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# An application of nature-inspired paradigms in the overall optimization of square and squared-off cascades to separate a middle isotope of tellurium

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ABSTRACT

Overall optimization of strategies for separation of middle components by squared-off cascades leads to more economical design due to reducing the cost of the plant. This research was conducted in order to compare the performance of squared-off cascades with different number of sections in the separation of middle isotope 125Te to 90% concentration level. The optimization algorithms used in this field are particle swarm optimization and sine cosine optimization. In these algorithms, a particular fitness function is employed to minimize the total interstage flow of the cascade by determining the optimal value for the total number of separation elements and maximizing the average recovery factor of all steps. Overall, the results showed the functionality of the proposed approach, and it could be used to separate middle components. The PSO algorithm performed better than SCA by 5% on average and outperformed some other algorithms.



## 1. Introduction

Tellurium-125 is one of the eight stable isotopes of tellurium, which is used in medical and pharmaceutical sciences (Tellurium-125 with a concentration of higher than 99.9% is converted to iodine-125) and nuclear physics research. Complexity of the separation process of this intermediate isotope from natural concentration to 99.9% makes it highly-expensive. One of the most essential and unique methods of separation of this isotope is gas centrifuge technology. Gas centrifuge machines can be arranged in different configurations such as tapered, square and Squared-off cascades. The tapered cascade (Fig. 1-a) is the most conventional and oldest isotope separation cascade, which has a higher efficiency than square and squared-off cascades. Maximum separation efficiency of the multi component mixtures separation can be achieved by Q-cascade and Matched abundance ratio or R cascade which is a particular case of this tapered cascade and has the best performance among all model cascades (Borisevich et al., 2011; Smirnov et al., 2010; Smirnov and Sulaberidze, 2013; Song et al., 2010; Sulaberidze et al., 2008b; Zeng et al., 2011; Zeng et al., 2012; Zeng et al., 2013; Zeng et al., 2014; Zeng et al., 2018; Imani et al., 2021b). There are some other model cascades which can be fundamental basis of cascade design in the multi component mixture separation such as quasi ideal cascade, matched-X cascade or pseudo binary cascades (Zeng et al., 2014). In all tapered cascades, the feed enters via the stage with the most gas centrifuge machines. The lighter and heavier components are enriched through the cascade to the last and first stages respectively and they are available in the head and tail streams of these stages. In tapered cascades, due to a different number of gas centrifuges in the stages, the feed stage entry is limited. In order to achieve the optimum operating conditions, the cascade feed should enter the stage with the most number of gas centrifuges (Imani et al., 2021c). Also, due to the limited separation factor of a single gas centrifuge, it is not possible to completely separate the isotopes with low natural concentration, particularly middle components, by one separation step and it is necessary to use the separation cascade more than once to obtain the desired concentration of the target isotope. For example, for the tellurium-125 with a natural concentration of 7.07%, it is required to separate this isotope to 90% by more than one separation step. Because of variety of the design and operational parameters in the separation steps, the feed stage number will be different in each step. Therefore, if tapered cascades are used, the cascade arrangement must change in each step requiring the construction of a different piping system and facilities for each step. To solve this problem, in the field of multi component mixture separation by gas centrifuge cascades and

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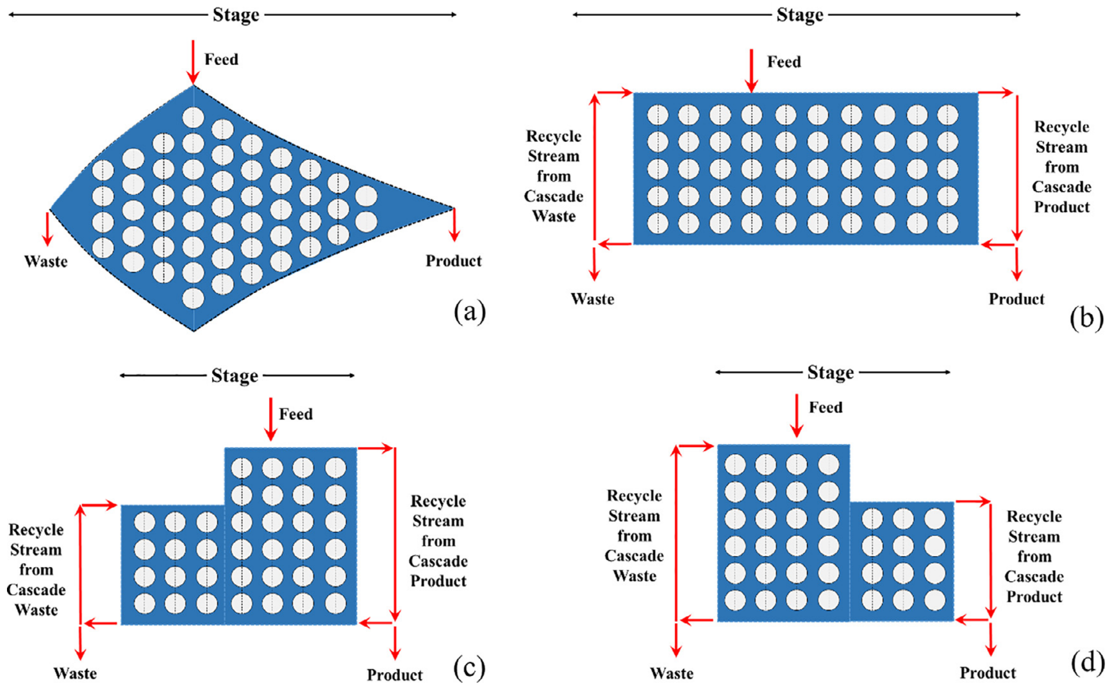


**Fig. 1.** Types of gas centrifuge cascades: (a) Tapered, (b) Square & (c-d) Squared-off.

using a single cascade for different separation steps, square and squared-off cascades have been recommended.

In the square cascades (Fig. 1-b) feed flow rate of all stages is equal. Therefore, cascade feed can be entered into any stages without the need to change the arrangement of the separating elements. Although, using square cascades reduce the fixed cost of industrial projects for the stable isotope separation by gas centrifuge method, their efficiency is lower than the tapered cascades due to the deviation of the ideal conditions in these type of cascades (Sulaberidze et al., 2008a; Wang et al., 2008). Therefore, to increase the efficiency of these cascades, multi-section squared-off cascades (Fig. 1-c and Fig. 1-d represent the two configurations of the two-section ones), that have a function between the square and tapered cascades, have been considered by researchers. Squared-off cascades with two, three or more sections are similar to several square cascades that are connected. Therefore, the feed flow rate of stages of each section is equal, and the cascade feed enters the section with the maximum feed flow rate (The most number of gas centrifuge machines belongs to this section). Hence, the schematic of the feed flow rate of stages resembles tapered cascades, and as the number of sections increases, its performance gets closer to the tapered cascade. On the other hand, considering that the feed flow rate of stages in the section where the cascade feed is entered is equal, therefore, the feed stage of the cascade can be any of the stages of that section. Obviously, in squared-off cascades, by reducing the number of sections, the flexibility of the feed stage increases and its performance gets closer to the square cascade. Therefore, the purpose of using a squared-off cascade is to benefit from the advantages of square and tapered cascades simultaneously.

Separation of a middle stable isotope of elements is of interest for many researchers. As mentioned above there are several steady state and transient methods for separation of a middle isotope such as common designs of matched abundance ratio cascades and Q-cascades, taking the output of a cascade from the stage where the target isotope has the highest concentration or multi flow cascade, and using cascades in series in a batch or continuous way to separate every isotope step by step from the lightest to the heaviest or multi cascade scheme (Cao et al., 2004; De La Garza et al., 1961; Imani et al., 2019; Imani et al., 2021a; Zeng et al., 2014; Zeng et al., 2018; Zeng et al., 2020).

It should be noted that in tellurium isotopes, the mass differences, as well as the concentration differences between the target component and the neighboring components, make the separation of isotopes difficult. There have been a few studies in this area, which include for example the study of Sosnin et al. on the separation of $^{120}$Te and $^{122}$Te to concentrations higher than 99%, where they proposed the Remainder Reduction method for this purpose (Sosnin et al., 2002). In another study by Sosnin et al. on the separation of $^{123}$Te to concentrations higher than 99%, these researchers presented a strategy that involved using four steps of a square cascade to increase the concentration of the isotope from 2.10% to 54.9% and then using a transient cascade system to increase the concentration to over 99% (Sosnin and Tcheltsov, 1999). Suvorov et al. also proposed a strategy with three steps of square cascade for enriching $^{123}$Te to a concentration of over 96% (Suvorov and Tcheltsov, 1993). In this study, for a separation strategy, all the parameters of cascades are optimized together, with the PSO or SCA method. This procedure includes finding the optimal parameters and selecting the optimal cascade to separate a middle component to a high concentration (up to a concentration level of at least 90%). In recent years, nature-inspired optimization algorithms have attracted great attention from researchers in analyzing optimization problems. These *meta*-heuristic algorithms have also found increasing use in the design and analysis of separation processes for binary and multi-component mixtures. A brief overview of these algorithms which are used in this field leads to methods of genetic algorithm, simulated annealing, particle swarm optimization (Khooshechin et al., 2021), ant colony optimization (Ezazi et al., 2020a), gray wolf optimization (Dadashzadeh et al., 2021; Imani et al., 2021a), artificial bee colony (Ezazi et al., 2020b), Harmony search (Mansourzadeh et al., 2019), TLBO (Mansourzadeh et al., 2018) and so on.

In this research, in order to optimize the problem and find the optimal parameters, two powerful optimization algorithms, PSO and SCA have been used and the results obtained have been compared with each other. Also, not much research is available to develop an appropriate strategy for the separation of intermediate isotopes of elements. In one research, Azizov et al. proposed a technique for optimizing a system of three square cascades for the simultaneous concentration of four components in a mixture under separation (Azizov et al., 2020). Khooshechin et al. have also investigated square separation cascades. Also, Borisevich et al. studied the separation of molybdenum isotopes using a four-section squared-off cascade (Borisevich et al., 2017). In all these researches, the optimization of square cascade parameters has been done step by step while finding the optimal parameters of all separation steps simultaneously has many advantages. For example, in overall optimization of steps, there is no need to know the desired concentration of product flow in each step. Moreover, the fitness function can be upgraded according to the needs of the user. In step by step optimization, only some parameters can reach to optimal values; however, in the overall optimization approach, all parameters in the objective function can reach the desired value. In previous studies, the overall optimization has not considered for the squared-off cascades and this paper examines this approach. For this purpose, a suitable objective function is defined in which the following aims such as obtaining the desired concentration and maximizing the final product can be achieved. Furthermore, it considers the step-by-step optimization goals, which are adjusting a setpoint for the average recovery of cascade in the steps while minimizing the total inter stage flow of the cascade. In this regard, an item in the form of multiplying the number of stages in the cascade and the number of machines in the stages for the square cascade and an item in the form of summation of multiplying the total number of stages in the sections and the number of machines of stages in each section for the squared-off cascade are added to the objective function. Therefore, in this research, the overall optimization approach for some configurations of squared-off cascade in the separation of middle isotope Tellurium 125 as an example with the help of two nature-inspired algorithms PSO and SCA is examined and the obtained results are presented in order to select the optimal cascade.

## 2. Theory

### 2.1. Definition of the problem and governing equations

In square cascades, input feed flow rates of the stages are kept constant by two recycle streams in the first and last stages and therefore this parameter is equal for all stages. Constant flow rates in all stages and recycle streams are two main factors for the design of the square cascades, and adjustment of these parameters makes the application of these cascades in stable isotopes separation practical. In a squared-off cascade, several square cascades are connected, which significantly increases the recovery efficiency and provides flexible operation compared to square cascades. It should be noted that each part of the squared-off cascade has the characteristics of a square cascade.

Fig. 2 shows the schematic of the square and squared-off cascade with two sections for example. For each stage, $L_n$, $L_n^{'}$ and $L_n^{''}$ are the flow rates of the feed, head and tail streams, respectively and $C_{i,n}$, $C_{i,n}^{'}$ and $C_{i,n}^{''}$ are concentrations of the component $i$ in the corresponding streams. $\varepsilon$ and $\varepsilon^{'}$ are the recycle streams, respectively. $\lambda$ is the intersection recycle ratio in the squared-off cascade.

All the variables and equations for square and squared-off cascades are listed in Table 1. These equations are written in their general form and therefore apply to square cascades as well as k-section squared-off cascades. Based on this table, the degrees of freedom for square and k-section squared-off cascades are 2 and k + 1, respectively. In order to simulate these cascades, unknown variables should be assumed as known variables. It is obvious that the value of these variables affects other parameters directly and, it is necessary to optimize these parameters beside other cascade parameters.

## 3. Optimization algorithms and the process of optimizing cascade parameters

In this study, we use two powerful optimization algorithms called the Particle swarm optimization, PSO and Sine cosine algorithm, SCA. Based on the NFL theorem that says all algorithms perform equal on all optimization problems, there are still problems that have not yet been solved, or they can be solved better by new algorithms. These two cases are the motivations of applying the two population-based optimization algorithms for the performance evaluation of them in the separation of stable isotopes in comparison with the current well-known algorithms.

The particle swarm optimization (PSO) algorithm, proposed by Kennedy and Eberhart in 1995. This algorithm is the outcome of the popularity several years after the invention of the Genetic algorithm. The PSO algorithm mimics the social and individual behav-

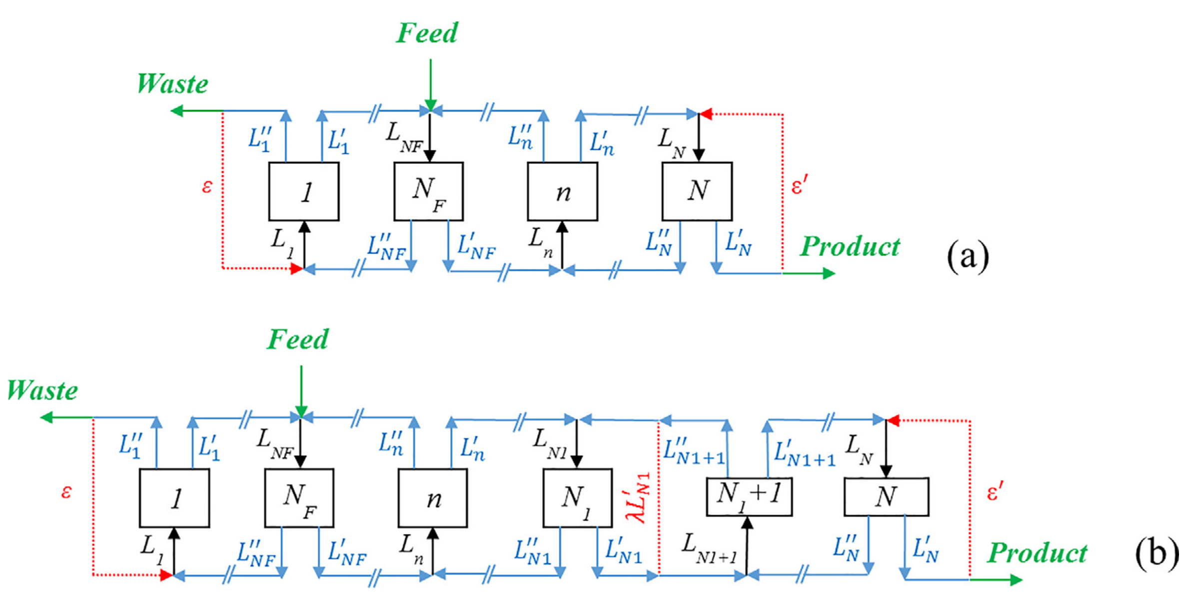


**Fig. 2.** The schematic of (a) square cascade and (b) two-section squared-off cascade streams.

**Table 1**
The independent equations and variables governing the square and k-section squared-off cascades.

| Equations | Quantity | Variables | Quantity |
|---|---|---|---|
| $L_n - L_n - L_n = \delta F \delta = \begin{cases} 1 & n = N_F \\ 0 & n \neq N_F \end{cases}$ | $N$ | $L'_n$ | $\boldsymbol{N}$ |
| $L_n - \theta_n L_n = 0$ | $N$ | $L''_n$ | $\boldsymbol{N}$ |
| $L_n - L_{n-1} - L_{n+1} = \delta F$ | $N$ | | |
| $L_1 - \varepsilon_1 - L_2 = \delta F$ | | $\theta_{\text{n}}$ | $\boldsymbol{N}$ |
| $L_n - L_{n-1} - L_{n+1} - \lambda_{sk-1,sk} L_n = \delta F$ $L_{sk-1} > L_{sk}$ Last Stage Node of Section s $k-1$ | | *Product* | 1 |
| $L_n - (1 - \lambda_{sk-1,sk}) L_{n-1} - L_{n+1} = 0$ $L_{sk-1} > L_{sk}$ First Stage Node of Section s $k$ | | *Waste* | 1 |
| $L_N - \varepsilon_N - L_{N-1} = \delta F$ | | $\theta_{\text{Cascade}}$ | 1 |
| $L_1 - \varepsilon_1 - Waste = 0$ | 1 | $\varepsilon$ | 1 |
| $L_N - \varepsilon_N - Product = 0$ | 1 | $\varepsilon$ | 1 |
| $\text{Product} - \theta_{Cascade} Feed = 0$ | 1 | $\lambda_{k-1,k}$ | $\boldsymbol{k} - 1$ |
| $L_n C_{i,n} - L_{n-1} C_{i,n-1} - L_{n+1} C_{i,n+1} = \delta F C_{i,F}\ \delta = \begin{cases} 1 n = N_F \\ 0 n \neq N_F \end{cases}$ | $N(N_c - 1)$ | $C_{i,n}$ | $\boldsymbol{NN_c}$ |
| $L_1 C_{i,1} - \varepsilon_1 C_{i,1} - L_2 C_{i,2} = \delta F C_{i,F}$ | $N(N_c - 1)$ | $C'_{i,n}$ | |
| $\begin{cases} [L_n C_{i,n} - L'_{n-1} C'_{i,n-1} - (1 - \lambda_{sk-1,sk}) L''_{n+1} C''_{i,n+1}] = 0 L_{sk-1} < L_{sk} \\ L_n C_{i,n} - L'_{n-1} C'_{i,n-1} - L''_{n+1} C''_{i,n+1} - \lambda_{sk-1,sk} L'_n C'_{i,n} = \delta F C_{i,F} L_{sk-1} > L_{sk} \end{cases}$ | | | |
| $\begin{cases} L_n C_{i,n} - L'_{n-1} C'_{i,n-1} - L''_{n+1} C''_{i,n+1} - \lambda_{sk-1,sk} L''_n C''_{i,n} = \delta F C_{i,F} L_{sk-1} < L_{sk} \\ L_n C_{i,n} - (1 - \lambda_{sk-1,sk}) L'_{n-1} C'_{i,n-1} + L''_{n+1} C''_{i,n+1} = 0 L_{sk-1} > L_{sk} \end{cases}$ | | | |
| $L_N C_{i,N} - \varepsilon'_N C'_{i,N} - L'_{N-1} C'_{i,N-1} = \delta F C_{i,F}$ | | | |
| $\frac{\frac{C_{i,n}}{C_{i,n}}}{\frac{C_{j,n}}{C_{j,n}}} = \alpha_0^{M_j - M_i} = \alpha_{ik,n} \quad i = j-1,\ j = 2, \ldots N_c$ | $N(N_c - 1)$ | | |
| $\sum_{i=1}^{N_c} C_{i,n} = \sum_{i=1}^{N_c} C_{i,n} = \sum_{i=1}^{N_c} C_{i,n} = 1$ | $3N$ | $C''_{i,n}$ | $\boldsymbol{NN_c}$ |
| Total | $3NN_c + 3N + 3$ | Total | $3\boldsymbol{NN_c} + 3\boldsymbol{N} + 4 + \boldsymbol{k}$ |

ior of herd of animals, schools of fishes, or flocks of birds in foraging (Bonebeau et al., 1999; Eberhart et al., 1996; Eberhart and Kennedy, 1995). Based on this algorithm, the position of each particle represents a point in the answer domain of the problem. Particles have memory and store the best position that they can find in the search domain. At each iteration, the new position of the particles is updated according to the current velocity vector and the best position found by that particle and the best particle in the group. One of the advantages of this algorithm is having memory; in other words, each particle benefits from its past information, while such a feature does not exist in other evolutionary optimization algorithms such as genetic algorithm, where the previous information of the problem disappears suddenly with population change. This algorithm is used to solve a wide range of optimization problems. With the investigations conducted, it has been found that PSO has discovered the better solutions area more quickly than other evolutionary algorithms and converges faster to the optimal solution (Adibifard et al., 2016; Khooshechin et al., 2021; Khoshahval et al., 2010; Mokhtari and Noroozi, 2015; Noroozi et al., 2013; Sharifi et a, 2012).

The SCA algorithm came to existence in 2016 by Mirjalili that uses the principle of trigonometric sine and cosine functions for updating the positions of individuals toward the optimal solution for tackling challenging problems in different fields of study. This algorithm has attracted great attention from researchers. Various optimization problems have widely used this algorithm, including Electrical engineering, Control engineering, Computer engineering, Classification, Image processing and other applications. This attention is due to its reasonable execution time, good convergence acceleration rate, and high efficiency compared to several well-regarded optimization algorithms (Gabis et al., 2021; Mirjalili, 2016).

The scope of this research is the performance investigation and comparison of the optimization results of these two algorithms in the separation of valuable $^{125}$Te isotope via overall optimization approach and thus readers are referred to the references (Bonebeau et al., 1999; Mirjalili, 2016) for additional information about detail description of these two algorithms. The flowchart of the optimization process is shown in Fig. 3.

### 3.1. Fitness function of square cascade

The defined fitness function consists of some terms. Here, the objective function for optimizing a square cascade for separating the isotope $^{125}$Te is defined as Equation (1). Since these terms do not have the same order, the coefficients $a_1$, $a_2$, $a_3$, and $a_4$ are adjusted by trial and error to make sure that all of these terms contribute equally to the optimization process.

$$Objective\ \ Function = a_1(N_{tot.} * N_{Mach.}) + \tag{1}$$

$$a_2(C_p - C_{pTarget}) + a_3(Sum(R) - 0.9 * n_{run}) + a_4(Product - Product_{Target})$$

In this objective function, the first term represents the minimization of total number of machines in the cascade which is equivalent of reduction in the total interstage flow of the cascade. This term makes the balance between total number of stages of the cascade, $N_{tot.}$ and number of machines in the single stage, $N_{Mach.}$ and the outcome is the optimal value for all of steps. The second term provides the desired concentration level of the target isotope in the final product stream, $C_{pTarget}$. The third term increases the average isotope recovery value of all step ($n_{run}$ is the number of steps), $R$, as much as possible to 90% and the fourth term increases the final product of the process to the target value of predetermined amount, $Product_{Target}$.

### 3.2. Fitness function of squared-off cascade

For separating the isotope $^{125}$Te by squared-off cascades, the objective function of optimization process is defined as Equation (2) in which, the second, third and fourth terms are the same of ones applied in the square cascade objective function. The optimal number of stages of the cascade obtained for the square cascade should be consider as the number of stages for the squared-off cascades and based on it, the first term of squared-off cascade objec-

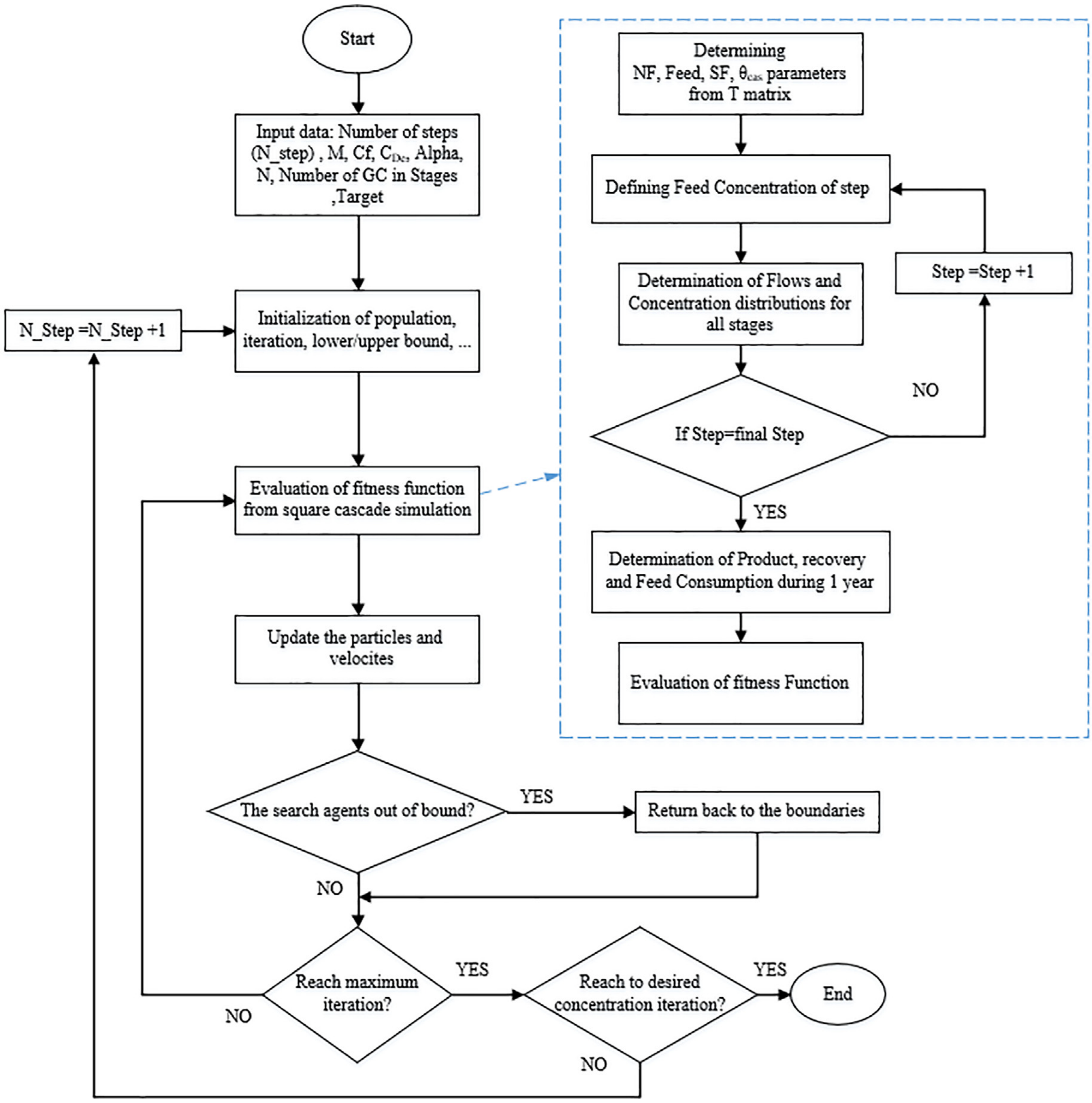


**Fig. 3.** The flowchart of the optimization algorithm.

tive function optimize the configuration of the sections in the 2, 3 and 4-section squared-off cascades. $N_{seci.}$ is the number of stages in the section $i$ and $N_{Mach.seci.}$ is the number of machines in that section.

$$ObjectiveFunction = a_1 \sum_{i=1}^{k} (N_{seci.} * N_{Mach.seci.}) + \quad (2)$$

$$a_2(C_p - C_{pTarget}) + a_3(Sum(R) - 0.9 * n_{run}) + a_4(Product - Product_{Target})$$

## 4. Results and discussion

### 4.1. Comparison of PSO and SCA algorithms with other optimization algorithms

To confirm the validity of the proposed code, the results are compared with the values reported in (Palkin, 2013). In addition, the results are compared with other optimization algorithms such as HS, DA, ALO, and SSA (Geem et al., 2001; Mirjalili, 2015a, 2015b, 2016; Mirjalili et al., 2017). In the introduced optimization problem, by varying the feed flow rate of gas centrifuges in each stage and stage cuts, the goal is to achieve the minimum total number of machines in the cascade. The cascade characteristics are presented in Table 2. By setting the objective function and performing optimization, Fig. 4 represents the average of 10 runs for each optimization algorithm. As can be seen, the PSO and SCA algorithms have minimum objective function values among other algorithms. Also, PSO algorithm has a maximum convergence rate to the other algorithms. In Table 3, the result of the best PSO run is compared with the result of Palkin. It can be seen that the results are very competitive and the PSO improved the final result by 1%. Due to the good results of PSO and SCA, they can be examined for designing the square and squared-off cascade.

### 4.2. Optimization of square cascade

In this study, the goal is to separate $^{125}Te$ with 90% concentration level in the final product using square and 2, 3 and 4-section squared-off cascades and performance comparison of two meta-

**Table 2**
The cascade characteristics for optimization in (Palkin, 2013).

| Unit separation factor | $\exp(1+\theta-\theta^2-\ln(f))^{(}1/3)$ | | |
|---|---|---|---|
| Parameter | Feed | Product | Waste |
| flow, g/s | 15.9 | 2.1 | 13.8 |
| Concentration of $^{235}U$ | 0.85 | 4.4 | 0.3 |
| Concentration of $^{234}U$ | 0.016 | 0.099 | 0.003 |
| Concentration of $^{232}U$ | 1.5e-7 | 1.1e-7 | 7.7e-9 |
| Concentration of $^{236}U$ | 0.35 | 1.29 | 0.2 |
| Concentration of $^{238}U$ | 98.784 | 94.211 | 99.497 |

heuristic paradigms in the optimization of these four cascades based on the particular defined fitness function is conducted. The specifications of the assumed machines are given in Fig. 5 which shows the unit separation factor as a function of the single machine feed flow rate.

It is assumed that Tellurium in the form of natural Tellurium hexafluoride with the characteristics listed in Table 4 is injected into the cascade as the feed of the first step. Also, the amount of final product is considered to be 20 kg per year as a constraint of the problem.

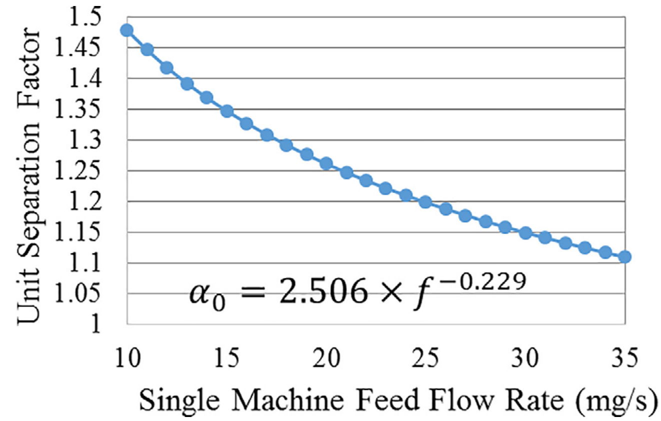


**Fig. 5.** Correlation of the unit separation factor and the single machine feed flow rate.

As previously mentioned, due to the fact that the target isotope of tellurium 125 is an intermediate isotope with low natural concentration, in order to achieve a concentration level above 90%,

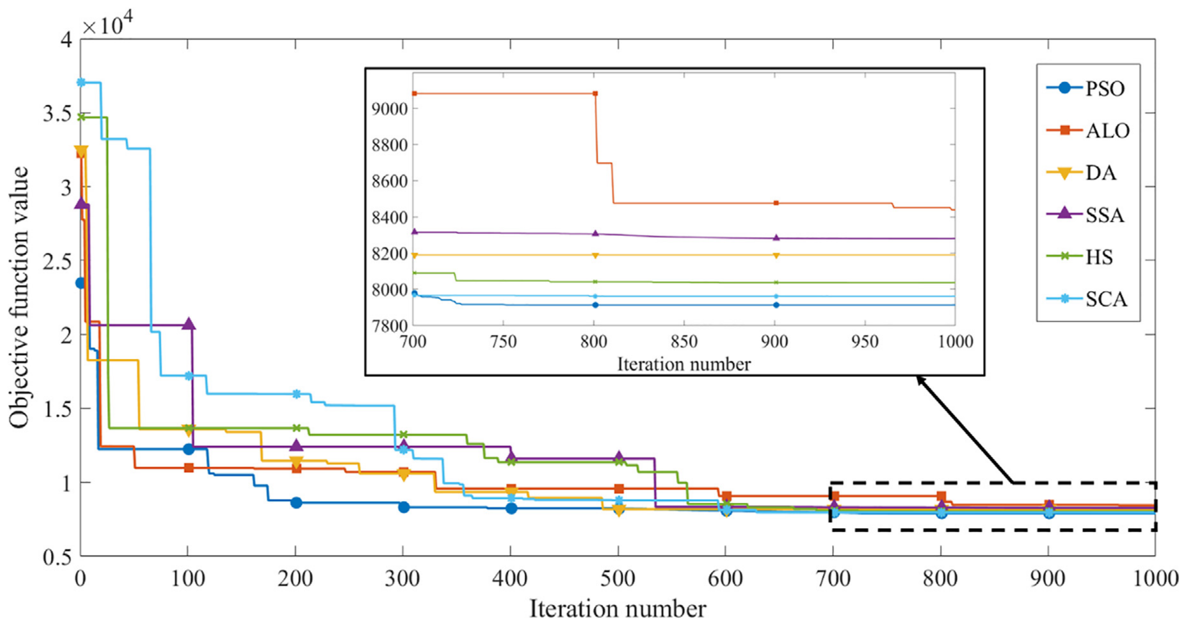


**Fig. 4.** Objective function value vs. iteration numbers for different optimization algorithms.

**Table 3**
Comparison of the optimal parameters of PSO and Palkin.

| Step No. | $N_{Mach.}$ ,$10^3$ (Palkin) | $N_{Mach.}$ , $10^3$ (PSO) | f, mg/s (Palkin) | f, mg/s (PSO) | $\theta_n$ (Palkin) | $\theta_n$ (PSO) |
|---|---|---|---|---|---|---|
| 1 | 0.4 | 0.365 | 64.3 | 68.62 | 0.47 | 0.4546 |
| 2 | 0.7 | 0.594 | 67.7 | 73.01 | 0.46 | 0.4223 |
| 3 | 0.94 | 0.849 | 69.5 | 73.53 | 0.46 | 0.4877 |
| 4 | 1.14 | 1.057 | 70.6 | 72.06 | 0.46 | 0.4211 |
| 5 | 1.29 | 1.231 | 71.8 | 70.70 | 0.46 | 0.4742 |
| 6 | 1.01 | 1.013 | 73.2 | 72.95 | 0.46 | 0.4702 |
| 7 | 0.78 | 0.850 | 73.8 | 72.00 | 0.45 | 0.4667 |
| 8 | 0.6 | 0.672 | 74.2 | 68.23 | 0.46 | 0.4225 |
| 9 | 0.44 | 0.479 | 74.6 | 69.94 | 0.46 | 0.4842 |
| 10 | 0.32 | 0.358 | 75.1 | 72.17 | 0.46 | 0.4545 |
| 11 | 0.21 | 0.239 | 75.9 | 69.67 | 0.46 | 0.4207 |
| 12 | 0.13 | 0.134 | 77.4 | 72.13 | 0.46 | 0.4907 |
| 13 | 0.06 | 0.072 | 80.5 | 65.61 | 0.47 | 0.4438 |
| | | | Sum | 8.02 | 7.912 | |

**Table 4**
Composition and mass numbers of natural Tellurium hexafluoride.

| Isotope No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $C_i^F$ (%) | 0.09 | 2.55 | 0.89 | 4.74 | 7.07 | 18.84 | 31.74 | 34.08 |
| $M_i$ | 120 | 122 | 123 | 124 | 125 | 126 | 128 | 130 |

more than one separation step is required. According to the maximum achievable concentration in the light and heavy streams of the cascade, in the first step of separation, it should be isolated from the light fraction (Imani et al., 2021a; Smirnov and Sulaberidze, 2014). According to the previous statements about solving all steps simultaneously, the number of steps required for the desired separation is determined by the algorithm. The results of square cascade optimization using two algorithms PSO and SCA are presented in Table 5. According to the objective function defined in the optimization section, the minimum total number of centrifuges obtained by the algorithms is 135 and 140 respectively in the form of 27 and 28 stage cascades. As shown, both algorithms have worked almost closely together in terms of finding the optimal number of stages, but PSO has outperformed in terms of finding the optimal values of other parameters such as total feed consumption, total recovery and parameter D. The parameter D is defined as Equations (3) or (4) depending on whether the target isotope is separated in the light or the heavy fraction of the cascade. Improving the $D$ is important because the closer the value of D to 1, the better the separation performance will be, which means fewer separation steps will be required.

$$D = \frac{P}{F}\sum_{i=1}^{k} C'_{i,N} + \frac{W}{F}\sum_{i=k+1}^{N_c} C''_{i,1} \tag{3}$$

$$D = \frac{P}{F}\sum_{i=1}^{k-1} C'_{i,N} + \frac{W}{F}\sum_{i=k}^{N_c} C''_{i,1} \tag{4}$$

In fact, both algorithms meet the problem conditions, but the PSO has performed better. In the following, both algorithms are used to optimize the squared-off cascade with different number of sections based on the number of stages obtained from square cascade optimization as the appropriate number of stages. The intended objective function is also in accordance with the function introduced in the previous section, in which, with the optimum number of stages of the square cascade, the number of stages in each section and the number of machines in the sections will also be optimized that the mathematical form of which is as follows.

$$\sum_{i=1}^{k} N_{seci.} = 27(PSO) \tag{5}$$

$$\sum_{i=1}^{k} N_{seci.} = 28(SCA)$$

### 4.3. Optimization of squared-off cascade

Tables 6 and 7 present the results of 2, 3, and 4-section squared-off cascade optimization using two PSO and SCA optimization algorithms. According to the presented results, as it is known, both optimization algorithms are able to pass the predetermined constraints for the objective function defined in the form of optimization items. For example, both algorithms meet the amount of 20 kg for total product, 90% concentration for the final product, and 90% for the average recovery factor of steps. This objective function considers both the step-by-step optimization approach and the overall optimization approach that has not been addressed before. The items intended for the amount of total product and the concentration of the final stream are consistent with the overall approach, while the terms of recovery and the total number of machines are optimized for all steps according to the step-by-step goals and in fact, the minimum number of machines obtained is the optimal number for each step.

According to the results in Tables 6, 7 and Figs. 6, 7, 8 and 9, the following items can be manifested as useful conclusions:

**Table 5**
Summary of the optimized square cascade results for separation of $^{125}$Te to 90%

| Type of Cascade (Algorithm) | Number of GCs | Step No. | Suitable Cascade Fraction | $C_5^P$ or $C_5^W$ | Feed (g/y) | Product (g/y) | Operational Period (d) | R (%) | D |
|---|---|---|---|---|---|---|---|---|---|
| Square (PSO Results) | 135 | 1 | Light | 0.4465 | 299,758 | 46,225 | 275 | 97.39 | 0.995 |
| | | 2 | Heavy | 0.9047 | 46,225 | 20,000 | 90 | 87.64 | 0.918 |
| | | | | | | | | Total | 84.93 |
| | 140 | 1 | Light | 0.3903 | 354,312 | 47,745 | 254 | 74.39 | 0.980 |
| | | 2 | Heavy | 0.9120 | 47,745 | 20,000 | 111 | 99.62 | 0.968 |
| | | | | | | | | Total | 73.81 |

**Table 6**
Summary of the optimized square, 2, 3 and 4-section squared-off cascade results for separation of $^{125}$Te to 90% using PSO algorithm.

| Type of Cascade | Step No. | Suitable Cascade Fraction | $C_5^P$ or $C_5^W$ | Feed (g/y) | Product (g/y) | Operational Period (d) | R (%) | D |
|---|---|---|---|---|---|---|---|---|
| Square | 1 | Light | 0.4465 | 299,758 | 46,225 | 275 | 97.39 | 0.995 |
| | 2 | Heavy | 0.9047 | 46,225 | 20,000 | 90 | 87.64 | 0.918 |
| | 1 | Light | 0.4384 | 294,822 | 44,073 | 339 | 92.69 | 0.993 |
| | 2 | Heavy | 0.9000 | 44,073 | 20,000 | 26 | 93.17 | 0.932 |
| 3-Section Squared-off | 1 | Light | 0.4402 | 294,150 | 44,431 | 343 | 94.04 | 0.993 |
| | 2 | Heavy | 0.9000 | 44,431 | 20,000 | 22 | 92.04 | 0.991 |
| | 1 | Light | 0.4116 | 294,002 | 44,584 | 289 | 91.22 | 0.993 |
| | 2 | Heavy | 0.9001 | 44,584 | 20,000 | 76 | 96.17 | 0.993 |

**Table 7**
Summary of the optimized square, 2, 3 and 4-section squared-off cascade results for separation of $^{125}$Te to 90% using SCA Algorithm.

| Type of Cascade | Step No. | Suitable Cascade Fraction | $C_5^P$ or $C_5^W$ | Feed (g/y) | Product (g/y) | Operational Period (d) | R (%) | D |
|---|---|---|---|---|---|---|---|---|
| Square | 1 | Light | 0.3903 | 354,312 | 47,745 | 254 | 74.39 | 0.980 |
| | 2 | Heavy | 0.9120 | 47,745 | 20,000 | 111 | 99.62 | 0.968 |
| | 1 | Light | 0.3988 | 341,253 | 47,176 | 338 | 78.00 | 0.983 |
| | 2 | Heavy | 0.9005 | 47,176 | 20,000 | 27 | 95.70 | 0.943 |
| 3-Section Squared-off | 1 | Light | 0.4248 | 324,501 | 47,594 | 339 | 88.14 | 0.989 |
| | 2 | Heavy | 0.9009 | 47,594 | 20,000 | 25 | 93.06 | 0.956 |
| | 1 | Light | 0.4171 | 316,886 | 45,577 | 292 | 83.39 | 0.985 |
| | 2 | Heavy | 0.9181 | 45,577 | 20,000 | 73 | 98.28 | 0.971 |

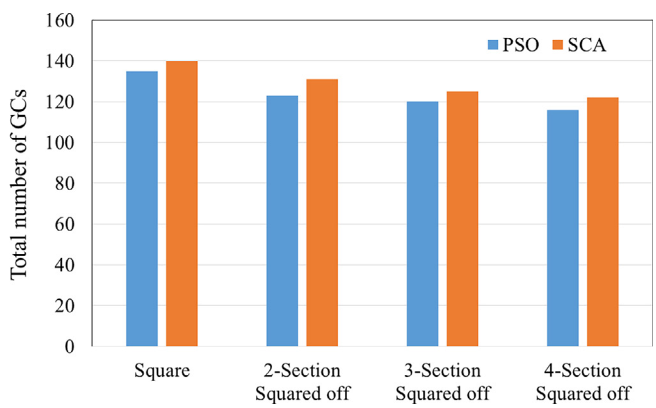


**Fig. 6.** Total number of gas centrifuges of optimized square, 2, 3 and 4-section squared-off cascades.

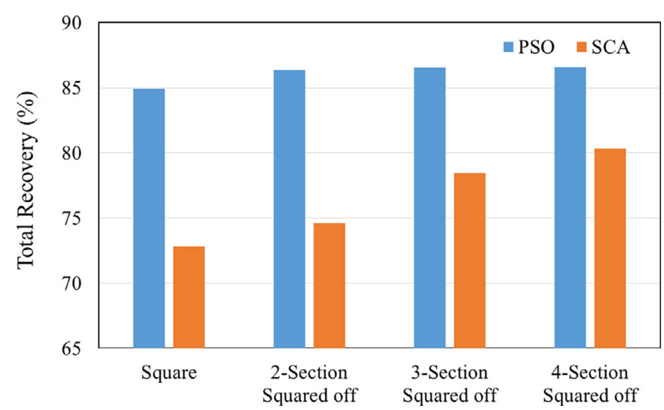


**Fig. 8.** Total recovery of optimized square, 2, 3 and 4-section squared-off cascades.

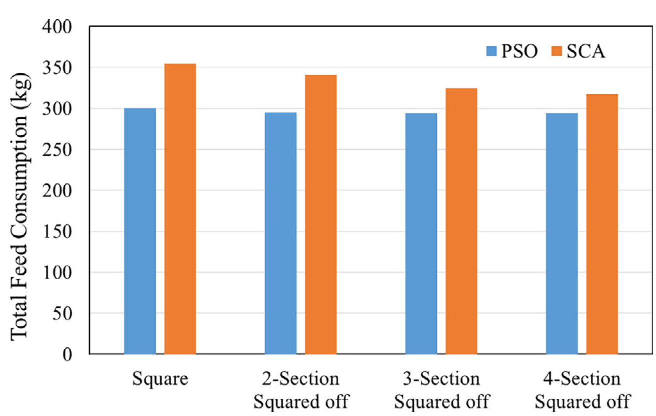


**Fig. 7.** Total feed consumption of optimized square, 2, 3 and 4-section squared-off cascades.

- Like the results obtained for the square cascade, the PSO algorithm has been more successful in finding the optimal parameters of the squared-off cascade. The total number of machines for squared-off cascades obtained from the two algorithms is shown in Fig. 6. For example, according to this figure, the minimization of the total number of machines of 2-section squared-off cascade with the help of PSO is about 7% better than SCA. The mentioned trend is true for all squared-off cascades with different number of sections.
- In Figs. 7 and 8, the total feed consumption and total recovery of the cascades for PSO and SCA methods have been shown respectively. According to these Figures, it is clear that converting the cascade from square to squared-off improves the most important parameters such as the total number of machines, total feed consumption and total recovery of the cascades. Considering the amount of product produced per year, it is observed that the cascade with more sections has consumed less feed.
- Fig. 9 represents the difference percentage of the total recovery factor between the cascades. For example, the first column shows this item between square and 2-section squared-off cascades. As the numbers of sections increases, the parameters considered in the cascade improve, but as the results show, the more the number of sections, the lower the improvement of the parameters. For example, according to the Figure, the total recovery factor between the 3 and 4-section squared-off cascades has increased by only 0.05%. It should also be noted that as the number of sections increases, more operational complexity is applied to the cascade. For example, we can mention the increase in the number of cascade control parameters such as stages cuts and intersection recycle ratios. With these interpretations, in order to select the appropriate cascade, it is necessary to strike a balance between the number of sections and the amount of operational complexity.

Table 8 presents the optimized values of all parameters for all steps of the four cascades. Also, based on the data in Table 9, it can be seen that for all four cascades, the stage cuts in the steps are in appropriate ranges in terms of operation. These cut values are optimized in such a way to make the cascade operational in terms of pressure constraints.

## 5. Conclusions

In this research a new methodology based on PSO and SCA algorithms was proposed to examine the performance of square and squared-off cascades in the separation of $^{125}$Te middle component via proposition of an overall procedure for the optimization of the separation strategy. In addition, a new objective function has been developed in line with the overall approach, in which, the goals

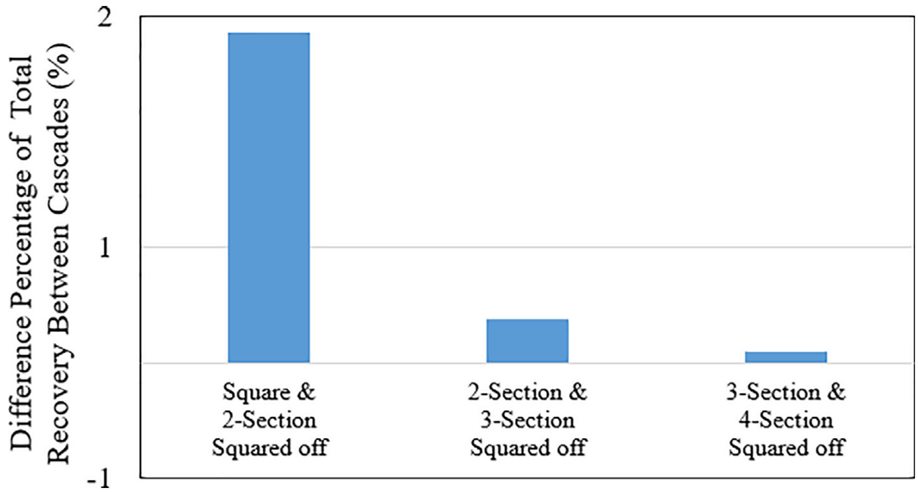


**Fig. 9.** Difference percentage of total recovery between cascades (PSO results).

**Table 8**
Optimized parameters of square, 2, 3 and 4-section squared-off cascades for separation of $^{125}Te$ to 90 % using PSO algorithm.

| | | Cascade Type | Square |
|---|---|---|---|
| | | N × GC | 27 × 5 |
| No | Variable parameter | Step 1 | Step 3 |
| 1 | Feed (mg/s) | 12.0.63 | 5.92 |
| 2 | $N_F$ | 13 | 26 |
| 3 | L (mg/s) | 51.98 | 60.48 |
| 4 | $\theta_{cascade}$ | 0.1542 | 0.5675 |
| | | Cascade Type | 2-Section Squared-off |
| | | $N_1 \times GC_1$ | 21 × 5 |
| | | $N_2 \times GC_2$ | 6 × 3 |
| No | Variable parameter | Step 1 | Step 2 |
| 1 | Feed (mg/s) | 10.04 | 20.26 |
| 2 | $N_F$ | 12 | 13 |
| 3 | $L_1$ (mg/s) | 60.76 | 115.45 |
| 4 | $L_2$ (mg/s) | 48.93 | 36.46 |
| 5 | $\theta_{cascade}$ | 0.1495 | 0.5462 |
| | | Cascade Type | 3-Section Squared-off |
| | | $N_1 \times GC_1$ | 2 × 2 |
| | | $N_2 \times GC_2$ | 9 × 4 |
| | | $N_3 \times GC_3$ | 16 × 5 |
| No | Variable parameter | Step 1 | Step 2 |
| 1 | Feed (mg/s) | 9.91 | 23.90 |
| 2 | $N_F$ | 12 | 17 |
| 3 | $L_1$ (mg/s) | 56.22 | 64.62 |
| 4 | $L_2$ (mg/s) | 68.81 | 121.03 |
| 5 | $L_3$ (mg/s) | 50.93 | 112.91 |
| 6 | $\theta_{cascade}$ | 0.1510 | 0.5499 |
| | | Cascade Type | 4-Section Squared-off |
| | | $N_1 \times GC_1$ | 6 × 3 |
| | | $N_2 \times GC_2$ | 7 × 5 |
| | | $N_3 \times GC_3$ | 8 × 6 |
| | | $N_3 \times GC_3$ | 6 × 3 |
| No | Variable parameter | Step 1 | Step 2 |
| 1 | Feed (mg/s) | 12.54 | 7.25 |
| 2 | $N_F$ | 17 | 19 |
| 3 | $L_1$ (mg/s) | 48.80 | 99.88 |
| 4 | $L_2$ (mg/s) | 74.61 | 174.20 |
| 5 | $L_3$ (mg/s) | 94.06 | 195.37 |
| 6 | $L_4$ (mg/s) | 45.57 | 97.49 |
| 7 | $\theta_{cascade}$ | 0.1438 | 0.5612 |

**Table 9**
Ranges of stage cut values in the steps of optimized square, 2, 3 and 4-section squared-off cascades.

| **Type of Cascade** | **Square** | **2-Section Squared-off** | **3-Section Squared-off** | **4-Section Squared-off** |
|---|---|---|---|---|
| **Step 1** | 0.39–0.64 | 0.43–0.51 | 0.43–0.51 | 0.38–0.51 |
| **Step 2** | 0.47–0.57 | 0.46–0.65 | 0.45–0.55 | 0.48–0.52 |

such as total product and final product concentration have been set in line with the overall approach. At the same time, the objects related to optimal average recovery factor and the best value for the total number of machines for all steps can be achieved in proportion to the approach. The obtained results show the promising capabilities of the proposed method via PSO algorithm. According to the results, it is demonstrated that the optimal number of stages to increase the concentration of $^{125}$Te to 90% is 27 and at least two steps are needed to achieve this concentration. The results also showed that the 4-section squared-off cascade has a better capacity, total number of machines and total recovery in terms of similar predetermined value for the amount of total production output than the 2 and 3-section counterparts as well as the square cascade. Therefore it can be concluded that the method is an appropriate and trustworthy one for usage into cases of multi component mixture separation cascades and it can be a reliable road map for operational applications of desired isotope separation as well as other components of different elements.

## CRediT authorship contribution statement

**Farzaneh Ezazi:** Methodology, Software, Writing – original draft, Visualization, Data curation. **Morteza Imani:** Conceptualization, Methodology, Writing – original draft, Software, Validation, Visualization, Writing – original draft. **Jaber Safdari:** Writing – review & editing, Supervision. **Mohammad Hasan Mallah:** Supervision, Formal analysis. **Seyedeh Leila Mirmohammadi:** Formal analysis.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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