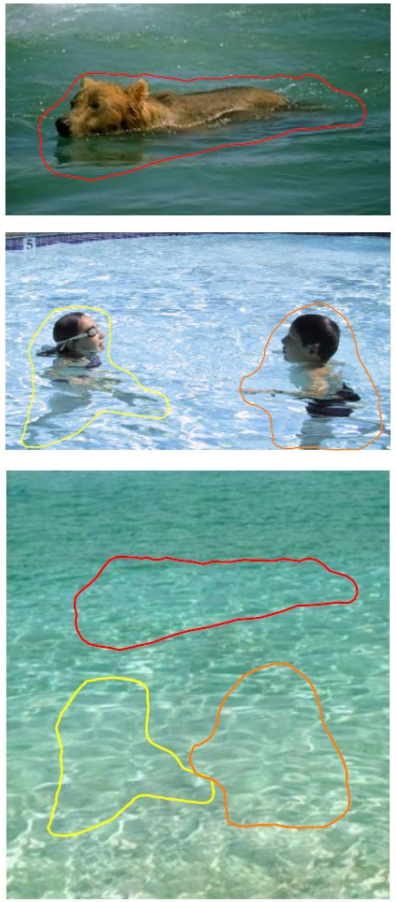
Blending

Nima Kalantari

Many slides from Alexei A. Efros, James Hayes, Rob Fergus

Goal

- Seamless copy of content from one image to another



sources/destinations

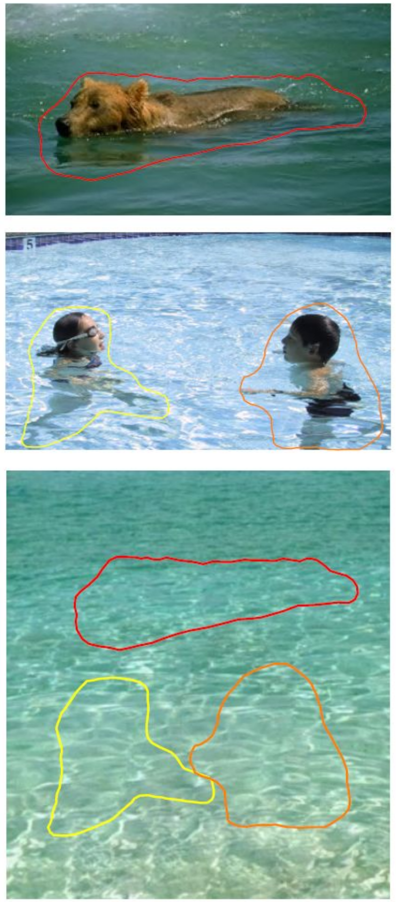
cloning

Gradient manipulation

- **Observation:**
  - Human visual system is sensitive to gradient
  - Gradient encodes edges and local contrast
- **Idea:**
  - Do your editing in the gradient domain
  - Reconstruct image from gradient
- Based on Perez et al. “Poisson Image Editing” SIGGRAPH 2003

Slide credit: F. Durand

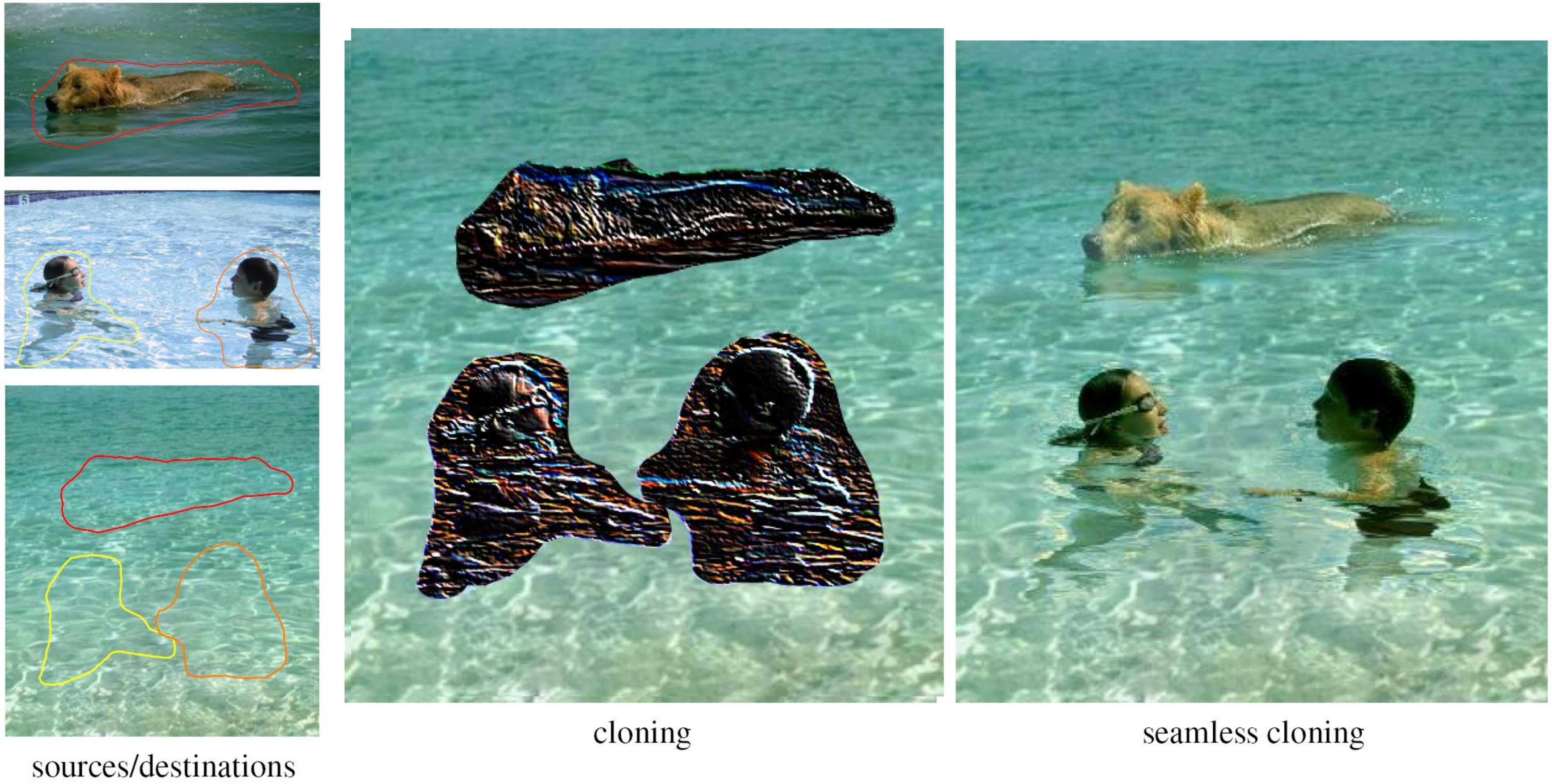
Goal



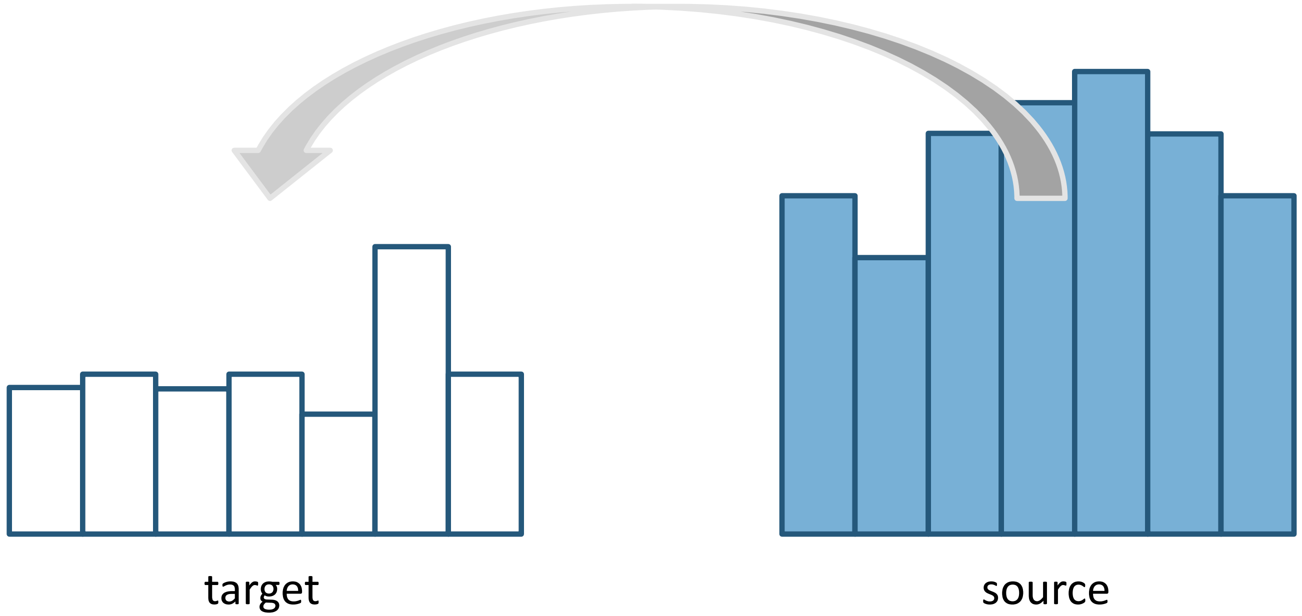
sources/destinations

cloning

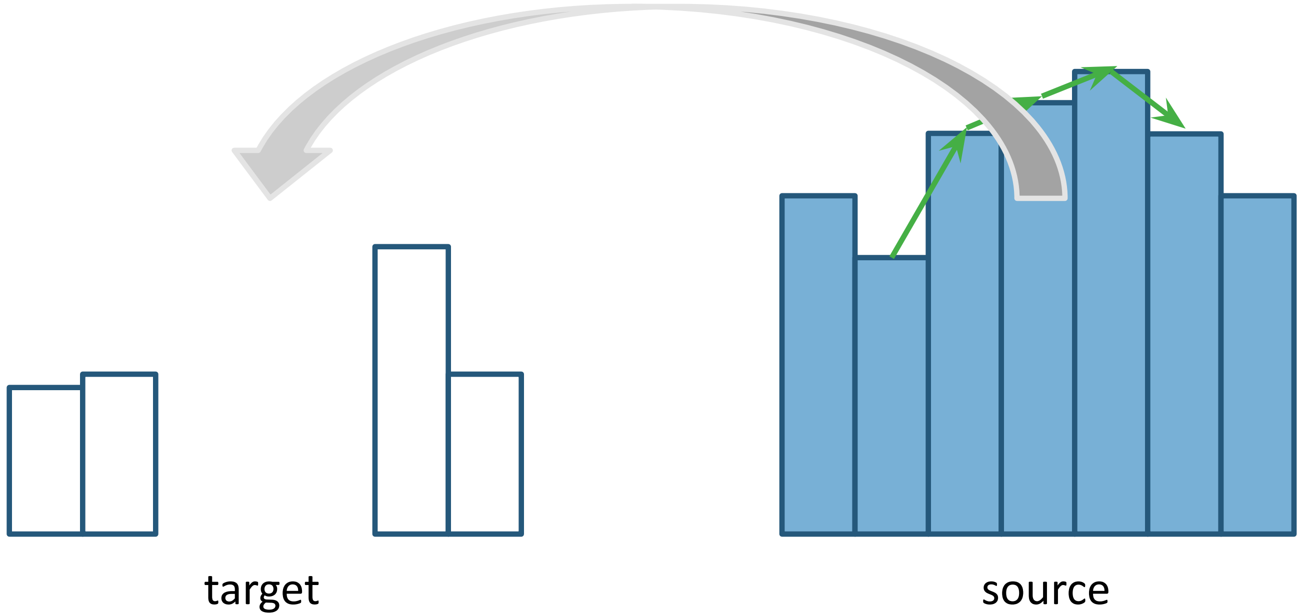
Gradient domain cloning



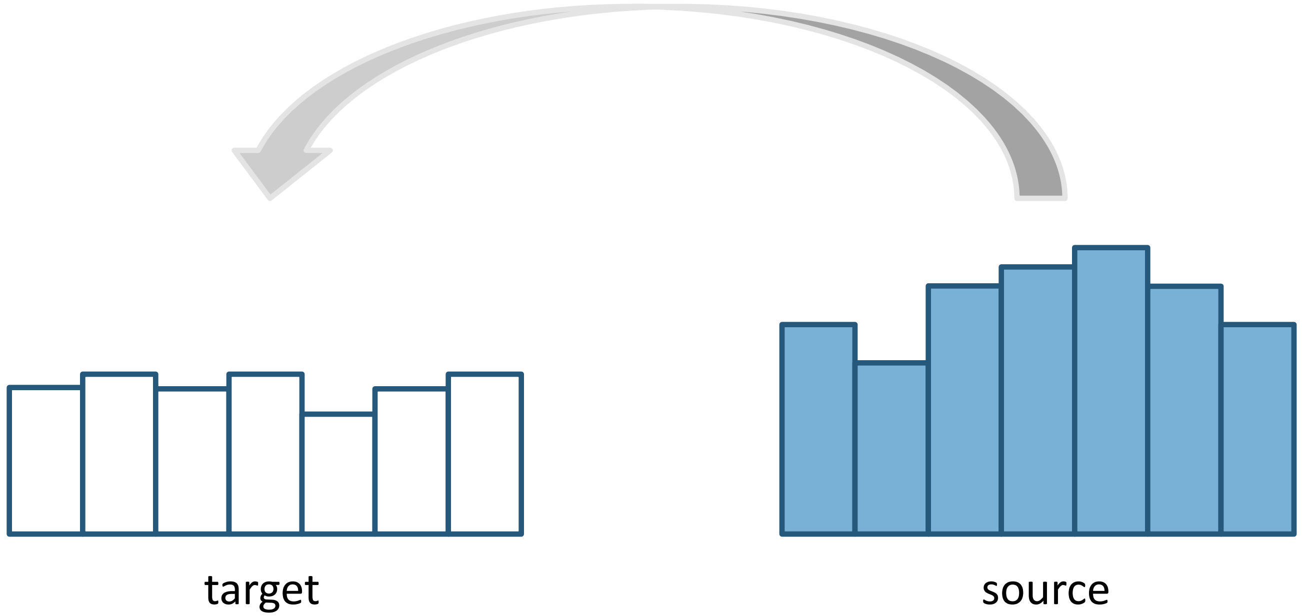
Gradient blending (1D) – Example 1



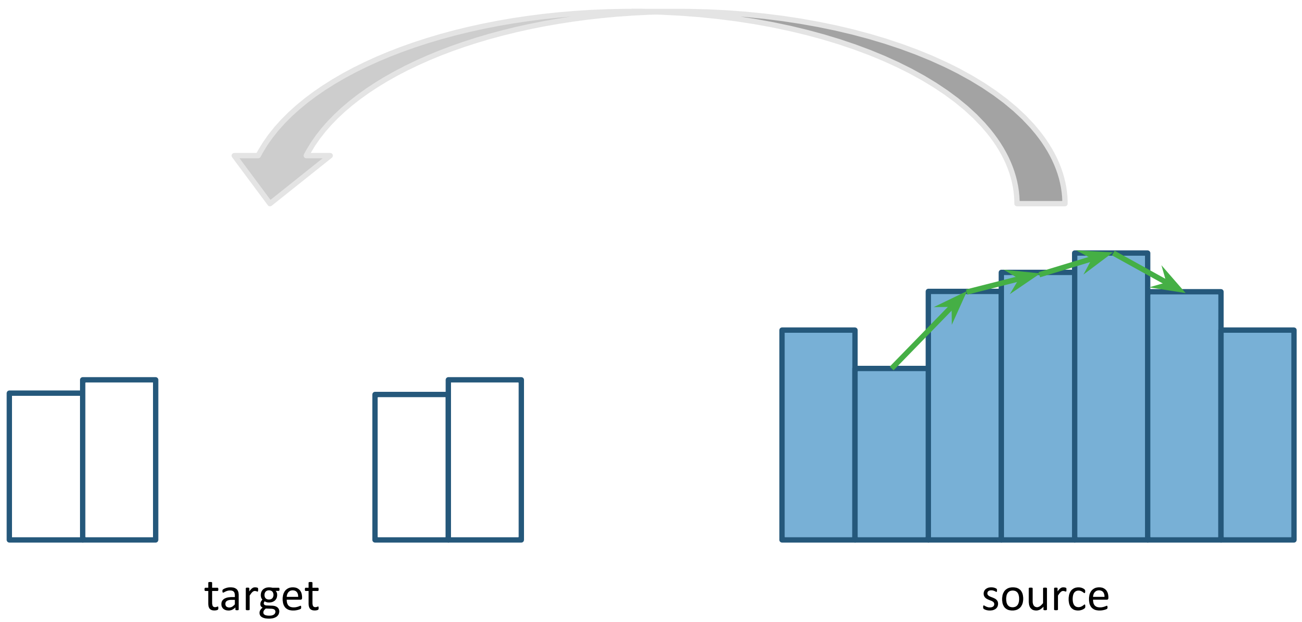
Gradient blending (1D) – Example 1



Gradient blending (1D) – Example 2



Gradient blending (1D) – Example 2

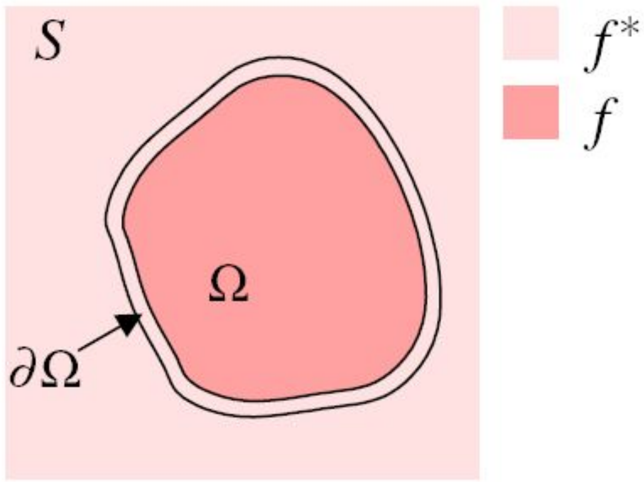


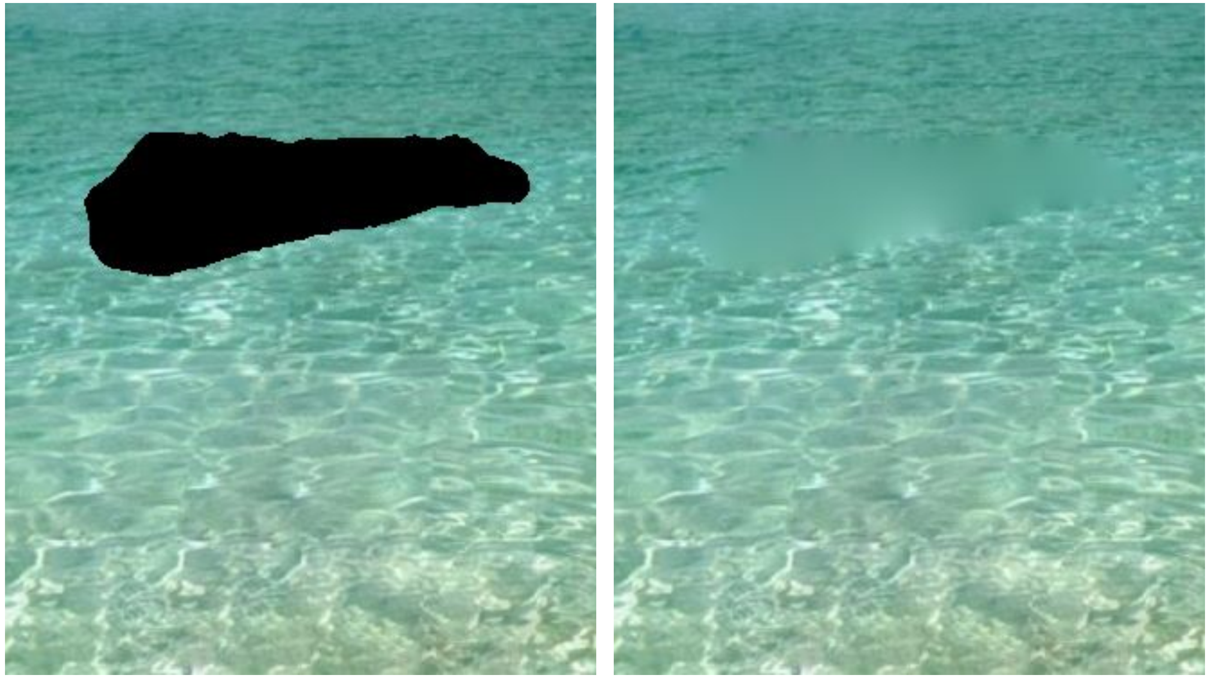
It is impossible to faithfully preserve the gradients

Membrane interpolation

- **Laplace equation (a.k.a. membrane equation)**

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

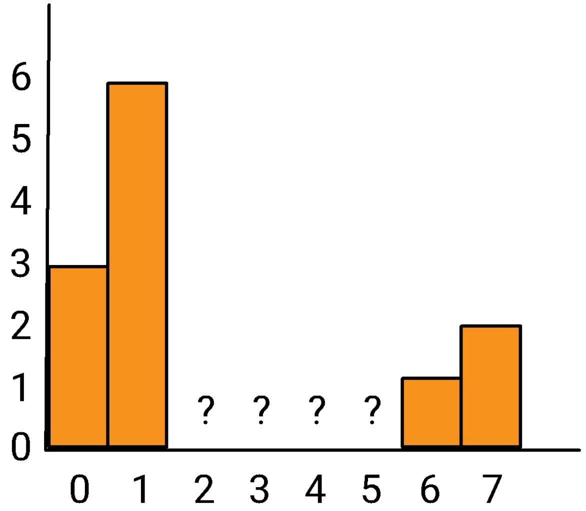




Slide credit: F. Durand

1D example: minimization

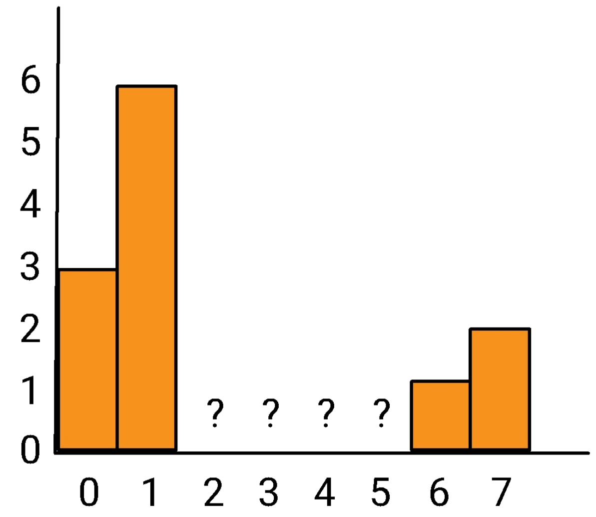
- **Minimize derivatives to interpolate**



Slide credit: F. Durand

1D example: minimization

- **Minimize derivatives to interpolate**

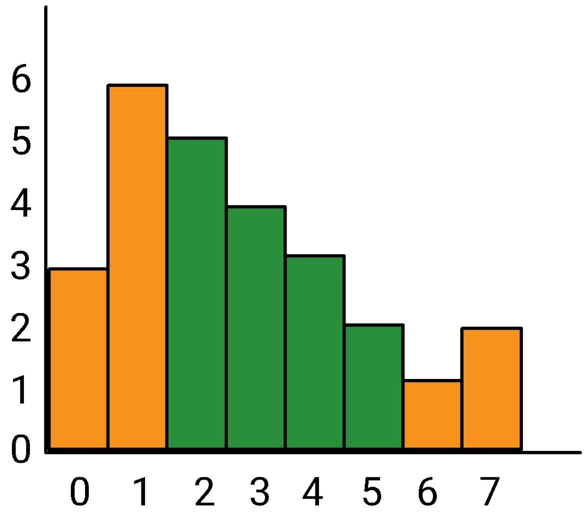


$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Slide credit: F. Durand

1D example

- Minimize derivatives to interpolate
- Pretty much says that second derivative should be zero
- (-1 2 -1) is a second derivative filter



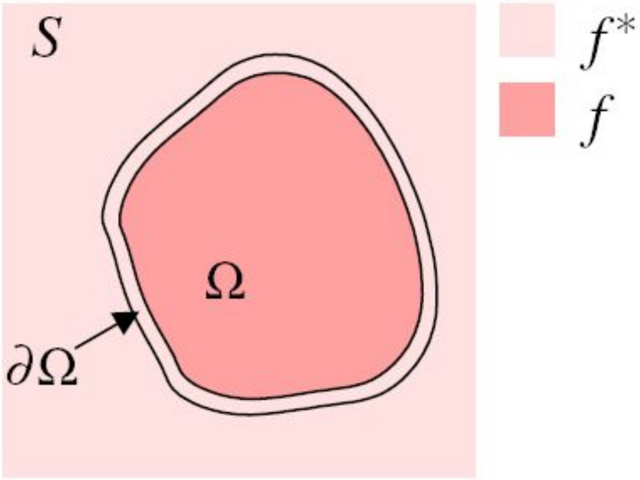
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

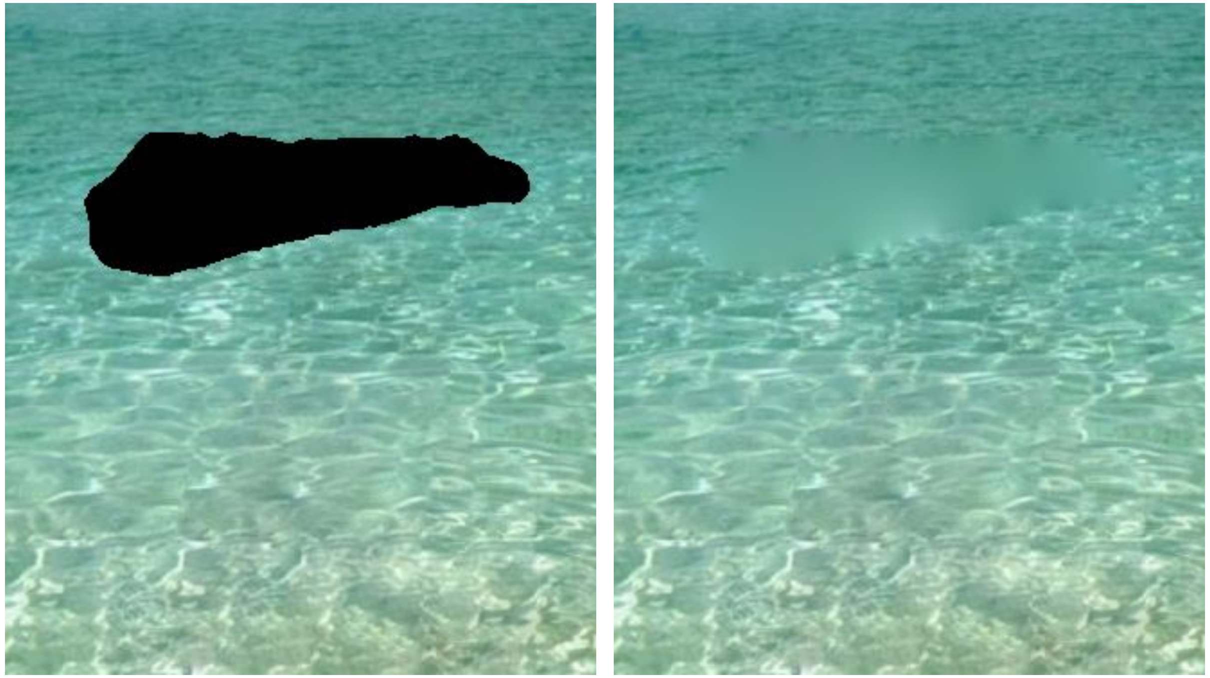
Slide credit: F. Durand

Membrane interpolation

- **Laplace equation (a.k.a. membrane equation)**

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$





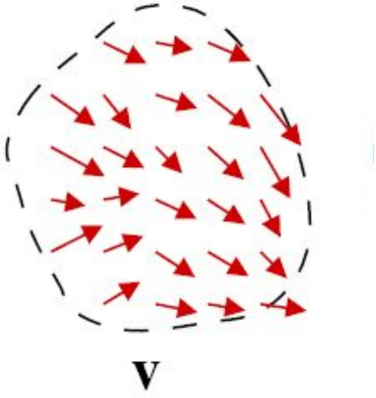
Slide credit: F. Durand

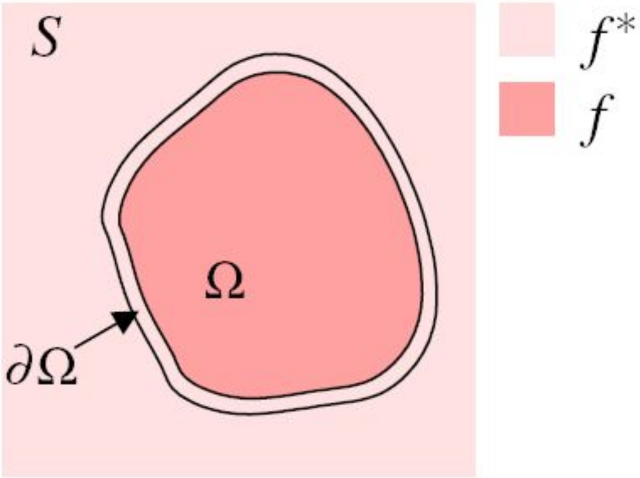
Seamless Poisson cloning

- **Given vector field *v* (pasted gradient), find the value of *f* in unknown region that optimize:**
- **Previously, v was null**

$$\min_f \iint_\Omega |\nabla f - \mathbf{v}|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

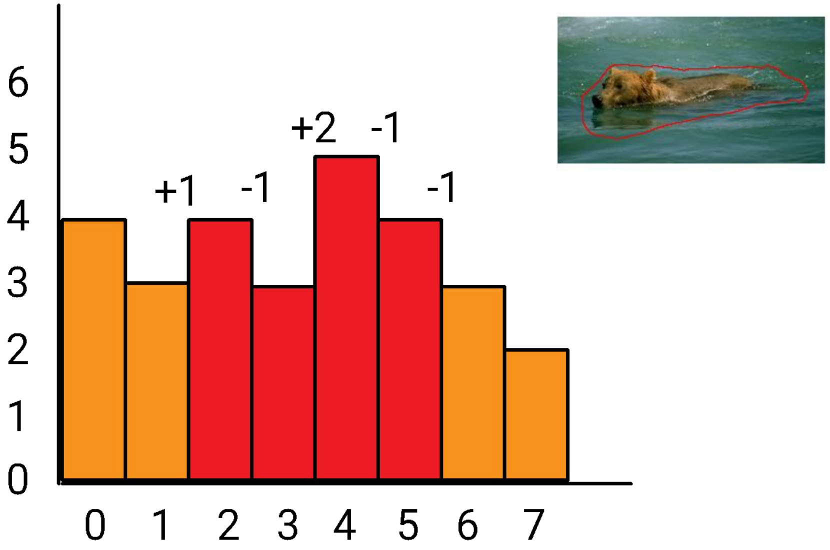
Pasted gradient

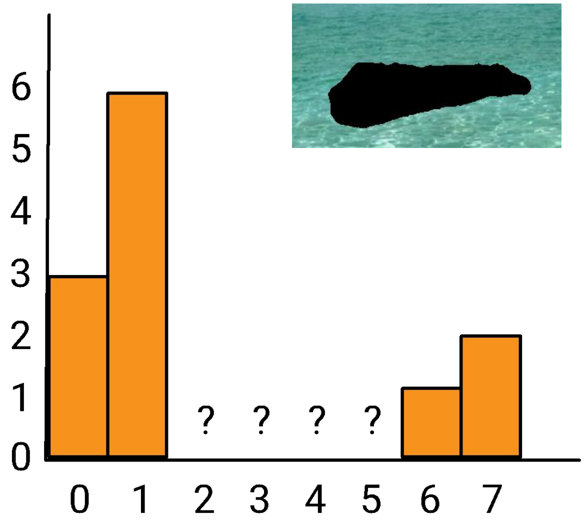




Slide credit: F. Durand

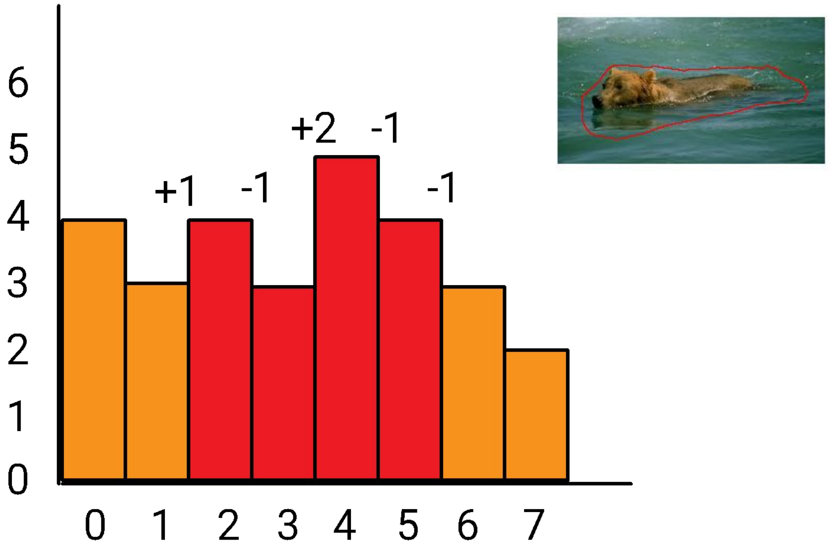
Discrete 1D example: minimization

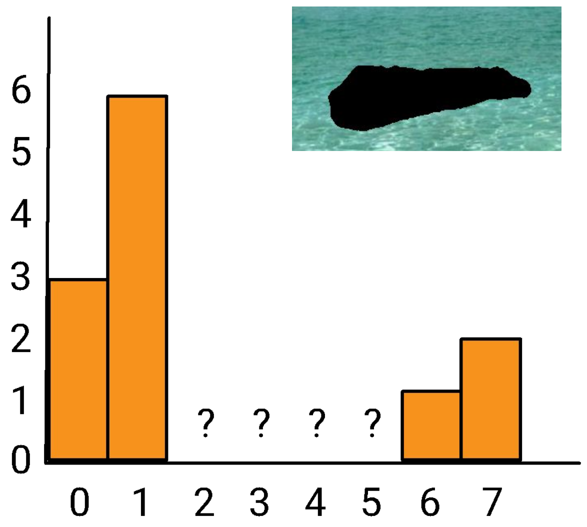




Slide credit: F. Durand

Discrete 1D example: minimization

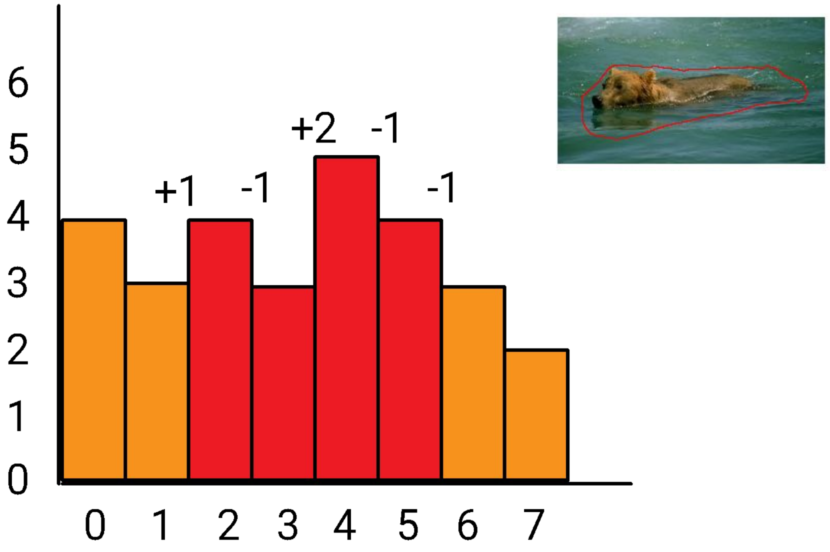


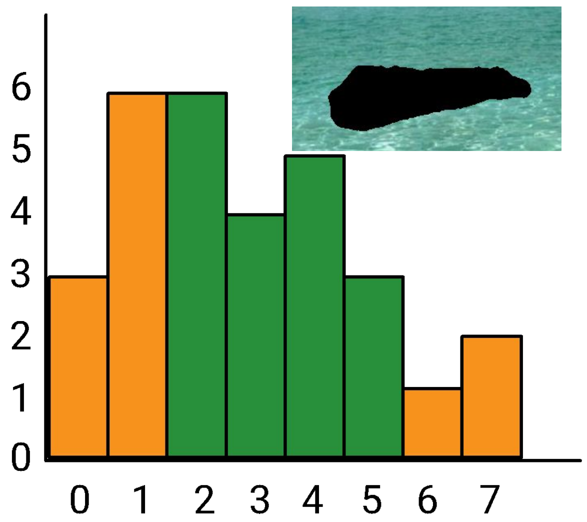


$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \\ -3 \\ 3 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Slide credit: F. Durand

Discrete 1D example: minimization





$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \\ -3 \\ 3 \\ 0 \\ 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 6 \\ 4 \\ 5 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

Slide credit: F. Durand

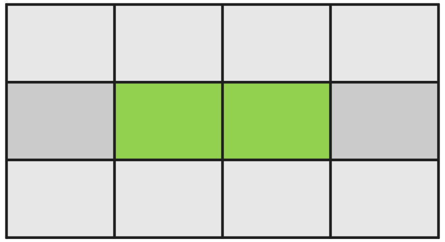
Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s



mask

?	?	?	?
?	?	?	?
?	?	?	?

output, x

What properties do we want x to have?

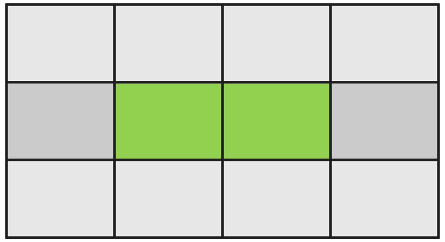
Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s



mask

?	?	?	?
?	?	?	?
?	?	?	?

output, x

(1) For unmasked pixels, $x_i = t_i$

(2) For masked pixels, we want the laplacian at x_i to match the laplacian at s_i

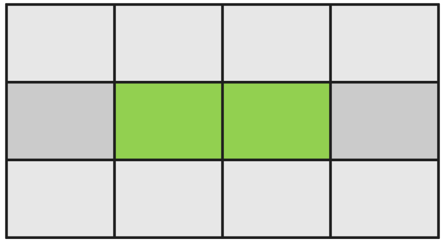
Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s



mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

$$x_1 = t_1$$
$$x_2 = t_2$$
$$x_3 = t_3$$
$$x_4 = t_4$$
$$4x_5 - x_4 - x_2 - x_6 - x_8 = 4s_5 - s_4 - s_2 - s_6 - s_8$$
$$x_6 = t_6$$
$$x_7 = t_7$$
$$4x_8 - x_7 - x_5 - x_9 - x_{11} = 4s_8 - s_7 - s_5 - s_9 - s_{11}$$
$$x_9 = t_9$$
$$x_{10} = t_{10}$$
$$x_{11} = t_{11}$$
$$x_{12} = t_{12}$$

Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s

mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

$$x_1 = 0.2$$
$$x_2 = 0.7$$
$$x_3 = 0.9$$
$$x_4 = 0.5$$
$$4x_5 - x_4 - x_2 - x_6 - x_8 = 0.2$$
$$x_6 = 0.9$$
$$x_7 = 0.2$$
$$4x_8 - x_7 - x_5 - x_9 - x_{11} = -1.6$$
$$x_9 = 0.8$$
$$x_{10} = 0.2$$
$$x_{11} = 0.7$$
$$x_{12} = 0.9$$

Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s

mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

$$
\begin{aligned}
x_1 &= 0.2 \\
x_2 &= 0.7 \\
x_3 &= 0.9 \\
x_4 &= 0.5 \\
4x_5 - x_4 - x_2 - x_6 - x_8 &= 0.2 \\
x_6 &= 0.9 \\
x_7 &= 0.2 \\
4x_8 - x_7 - x_5 - x_9 - x_{11} &= -1.6 \\
x_9 &= 0.8 \\
x_{10} &= 0.2 \\
x_{11} &= 0.7 \\
x_{12} &= 0.9
\end{aligned}
$$

In this simple case, we could solve for everything by hand.

$$
\begin{aligned}
4x_5 - 0.5 - 0.7 - 0.9 - x_8 &= 0.2 \\
4x_8 - 0.2 - x_5 - 0.8 - 0.7 &= -1.6
\end{aligned}
$$

$$
\begin{aligned}
4x_5 - x_8 &= 2.3 \\
4x_8 - x_5 &= 0.1
\end{aligned}
$$

$$
\begin{aligned}
x_5 &= 0.62 \\
x_8 &= 0.18
\end{aligned}
$$

Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s

mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

1											
	1										
		1									
			1								
	-1		-1	4	-1		-1				
					1						
						1					
				-1		-1	4	-1		-1	
								1			
									1		
										1	
											1

A

*

?
?
?
?
?
?
?
?
?
?
?
?

x

=

.2
.7
.9
.5
.2
.9
.2
-1.6
.8
.2
.7
.9

b

Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s

mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

$$A * x = b$$

$$A^{-1}A * x = A^{-1}b$$

$$x = A^{-1}b$$

1											
	1										
		1									
			1								
	-1		-1	4	-1		1				
					1						
						1					
				-1		-1	4	-1		-1	
								1			
									1		
										1	
											1

A

*

?
?
?
?
?
?
?
?
?
?
?
?

x

=

.2
.7
.9
.5
.2
.9
.2
-1.6
.8
.2
.7
.9

b

Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s

mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

?	?	?	?
?	?	?	?
?	?	?	?

output, x

1	4	7	10
2	5	8	11
3	6	9	12

Pixel indexing

1											
	1										
		1									
			1								
	-1		-1	4	-1		-1				
					1						
						1					
				-1		-1	4	-1		-1	
								1			
									1		
										1	
											1

A

*

.2
.7
.9
.5
.62
.9
.2
.18
.8
.2
.7
.9

x

=

.2
.7
.9
.5
.2
.9
.2
-1.6
.8
.2
.7
.9

b

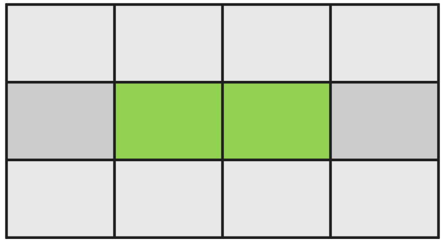
Simple 2d example

.2	.5	.2	.2
.7	.7	.7	.7
.9	.9	.8	.9

target, t

.8	.6	.6	.6
.6	.6	.2	.6
.6	.8	.6	.6

source, s



mask

0	-1	0
-1	4	-1
0	-1	0

Laplacian

.2	.5	.2	.2
.7	.62	.18	.7
.9	.9	.8	.9

output, x

1	4	7	10
2	5	8	11
3	6	9	12

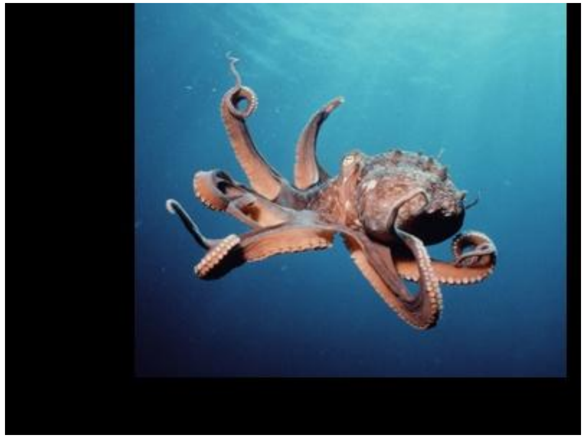
Pixel indexing

.2
.7
.9
.5
.62
.9
.2
.18
.8
.2
.7
.9

x

Example

target



source

mask

no blending

gradient domain blending

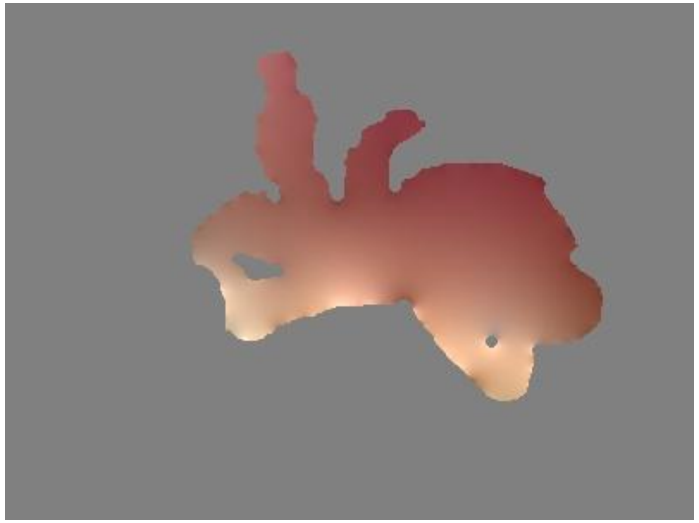
What's the difference?

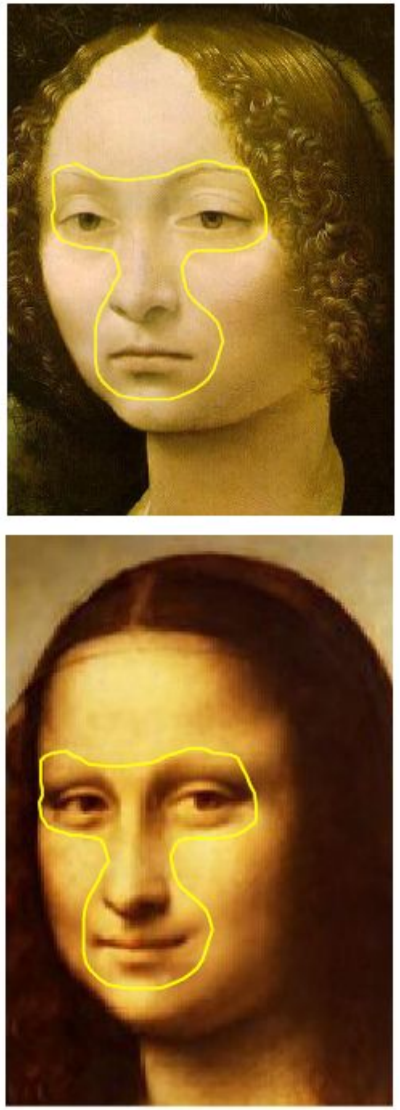

gradient domain blending

-

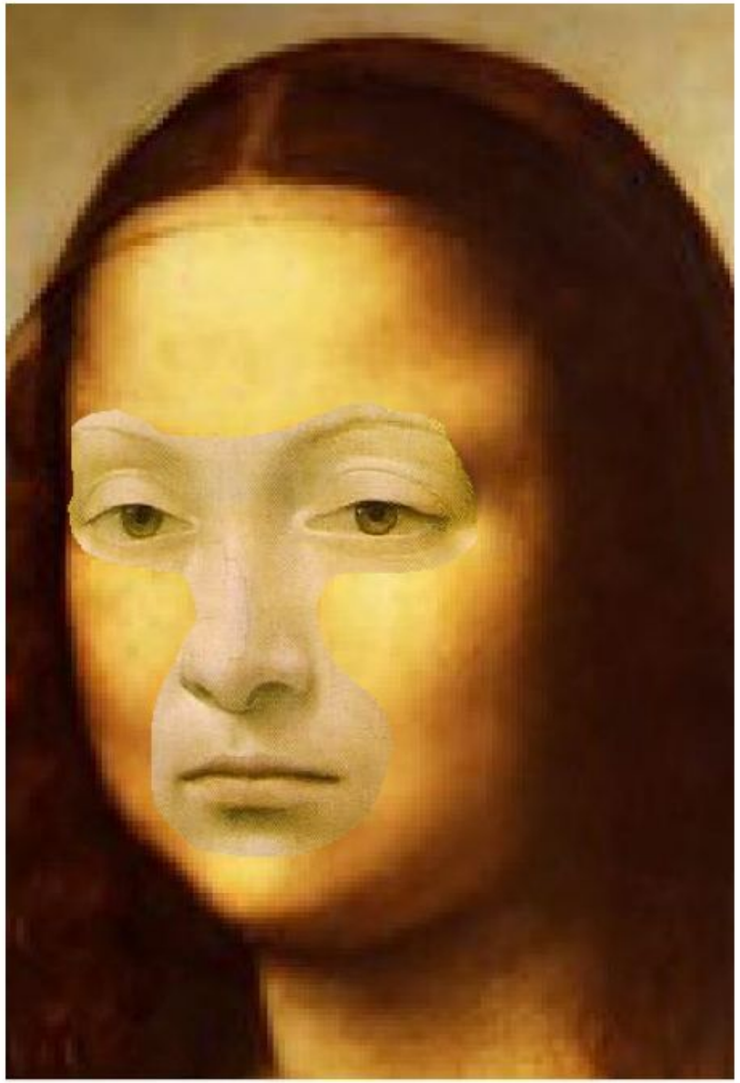

no blending

=

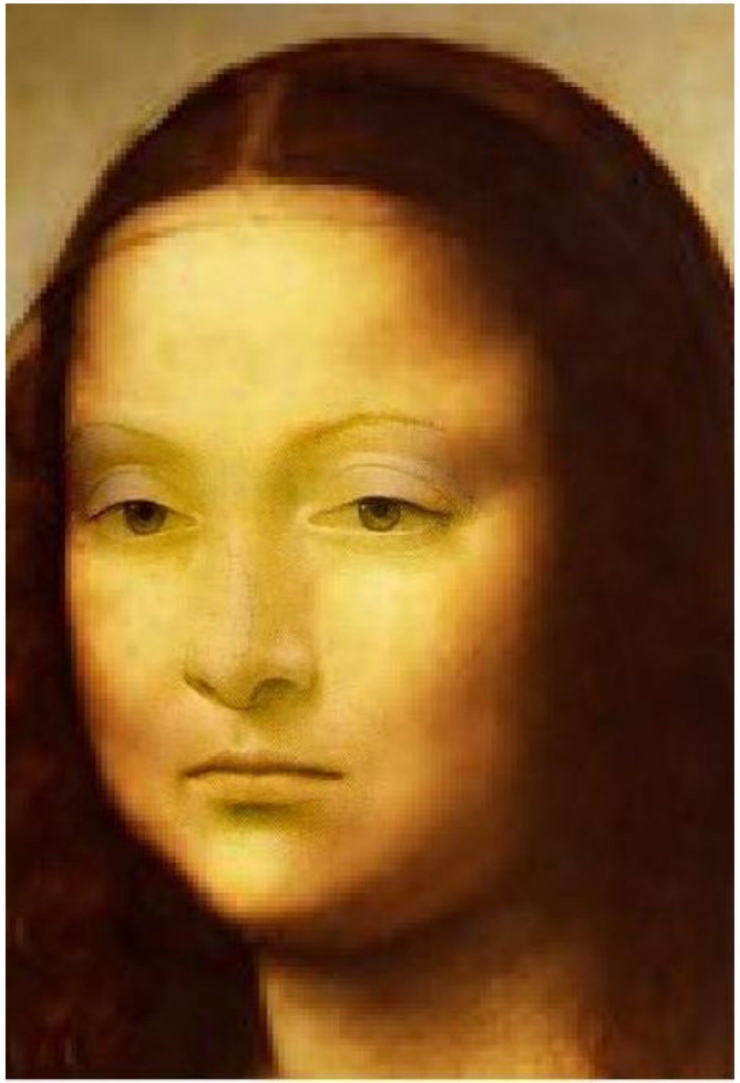




source/destination



cloning



seamless cloning

Perez et al. SIGGRAPH 03

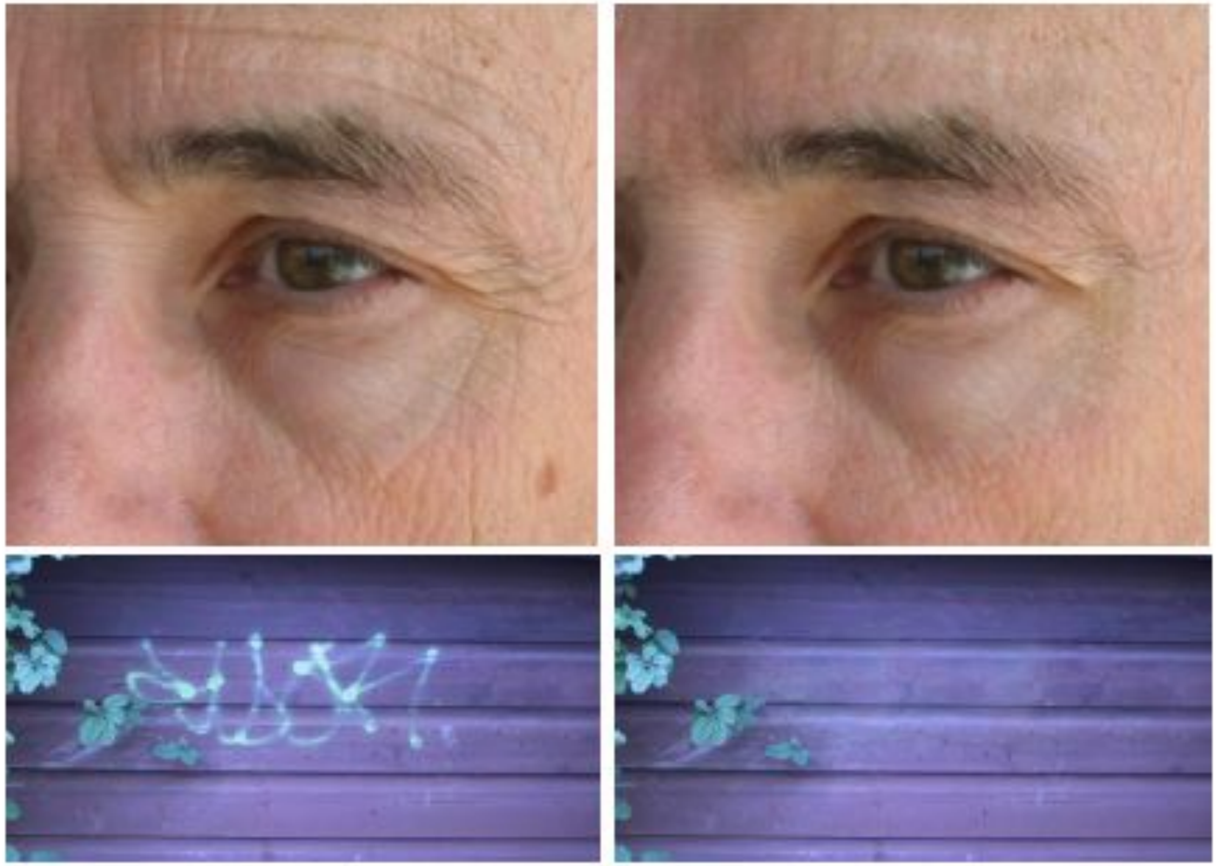


Figure 2: **Concealment**. By importing seamlessly a piece of the background, complete objects, parts of objects, and undesirable artifacts can easily be hidden. In both examples, multiple strokes (not shown) were used.

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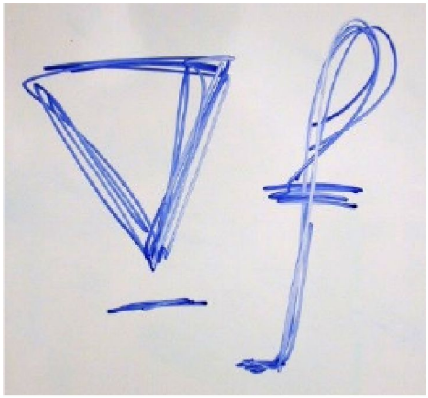
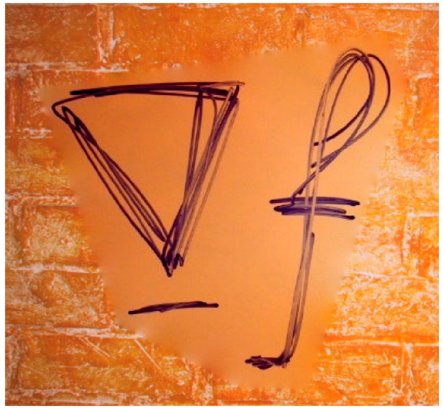
Mixing gradients





Source

Average

Mixed