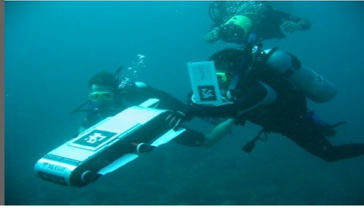


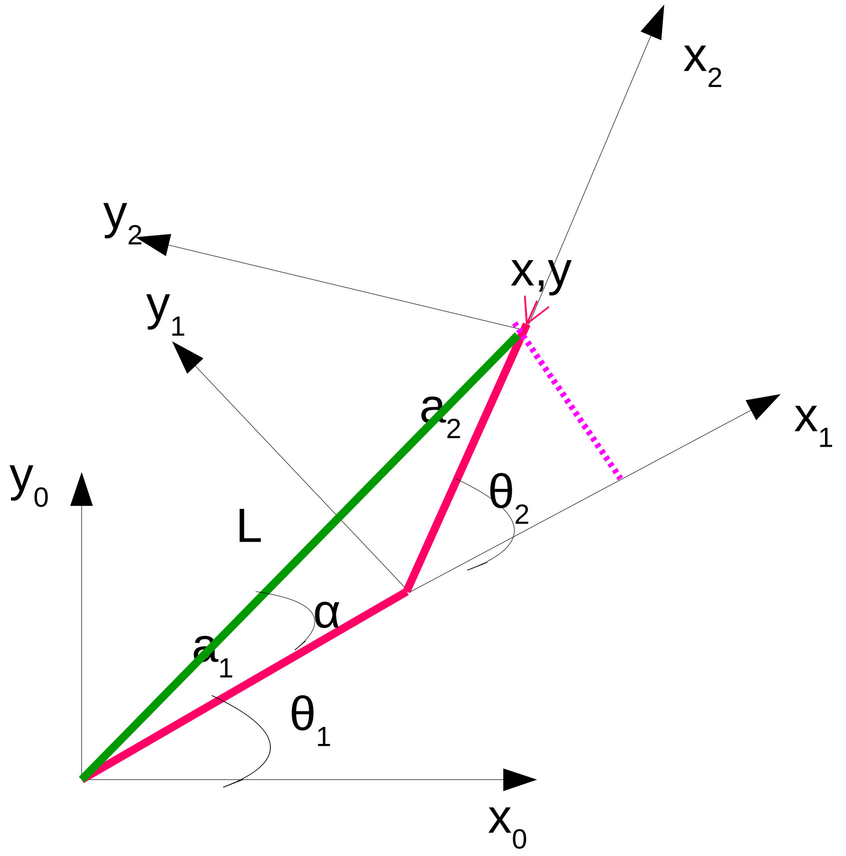


CSci5551 Introduction to Intelligent Robotics Systems

Forward Kinematics

College of Science & Engineering
University of Minnesota

Inverse Kinematics of the Two Link Arm



College of Science & Engineering
University of Minnesota

Inverse Kinematics of the Two Link Arm

$$x^2 + y^2 = L^2$$

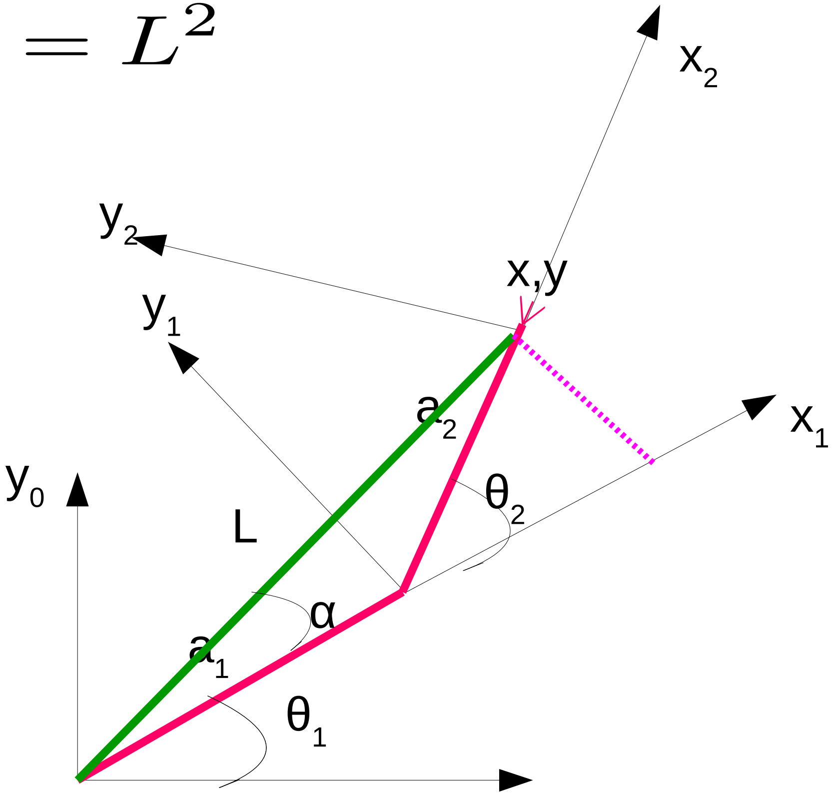
$$a_1^2 + a_2^2 - 2a_1 a_2 cos(\pi - \theta_2) = L^2$$

$$cos\theta_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$Let\ D = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$sin\theta_2 = \pm\sqrt{1 - D^2}$$

$$\theta_2 = tan^{-1} \pm \frac{\sqrt{1 - D^2}}{D}$$

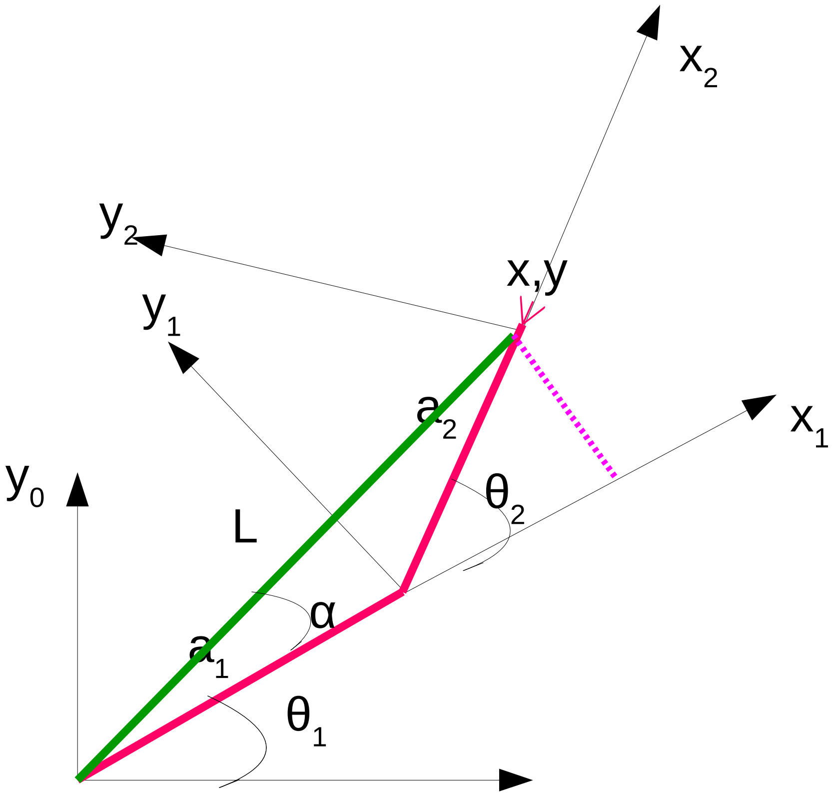


Inverse Kinematics of the Two Link Arm

$$\theta_1 + \alpha = tan^{-1}\left(\frac{y}{x}\right)$$

$$\alpha = tan^{-1}\frac{a_2 sin\theta_2}{a_1 + a_2 cos\theta_2}$$

(How did we get that last expression?)



Velocity Kinematics

- Suppose we would like the tool to follow a path at constant velocity
- Need the relationship between the tool velocity and joint velocities

Velocity Kinematics

- Tool coordinates

  $x=a_1\cos(\theta_1)+a_2\cos(\theta_1+\theta_2)$
  $y=a_1\sin(\theta_1)+a_2\sin(\theta_1+\theta_2)$

- Velocity equations

$$\dot{x} = -a_1 sin\theta_1 \times \dot{\theta}_1 - a_2 \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$
$$\dot{y} = -a_1 cos\theta_1 \times \dot{\theta}_1 + a_2 cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$

Velocity Kinematics

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix} \dot{\theta}$$

$$\dot{\mathbf{x}} = J\dot{\theta}$$

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

is the Jacobian matrix.

The inverse problem?

$$\dot{\theta} = J^{-1}\dot{\mathbf{x}}$$

$$J^{-1} = \frac{1}{\Delta}\begin{bmatrix} a_2 c_{12} & a_2 s_{12} \\ -a_1 c_1 - a_2 c_{12} & -a_1 s_1 - a_2 s_{12} \end{bmatrix}$$

$$\Delta = a_1 a_2 s_2$$

Frames to links?

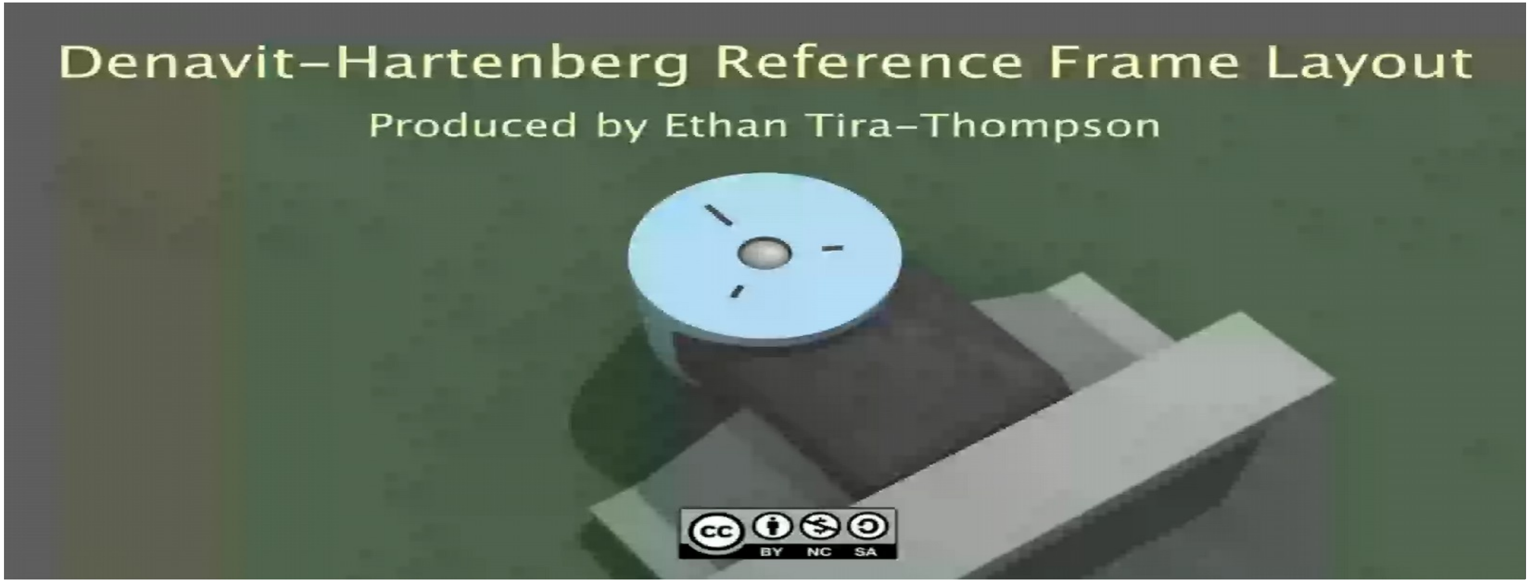
- Denavit-Hartenberg (DH) Convention
- To assess effect of change in one link on other links
- Forward Kinematics

- In the DH convention, each homogeneous transformation $A_{i,i+1}$ is represented by four basic (current frame) transformations

$$A_{i,i+1} = Rot_{z,\theta} Trans_{z,d} Trans_{x,a} Rot_{x,\alpha}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

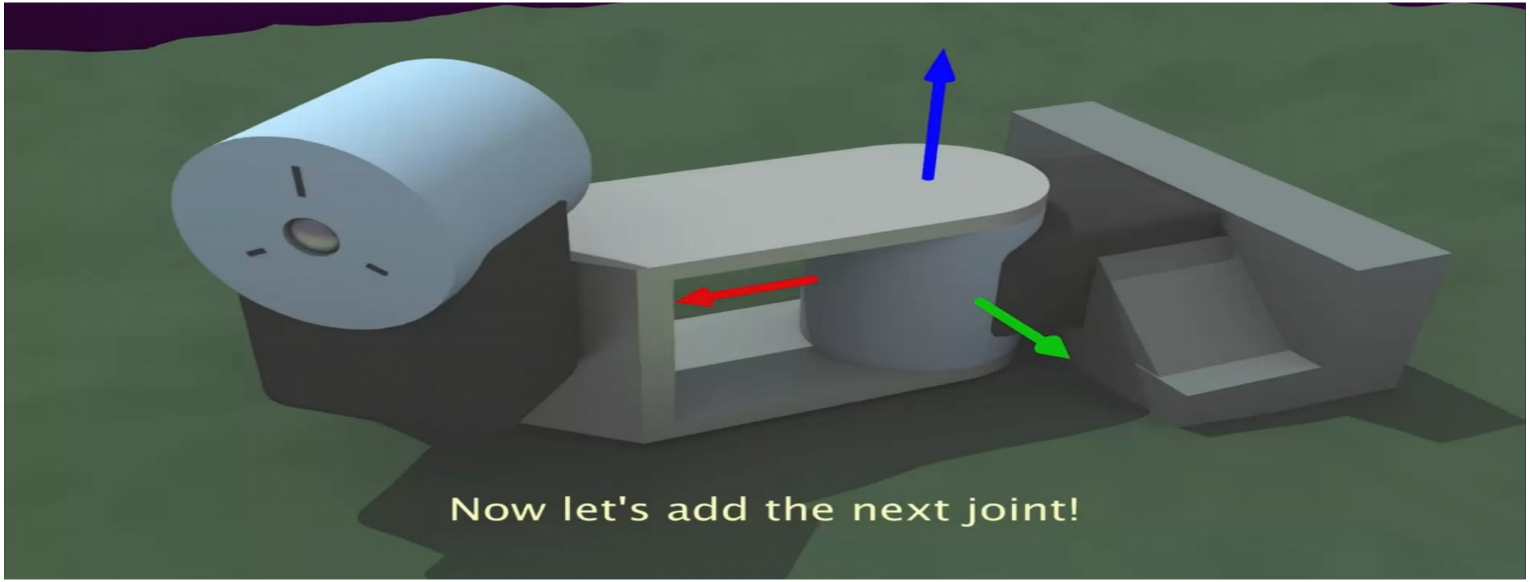
DHC Step 1

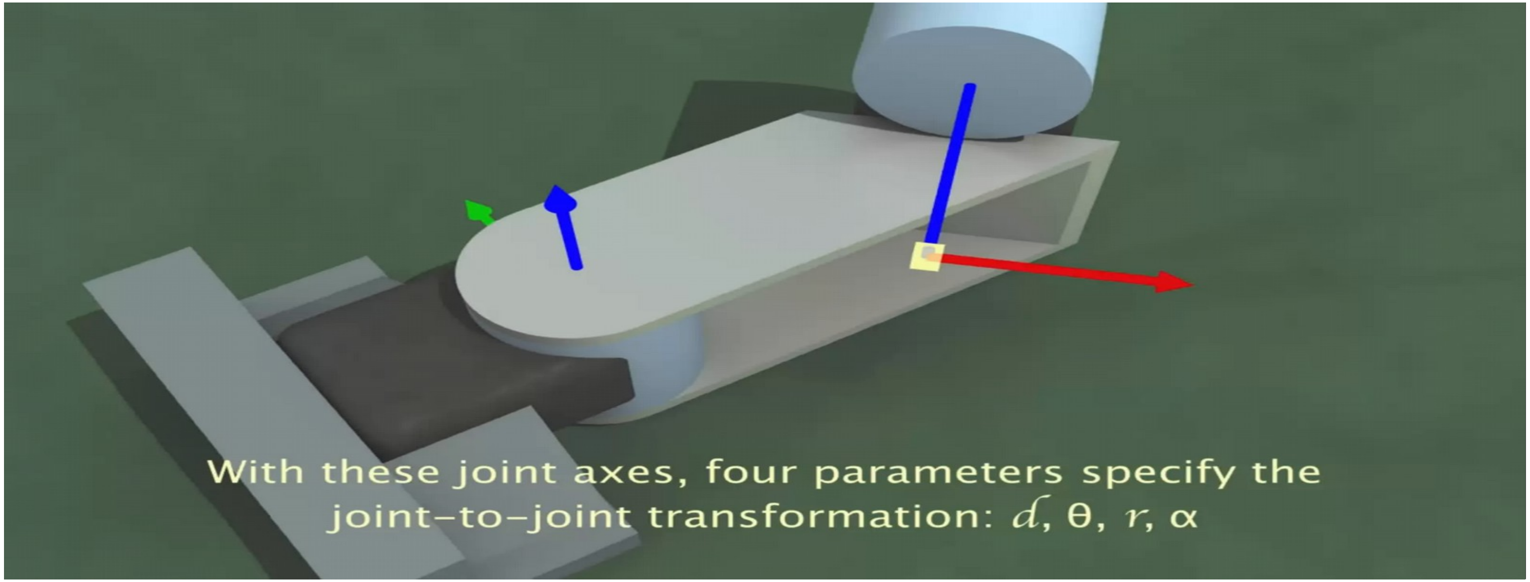


From TekkotsuRobotics :https://youtu.be/rA9tm0gTln8
http://www.tekkotsu.org/Kinematics.html

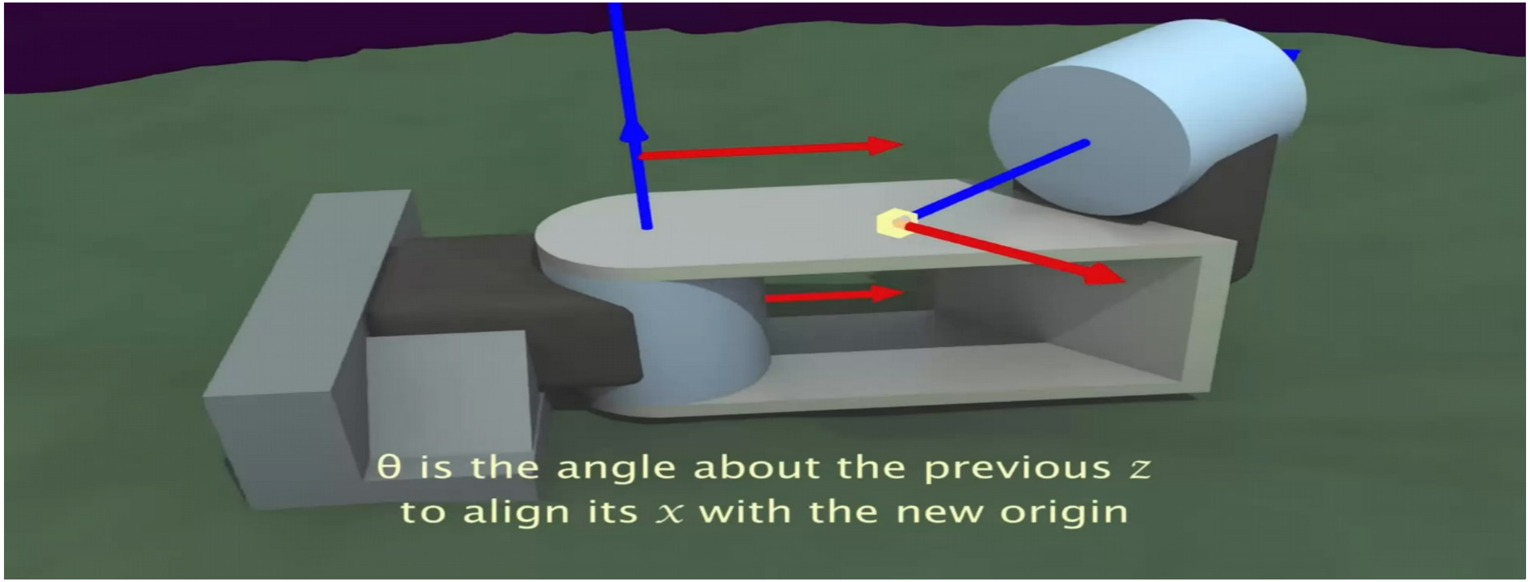
COLLEGE OF Science & Engineering
UNIVERSITY OF MINNESOTA

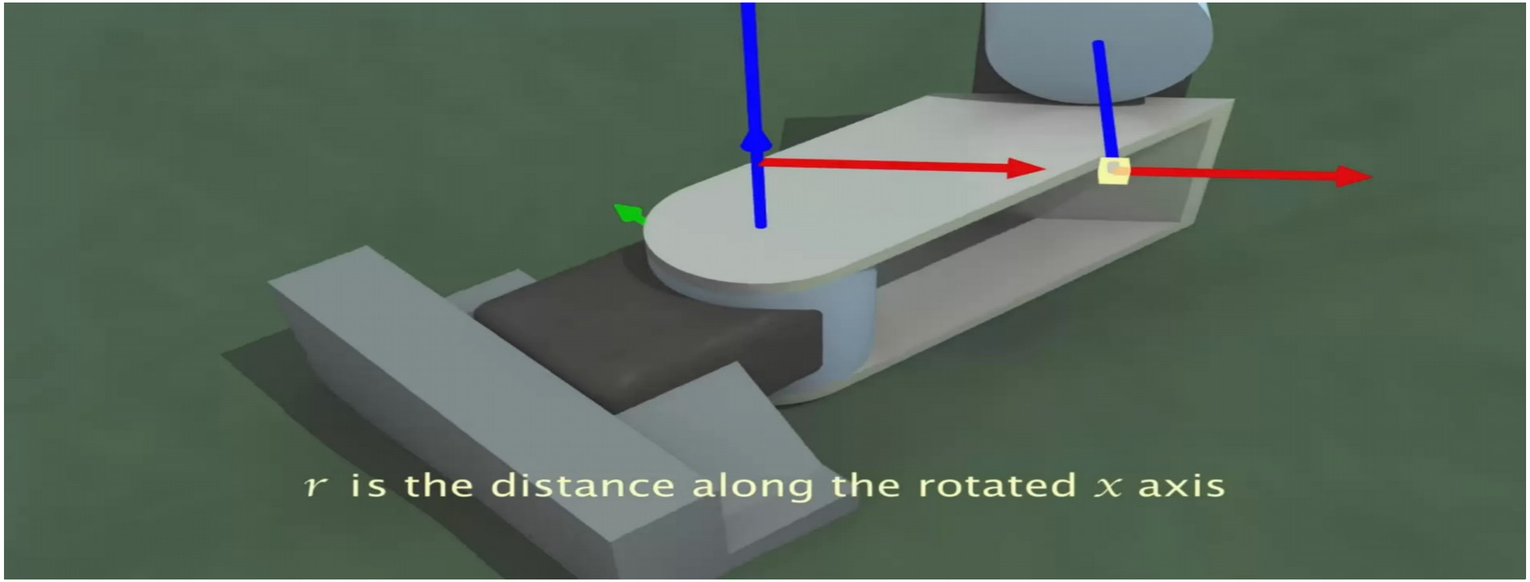
DHC Step 2

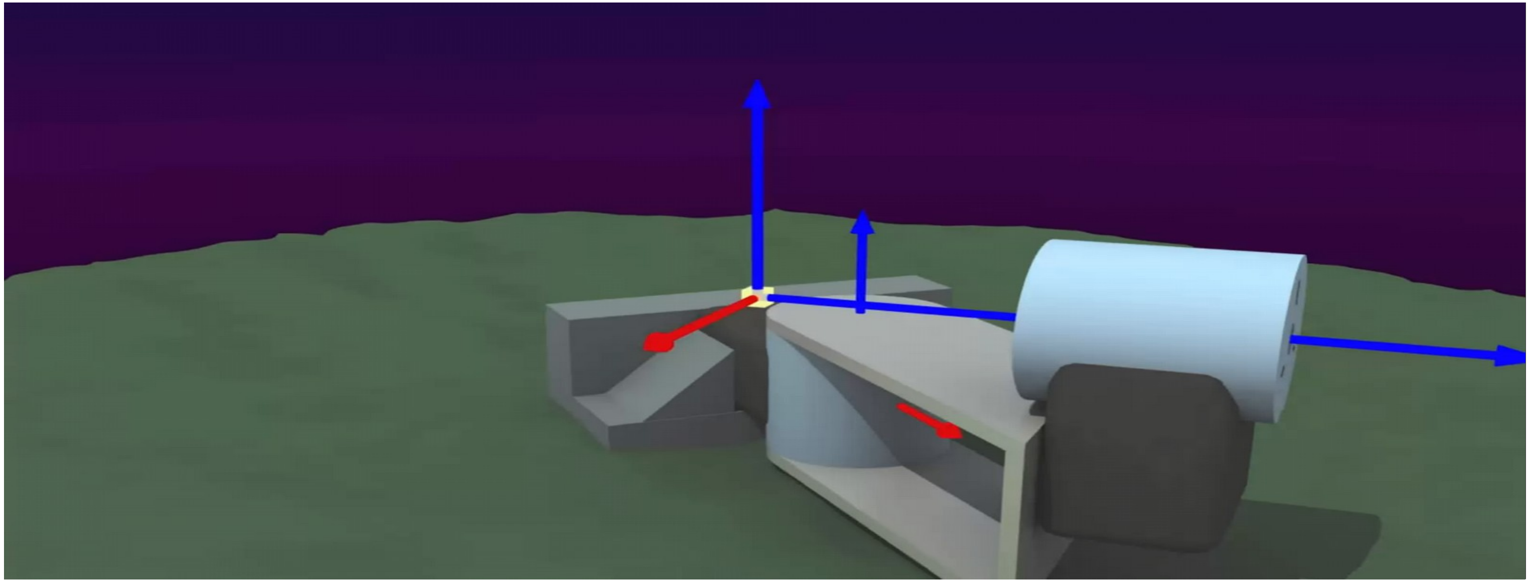




d here is **a** in the book/slides


θ is the angle about the previous z
to align its x with the new origin


r is the distance along the rotated x axis



Transformation between i-1 and i

- Four successive elementary transformations are required to relate the *i*-th coordinate frame to the (*i*-1)-th coordinate frame:
  - Rotate about the Z_{i-1} axis an angle of θ_i to align the X_{i-1} axis with the X_i axis.
  - Translate along the Z_{i-1} axis a distance of d_i, to bring X_{i-1} and X_i axes into coincidence.
  - Translate along the X_i axis a distance of a_i to bring the two origins O_{i-1} and O_i as well as the X axis into coincidence.
  - Rotate about the X_i axis an angle of α_i (in the right-handed sense), to bring the two coordinates into coincidence.

DH Parameters

Parameter	Axis	Description
θ_i	z_{i-1}	Joint Angle - Variable for revolute joints.
d_i	z_{i-1}	Link Offset - Variable for prismatic joints.
a_i	x_i	Link Length - Constant perpendicular distance between z_{i-1} and z_i.
α_i	x_i	Link Twist - Constant angle between z_{i-1} and z_i.

COLLEGE OF Science & Engineering
UNIVERSITY OF MINNESOTA

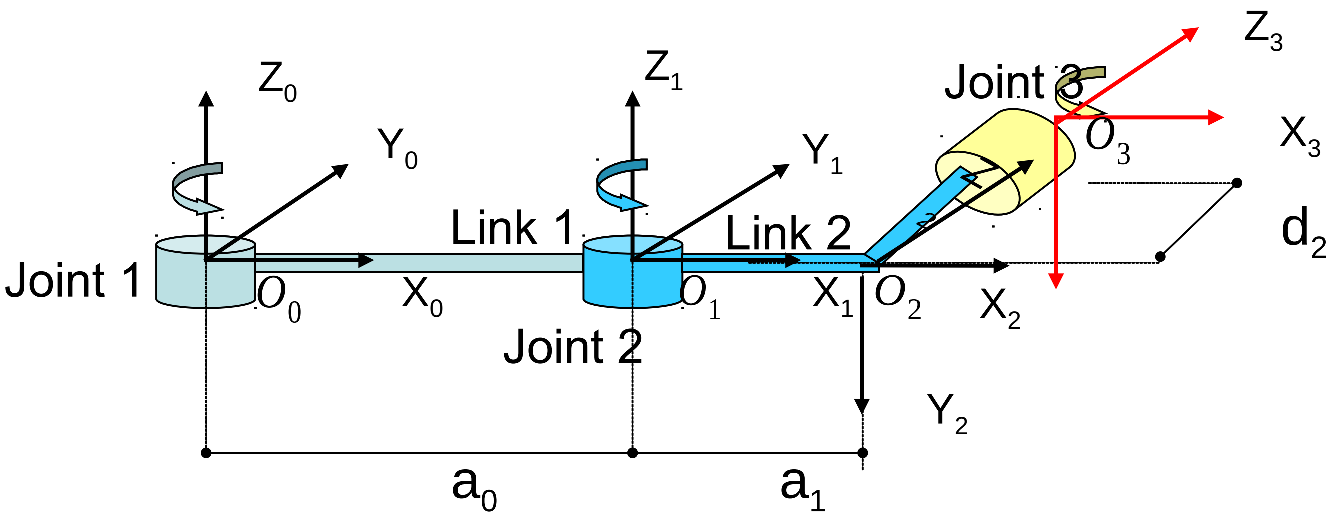
Denavit-Hartenberg Convention

- Number the joints from 1 to n starting with the base and ending with the end-effector.
- *Establish the base coordinate system.* Establish a right-handed orthonormal coordinate system (X_0, Y_0, Z_0) at the supporting base with Z_0 axis lying along the axis of motion of joint 1.
- *Establish joint axis.* Align the Z_i with the axis of motion (rotary or sliding) of joint i+1.
- *Establish the origin of the ith coordinate system.* Locate the origin of the i_{th} coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- *Establish X_i axis.* Establish $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$ or along the common normal between the Z_{i-1} & Z_i axes when they are parallel.
- *Establish Y_i axis.* Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- Find the link and joint parameters

College of Science & Engineering
University of Minnesota

Example

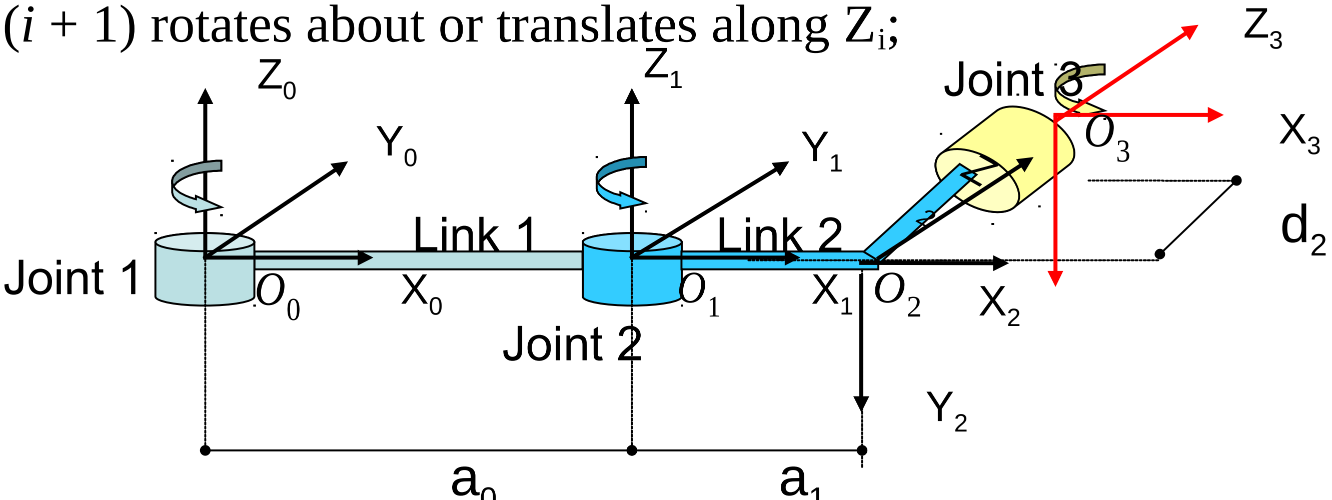
- 3 Revolute Joints



College of Science & Engineering
University of Minnesota

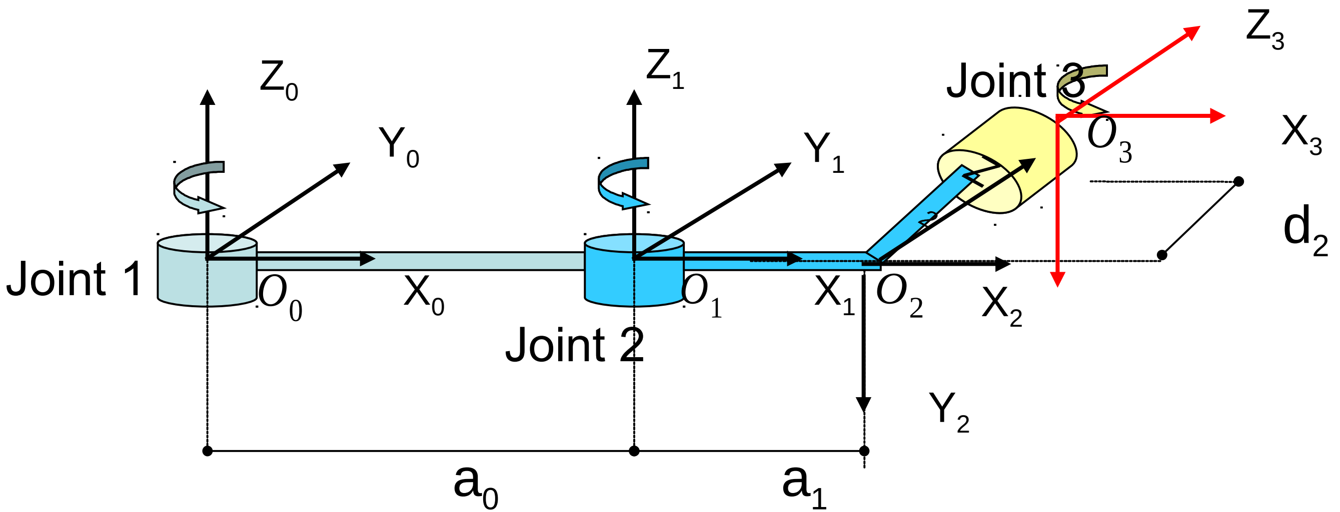
Link Coordinate Frames

- *Assign Link Coordinate Frames:*
  - To describe the geometry of robot motion, we assign a Cartesian coordinate frame (O_i, X_i,Y_i,Z_i) to each link, as follows:
    - establish a right-handed orthonormal coordinate frame O_0 at the supporting base with Z_0 lying along joint 1 motion axis.
    - the Z_i axis is directed along the axis of motion of joint (i + 1), that is, link (i + 1) rotates about or translates along Z_i;



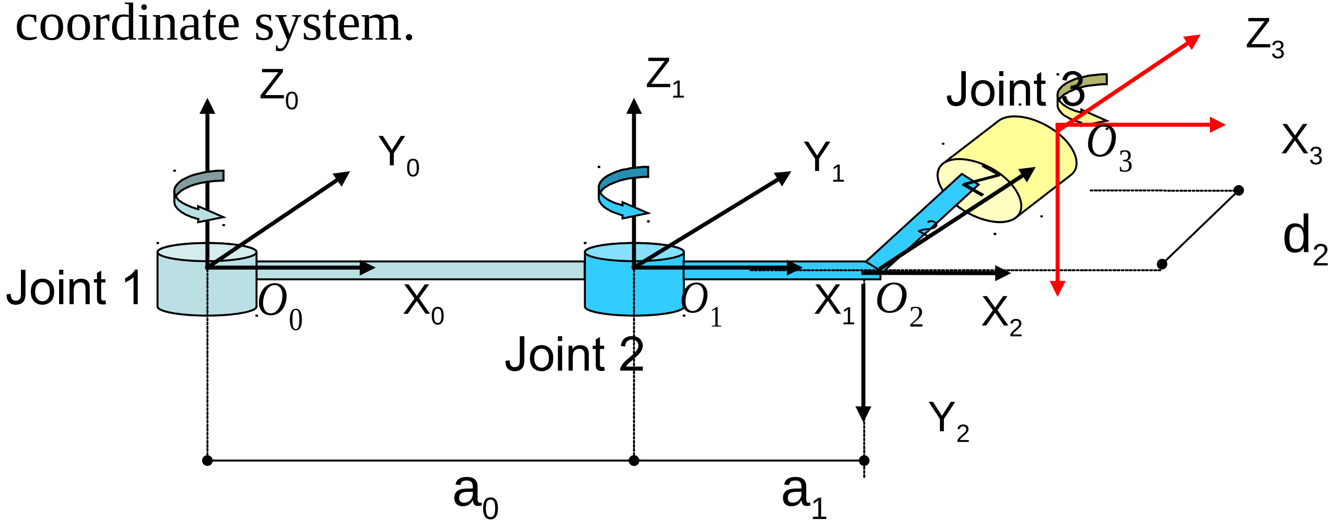
Link Coordinate Frames

- Locate the origin of the ith coordinate at the intersection of the Z_i & Z_{i-1} or at the intersection of common normal between the Z_i & Z_{i-1} axes and the Z_i axis.
- the X_i axis lies along the common normal from the Z_{i-1} axis to the Z_i axis $X_i = \pm(Z_{i-1} \times Z_i)/\|Z_{i-1} \times Z_i\|$, (if Z_{i-1} is parallel to Z_i, then X_i is specified arbitrarily, subject only to X_i being perpendicular to Z_i);



Link Coordinate Frames

- Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
  - The hand coordinate frame is specified by the geometry of the end-effector. Normally, establish Z_n along the direction of Z_{n-1} axis and pointing away from the robot; establish X_n such that it is normal to both Z_{n-1} and Z_n axes. Assign Y_n to complete the right-handed coordinate system.



Link and Joint Parameters

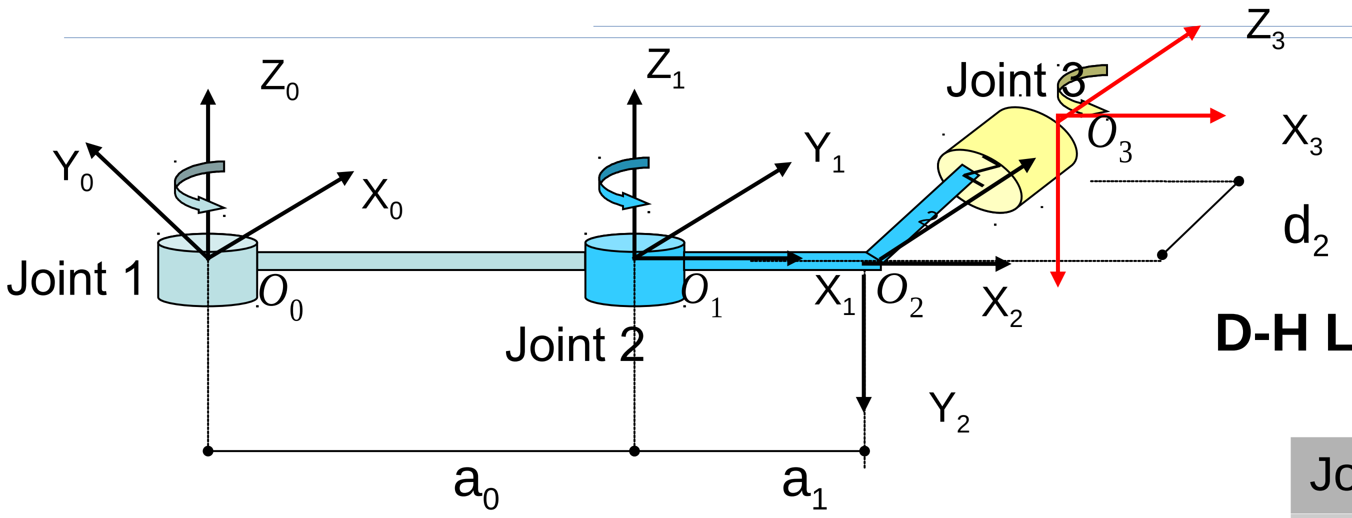
- *Joint angle* θ_i : the angle of rotation from the X_{i-1} axis to the X_i axis about the Z_{i-1} axis. It is the joint variable if joint i is rotary.

- *Joint distance* d_i : the distance from the origin of the (i-1) coordinate system to the intersection of the Z_{i-1} axis and the X_i axis along the Z_{i-1} axis. It is the joint variable if joint i is prismatic.

- *Link length* a_i: the distance from the intersection of the Z_{i-1} axis and the X_i axis to the origin of the ith coordinate system along the X_i axis.

- *Link twist angle* α_i: the angle of rotation from the Z_{i-1} axis to the Z_i axis about the X_i axis.

Example



D-H Link Parameter Table

Joint	θ	*d*	*a*	α
1	θ_1^*	0	a_0	0
2	θ_2^*	0	a_1	90

α_i : rotation angle from Z_{i-1} to Z_i about X_i

a_i : distance from intersection of Z_{i-1} & X_i to origin of i coordinate along X_i

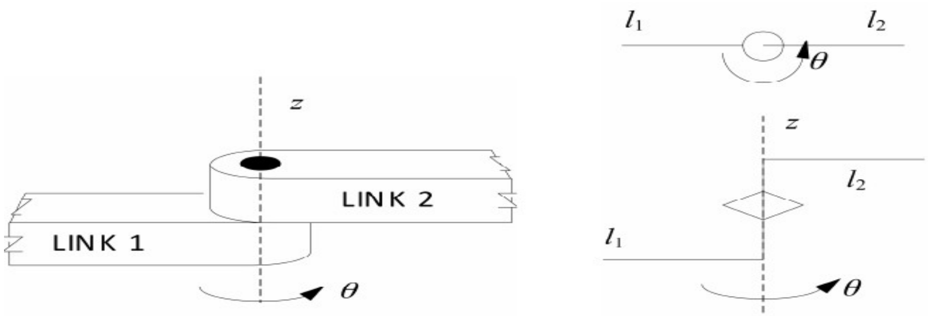
d_i : distance from origin of (i-1) coordinate to intersection of Z_{i-1} & X_i along Z_{i-1}

θ_i : rotation angle from X_{i-1} to X_i about Z_{i-1}

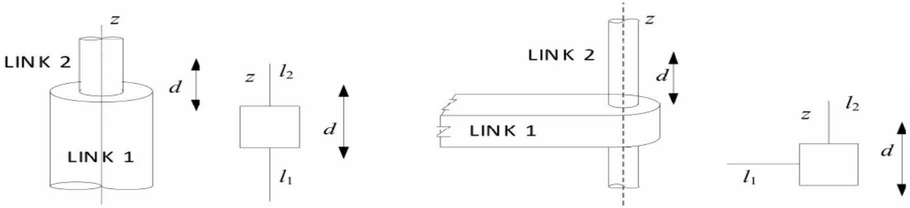
COLLEGE OF Science & Engineering
UNIVERSITY OF MINNESOTA

Joints

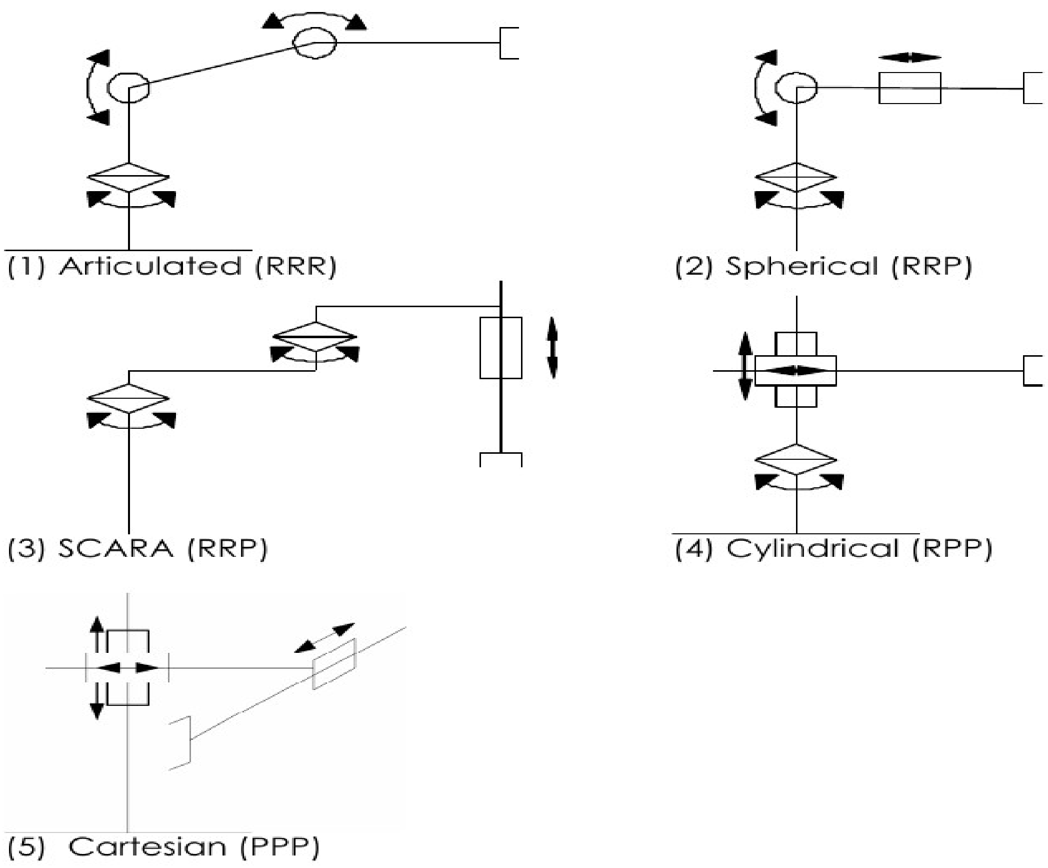
- Revolute
  - Rotation-only



- Prismatic
  - Translation-only



Examples


(1) Articulated (RRR)
(2) Spherical (RRP)
(3) SCARA (RRP)
(4) Cylindrical (RPP)
(5) Cartesian (PPP)