Lecture 1 : Applications of Bayes Formula

Exercise 1 (Light bulbs).
A light bulb manufacturing factory produces 3 types of light bulbs. The probabilities that the lifetime of the bulbs of type 1, 2 and 3 will exceed 100 hours are respectively 0.7, 0.4 and 0.3. We suppose that 30% of the manufactured bulbs are of type 1, 20% are of type 2 and 50% are of type 3.

1. What is the probability that a randomly drawn light bulb will last more than 100 hours ?
2. Given that a bulb has lasted more than 100 hours, what is the probability that it comes from type i, $i = 1, 2, 3$?

Exercise 2 (False positives).
False positives are a problem in any kind of medical test : no test is perfect, and sometimes the test will incorrectly report a positive result. For example, if a test for a particular disease is performed on a patient, then there is a chance (usually small) that the test will return a positive result even if the patient does not have the disease. The problem lies, however, not just in the chance of a false positive prior to testing, but determining the chance that a positive result is in fact a false positive.

Suppose that a test for a particular disease has a very high success rate :

- if a tested patient has the disease, the test accurately reports this, a "positive", 99% of the time,
- if a tested patient does not have the disease, the test accurately reports that, a "negative", 95% of the time .

Suppose also, however, that only 0.1% of the population have that disease.

Calculate the probability that, given the test was positive, that it is a false positive.

Exercise 3 (Which urn does the ball come from ?).
We have two urns filled with balls. The first urn contains 10 black balls and 30 white balls, the second has 20 of each color. We choose at random one of the urns, and, from this urn, a ball is drawn at random. The ball is white. What is the probability that this ball was drawn from the first urn ?

Exercise 4 (Two-headed coin).
We have N coins in a box, one has heads at both sides. Assume that a coin is taken at random from the box, and without inspecting, it is flipped k times. All the k times the coin landed up heads.

1. Give, as a function of N and k, the probability that the two-headed coin was tossed.
2. Plot a graph of these probabilities for $N = 10^6$ and $k = 1, \ldots, 30$.