A1 Project 1

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Projekt 1: Wind Power Forecast

Descriptive Statistics

```r
D <- read.table("tuno.txt", header=TRUE, sep=" ",
                as.is=TRUE)

D$date <- as.Date("2003-01-01")-1+D$r.day
D$pow.obs.norm <- D$pow.obs/5000
```

Read the data tuno.txt into R

Make a graphical presentation of data or parts of the data, and present some summary statistics.

Summary statistics:

```r
## Dimensions of D (number of rows and columns)
dim(D)
```

```
## [1] 288   8
```

```r
##  Column/variable names
names(D)
```

```
## [1] "r.day"        "month"        "day"          "pow.obs"      "ws30"
## [6] "wd30"         "date"         "pow.obs.norm"
```

```r
## The first rows/observations
head(D)
```

```
##   r.day month day   pow.obs     ws30      wd30       date pow.obs.norm
## 1     1     1   1  243.0278 6.723611 4.0343405 2003-01-01   0.04860556
## 2     2     1   2 2780.0137 4.272603 2.1365208 2003-01-02   0.55600274
## 3     3     1   3 2118.6164 4.272603 1.6240318 2003-01-03   0.42372329
## 4     4     1   4 1660.8767 6.541096 0.2269022 2003-01-04   0.33217534
## 5     5     1   5 1872.7945 9.713699 5.3161852 2003-01-05   0.37455890
## 6     6     1   6 3212.2603 8.161644 0.9522963 2003-01-06   0.64245205
```

```r
## The last rows/observations
tail(D)
```

```
##     r.day month day    pow.obs      ws30      wd30       date pow.obs.norm
## 283   299    10  26  787.0000  9.323288 0.3152175 2003-10-26   0.15740000
## 284   300    10  27 1869.6438 11.280137 5.2411088 2003-10-27   0.37392877
## 285   301    10  28 2551.5205 12.623973 4.7614043 2003-10-28   0.51030411
## 286   302    10  29 2564.5616 11.154795 3.6750237 2003-10-29   0.51291233
## 287   303    10  30  449.5205  5.714384 3.0080934 2003-10-30   0.08990411
## 288   304    10  31  781.8082  6.102740 3.0877370 2003-10-31   0.15636164
```

```r
## Selected summary statistics
summary(D)
```

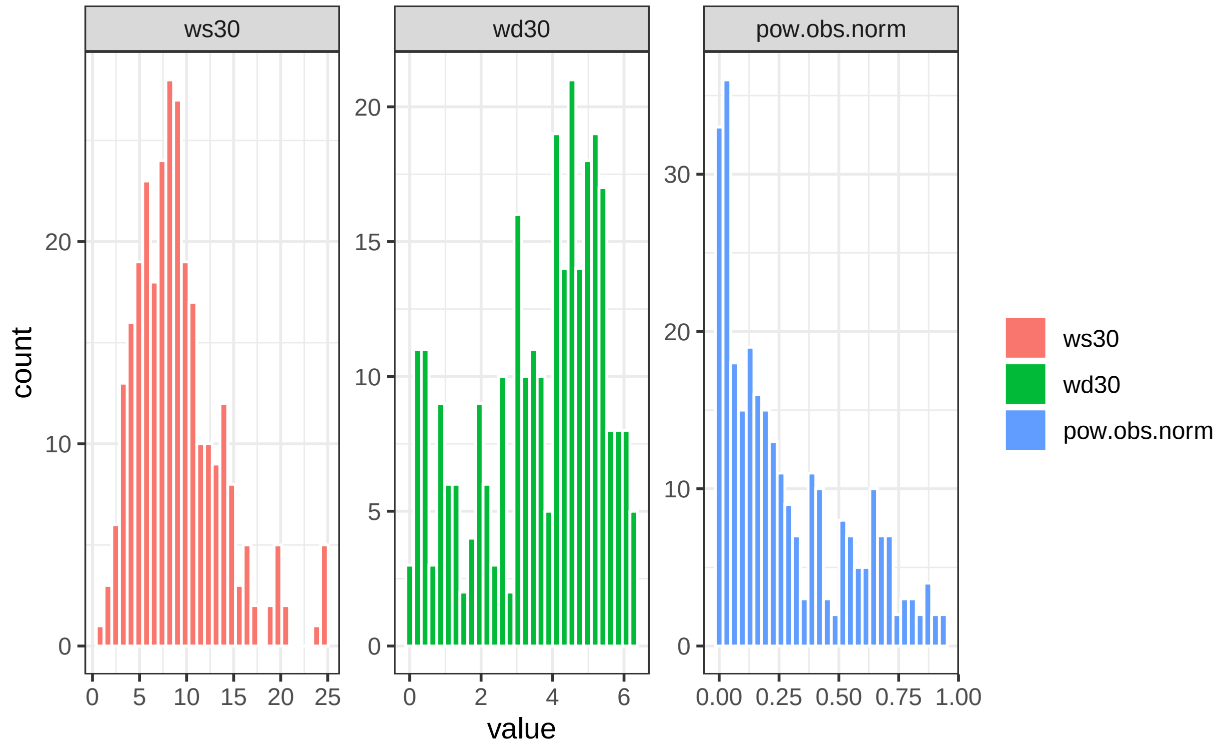
```
##      r.day            month             day           pow.obs
##  Min.   :  1.00   Min.   : 1.000   Min.   : 1.00   Min.   :   0.123
##  1st Qu.: 78.75   1st Qu.: 3.000   1st Qu.: 8.00   1st Qu.: 254.158
##  Median :156.50   Median : 6.000   Median :15.00   Median : 964.123
##  Mean   :154.30   Mean   : 5.594   Mean   :15.47   Mean   :1381.196
##  3rd Qu.:229.25   3rd Qu.: 8.000   3rd Qu.:23.00   3rd Qu.:2196.579
##  Max.   :304.00   Max.   :10.000   Max.   :31.00   Max.   :4681.062
##       ws30             wd30               date             pow.obs.norm
##  Min.   : 1.139   Min.   :0.000095   Min.   :2003-01-01   Min.   :0.0000247
##  1st Qu.: 5.779   1st Qu.:2.474999   1st Qu.:2003-03-19   1st Qu.:0.0508315
##  Median : 8.498   Median :4.079297   Median :2003-06-05   Median :0.1928247
##  Mean   : 9.112   Mean   :3.602390   Mean   :2003-06-03   Mean   :0.2762392
##  3rd Qu.:11.202   3rd Qu.:4.945443   3rd Qu.:2003-08-17   3rd Qu.:0.4393158
##  Max.   :24.950   Max.   :6.274642   Max.   :2003-10-31   Max.   :0.9362123
```

```r
## Another type of summary of the dataset
str(D)
```

```
## 'data.frame':    288 obs. of  8 variables:
##  $ r.day       : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ month       : int  1 1 1 1 1 1 1 1 1 1 ...
##  $ day         : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ pow.obs     : num  243 2780 2119 1661 1873 ...
##  $ ws30        : num  6.72 4.27 4.27 6.54 9.71 ...
##  $ wd30        : num  4.034 2.137 1.624 0.227 5.316 ...
##  $ date        : Date, format: "2003-01-01" "2003-01-02" ...
##  $ pow.obs.norm: num  0.0486 0.556 0.4237 0.3322 0.3746 ...
```

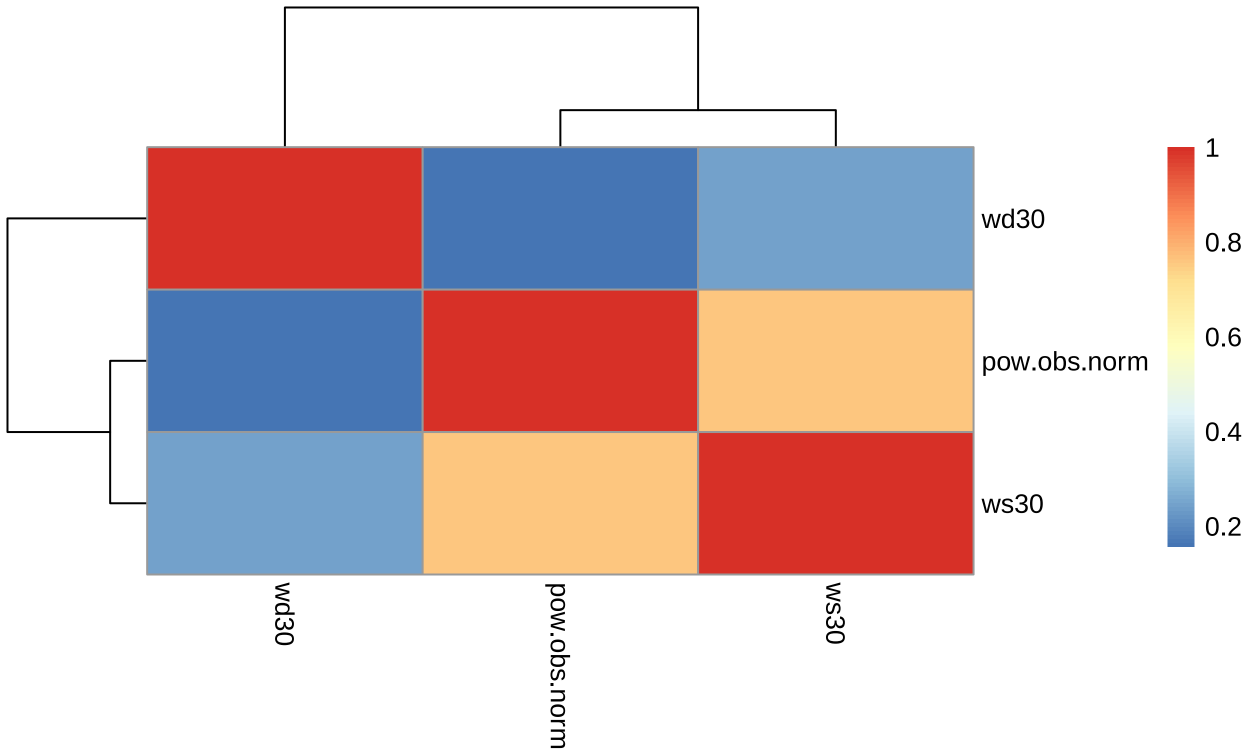
Visualization of the three relevant variables:

```r
meltD <- D %>%
  dplyr::select(-r.day, -month, -day, -pow.obs) %>%
  melt(id.vars = "date")

ggplot(meltD)+
  geom_histogram(aes(x = value, fill = variable), colour = "white")+
  facet_wrap(~ variable, scales = "free")+
  theme_bw()+
  labs(fill = "")
```

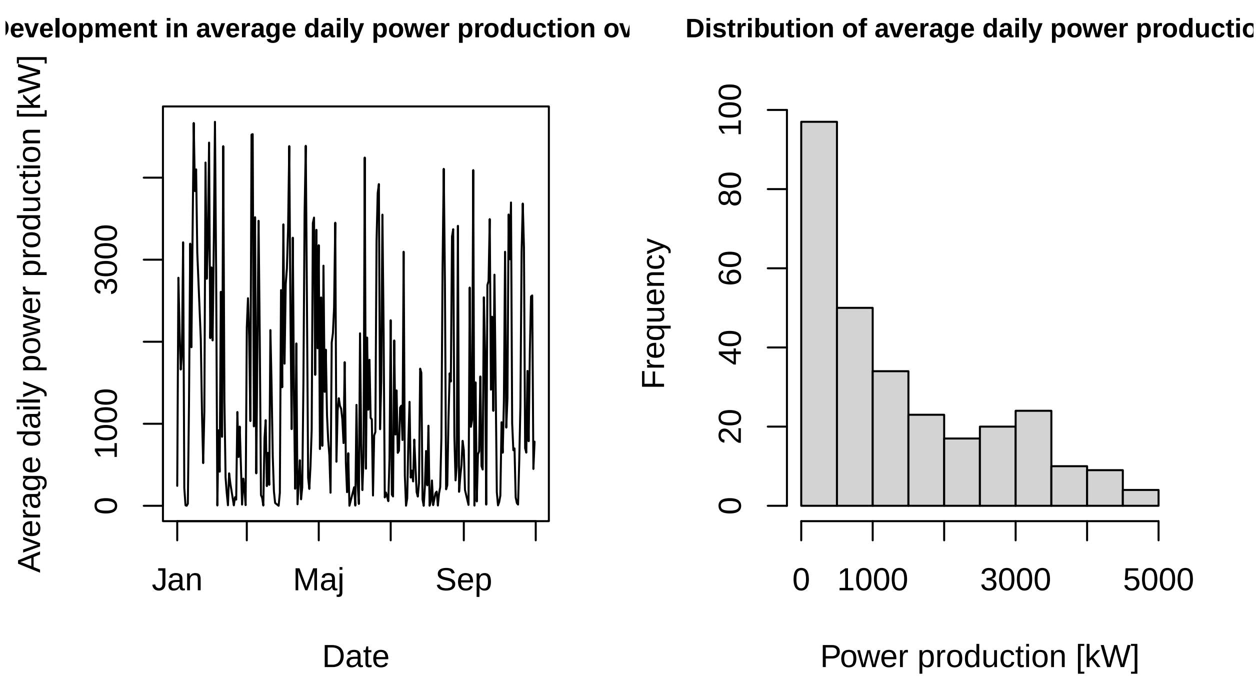


Correlation analysis

```
D %>%
  dplyr::select(pow.obs.norm, wd30, ws30) %>%
  cor() %>%
  pheatmap()
```

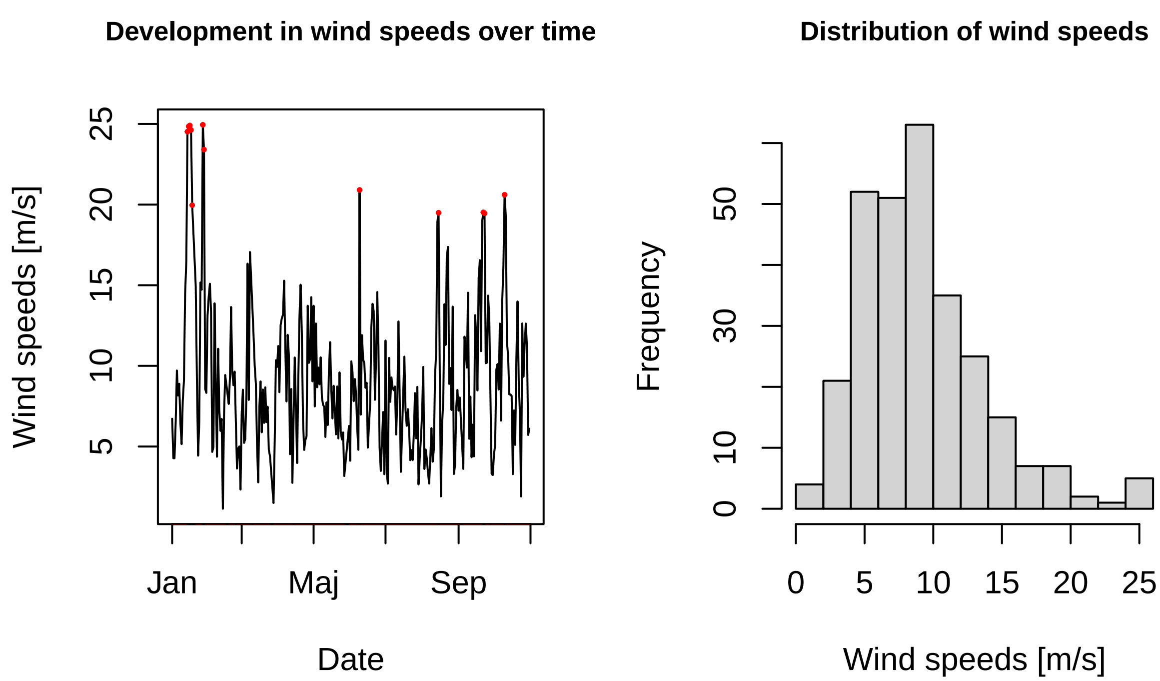



```
par(mfrow=c(1,2))
plot(D$date, D$pow.obs, type = 'l', xlab="Date", ylab="Average daily power production [kW]",
     main = 'Development in average daily power production over time', cex.main = 0.8, col=1)
hist(D$pow.obs, xlab="Power production [kW]", main='Distribution of average daily power production', ce
```



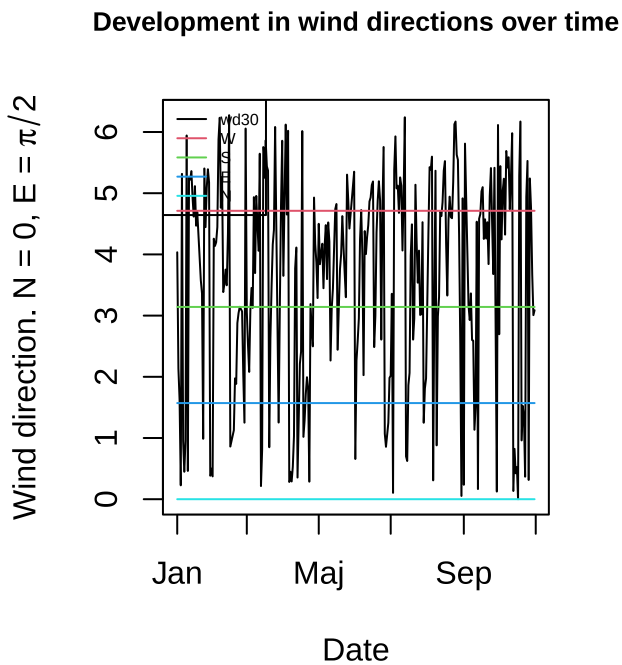
Outlier analysis

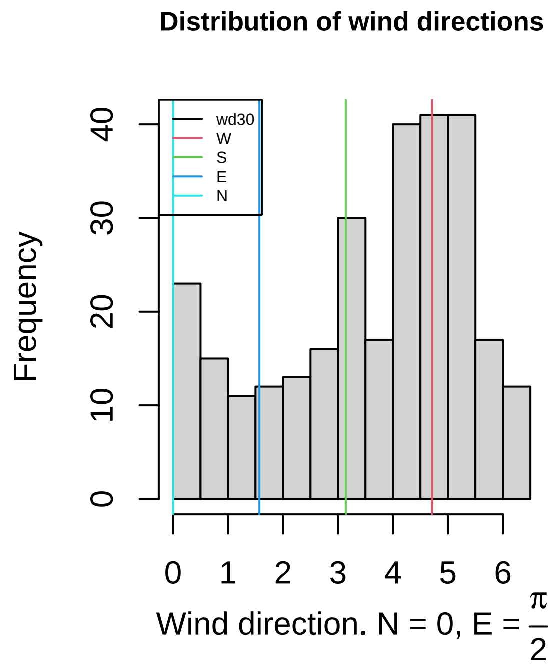
```
outlierFUN <- function(data, quantiles){
  v <- quantile(x = data, probs = quantiles)
  IQR <- v[2] - v[1]
  outliers <- ( ( data < (v[1] - 1.5 * IQR) ) | ( data > (v[2] + 1.5 * IQR) ) ) * data
  return (outliers)
}
D$outlierws30 <- outlierFUN(data = D$ws30, quantiles = c(0.25,0.75))
###               ###
par(mfrow=c(1,2))
plot(D$date, D$ws30, type = 'l', xlab="Date", ylab="Wind speeds [m/s]", cex.main = 0.8, col=1,
     main='Development in wind speeds over time')
points(x = D$date ,y = D$outlierws30, type = 'p', pch = 19, col = "red", cex = 0.25)
hist(D$ws30, xlab="Wind speeds [m/s]", main='Distribution of wind speeds', cex.main = 0.8)
```




```
par(mfrow=c(1,2))
plot(D$date, D$wd30, type = 'l', xlab="Date", ylab=expression(paste("Wind direction. N = 0, E = ", pi/2
     main='Development in wind directions over time', cex.main = 0.8)
lines(D$date, replicate(length(D$date), 3*pi/2), type='l', col=2) #W
lines(D$date, replicate(length(D$date), pi), type='l', col=3) #S
lines(D$date, replicate(length(D$date), pi/2), type='l', col=4) #E
lines(D$date, replicate(length(D$date), 0), type='l', col=5) #N
legend('topleft', legend = c('wd30', 'W', 'S', 'E', 'N'), col = 1:5, lty = 1, cex = 0.5)
#
hist(D$wd30, xlab=expression(paste('Wind direction. N = 0, E = ', frac(pi,2))),
     main='Distribution of wind directions', cex.main = 0.8, freq = TRUE) #####hist to show that wind r
abline(v = 3*pi/2, col=2)
```

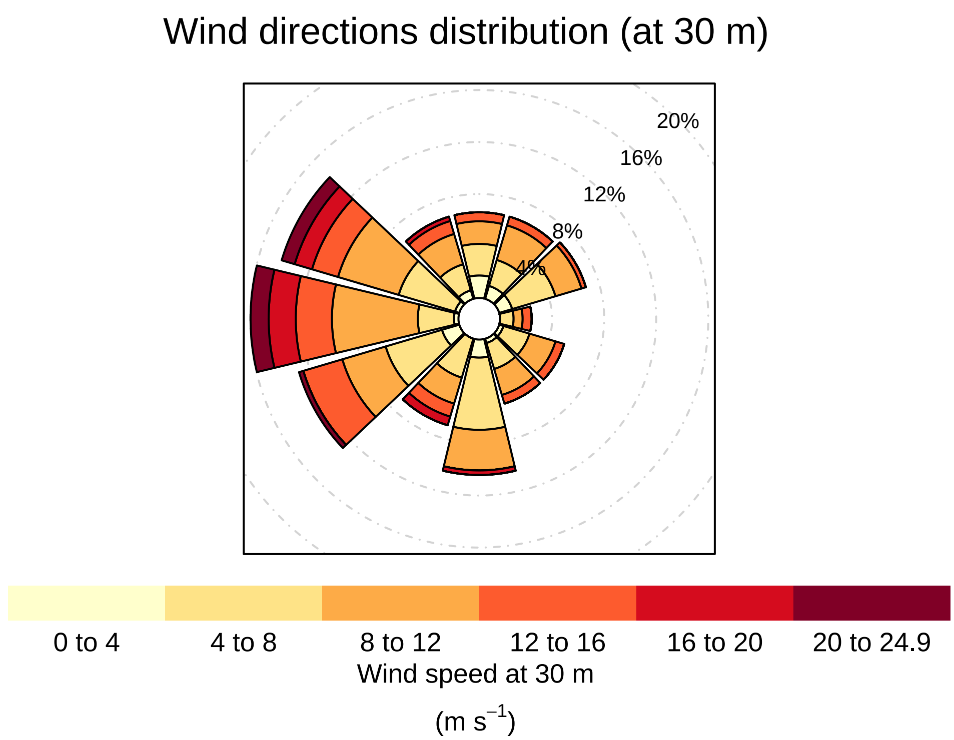
```
abline(v = pi, col=3)
abline(v = pi/2, col=4)
abline(v = 0, col=5)
legend('topleft', legend = c('wd30', 'W', 'S', 'E', 'N'), col = 1:5, lty = 1, cex = 0.5)
```





Wind rose

```
D$wd30dg <- D$wd30 * 180/pi
###wind d
intv <- 4
#Dwinter <- D[1:54,] #for testing the season plots above :) D$date[55] = 2003-03-01
#Dsummer <- D[140:230,] #for testing the season plots above :) D$date[140] = 2003-06-01
windRose(D, ws = "ws30", wd = "wd30dg", ws2 = NA, wd2 = NA,
         ws.int = intv, angle = 30, type = "default", bias.corr = TRUE, cols
         = "heat", grid.line = list(value=4, lty=4, col="lightgrey"), width = 1, seg = NULL, auto.text
         = TRUE, breaks = round(max(D$ws30)/intv), offset = 10, normalise = FALSE, max.freq =
           NULL, paddle = FALSE, key.header = "Wind speed at 30 m", key.footer = "(m/s)",
         key.position = "bottom", key = list(height=2), dig.lab = 3, statistic =
           "prop.count", pollutant = NULL, annotate = FALSE, angle.scale =
           45, border = "black", main="Wind directions distribution (at 30 m)",
         cex.main=0.75)
```



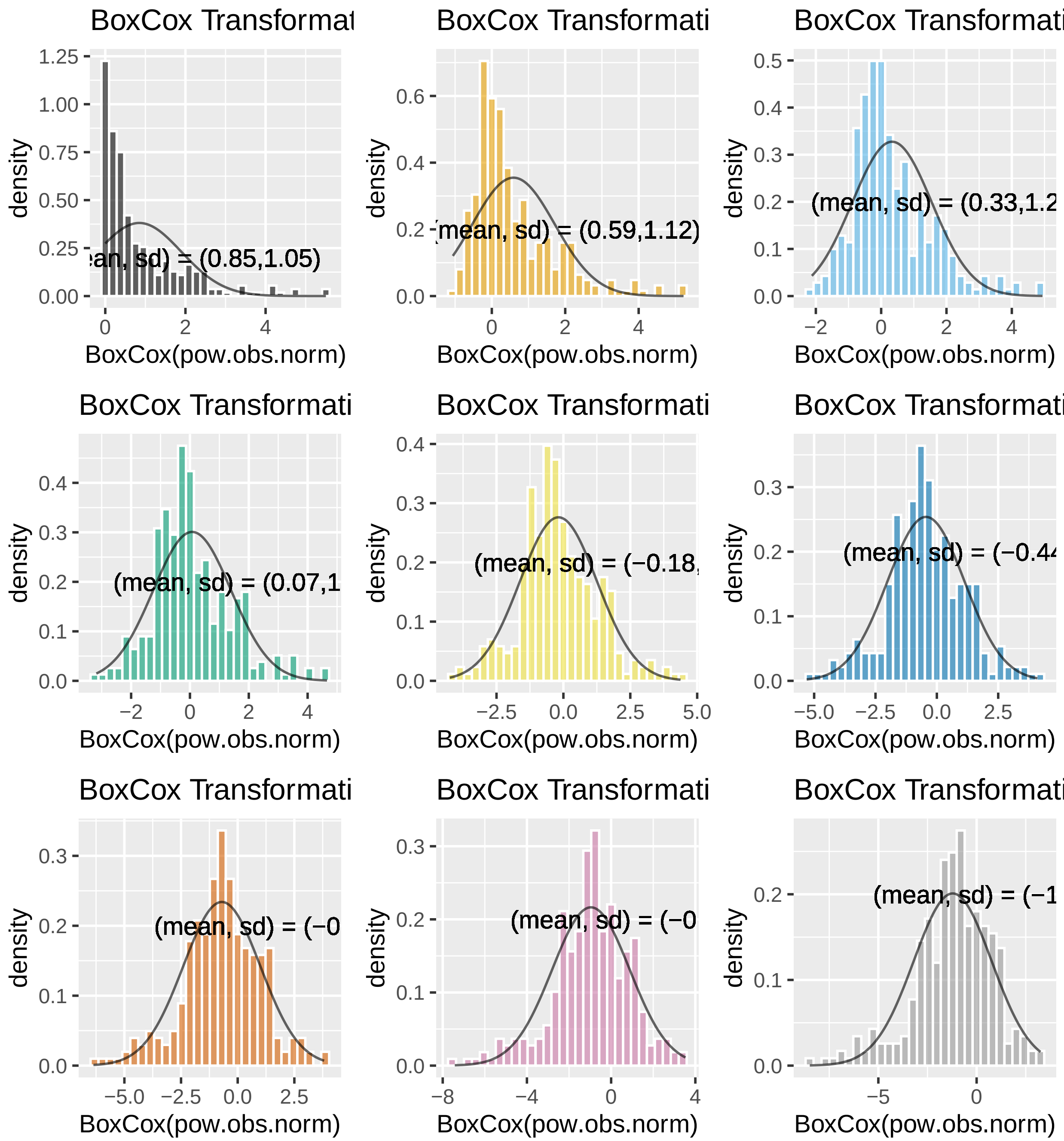
Simple Models

```
source("testDistribution.R")
```

Fit different probability density models to wind power, wind speed and wind direction data. You might consider different models e.g. beta, gamma, log normal, and different transformations e.g. (for wind power). It is important that you consider if the distributions/transformations are reasonable for the data that you try to model. Two approaches: 1) Box-Cox transformation 2) fit distributions directly to the standardized pow.obs.

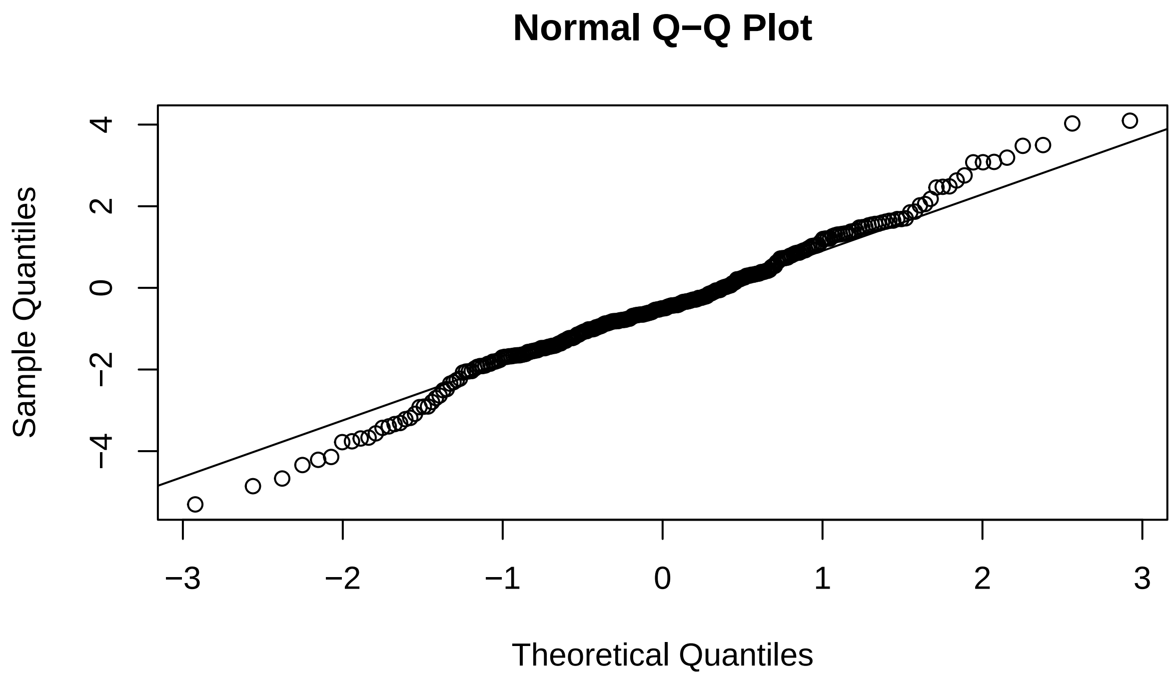
```
#Box-Cox transformation of pow.obs.norm
#Examine different transformations and the achieved fit when fitting a normal
#distribution

lambda <- c(0.0, 0.05, 0.10, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4)
pal <- palette.colors(length(lambda))
BoxCoxPlot <- list()
for (i in 1:length(lambda)){
  xData <- 2*log(D$pow.obs.norm^lambda[i]/(1-D$pow.obs.norm)^(1-lambda[i]))
  n <- nlminb(start = c(-1,1)
              , objective = testDistribution
              , x = xData
              , distribution = "normal")

  #simData <- rnorm(n = length(D$pow.obs.norm), mean = n$par[1], sd = n$par[2])
  #D$sim <- simData
```

```
  D$BoxCox <- xData

  BoxCoxPlot[[paste0(lambda[i])]] <- ggplot(D)+
    #geom_histogram(aes(x = sim, y = ..density..)
    #               , colour = "white"
    #               , alpha = 0.5
    #               , fill = "black")+
    geom_histogram(aes(x = BoxCox, y = ..density..)
                   , colour = "white"
                   , alpha = 0.6
                   , fill = pal[[i]])+
    labs(x = "BoxCox(pow.obs.norm)")+
    ggtitle(paste0("BoxCox Transformation of Windpower, ", expression(lambda), " = ", lambda[i]))+
    #geom_text(x = 2, y = 0.23, label = paste0("NLL = ", round(n$objective,2)))+
    geom_text(x = 2, y = 0.2, label = paste0("(mean, sd) = (", round(n$par[1],2), ",", round(n$par[2],2
    stat_function(fun = dnorm, n = length(D$pow.obs.norm), args = list(mean = n$par[1], sd = n$par[2]),
}

grid.arrange(grobs = BoxCoxPlot)
```



From here it can be seen that a box-cox transformation with lambda $= 0.25$ might be appropriate. It is however not a good approximation as can be seen from the qqplot below.

```
xData <- 2*log(D$pow.obs.norm^0.25/(1-D$pow.obs.norm)^(1-0.25))
qqnorm(xData)
qqline(xData)
```



Approach 2) Fit an exponential, gamma and beta distribution to the observed wind power data.

```r
par.exp <- nlminb(start = 0.2, objective = testDistribution,
                  distribution = "exponential",
                  x = D$pow.obs.norm)

par.exp$objective
```

```
## [1] -82.50862
```

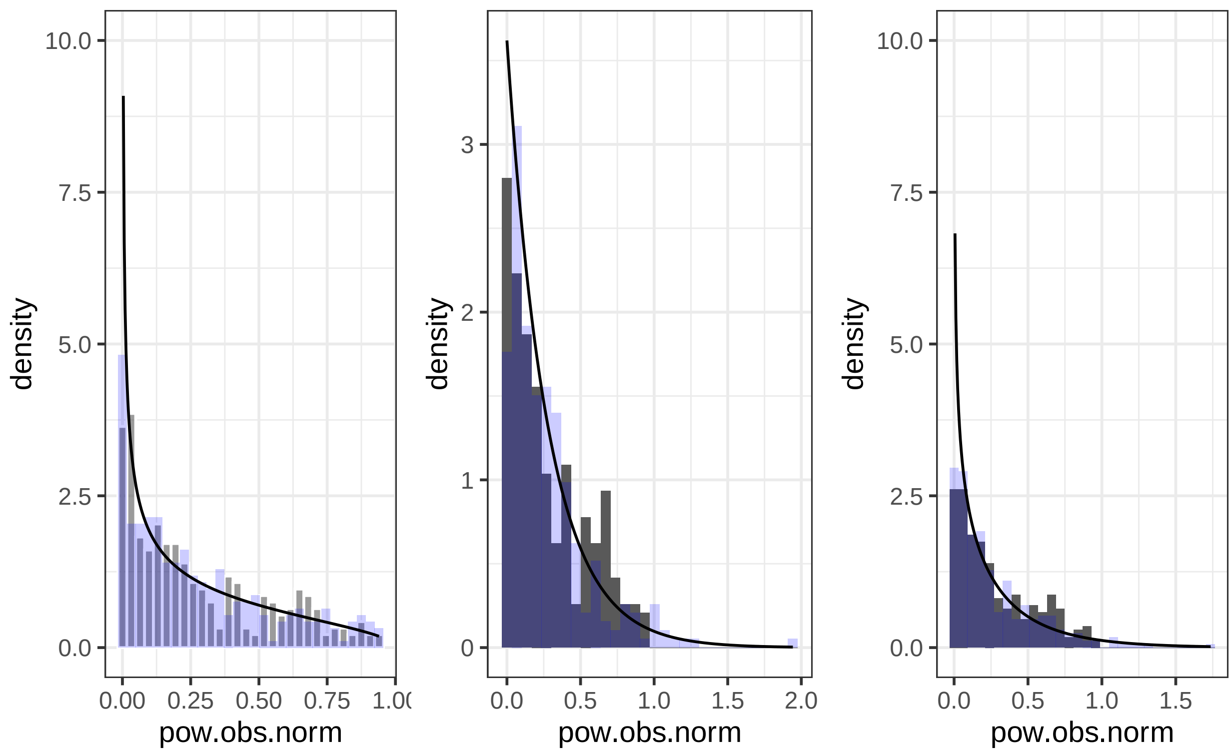
```r
par.beta <- nlminb(start = c(2,5)
                   , objective = testDistribution
                   , distribution = "beta"
                   , x = D$pow.obs.norm
                   , lower = c(0,0.8))
par.beta$objective
```

```
## [1] -121.6618
```

```r
par.gamma <- nlminb(start = c(2,5)
                    ,objective = testDistribution
                    ,distribution = "gamma"
                    ,x = D$pow.obs.norm)
par.gamma$objective
```

```
## [1] -97.38174
```

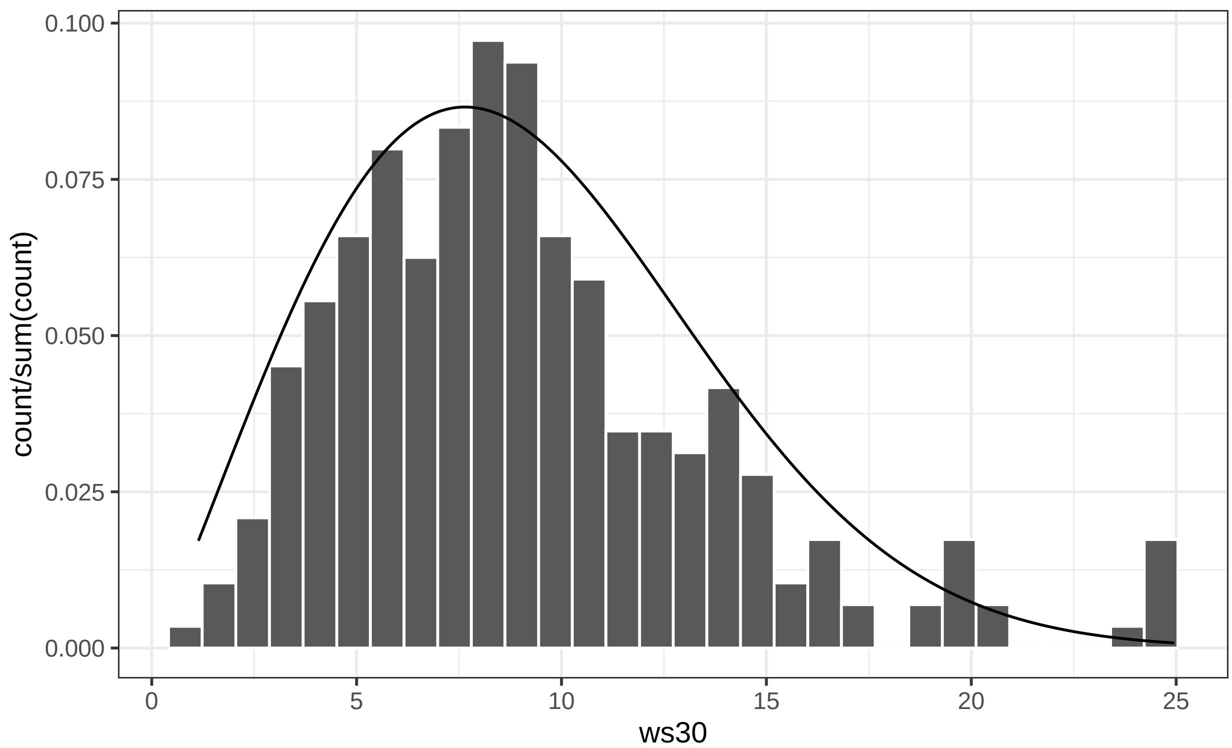
```r
#Sampling from the found beta distribution
D$simdata <- rbeta(length(D$pow.obs.norm), shape1 = par.beta$par[1]
                   ,shape2 = par.beta$par[2])
sam.plot.pow.beta <- ggplot(D)+
  geom_histogram(aes(x = pow.obs.norm, y =..density..), colour = "white", alpha = 0.6)+
  geom_histogram(aes(x = simdata, y =..density..), alpha = 0.2, fill = "blue")+
  theme_bw()+
  ylim(c(0,10))+
  stat_function(fun = dbeta, n = length(D$pow.obs.norm), args = list(shape1 = par.beta$par[1],shape2 = p

#Sampling from the found exp distribution
D$simdata <- rexp(length(D$pow.obs.norm), rate = par.exp$par)
sam.plot.pow.exp <- ggplot(D)+
  geom_histogram(aes(x = pow.obs.norm, y = ..density..))+
  geom_histogram(aes(x = simdata, y = ..density..)
                 , alpha = 0.2, fill = "blue")+
  theme_bw()+
  stat_function(fun = dexp, n = length(D$pow.obs.norm), args = list(rate = par.exp$par))

#Sampling from the found gamma distribution
D$simdata <- rgamma(length(D$pow.obs.norm), shape = par.gamma$par[1], rate = par.gamma$par[2])
sam.plot.pow.gamma <- ggplot(D)+
  geom_histogram(aes(x = pow.obs.norm, y = ..density..))+
  geom_histogram(aes(x = simdata, y = ..density..)
                 , alpha = 0.2, fill = "blue")+
  theme_bw()+
  ylim(c(0,10))+
  stat_function(fun = dgamma, n = length(D$pow.obs.norm), args = list(shape = par.gamma$par[1], rate = p

grid.arrange(sam.plot.pow.beta, sam.plot.pow.exp, sam.plot.pow.gamma, ncol = 3)
```




```
#Remove simulated data from the data frame
D <- D %>%
  dplyr::select(-simdata)
```

For wind speed distributions it is common practice to use the weibull distribution.

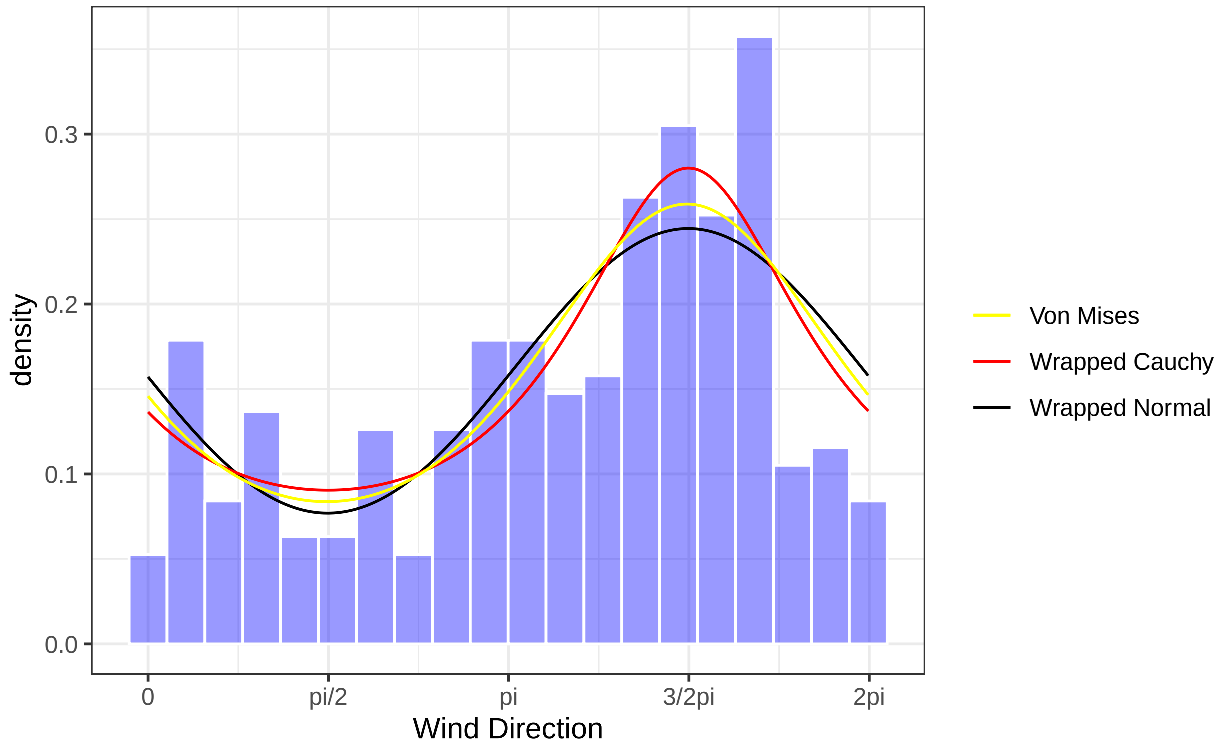
```
par.ws30 <- nlminb(start = c(1,1), objective = testDistribution
                   , x = D$ws30
                   , distribution = "weibull"
                   , lower = c(0,0))


ggplot(D)+
  geom_histogram(aes(x = ws30, y = ..count../sum(..count..))
                 , colour = "white"
                 , bins = 30)+
  theme_bw()+
  stat_function(fun = dweibull, n = dim(D)[1], args = list(shape = par.ws30$par[1], scale = par.ws30$par
```



Wind direction are supplied as radians in the dataset, and thus it is appropriate to fit circular distributions to this variable. Here we examine a circular normal distribution, wrapped cauchy and a von Mises distribution.

```
nll.wrappedNormal <- function(p,x){
  nll <- -sum(log(dwrappednormal(x, mu = circular(p[1]), rho = NULL, sd = p[2])))
  return(nll)
}

nll.wrappedCauchy <- function(p,x){
  nll <- -sum(log(dwrappedcauchy(x, mu = circular(p[1]), rho = p[2])))
  return(nll)
}

nll.vonMises <- function(p,x){
  nll <- -sum(dvonmises(x, mu = circular(p[1]), kappa = p[2], log = T))
  return(nll)
}

wrapped.par <- nlminb(start = c(2,1), objective = nll.wrappedNormal, x = D$wd30)
wrapped.cauc.par <- nlminb(start = c(1,1/10000), lower = c(-Inf, 1/10000), upper = c(Inf, 1),
                           objective = nll.wrappedCauchy, x = D$wd30)
wrapped.vonMises <- nlminb(start = c(0,1), objective = nll.vonMises, x = D$wd30, lower = c(-1000, 0))

ggplot(D)+
  theme_bw()+
  #geom_density(aes(x = wd30.centered, y = ..density..), alpha = .8, colour = "white", fill = "red", co
  geom_histogram(aes(x = wd30, y = ..density..), colour = "white", alpha = .4, fill = "blue", bins = 20
  scale_x_continuous(breaks = c(0,pi/2,pi,3/2*pi,2*pi)
                     , labels =c("0", "pi/2", "pi", "3/2pi", "2pi"))+
```

```r
  #stat_function(fun = dnorm, n = dim(D)[1], args = list(mean = par.wd30$par[1], sd = par.wd30$par[2]))
  stat_function(fun = dwrappednormal, n = dim(D)[1], args = list(mu = wrapped.par$par[1], sd = wrapped.p
  stat_function(fun = dwrappedcauchy, n = dim(D)[1], args = list(mu = wrapped.cauc.par$par[1],rho = wrap
  #stat_function(fun = dwrappedcauchy, n = dim(D)[1], args = list(mu = -1.5748695, rho = 0.2751607), aes
  stat_function(fun = dvonmises, n = dim(D)[1], args = list(mu = wrapped.vonMises$par[1], kappa = wrappe
  labs(x = "Wind Direction", colour = "")+
  scale_colour_manual(values = c("yellow", "red", "black", "blue"))
```



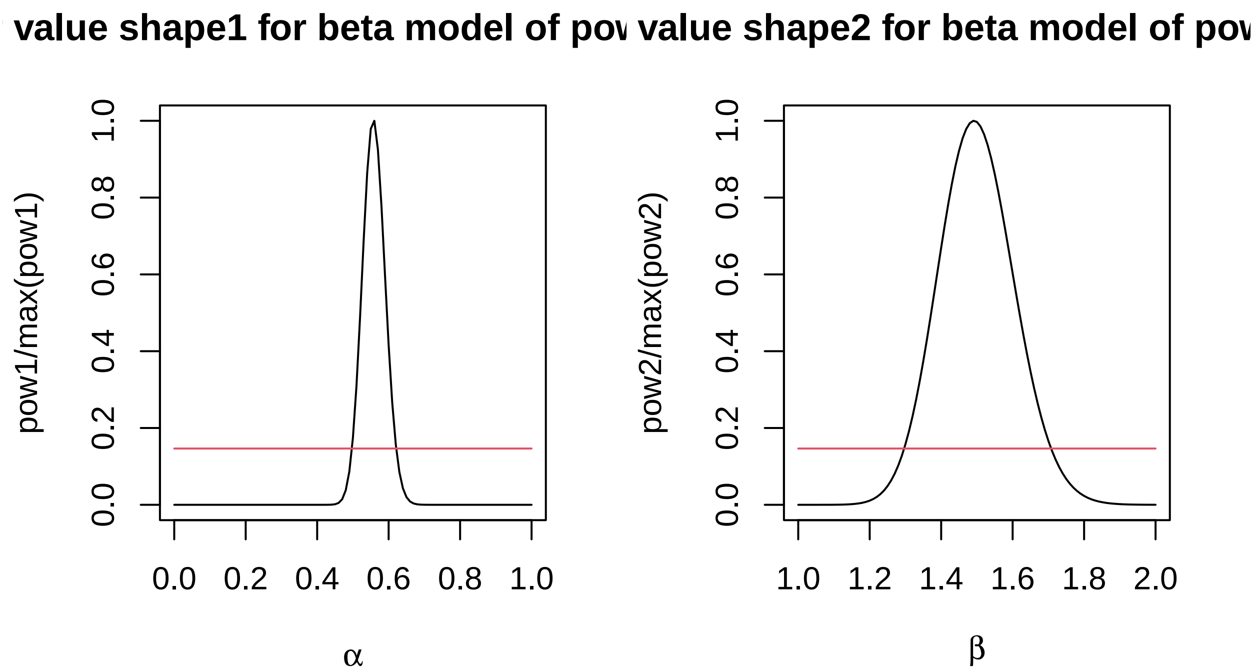

```r
#Calculate AICs
print(paste0("AIC wrapped normal: ", round(-2*log(exp(-wrapped.par$objective     ))+2*2,4), "|"
            ,"AIC wrapped cauchy: ", round(-2*log(exp(-wrapped.cauc.par$objective))+2*2,4), "|"
            ,"AIC von Mises: "     , round(-2*log(exp(-wrapped.vonMises$objective))+2*2,4)))
```

```
## [1] "AIC wrapped normal: 1020.5519|AIC wrapped cauchy: 1018.1731|AIC von Mises: 1019.3436"
```

```r
## CI ## WIND POWER
par(mfrow=c(1,1))
alpha <- 0.05
c <- exp(-0.5 * qchisq(1-alpha, df = 1))
#likelihood-based
mle.pow <- par.beta$par

pow.fun <- function(shape1, shape2, data){
  return( prod( dbeta(x = data, shape1 = shape1, shape2 = shape2, log = F) ) )
}
```

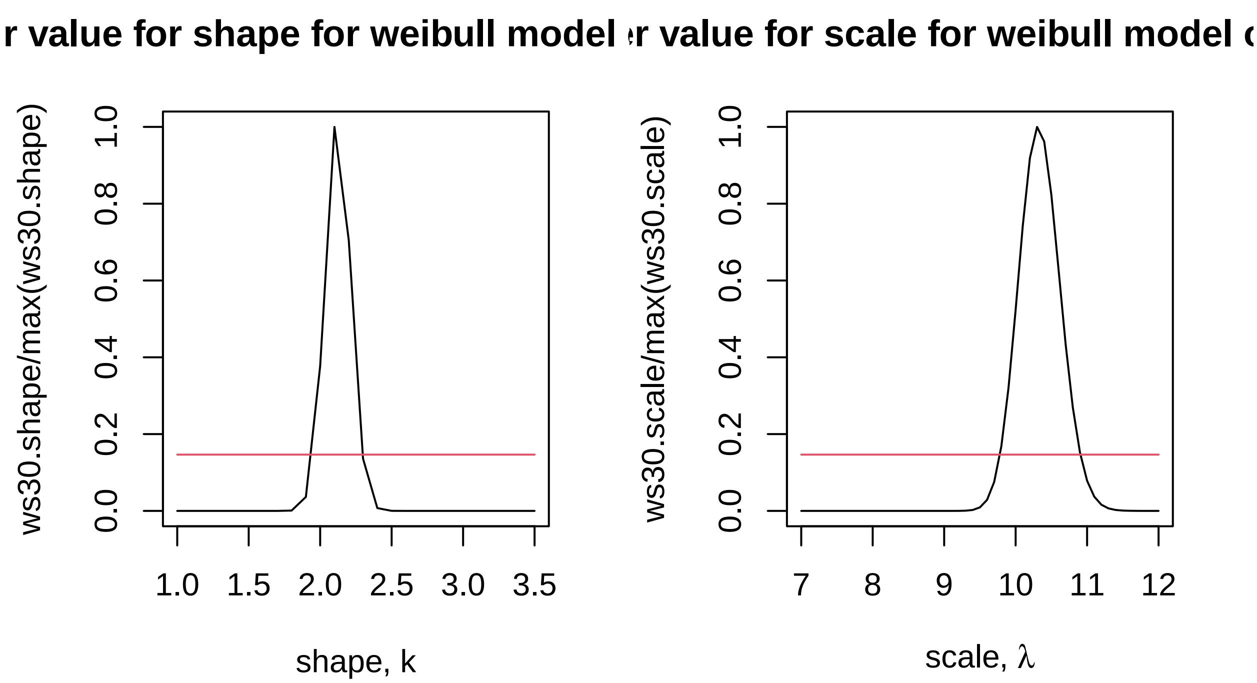
```r
l.pow.fun <- function(shape1, shape2, data){
  return( sum( dbeta(x = data, shape1 = shape1, shape2 = shape2, log = T) ) )
}

CIfun.pow <- function(y, first = T){##### T for shape, F for scale
  if(first){
    return( sum( dbeta(x = D$pow.obs.norm, shape1 = mle.pow[1], shape = mle.pow[2], log = T) ) -
      sum( dbeta(x = D$pow.obs.norm, shape1 = y, shape2 = mle.pow[2], log = T) ) -
      0.5 * qchisq(1-alpha, df = 1) )
  } else {
    return( sum( dbeta(x = D$pow.obs.norm, shape1 = mle.pow[1], shape = mle.pow[2], log = T) ) -
      sum( dbeta(x = D$pow.obs.norm, shape1 = mle.pow[1], shape2 = y, log = T) ) -
      0.5 * qchisq(1-alpha, df = 1) )
  }
}

par(mfrow=c(1,2))
shape1s <- seq(0, 1, by = 0.01)
pow1 <- sapply(X = shape1s, FUN = pow.fun, data = D$pow.obs.norm, shape2 = mle.pow[2])
plot(shape1s, pow1/max(pow1), col = 1, type = "l", xlab = expression(paste(alpha)),
     main = "Parameter value shape1 for beta model of power production")
CI.pow1 <- c(uniroot(f = CIfun.pow, interval = c(0, mle.pow[1]), first = T)$root,
            uniroot(f = CIfun.pow, interval = c(mle.pow[1], 1), first = T)$root)
lines(range(shape1s), c*c(1,1), col = 2)

shape2s <- seq(1, 2, by = 0.01)
pow2 <- sapply(X = shape2s, FUN = pow.fun, data = D$pow.obs.norm, shape1 = mle.pow[1])
plot(shape2s, pow2/max(pow2), col = 1, type = "l", xlab = expression(paste(beta)),
     main = "Parameter value shape2 for beta model of power production")
CI.pow2 <- c(uniroot(f = CIfun.pow, interval = c(1, mle.pow[2]), first = F)$root,
             uniroot(f = CIfun.pow, interval = c(mle.pow[2], 2), first = F)$root)
lines(range(shape2s), c*c(1,1), col = 2)
```

Conclude on the most appropriate model for each variable, also report parameters including assessment of their uncertainty. For models that does not include a transformation you should also give an assessment of the uncertainty of the expected value in the model.



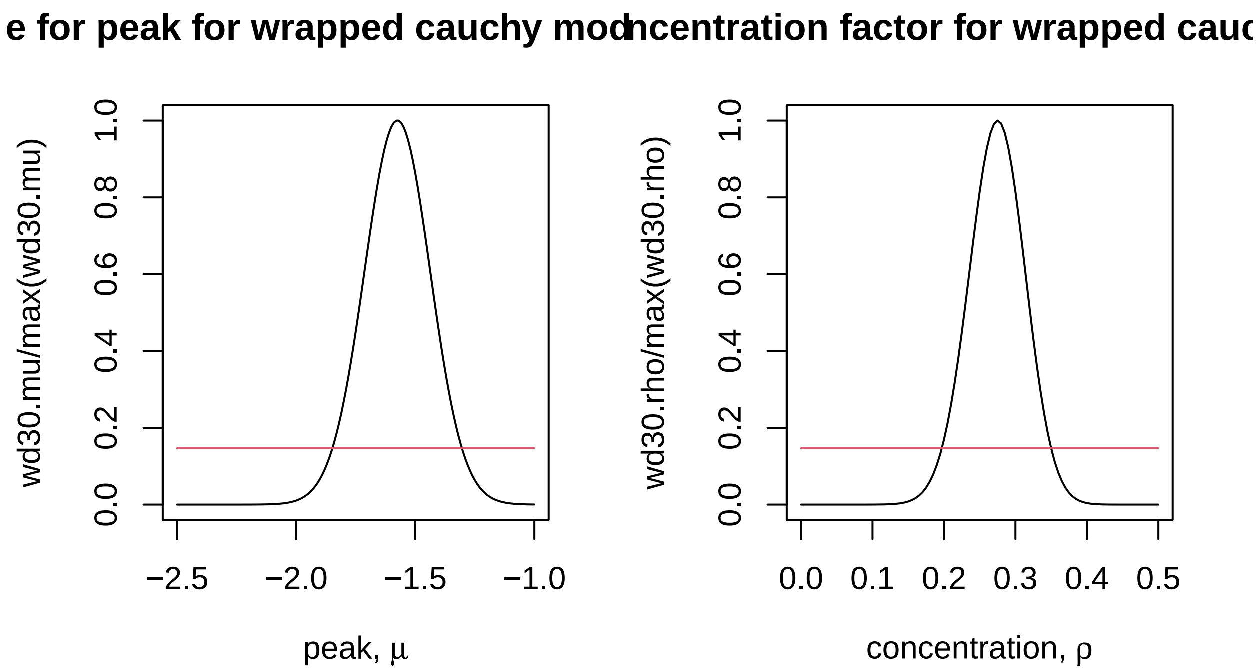

```
#wald
n <- dim(D)[1]
H.pow.shape1 <- hessian(l.pow.fun, mle.pow[1], shape2 = mle.pow[2], data = D$pow.obs.norm)
V.pow.shape1 <- as.numeric(-1/H.pow.shape1)
H.pow.shape2 <- hessian(l.pow.fun, mle.pow[2], shape1 = mle.pow[1], data = D$pow.obs.norm)
V.pow.shape2 <- as.numeric(-1/H.pow.shape2)
wald.pow.shape1 <- mle.pow[1] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.pow.shape1)
wald.pow.shape2 <- mle.pow[2] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.pow.shape2)

## CI ## WIND SPEED
par(mfrow=c(1,2))
#likelihood-based
mle.ws30.weib <- par.ws30$par

ws30.fun <- function(shape, scale, data){#####
  prod(dweibull(x = data, shape = shape, scale = scale, log = F)*2)#to not get full zeros
}

l.ws30.fun <- function(shape, scale, data){#####
  sum(dweibull(x = data, shape = shape, scale = scale, log = T))
}

CIfun.ws30 <- function(y, shape = T){##### T for shape, F for scale
  if(shape){
    sum(dweibull(x = D$ws30, shape = mle.ws30.weib[1], scale = mle.ws30.weib[2], log = T)) -
      sum(dweibull(x = D$ws30, shape = y, scale = mle.ws30.weib[2], log = T)) -
      0.5 * qchisq(1-alpha, df = 1)
  } else {
    sum(dweibull(x = D$ws30, shape = mle.ws30.weib[1], scale = mle.ws30.weib[2], log = T)) -
```

```
      sum(dweibull(x = D$ws30, shape = mle.ws30.weib[1], scale = y, log = T)) -
      0.5 * qchisq(1-alpha, df = 1)
  }
}
shapes <- seq(1, 3.5, by = 0.1)
ws30.shape <- sapply(X = shapes, FUN = ws30.fun, scale = mle.ws30.weib[2], data = D$ws30)
plot(shapes, ws30.shape/max(ws30.shape), col = 1, type = "l", xlab = "shape, k",
     main = "Parameter value for shape for weibull model of wind speed")
CI.ws30.shape <- c(uniroot(f = CIfun.ws30, interval = c(1, mle.ws30.weib[1]), shape = T)$root,
                   uniroot(f = CIfun.ws30, interval = c(mle.ws30.weib[1], 3.5), shape = T)$root)
lines(range(shapes), c*c(1,1), col = 2)

scales <- seq(7, 12, by = 0.1)
ws30.scale <- sapply(X = scales, FUN = ws30.fun, shape = mle.ws30.weib[1], data = D$ws30)
plot(scales, ws30.scale/max(ws30.scale), col = 1, type = "l", xlab = expression(paste("scale, ", lambda
     main = "Parameter value for scale for weibull model of wind speed")
CI.ws30.scale <- c(uniroot(f = CIfun.ws30, interval = c(7, mle.ws30.weib[2]), shape = F)$root,
                   uniroot(f = CIfun.ws30, interval = c(mle.ws30.weib[2], 12), shape = F)$root)
lines(range(scales), c*c(1,1), col = 2)
```




```
#wald
n <- dim(D)[1]
H.ws30.shape <- hessian(l.ws30.fun, mle.ws30.weib[1], scale = mle.ws30.weib[2], data = D$ws30)
V.ws30.shape <- as.numeric(-1/H.ws30.shape)
H.ws30.scale <- hessian(l.ws30.fun, mle.ws30.weib[2], shape = mle.ws30.weib[1], data = D$ws30)
V.ws30.scale <- as.numeric(-1/H.ws30.scale)
wald.ws30.shape <- mle.ws30.weib[1] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.ws30.shape)
wald.ws30.scale <- mle.ws30.weib[2] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.ws30.scale)
```

```r
## CI ## WIND DIRECTION
par(mfrow=c(1,2))
#likelihood-based
mle.wd30 <- wrapped.cauc.par$par

wd30.fun <- function(mu, rho, data){#####
  prod(dwrappedcauchy(x = data, mu = mu, rho = rho))
}

l.wd30.fun <- function(mu, rho, data){#####
  sum( log( dwrappedcauchy(x = data, mu = mu, rho = rho) ) )
}

CIfun.wd30 <- function(y, mu = T){##### T from mean, F for sigma
  if(mu){
    return( sum( log( dwrappedcauchy(x = D$wd30, mu = mle.wd30[1], rho = mle.wd30[2]) ) ) -
      sum( log( dwrappedcauchy(x = D$wd30, mu = y, rho = mle.wd30[2]) ) ) -
      0.5 * qchisq(1-alpha, df = 1) )
  } else {
    return( sum( log( dwrappedcauchy(x = D$wd30, mu = mle.wd30[1], rho = mle.wd30[2]) ) ) -
      sum( log( dwrappedcauchy(x = D$wd30, mu = mle.wd30[1], rho = y) ) ) -
      0.5 * qchisq(1-alpha, df = 1) )
  }
}

mus <- seq(-2.5, -1, by = 0.01)
wd30.mu <- sapply(X = mus, FUN = wd30.fun, rho = mle.wd30[2], data = D$wd30)
plot(mus, wd30.mu/max(wd30.mu), col = 1, type = "l", xlab = expression(paste("peak, ", mu)),
     main = "Parameter value for peak for wrapped cauchy model of wind direction")
CI.wd30.mu <- c(uniroot(f = CIfun.wd30, interval = c(-2.5, mle.wd30[1]), mu = T)$root,
                uniroot(f = CIfun.wd30, interval = c(mle.wd30[1], -1), mu = T)$root)
lines(range(mus), c*c(1,1), col = 2)

rhos <- seq(0, 0.5, by = 0.005)
wd30.rho <- sapply(X = rhos, FUN = wd30.fun, mu = mle.wd30[1], data = D$wd30)
plot(rhos, wd30.rho/max(wd30.rho), col = 1, type = "l", xlab = expression(paste("concentration, ", rho))
     main = "Parameter value for concentration factor for wrapped cauchy model of wind direction")
CI.wd30.rho <- c(uniroot(f = CIfun.wd30, interval = c(0, mle.wd30[2]), mu = F)$root,
                   uniroot(f = CIfun.wd30, interval = c(mle.wd30[2], 0.5), mu = F)$root)
lines(range(rhos), c*c(1,1), col = 2)
```




```r
#wald
n <- dim(D)[1]
H.wd30.mu <- hessian(l.wd30.fun, mle.wd30[1], rho = mle.wd30[2], data = D$wd30)
V.wd30.mu <- as.numeric(-1/H.wd30.mu)
H.wd30.rho <- hessian(l.wd30.fun, mle.wd30[2], mu = mle.wd30[1], data = D$wd30)
V.wd30.rho <- as.numeric(-1/H.wd30.rho)
wald.wd30.mu <- mle.wd30[1] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.wd30.mu)
wald.wd30.rho <- mle.wd30[2] + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.wd30.rho)

#All CIs of parameters
round( rbind( CI.pow1, wald.pow.shape1, CI.pow2, wald.pow.shape2, mle.pow,
              CI.ws30.shape, wald.ws30.shape, CI.ws30.scale, wald.ws30.scale, mle.ws30.weib,
              CI.wd30.mu, wald.wd30.mu, CI.wd30.rho, wald.wd30.rho, mle.wd30 ), digits = 3 )
```

```
##                    [,1]   [,2]
## CI.pow1            0.497  0.621
## wald.pow.shape1    0.495  0.619
## CI.pow2            1.296  1.708
## wald.pow.shape2    1.286  1.697
## mle.pow            0.557  1.492
## CI.ws30.shape      1.954  2.295
## wald.ws30.shape    1.952  2.294
## CI.ws30.scale      9.781 10.906
## wald.ws30.scale    9.756 10.879
## mle.ws30.weib      2.123 10.318
## CI.wd30.mu        -1.848 -1.304
## wald.wd30.mu      -1.845 -1.305
## CI.wd30.rho        0.197  0.350
## wald.wd30.rho      0.199  0.352
```

```
## mle.wd30        -1.575  0.275
```

```
alpha <- par.beta$par[1]; beta <- par.beta$par[2]
#Beta: E[X] = alpha/(alpha + beta), Var[X] = alpha*beta/((alpha+beta)^2*(alpha+beta+1))
E.pow.obs <- alpha/(alpha + beta)
CI.E.pow.obs <- alpha/(alpha + beta) + c(-1,1) * qnorm(1-alpha/2) * alpha*beta/((alpha+beta)^2*(alpha+be
#(CI.E.pow.obs <- mean(D$pow.obs.norm) + c(-1,1) * qnorm(1-alpha/2) * sd(D$pow.obs.norm) / dim(D)[1])

#Weibull: E[X] = lambda * gamma(1+1/k); Var[X] = lambda^2*(gamma(1+2/k) - (gamma(1+1/k))^2)
#par.ws30$par[2]*gamma(1+1/par.ws30$par[1]) #mean = lambda * Gamma(1 + 1/k); lambda = scale, k = shape
scale <- par.ws30$par[2]; shape <- par.ws30$par[1]
E.ws30 <- scale*gamma(1+1/shape)
V.ws30 <- scale^2*(gamma(1+2/shape) - (gamma(1+1/shape))^2)
CI.E.ws30 <- E.ws30 + c(-1,1) * qnorm(1-alpha/2) * sqrt(V.ws30) / dim(D)[1] #according to Central Limit
#(CI.E.ws30 <- mean(D$ws30) + c(-1,1) * qnorm(1-alpha/2) * sd(D$ws30) / dim(D)[1])

#Wrapped Cauchy: E[X] = mu, Var[X] = 1 - exp(-gamma)
#relationship between rho and gamma: gamma = -ln(rho)
mu <- wrapped.cauc.par$par[1]; gamma = -log(wrapped.cauc.par$par[2])
(E.wd30 <- mu)
```

```
## [1] -1.574857
```

```
(V.wd30 <- 1 - exp(-gamma)) #or 1 - rho
```

```
## [1] 0.7248408
```

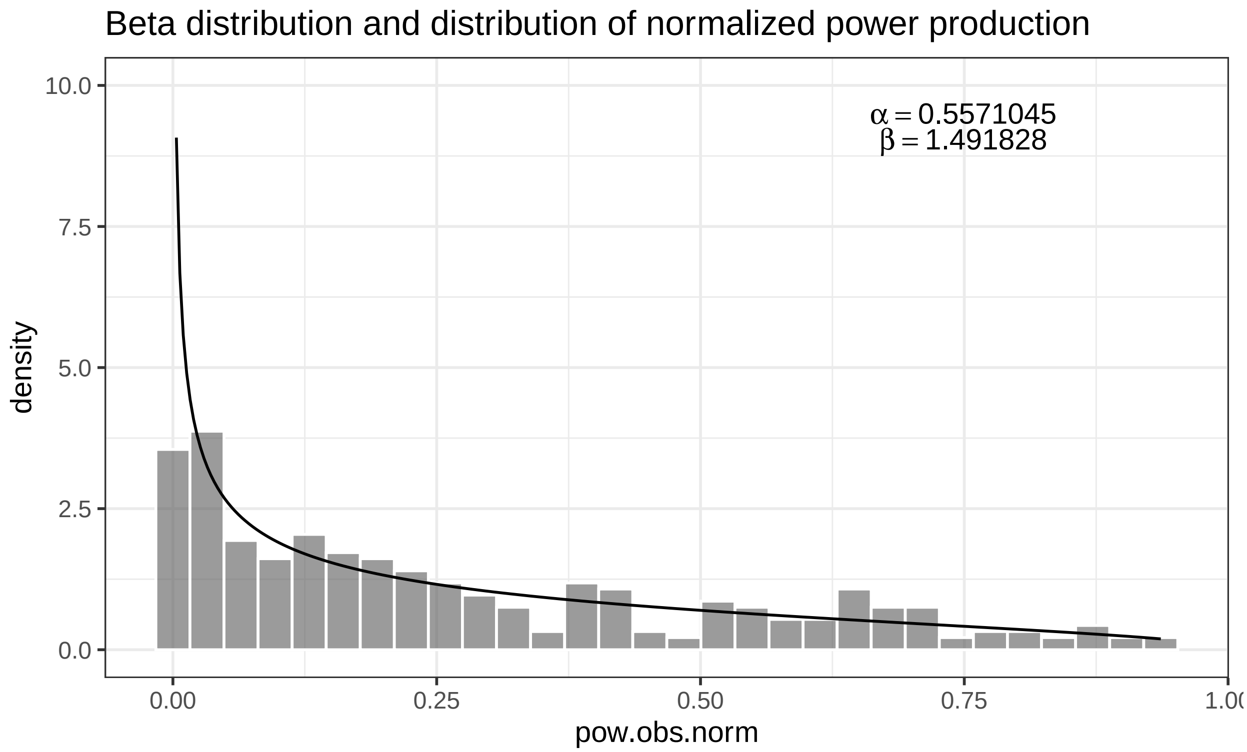
```
(CI.E.wd30 <- E.wd30 + c(-1,1) * qnorm(1-alpha/2) *V.wd30 / dim(D)[1]) #according to Central Limit Theo
```

```
## [1] -1.576335 -1.573380
```

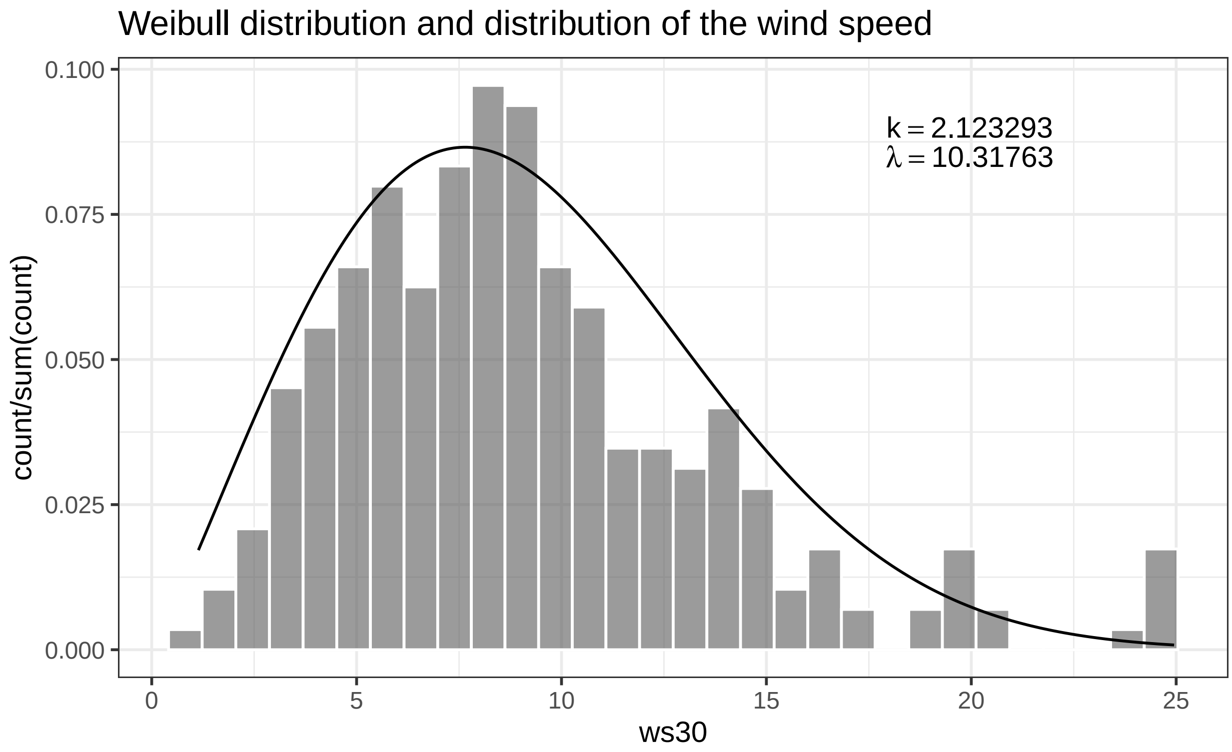
```
#(CI.E.wd30 <- mle.wd30[1] + c(-1,1) * qnorm(1-alpha/2) * sd(D$wd30) / dim(D)[1]) #mean(D$wd30) instead

round(rbind(c(CI.E.pow.obs[1], 1/par.exp$par, CI.E.pow.obs[2]) , c(CI.E.ws30[1], E.ws30, CI.E.ws30[2])
```

```
##          [,1]     [,2]     [,3]
## [1,]  0.27177  0.27624  0.27203
## [2,]  9.12849  9.13771  9.14694
## [3,] -1.57634 -1.57486 -1.57338
```

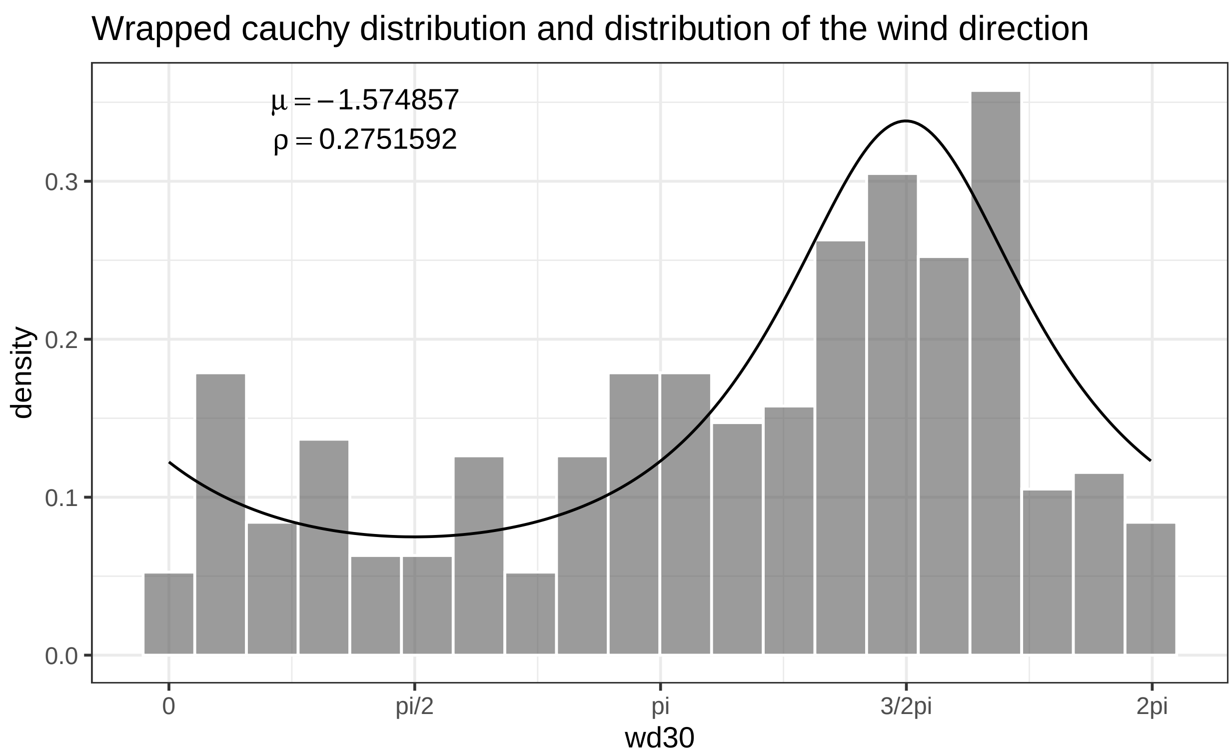
```
par(mfrow=c(1,3))

temp1 <- paste("alpha == ", mle.pow[1]) #par.beta$par[1]
temp2 <- paste("beta == ", mle.pow[2]) #par.beta$par[2]
temp <- c(temp1, temp2)

ggplot(D)+
  geom_histogram(aes(x = pow.obs.norm, y = ..density..), colour='white', alpha=0.6, bins=30)+
  theme_bw()+
  stat_function(fun = dbeta, n = dim(D)[1], args = list(shape1 = mle.pow[1], shape2 = mle.pow[2]))+
  ylim(c(0,10))+
  annotate( "text", x = 4/5*max(D$pow.obs.norm), y = c(9.5, 9.0), label = temp, parse = T  ) +
  ggtitle("Beta distribution and distribution of normalized power production")
```




```
temp1 <- paste("k == ", mle.ws30.weib[1]) #par.ws30$par[1]
temp2 <- paste("lambda == ", mle.ws30.weib[2]) #par.ws30$par[2]
temp <- c(temp1, temp2)

ggplot(D)+
  geom_histogram(aes(x = ws30, y = ..count../sum(..count..)) , colour = "white", alpha=0.6, bins = 30)+
  theme_bw()+
  stat_function(fun = dweibull, n = dim(D)[1], args = list(shape = par.ws30$par[1], scale = par.ws30$pa
  annotate( "text", x = 4/5*max(D$ws30), y = c(0.09,0.085), label = temp, parse = T  ) +
  ggtitle("Weibull distribution and distribution of the wind speed")
```




```r
temp1 <- paste("mu ==", mle.wd30[1]) #wrapped.cauc.par$par[1]
temp2 <- paste("rho ==", mle.wd30[2]) #wrapped.cauc.par$par[2]
temp <- c(temp1, temp2)

ggplot(D)+
  theme_bw()+
  geom_histogram(aes(x = wd30, y = ..density..), colour = "white", alpha = 0.6, bins = 20)+
  scale_x_continuous(breaks = c(0,pi/2,pi,3/2*pi,2*pi)
                     , labels =c("0", "pi/2", "pi", "3/2pi", "2pi"))+
  stat_function(fun = dwrappedcauchy, n = dim(D)[1], args = list(mu = wrapped.cauc.par$par[1], rho = 0.3
  annotate( "text", x = 1/5*max(D$wd30), y = c(0.35, 0.325), label = c(temp1,temp2), parse = T  ) +
  ggtitle("Wrapped cauchy distribution and distribution of the wind direction")
```


Wrapped cauchy distribution and distribution of the wind direction
μ = −1.574857
ρ = 0.2751592
density
0.0
0.1
0.2
0.3
0
pi/2
pi
3/2pi
2pi
wd30