AI Assignment 1

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1 I.1

5 card poker hands from deck of 52 cards.
a) There are $\binom{52}{5}$ atomic events (5-card hands)
b) The probability of each event is $\binom{52}{5}^{-1}$
c) Royal straight flush is 10, J, Q, K, A of the same color. Since there are 4 colors, there are exactly 4 hands that match this description. Therefore, the probability of being dealt a royal straight flush is $\frac{4}{\binom{52}{5}}$.
To get four of a kind, we need to get all cards of the same value. Since there are 13 unique values, there are only 13 hands that fits the requirements. The probability is $\frac{13}{\binom{52}{5}}$.

2 I.2

a) We can think of this as that we select a random card at first (with probability 1). Then, the probability of the next card having the same value is $\frac{3}{51} \approx 5.9\%$.
b) We can write the question as

$$P(Pair \mid DifferentSuit) = \frac{P(DifferentSuit \mid Pair)P(Pair)}{P(DifferentSuit)}$$

It is obvious that $P(DifferentSuit \mid Pair) = 1$. The equation simplifies to $P(Pair \mid DifferentSuit) = \frac{P(Pair)}{P(DifferentSuit)}$.
Since we calculated $P(Pair)$ in a), all we need to do is find $P(DifferentSuit)$. We know that $P(DifferentSuit) = 1 - P(SameSuit) = 1 - \frac{1}{4} = \frac{3}{4}$. Which gives $P(Pair \mid DifferentSuit) = \frac{\frac{3}{51}}{\frac{3}{4}} = \frac{4}{51} \approx 7.8\%$.

3 I.3

1) Yes, if the occurrence of B makes A more likely, the occurrence of A make B more likely. This can mathematically be written shown by:

$$P(A \mid B) > P(A) \text{ or } \frac{P(A \mid B)}{P(A)} = \phi > 1, P(A) \neq 0$$

We have $P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$ and $P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$

If we rewrite $P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$ as $P(B \mid A) = \phi P(B) > P(B)$ we can observe that the statement above is true.

2)
$P(S = 0) = \frac{6}{10}$
$P(S = 1) = \frac{4}{10}$
$P(R = 1 \mid S = 0) = (R = 0 \mid S = 1) = \frac{1}{3}$
$P(R = 0|R = 0) = P(R = 1|S = 1) = \frac{2}{3}$ We want to know $P(S = 0|R = 0)$. By Bayes Theorem we have

$$P(S = 0 \mid R = 0) = \frac{P(R = 0 \mid S = 0)P(S = 0)}{P(R = 0)} \tag{1}$$

There are two ways the signal we receive can be zero. Either we sent a 1 and received a zero, or we sent a zero and received a zero. Thus, we write

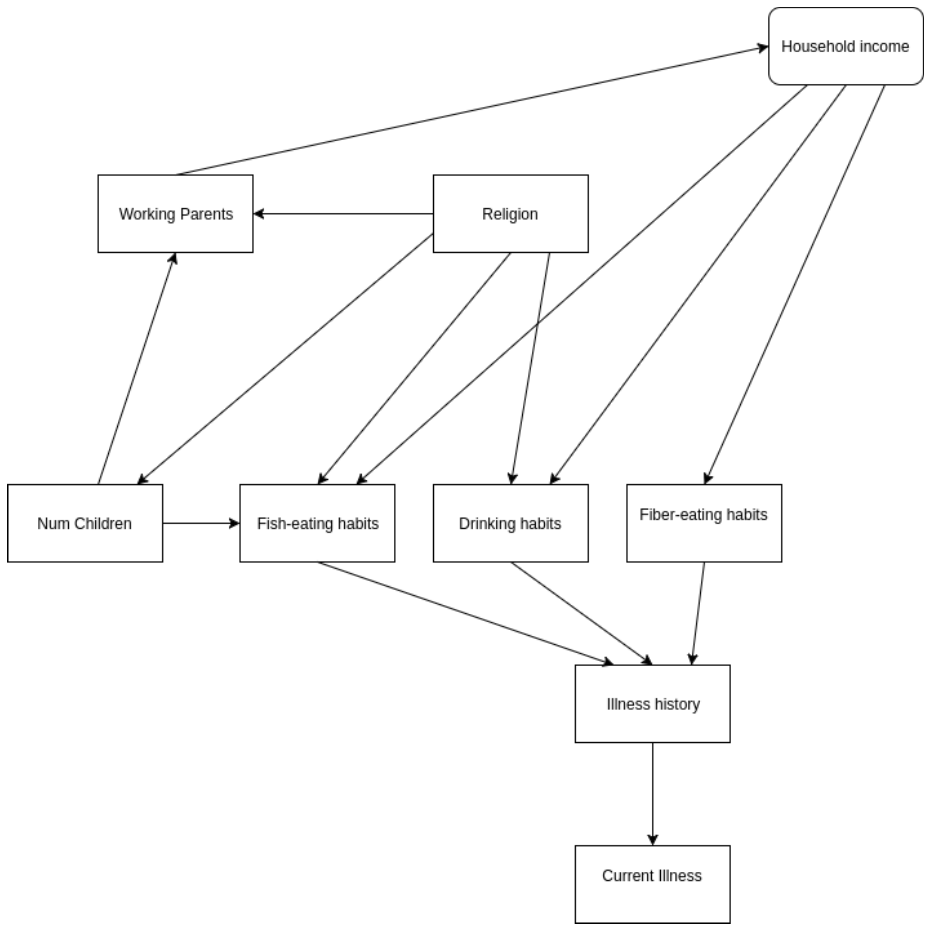
$$P(R = 0) = P(R = 0 \mid S = 0)P(S = 0) + P(R = 0 \mid S = 1)P(S = 1) = \frac{2}{3} * \frac{3}{5} + \frac{1}{3} * \frac{2}{5} = \frac{8}{15} \tag{2}$$

Inserted into 1 gives $P(S = 0 \mid R = 0) = \frac{\frac{2}{3} * \frac{3}{5}}{\frac{8}{15}} = 0.75 = 75\%$

4 I.4

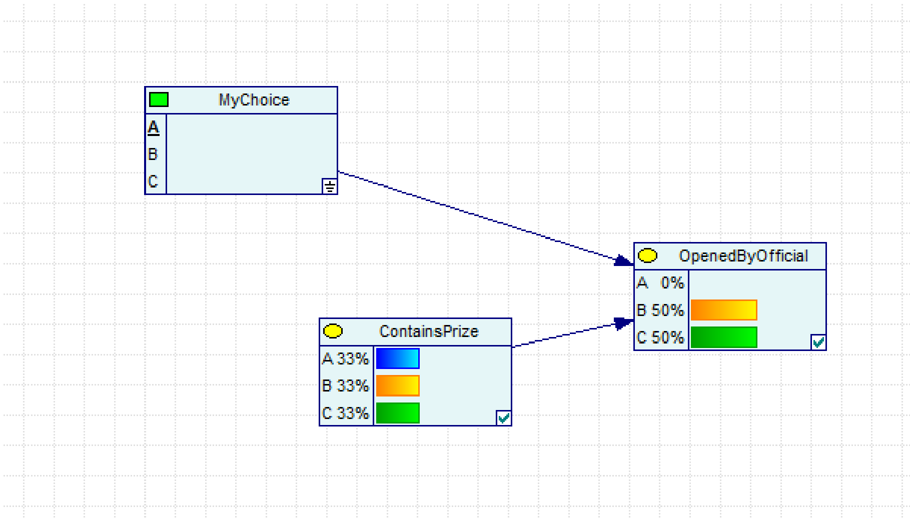
The drawing of the Bayesian network is below. We assume that ones religion influences fish eating habits (catholics often eat fish on fridays), and drinking habits. We also assume that household income directly affects eating and drinking habits, since it is usually true that households with more income eath more healthy (but may also drink more often). All dietary habits influence ones illness history, for better or for worse, which again influences the probability that one is ill right now. We also assume that the number of children you have influences how often you eat fish, since a lot of children does not enjoy eating it, and that may influence how often the parents makes it for dinner.

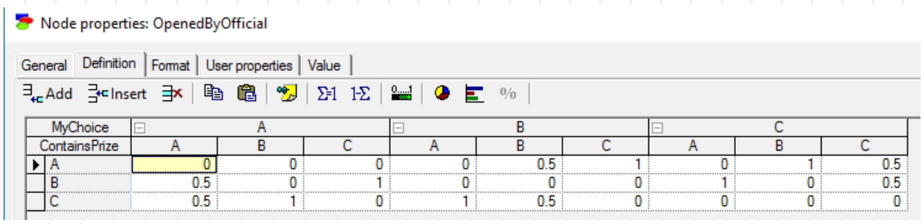
There is a question to whether or not NumChildren should directly influence drinking habits, since parents may abstain from alcohol when having children, and some may drink more to handle the stress. However, this is not included in our model. Religion and number of children influences working parents, which again influences household income. This is reasonable since religious families often have larger families, and traditionally the mother taking care of the kids while the father is working. Graphics included below.



5 III - Monty Hall Problem

We observe that you should switch doors after the official opens a door. In the example below there is a 67% chance that the prize is behind door B, given that you chose door A and the official opened door C. The reason for this is that the act of the official opening the door gives you information about where the prize is (or where it is not). With the initial guess, the probability of choosing the door with the prize is $\frac{1}{3}$. When the door is revealed, you know that the prize is behind one of the other doors. It is either behind your door, with probability $\frac{1}{3}$, or it is behind the other door, now with probability $\frac{2}{3}$.





MyChoice	A			B			C		
ContainsPrize	A	B	C	A	B	C	A	B	C
A	0	0	0	0	0.5	1	0	1	0.5
B	0.5	0	1	0	0	0	1	0	0.5
C	0.5	1	0	1	0.5	0	0	0	0