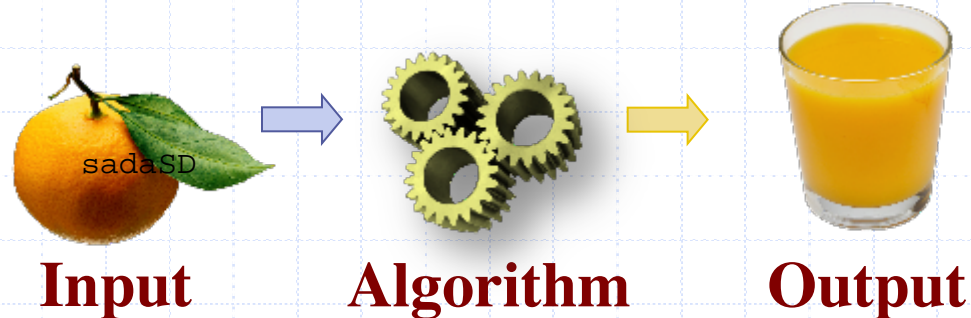
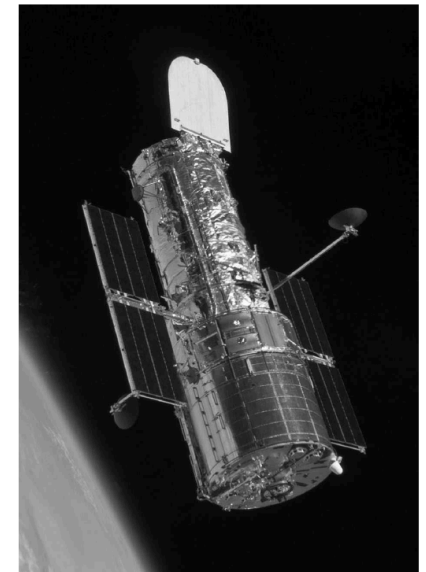
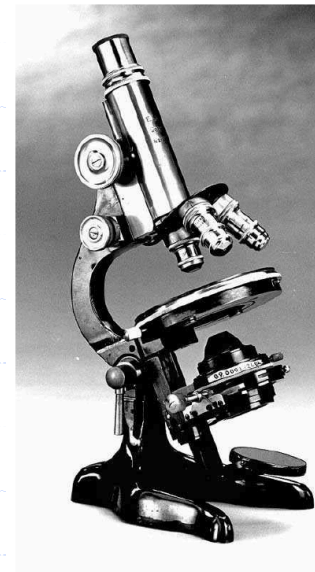


# Analysis of Algorithms



# Scalability

- ❑ Scientists often have to deal with differences in scale, from the microscopically small to the astronomically large.
- ❑ Computer scientists must also deal with scale, but they deal with it primarily in terms of data volume rather than physical object size.
- ❑ **Scalability** refers to the ability of a system to gracefully accommodate growing sizes of inputs or amounts of workload.



Microscope: U.S. government image, from the N.I.H. Medical Instrument Gallery, DeWitt Stetten, Jr., Museum of Medical Research. Hubble Space Telescope: U.S. government image, from NASA, STS-125 Crew, May 25, 2009.

# Algorithms and Data Structures

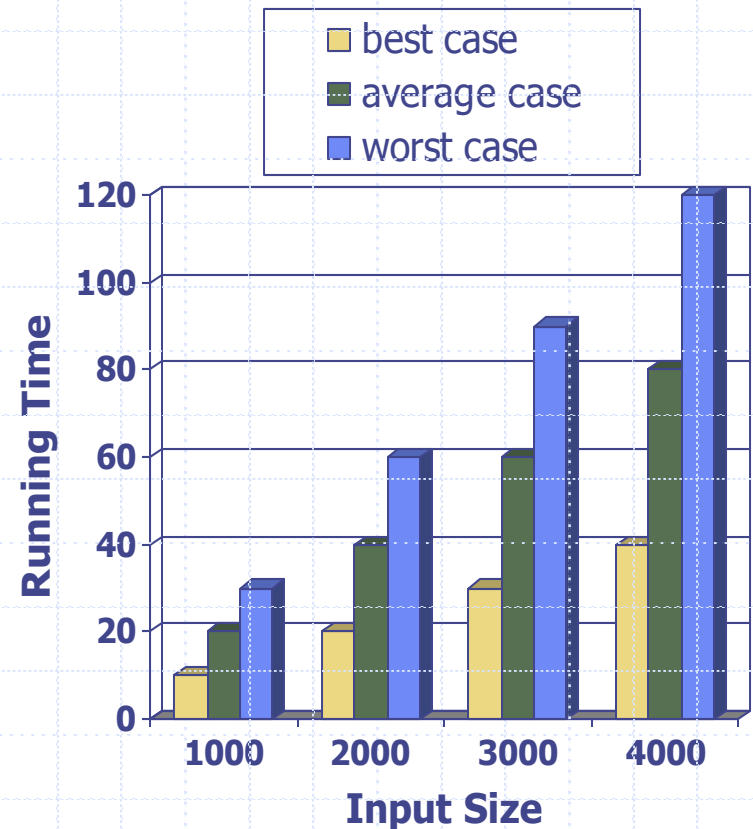
- An **algorithm** is a step-by-step procedure for performing some task in a finite amount of time.
  - Typically, an algorithm takes input data and produces an output based upon it.



- A **data structure** is a systematic way of organizing and accessing data.

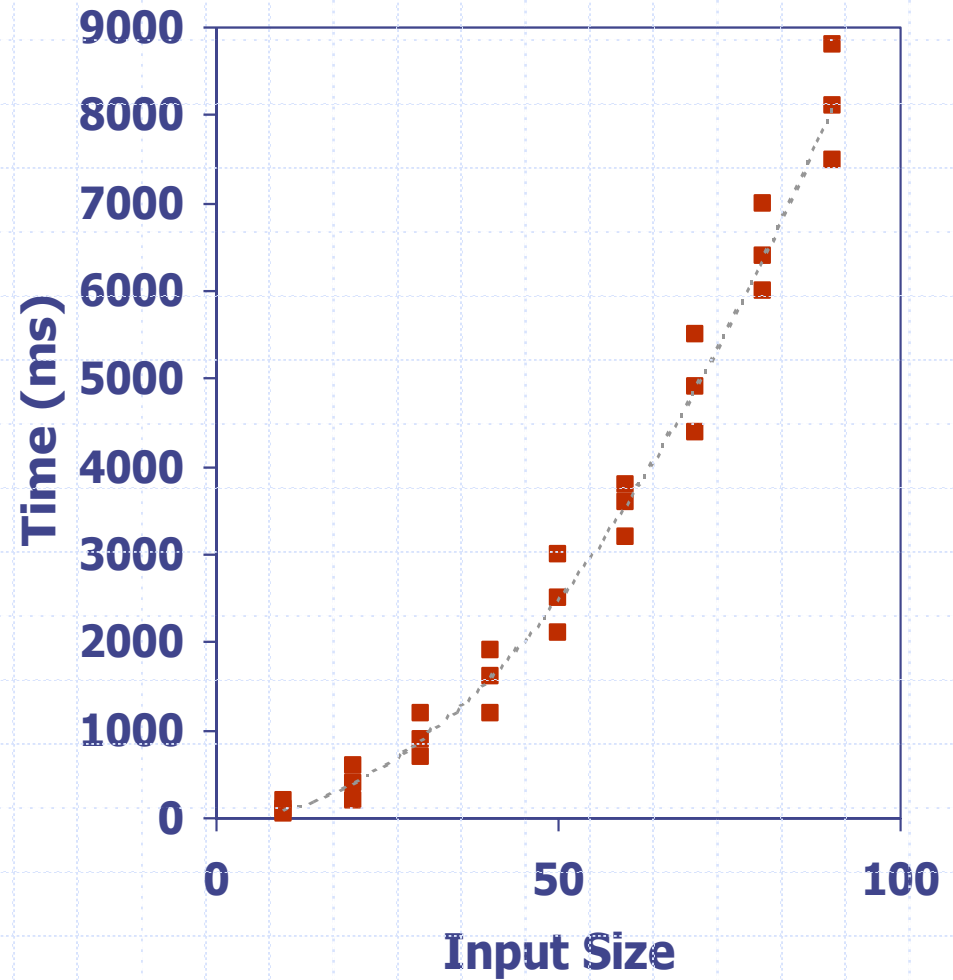
# Running Time

- Most algorithms transform input objects into output objects.
- The running time of an algorithm typically grows with the input size.
- Average case time is often difficult to determine.
- We focus primarily on the **worst case running time**.
  - Easier to analyze
  - Crucial to applications such as games, finance and robotics



# Experimental Studies

- Write a program implementing the algorithm
- Run the program with inputs of varying size and composition, noting the time needed:
- Plot the results



# Limitations of Experiments

- ❑ It is necessary to implement the algorithm, which may be difficult
- ❑ Results may not be indicative of the running time on other inputs not included in the experiment.
- ❑ In order to compare two algorithms, the same hardware and software environments must be used



# Theoretical Analysis



- ❑ Uses a high-level description of the algorithm instead of an implementation
- ❑ Characterizes running time as a function of the input size,  $n$
- ❑ Takes into account all possible inputs
- ❑ Allows us to evaluate the speed of an algorithm independent of the hardware/software environment

# Pseudocode

- ❑ High-level description of an algorithm
- ❑ More structured than English prose
- ❑ Less detailed than a program
- ❑ Preferred notation for describing algorithms
- ❑ Hides program design issues



# Pseudocode Details

## □ Control flow

- **if ... then ... [else ...]**
- **while ... do ...**
- **repeat ... until ...**
- **for ... do ...**
- Indentation replaces braces

## □ Method declaration

**Algorithm** *method* (*arg* [, *arg*...])

**Input** ...

**Output** ...

## □ Method call

*method* (*arg* [, *arg*...])

## □ Return value

**return** *expression*

## □ Expressions:

← Assignment

= Equality testing

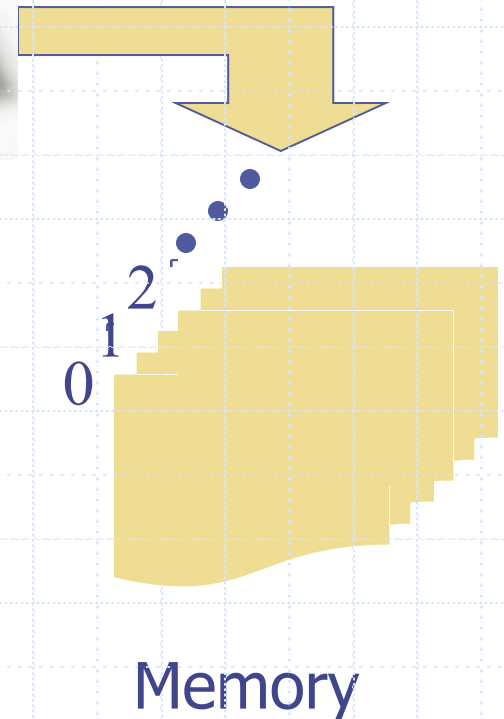
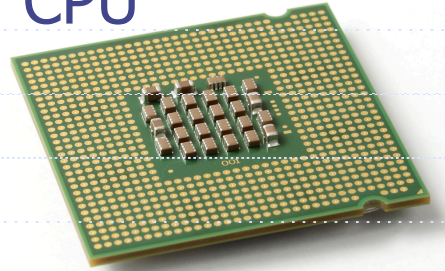
$n^2$  Superscripts and other  
mathematical  
formatting allowed

# The Random Access Machine (RAM) Model

A **RAM** consists of

- A **CPU**
- An potentially unbounded bank of **memory** cells, each of which can hold an arbitrary number or character
- Memory cells are numbered and accessing any cell in memory takes unit time

CPU



# Primitive Operations



- Basic computations performed by an algorithm
  - Identifiable in pseudocode
  - Largely independent from the programming language
  - Assumed to take a constant amount of time in the RAM model
- Examples.
    - Evaluating an expression
    - Assigning a value to a variable
    - Indexing into an array
    - Calling a method
    - Returning from a method