High Performance Computing
USN-HCMV , Paris 13
Joint Master 2022

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Worksheet 1

Let A be the matrix of size 100 :
$A = SDS^{-1}$ with $S = (1, \beta)$ an upper bidiagonal matrix.
$\beta = 0.9$ and $D = \alpha * diag(1, 2, ..., 100)$ and $\alpha = 2$

To check the validity of your algorithms, set :

- $x_{exact} = rand(100, 1)$
- $b = Ax_{exact}$
- if x is your computed solution, check the error $||x - x_{exact}||_2$

Exercise 1

Write A as A = D - E - F
Use Matlab functions diag, triu, tril
Solve $Ax = b$ (and check) with the following methods and comment the efficiency of these different methods.

- Jacobi
- Gauss Seidel $(D - E)x = Fx + b$
- Backward Gauss Seidel $(D - F)x = Ex + b$
- Simple projection method : Steepest Descent
- Successive Over Relaxation (SOR) $\omega A = (D - \omega E) - (\omega F + (1 - \omega)D)$. Study convergence as a function of ω

Exercise 2

- Can you solve $Ax = b$ with a Conjugate gradient ? Why ?
- Modify A and b so that you can find the solution x with a Conjugate gradient and solve it.
- Solve the modified problem with Jacobi, Gauss Seidel, Steepest Descent, SOR and compare the costs

Exercise 3

Apply these different solvers to the Laplace equation discretised with 2^{nd} order Finite Differences in dimension 1 and 2 (See Prof. Halpern's course) with Dirichlet boundary conditions. Check that the discretisation provides a second order numerical solution. (Use manufactured solutions).

Write a small report containing all your results and analysis.