

CSCE 448/748 – Computational Photography

Camera and Image Formation

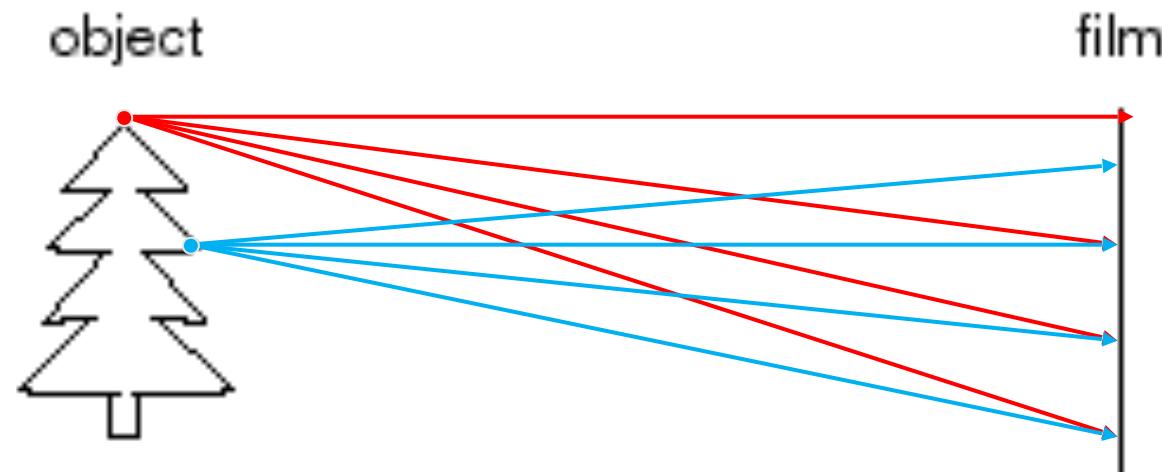
Nima Kalantari

Outline

- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

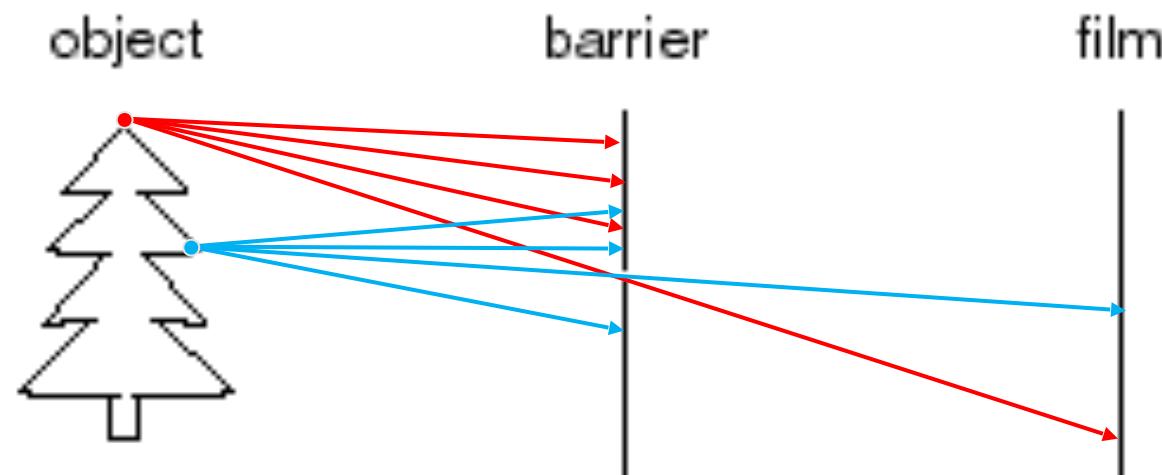
How do we see the world?

- **Let's design a camera**
 - **Put a piece of film in front of an object**
 - **Do we get a reasonable image?**

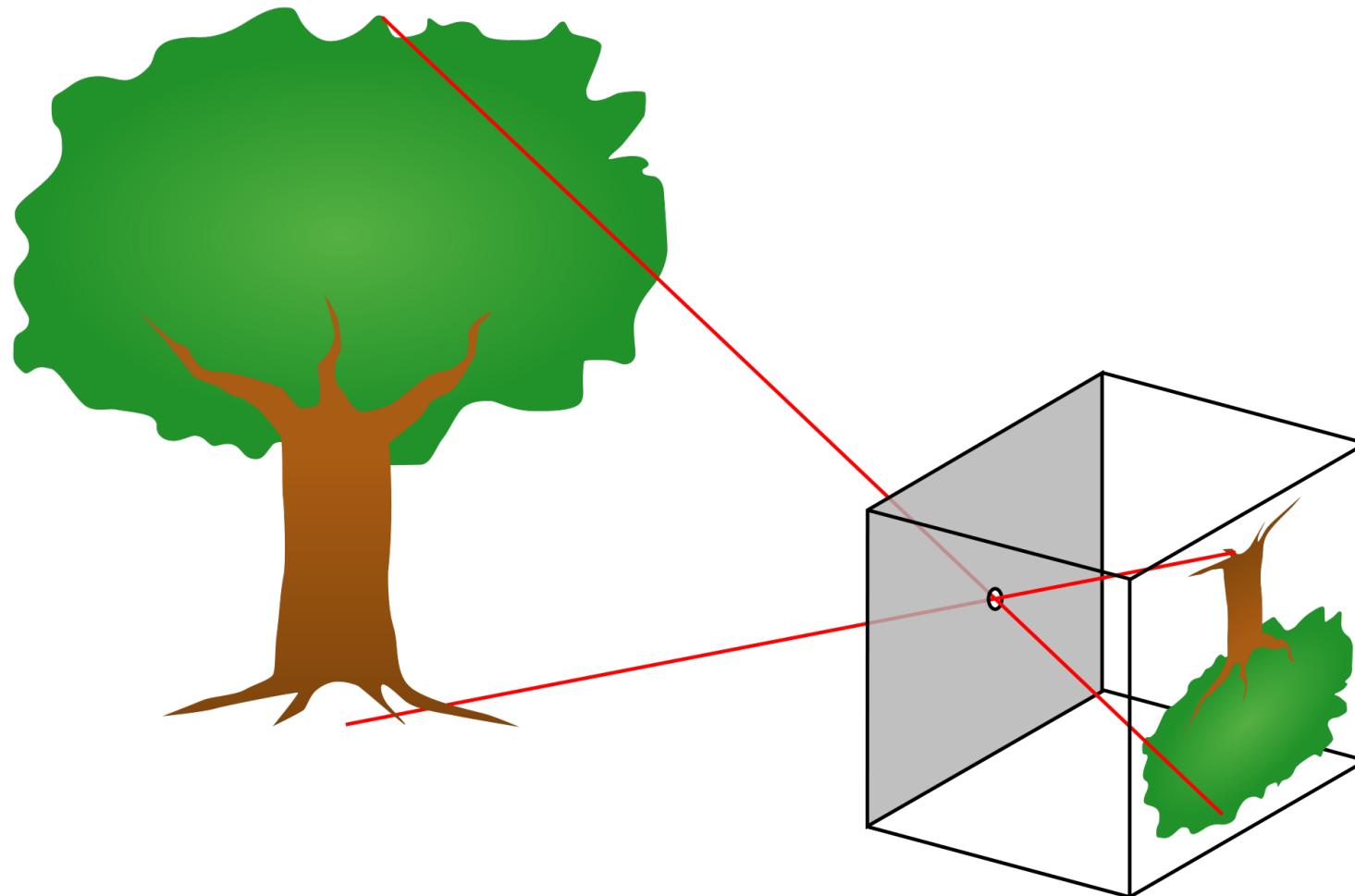


Pinhole camera

- **Add a barrier to block off most of the rays**
 - **This reduces blurring**
 - **The opening known as the aperture**
 - **How does this transform the image?**



Pinhole camera



Pinhole camera model

➤ Pinhole model:

- Captures pencil of rays – all rays through a single point
- The point is called Center of Projection (COP)
- The image is formed on the Image Plane (film/sensor)
- Image distance d is distance from COP to Image Plane

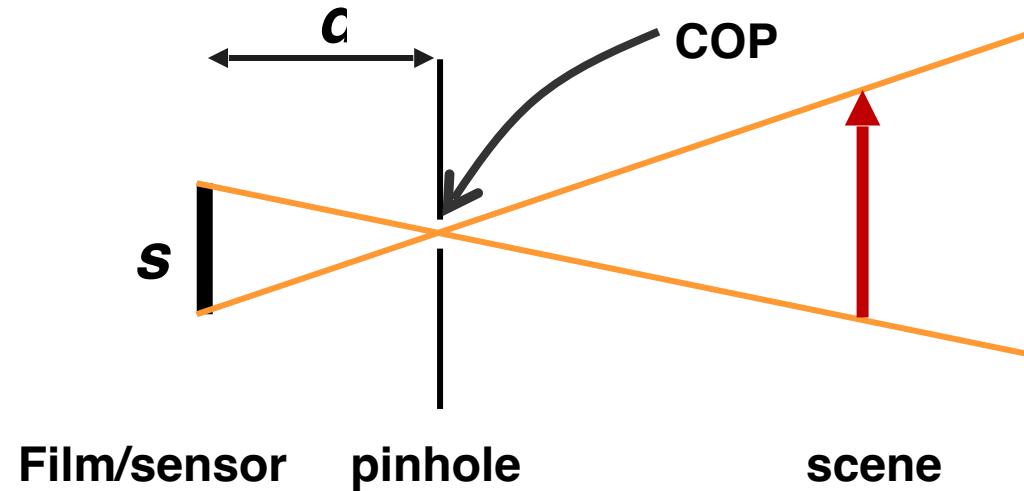
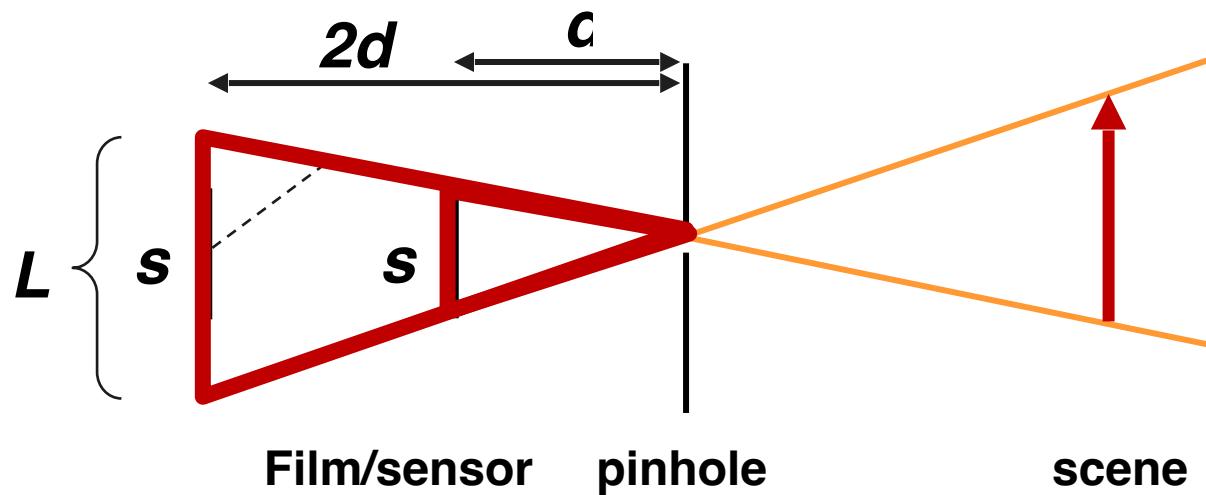


Image distance: pinhole optics

- What happens when the image distance is doubled?
 - Projected object height **is doubled**

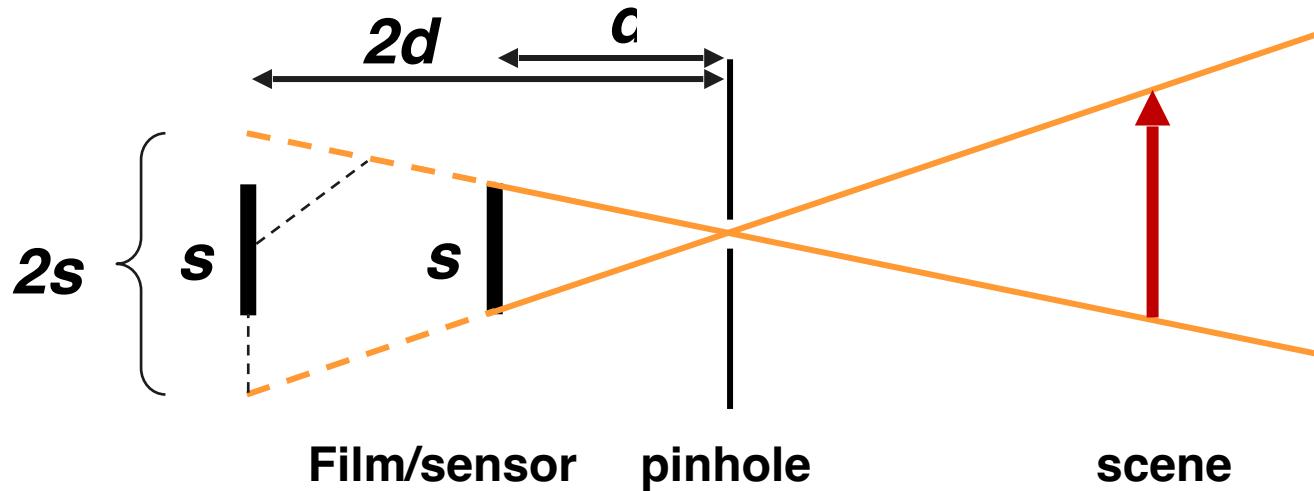
s = projected object height on sensor at distance d
L = projected object height on sensor at distance 2d



$$\frac{L}{s} = \frac{2d}{d} = 2$$

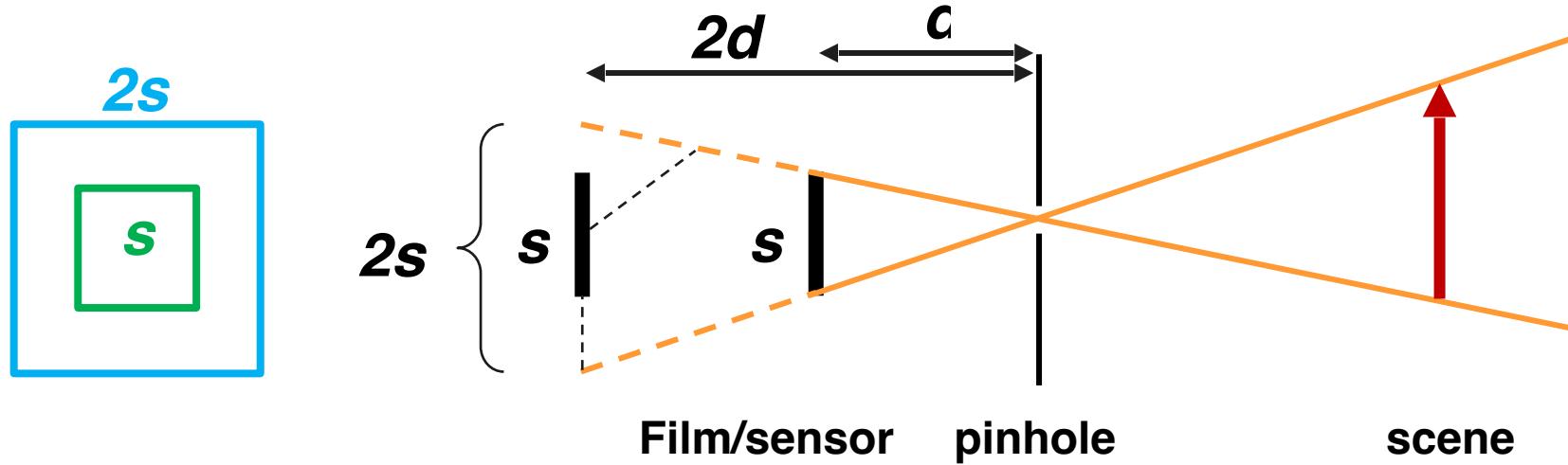
Image distance: pinhole optics

- What happens when the image distance is doubled?
 - Projected object height **is doubled**
 - Amount of light gathered



Amount of light gathered

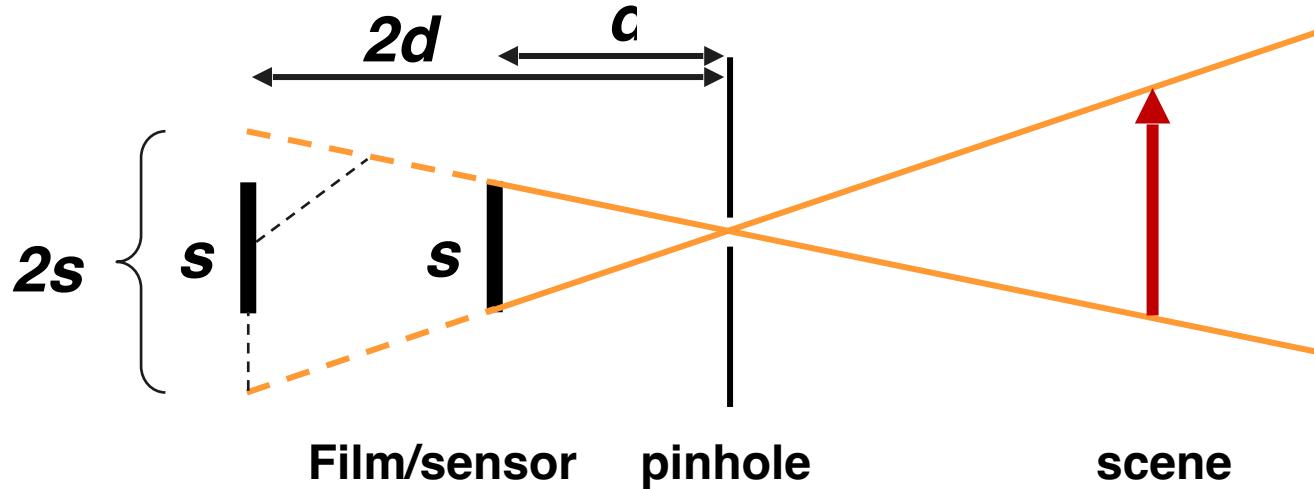
- Let's say a sensor of size s at distance d receives 1 unit of light
- How much light sensor of size $2s$ at distance $2d$ receives?
 - 1 unit of light
- How about a sensor of size s at distance $2d$?



$$\frac{s^2}{(2s)^2} = \frac{x}{1}$$
$$x = \frac{1}{4}$$

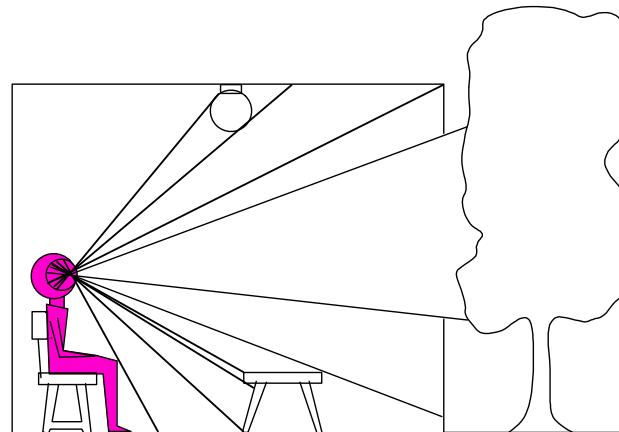
Image distance: pinhole optics

- What happens when the image distance is doubled?
 - Projected object height **is doubled**
 - Amount of light gathered **is divided by 4**



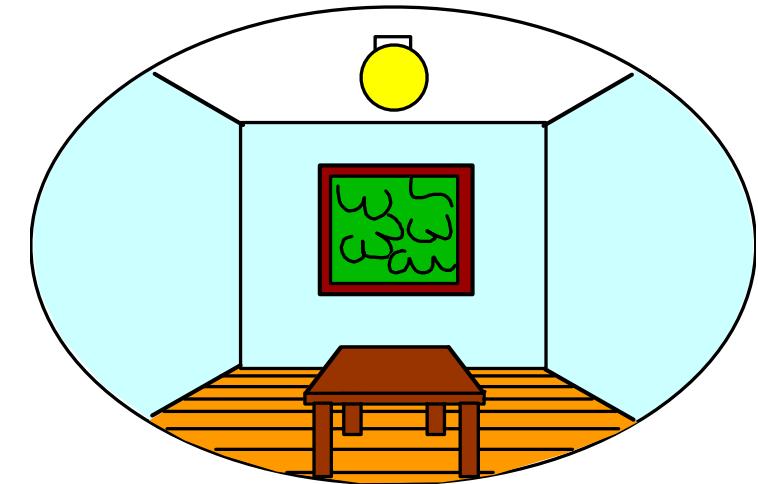
Dimensionality Reduction Machine (3D to 2D)

- But there is a problem ...



Point of observation

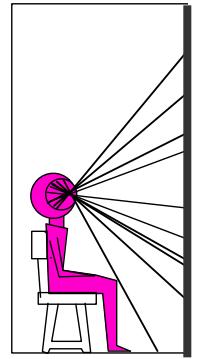
3D world



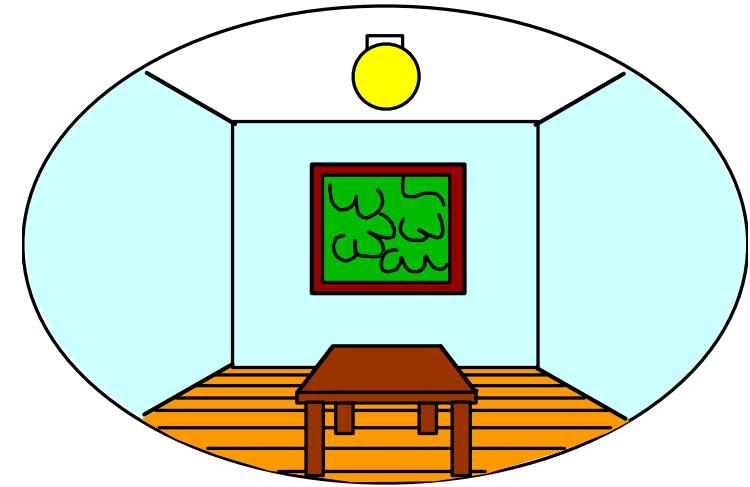
2D image

How we see the world

Painted backdrop



3D world



2D image

Fooling the eye



CoolOpticalIllusions.com

JULIAN BEEVER

Fooling the eye

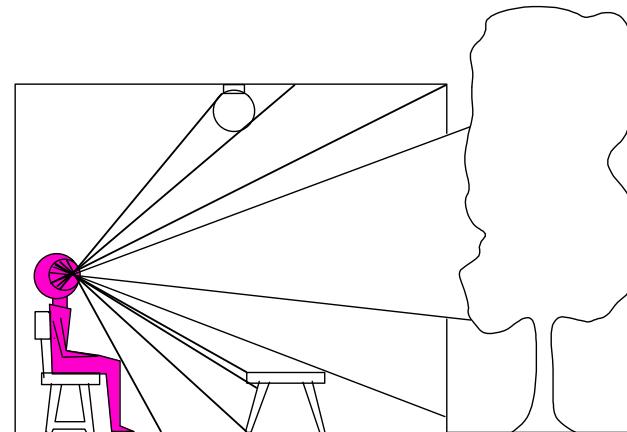


Making of 3D sidewalk art: <http://www.youtube.com/watch?v=3SNYtd0Ayt0>

Dimensionality Reduction Machine (3D to 2D)

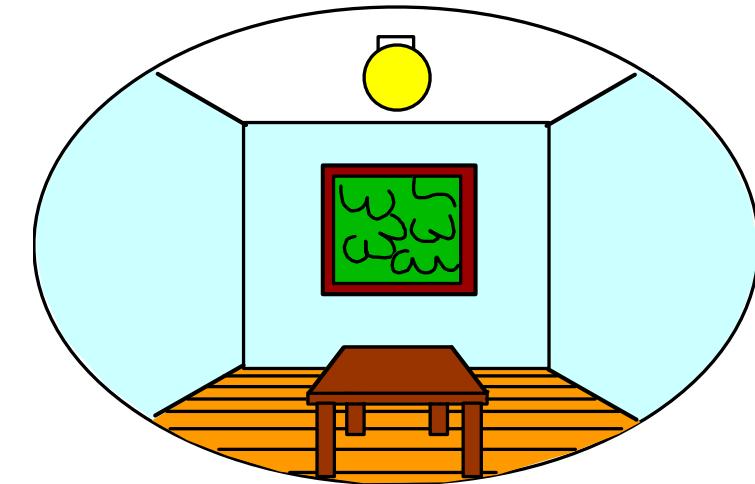
➤ What have we lost?

- Depth
- lengths
- Angles



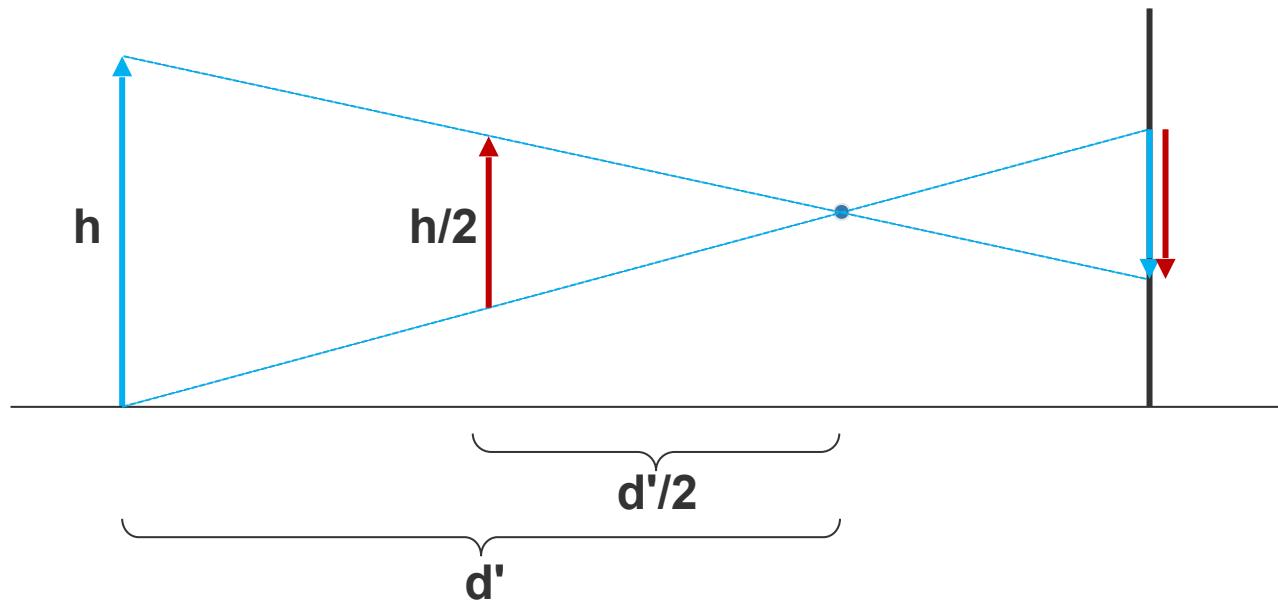
Point of observation

3D world

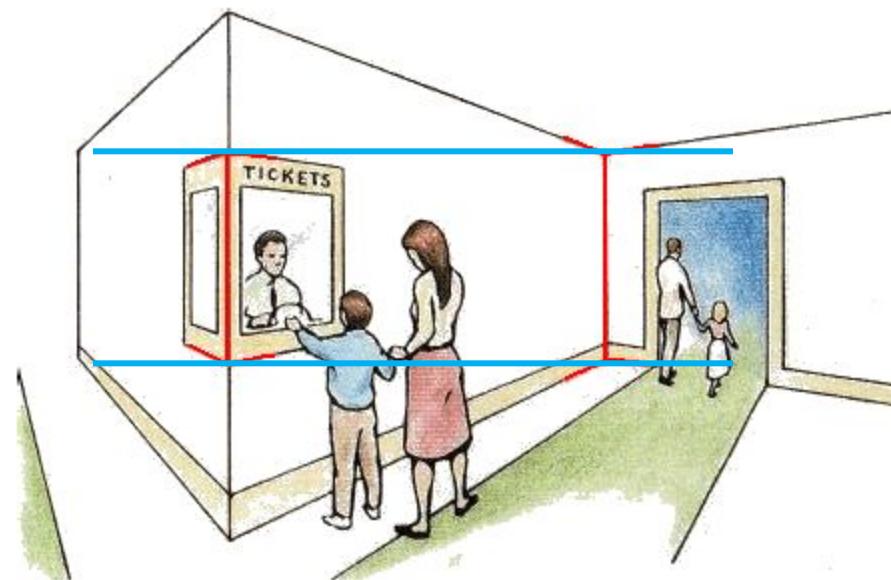
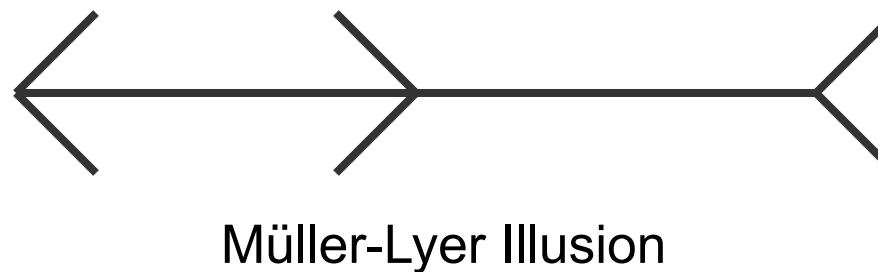


2D image

Lengths can't be trusted...

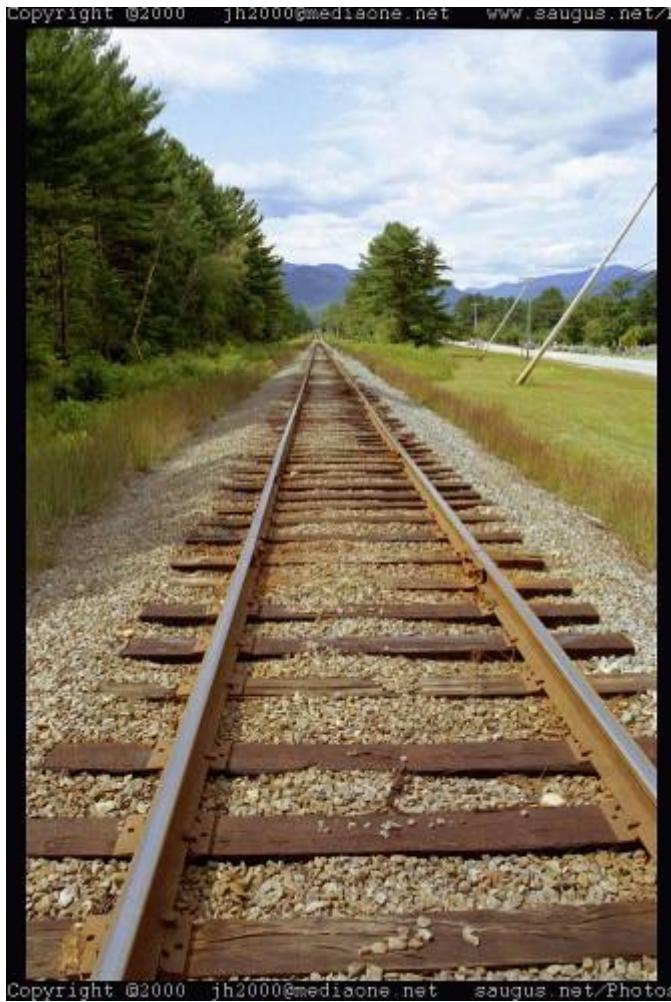


...but humans adapt!



We do not make measurements in the image plane

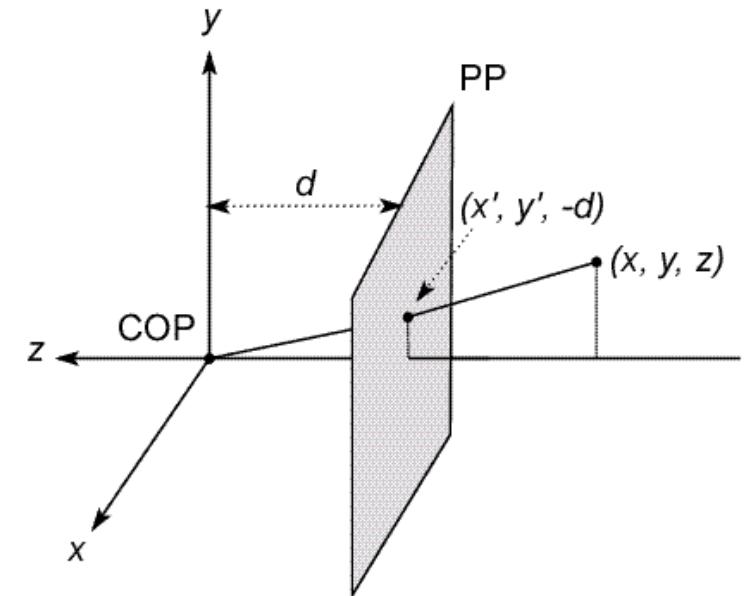
Angles can't be trusted...



Modeling projection (perspective)

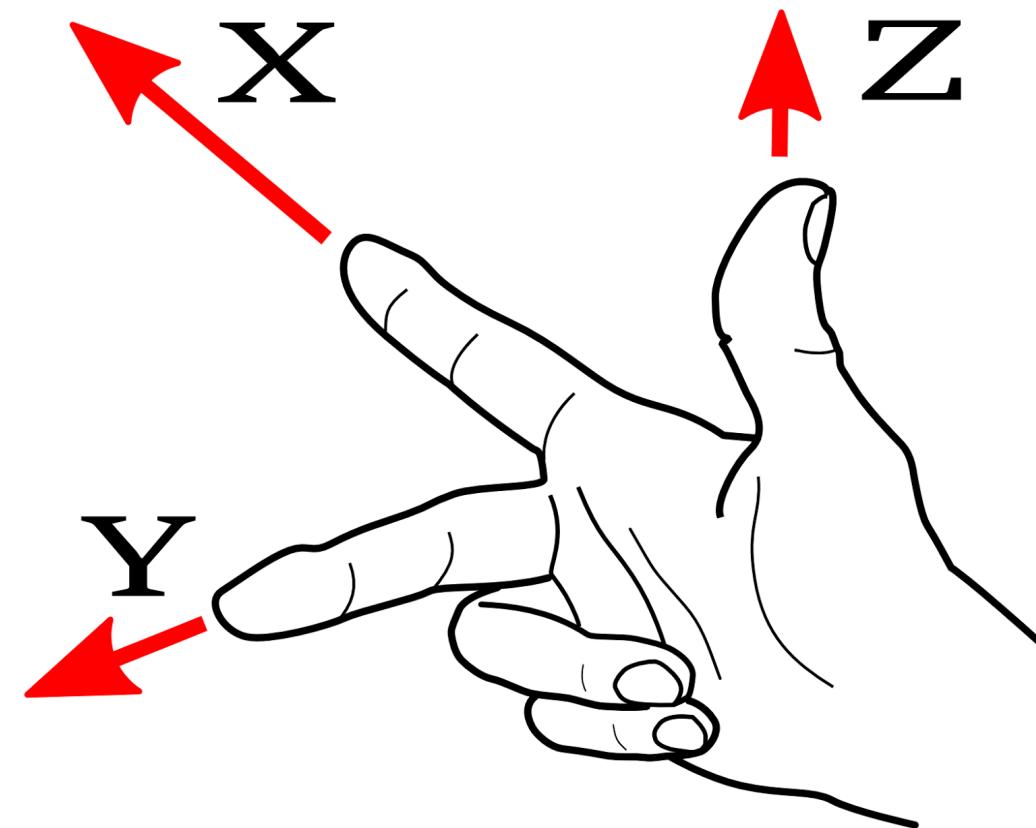
➤ The coordinate system

- We will use the pinhole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- Put the image plane (Projection Plane) in front of the COP
 - Why?
- The camera looks down the negative z axis
 - We need this if we want right-handed-coordinates

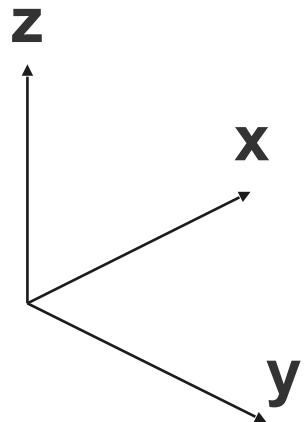


Left- and right-hand coordinates

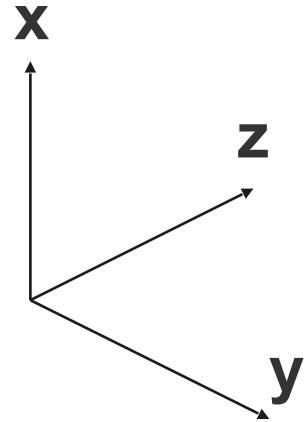
- With left hand: left hand coordinate
- With right hand: right hand coordinate



Examples



Left-handed



Right-handed

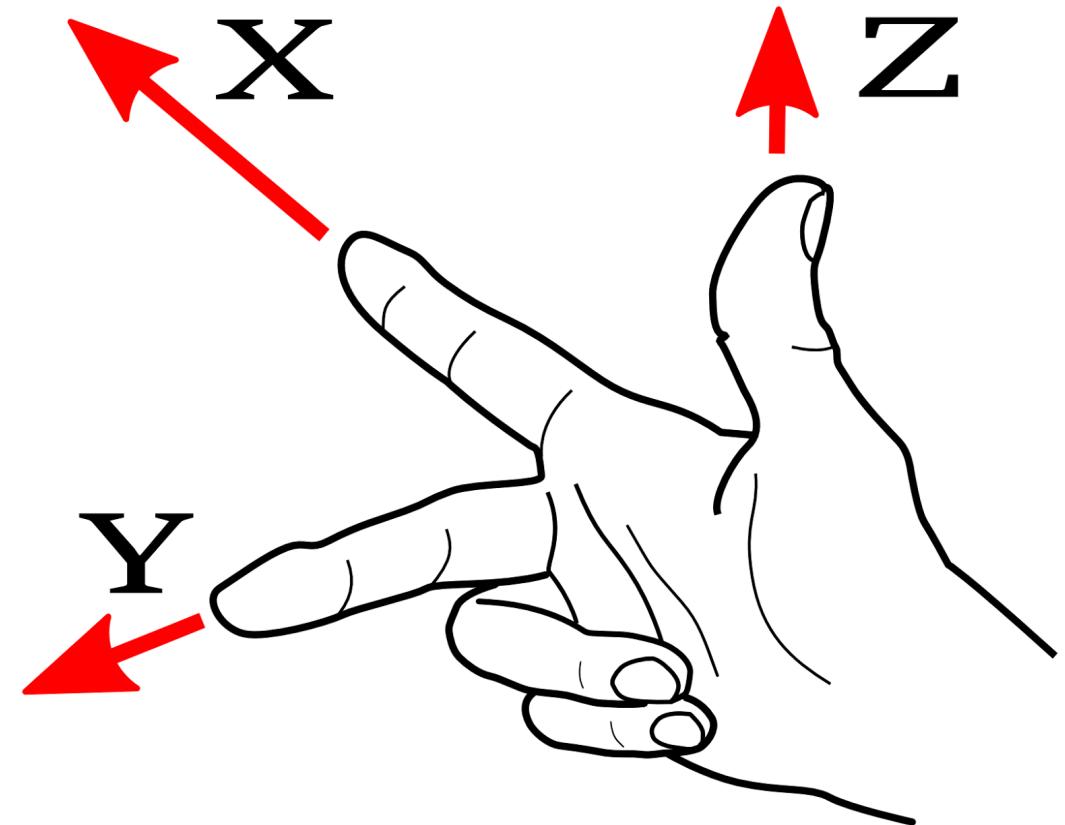


Image from Drew Noaks

Modeling projection (perspective)

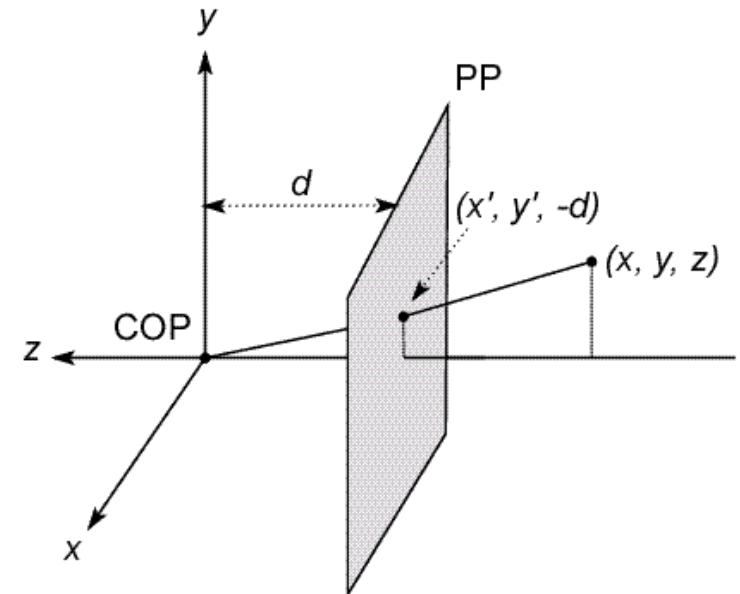
➤ Projection equations

- Compute intersection with PP of ray from (x, y, z) to COP
- Derived using similar triangles

$$(x, y, z) \rightarrow \left(-d \frac{x}{z}, -d \frac{y}{z}, -d \right)$$

- We get the projection by ignoring the last coordinate:

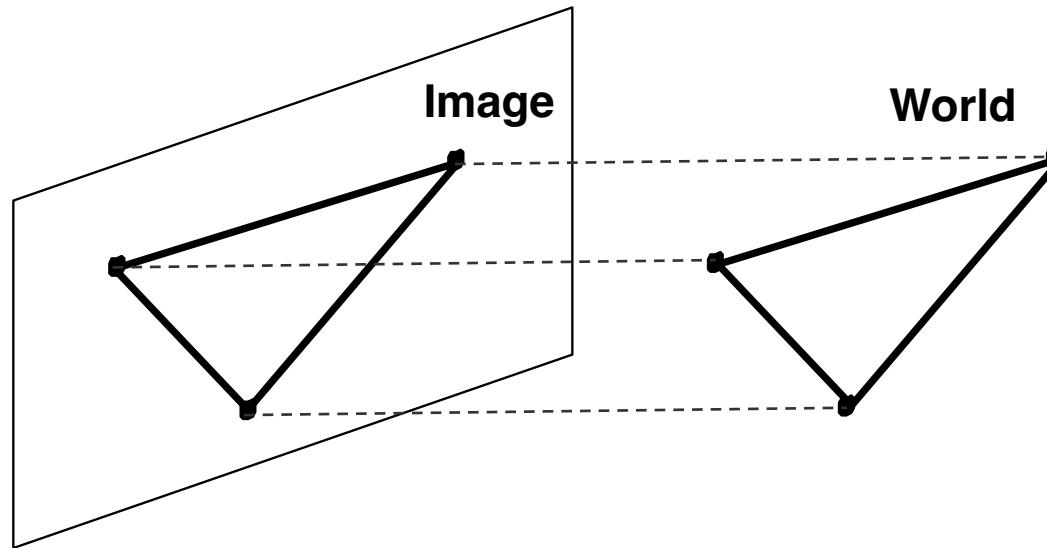
$$(x, y, z) \rightarrow \left(-d \frac{x}{z}, -d \frac{y}{z} \right)$$



Orthographic Projection

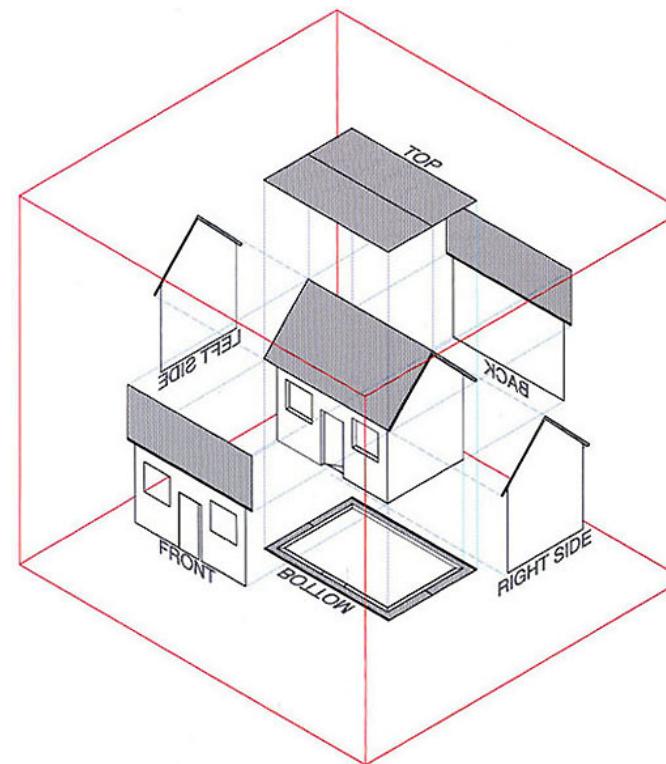
- Drop the z coordinate
- Also called “parallel projection”

- $x' = x$
- $y' = y$



Orthographic Projection

- Parallel lines remain parallel
- Application in technical drawing
- Projection in real cameras is perspective!

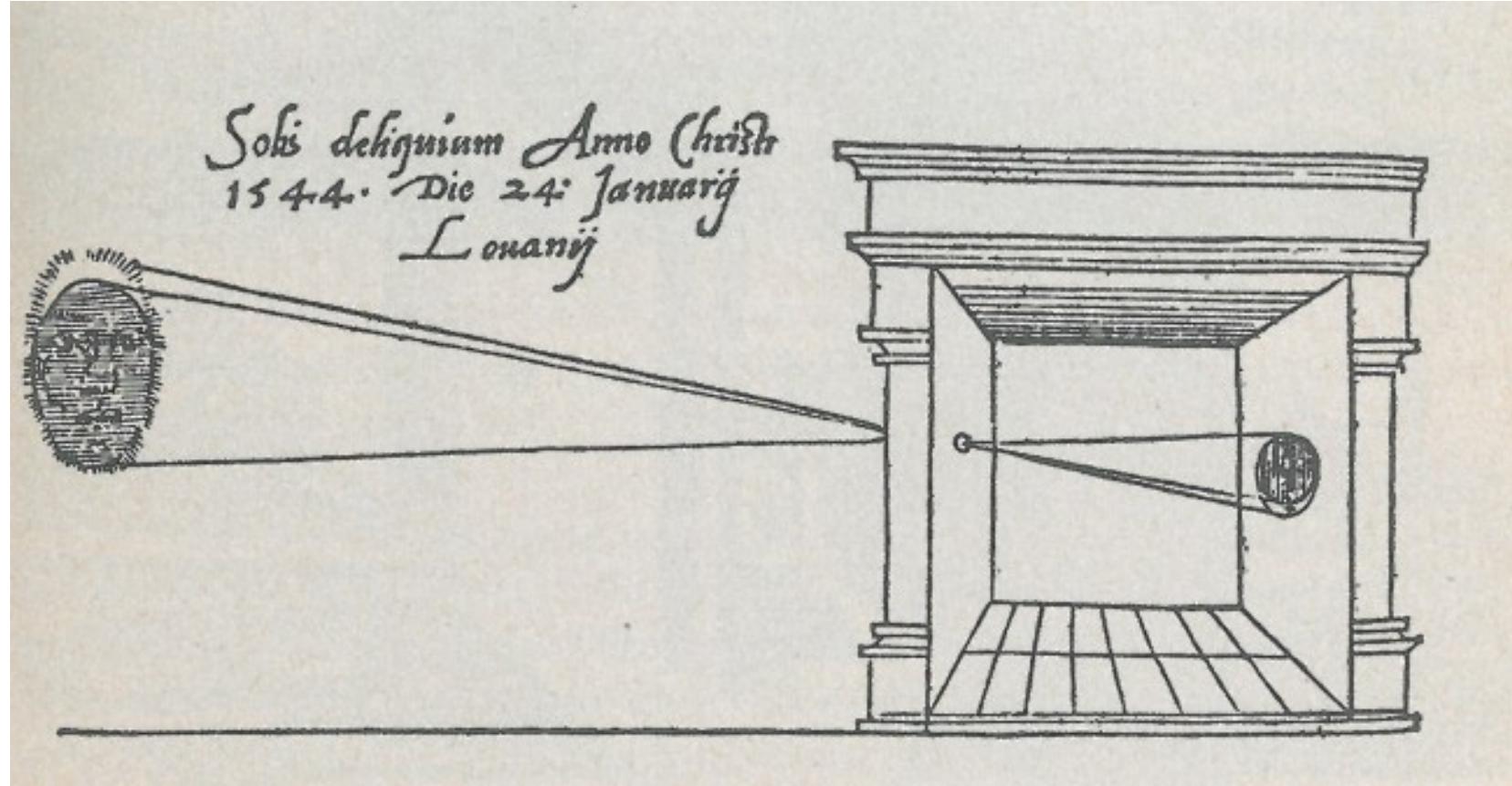


Building a real camera



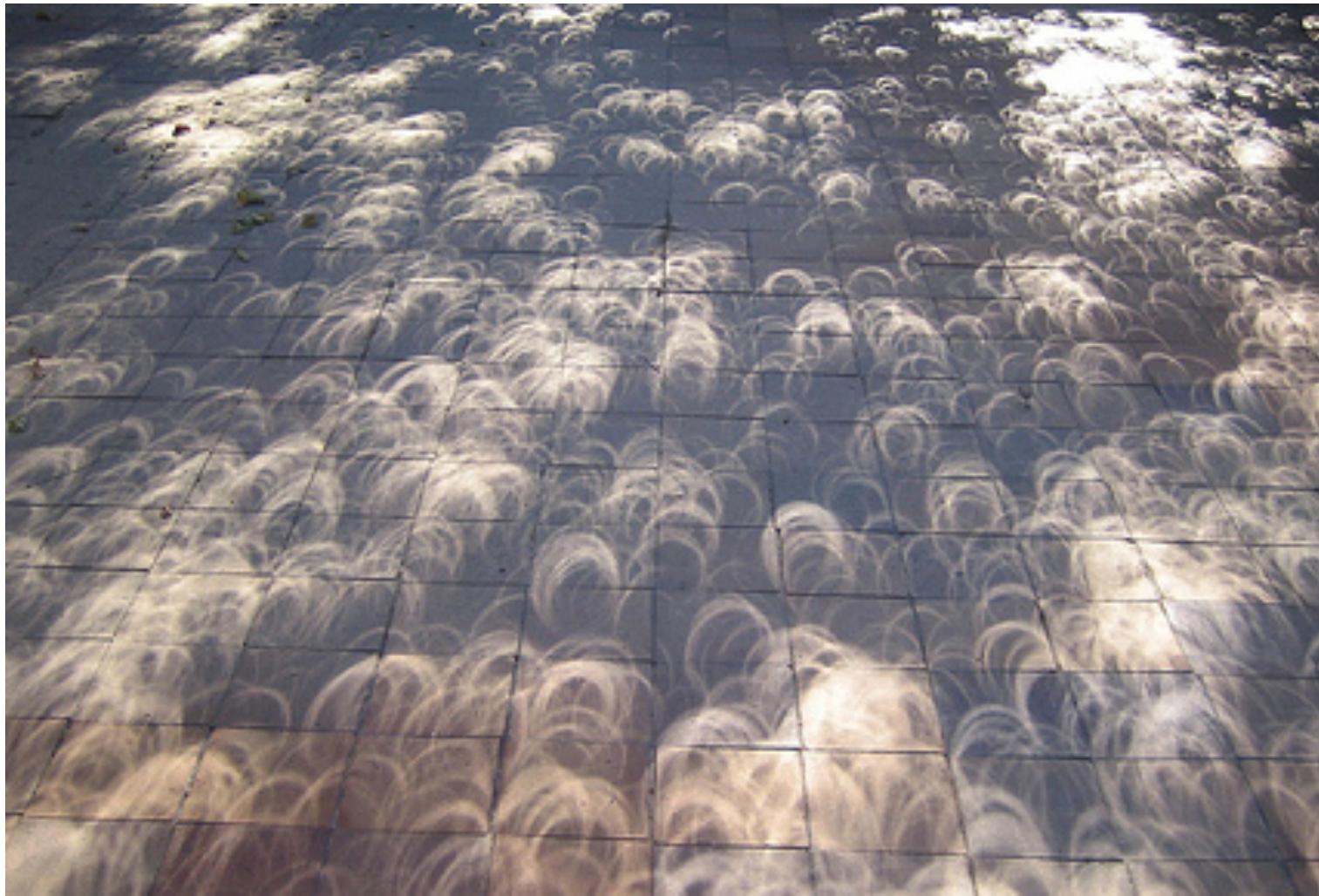
Camera Obscura

First published picture of camera obscura (1545)



Depth of the room is the image distance

Pinhole cameras everywhere



Tree shadow during a solar eclipse

Photo credit: Nils van der Burg <http://www.physicstogo.org/index.cfm>

Accidental pinhole cameras

- Accidental Pinholes produce images that are usually misinterpreted as shadows



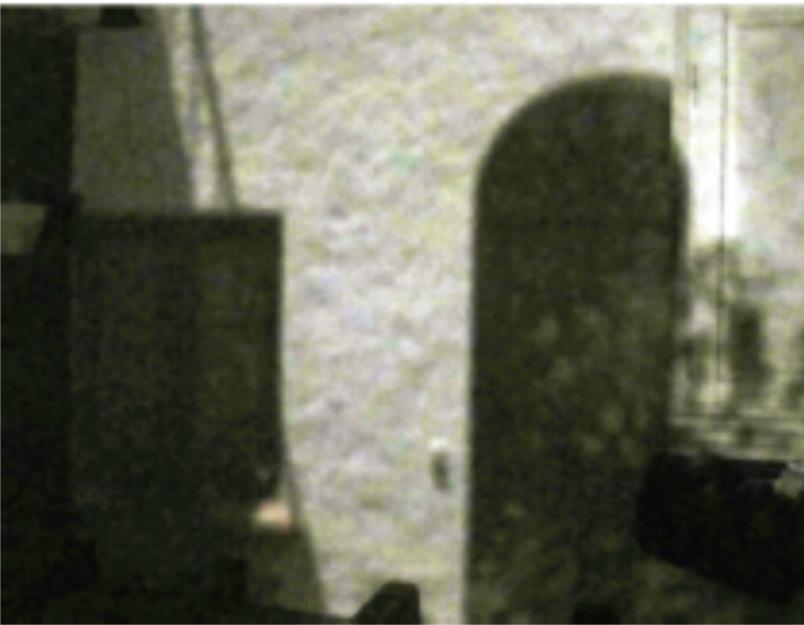
Torralba and Freeman, CVPR'12







Window turned into a pinhole



View outside

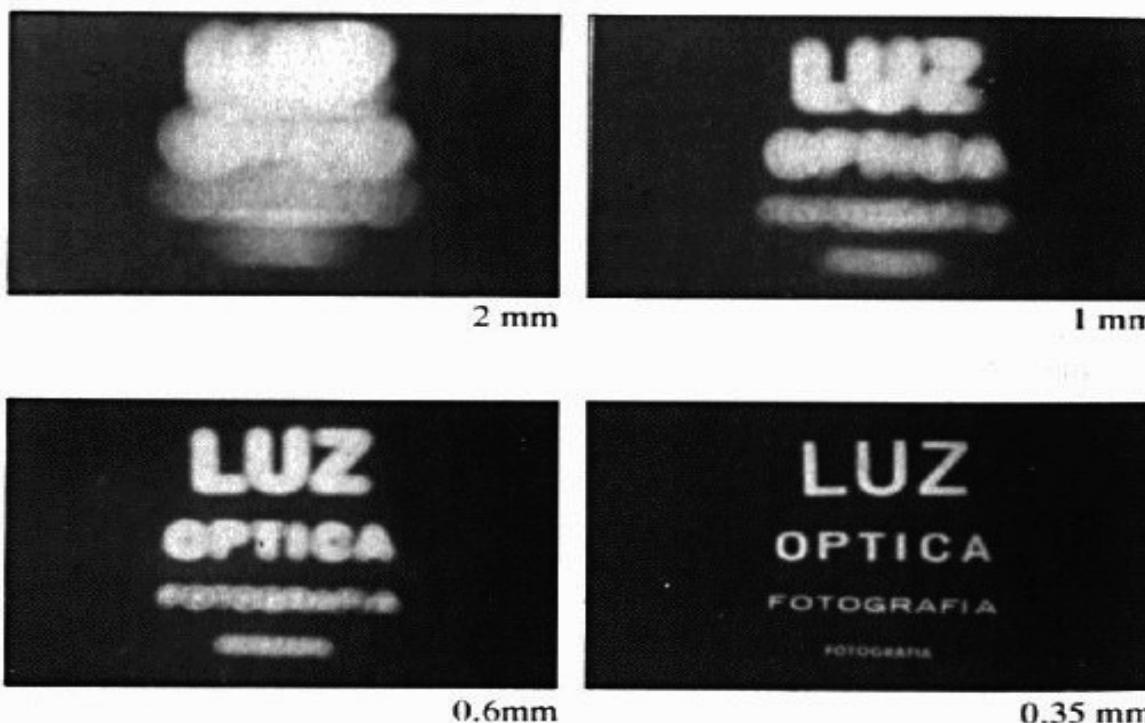


Home-made pinhole camera

- Why is it so blurry?



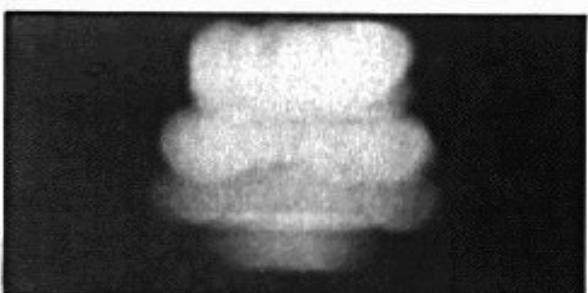
Shrinking the aperture



Shrinking the aperture

- **Why not make the aperture as small as possible?**
 - **Problem 1: Less light gets through**
 - **We can increase the exposure, but causes motion blur**
 - **Problem 2: Diffraction effects**

Shrinking the aperture



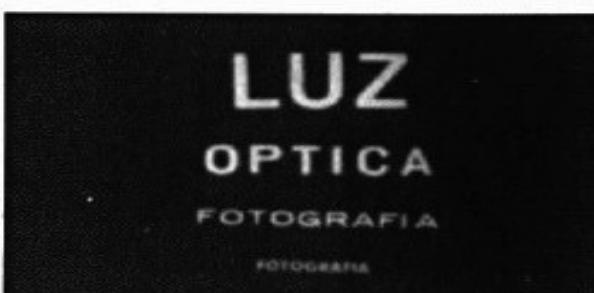
2 mm



1 mm



0.6mm



0.35 mm



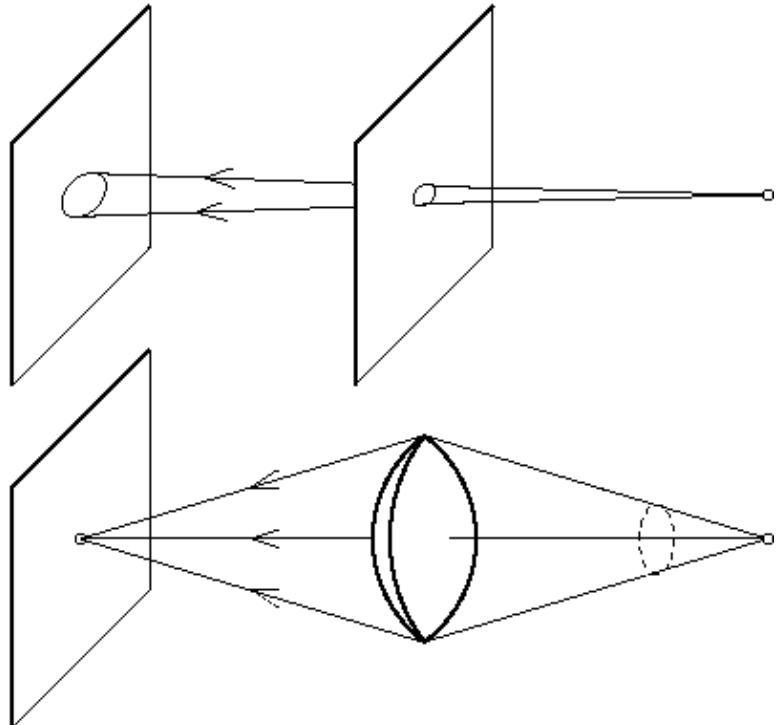
0.15 mm



0.07 mm

Use a lens

- Make the aperture larger, but use a lens to focus the light
- Refraction to the rescue



film/sensor



From Photography, London et al.

Assignment 1

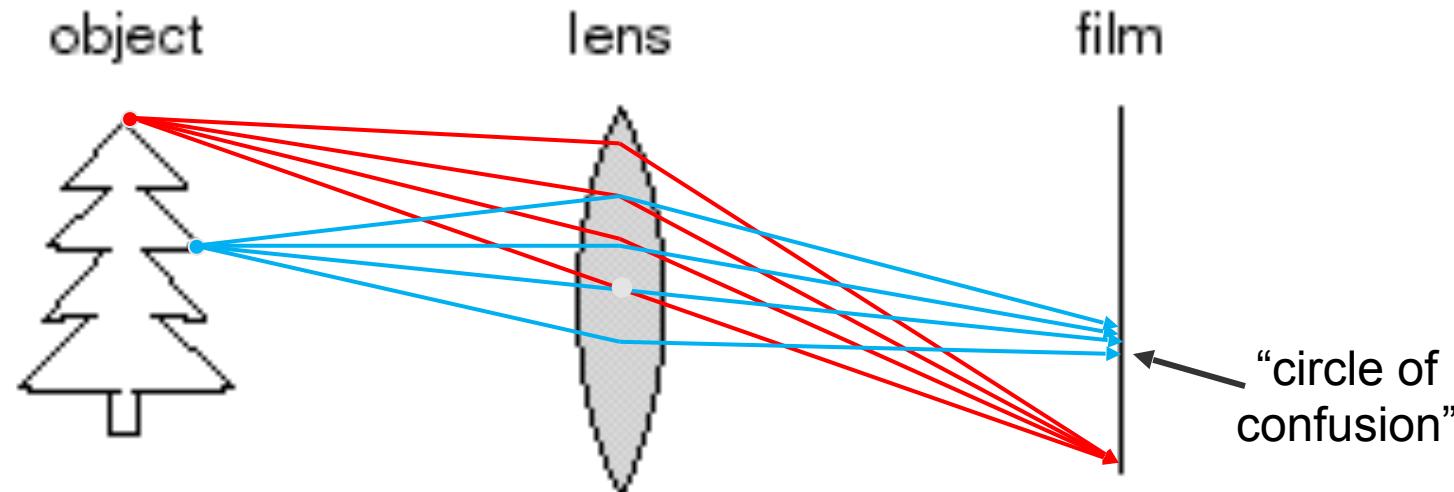
- **Building a pinhole camera using a cardboard box**
- **Requires a digital camera with the ability to capture long exposure images (15 – 30 seconds)**
- **There are apps that allow capturing images with long exposure**
- **Teams of up to 3**
- **Only one member submit the report via Canvas**
- **All team members receive the same grade**

Outline

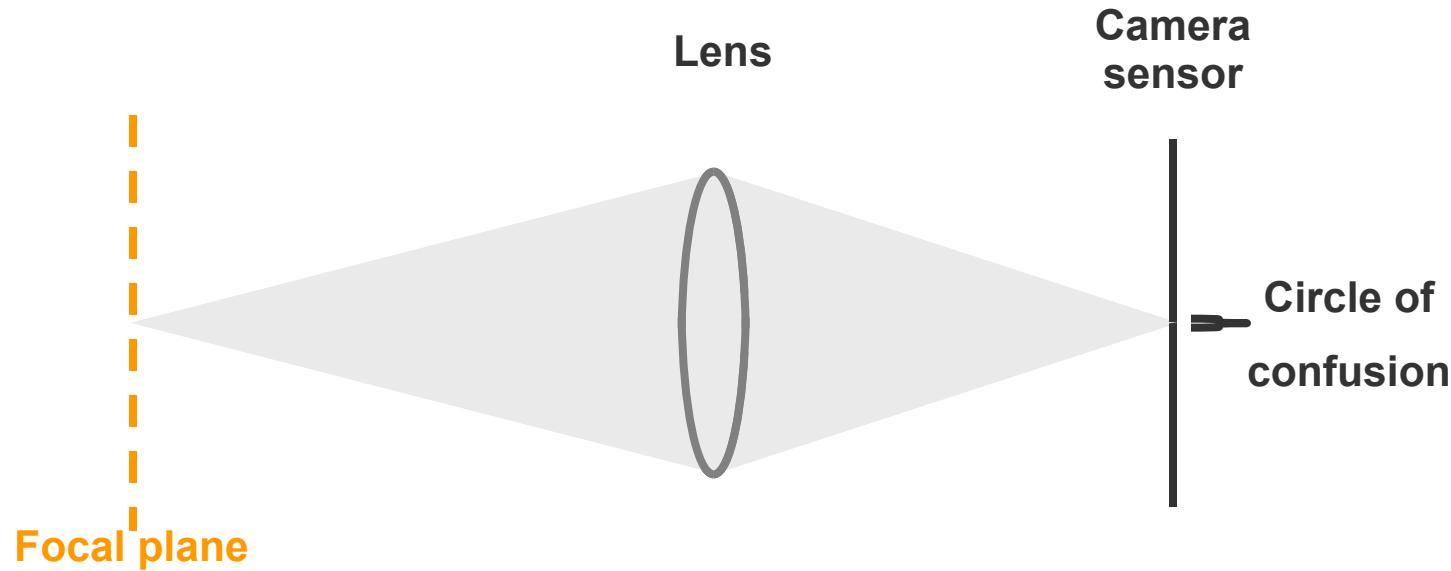
- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

Focus and Defocus

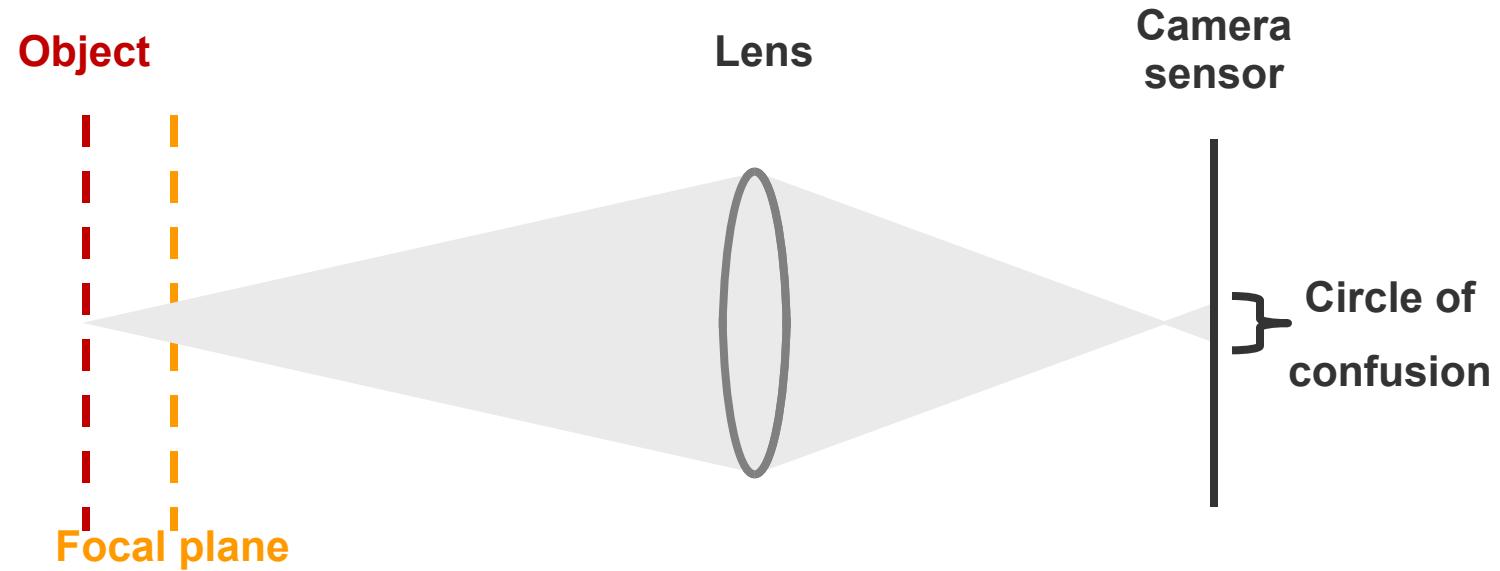
- A lens focuses light onto the film
 - A specific distance at which objects are “in focus”
 - Other points project to a “circle of confusion” in the image



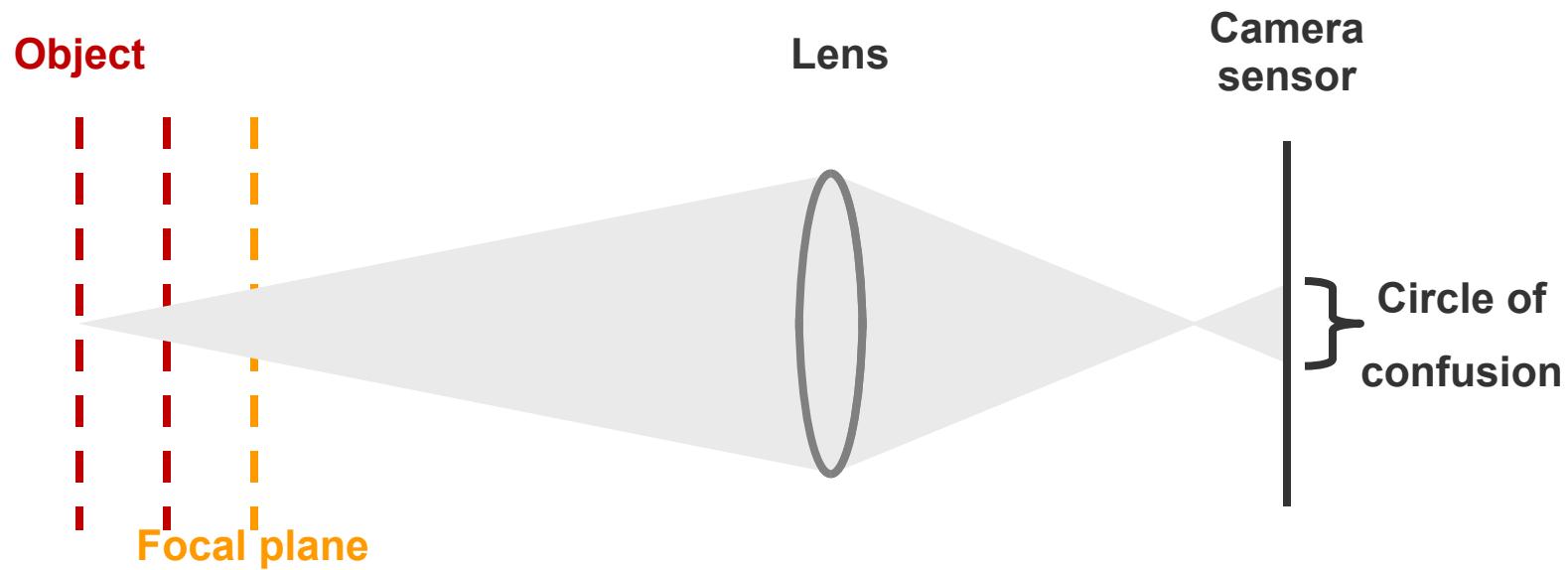
Lens and defocus



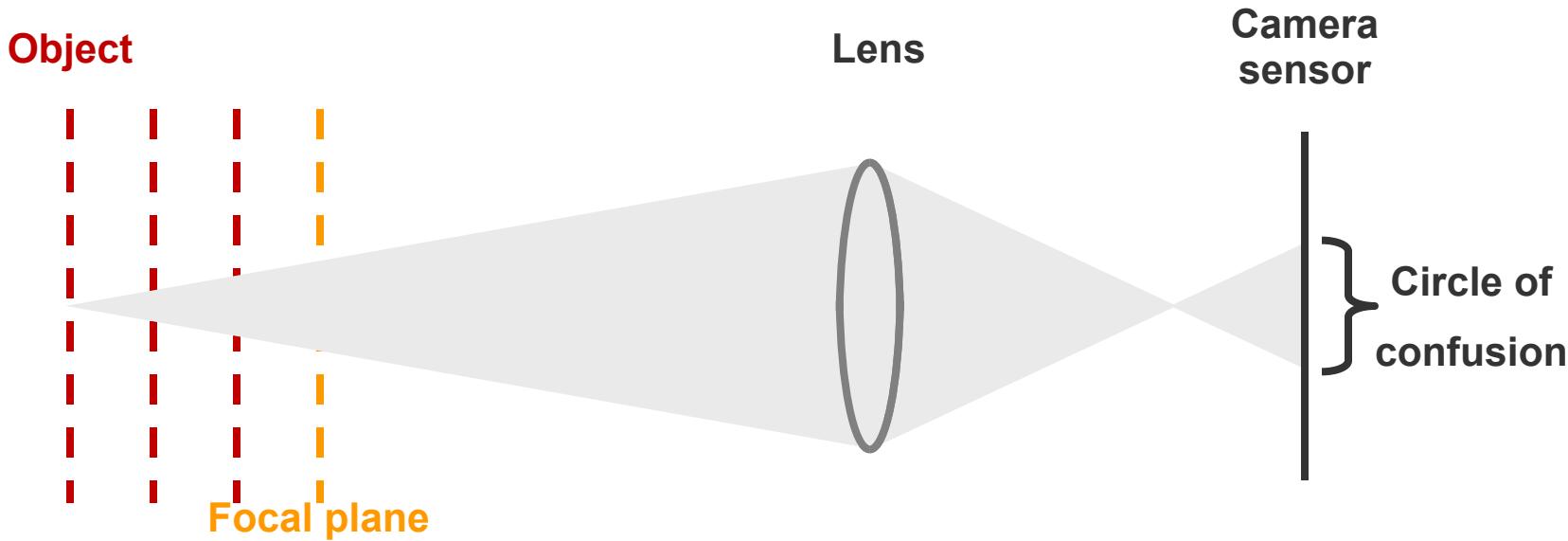
Lens and defocus



Lens and defocus

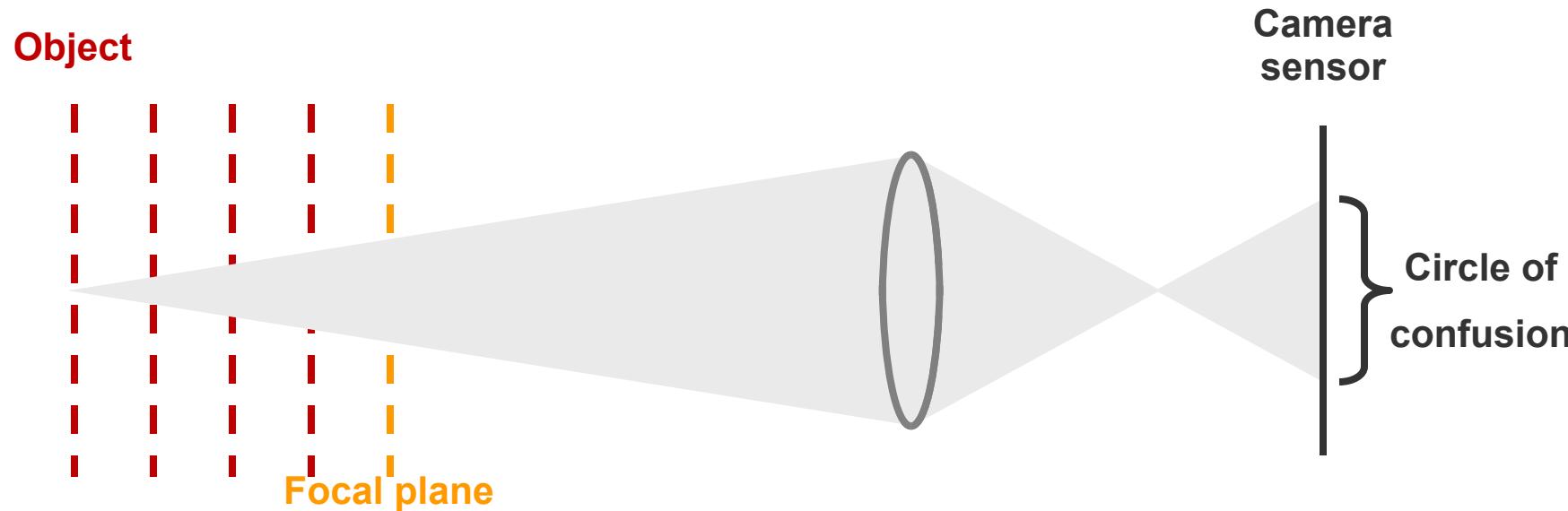


Lens and defocus



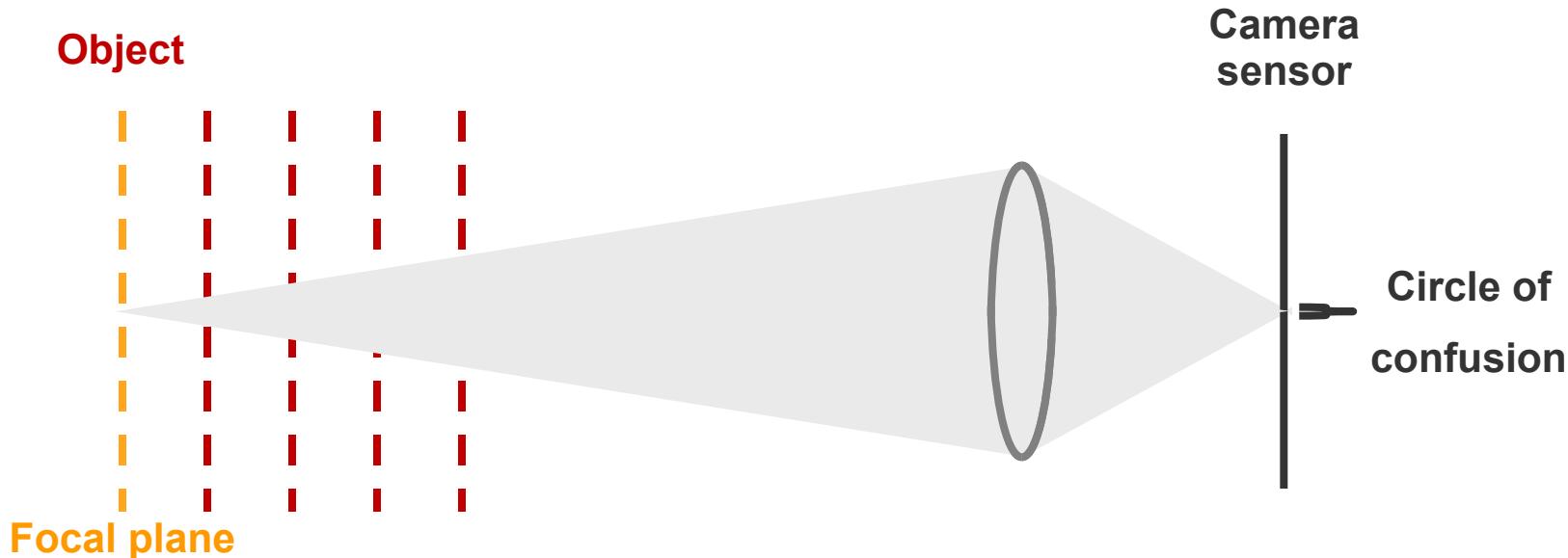
Lens and defocus

- What if we want to push the focal plane to where the object is?
 - Move the sensor closer to the lens



Lens and defocus

- **What if we want to push the focal plane to where the object is?**
 - **Move the sensor closer to the lens**
- **What about focusing to infinity?**
 - **Move the sensor to the focal length**



Focal Length

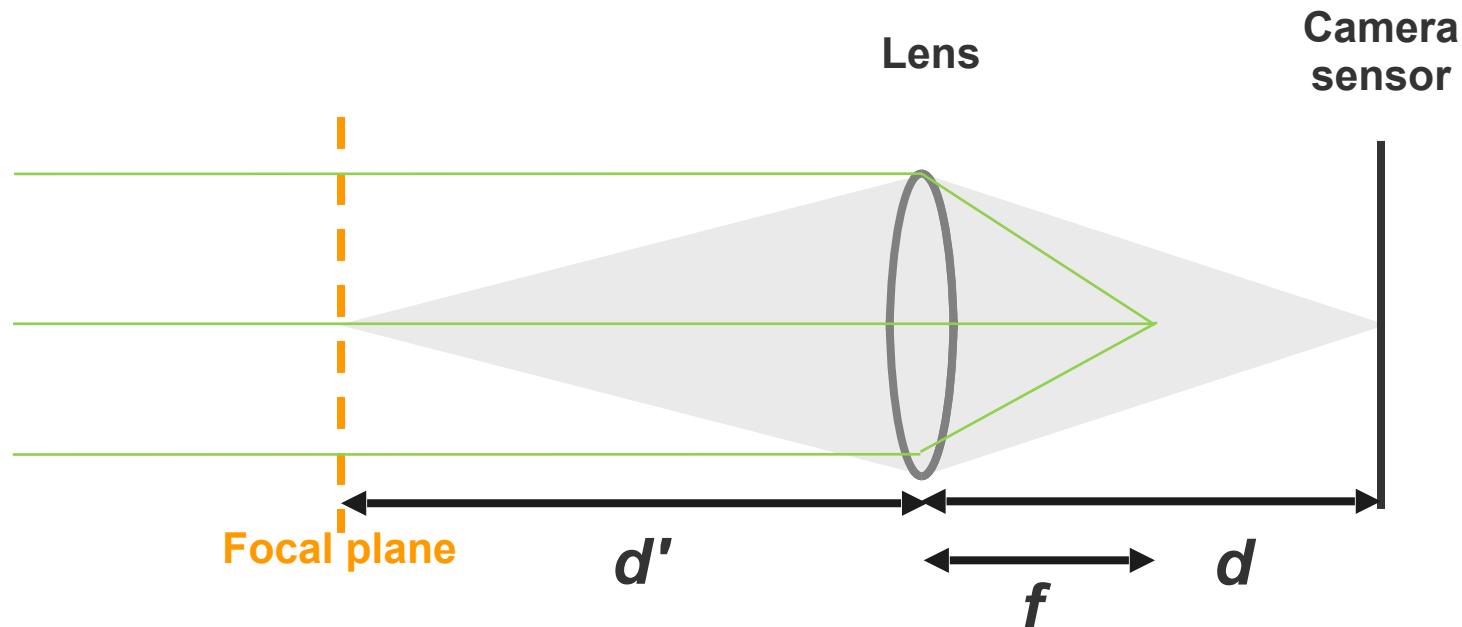
- The distance between sensor and lens when the camera is focused to infinity
- Focal length is the property of the lens



Terminology

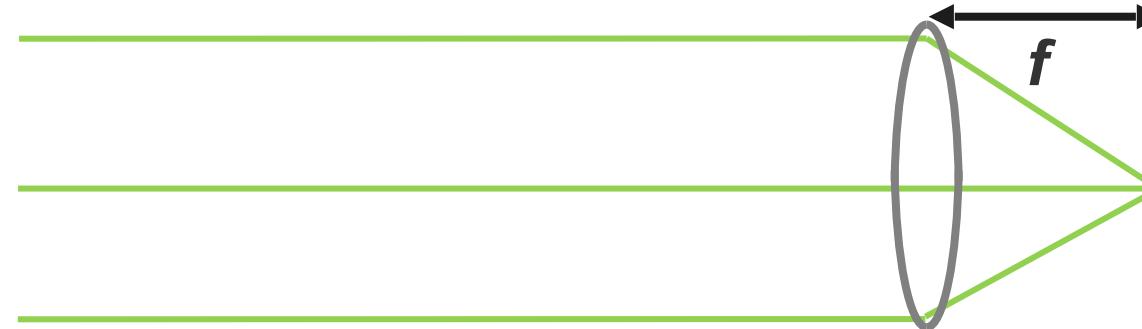
- **Focal length (f)**
- **Image distance (d)**
- **Object distance (d')**

What is the relationship between them?

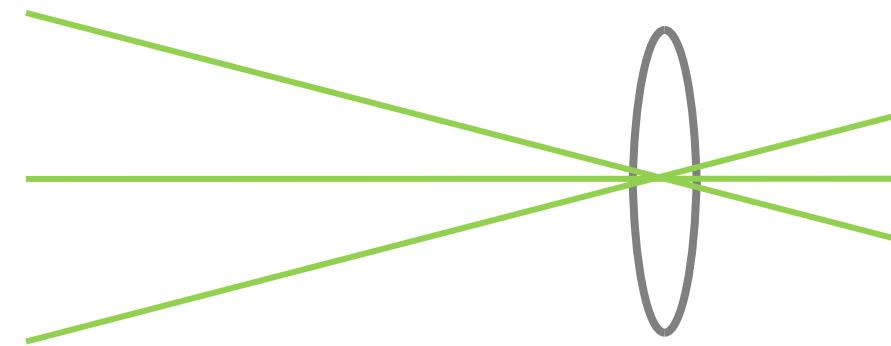


Thin lens optics

- **Thickness of the lens negligible compared to radii of the curvature**
- **Simplification of geometrical optics for well-behaved lenses**
- **All parallel rays converge to a point at the focal length f**

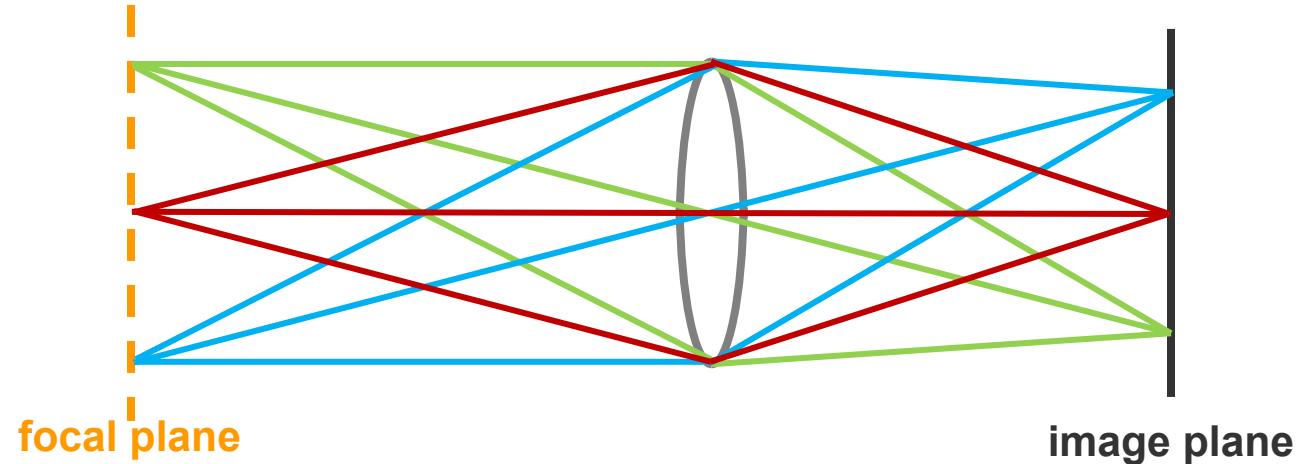


- **All rays going through the center are not deviated**



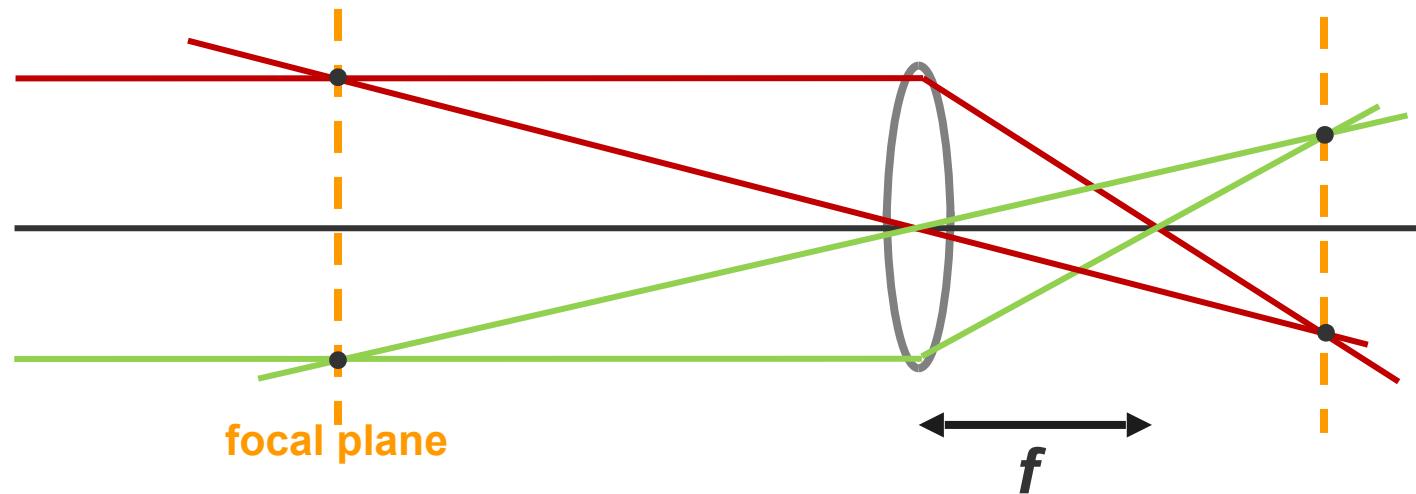
Focusing

- There is a focal plane for a given image plane
 - All rays coming from a point on the focal plane are focused to a point on the image plane

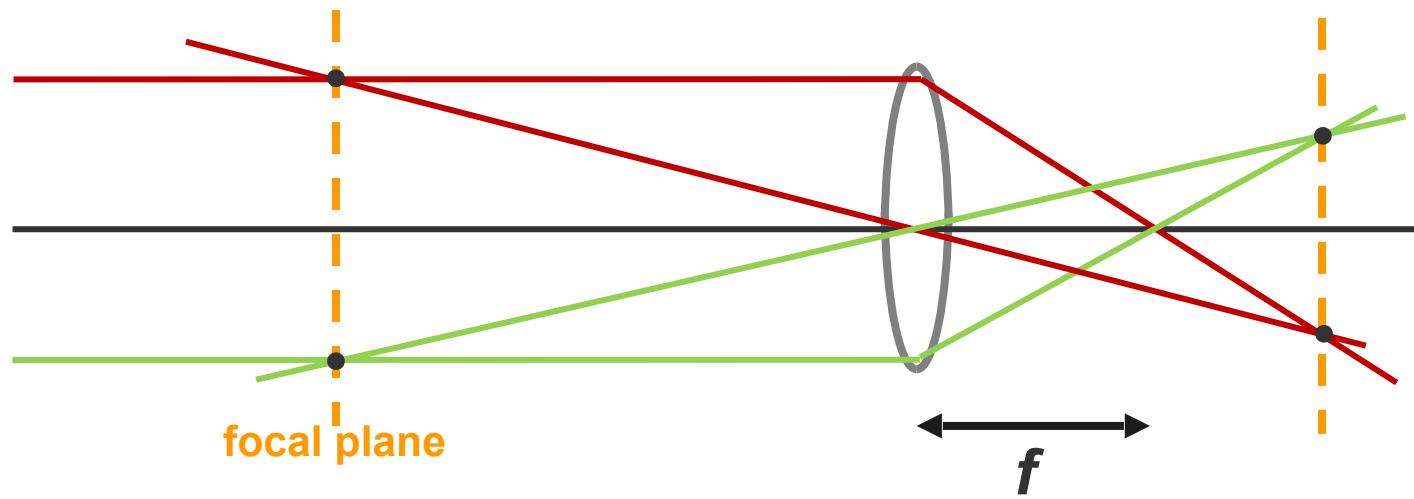


How to trace rays

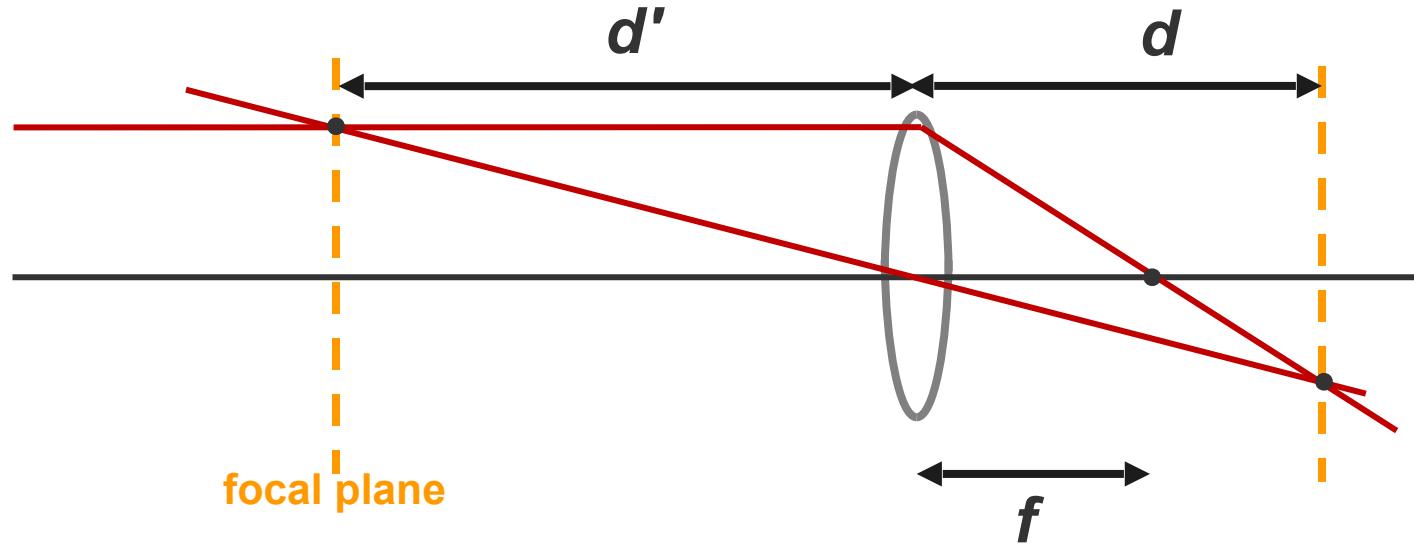
- **Pick a focal plane**
- **Pick a focal length and trace the parallel lines**
- **Trace the rays through the center**



Thin lens formula

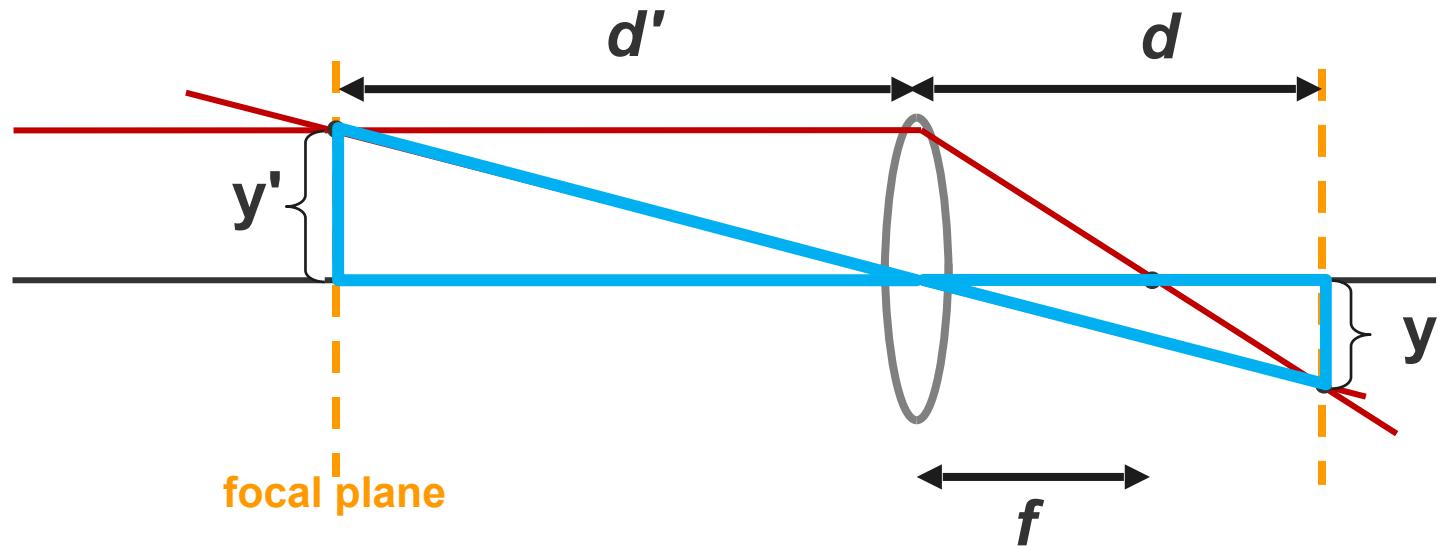


Thin lens formula



Thin lens formula

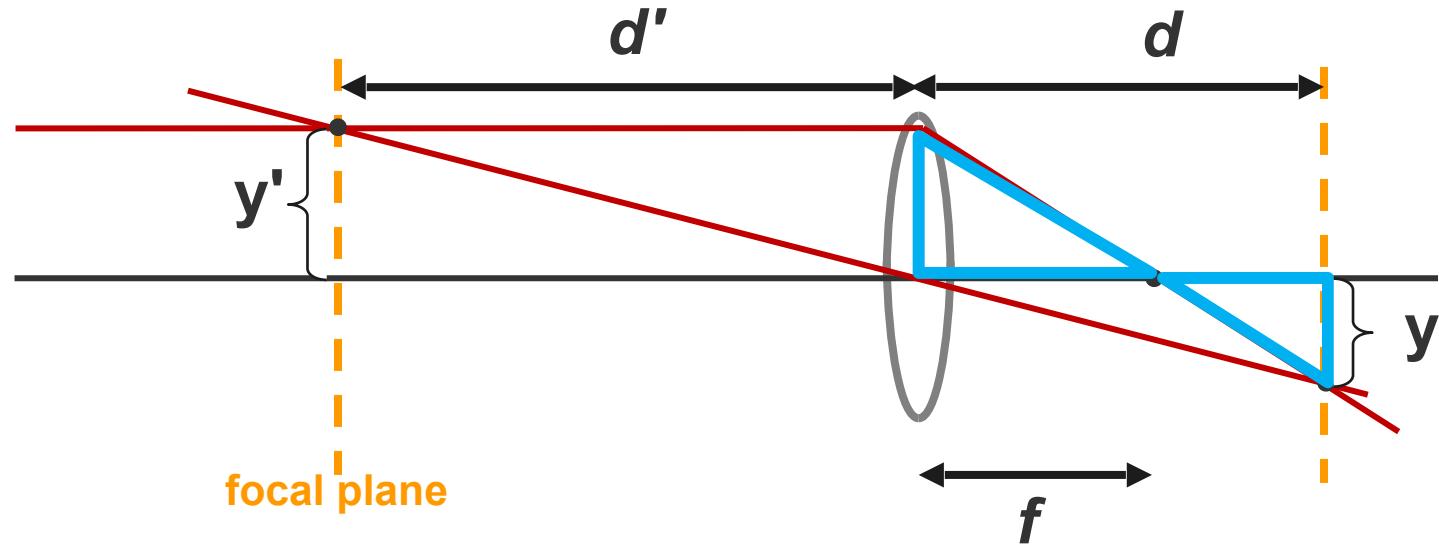
$$\frac{y'}{y} = \frac{d'}{d}$$



Thin lens formula

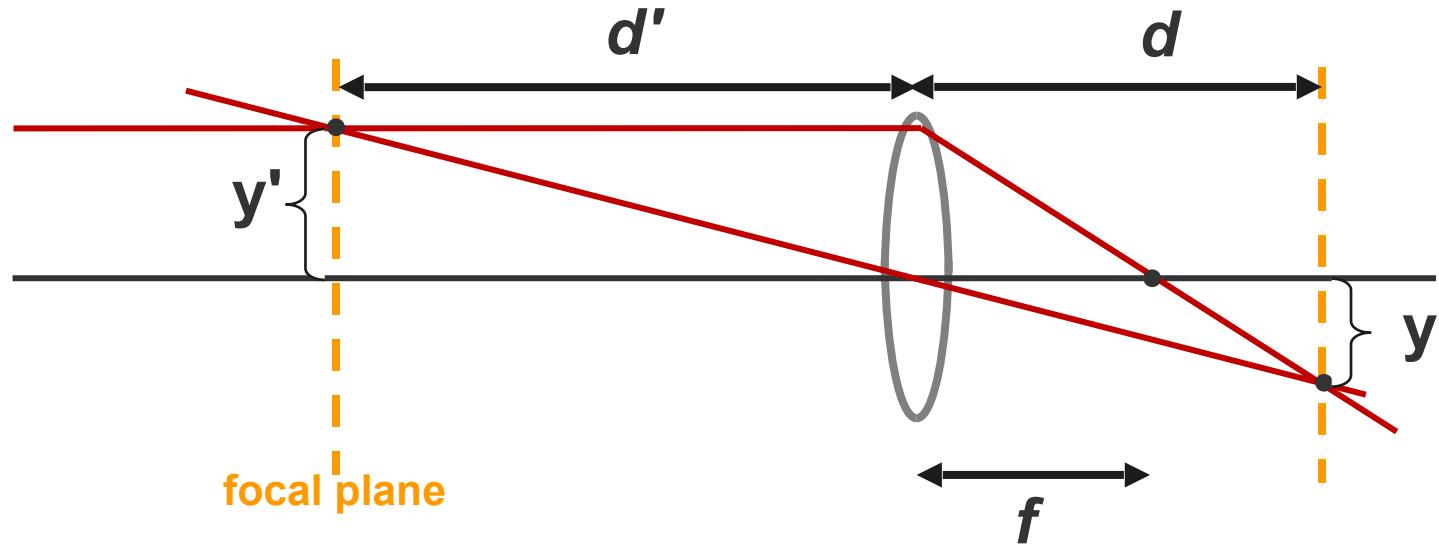
$$\frac{y'}{y} = \frac{d'}{d}$$

$$\frac{y'}{y} = \frac{f}{d - f}$$



Thin lens formula

$$\left. \begin{aligned} \frac{y'}{y} &= \frac{d'}{d} \\ \frac{y'}{y} &= \frac{f}{d-f} \end{aligned} \right\} \quad \boxed{\frac{1}{d'} + \frac{1}{d} = \frac{1}{f}}$$



Outline

- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

Depth of field



Depth of field

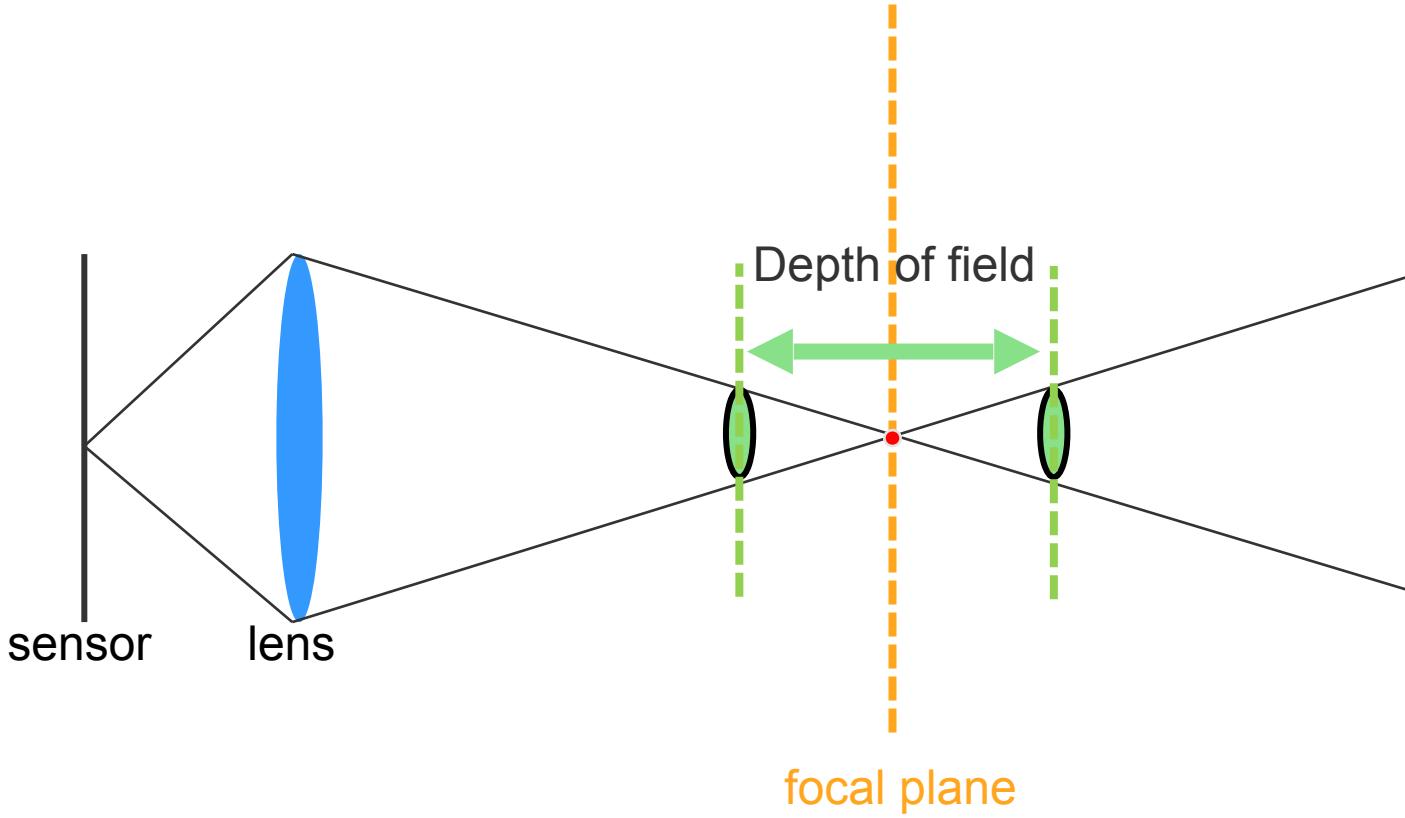


Depth of field



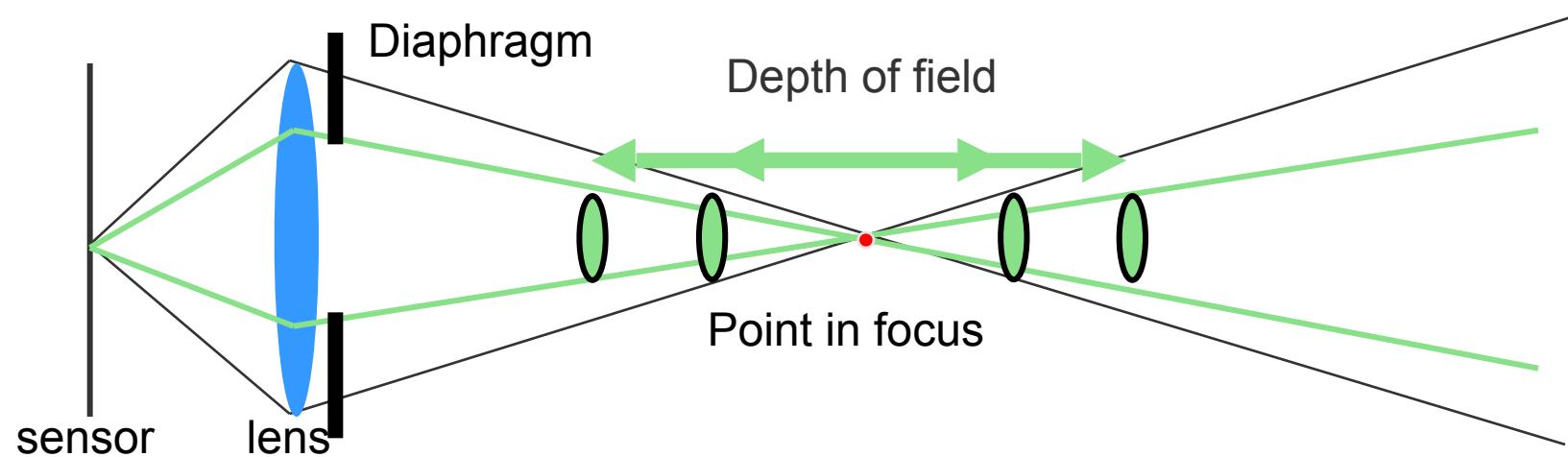
Depth of field

- We allow for some tolerance



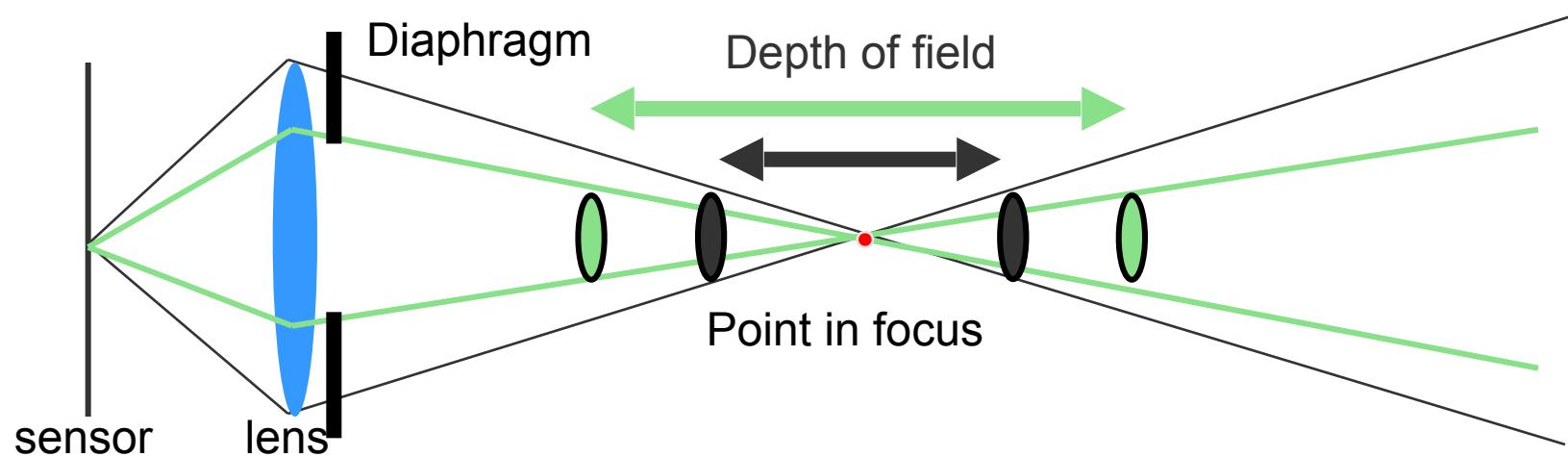
Controlling depth of field

- How can we increase the depth of field?
 - By reducing the size of the aperture



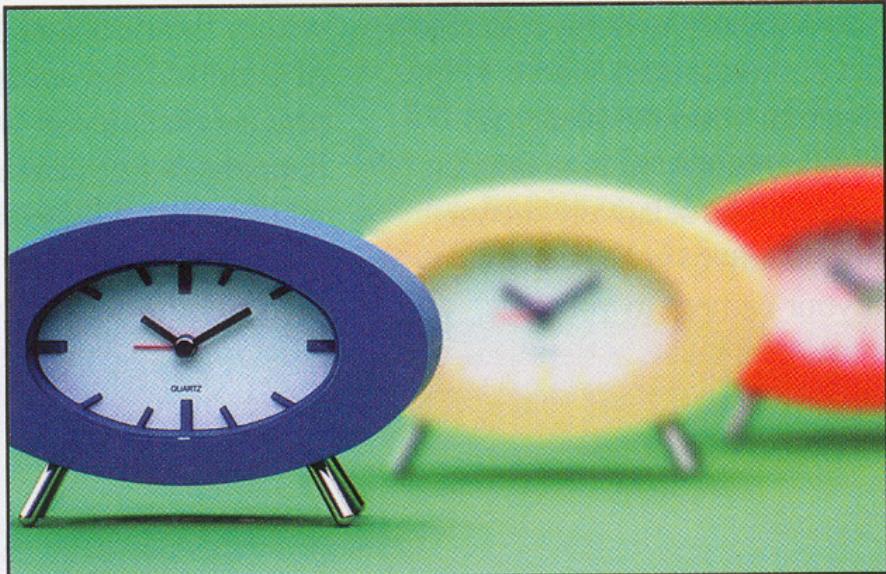
Controlling depth of field

- How can we increase the depth of field?
 - By reducing the size of the aperture
- If we decrease the aperture diameter by a factor of 2
 - The DoF increases by a factor of 2 (similar triangles)



Depth of field

LESS DEPTH OF FIELD

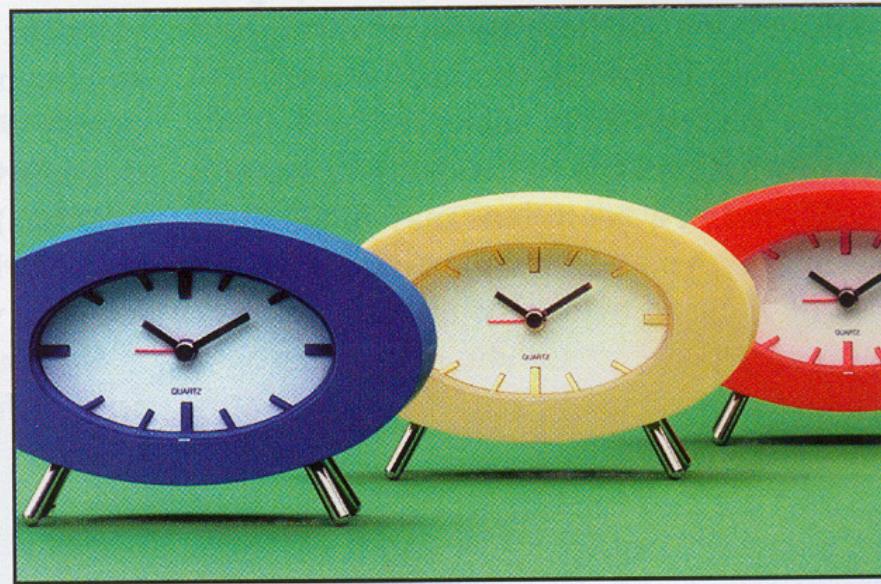


Wider aperture

f/2

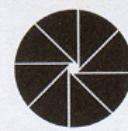


MORE DEPTH OF FIELD



Smaller aperture

f/16

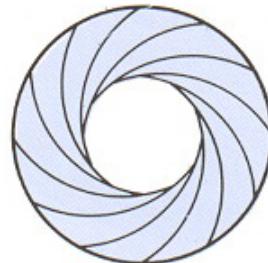


Aperture

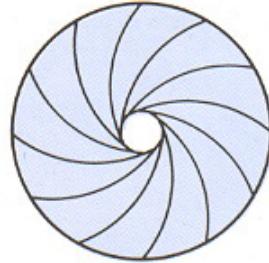
- Diameter of the lens opening (controlled by diaphragm)
- Expressed as a fraction of focal length, in f-number
 - f/2.0 on a 50mm means that the aperture is 25mm
 - f/2.0 on a 100mm means that the aperture is 50mm
- Small f number = big aperture



Full aperture



Medium aperture



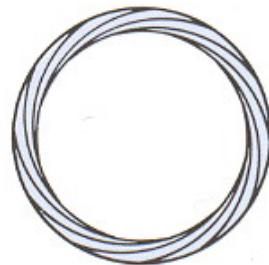
Stopped down

$$D = \frac{f}{N}$$

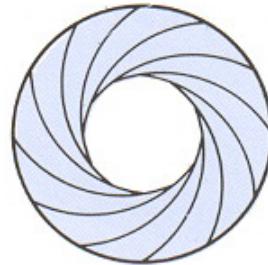
↓
Focal length
↑
Aperture diameter
↑
F-number

Aperture

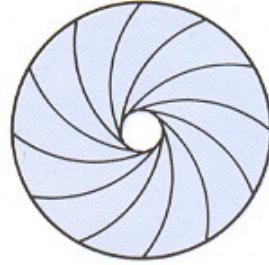
- What happens to the area of the aperture when going from f/2.0 to f/4.0?
 - Divided by 4
- Typical f numbers are
 - f/2.0, f/2.8, f/4, f/5.6, f/8, f/11, f/16, f/22, f/32
 - See the pattern?



Full aperture



Medium aperture



Stopped down

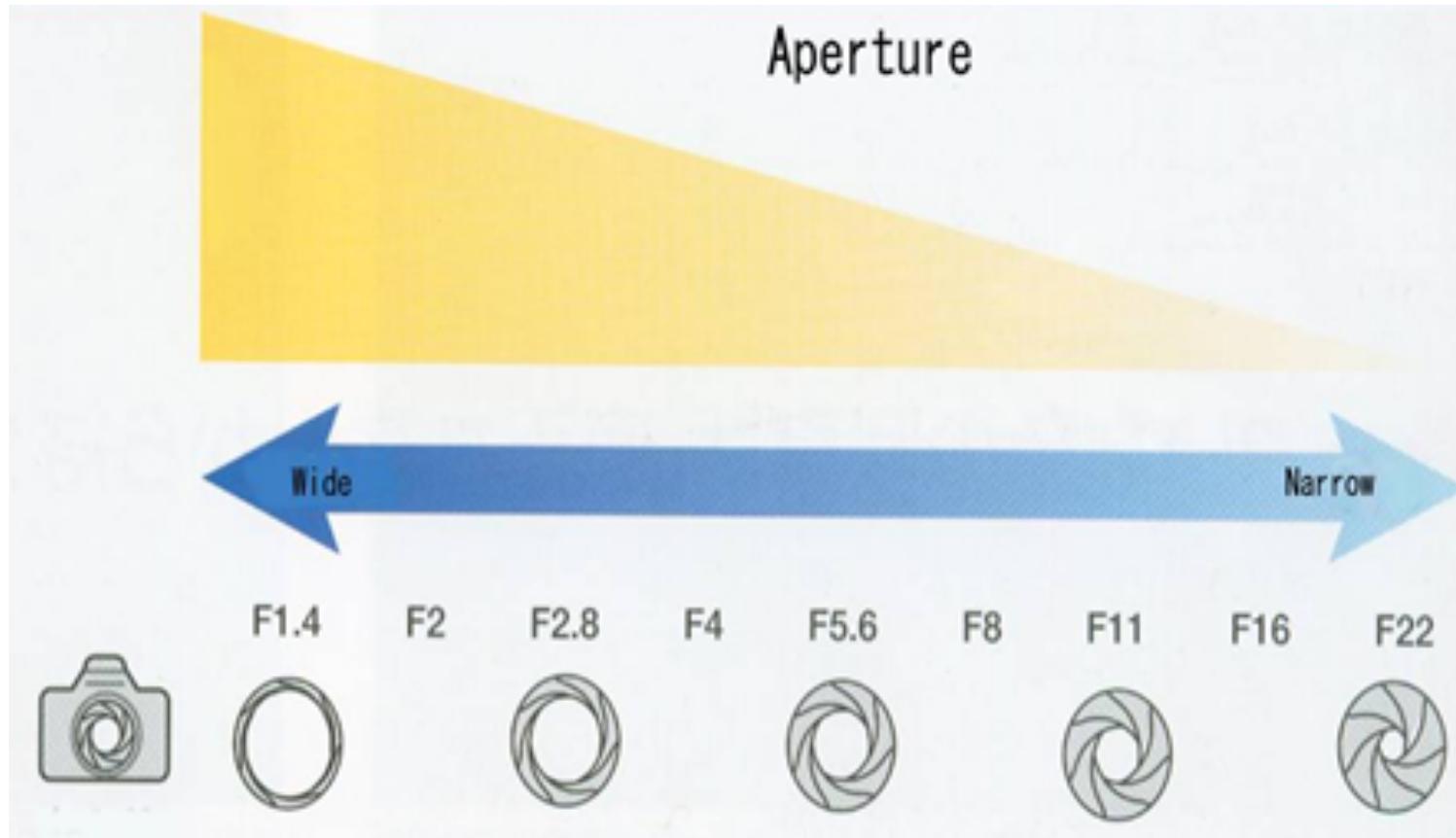
$$D = \frac{f}{N}$$

Focal length
↓
↑ Aperture diameter ↑ F-number

A mathematical equation showing the relationship between focal length (f), aperture diameter (D), and F-number (N). Arrows point from the text labels to the corresponding variables in the equation.

$$\text{Area} = \pi \left(\frac{f}{2N} \right)^2$$

F-number: focal length/aperture diameter



Varying the aperture



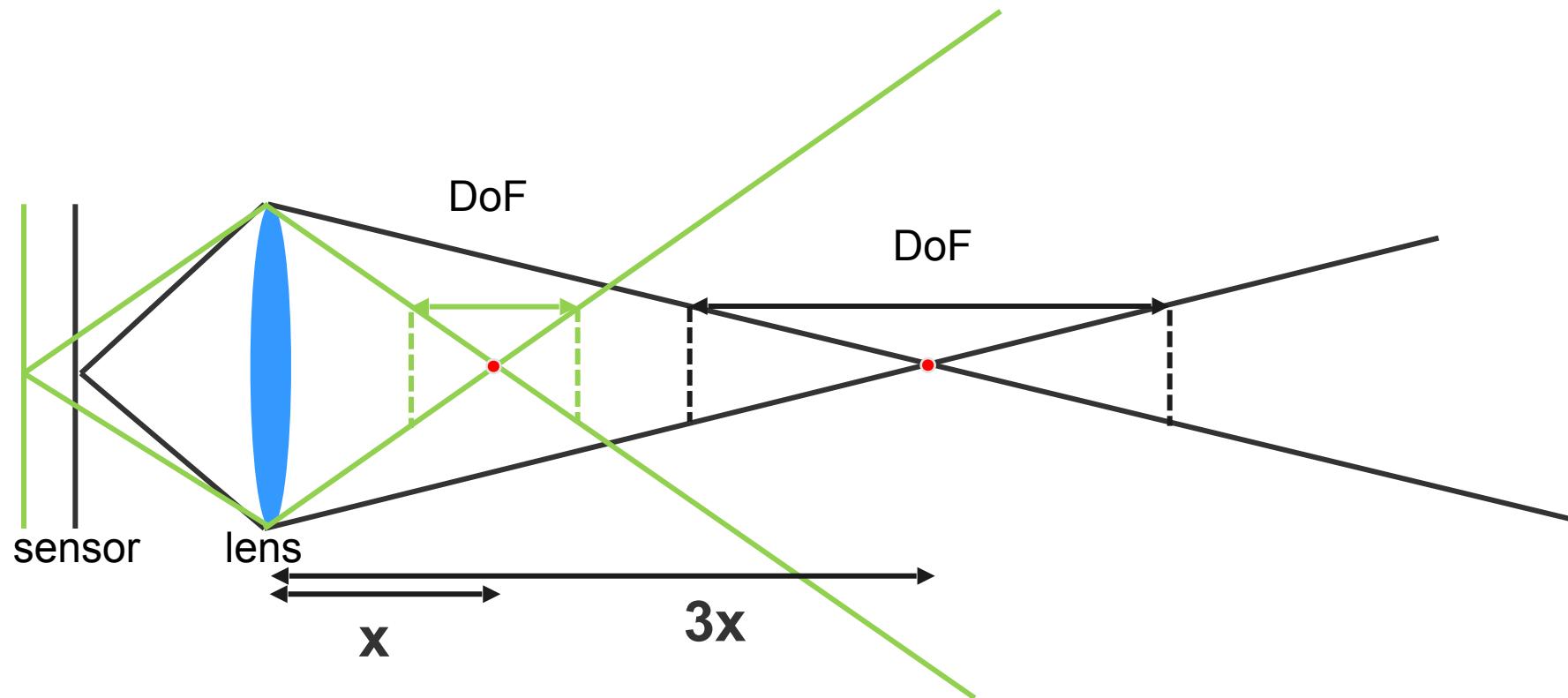
Wide aperture = small DOF



Narrow aperture = large DOF

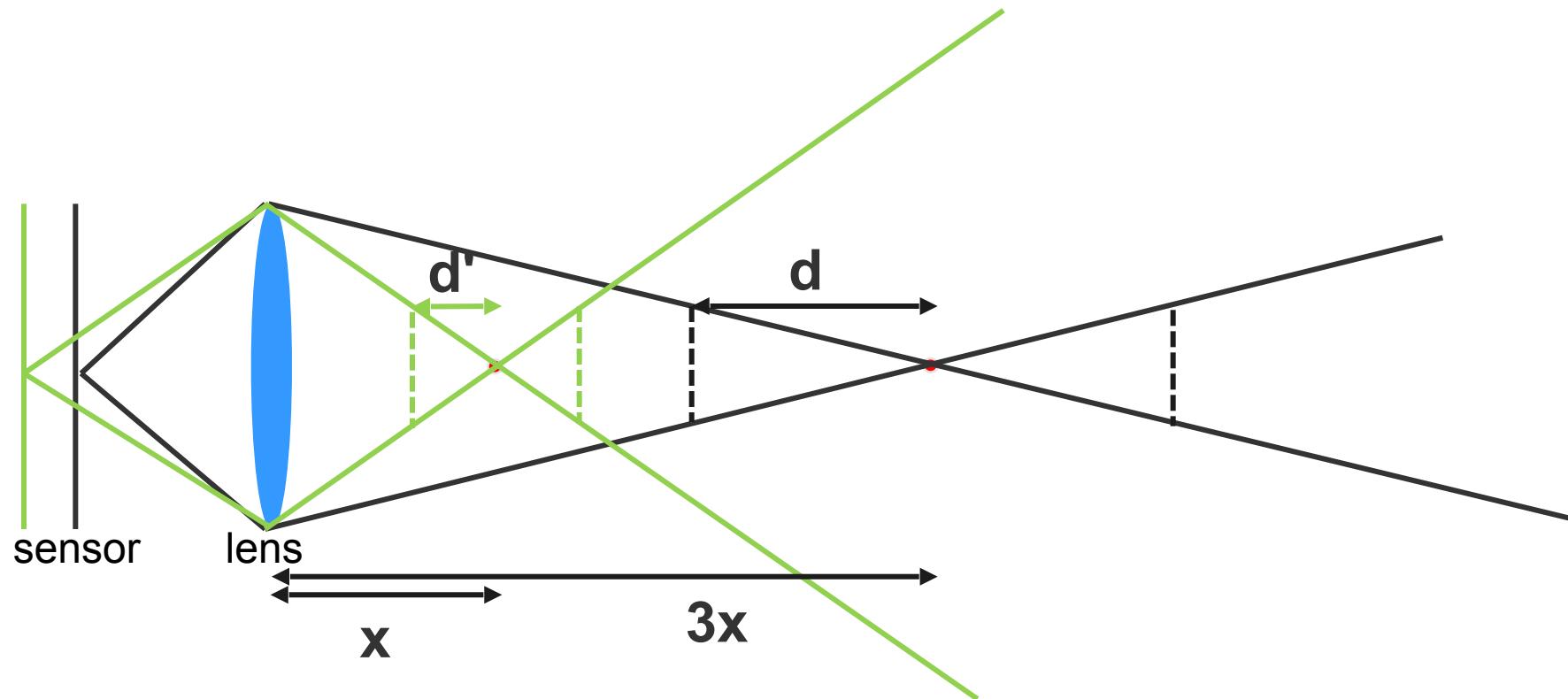
Depth of field & focusing distance

- What happens when we divide focusing distance by three?



Depth of field & focusing distance

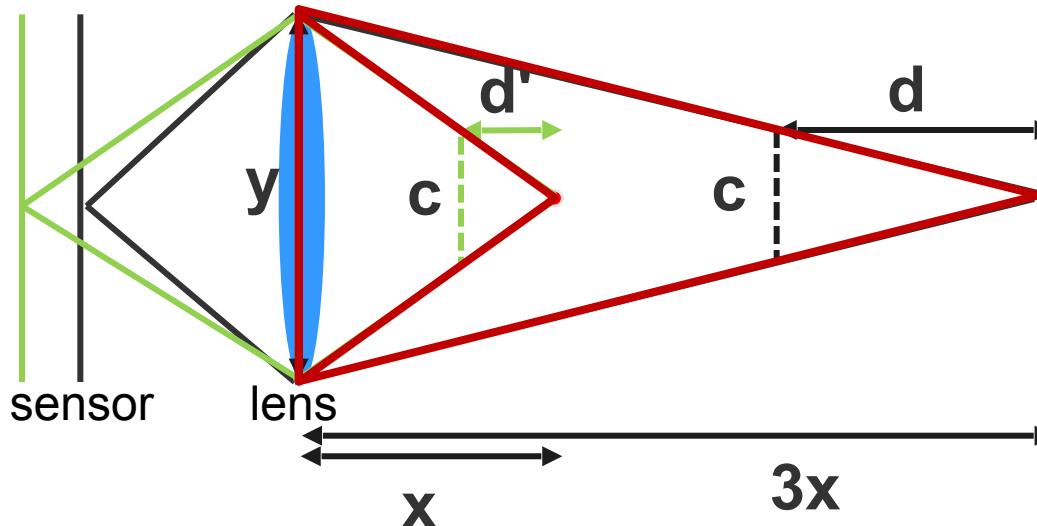
- What happens when we divide focusing distance by three?



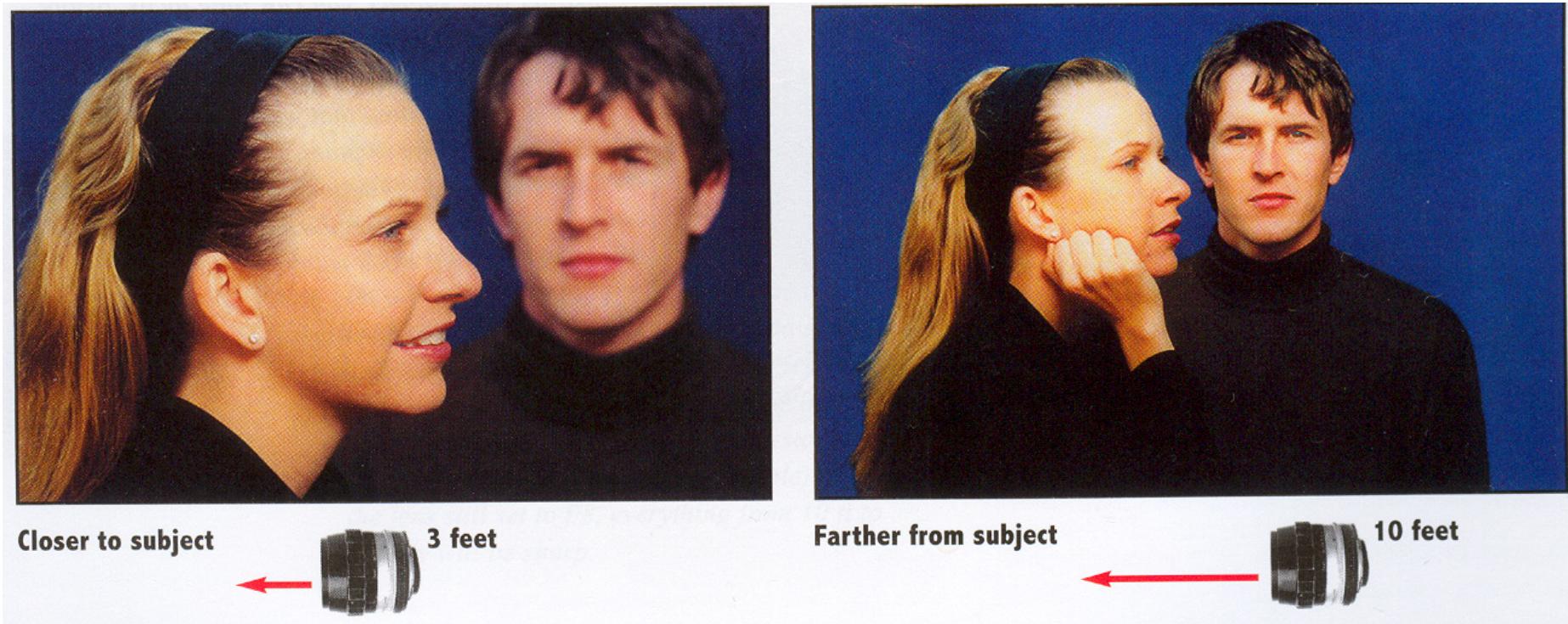
Depth of field & focusing distance

- What happens when we divide focusing distance by three?
 - Similar triangles => divided by three as well

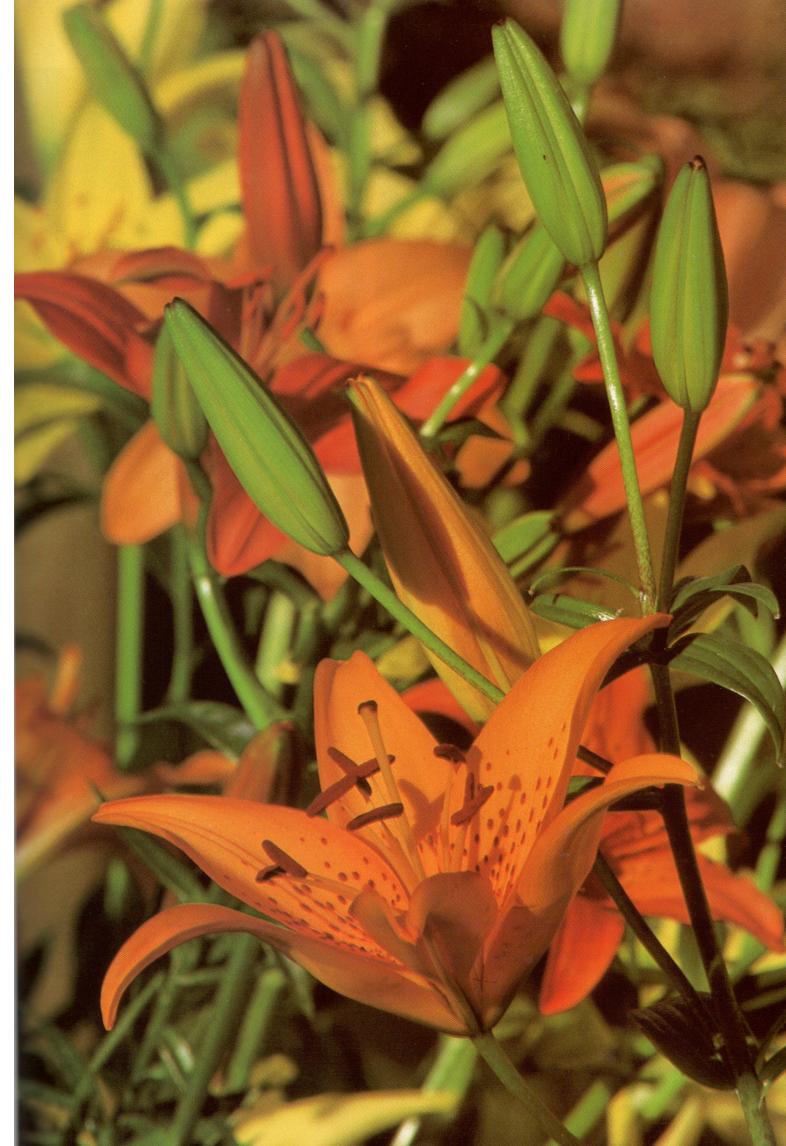
$$\frac{y}{c} = \frac{x}{d'} \quad \frac{y}{c} = \frac{3x}{d} \quad \longrightarrow \quad d = 3d'$$



Depth of field & focusing distance



Nice Depth of Field effect



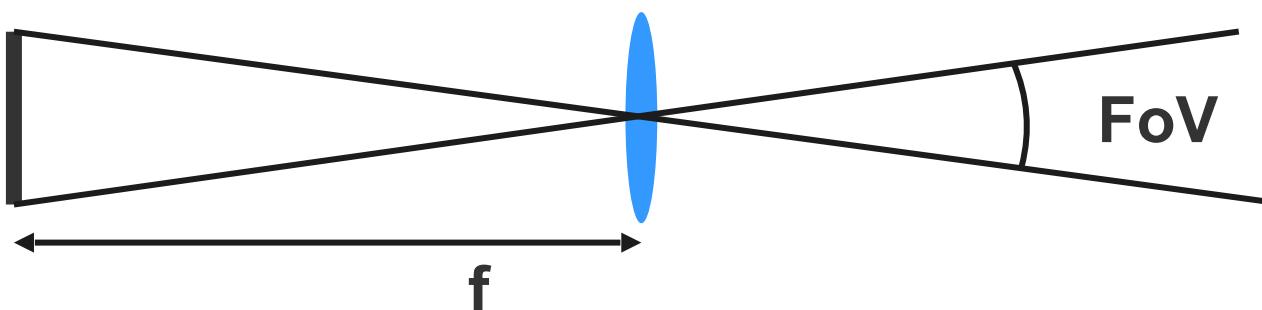
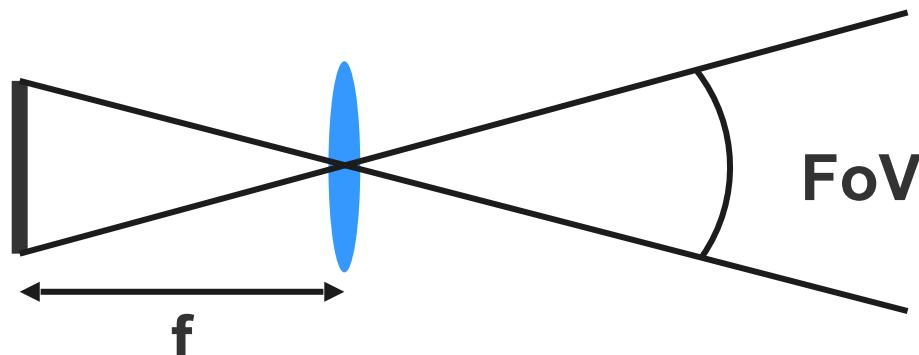
From Peterson, Understanding Exposure

Outline

- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

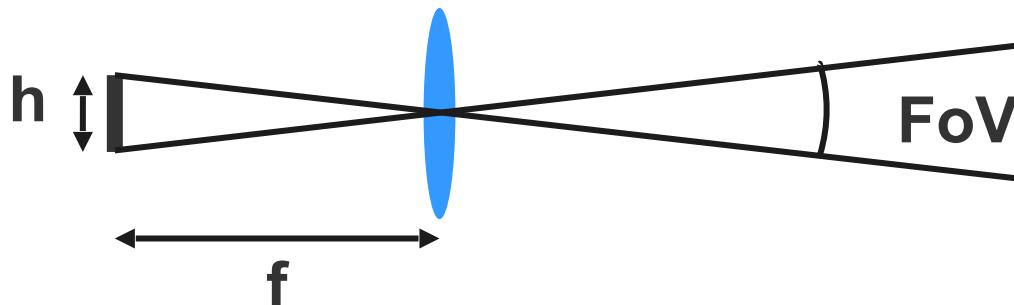
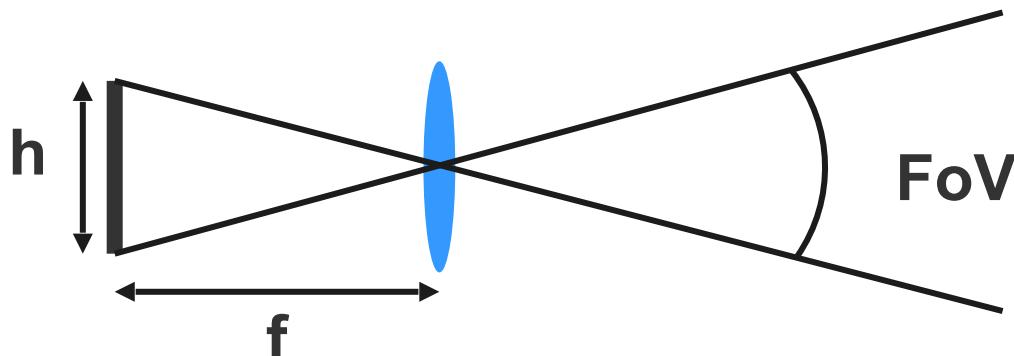
Field of View

- The angle through which the camera can see the world
- What happens if we increase the focal length?
 - FoV decreases



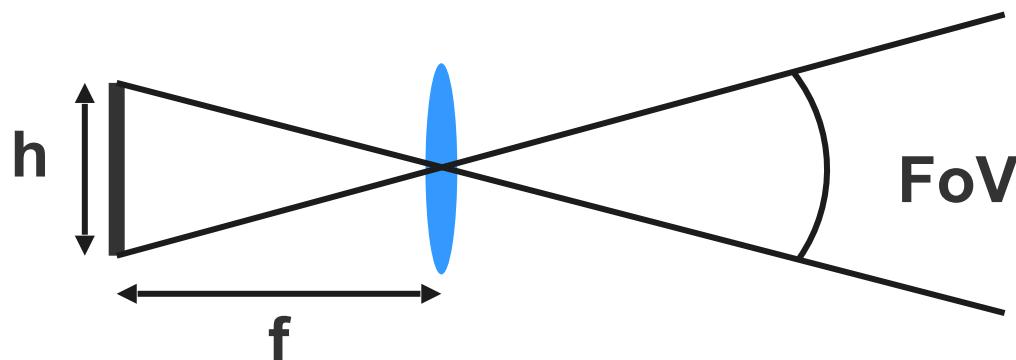
Field of View

- The angle through which the camera can see the world
- What if we decrease the sensor size
 - FoV decreases

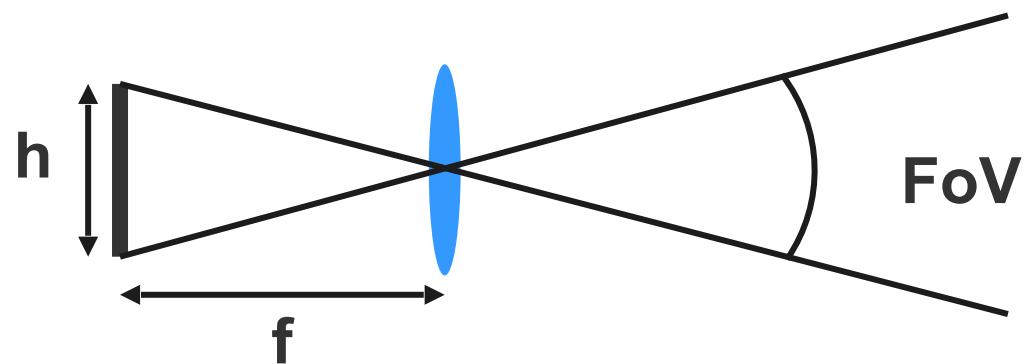


Field of View

- Depends on two factors
 - Inversely proportional to focal length
 - Directly proportional to sensor size



Field of View



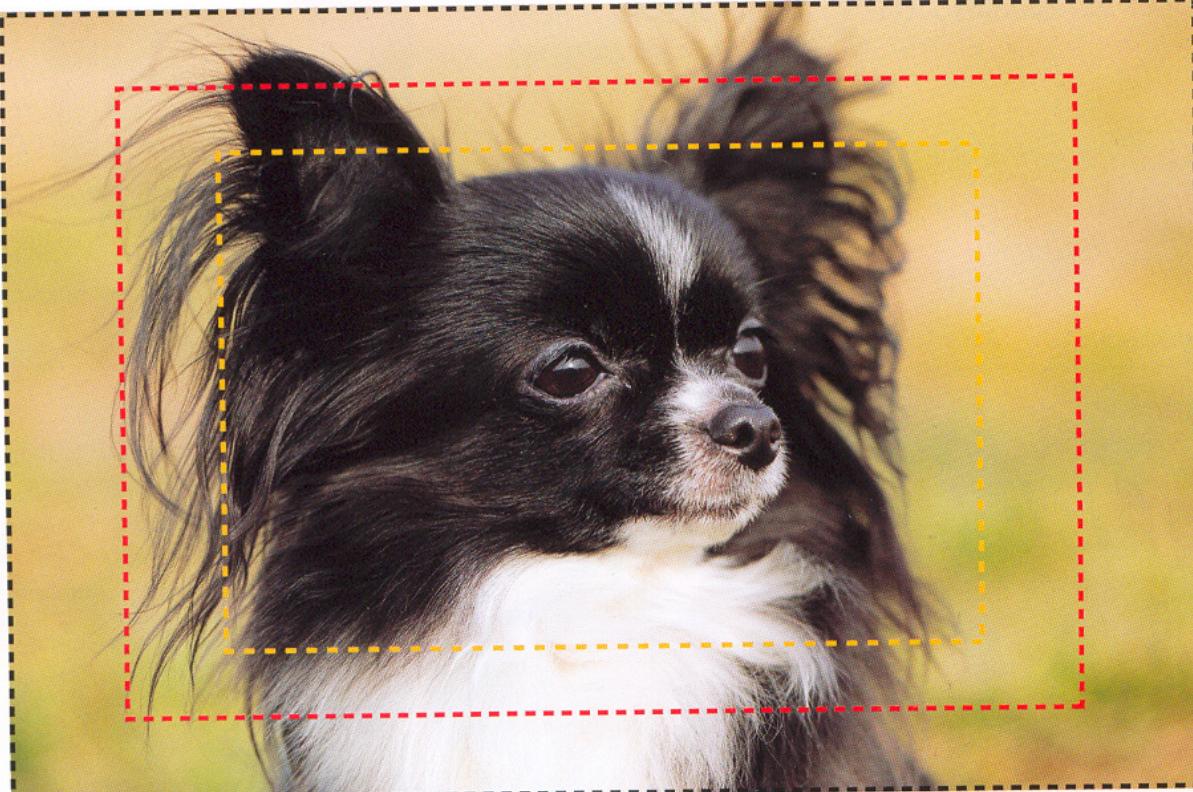
$$\text{FoV} = 2 \arctan\left(\frac{h}{2f}\right)$$

Effect of sensor size

Sensor size

➤ Similar to cropping

35mm full size and digital shooting range image size (picture dimensions) and lens selection



— EOS-1Ds / — EOS-1D/ — EOS 10D



EOS-1D

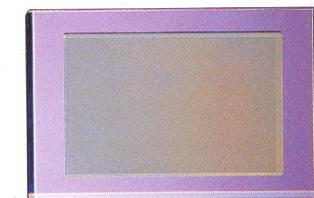


EOS 10D

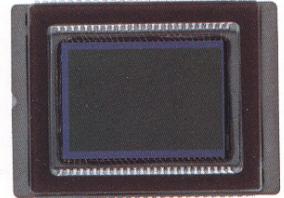
(The EOS Kiss Digital/EOS DIGITAL Rebel/EOS 300D DIGITAL SLR camera has the same image size as the EOS 10D.)



EOS-1Ds : 35.8 x 23.8mm

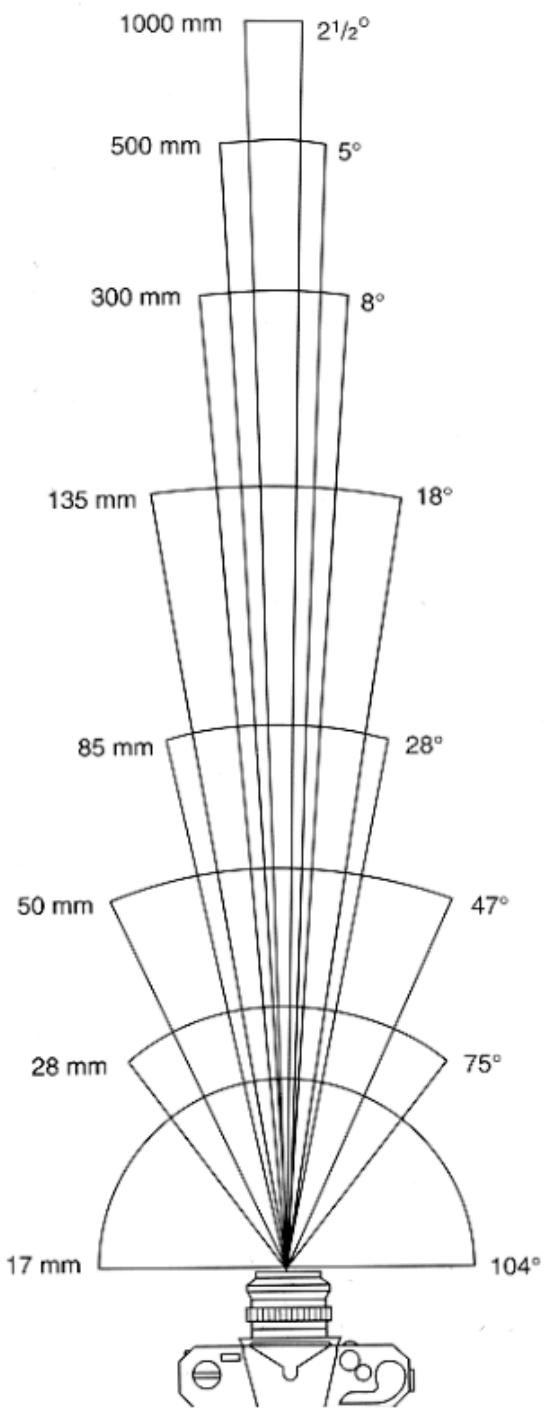


EOS-1D : 28.7 x 19.1mm

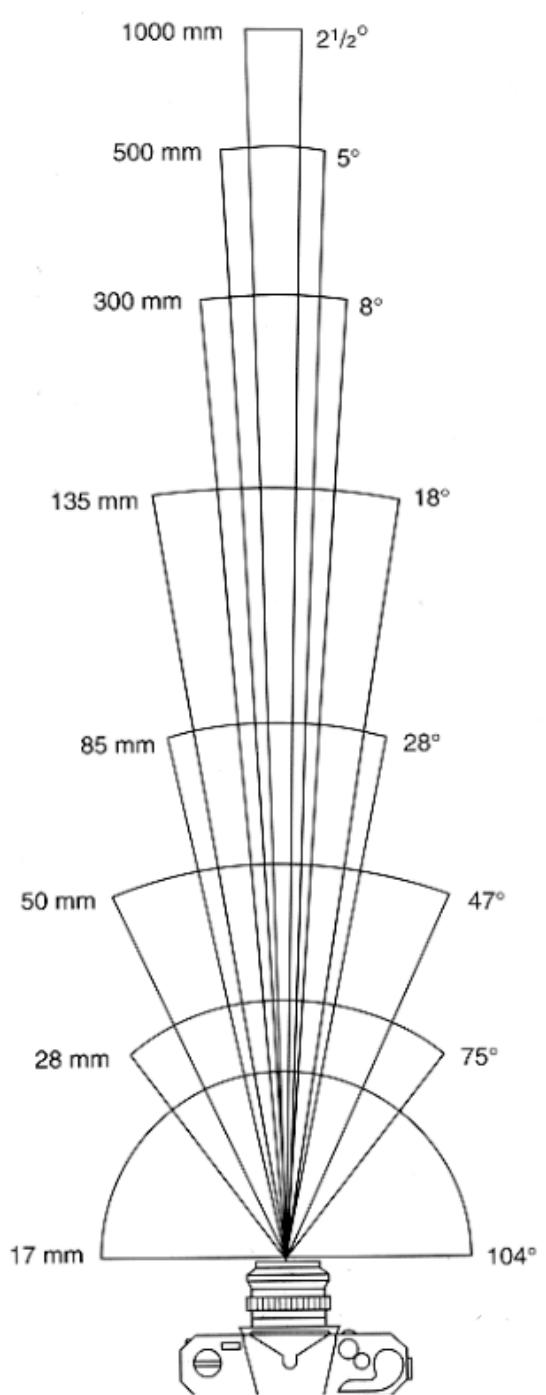


EOS 10D : 22.7 x 15.1mm

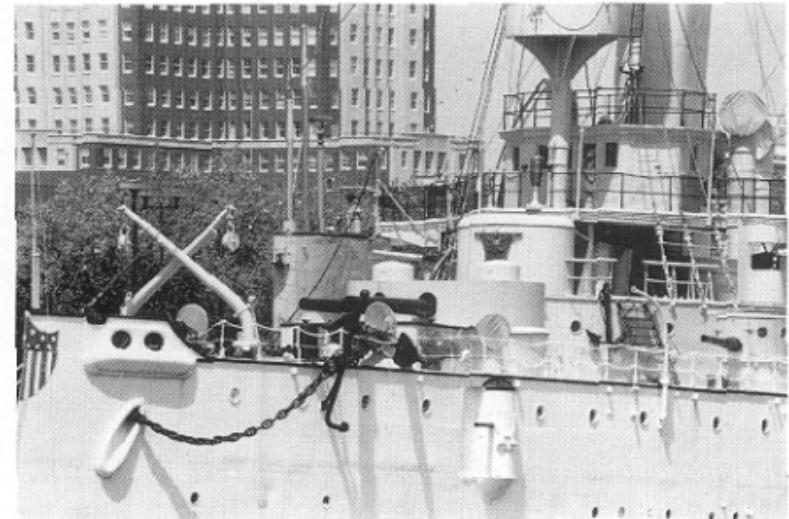
Effect of focal length



From London and Upton



135mm



300mm



500mm



From London and Upton

Expensive toys...



Sigma 200-500mm F2.8 EX DG lens

What does 1600mm lens look like?

<http://www.digitapixels.net/varia/the-web/sigma-200-500mm-f28-ex-dg-lens-on-the-field/>



800mm f5.6 L IS



600mm f4 L IS II



200-400mm f4 L IS



500mm f4 L IS II



400mm f2.8 L IS II

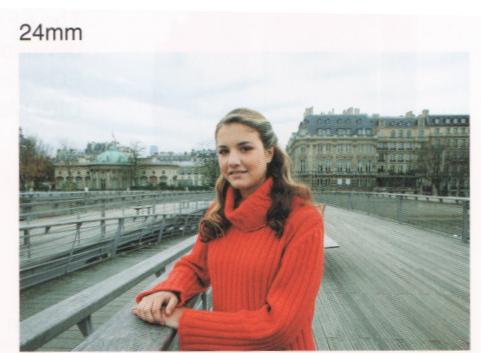


300mm f2.8 L IS II

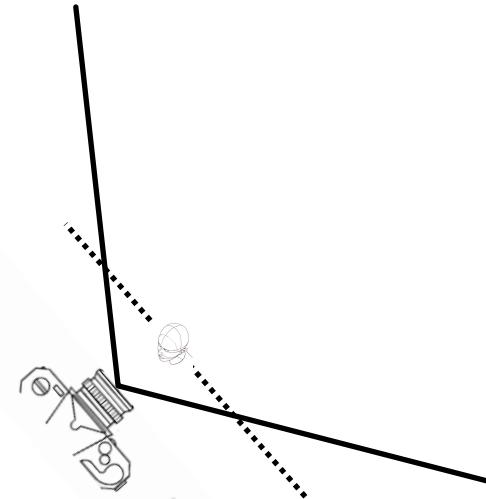
http://dancarrphotography.com/blog/wp-content/uploads/2011/05/Canon_super_tele_comparison.jpg

Focal length and perspective

- Increase the focal length
- Move away from the subject
- Subject remains the same size
- Background moves closer



Perspective Composition



16 mm (110°)

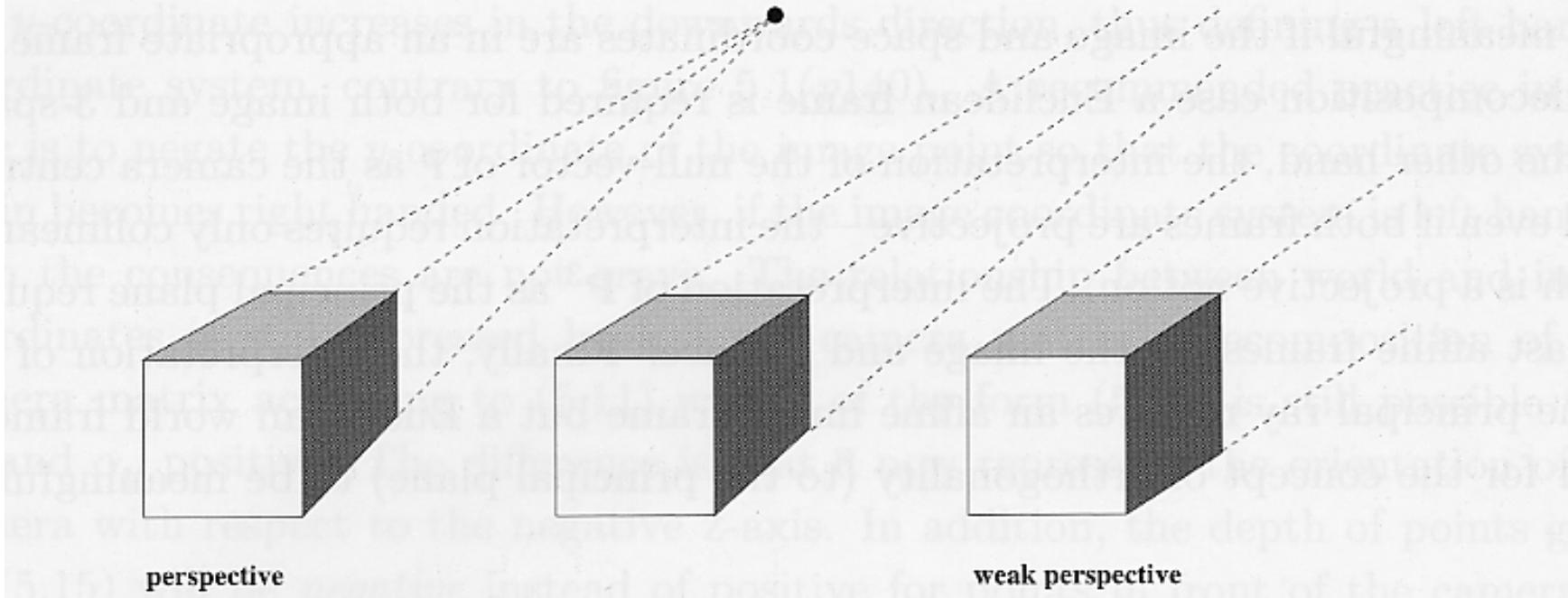
Up close and zoomed wide
With short focal length

Perspective Composition



200 mm (12°)

Walk back and zoom in
with long focal length

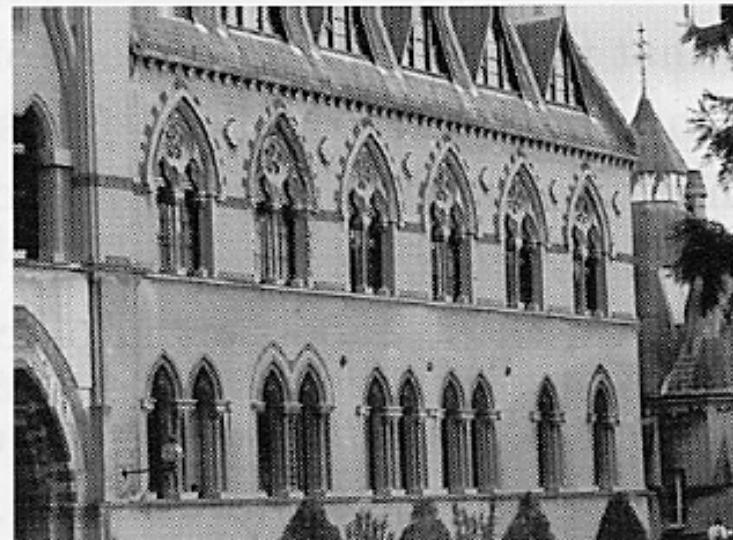
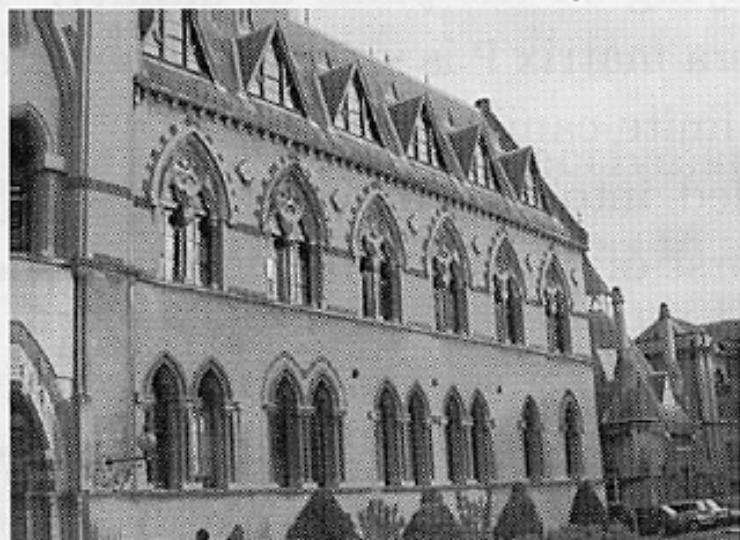


perspective

weak perspective

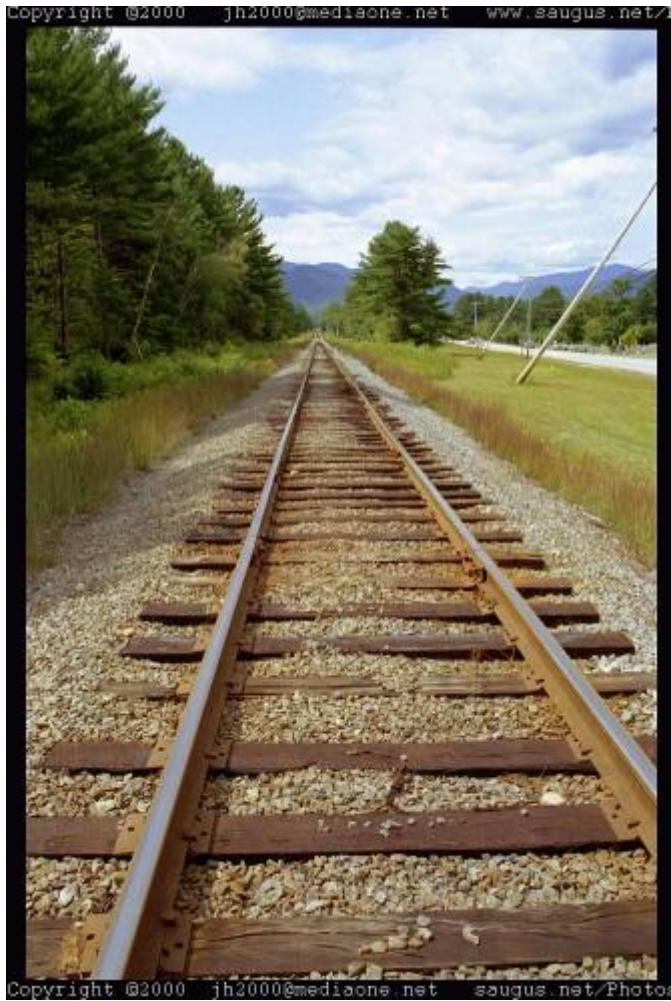
— increasing focal length —————→

— increasing distance from camera —————→



From Zisserman & Hartley

Funny things happen...



Copyright @2000 jh2000@mediacone.net www.saugus.net/Photos

Modeling projection (perspective)

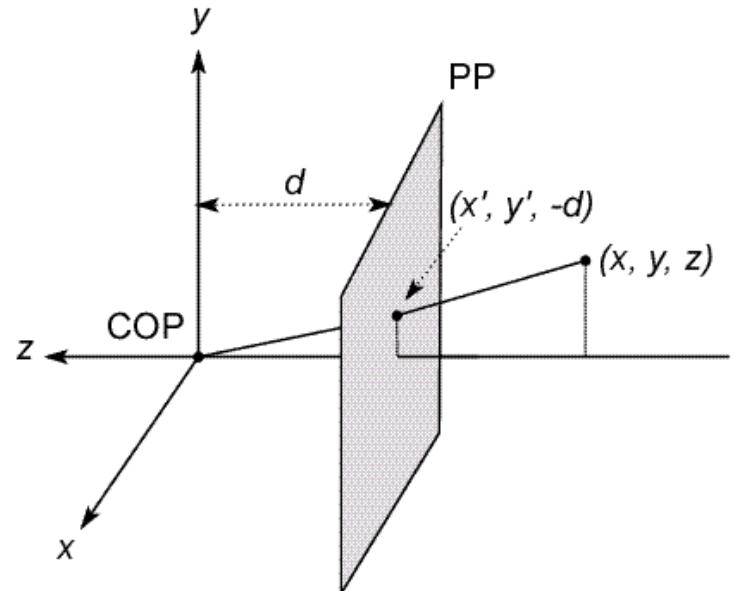
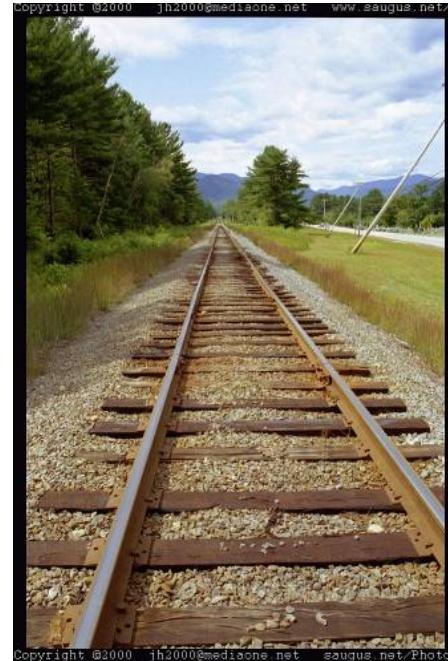
➤ Projection equations

- Compute intersection with PP of ray from (x, y, z) to COP
- Derived using similar triangles

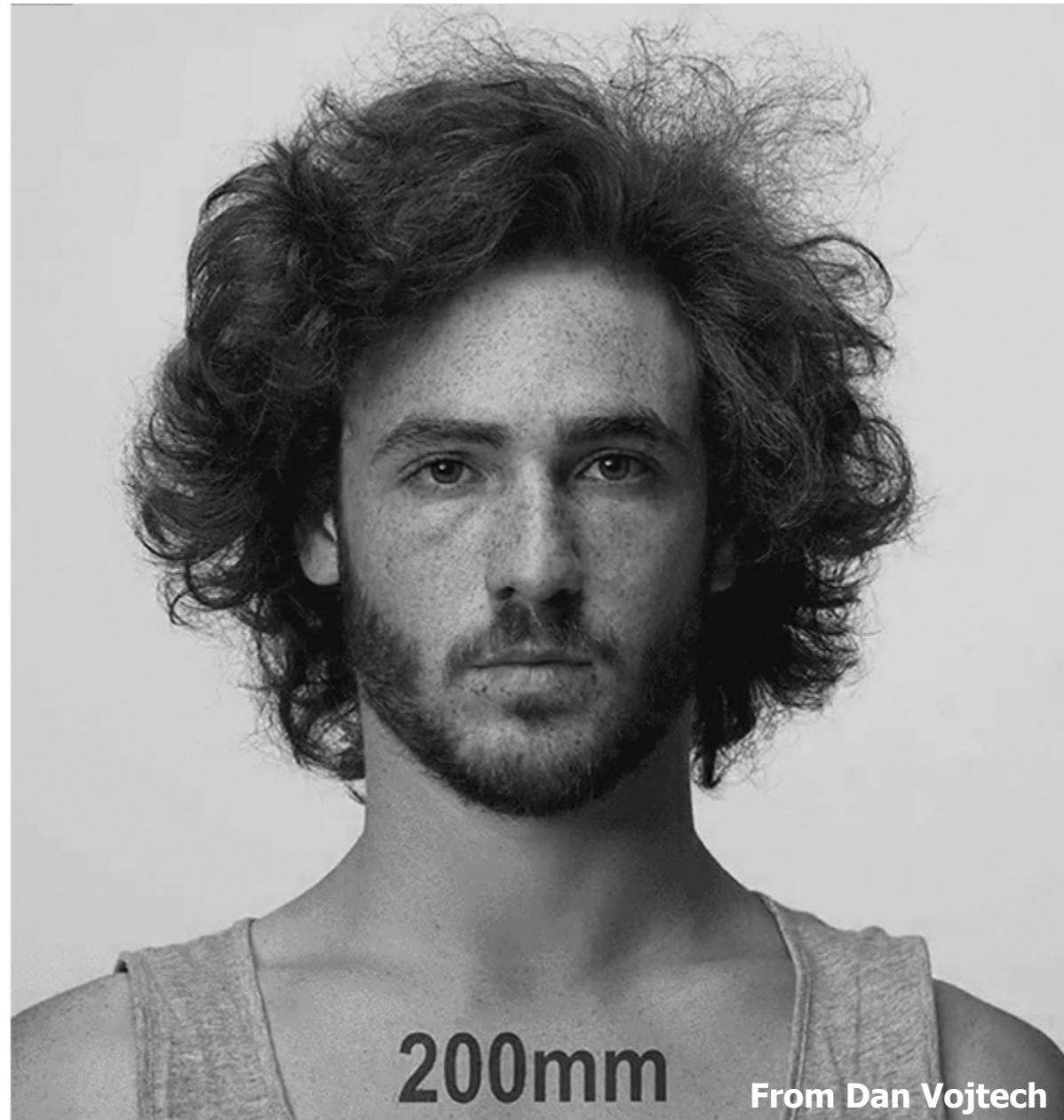
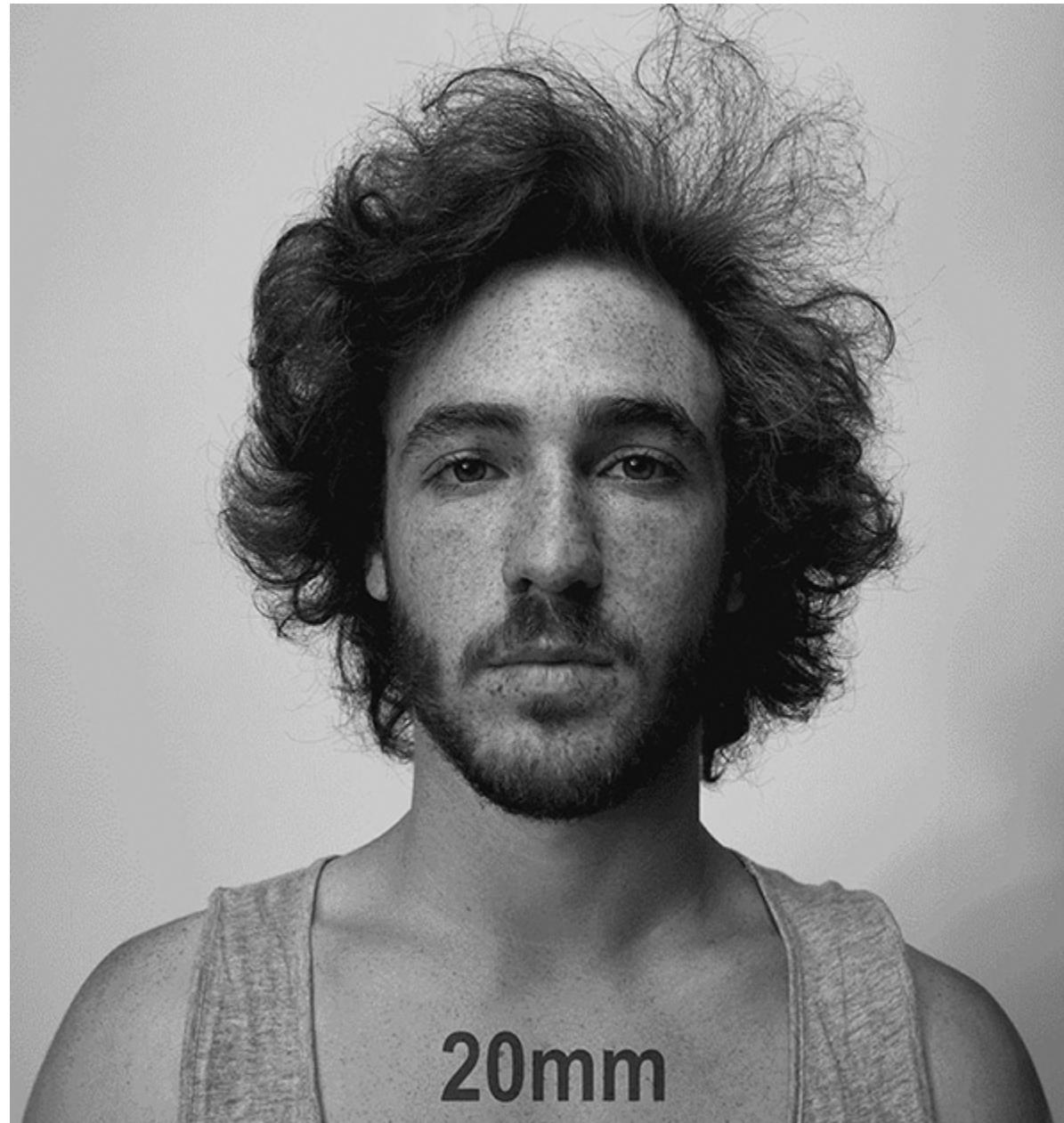
$$(x, y, z) \rightarrow \left(-d \frac{x}{z}, -d \frac{y}{z}, -d \right)$$

- We get the projection by ignoring the last coordinate:

$$(x, y, z) \rightarrow \left(-d \frac{x}{z}, -d \frac{y}{z} \right)$$



Field of View / Focal Length



From Dan Vojtech

Fun with Focal Length (Jim Sherwood)

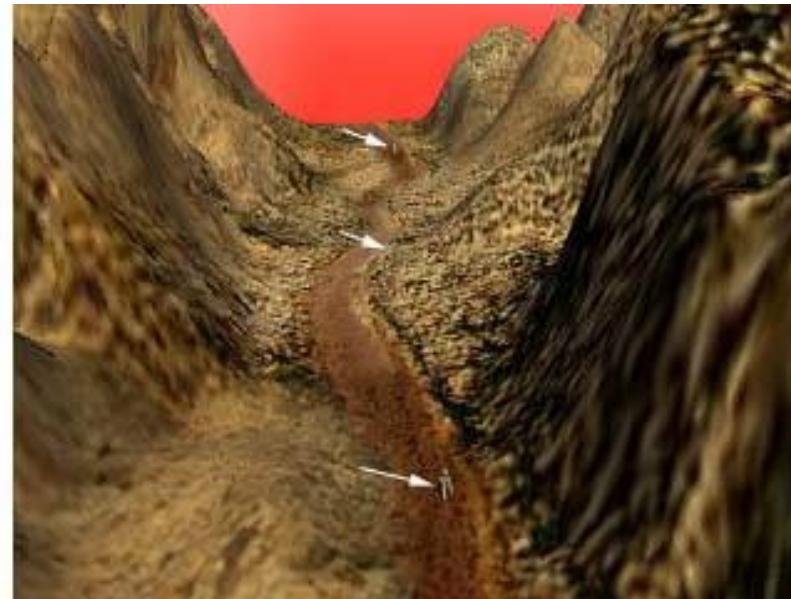


Figure 5.1

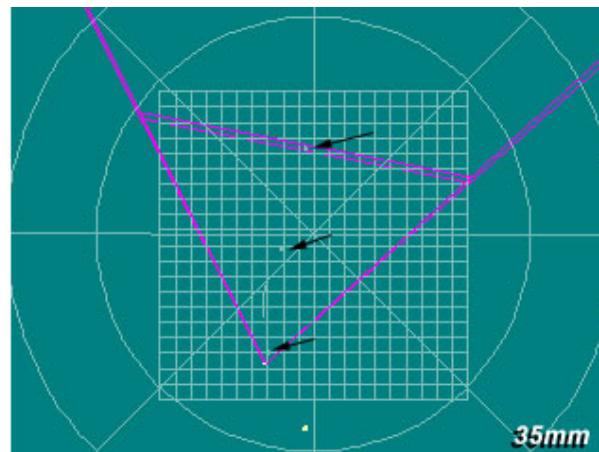
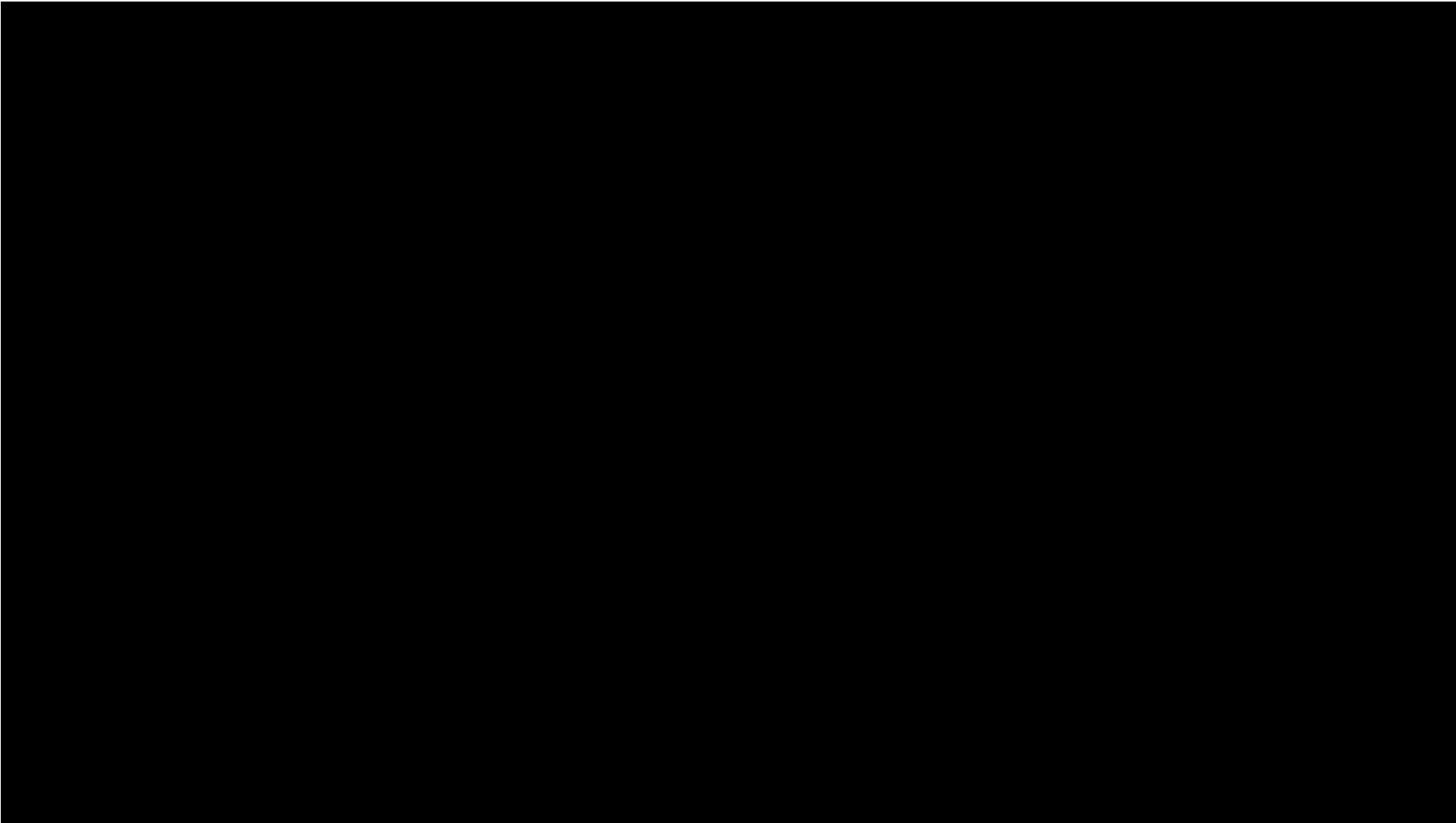


Figure 5.2

Dolly Zoom ("Vertigo Shot")



Outline

- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

Exposure

- **The amount of light to sensor/film**
- **Two main parameters:**
 - **Shutter speed**
 - **Aperture (area of lens)**

Shutter speed

- **Controls how long the film/sensor is exposed**
- **Pretty much linear effect on exposure**
- **Usually in fraction of a second:**
 - **1/30, 1/60, 1/125, 1/250, 1/500**
 - **Get the pattern ?**

Main effect of shutter speed (motion blur)

Slow shutter speed



Fast shutter speed



Shutter speed

- **Controls how long the film/sensor is exposed**
- **Pretty much linear effect on exposure**
- **Usually in fraction of a second:**
 - **1/30, 1/60, 1/125, 1/250, 1/500**
 - **Get the pattern ?**
- **On a normal lens, normal humans can hand-hold down to 1/60**
 - **In general, the rule of thumb says that the limit is the inverse of focal length, e.g., 1/500 for a 500mm**

Effect of shutter speed

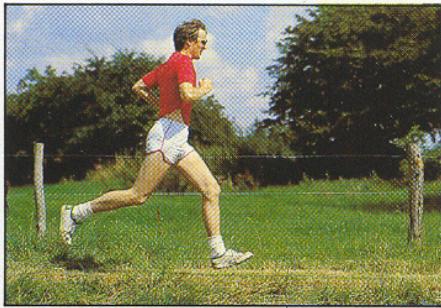
➤ Freezing motion

Walking people



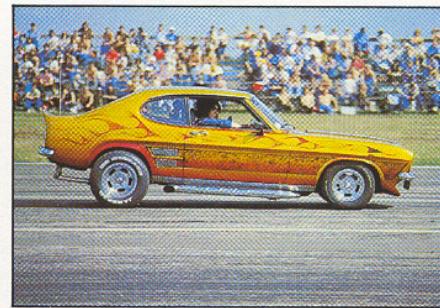
1/125

Running people



1/250

Car



1/500

Fast train

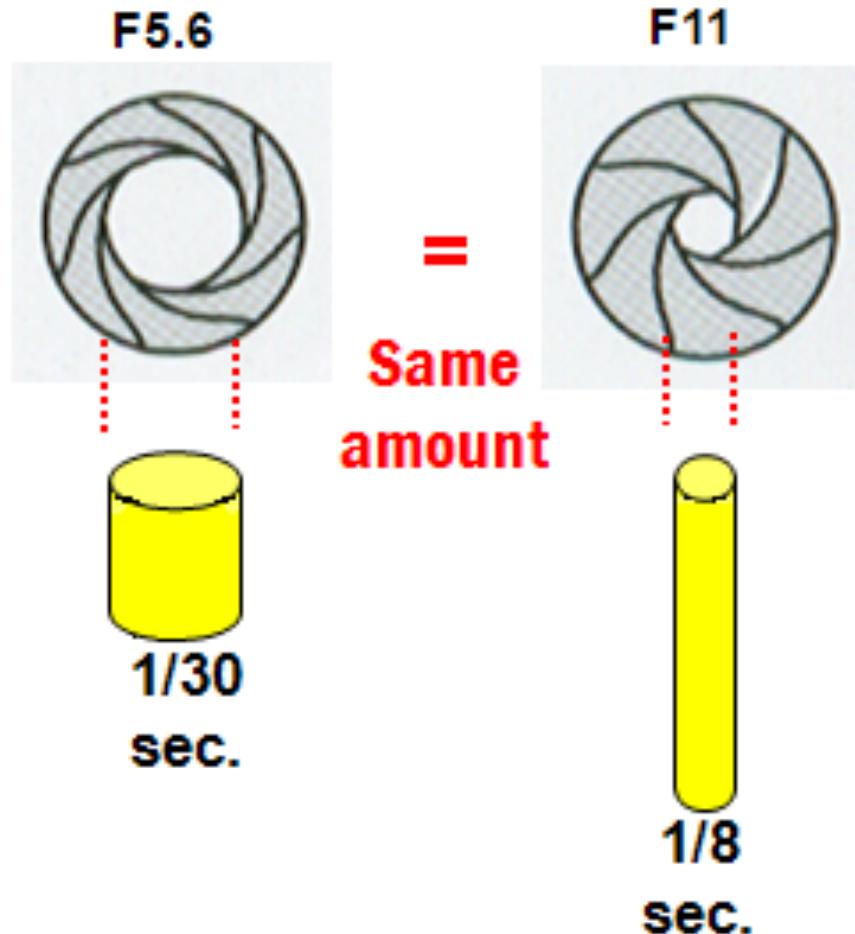


1/1000

Exposure

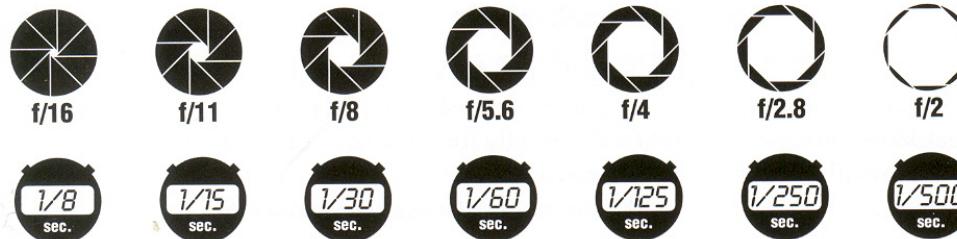
- **Get the right amount of light to sensor/film**
- **Two main parameters:**
 - **Shutter speed**
 - **Aperture (area of lens)**
- **Exposure ~ shutter time x lens area**

Exposure: shutter speed vs. aperture

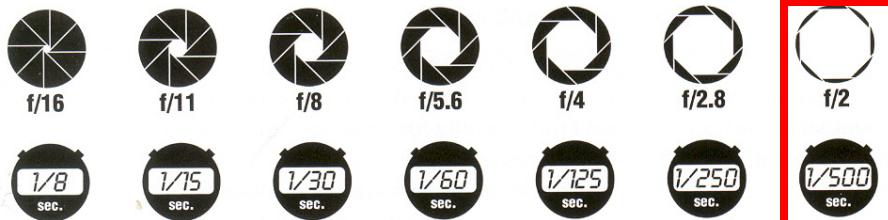


Reciprocity

- Assume we know how much light we need
- We have several choices for shutter speed/aperture pairs



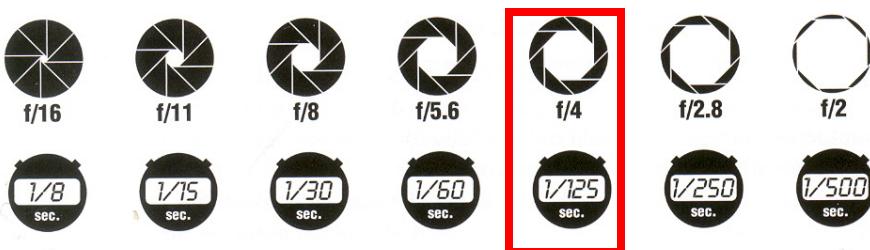
- What will guide our choice of a shutter speed?
 - Freeze motion vs. motion blur, camera shake
- What will guide our choice of an aperture?
 - Depth of field, diffraction limit
- Often, we must compromise
 - Open more to enable faster speed (but shallow DoF)



Large aperture (shallow depth of field), fast shutter speed (motion sharp). A fast shutter speed (1/500 sec) stops the motion of the pigeons so completely that the flapping wings are frozen. But the wide aperture (f/2) needed gives so little depth of field that the background is now out of focus.

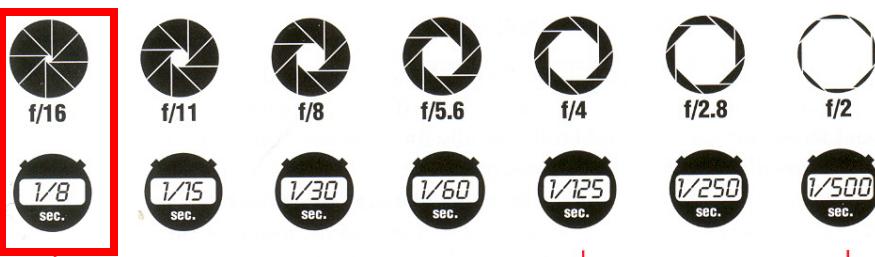


Medium aperture (moderate depth of field), medium shutter speed (some motion sharp). A medium aperture ($f/4$) and shutter speed (1/125 sec) sacrifice some background detail to produce recognizable images of the birds. But the exposure is still too long to show the motion of the birds' wings sharply.



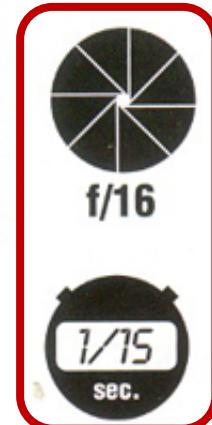
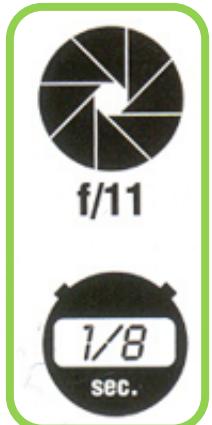


Small aperture (deep depth of field), slow shutter speed (motion blurred). In this scene, a small aperture ($f/16$) produced great depth of field; the nearest paving stones as well as the farthest trees are sharp. But to admit enough light, a slow shutter speed ($1/8$ sec) was needed; it was too slow to show moving pigeons sharply. It also meant that a tripod had to be used to hold the camera steady.



Exposure

- A stop is doubling or halving of the amount of light
 - Can be controlled by both aperture and shutter speed
- To increase the exposure by 1 stop
 - Increase the aperture by a factor of 2
 - Increase the shutter time by a factor of 2





More fun



<http://vimeo.com/14958082>

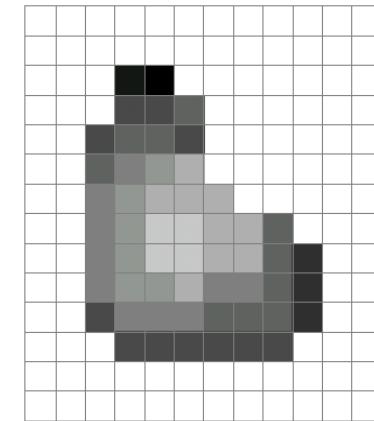
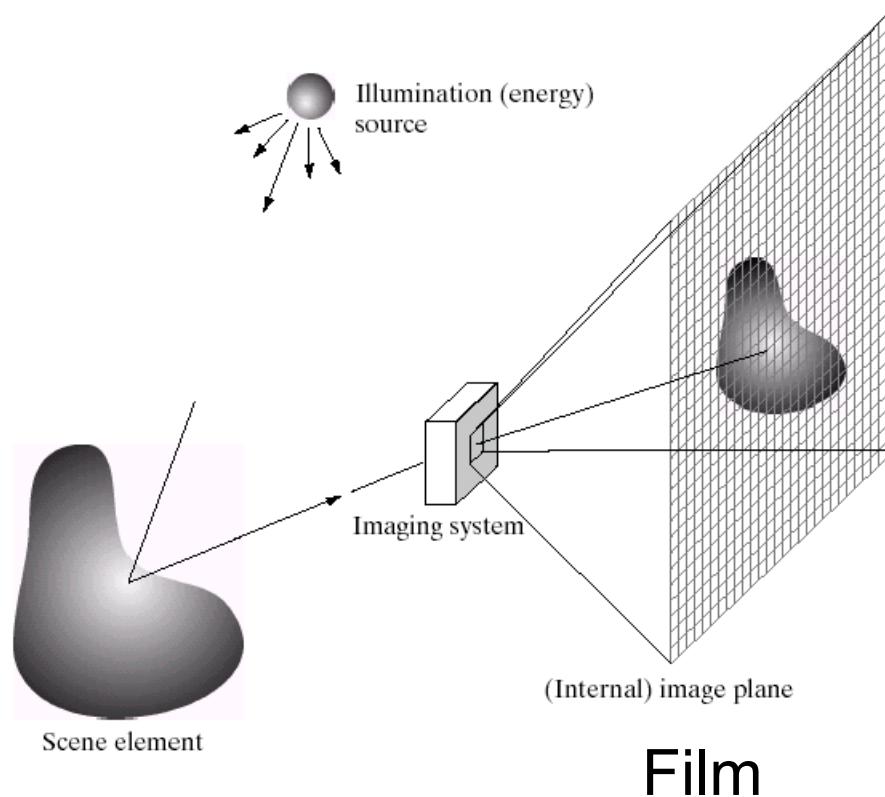
Pixelstick



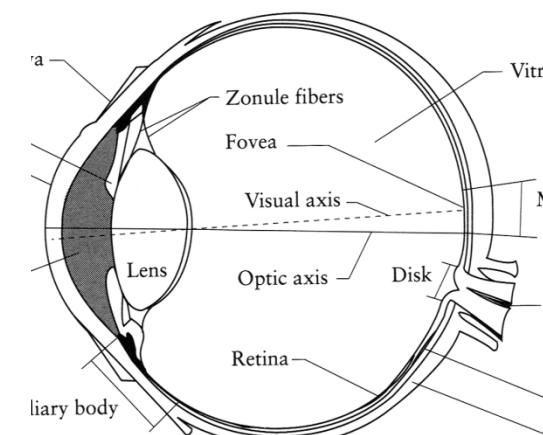
Outline

- **Pinhole camera**
- **Lens**
- **Depth of field**
- **Field of view**
- **Exposure**
- **Digital camera**

Image Formation



Digital Camera



The Eye

Digital camera

- A digital camera replaces film with a sensor array
 - Each cell in the array is light-sensitive diode that converts photons to electrons
 - Two common types
 - Charge Coupled Device (CCD)
 - CMOS



Sampling and Quantization

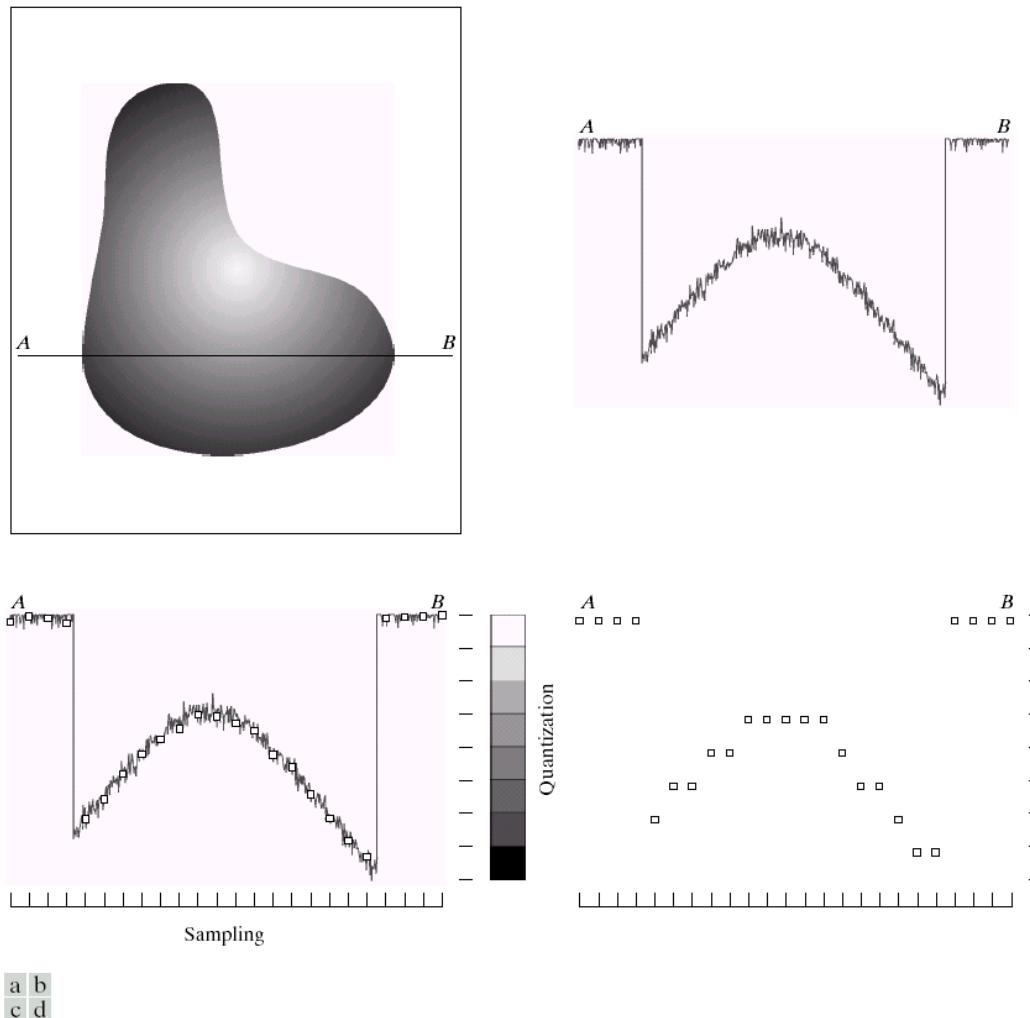


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sensor Array

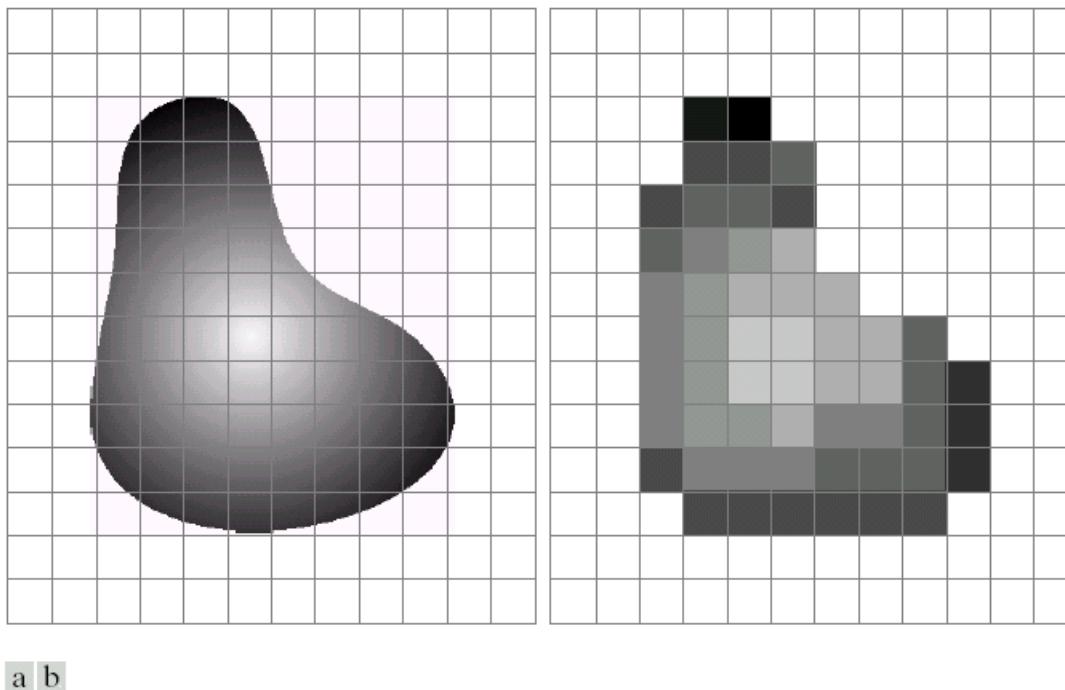


FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Global vs rolling shutter

- **Global: Taking a snapshot of the scene at a single instant**
- **Rolling: Scanning across the scene vertically or horizontally**

Rolling shutter (focal plane shutter)



Other shutter systems

- **Also have leaf shutters**
 - **Circular iris that closes**
- **Global electronic shutter**
 - **Different circuit design that exposes all pixels with the same time duration**
- **Mixtures of physical and electronic shutter**
 - **E.g., electronic starts exposure, physical shutter ends exposure**



London

Rolling shutter



Credit: smarter everyday

Rolling shutter



Credit: smarter everyday