Practical Assignment 1: Finite Difference Method

Deadlline 31/07/2020

Let $\Omega = (0,1) \subset \mathbb{R}$ and $f \in L^2(\Omega)$.

$$-u_{xx} = f(x) \quad \text{in } \Omega \tag{0.1}$$

1. Dirichlet boundary condition

a. Solve equation (0.1) with uniform mesh subject to a Dirichlet boundary condition:

$$u(0) = \alpha, \quad u(1) = \beta$$

b. Solve equation (0.1) subject to a Dirichlet boundary condition:

$$u(0) = 0, \quad u(1) = 0$$

with non uniform mesh $x_i = 1 - \cos\frac{\pi i}{2N}$ for $i = 0, \cdots, N$.

2. Dirichlet-Neumann boundary condition

Solve equation (0.1) with uniform mesh subject to a Dirichlet Neumann boundary condition:

$$u'(0) = \alpha, \quad u(1) = \beta$$

3. Neumann boundary condition

Solve equation (0.1) with uniform mesh subject to a Neumann boundary condition:

$$u'(0) = \alpha, \quad u'(1) = \beta$$

with condition $\int_0^1 f(x)dx = \alpha - \beta$ and $\int_0^1 u(x)dx = 0$