# Conditional Independence and Baye’s Rule

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# Conditional Independence

- Definition
  - Two events J and M are said to be conditional independent given A ➔

$$P(J \cap M \mid A) = P(J \mid A)P(M \mid A)$$

$$\frac{P(J \cap M \mid A)}{P(M \mid A)} = P(J \mid A) \Rightarrow$$

$$\frac{P(J \cap M \cap A)/P(A)}{P(M \cap A)/P(A)} \Rightarrow$$

Regular properties on probability are still valid on conditional probability

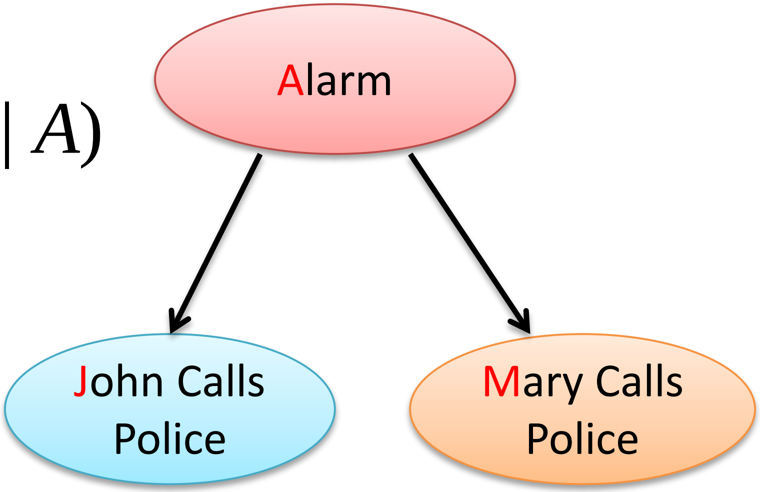
$$P(J \mid A \cap M) = P(J \mid A)$$

# Conditional Independence

- Once you know the alarm is ringing, then whether 盧梅桂 hears the alarm doesn't affect whether 江宗翰 hears the alarm.

$$P(J \mid A \cap M) = P(J \mid A) \text{ or}$$

$$P(J \cap M \mid A) = P(J \mid A)P(M \mid A)$$



# Challenge!!

- If event A and B are independent, are they also conditional independent given any event C?

P(appear in exam| challenge problem) is high

# Safety Helmet Problem

- Minister says: Among the people died in motorcycle accidents, 95% of them do not wear a safety helmet.
- Q: should we ask riders to wear safety helmets?
- Problem?

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- Solution?

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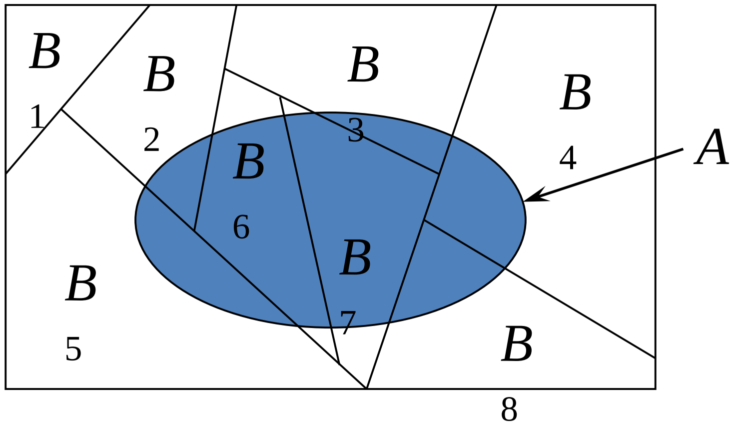
# Bayes' Theorem (or Baye's Rule)

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

- Suppose $S = \bigcup_{i=1}^{n} B_i, B_i \cap B_j = \phi \text{ for } i \neq j$ , then using total probability, P(A) can be derived as $\sum_{i=1}^{n} P(A|B_i)P(B_i)$ so Baye's Theorem can be written as

$$P(B|A) = \frac{P(A|B) * P(B)}{\sum_{i=1...n} P(B_i)P(A|B_i)}$$



# Bayes’ Theorem (or Baye’s Rule)

- Usage Scenario1：It’s much easier to estimate P(A|B) than P(B|A)
- Usage Scenario2: When A occurs, and there are some possible B’s that can happen, and you want to know which Bi is more likely.
  - Example1: when you saw a person bringing an umbrella (A), is it more likely the weather is sunny (B1) or rainy (B2)?
  - Example2: you know a person participating CSIE night, is it more likely the weather is sunny (B1) or rainy (B2)?
    - A: participating in CSIE-night
    - B1: Probability grade =A+
    - B2: Probability grade =B
  - P(A|Bi) is easier to estimate than P(Bi|A)
  - P(B): your priori knowledge about the possibility of each B
  - P(A): independent of B→ not important for this usage

# More on Baye’s Rule

$$P(B \mid A) = \frac{P(A \mid B) * P(B)}{P(A)}$$

- P(B): prior probability
- P(A|B): Likelihood  probability
- P(B|A) : posterior probability

# Example:

| Radar (A) / Airplane(B) | Low | Medium | High |
|---|---|---|---|
| Absent | P(Absent ∩Low)=0.45 | 0.2 | 0.05 |
| Present | 0.02 | 0.08 | 0.2 |

Absent

$$P(B\,|\,A) = \frac{P(A\,|\,B) * P(B)}{P(A)}$$

Low

- P(B): prior probability
  - P(Absent) = 0.7
- P(AlB): Likelihood probability
  - P(LowlAbsent) = 0.45/0.7=0.64
- P(A): marginal likelihood
  - P(Low)= 0.47
- P(B|A) : posterior probability
  - P(Absent|Low)= 0.64*0.7/0.47=0.96

徐敬能

# Example: disease testing

- A company announce a disease (occur rate= 1%) testing product. The performance looks like:
  - P(Test= + | Disease=true) = 0.95
  - P(Test= - | Disease=false) = 0.95

徐維謙

- If we found a positive test, what is the probability that the subject has this disease?
  - P(Disease=true| Test= +)? (ans: 0.161)
- Bayes's rule tells us

$$P(Disease = true \mid Test = +) = \frac{P(Test = + \mid Disease = true) * P(Disease = true)}{P(Test = +)}$$

=0.95*0.01/(0.95*0.01+0.05*0.99)=16%

# Challenging Questions

How to make such testing useful?

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- Let's try this 2 times. If the person have received two positive tests, what is the chance that he/she really has the disease?

$$P(Disease = true \mid Test = {++}) = \frac{P(Test = {++} \mid Disease = true) * P(Disease = true)}{P(Test = {++})}$$