Introduction to CUDA Parallel Programming

Homework Assignment 3

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1 README

This file is `report.pdf`. `src/` folder contains the source code. `result/` folder contains the execution results.

In `src/` folder, executing `make` to compile the program.

To execute the program, run `./poisson` and follow the instructions to enter the GPU ID, size L of the 3D lattice, number of threads per block, and whether to compute the solution vector with CPU/GPU/both. The program will solve the poisson equation and output the GPU/CPU error, total iterations, execution time, and RMSE compared with the Columb's law to standard output and produce two field configuration files `phi_CPU.dat` and `phi_GPU.dat`.

In `result/` folder, `phi_CPU_[L].dat` is the field configuration for cube size LxLxL computed by CPU. `phi_GPU_[L].dat` is the field configuration for cube size LxLxL computed by GPU. The file content in `phi_CPU_[L].dat` and `phi_GPU_[L].dat` are almost the same except for the first line.

2 Analysis

To solve the Poisson equation $\bigtriangledown^2\phi = -\frac{\rho}{\epsilon_0}$ on the 3D lattice with a point charge q at (x_0, y_0, z_0), $\rho(x,y,z) = q\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$

$$\begin{aligned}
&\bigtriangledown^2 \phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -\frac{\rho(x,y,z)}{\epsilon_0} \\
\Rightarrow &[\phi(x+a,y,z) + \phi(x-a,y,z) + \phi(x,y+a,z) + \phi(x,y-a,z) \\
&\quad + \phi(x,y,z+a) + \phi(x,y,z-a) - 6\phi(x,y,z)]/a^2 = -\frac{q}{\epsilon_0}\frac{\delta_{x,x_0}}{a}\frac{\delta_{y,y_0}}{a}\frac{\delta_{z,z_0}}{a}
\end{aligned}$$

In the lattice unit $a = 1$,

$$\begin{aligned}\phi(x,y,z) =& \frac{1}{6}[\phi(x+1,y,z) + \phi(x-1,y,z) + \phi(x,y+1,z) + \phi(x,y-1,z) \\ &+ \phi(x,y,z+1) + \phi(x,y,z-1) + \frac{q}{\epsilon_0}\delta_{x,x_0}\delta_{y,y_0}\delta_{z,z_0}]\end{aligned}$$

Columb's law:

$$\begin{aligned}\phi &= \frac{q}{4\pi\epsilon_0 r} \\ \Rightarrow \phi(x,y,z) &= \frac{q}{4\pi\epsilon_0\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}\end{aligned}$$

3 Result

Except for the point at which the point charge is and the boundaries, we can use the Columb's law to compute the potential. We get the following table.

L	RMSE
8	2.21e+09
16	1.02e+09
32	4.89e+08
64	2.40e+08

4 Discussion

From the above table, we can see that the RMSE decreases as L increases. Therefore, when $L >> 1$, the potential will approach the Columb's law.