A2 Project 1

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Setting working directories and loading the data. Data is also normalized.

```
library(ggplot2)
library(numDeriv)

if (Sys.getenv("LOGNAME") == "mortenjohnsen"){
  setwd("/Users/mortenjohnsen/OneDrive - Danmarks Tekniske Universitet/DTU/9. Semester/02418 - Statisti
} else {
  setwd("~/Documents/02418 Statistical Modelling/Assignments/Assignment 1/Project-1")
  #setwd("~/Documents/02418 Statistical Modelling/Assignments/Assignment 1/Project-1/Project 2")
  #source("testDistribution.R")
}

D <- read.table("tuno.txt", header=TRUE, sep=" ",
                as.is=TRUE)

D$date <- as.Date("2003-01-01")-1+D$r.day
D$pow.obs.norm <- D$pow.obs/5000
```

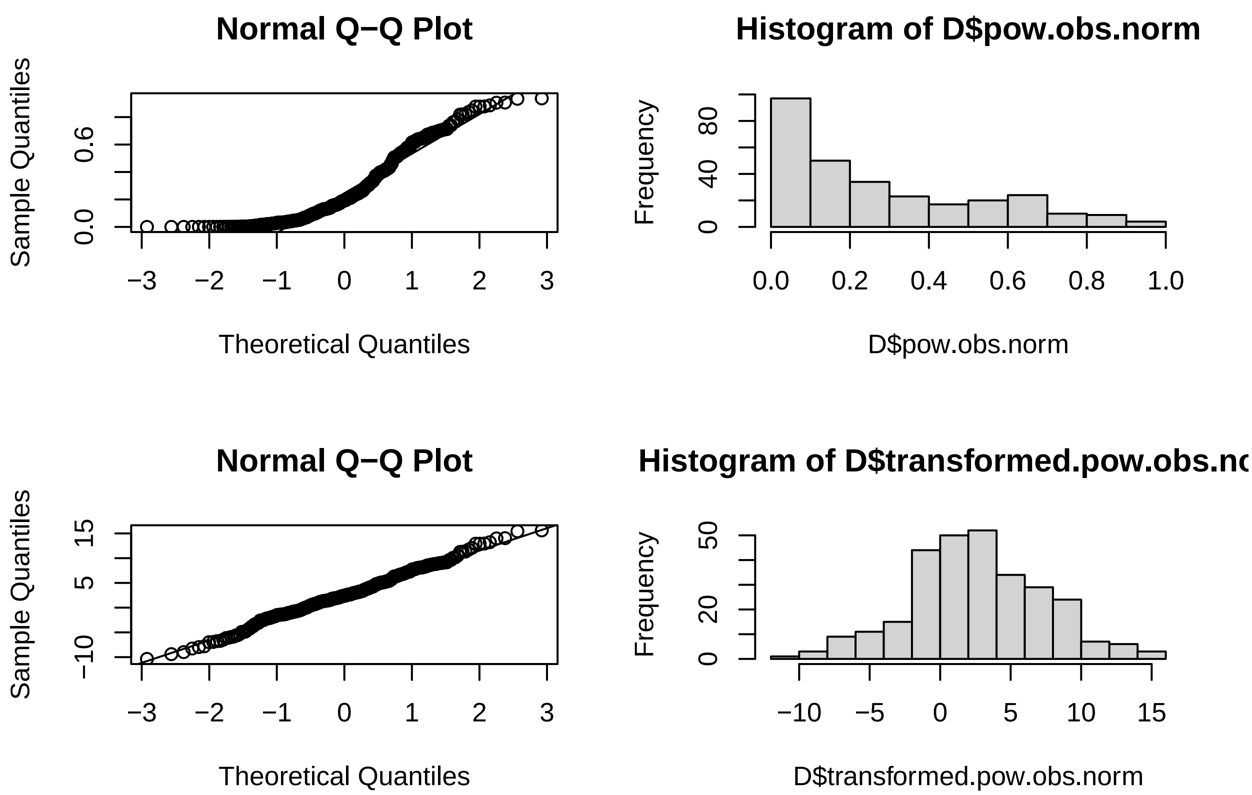
Data is transformed to be normal using transformation 1 in description of assignment 1:

$$y^{(\lambda)} = \frac{1}{\lambda} log\left(\frac{y^{\lambda}}{1-y^{\lambda}}\right); \lambda > 0$$

Below can be seen qq-plots comparing the normalized data before and after transformation to allow for a better understanding of the motivation behind the transformation.

```
#lambdas <- seq(-0.5,0.5,by=0.01)
#library(MASS)
#boxcox(lm(D$pow.obs.norm~1), lambda=lambdas)

#Define transformation function
Trans.eq1 <- function(lambda, y){
  y_lambda <- 1/lambda * log(y^lambda/(1-y^lambda))#, lambda > 0
  return(y_lambda)
}
#Optimization function
#Måske er det bedre at lave nogle undersøgelser selv frem for bare at optimere lambda (Overvej til sene
#se kode fra lecture 4 linje 5-73
lambda_NLL <- function(lambda, x = D$pow.obs.norm){
  y <- Trans.eq1(lambda, x)
  NLL <- -as.numeric(shapiro.test(y)$statistic)
```

```
  return(NLL)
}
lambda.hat <- nlminb(start = 0.2, objective = lambda_NLL)
#round to two decimal points.
lambda <- round(lambda.hat$par, 2)
D$transformed.pow.obs.norm <- Trans.eq1(lambda, D$pow.obs.norm)

#Check qqplot:
par(mfrow = c(2,2))
qqnorm(D$pow.obs.norm)
qqline(D$pow.obs.norm)
hist(D$pow.obs.norm)
qqnorm(D$transformed.pow.obs.norm)
qqline(D$transformed.pow.obs.norm)
hist(D$transformed.pow.obs.norm)
```



Two models will be produced: a non-normal beta or gamma regression model or the normalized data and a normal linear regression model of the transformed, normalized data.

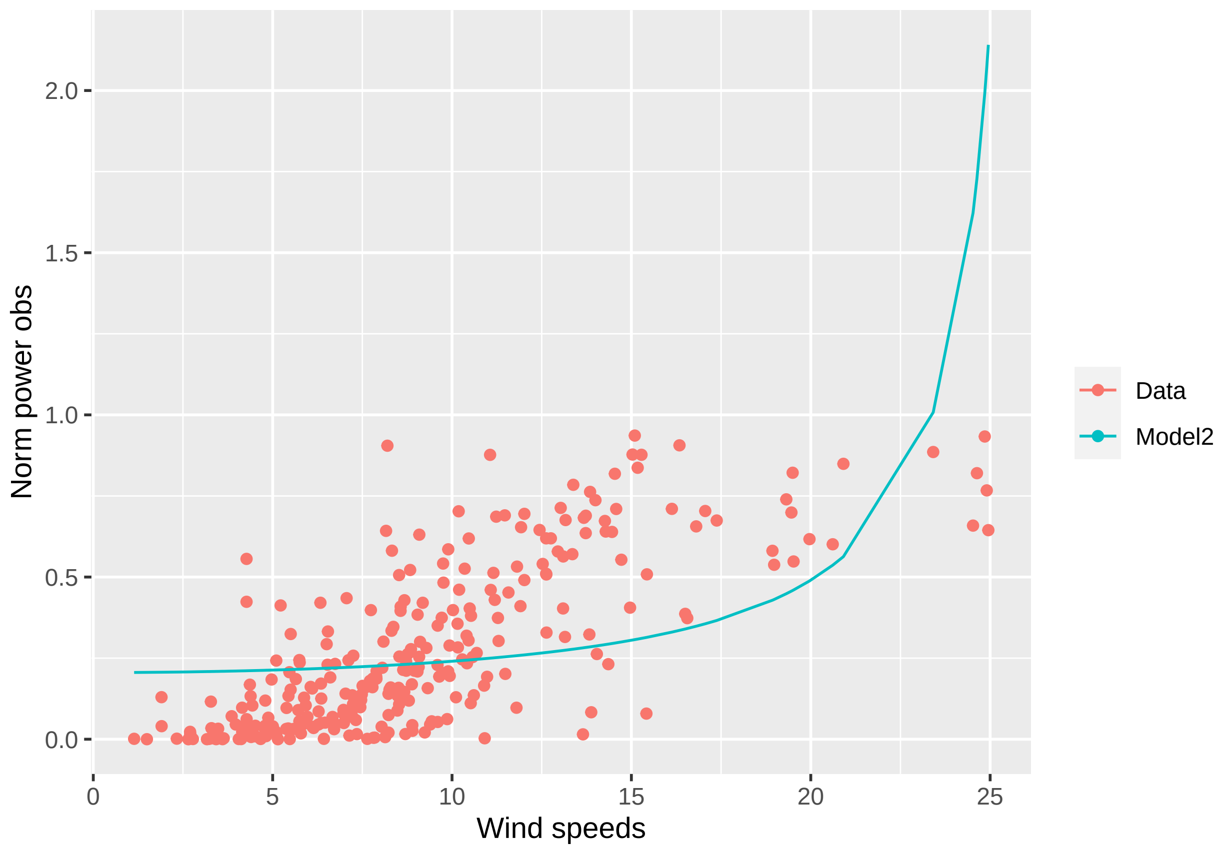
First the non-normal:

```
#gamma's pretty bad. Maybe beta would be better?
glmfit2 <- glm( (pow.obs.norm ~ I(ws30^2)), data = D, family = Gamma)
summary(glmfit2)
```

```
##
```

```
## Call:
## glm(formula = (pow.obs.norm ~ I(ws30^2)), family = Gamma, data = D)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -4.0096  -1.1870  -0.2219   0.5052   1.7855
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.8662199  0.2590041   18.79   <2e-16 ***
## I(ws30^2)   -0.0070668  0.0005627  -12.56   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for Gamma family taken to be 0.7449897)
##
##     Null deviance: 503.32  on 287  degrees of freedom
## Residual deviance: 447.39  on 286  degrees of freedom
## AIC: -223.72
##
## Number of Fisher Scoring iterations: 6
```

```r
D$glmpred2 <- ( glmfit2$coefficients[1] + glmfit2$coefficients[2] * D$ws30^2)^(-1)

ggplot(data = D)+
  geom_point(aes(x=ws30, y=pow.obs.norm, colour="Data"))+
  geom_line(aes(x=ws30, y=glmpred2, colour="Model2"))+
  labs(x = "Wind speeds", y="Norm power obs", colour = "")
```



This can also be done by hand. See p. 163-65 for reference. Exponential family regression models estimate the log-likelihood contribution of an outcome y_i as seen below:

$$logL(\theta_i, \phi) = \frac{y_i \theta_i - A(\theta_i)}{\phi} + c(y_i, \phi)$$

Here, $A(\theta_i)$ is defined as such that its derivative is the mean $E_{y_1} = A'(\theta_i) = \mu_i$. ϕ is the dispersion parameter, where $var(y_i) = \phi A''(\theta_i) = \phi v(\mu_i)$. To use this general exponential family for regression analysis we only have to specify $A(\theta_i)$ and a link function $h(\mu_i)$. The reason as to why $c(y_i, \phi)$ can be neglected is that the term disappears when determining the score function anyways.??? KOMMENTAR

Now to the normal linear model:

```
transfit1 <- lm( (D$transformed.pow.obs.norm ~ ws30), data = D ) #good for comparison
transfit12 <- lm( (D$transformed.pow.obs.norm ~ ws30 + I(ws30^2)), data = D) #brilliant
transfit2 <- lm( (D$transformed.pow.obs.norm ~ I(ws30^2)), data = D ) #1. jeg har ladt disse to stå for
transfit3 <- lm( (D$transformed.pow.obs.norm ~ I(ws30^3)), data = D ) #2.
#transfitS <- lm( (D$transformed.pow.obs.norm ~ ws30 + I(ws30^2) + Season), data = D )
summary(transfit1)
```

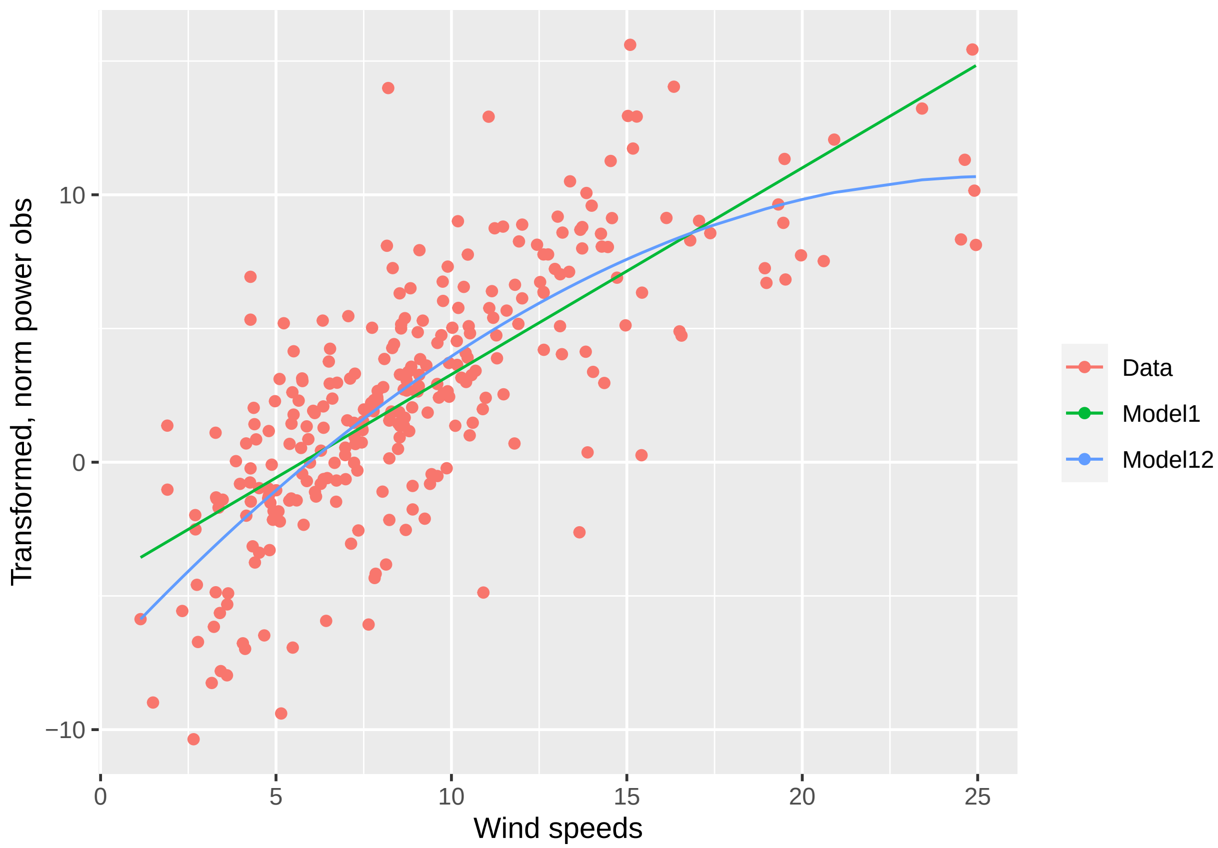
```
##
## Call:
## lm(formula = (D$transformed.pow.obs.norm ~ ws30), data = D)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.9294 -1.6153  0.2488  1.9131 12.1004
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.44248    0.41818  -10.62   <2e-16 ***
## ws30         0.77250    0.04106   18.81   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.171 on 286 degrees of freedom
## Multiple R-squared:  0.5531, Adjusted R-squared:  0.5516
## F-statistic:   354 on 1 and 286 DF,  p-value: < 2.2e-16
```

```
summary(transfit12)
```

```
##
## Call:
## lm(formula = (D$transformed.pow.obs.norm ~ ws30 + I(ws30^2)),
##     data = D)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5977 -1.5002  0.0378  1.7962 11.6778
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.450337   0.739811 -10.071  < 2e-16 ***
## ws30         1.418150   0.138965  10.205  < 2e-16 ***
## I(ws30^2)   -0.027717   0.005719  -4.846 2.07e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.053 on 285 degrees of freedom
## Multiple R-squared:  0.5871, Adjusted R-squared:  0.5842
## F-statistic: 202.7 on 2 and 285 DF,  p-value: < 2.2e-16
```

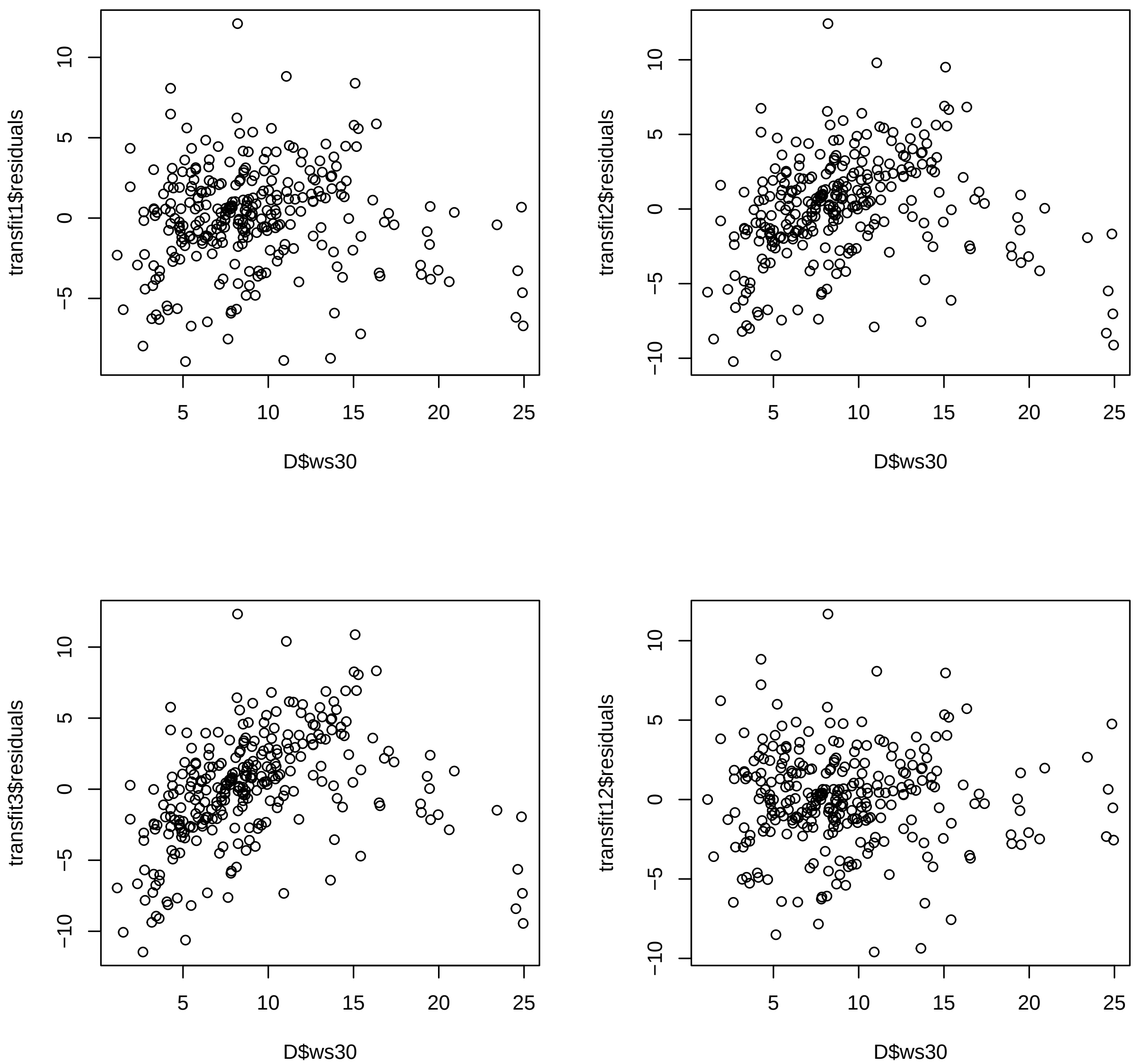
```
D$transformed.pred1 <- transfit1$coefficients[1] + transfit1$coefficients[2] * D$ws30
D$transformed.pred12 <- transfit12$coefficients[1] + transfit12$coefficients[2] * D$ws30 + transfit12$c

#D$transformed.predS <- transfitS$coefficients[1] + transfitS$coefficients[2] * D$ws30 + transfitS$coef
#summary(transfitS)

ggplot(data = D)+
  geom_point(aes(x=ws30, y=transformed.pow.obs.norm, colour="Data"))+
  geom_line(aes(x=ws30, y=transformed.pred1, colour="Model1"))+
  geom_line(aes(x=ws30, y=transformed.pred12, colour="Model12"))+
  #geom_line(aes(x=ws30, y=transformed.predS, colour="ModelS"))+
  labs(x = "Wind speeds", y="Transformed, norm power obs", colour = "")
```




```
summary(lm( (D$transformed.pow.obs.norm ~ ws30 + I(ws30^2) + wd30), data = D)) #simply a check with the
```

```
##
## Call:
## lm(formula = (D$transformed.pow.obs.norm ~ ws30 + I(ws30^2) +
##     wd30), data = D)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.6060 -1.4739  0.0307  1.7897 11.6787
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.496543   0.801711  -9.351  < 2e-16 ***
## ws30         1.416925   0.139440  10.162  < 2e-16 ***
## I(ws30^2)   -0.027728   0.005729  -4.840 2.14e-06 ***
## wd30         0.016252   0.107570   0.151     0.88
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.058 on 284 degrees of freedom
## Multiple R-squared:  0.5872, Adjusted R-squared:  0.5828
## F-statistic: 134.6 on 3 and 284 DF,  p-value: < 2.2e-16
```

Tjek af modellerne:



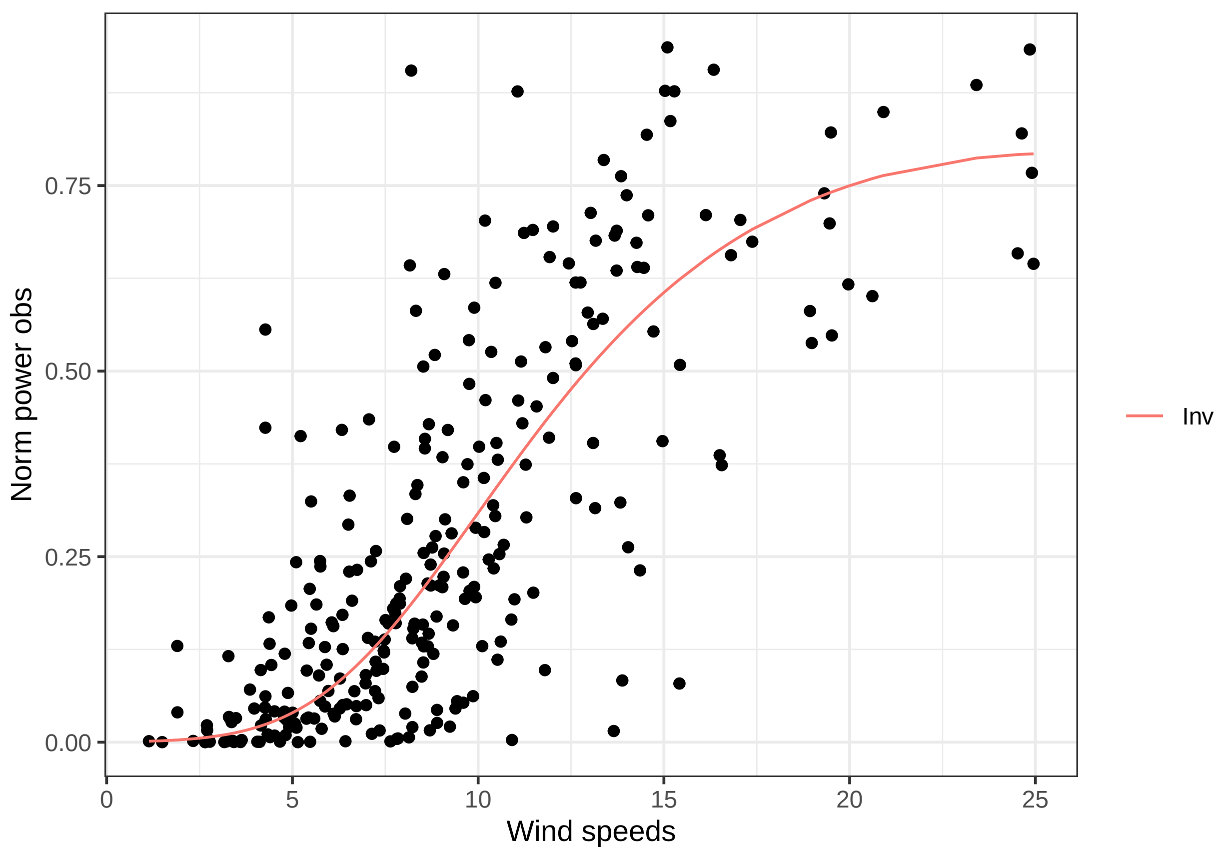
Normal linear model by hand and by optimisation:

```
D$base <- 1 #slide 15 for all of this
X <- as.matrix(data.frame(D$base, D$ws30, D$ws30^2))
Y <- D$transformed.pow.obs.norm
betas <- solve( (t(X)%*%X) ) %*% t(X) %*% Y

NLL.norm.lin <- function(p, regr1, regr2, response){
  mu <- p[1] + p[2]*regr1 + p[3]*regr2
  sigma <- sqrt( 1/length(response) * sum( (response - mu)^2  ) )
  return( -sum( dnorm( x = response, mean = mu, sd = sigma, log = T ) )  )
}
norm.lin.par <- nlminb(c(1,1,1), objective = NLL.norm.lin, regr1 = D$ws30, regr2 = D$ws30^2, response =
```

```r
round( rbind(transfit12$coefficients,betas[1:3],norm.lin.par$par), digits=6)
```

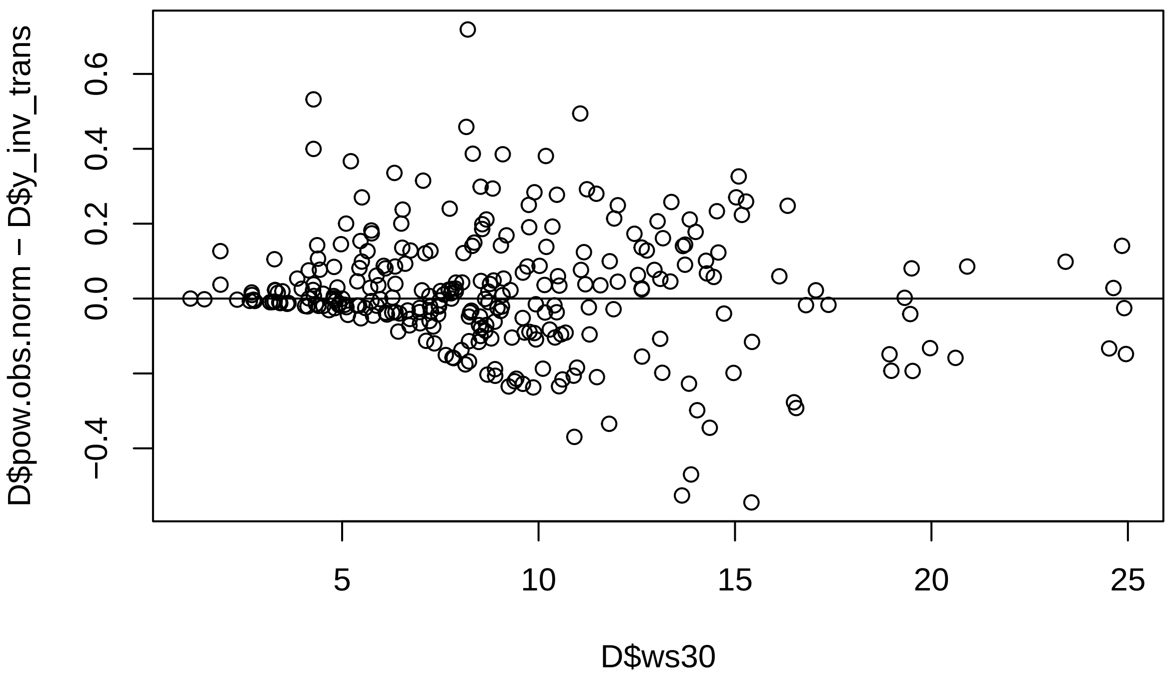
```
##       (Intercept)     ws30 I(ws30^2)
## [1,]   -7.450337 1.418150 -0.027717
## [2,]   -7.450337 1.418150 -0.027717
## [3,]   -7.450346 1.418151 -0.027717
```

```r
y_p <- D$transformed.pred12
#y_pS <- D$transformed.predS
D$y_inv_trans <- 1/( exp(y_p*lambda)+1 )^(1/lambda) * exp(y_p)
#D$Season_inv_trans <- 1/( exp(y_pS*lambda)+1 )^(1/lambda) * exp(y_pS)

#D$Season <- 0
#D$Season[D$month > 5] <- 1

ggplot(data = D)+
  geom_point(aes(x=ws30, y=pow.obs.norm))+
  geom_line(aes(x=ws30, y=y_inv_trans, colour="Inv"))+
  #geom_line(aes(x=ws30, y=Season_inv_trans, colour="InvS"))+
  labs(x = "Wind speeds", y="Norm power obs", colour = "")+
  theme_bw()
```




```r
  #scale_shape_manual(values = c(1:10))+
  #scale_colour_manual(values = c("blue", "yellow", "black"))
```

```r
plot(D$ws30, D$pow.obs.norm-D$y_inv_trans)
abline(h=0)
```



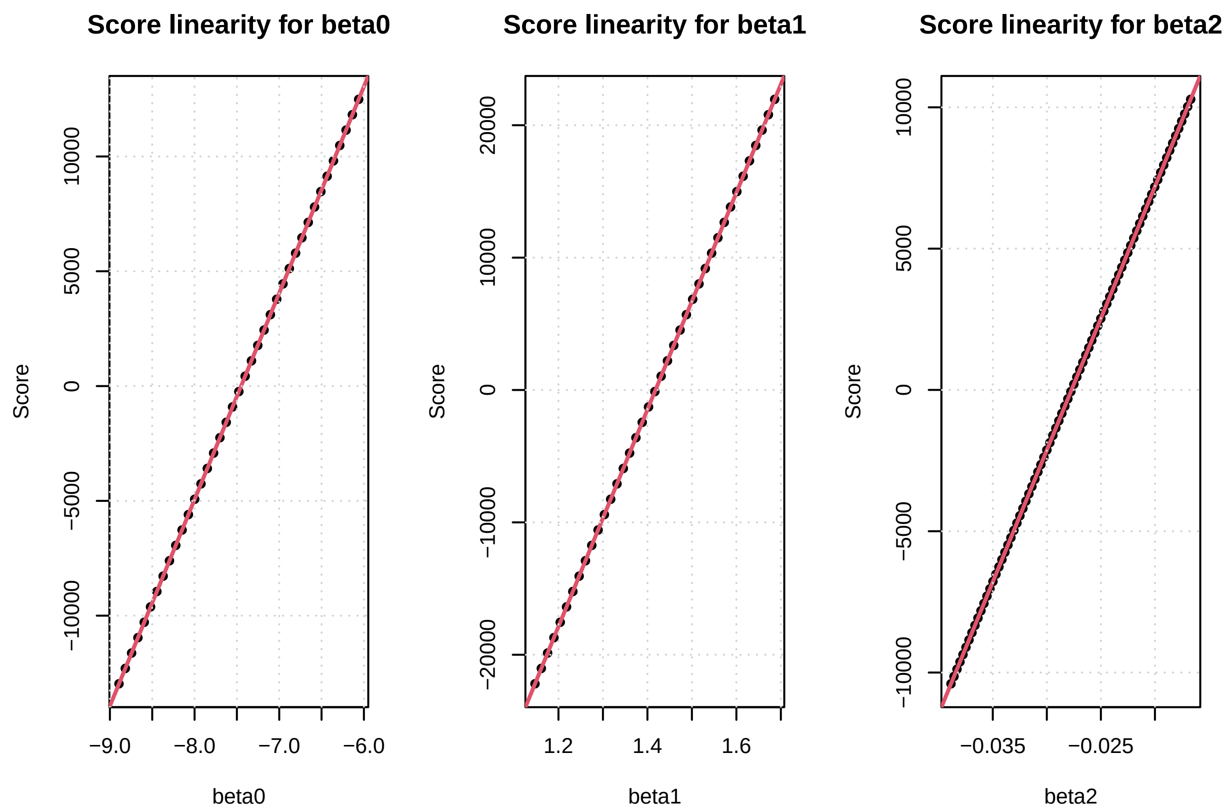
Uncertainties of parameters (in transformed domain)

```r
norm.lin.sds <- sqrt(diag(solve(hessian( func = NLL.norm.lin, x = norm.lin.par$par, regr1 = D$ws30, reg
Wald.CI.upper <- norm.lin.par$par + qnorm(0.975) * norm.lin.sds
Wald.CI.lower <- norm.lin.par$par - qnorm(0.975) * norm.lin.sds

cat("beta_0 = ", norm.lin.par$par[1], "95% CI [",Wald.CI.lower[1],", ",Wald.CI.upper[1],"]"
    ,"\nbeta_1 = ", norm.lin.par$par[2], "95% CI [",Wald.CI.lower[2],", ",Wald.CI.upper[2],"]"
    ,"\nbeta_2 = ", norm.lin.par$par[3], "95% CI [",Wald.CI.lower[3],", ",Wald.CI.upper[3],"]"
    )
```

```
## beta_0 =  -7.450346 95% CI [ -8.892549 ,  -6.008142 ]
## beta_1 =  1.418151 95% CI [ 1.147252 ,  1.68905 ]
## beta_2 =  -0.02771658 95% CI [ -0.03886543 ,  -0.01656773 ]
```

```r
#Test regularity
score.nll.norm <- function(theta, regr1, regr2, response, sigma){

  beta0 <- theta[1]
  beta1 <- theta[2]
  beta2 <- theta[3]
  n <- length(response)
```

```r
  theta <- beta0 + beta1 * regr1 + beta2 * regr2
  #sigma <- sqrt( 1/length(response) * sum( (response - theta)^2 ) )
  score <- n/sigma^2 * (mean(response) - theta)#p. 213
  return(-sum(score))
}

beta0.test.range <- seq(Wald.CI.lower[1], Wald.CI.upper[1], abs(0.01*norm.lin.par$par[1]))
beta1.test.range <- seq(Wald.CI.lower[2], Wald.CI.upper[2], abs(0.01*norm.lin.par$par[2]))
beta2.test.range <- seq(Wald.CI.lower[3], Wald.CI.upper[3], abs(0.01*norm.lin.par$par[3]))

sigma.s <- sqrt( 1/dim(D)[1] * sum ( (D$transformed.pow.obs.norm - ( norm.lin.par$par[1] + norm.lin.par$

res0 <- apply(cbind(beta0.test.range, norm.lin.par$par[2], norm.lin.par$par[3]), MARGIN = 1, FUN = score

res1 <- apply(cbind(norm.lin.par$par[1], beta1.test.range, norm.lin.par$par[3]), MARGIN = 1, FUN = score

res2 <- apply(cbind(norm.lin.par$par[1], norm.lin.par$par[2], beta2.test.range), MARGIN = 1, FUN = score

par(mfrow = c(1,3))
plot(beta0.test.range, res0, main = "Score linearity for beta0"
     ,xlab = "beta0", ylab = "Score", pch = 16)
abline(lm(res0 ~ beta0.test.range), col = 2, lwd = 2)
grid()

plot(beta1.test.range, res1, main = "Score linearity for beta1"
     ,xlab = "beta1", ylab = "Score", pch = 16)
abline(lm(res1 ~ beta1.test.range), col = 2, lwd = 2)
grid()

plot(beta2.test.range, res2, main = "Score linearity for beta2"
     ,xlab = "beta2", ylab = "Score", pch = 16)
abline(lm(res2 ~ beta2.test.range), col = 2, lwd = 2)
grid()
```



Interpretation of parameters

```
transfit12$coefficients
```

```
## (Intercept)        ws30   I(ws30^2)
## -7.45033702  1.41814978 -0.02771652
```

```
beta_0 <- transfit12$coefficients[1]
( offset_inv <- 1/( exp(beta_0*lambda)+1 )^(1/lambda) * exp(beta_0) )
```

```
##  (Intercept)
## 0.0003463096
```

The regression model used is a normal model with data transformation. The first parameter is the intercept with the y-axis and can be transformed back to the original domain yielding an offset value of 0.00035. This is as expected since there should be no power production at 0 wind speed. The other two parameters are slope coefficients. The first is positive and describes the relationship between the wind speed and the power production. According to this model it should increase linearly (in the transformed domain). The second coefficient is negative and implies a negative proportionality to the square of the wind speed (in the transformed domain).