Vietnam **HCM**

Lab7: LQR Problems

Exercise 1: Consider the following system:

$$\ddot{x} + x = u, \qquad x(0) = 0,\ \dot{x}(0) = 1.$$

1. Use any ODE solver when no control is applied (i.e. $u = 0$)
   Use for example "ode45" and plot the phase portrait.

2. We want to stabilize the system near the origin by minimizing the following cost function:

   $$J(x, u) := \int_0^{+\infty} (x(t)^2 + \dot{x}(t)^2 + u(t)^2)dt.$$

   Show that the solution of the Riccati correspondant equation is:

   $$-\begin{pmatrix} -a\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & -a \end{pmatrix}$$

   where $a = \sqrt{2\sqrt{2} - 1}$.

3. Compute the optimal control.

4. Suppose that we know the exact optimal state solution:

   $$x(t) = \frac{2}{b} e^{-\frac{a}{2}t} \sin \frac{b}{2} t$$

   where $b = \sqrt{2\sqrt{2} + 1}$.
   Solve numerically the problem and compare your results to the exact solution.

Exercise 2: Consider the following system:

$$\dot{x} = y + u_1,\ \dot{y} = u_2.$$

1. Solve the optimal control problem corresponding to the following cost function:

   $$J(x, u) := \int_0^{+\infty} (x(t)^2 + y(t)^2 + u_1(t)^2 + u_2(t)^2)dt.$$

2. Test different initial points and comment your results.