Lab1 : Optimization & equations in 1D

1 Bissection method

Write a matlab function with the following calling sequence

$$[x, v, it, nf] = BM(f, a, b, Tol, itmax)$$

Where

— f is an m function,
— $a,\ b\ (b > a)$ are two real numbers such that $f(a)f(b) < 0$
— Tol is some fixed tolerance or precision
— $itmax$ is the maximum nuber of iterations, and
— x is an output corresponding to an approximate solution of "$f(x) = 0$"
— nf is the number of function's evaluations
— it is the number of iterations.

This matlab function corresponds to the bissection method :

— $it = 0$,
— while $it < itmax$
  — $x = \frac{a+b}{2}$
  — if $|f(x)| < Tol$ or $|\frac{b-a}{2}| < Tol$, STOP, Else
  — $it = it + 1$, if $f(x)f(a) > 0$ then $a = x$, else $b = x$.

Test this functions for

1. $f(x) = x^3 - 2x - 1, \quad a = 1$ and $b = 2$
2. $f(x) = \sin(x)\cos(x^2) + x(x - \frac{\pi}{2})$, $a =?$ and $b =?$.

2 Newton method

Write a matlab function with the following calling sequence

$$[x, v, it, nf] = Newton(fgradf, x_0, Tol, itmax)$$

corresponding to the following algorithm

$$x^{k+1} := x - \frac{f(x)}{f'(x)}$$

where "frgradf" is an m function that returns $f(x)$ and $f'(x)$.
Test the newton function on several examples.

3 Minimization of unimodular functions (the Golden Section Method)

Using the same inputs as for B.M, write a matlab function corresponding to the following algorithm
- $it = 0$, $p = \frac{\sqrt{5}-1}{2}$, $c = b - p(b-a)$ and $d = a + p(b-a)$
- while $it < itmax$ and $|c - d| > Tol$
  - $f_c := f(c)$,$f_d := f(d)$, $it = it + 1$
  - if $f_c < f_d$
    - $b := d$
    - $d := c$ and $f_d := f_c$
    - $c := b - p(b-a)$
    - $f_c := f(c)$
  - else
    - $a := c$
    - $c := d$ and $f_c := f_d$
    - $d := a + p(b-a)$
    - $f_d := f(d)$

Test this functions on several examples.

4 Line search Methods

Let $f : \mathbb{R}^n \to \mathbb{R}$, $x^0 \in \mathbb{R}^n$ and d a descent direction for f at x^0, i.e :

$$\nabla f(x^0)'d < 0.$$

How to compute a step-size $\tilde{\alpha}$ such that

$$f(x^0 + \tilde{\alpha}d)" < "f(x^0)"?$$

There are different ways and methods to get such $\tilde{\alpha}$.
- Exact line search $\tilde{\alpha} := \text{argmin}\{f(x^0 + \alpha d), \alpha \geq 0\}$.
  Usually it is impossible to compute an excact line search.
- Armijo "Backtracking"
  - choose $\beta \in (0;1)$, $\alpha \in (0;1/2)$ and $t_0 > 0$,
  - $t := t_0$, repeat $t := \beta t$, until

$$f(x^0 + td) < f(x^0) + \alpha t \nabla f(x^0)'d.$$

  - $\tilde{\alpha} := t$
- ...

Write a matlab function corresponding to the Armijo strategy and present different tests when f is quadratic convex, when f is convex and when f is non-convex.