Theoretical Assignment 3: Finite Volume Method

Deadline: 06/01/2020

1. If $u \in \mathbb{C}^2([0,1],\mathbb{R})$, there exists $C \in \mathbb{R}_+$ only depending on u such that

$$|\tau_i^n| = |K_i^n + R_{i-1/2}^n - R_{i+1/2}^n| \leq C(h^2 + k) \quad \forall i = 2, \cdots, N-1$$

where

$$K_i^n = \int_{|T_i|} u_t(x, t_n)dx - \frac{|T_i|(u(x_i, t_{n+1}) - u(x_i, t_n))}{k}$$

$$R_{i-1/2}^n = u_x(x_{i-1/2}, t_n)dx - \frac{u(x_i, t_n) - u(x_{i-1}, t_n)}{|D_{i-1/2}|}$$

$$R_{i+1/2}^n = u_x(x_{i+1/2}, t_n)dx - \frac{u(x_{i+1}, t_n) - u(x_i, t_n)}{|D_{i+1/2}|}|$$

2. If $u \in \mathbb{C}^2([0,1],\mathbb{R})$, there exists $C \in \mathbb{R}_+$ only depending on u such that

$$|\tau_i^{n+1}| = |S_i^{n+1} + R_{i-1/2}^{n+1} - R_{i+1/2}^{n+1}| \leq C(h^2 + k) \quad \forall i = 2, \cdots, N-1$$

where

$$S_i^{n+1} = \int_{|T_i|} u_t(x, t_{n+1})dx - \frac{|T_i|(u(x_i, t_{n+1}) - u(x_i, t_n))}{k}$$

$$R_{i-1/2}^{n+1} = u_x(x_{i-1/2}, t_{n+1})dx - \frac{u(x_i, t_{n+1}) - u(x_{i-1}, t_{n+1})}{|D_{i-1/2}|}$$

$$R_{i+1/2}^{n+1} = u_x(x_{i+1/2}, t_{n+1})dx - \frac{u(x_{i+1}, t_{n+1}) - u(x_i, t_{n+1})}{|D_{i+1/2}|}|$$

3. If $u \in \mathbb{C}^2([0,1],\mathbb{R})$, there exists $C \in \mathbb{R}_+$ only depending on u such that

$$|\tau_i^{n+1/2}| = |K_i^n - S_i^{n+1} + \frac{R_{i-1/2}^n - R_{i+1/2}^n}{2} + \frac{R_{i-1/2}^{n+1} - R_{i+1/2}^{n+1}}{2}| \leq C(h^2 + k^2) \quad \forall i = 2, \cdots, N-1$$

where

$$S_i^{n+1} = \int_{|T_i|} u_t(x, t_{n+1})dx - \frac{|T_i|(u(x_i, t_{n+1}) - u(x_i, t_n))}{k}$$

$$R_{i-1/2}^{n+1} = u_x(x_{i-1/2}, t_{n+1})dx - \frac{u(x_i, t_{n+1}) - u(x_{i-1}, t_{n+1})}{|D_{i-1/2}|}$$

$$R_{i+1/2}^{n+1} = u_x(x_{i+1/2}, t_{n+1})dx - \frac{u(x_{i+1}, t_{n+1}) - u(x_i, t_{n+1})}{|D_{i+1/2}|}|$$