Finite Volume Method: Theoretical Assignment 2

December 8, 2020

Deadline: 22/12/2020

Let convex set $\Omega \subset \mathbb{R}$ and $f \in L^2(\Omega)$, we consider the equation

$$-\Delta u = f(x) \quad \text{ in } \Omega$$

subject to a Dirichlet boundary condition:

$$u(x) = u_d \text{ on } \Gamma$$

1. How to discretize the previous system equation by finite volume method with admissible mesh $(T_i)_{i\in[1,I]}$, $\bigcup_{i=1}^{I} T_i = \Omega$ such that $(u_i)_{i\in[0,I+I^\Gamma]}$, is discrete solution, satisfy

$$-d(g(u)) = f$$

where $f = \{f_i\}_{i\in[1,N]}$ and f_i is mean-value of f in T_i, with boundary value u_k, $k \in [I+1, I+I^\Gamma]$, and $k \in V(i)$

$$u_k = u(x_k) \text{ or } u_k = \frac{1}{l_{ik}} \int_{i|k} u(\sigma) d\sigma$$

which one is better.

2. We suppose that f and u_d are positive on Ω. Prove that

$$u_i \geq 0 \qquad \forall i \in [1, I]$$

3. Let $(w_i)_{i\in[I+I^\Gamma]}$ and $(v_{ik})_{ik\in E}$ be given. There holds

$$(dv, w)_T = -2(v, gu)_D + (v, \gamma w)_\Gamma$$

4. For any $(w_i)_{i\in[1,I+i^\Gamma]}$ with $w_k = 0$ for all $k \in [I+1, I+I^\Gamma]$, there holds

$$2(g(u), g(w))_D = (f, w)_T$$

5. For any $(w_i)_{i\in[1,I+I^\Gamma]}$ with $w_k = 0$ for all $k \in [I+1, I+I^\Gamma]$, there holds

$$2(\delta u, g(w))_D = (f, w)_T$$

6. There exists constant $C > 0$(depending on u) such that

$$\left| \frac{1}{l_{ik}} \int_{l_{ik}} (\nabla u(\sigma) - \nabla u(\mathrm{x}_{ik})) \cdot n_{ik}(\sigma) d\sigma \right| \leq C l_{ik}$$

$$\left| \frac{1}{d_{ik}} \int_{[x_i,x_k]} (\nabla u(\sigma) - \nabla u(\mathrm{x}_{ik})) \cdot n_{ik}(\sigma) d\sigma \right| \leq C d_{ik}$$

7. We assume that there exists $C_1 > 0$ such that

$$\|w\|_{0,T} \leq C_1 |w|_{1,D}$$

with $(w_i)_{i\in[I+I^{\Gamma}]}$ and $w_k = 0$ for all $k \in [I, I+I^{\Gamma}]$. We prove that there exit C_2

$$|u|_{1,D} \leq C_2 \|f\|_{0,T}$$

with $u_d = 0$.