A2 Project 2

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Projekt 2: Survival Data

Analysis of the Binary Data

Read the data Logistic.txt into R.

Fit a logistic regression model for the binary outcome AIDS="yes" versus AIDS="no" with the explanatory variable treatment with AZT (Yes, NO). Present the odds ratio for the effect of AZT on AIDS with 95% confidence interval and interpret the result in words

The logistic regression model is given by the likelihood function:

$$L(\theta) = \prod_{i} \left(\frac{\theta_i}{1 - \theta_i}\right)^{y_i} (1 - \theta_i)$$

Here θ is calculated as:

$$\theta_i = \frac{e^{\beta_0 + \beta_1 t_{AZT}}}{1 + e^{\beta_0 + \beta_1 t_{AZT}}}$$

```
nll.log <- function(theta, event, n, treatment){
  beta0 <- theta[1]
  beta1 <- theta[2]
  theta <- exp(beta0 + beta1 * treatment)/(1 + exp(beta0 + beta1 * treatment))
  nll <- -sum(log(theta)*event + log(1-theta)*(n - event))
  return(nll)</pre>
```

```
}
# For AZT treatment:
log.reg <- nlminb(start = c(1,1), objective = nll.log, event = log.data$AIDS_yes, n = log.data$n, treat
beta0 <- log.reg$par[1]</pre>
beta1 <- log.reg$par[2]</pre>
logistic \leftarrow data.frame("AZT" = c(rep(1,170), rep(0,168))
                                                      "AIDS_yes" = c(rep(c(1,0),c(25,170-25)), rep(c(1,0), c(44, 168-44))))
fit.glm <- glm(AIDS_yes ~ AZT, data = logistic, family = binomial)</pre>
print(cat(pasteO("with glm model: ", coef(fit.glm)
     ,"\nBy hand (according to slide 19 lect 4): "
    ,"\nbeta_0 = ", log.reg$par[1], ", beta_1 = ", log.reg$par[2])))
## with glm model: -1.03609193168383
## By hand (according to slide 19 lect 4):
## beta_0 = -1.03609192753506, beta_1 = -0.721765976832429 with glm model: -0.721765985868547
## By hand (according to slide 19 lect 4):
## beta_0 = -1.03609192753506, beta_1 = -0.721765976832429NULL
Calculate 95% wald confidence interval for \beta_1 and subsequently for the odds ratio. Hereafter we justify the
use of the Wald CI by examining whether the score function show linearity around the MLE (p. ).
sd \leftarrow sqrt(diag(solve(hessian(func = nll.log, x = c(beta0, beta1), event = log.data$AIDS_yes, n = log.dataa$AIDS_yes, n = log.dataaa_yes, n = log.dataa_yes, n = lo
sd.beta0 <- sd[1]</pre>
sd.beta1 <- sd[2]
beta0.wald.CI \leftarrow beta0 + c(-1,1)*dnorm(0.975)*sd.beta0
beta1.wald.CI \leftarrow beta1 + c(-1,1)*dnorm(0.975)*sd.beta1
#Test regularity
score.nll.log <- function(theta){</pre>
    event <- log.data$AIDS_yes
    n <- log.data$n
    treatment \leftarrow c(1,0)
    beta0 <- theta[1]</pre>
    beta1 <- theta[2]</pre>
    theta <- exp(beta0 + beta1 * treatment)/(1 + exp(beta0 + beta1 * treatment))
    score <- -sum(event/theta - (n - event)/(1 - theta))</pre>
    return(score)
}
beta0.test.range <- seq(beta0.wald.CI[1], beta0.wald.CI[2], abs(0.01*beta0))
```

beta1.test.range <- seq(beta1.wald.CI[1], beta1.wald.CI[2], abs(0.01*beta1))

Score linearity for β_0 Score linearity for β₁ 10 0 2 2 Score Score 0 0 5 -5 -10 -15 -1.04-0.72-1.08-1.00-0.78-0.66 β_0 β_1

Odds ratio = $exp(\beta_1) = 0.4858934$, 95% CI [0.4534386, 0.5206711]. Thus for the individuals receiving the AZT treatment, the odds of developing AIDS or dying is reduced by a factor 0.486 95% CI [0.45, 0.52].

Test the hypothesis of no effect of AZT on AIDS using:

- The likelihood ratio test
- The Wald test
- The score test

Likelihood ratio test Calculate the objective value based on no treatment effect:

Run the LRT (assuming regularity of the likelihoods (page 36)):

The log-ratio test show that the effect of the treatment is significant on a significance level of a

Wald test statistic Calculating the wald test statistic (p. 156, or p. 42 with $\theta_0 = 0$).

The wald test show that the effect of the treatment is significant on a significance level of alpha

```
## The wald test statistic = -2.589515
## p-value = 0.009611115
```

The score test Calculating the score function for the logistic regression model.

$$S(\theta) = \frac{1}{\theta} \sum_{i} y_i + \frac{1}{1+\theta} \sum_{i} 1 - y_i$$

```
#ikke færdig
score.nll.log(log.reg$par) / sqrt(solve(hessian(func = score.nll.log, x = log.reg$par)))
## [,1] [,2]
## [1,] NaN 2.628648e-05
## [2,] 2.628648e-05 NaN
```

Analysis of the survival time data

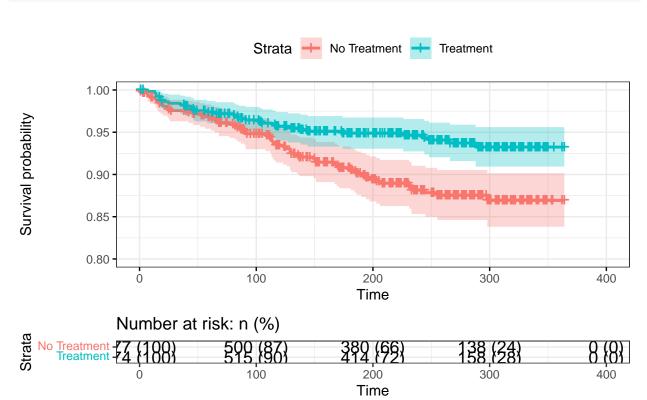
Descriptive statistics

Read the data actg320.txt into R Read in the data:

How many patients got AIDS or died in the two treatment groups? And how long was the total follow-up time in the two groups?

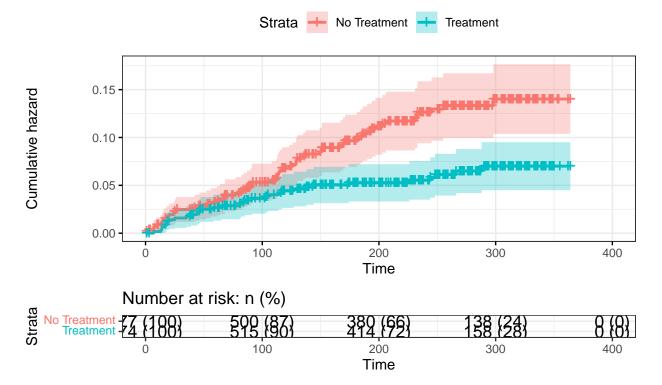
```
## # A tibble: 2 x 5
       tx `Got AIDS or DIED` Proportion `Participants Given the Treatment` Total~1
                                   <dbl>
##
   <int>
                        <int>
                                                                       <int>
                                                                               <int>
## 1
        0
                           63
                                  0.109
                                                                         577 128991
## 2
                           33
                                  0.0575
                                                                         574 135950
## # ... with abbreviated variable name 1: `Total follow up time`
```

Plot the survival functions in the two treatment groups, which group seems to be doing best? The treatment group seems to be doing the best (consider making this plot by hand instead of using survfit).



```
ggsurvplot(kaplan.meier
    , data = actg
    , conf.int = T
    , risk.table = "abs_pct"
    ,ggtheme = theme_bw()
    ,fun = "cumhaz"
    ,legend.labs = c("No Treatment", "Treatment"))
```

Plot the cumulative incidence functions for the two groups, which plot would you prefer?



Compare the survival in the two treatment groups using a log-rank test. See slides 31-33 from week 6 for calculations by hand.

```
survdiff(Surv(time, event) ~ tx, data = actg)
## Call:
## survdiff(formula = Surv(time, event) ~ tx, data = actg)
##
          N Observed Expected (O-E)^2/E (O-E)^2/V
##
## tx=0 577
                  63
                          47.1
                                    5.37
                                              10.5
                  33
                          48.9
##
  tx=1 574
                                    5.17
                                              10.5
##
    Chisq= 10.5 on 1 degrees of freedom, p= 0.001
```

p-value of 0.001, thus there is a significant difference between the two treatment groups based on a significance level of 0.05.

Parametric survival models

Fit parametric survival models containing treatment (tx) and CD4 count (cd4) as explanatory variables

• Try using the exponential, Weibull and log-logistic models, which one gave the best fit (and why)?

```
exp.model <- survreg(Surv(time, event) ~ tx + cd4, data = actg, dist = "exponential")
weibull.model <- survreg(Surv(time, event) ~ tx + cd4, data = actg, dist = "weibull")</pre>
loglogistic.model <- survreg(Surv(time, event) ~ tx + cd4, data = actg, dist = "loglogistic")</pre>
summary(exp.model)
##
## Call:
## survreg(formula = Surv(time, event) ~ tx + cd4, data = actg,
       dist = "exponential")
##
                 Value Std. Error
                                    Z
## (Intercept) 6.71473 0.15647 42.9 < 2e-16
               0.66680
                           0.21489 3.1 0.0019
               0.01609 0.00251 6.4 1.5e-10
## cd4
##
## Scale fixed at 1
##
## Exponential distribution
## Loglik(model) = -819.9 Loglik(intercept only) = -856.6
## Chisq= 73.36 on 2 degrees of freedom, p= 1.2e-16
## Number of Newton-Raphson Iterations: 7
## n= 1151
nll.exp <- function(theta</pre>
                     , time = actg$time
                     , event = actg$event
                     , treatment = actg$tx
                     , cd4 = actg$cd4){
  beta0 <- theta[1]
  beta1 <- theta[2]</pre>
  beta2 <- theta[3]</pre>
 h <- exp(- beta0 - beta1 * treatment - beta2 * cd4)
  H <- time/exp(beta0 + beta1 * treatment + beta2 * cd4)</pre>
 nll <- -sum(event*log(h) - H)</pre>
  return(nll)
}
exp.model.manual <- nlminb(start = c(1,1,1), objective = nll.exp)</pre>
sd.exp.manual <- sqrt(diag(solve(hessian(func = nll.exp, x = exp.model.manual$par))))</pre>
exp.model.manual.AIC <- - 2 * exp.model.manual$objective - 2 * 3</pre>
```

Weibull model Use: slide 10 for f(t), S(t), h(t) and H(t) relationships, slide 19 for general log-likelihood expression for censored data, slide 55 for S(t) and z, slide 60 for h(t).

```
summary(weibull.model)
```

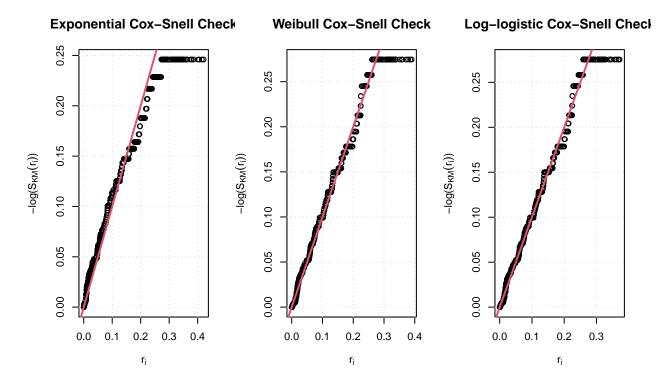
```
##
## Call:
## survreg(formula = Surv(time, event) ~ tx + cd4, data = actg,
## dist = "weibull")
```

```
Value Std. Error
                                    Z
0.84013
                         0.28582 2.94 0.0033
              0.02063
                         0.00379 5.44 5.3e-08
## cd4
## Log(scale) 0.24834
                         0.09715 2.56 0.0106
##
## Scale= 1.28
##
## Weibull distribution
## Loglik(model) = -816.3 Loglik(intercept only) = -852.9
## Chisq= 73.2 on 2 degrees of freedom, p= 1.3e-16
## Number of Newton-Raphson Iterations: 9
## n= 1151
nll.wei <- function(theta
                   , time = actg$time
                    , event = actg$event
                    , treatment = actg$tx
                    , cd4 = actg$cd4){
 sigma <- exp(theta[1]) #estimate scale parameter on log-scale</pre>
 beta0 <- theta[2]
 beta1 <- theta[3]
 beta2 <- theta[4]
 h < -1/sigma * time^(1/sigma - 1) * exp(-1/sigma * (beta0 + beta1 * treatment + beta2 * cd4))
 H <- time^(1/sigma) * exp(-1/sigma * (beta0 + beta1 * treatment + beta2 * cd4))
 nll <- -sum(event*log(h) - H)</pre>
 return(nll)
}
weibull.model.manual \leftarrow nlminb(start = c(1,1,1,1), objective = nll.wei)
sd.weibull.manual <- sqrt(diag(solve(hessian(func = nll.wei, x = weibull.model.manual$par))))</pre>
weibull.model.manual.AIC <- 2 * weibull.model.manual$objective + 2 * length(weibull.model.manual$par) #
summary(loglogistic.model)
##
## Call:
## survreg(formula = Surv(time, event) ~ tx + cd4, data = actg,
      dist = "loglogistic")
##
                Value Std. Error
                                     Z
                         0.25453 26.82 < 2e-16
## (Intercept) 6.82584
## tx
              0.84295
                         0.28980 2.91 0.0036
              0.02080
                         0.00375 5.55 2.9e-08
## Log(scale) 0.20259
                         0.09558 2.12 0.0340
## Scale= 1.22
##
## Log logistic distribution
## Loglik(model) = -815.8
                        Loglik(intercept only) = -852.7
## Chisq= 73.73 on 2 degrees of freedom, p= 9.8e-17
## Number of Newton-Raphson Iterations: 6
## n= 1151
```

```
nll.loglog <- function(theta</pre>
                       , time = actg$time
                       , event = actg$event
                       , treatment = actg$tx
                       , cd4 = actg$cd4){
  sigma <- theta[1]</pre>
  beta0 <- theta[2]</pre>
  beta1 <- theta[3]</pre>
  beta2 <- theta[4]
  z <- (log(time) - (beta0 + beta1 * treatment + beta2 * cd4))/sigma
  h \leftarrow \exp(z)/(1 + \exp(z)) * 1/\text{sigma} * 1/\text{time}
  S \leftarrow 1 / (1 + \exp(z))
  H \leftarrow -log(S)
  \#H \leftarrow time^{(1/sigma)} * exp(-1/sigma * (beta0 + beta1 * treatment + beta2 * cd4))
  nll <- -sum(event*log(h) - H)</pre>
  return(nll)
}
loglogistic.model.manual <- nlminb(start = c(1,1,1,1), objective = nll.loglog)</pre>
sd.loglogistic.manual <- sqrt(diag(solve(hessian(func = nll.loglog, x = loglogistic.model.manual$par)))
loglogistic.model.manual.AIC <- 2 * loglogistic.model.manual$objective + 2 * length(loglogistic.model.m
```

Examining the fits of the models through the cox-snell residuals

```
par(mfrow = c(1,3))
actg$cox.snell.exp <- actg$time * exp(- exp.model$linear.predictors)</pre>
fit.exp <- survfit(Surv(cox.snell.exp, event == 1) ~ 1, data = actg)</pre>
plot(fit.exp$time, -log(fit.exp$surv), main = "Exponential Cox-Snell Check"
     ,xlab = TeX("$r i$")
     ,ylab = TeX("-log(\$S_{KM}(r_i)\$)"))
grid()
abline(a=0, b=1, col = 2, lwd = 2)
#slide 51 week 7 for weibull cox-snell
beta <- t(t(weibull.model.manual$par[2:4]))</pre>
x <- as.matrix(cbind(1, actg$tx, actg$cd4))</pre>
y.pred <- x %*% beta
y <- log(actg$time)
sigma <- exp(weibull.model.manual$par[1])</pre>
actg$r <- exp((y - y.pred)/sigma)</pre>
#slide 35 week 7 for the plot. [r_i, log(S_KM(r_i))], should yield a linear relationship with slope = 1
fit.wei <- survfit(Surv(r, event == 1) ~ 1, data = actg)</pre>
#the output from the survfit function: time = r_i (as we input r_i as time).
#the 'surv' output is the kaplan meier survival function of r_i:
plot(fit.wei$time, -log(fit.wei$surv), main = "Weibull Cox-Snell Check"
     ,xlab = TeX("$r_i$")
     ,ylab = TeX("-log($S_{KM}(r_i)$)"))
grid()
abline(a=0, b=1, col = 2, lwd = 2)
```



Weibull and log-logistic seem quite similar. For both we still see a heavy tail, which is not accounted for in the model.

Use AIC to compare the models (exp excluded due to poor cox-snell fit):

```
## Weibull Regression Model AIC: 1640.671
## Log-logistic Regression Model AIC: 1639.655
```

Here, the loglog model has the lower AIC, however we stick to the Weibull regression model as both these models have scale > 1, and thus show that the survival curve behaviour is monotonously decrease, and as such the loglog model has no modelwise advantage as compared to the weibull model. (Dobbelttjek dette)

```
weibull.upper <- as.numeric(weibull.model.manual$par + qnorm(0.975)*sd.weibull.manual)
weibull.lower <- as.numeric(weibull.model.manual$par - qnorm(0.975)*sd.weibull.manual)</pre>
```

```
cat("beta_0 = ", weibull.model.manual$par[2], "95% CI [",weibull.lower[2],", ",weibull.upper[2],"]"
   ,"\nbeta_1 = ", weibull.model.manual$par[3], "95% CI [",weibull.lower[3],", ",weibull.upper[3],"]"
   ,"\nbeta_2 = ", weibull.model.manual$par[4], "95% CI [",weibull.lower[4],", ",weibull.upper[4],"]"
   ,"\nscale = ", exp(weibull.model.manual$par[1]), "95% CI [",exp(weibull.lower[1]),", ",exp(weibull.")
```

Using the survival model you chose, make a table of estimates and their 95% confidence intervals

```
## beta_0 = 7.05743 95% CI [ 6.562829 , 7.55203 ]
## beta_1 = 0.8401271 95% CI [ 0.2799285 , 1.400326 ]
## beta_2 = 0.02063062 95% CI [ 0.01319748 , 0.02806376 ]
## scale = 1.281897 95% CI [ 1.059638 , 1.550775 ]
```

Table format:

```
tibble("parameter" = c("beta_0", "beta_1", "beta_2", "sigma")
   , "theta hat" = c(weibull.model.manual$par[2:4], exp(weibull.model.manual$par[1]))
   , "lower CI (2.5%)" = c(weibull.lower[2:4], exp(weibull.lower[1]))
   , "upper CI (97.5%)" = c(weibull.upper[2:4], exp(weibull.upper[1])))
```

```
## # A tibble: 4 x 4
    parameter `theta hat` `lower CI (2.5%)` `upper CI (97.5%)`
##
     <chr>
                    <dbl>
                                       <dbl>
                                                          <dbl>
## 1 beta_0
                    7.06
                                      6.56
                                                         7.55
## 2 beta_1
                    0.840
                                      0.280
                                                         1.40
## 3 beta_2
                    0.0206
                                      0.0132
                                                         0.0281
## 4 sigma
                    1.28
                                      1.06
                                                         1.55
```

 θ does not span 1, and thus the estimated weibull distribution is significantly different from simply using an exponential distribution.

Double-check whether the regularity assumption which is necessary for the use of Wald confidence intervals is appropriate, by examining the linearity of the score function at the MLE estimates:

Using your model compute the time ratio for the treatment effect. Similarly, compute the time ratio for the effect of increasing the CD4 count with 50. In both cases unceartainty evaluation (e.g. confidence intervals) should be included. Interpret the results in words For the treatment effect:

```
beta1 <- weibull.model.manual$par[3]
cat("Time Ratio for the treatment effect: TR(tx = 1, tx = 0) = ", exp(beta1), " 95% CI [",exp(c(weibull</pre>
```

Time Ratio for the treatment effect: TR(tx = 1, tx = 0) = 2.316661 95% CI [1.323035 , 4.056521]

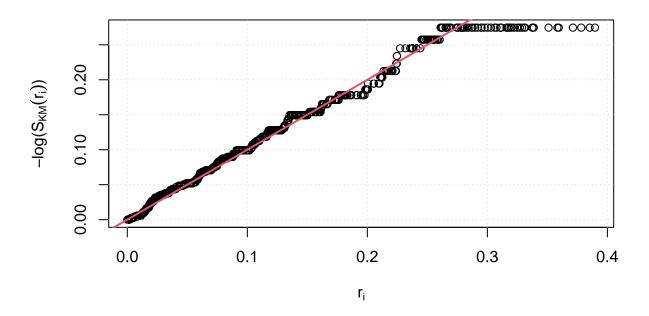
Based on this calculation of the time ratio of the treatment effect, it is evident that the treatment increase the median survival time by a factor 2.32 95% CI [1.32; 4.06].

For increasing the cd4 count by 50:

Here we see that by increasing the cd4 count by 50, the median survival time is increased by a factor 2.8 95% CI [1.93,4.07].

Assess the goodness of fit of this model using a plot based on the Cox Snell residuals This plot was already made earlier, but here it is again.

Weibull Cox-Snell Check

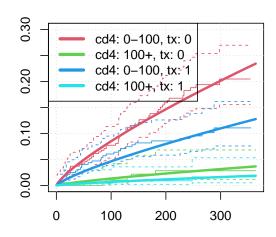


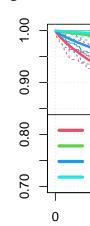
```
actg$cd4.cats <- cut(actg$cd4, breaks = c(0,100,max(actg$cd4)))
survival.functions <- survfit(Surv(time, event) ~ tx + cd4.cats, data = actg)
actg$cd4.category.num <- 0</pre>
```

```
actg$cd4.category.num[actg$cd4 < 100] <- 1</pre>
actg$cd4.category.num[actg$cd4 >= 100] <- 2</pre>
wei.survival <- function(theta, time, tx, cd4){</pre>
  beta0 <- theta[1]</pre>
  beta1 <- theta[2]</pre>
  beta2 <- theta[3]</pre>
  scale <- theta[4]</pre>
  out <- exp(-(time/exp(beta0 + beta1*tx + beta2*cd4))^(1/scale))</pre>
  return(out)
}
#for fixed values
step.size <- 0.5
theta <- c(weibull.model$coefficients,weibull.model$scale)</pre>
time.steps <- seq(0.5,max(actg$time), step.size) #lader den lige starte i 0.5 for at undgå bøvl med h.c
first.sim <- wei.survival(theta, time.steps, 0, median(actg$cd4[actg$cd4 < 100 & actg$tx == 0]))
second.sim <- wei.survival(theta, time.steps, 0, median(actg$cd4[actg$cd4 > 100 & actg$tx == 0]))
third.sim <- wei.survival(theta, time.steps, 1, median(actg$cd4[actg$cd4 < 100 & actg$tx == 1]))
fourth.sim <- wei.survival(theta, time.steps, 1, median(actg$cd4[actg$cd4 > 100 & actg$tx == 1]))
\#wei.survival(c(weibull.model\$coefficients, weibull.model\$scale), actg\$time, actg\$tx, actg\$cd4)
par(mfrow = c(1,2))
plot(survival.functions, cumhaz = T, conf.int = T, col = 2:5, ylim = c(0,0.3), lwd = 1, main = "Weibull
f.weibull.t <- function(t, theta, x, scale){</pre>
  shape <- as.vector(theta %*% x)</pre>
  out <- shape/scale * (t/scale)^(shape-1) * exp(-(t/scale)^shape)</pre>
  return(out)
}
h.weibull.t <- function(t, theta, treatment, cd4, step.size){
  sigma <- exp(theta[1])</pre>
  beta0 <- theta[2]</pre>
  beta1 <- theta[3]</pre>
  beta2 <- theta[4]
  out <- cumsum(1/sigma * t^(1/sigma - 1) * exp(-1/sigma * (beta0 + beta1 * treatment + beta2 * cd4))*s
  return(out)
}
lines(time.steps, h.weibull.t(time.steps, weibull.model.manual$par, 0, median(actg$cd4[actg$cd4 < 100 &
lines(time.steps, h.weibull.t(time.steps, weibull.model.manual$par, 0, median(actg$cd4[actg$cd4 > 100 &
lines(time.steps, h.weibull.t(time.steps, weibull.model.manual$par, 1, median(actg$cd4[actg$cd4 < 100 &
lines(time.steps, h.weibull.t(time.steps, weibull.model.manual$par, 1, median(actg$cd4[actg$cd4 > 100 &
legend(legend = c("cd4: 0-100, tx: 0", "cd4: 100+, tx: 0", "cd4: 0-100, tx: 1", "cd4: 100+, tx: 1"), lt
plot(survival.functions, conf.int = T, col = 2:5, ylim = c(1,0.7), lwd = 1, main = "Weibull Regression I
grid()
lines(time.steps, first.sim, col = 2 , lwd = 3)
lines(time.steps, second.sim, col = 3, lwd = 3)
```

```
lines(time.steps, third.sim, col = 4 , lwd = 3)
lines(time.steps, fourth.sim, col = 5, lwd = 3)
legend(legend = c("cd4: 0-100, tx: 0", "cd4: 100+, tx: 0", "cd4: 0-100, tx: 1", "cd4: 100+, tx: 1"), lt
```

Regression Models and Kaplan Meier Cumul Regression Mo





Give a graphical presentation of your model