Linear Algebra
Final Project

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Derivative of the cost function for calculating the gradient

$$\Phi(x) = \sigma(w^2 x + b) \qquad , \qquad \sigma(x) = \frac{1}{1+\mathrm{e}^{-x}}$$

$$cost(w,b) = \sum_{i=1}^{N} (\Phi(x_i) - y_i)^2$$

$$\Rightarrow \frac{dc}{dw} = \sum_{i=1}^{N} 2\frac{d(\Phi(x_i)-y_i)}{dw}(\Phi(x_i) - y_i)$$

$$\frac{d(\Phi(x_i)-y_i)}{dw} = \frac{d(\frac{1}{1+\mathrm{e}^{-(w^T x+b)}})}{dw} - \frac{dy_i}{dw}$$

$$= (-1)\frac{d(1+\mathrm{e}^{-(w^T x+b)})}{dw}(\frac{1}{1+\mathrm{e}^{-(w^T x+b)}})^2 - 0$$

$$= -(0 + 0 - x\mathrm{e}^{-(w^T x+b)})(\frac{1}{1+\mathrm{e}^{-(w^T x+b)}})^2$$

$$= x\mathrm{e}^{-(w^T x+b)}(\frac{1}{1+\mathrm{e}^{-(w^T x+b)}})^2$$

To calculate $\frac{dc}{db}$, we take the derivative in the same way and we will have:

$$\frac{dc}{db} = \mathrm{e}^{-(w^T x+b)}(\frac{1}{1+\mathrm{e}^{-(w^T x+b)}})^2$$