Theoretical Assignment 2: Convection Diffusion Equations on 1D

Deadline:28/08/2020

We consider the convection diffusion equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1$$

where α is the diffusion coefficient. The initial condition is

$$u(x, 0) = u_0(x)$$

The boundary condition is

$$u(a, t) = \phi_a(t), \quad u(b, t) = \phi_b(t)$$

1 Write Forward Euler method with the Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term and find stable condition with $\alpha = 0$

2 Write Crank-Nicolson method with the Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term and find stable condition with $\alpha = 0$

3 Write Forward Euler method with the Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term, central difference scheme for diffusion term and find stable condition with $\alpha \neq 0$

4 Write Crank-Nicolson method with the Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term, central difference scheme for diffusion term and find stable condition with $\alpha \neq 0$