

Written examination: 3. June 2024

Course name and number: **Introduction to Statistics (02402)**

Duration: 4 hours

Aids and facilities allowed: All

The questions were answered by

\_\_\_\_\_  
(student number)

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(signature)

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(table number)

This exam consists of 30 questions of the “multiple choice” type, which are divided between 14 exercises. To answer the questions, you need to fill in the “multiple choice” form on exam.dtu.dk.

5 points are given for a correct “multiple choice” answer, and  $-1$  point is given for a wrong answer. ONLY the following 5 answer options are valid: 1, 2, 3, 4, or 5. If a question is left blank or an invalid answer is entered, 0 points are given for the question. Furthermore, if more than one answer option is selected for a single question, which is in fact technically possible in the online system, 0 points are given for the question. The number of points needed to obtain a specific mark or to pass the exam is ultimately determined during censoring.

**The final answers should be given by filling in and submitting the form.**  
**The table provided here is ONLY an emergency alternative.**  
**Remember to provide your student number if you do hand in on paper.**

<b>Exercise</b>	I.1	I.2	II.1	III.1	III.2	IV.1	IV.2	V.1	V.2	V.3
<b>Question</b>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<b>Answer</b>										

<b>Exercise</b>	VI.1	VI.2	VII.1	VIII.1	VIII.2	VIII.3	IX.1	IX.2	X.1	XI.1
<b>Question</b>	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
<b>Answer</b>										

<b>Exercise</b>	XI.2	XI.3	XII.1	XII.2	XIII.1	XIII.2	XIII.3	XIV.1	XIV.2	XIV.3
<b>Question</b>	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
<b>Answer</b>										

The exam paper contains 24 pages.

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**Multiple choice questions:** Note that in each question, one and only one of the answer options is correct. Furthermore, not all the suggested answers are necessarily meaningful. Always remember to round your own result to the number of decimals given in the answer options before you choose your answer. Also remember that there may be slight discrepancies between the result of the book's formulas and corresponding built-in functions in R.

### Exercise I

Consider the following hypothesis test:

$$H_0 : \mu = 18$$

$$H_1 : \mu \neq 18$$

A sample with  $n = 48$  observations provided a sample mean  $\bar{x} = 17$  and a sample standard deviation  $s = 4.5$ . It is assumed that the sample is collected from a normal distributed population.

#### Question I.1 (1)

What is the value of the test statistic ( $t_{\text{obs}}$ ) and what are the critical values with significance level  $\alpha = 0.05$  (both answers must be correct)?

- 1 ☐  $t_{\text{obs}} = -1.54$ .  $t_{0.025} = -2.012$  and  $t_{0.975} = 2.012$
- 2 ☐  $t_{\text{obs}} = -2.01$ .  $t_{0.025} = -2.685$  and  $t_{0.975} = 2.685$
- 3 ☐  $t_{\text{obs}} = -1.96$ .  $t_{0.025} = -1.678$  and  $t_{0.975} = 1.678$
- 4 ☐  $t_{\text{obs}} = -1.89$ .  $t_{0.025} = -2.012$  and  $t_{0.975} = 2.012$
- 5 ☐ Cannot be calculated without more information.

#### Question I.2 (2)

From another sample of  $n = 45$  observations, the value of the test statistic ( $t_{\text{obs}}$ ) is -1.74. Compute the  $p$ -value and draw a conclusion using significance level  $\alpha = 0.05$  (both must be correct).

- 1 ☐ The  $p$ -value is 0.0889. We do not reject the null hypothesis.
- 2 ☐ The  $p$ -value is 0.0805. We reject the null hypothesis.
- 3 ☐ The  $p$ -value is 0.0560. We do not reject the null hypothesis.

4 ☐ The  $p$ -value is 0.1339. We do not reject the null hypothesis.

5 ☐ We cannot draw a conclusion.

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<b>Exercise II</b>
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A random sample of  $n = 30$  observations is collected and the sample mean is estimated to be  $\bar{x} = 1.01$  and the sample standard deviation  $s = 0.09$ .

**Question II.1 (3)**

What is the 95% confidence interval for the standard deviation?

- 1 ☐ [0.06, 0.13]
- 2 ☐ [0.24, 0.40]
- 3 ☐ [0.09, 0.13]
- 4 ☐ [0.05, 0.09]
- 5 ☐ [0.072, 0.121]

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**Exercise III**

The outcome of an experiment is described by the random variable  $X$ , where  $X$  has the following density function:

$x$	0	1	2	3	4
$f(x)$	0.17	0.22	0.28	0	0.33

**Question III.1 (4)**

What is the probability  $P(X < 3)$

- 1 ☐ This cannot be deduced from the information provided
- 2 ☐ 0.28
- 3 ☐ 0.33
- 4 ☐ 0.40
- 5 ☐ 0.67

**Question III.2 (5)**

The mean of  $X$  is 2.10. What is the variance of  $X$ ?

- 1 ☐ 1.49
- 2 ☐ 2.10
- 3 ☐ 2.21
- 4 ☐ 4
- 5 ☐ 6.62

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**Exercise IV**

A survey has been conducted to assess the concentration (we shall disregard the unit) of a chemical compound in the soil in four different locations. The measured concentrations from the survey are given in the below table:

Location 1	Location 2	Location 3	Location 4
253.7	261.1	257.9	244.1
241.2	244.2	263.5	244.9
255.8	250.5	258.6	243.9
249.3	264.9		247.1
	259.3		

The overall average concentration found across all four locations is 252.5 and the average concentrations for each location are listed below:

Location	1	2	3	4
Average concentration	250.0	256.0	260.0	245.0

We then fit a one-way ANOVA model on the form

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij},$$

where the errors  $\varepsilon_{ij}$  are independent and follow normal distributions with mean 0 and variance  $\sigma^2$ . The overall mean is  $\mu$  and the effect of Location  $i$  is  $\alpha_i$ . The ANOVA table associated with the model shows that the total sum of squares,  $SST$ , is 915.92 and the location sum of squares,  $SS(\text{Location})$ , is 480.00.

**Question IV.1 (6)**

What is the estimate of the effect of Location 1?

- 1 ☐ -2.5
- 2 ☐ 2.5
- 3 ☐ 6.0
- 4 ☐ 250.0
- 5 ☐ 252.5

**Question IV.2 (7)**

What is the estimate of the error standard deviation?

1 ☐  $\hat{\sigma} = 6.027$

2 ☐  $\hat{\sigma} = 6.325$

3 ☐  $\hat{\sigma} = 7.814$

4 ☐  $\hat{\sigma} = 20.879$

5 ☐  $\hat{\sigma} = 36.327$

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### Exercise V

12 observations of perfluorooctanesulfonic acid (PFOS) at six different concentrations were analysed using a new experimental testing method SPETT.

PFOS concentrations are measured in mg/kg. The data was read into R by:

```
# PFOS concentrations
x <- c(0, 0, 2, 2, 4, 4, 6, 6, 8, 8, 12, 12)
# SPETT values
y <- c(16, 116, 1170, 841, 2287, 2012, 2653, 3333, 4270, 3999, 5750, 5407)
```

The linear regression model

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \text{ where } \varepsilon_i \sim N(0, \sigma^2) \text{ and i.i.d. for } i = 1, \dots, 12,$$

was set up, where  $Y_i$  is the SPETT value and  $x_i$  the PFOS concentration of the  $i$ 'th observation.

Note that in the remaining of the exercise, the normal distribution and i.i.d. assumptions of the errors are implicit (hence not written with the model).

#### Question V.1 (8)

What is the estimate of  $\beta_1$ ?

- 1 ☐ 160.7
- 2 ☐ 467.6
- 3 ☐ 511.0
- 4 ☐ 1020.7
- 5 ☐ 259.0

#### Question V.2 (9)

We wish to test the hypothesis  $H_0 : \beta_0 = 0$ , as this would indicate whether the expected SPETT value is zero for a PFOS concentration of zero. We use a significance level of  $\alpha = 0.05$ .

Which of the following statements is correct?

- 1 ☐ We do not reject the null hypothesis, since  $|\hat{\beta}_0| > 1.96$ , where 1.96 is the 95% quantile in a normal distribution.



- 2 ☐ We do not reject the null hypothesis, since the  $p$ -value is 0.23.
- 3 ☐ We do not reject the null hypothesis, since the  $p$ -value is 0.0027.
- 4 ☐ We reject the null hypothesis, since the  $p$ -value is 0.0027.
- 5 ☐ We reject the null hypothesis, since the  $p$ -value is 0.23.

**Question V.3 (10)**

Researchers would like to know the uncertainty of the SPETT value for a new observation with PFOS concentration 7 mg/kg. What is the 95% prediction interval at this concentration?

- 1 ☐ [2829, 4039]
- 2 ☐ [3253, 3615]
- 3 ☐ [487, 522]
- 4 ☐ [1898, 3099]
- 5 ☐ [2653, 4270]

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### Exercise VI

In a high quality paper factory the production occurs in batches. In a quality check experiment, batches were randomly selected, i.e. the quality is independent between the selected batches. In the first quality check 20 out of 85 batches were found to not live up to the quality requirement.

#### Question VI.1 (11)

What is the confidence interval calculated at a significance level  $\alpha = 10\%$  for the proportion not living up to the quality requirements (note, use formula to get the correct answer)?

- 1 ☐ [0.084, 0.387]
- 2 ☐ [0.128, 0.342]
- 3 ☐ [0.153, 0.342]
- 4 ☐ [0.160, 0.311]
- 5 ☐ [0.176, 0.294]

#### Question VI.2 (12)

A new quality check is planned. The factory wants a 95% confidence interval with an expected width of 0.1. As a guess of the population proportion, the observed proportion from the first quality check is used (i.e. 20 out of 85 batches).

What is the minimum number of batches which must be taken for the new check to achieve this accuracy?

- 1 ☐  $n = 18$
- 2 ☐  $n = 70$
- 3 ☐  $n = 195$
- 4 ☐  $n = 277$
- 5 ☐  $n = 385$

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## Exercise VII

At a Danish company, a study was designed to investigate whether the online training modules for learning the concepts of sustainability improve trainees knowledge. The trainees participated in a quiz before and after completing the activities of the online training module. The two quizzes are referred to as pre-test and post-test. Six trainees' scores in the pre-test (denoted **pre**) and post-test (denoted **post**) are shown in the table below.

Trainee	1	2	3	4	5	6
pre	41	46	35	49	33	42
post	42	47	43	55	28	49

The following code is now run in R for input data:

```
pre <- c(41, 46, 35, 49, 33, 42)
post <- c(42, 47, 43, 55, 28, 49)
```

### Question VII.1 (13)

Assuming the samples are from normal distributed populations and the improvement is measured by a difference in the mean of the populations, which of the following codes will result in the desired hypothesis test after reading in the data?

1 ☐ `t.test(mean(pre), mean(post))`

2 ☐ `t.test(post, pre, var.equal=TRUE)`

3 ☐ `t.test(pre, post)`

4 ☐ `t.test(pre-post)`

5 ☐ `t.test(sd(pre), sd(post))`

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### Exercise VIII

It is assumed that there are 2.5 forest fires on average on a hot summer day, and that the number of forest fires per day follows a Poisson distribution.

#### Question VIII.1 (14)

What is the probability that there are at least five forest fires on a hot summer day?

- 1 ☐ 0.004
- 2 ☐ 0.067
- 3 ☐ 0.109
- 4 ☐ 0.175
- 5 ☐ 0.762

#### Question VIII.2 (15)

Let the random variable  $X$  represent the number of forest fires in a period of seven consecutive hot summer days. Which distribution does  $X$  follow?

- 1 ☐ A Poisson distribution with mean 17.5.
- 2 ☐ A binomial distribution with  $n = 49$  and  $p \approx 0.36$ .
- 3 ☐ A Normal distribution with mean 17.5 and standard deviation 7.
- 4 ☐ An exponential distribution with rate  $\approx 0.57$ .
- 5 ☐ None of the above.

#### Question VIII.3 (16)

It is estimated that 78% of forest fires could have been avoided if simple precautions had been met. If the fire department one day reports five forest fires, what is the probability that all of these forest fires could have been avoided if the simple precautions had been met?

- 1 ☐ 0.0005
- 2 ☐ 0.156
- 3 ☐ 0.175

$$4 \square 0.289$$

$$5 \square 0.711$$

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### Exercise IX

30 butterflies were captured and wing lengths were measured in cm. Some summary statistics of the sample (stored in `length_cm`) are shown below.

```
round(summary(length_cm),2)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##      3.06   3.72   3.93   4.03   4.32   5.37 

round(sd(length_cm),2)

## [1] 0.49
```

#### Question IX.1 (17)

Perform parametric bootstrapping and simulate 10000 samples assuming that the population from which the sample was taken follows a normal distribution. Which of the following is the correct 99% confidence interval for the median of the wing length?

HINT: If you do the "default" coding from the book, you will get the correct result with `set.seed(2023)`.

- 1 ☐ [3.46 cm, 3.93 cm]
- 2 ☐ [3.38 cm, 4.00 cm]
- 3 ☐ [2.81 cm, 5.48 cm]
- 4 ☐ [3.76 cm, 4.31 cm]
- 5 ☐ [3.90 cm, 4.17 cm]

#### Question IX.2 (18)

The following code was run:

```
median_cm <- 3.93
mean_cm <- 4.03
sd_cm <- 0.49
n <- length(length_cm)
k <- 10000
```

Which of the following answers makes use of non-parametric bootstrapping to estimate the 95% confidence interval for the first quartile (25th percentile) of the wing lengths in cm?

1 ☐

```
fun <- function(x) quantile(x, 0.25, type=2)
sim_samples <- replicate(k, sample(length_cm, n, replace = TRUE))
sim_stats <- apply(sim_samples, 2, fun)
quantile(sim_stats, c(0.005, 0.995))
```

2 ☐

```
fun <- function(x) quantile(x, 0.25, type=2)
sim_samples <- replicate(k, rnorm(n, mean_cm, sd_cm))
sim_stats <- apply(sim_samples, 2, fun)
quantile(sim_stats, c(0.025, 0.975))
```

3 ☐

```
fun <- function(x) quantile(x, 0.75, type=2)
sim_samples <- replicate(k, sample(length_cm, n, replace = TRUE))
sim_stats <- apply(sim_samples, 2, fun)
quantile(sim_stats, c(0.01, 0.99))
```

4 ☐

```
fun <- function(x) quantile(x, 0.75, type=2)
sim_samples <- replicate(k, rnorm(n, mean_cm, sd_cm))
sim_stats <- apply(sim_samples, 2, fun)
quantile(sim_stats, c(0.01, 0.99))
```

5 ☐

```
fun <- function(x) quantile(x, 0.25, type=2)
sim_samples <- replicate(k, sample(length_cm, n, replace = TRUE))
sim_stats <- apply(sim_samples, 2, fun)
quantile(sim_stats, c(0.025, 0.975))
```

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### Exercise X

A study involves two populations: a population of systems analysts using a current technology and a population of systems analysts using a new software package. The assumption of normal distributed population is fulfilled for both populations. In terms of the time required to complete a system design project, the population means are as follows:

$\mu_1$  is the mean project completion time for systems analysts using the current technology.

$\mu_2$  is the mean project completion time for systems analysts using the new software package.

The researcher in charge of the new software package would like to know if the new software package will provide a shorter mean project completion time. The hypothesis which must be tested is

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 - \mu_2 \neq 0$$

The following summary statistics are provided:

- Sample sizes:  $n_1 = 12$ ,  $n_2 = 12$
- Sample means:  $\bar{x}_1 = 325$  hours,  $\bar{x}_2 = 286$  hours
- Sample standard deviations:  $s_1 = 40$ ,  $s_2 = 44$

### Question X.1 (19)

What is value of the  $t$ -test statistic ( $t_{obs}$ ) and the degree of freedom (both answers must be correct)?

1 ☐  $t_{obs} = 1.80$  and  $df = 16.8$

2 ☐  $t_{obs} = 2.27$  and  $df = 21.8$

3 ☐  $t_{obs} = 2.91$  and  $df = 20.8$

4 ☐  $t_{obs} = 3.12$  and  $df = 19.8$

5 ☐  $t_{obs} = 2.02$  and  $df = 21.8$

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### Exercise XI

In a laboratory, one wants to dilute a stock solution A of concentration  $C_A$  to obtain a solution B of concentration  $C_B$ . The following rule is applied:

$$C_B = \frac{C_A V_A}{V_B}$$

Here,  $V_A$  and  $V_B$  represent the volumes of the initial stock solution A and the resulting solution B, respectively.  $C_A$ ,  $C_B$ ,  $V_A$  and  $V_B$  are all random variables.

#### Question XI.1 (20)

Which of the following formulas can be utilized to approximate the standard deviation of solution B's concentration ( $\sigma_{C_B}$ ) via the non-linear error propagation rule?

- 1 ☐  $\sigma_{C_B} = \sqrt{\frac{\left(\frac{\partial C_B}{\partial C_A} \sigma_{C_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_A} \sigma_{V_A}\right)^2}{\left(\frac{\partial C_B}{\partial V_B} \sigma_{V_B}\right)^2}}$
- 2 ☐  $\sigma_{C_B} = \left(\frac{\partial C_B}{\partial C_A} \sigma_{C_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_A} \sigma_{V_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_B} \sigma_{V_B}\right)^2$
- 3 ☐  $\sigma_{C_B} = \sqrt{\left(\frac{\partial C_B}{\partial C_A} \sigma_{C_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_A} \sigma_{V_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_B} \sigma_{V_B}\right)^2}$
- 4 ☐  $\sigma_{C_B} = \frac{\left(\frac{\partial C_B}{\partial C_A} \sigma_{C_A}\right)^2 + \left(\frac{\partial C_B}{\partial V_A} \sigma_{V_A}\right)^2}{\left(\frac{\partial C_B}{\partial V_B} \sigma_{V_B}\right)^2}$
- 5 ☐  $\sigma_{C_B} = \left(\frac{\partial C_B}{\partial C_A} \sigma_{C_A}\right) + \left(\frac{\partial C_B}{\partial V_A} \sigma_{V_A}\right) + \left(\frac{\partial C_B}{\partial V_B} \sigma_{V_B}\right)$

#### Question XI.2 (21)

Let  $X_i$  be a random variable. The following code is run in R to draw  $k$  realizations of  $X_i$  from a distribution:

```
x <- rnorm(k)^2 + rnorm(k)^2
```

Which of the following statements is correct?

- 1 ☐  $X_i$  follows a  $\chi^2$ -distribution with 1 degree of freedom, and thus has  $q_{0.25} = 0.102$ .
- 2 ☐  $X_i$  follows a standard normal distribution with mean 0 and variance 1, and thus has  $q_{0.25} = -0.674$ .

- 3 ☐  $X_i$  follows a  $\chi^2$ -distribution with 2 degrees of freedom, and thus has  $q_{0.25} = 0.575$ .
- 4 ☐  $X_i$  follows a  $\chi^2$ -distribution with 3 degrees of freedom, and thus has  $q_{0.25} = 1.213$ .
- 5 ☐  $X_i$  follows a normal distribution with mean 0 and variance 3, and thus has  $q_{0.25} = -1.168$ .

**Question XI.3 (22)**

Which of the following statements is NOT true?

- 1 ☐ Non-parametric bootstrapping is a re-sampling technique used to estimate the variability of a parameter without making any assumptions about the underlying population.
- 2 ☐ Non-parametric bootstrapping involves repeatedly sampling with replacement from the original sample to create many new, simulated samples of the same size as the original sample.
- 3 ☐ Non-parametric bootstrapping should be preferred over parametric bootstrapping if you know the distribution of the population.
- 4 ☐ Non-parametric bootstrapping can be applied to estimate the 95% confidence interval of a sample mean.
- 5 ☐ Non-parametric bootstrapping can be used to estimate confidence intervals for a parameter and to test hypotheses.

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## Exercise XII

A certain industrial process depends on the pH value and the amount of catalyst in a solution. The relationship between output per hour (kg/h) (**efficiency**), pH (**ph**), and the amount of catalyst used (**catalyst**) is to be investigated using the following multiple linear regression model

$$\text{efficiency}_i = \beta_0 + \beta_1 \cdot \text{ph}_i + \beta_2 \cdot \text{catalyst}_i + \varepsilon_i,$$

where the  $\varepsilon_i$  are independent and  $N(0, \sigma^2)$ -distributed. R output from fitting the model to the data available is shown below:

```
##
## Call:
## lm(formula = efficiency ~ ph + catalyst)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.6360  -2.4232  -0.5899   1.7566  16.8564
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   39.660      14.652   2.707  0.01496 *
## ph            -4.059       1.532  -2.649  0.01687 *
## catalyst       4.593       1.247   3.683  0.00184 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.34 on 17 degrees of freedom
## Multiple R-squared:  0.5492, Adjusted R-squared:  0.4962
## F-statistic: 10.36 on 2 and 17 DF,  p-value: 0.001145
```

### Question XII.1 (23)

Look at the R output above. Which of the following statements is correct, given a significance level of  $\alpha = 1\%$ ?

- 1 ☐ The effect of catalyst on efficiency is not significant, because the  $p$ -value is less than 0.01.
- 2 ☐ The amount of catalyst have a significant effect on efficiency, while the pH does not.
- 3 ☐ Both the amount of catalyst and the pH are significant, because the  $p$ -values are less than 0.05.
- 4 ☐ Neither the amount of catalyst nor the pH are significant, because the  $p$ -values are less than 0.05.
- 5 ☐ The model intercept is significant, because the  $p$ -value of 0.0150 is greater than 0.01.

**Question XII.2 (24)**

Look at the same R output as above. What effect does an increase of two units of catalyst have on the expected hourly output assuming pH level is kept constant at 4?

- 1 ☐ The expected hourly output increases by 3.87 kg per hour.
- 2 ☐ The expected hourly output decreases by 4.06 kg per hour.
- 3 ☐ The expected hourly output increases by 4.59 kg per hour.
- 4 ☐ The expected hourly output increases by 9.19 kg per hour.
- 5 ☐ The expected hourly output remains constant.

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### Exercise XIII

In an experiment, five different chemical substances were mixed with soil in equivalent concentrations. The same type of plants were grown next to each other in each of the five mixes, hence the plants were exposed to the same growing conditions except for the different chemicals in the soil.

When the plants were matured, they were harvested for each of the five different chemical-mixed soil conditions and it was counted how many of them had a trace of the chemicals.

The recorded counts for each of the chemicals were:

	A	B	C	D	E
Trace	12	14	18	5	15
No trace	38	35	35	42	35

The R code for reading in the data is given by:

```
tbl1 <- matrix(c(12, 14, 18, 5, 15,  
                 38, 35, 35, 42, 35), nrow=2, byrow=TRUE)
```

#### Question XIII.1 (25)

Under the null hypothesis of no difference in proportion of plants that had no trace of chemicals

$$H_0 : p_i = p, i = 1, \dots, 5$$

what is the expected number of plants with no trace for chemical C?

1 ☐ 12.1

2 ☐ 12.6

3 ☐ 39.4

4 ☐ 50.2

5 ☐ 53.1

#### Question XIII.2 (26)

What is the conclusion of the hypothesis test using significance level of  $\alpha = 0.05$  for the null hypothesis that there is no difference in proportion of plants

$$H_0 : p_i = p, i = 1, \dots, 5$$

with trace of chemical (both argument and conclusion must be correct)?

- 1 ☐ The relevant  $p$ -value is 0.041, however the rule of thumb for validity is not ok. Therefore, no conclusion can be drawn.
- 2 ☐ The relevant  $p$ -value is 0.041 and the rule of thumb for validity is checked ok. Therefore, a significant difference in uptake of the five chemicals is detected.
- 3 ☐ The relevant  $p$ -value is 0.083, however the rule of thumb for validity is not checked ok. Therefore, no conclusion can be drawn.
- 4 ☐ The relevant  $p$ -value is 0.083 and the rule of thumb for validity is checked ok. Therefore, it is accepted that no difference in uptake of the five chemicals is detected.
- 5 ☐ The relevant  $p$ -value is 0.021 and the rule of thumb for validity is checked ok. Therefore, a significant difference in uptake of the five chemicals is detected.

### Question XIII.3 (27)

In another experiment, seeds from 200 different plant types were selected. For each plant type, a particular gene modification was made to some seeds. The regular and the modified seeds were grown pairwise next to each other and thus exposed to the same conditions. After the plants were matured, their quality for sale was assessed. The resulting counts were

	Low	Medium	High
Low	25	9	19
Medium	23	21	23
High	28	33	19

where the regular seed categories are the rows and the modified seed categories are the columns.

It is desired to carry out a test for independence of the two variables on significance level  $\alpha = 0.05$ . The validity of the test has been checked ok. What is the result of the usual test (both conclusion and arguments must be correct)?

- 1 ☐ The relevant  $p$ -value is 0.048, thus it is accepted that that the variables are independent.
- 2 ☐ The relevant  $p$ -value is 0.048, thus it is concluded that that the variables are not independent.
- 3 ☐ The relevant  $p$ -value is 0.063, thus it is concluded that that the variables are not independent.
- 4 ☐ The relevant  $p$ -value is 0.063, thus it is accepted that that the variables are independent.
- 5 ☐ None of the answers above are correct.

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**Exercise XIV**

Some researchers examine the effects of different types of software and different types of hardware on the total runtime of a computer program. The researchers have applied a two-way ANOVA model and have obtained the following ANOVA table, where some numbers have been replaced with letters:

Source	DF	Sum Sq	Mean Sq	Test statistic	$p$ -value
Software	5	$SS(S)$	$MS(S)$	$F_1$	0.432
Hardware	4	$SS(H)$	$MS(H)$	$F_2$	0.036
Residual	20	$SSE$	$MSE$		

**Question XIV.1 (28)**

What can be deduced about the relationship between  $F_1$  and  $F_2$  from the information provided in the ANOVA table above?

- 1 ☐  $F_1 > F_2$
- 2 ☐  $0.036 \cdot F_1 = 0.432 \cdot F_2$
- 3 ☐  $F_1 = F_2$
- 4 ☐  $F_1 < F_2$
- 5 ☐ Nothing can be deduced about the relationship between  $F_1$  and  $F_2$  with provided information.

**Question XIV.2 (29)**

Which of the following statements is correct?

- 1 ☐ Using a significance level of 1%, we find that both the software type and the hardware type have significant effect on the runtime of the computer program.
- 2 ☐ Using a significance level of 5%, we find that both the software type and the hardware type have significant effect on the runtime of the computer program.
- 3 ☐ Using a significance level of 10%, we find that neither the software type nor the hardware type have significant effect on the runtime of the computer program.
- 4 ☐ Using a significance level of 5%, we find that the software type has significant effect on the runtime of the computer program, while the hardware type does not.

- 5 ☐ Using a significance level of 1%, we find that neither the software type nor the hardware type have significant effect on the runtime of the computer program.

**Question XIV.3 (30)**

The researchers now want to perform post-hoc analysis and have selected a significance level  $\alpha$ . The researchers want to make pairwise comparisons of the mean runtime for all the different hardware types, and in order to control the type I error rate, the researchers want to use the Bonferroni correction. What Bonferroni corrected significance level should the researchers use?

- 1 ☐  $\alpha/4$   
2 ☐  $\alpha/5$   
3 ☐  $\alpha/6$   
4 ☐  $\alpha/10$   
5 ☐  $\alpha/15$

The exam is finished. Enjoy the summer!