Subject: Date: / /

مشتق تابع $Cost^{(w,b)}$ برای محاسبه گرادیان

$$\Phi(x) = \sigma(w^T x + b) \quad , \sigma(x) = \frac{1}{1+e^{-x}}$$

$$Cost(w,b) = \sum_{i=1}^{N} (\Phi(x_i) - y_i)^2$$

$$\Rightarrow \frac{\partial c}{\partial w} = \sum_{i=1}^{N} 2\frac{\partial(\Phi(x_i) - y_i)}{(1)\ \partial w} \cdot (\Phi(x_i) - y_i)$$

$$\Rightarrow (1)\ \frac{\partial(\Phi(x_i) - y_i)}{\partial w} = \frac{\partial\left(\frac{1}{1+e^{-(w^Tx+b)}}\right)}{\partial w} - \underbrace{\frac{\partial y_i}{\partial w}}_{0}$$

$$\Rightarrow (-1)\frac{\partial(1+e^{-(w^Tx+b)})}{\partial w}\left(\frac{1}{1+e^{-w^Tx+b}}\right)^2 - 0 = -1(0 + 0 + -w^T e^{-(w^Tx+b)})\left(\frac{1}{1+e^{-w^Tx+b}}\right)^2$$

$$\Rightarrow + w^T e^{-(w^Tx+b)}\left(\frac{1}{1+e^{-w^Tx+b}}\right)^2$$

برای $\frac{\partial C}{\partial b}$ نیز همین است با این تفاوت که به جای w^T ، 1 قرار می‌گیرد.

$$\frac{\partial c}{\partial b} = 1 e^{-(w^Tx+b)}\left(\frac{1}{1+e^{-w^Tx+b}}\right)^2$$