FVM: Practical Assignment 1

October 13, 2020

Deadline: 28/10/2020

Given a 1D Poisson problem on $\Omega = (0, 1)$

$$-u''(x) + vu_x = f(x),\ x \in \Omega\ (*)$$

1. Dirichlet boundary condition with velocity $v = 0.3$

a) Solve equation (*) subject to homogeneous Dirichlet boundary condition

$$u(0) = a, u(1) = b$$

by finite volume method on a regular grid and the control point is the mid point of each control volume ($x_i = (x_{i-1/2} + x_{i+1/2})/2$).

b) Solve equation (*) with regular grid and the control point is $1/3$ from the left of each control volume ($x_i = 2/3x_{i-1/2} + 1/3x_{i+1/2}$) and $1/3$ from the right of each control volume ($x_i = 1/3x_{i-1/2} + 2/3x_{i+1/2}$) .

c) How to approximate the mean-value of f over T_i and compare some ways approximation.

d) solve equation (*) with singular grid (not uniform grid).

2. Neumann boundary condition with velocity $v = 0$

Solve equation (*) subject to homogeneous Neumann boundary condition

$$u'(0) = 0, u'(1) = 0 \text{ with } \int_0^1 f(x)dx = 0 \text{ and } \int_0^1 u(x)dx = 0$$

by finite volume method on a regular grid and singular grid with the control point be the mid point of each control volume ($x_i = (x_{i-1/2} + x_{i+1/2})/2$).