Practical Assignment 2: Diffusion Convection Equations on 1D

Deadline:28/08/2020

We consider the convection diffusion equation

$$\frac{\partial u}{\partial t} + 2\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}, \quad t > 0, \quad 0 < x < 1$$

where ϵ is the diffusion coefficient. The initial condition is

$$u(x,0) = u_0(x)$$

The boundary condition is

$$u(a,t) = \phi_a(t), \quad u(b,t) = \phi_b(t)$$

Initial condition: $u_0(x) = \sin(4\pi x)$
Boundary condition: $u(0,t) = u(1,t) = \sin(8\pi t)$
Exact solution: $u(x,t) = e^{-\frac{16\pi^2}{\alpha}t}\sin(4\pi(x-2t))$
We implement and compare the following methods

1 $\alpha = 0$

1.1 Forward Euler with Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term

1.2 Crank-Nicolson method with Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term

1.3 Compare previous methods? (accuracy, stability,computing time).

2 $\alpha = 10^{-4}$

2.1 Forward Euler with Lax-Friedrich, Lax-Wendroff, and upwind schemes convection term

2.2 Crank-Nicolson method with Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term

2.3 Compare previous methods? (accuracy, stability,computing time).

3 $\alpha = \frac{1}{16}$

3.1 Forward Euler with Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term

3.2 Crank-Nicolson method with Lax-Friedrich, Lax-Wendroff, and upwind schemes for convection term

3.3 Compare previous methods? (accuracy, stability,computing time).