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A Linear Model for Dynamic Generation Expansion Planning Considering Loss Of Load Probability

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***Abstract*— Computation of Loss Of Load Probability (LOLP) is a challenge in Generation Expansion Planning (GEP) problem. This paper presents a dynamic GEP model considering LOLP as a reliability criterion. The objective function of the proposed dynamic GEP model includes the discounted total costs of investment cost, operation cost, and maintenance cost. The proposed LOLP constrained GEP problem is formulated as a mixed integer nonlinear programming (MINLP) problem. The MINLP formulation of GEP problem is then converted to a mixed integer linear programming (MIP) problem by applying several approximations to LOLP constraint. The utilized approximations are divided into two clusters for low and high order outages. A test case containing 12 types of installed and 5 types of candidate plants with a 14-year planning horizon is used to validate the efficiency of the proposed dynamic GEP problem. The developed MIP-based GEP model is solved using CPLEX solver and the obtained results are compared with the conventional LOLP-constrained GEP model.**

***Index Terms*—Capacity outage, generation expansion planning, loss of load probability, optimization, reliability.**

NOMENCLATURE

A. Indices

g	Index of plants
k	Index of all units
t	Index of planning stages

B. Sets

Ω_g	Set of all plants
Ω_k	Set of all units
Ω_e	Set of existing plants
Ω_t	Set of planning stages
Ω_{new}	Set of candidate plants

C. Scalars

R^{low}/R^{up}	Desired lower/upper bounds of reserve margin in percentage of yearly peak load
L^{avg}/L^{min}	Average/minimum loads as fraction of the forecasted peak load L_t^p

D. Parameters

d	Outage degree of the desired approximations
CL_{new}	Unit construction limit or tunnel limit of candidate plant new per stage
C_g	Generation capacity of plant g in MW
F_g	Forced Outage Rate (FOR) of plant g
$I_{t,new}$	Investment cost of candidate plant new in \$/kW at stage t
P_{tg}	Production cost of plant g in \$/kWh at stage t
M_{tg}	Maintenance cost of plant g in \$/kW – month at stage t
L_t^p	Forecasted annual peak load at stage t (MW)
$full_t$	Sum of capacities of all existing plus candidate units at stage t (MW)
AF_t	Average FORs of all existing plus potential candidate units at stage t
AC_t	Average capacity of all existing plus potential candidate units at stage t (MW)
NU_t	Number of units with average capacity AC_t needed to supply load and reserve at stage t
j_t	Approximated product of availabilities of all existing plus potential candidate units at stage t
$LOLP^c$	Upper bound of desired LOLP at each stage

E. Variables

Z	Objective variable
PG_{tg}	Power generation of plant g at stage t (MW)
IC_t	Installed capacity at stage t (MW)
$ST_{t,d}$	Approximated LOLP caused by d simultaneous outages at stage t
$LOLP_t$	Loss of load probability at stage t
$N_{t,g}$	Cumulative number of units of the plant type g at stage t
$U_{t,k}$	Binary variable indicating the existence of unit k at stage t. 1 if it exists and 0 otherwise
$D_{t,k}$	Binary variable indicating the violation of peak supply due to single outage of unit k.

I. INTRODUCTION

GENERATION Expansion Planning (GEP) plays an important role in the long-term economic and adequate design of power system which is concerned with determining

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the decisions to construct new power plants in order to reliably supply the forecasted load over a specific planning horizon. GEP models are divided into two classes of static and dynamic. Static GEP model is a single period GEP problem which introduces less computational complexities to the problem, whereas dynamic GEP is a multiperiod planning problem with higher computational complexities. Dynamic GEP as a challenging problem is used to determine the number, size, type of technology and the time at which new generating units must be added to the power system. In a vertically integrated power system with a centralized planner, the optimal GEP configuration can be found by minimizing the discounted total costs including investment cost, operation cost and maintenance cost. However, in a market-based power system the objective function will be to maximize the profit. The results of centralized least-cost GEP problem, can be useful either to obtain the optimal expansion plan in a centralized planning approach or to guide the market participants and regulators through the selection of plant types for construction or to help them participate in capacity market efficiently [1, 2]. This problem is usually constrained by available budget and physical limitations applied to the power system or generation units such as reliability, capacity mix, and adequate load supply. Reliability is divided to Adequacy and Security and it should be noted that throughout this paper, by reliability the authors mean adequacy. Due to economic reasons, constructing a fully reliable power system is practically impossible. However, some reliability criteria such as Loss of Load Probability (LOLP) and Loss of Load Expectation (LOLE) can be kept below a standard value during the planning horizon. To this end the reliability constrained GEP is introduced.

As an individual problem, the LOLP criterion can be calculated accurately using the Capacity Outage Probability Table (COPT) for a given configuration of installed generating units. However, the integration of COPT-based formulation of LOLP criterion in an optimization problem such as GEP, results in an inherently discrete and highly nonlinear optimization problem that needs much computational and time effort to obtain the global optimum solution. For large scale power systems, such a reliability constrained MINLP formulation of GEP problem may fail to give a feasible solution. Therefore, different approaches have been proposed to solve this problem needing less computational complexity.

Several approaches have been proposed for the GEP problem. The proposed methods for GEP models are mainly divided into two clusters: 1) heuristic methods such as Ant Colony Optimization (ACO) [3], Genetic Algorithm (GA) [4], [5], Honey Bee Algorithm [6], and PSO [7, 8] and 2) analytic methods like Hierarchy Approach [9], Decision Tree [10], and mixed integer programming [11, 12].

Reliability-constrained GEP is a subclass of GEP in which the reliability constraints are also included. The proposed approaches in which the LOLP criterion is considered are divided into two categories: 1) approaches that merely defined LOLP limit without presenting the formulation [13-16] and 2) those that presented a clear formulation for LOLP computation [17-21]. An improved GA has been proposed in [13] and the results of the proposed method have been compared to different methods such as Dynamic Programming and GA. A game-theoretic model for GEP has been developed in [14] for competitive generation expansion. In [16], an improved Evolutionary Programming (EP) has been proposed to deal with the highly-nonlinear GEP problem.

As mentioned above, specific LOLP formulations have been proposed in some literatures. A dynamic GEP with non-linear LOLP formulations based on cumulant method has been proposed in [17] and the resultant MINLP model has been solved by GA and an optimization software package namely LINGO. A linear reliability calculation method, called the conventional method throughout this paper, was first proposed in [18] and has been utilized in several GEP-related researches [19-21]. In [19], an integrated renewable-conventional multi-objective GEP model has been proposed utilizing the conventional method for LOLP calculations. The same formulation for conventional LOLP calculation could be found in [20, 21].

A review through literature reveals that accurate reliability assessment and in more specific, LOLP criterion calculation, has usually created complexity and nonlinearity in reliability constrained GEP models. In most of previous GEP models no efficient LOLP formulation has been proposed especially for higher order outages. In order to overcome this obstacle, few approaches in the literature have approximated LOLP with a low accuracy formulation. The gap this paper intends to fill is developing a linear approximation of LOLP-constrained GEP model.

In this paper, the dynamic GEP problem is modeled as an MIP problem using several approximations. The contributions of this paper are as follows: 1- A new accurately approximated formulation is proposed for LOLP calculation in optimization model of GEP problem, resulting in lower costs while maintaining the desired reliability of the system with negligible error. The main difference in LOLP calculation of the proposed method with respect to conventional methods is that the load aspect of the LOLP formulation is also considered in the proposed method. 2- This method also provides two approximated formulations with different levels of accuracy which results in a flexible mixed-integer linear formulation for reliability-constrained GEP model and allows the planner to compromise between accuracy and the system hardware requirements. 3- The other advantage of the proposed method is that by linearizing the inherent MINLP model to a MIP one, the global optimum expansion plan can be found efficiently using available MIP solver packages like CPLEX.

The rest of the paper is organized as follows. In the next section, the general formulation of GEP problem is presented. In section III, the novel proposed approximation for LOLP calculation is described. The simulation results for a dynamic GEP test case are described in section IV. Finally, the paper is concluded in section V.

II. General GEP Formulation

In this part, the general formulation of the GEP problem including the objective function and the related constraints are presented. The objective function of the GEP model is expressed as follows:

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$$\begin{aligned} \text{Min} \;\; z \;\; = & \sum_{t\in\Omega_t}\sum_{j\in\Omega_{new}} I_{tj}\cdot C_j\cdot\left(N_{tj}-N_{t-1,j}\right) \\ & +\sum_{t\in\Omega_t}\sum_{g\in\Omega_g} P_{tg}\cdot PG_{tg} \\ & +\sum_{t\in\Omega_t}\sum_{g\in\Omega_g} M_{tg}\cdot C_g\cdot N_{tg} \end{aligned} \tag{1}$$

The objective function includes the discounted total costs of investment cost, operation cost, and maintenance cost.

The objective function of GEP problem is optimized subject to different constraints including capacity constraint, reserve constraint, limits of new installation units and reliability constraints. The power balance and reserve constraint at each stage of the planning horizon are satisfied by (2) and (3) respectively.

$$\sum_{g\in\Omega_g} PG_{gt} \geq L^{avg}\cdot L_t^p \;;\;\; \forall\, t\in\Omega_t \tag{2}$$

$$(1+R^{low})\times L_t^p \leq IC_t \leq (1+R^{up})\times L_t^p \;\;;\;\; \forall\, t\in\Omega_t \tag{3}$$

where:

$$IC_t = \sum_{k\in\Omega_k} C_k\cdot U_{t,k} \;\;;\;\; \forall\, t\in\Omega_t \tag{4}$$

The reliability model considered in this paper is of Hierarchical Level 1 (HL1) class so a single-bus system where all generation and load are connected to the same bus is considered.

The tunnel constraint is presented by (5). Tunnel constraint enforces the limit on the number of units that can be built from each plant type at each stage.

Note that variable N is the cumulative number of constructed units (i.e. newly installed) at stage t.

$$N_t^{new} - N_{t-1}^{new} \leq CL_{new} \;\;;\;\; \forall\, t\in\Omega_t \tag{5}$$

Constraint (6) is considered to exert the power generation limit of units.

$$PG_{t,k} \leq C_k\cdot U_{t,k} \;\;;\;\; \forall\, t\in\Omega_t \tag{6}$$

Finally, constraint (7) enforces the upper limit of LOLP which must be satisfied at each stage of the planning horizon. The calculation of this constraint will be discussed in detail in the following section.

$$LOLP_t \leq LOLP^c \;\;;\;\; \forall\, t\in\Omega_t \tag{7}$$

III. Proposed LOLP Formulation

As discussed before, the complete and detailed LOLP calculation in the GEP optimization problem is challenging and time-consuming. In this section, the basic method for LOLP evaluation and a two-stage approximation process are proposed which not only make the calculations easier and faster but also hold an acceptable approximation of the accurate LOLP value.

In order to achieve a better understanding of the process of the proposed method, the flowchart diagram of the method is presented in Fig. 1. Based on this flowchart, the first step is entering the required data regarding power system and generation units as inputs of the problem. The program is run with an initial guess for assignment of outage orders to each of the approximation types of the proposed method. If the results were not satisfactory regarding computation times or reliability, the outage order assignments can be flexibly modified to obtain an optimum result.

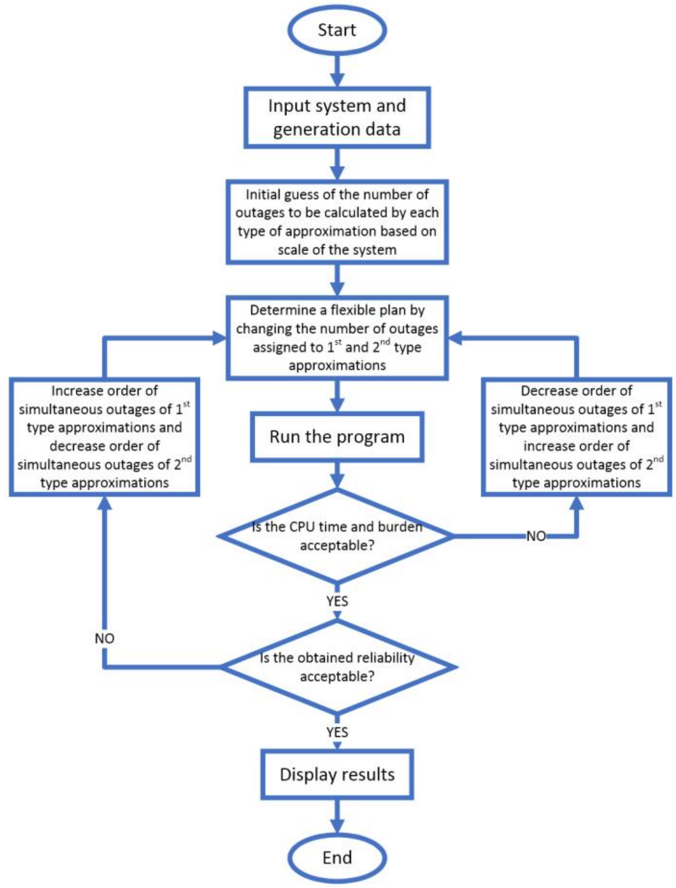

Fig. 1. Flowchart of the proposed method

A. General LOLP Formulation

The method used for LOLP evaluation in this paper is based on the cumulative method presented in [22]. Fig. 2 illustrates the Load Duration Curve (LDC) modeled as a constant slope linear function connecting the peak load to the base load. IP_t is the individual probability of every 1 MW of load at stage t based on the LDC shown in Fig. 2 and is obtained by (8).

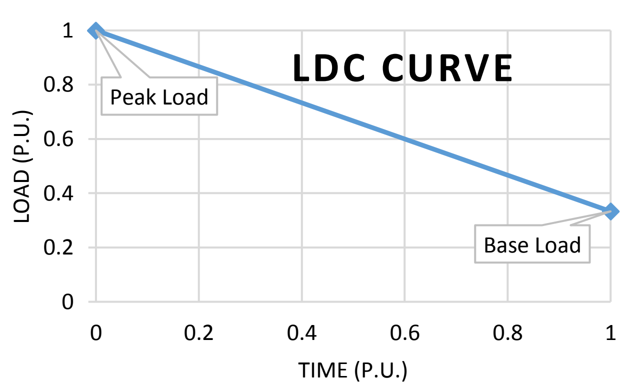

Fig. 2. Typical linear model for Load Duration Curve (LDC)

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$$IP_t = \frac{1}{(1 - L^{min}) \times L_t^p} \;\; ; \;\; \forall\, t \in \Omega_t \tag{8}$$

Based on these assumptions, LOLP can be defined as the summation of the probability of an outage times the cumulative probability of the amount of load not supplied or in other words, the probability of the load to be more than the available capacity after the outage. In this regard, the general LOLP formulation is given in (9).

$$LOLP_t = A + B + \cdots \tag{9}$$

where:

$$A = \sum_{k_1 \in \Omega_k} \left[\left(U_{t,k_1} . D_{t,k_1} . F_{k_1} . \prod_{k \neq k_1} \left(1 - U_{t,k} \times F_k\right) \right) \times \left(\left(L_t^p + C_{k_1} - IC_t\right) . IP_t . H_{t,k} + \left(1 - H_{t,k}\right) \right) \right] \tag{10}$$

$$B = \sum_{k_1 \in \Omega_k} \sum_{k_2 \in \Omega_k} \left[\left(U_{t,k_1} . U_{t,k_2} . D_{t,k_1,k_2} . F_{k_1} . F_{k_2} \times \prod_{k \neq k_1, k_2} \left(1 - U_{t,k} \times F_k\right) \right) \times \left(\left(L_t^p + C_{k_1} + C_{k_2} - IC_t\right) . IP_t . H_{t,k} + \left(1 - H_{t,k}\right) \right) \right] \tag{11}$$

In (9), the term $\mathrm{U_{t,k_1}} . \mathrm{F_{k_1}} . \prod_{k \neq k_1} (1 - \mathrm{U_{t,k}} \times \mathrm{F_k})$ is the probability of a generation capacity outage and the second term $(\mathrm{L_t^p} + \mathrm{C_{k_1}} - \mathrm{IC_t}) . \mathrm{IP_t} . \mathrm{H_{t,k}} + (1 - \mathrm{H_{t,k}})$ is the probability of the load to be more than the available capacity after the outage. A and B are dedicated to the 1st and 2nd-order outages, respectively (i.e. single and double outages). Higher order outages can be formulated in the same way.

The conventional method for integrating LOLP calculations in an optimization problem which was first presented in [18] lacks the load aspect of the calculations. The main difference between the proposed and conventional method in a GEP problem is that in the conventional method [20], only the generation outage aspect of the LOLP formulation is considered, i.e. the term $(\mathrm{U_{t,k_1}} . \mathrm{F_{k_1}} . \prod_{k \neq k_1} (1 - \mathrm{U_{t,k}} \times \mathrm{F_k}))$, but in the proposed method, the load aspect of the LOLP formulation, i.e. $((\mathrm{L_t^p} + \mathrm{C_{k_1}} - \mathrm{IC_t}) . \mathrm{IP_t} . \mathrm{H_{t,k}} + (1 - \mathrm{H_{t,k}}))$ is also considered. In fact, since the load aspect of the LOLP formulation is not considered in the conventional method, the probability of existence of a given load state was neglected and the GEP model is optimized considering the peak load of each planning year. In order to obtain the binary values D_t that indicate the loss of load caused by different outages, (12) is applied [16].

$$\frac{\left(L_t^p + C_k - IC_t\right)}{full_t} \le D_{t,k} \le 1 + \frac{\left(L_t^p + C_k - IC_t\right)}{full_t} ; \forall t \in \Omega_t \tag{12}$$

$D_{t,k}$ will be equal to 1 if installed capacity is less than the peak load at stage t after the outage of unit k, and 0 otherwise. The constraint given in (12) is valid for single outages, but higher order outages can be obtained in the same way.

Since the probability of load to be more than base load is equal to 1, binary values H_t are considered to force the probability of lost load to be 1 in case there is a capacity outage which results in the remained generation capacity to be less than base load. These binaries are obtained using (13).

$$\frac{IC_t - L^{min} L_t^p - C_k}{full_t} \le H_{t,k} \le 1 + \frac{IC_t - L^{min} L_t^p - C_k}{full_t} \tag{13}$$

$H_{t,k}$ will be equal to 1 if installed capacity is more than the base load at stage t after the outage of unit k, and 0 otherwise. The constraint given in (13) is valid for single outages, but higher order outages can be obtained in the same way. It should be noted that, in a large-scale power system it is very unlikely that $H = 0$, since this condition requires very high number of outages which have very low probabilities and thus, a negligible impact on LOLP. Therefore, H is assumed equal to 1 with a good approximation for the rest of the paper. Replacing $H = 1$ in (9) will result in (14).

$$LOLP_t = A' + B' + \cdots \tag{14}$$

where:

$$\mathrm{A}' = \sum_{\mathrm{k_1} \in \Omega_\mathrm{k}} \left[\left(\mathrm{U_{t,k_1}} . \mathrm{D_{t,k_1}} . \mathrm{F_{k_1}} . \prod_{\mathrm{k} \neq \mathrm{k_1}} \left(1 - \mathrm{U_{t,k}} \times \mathrm{F_k}\right) \right) \times \left(\left(\mathrm{L_t^p} + \mathrm{C_{k_1}} - \mathrm{IC_t}\right) . \mathrm{IP_t} \right) \right] \tag{15}$$

$$\mathrm{B}' = \sum_{\mathrm{k_1} \in \Omega_\mathrm{k}} \sum_{\mathrm{k_2} \in \Omega_\mathrm{k}} \left[\left(\mathrm{U_{t,k_1}} . \mathrm{U_{t,k_2}} . \mathrm{D_{t,k_1,k_2}} . \mathrm{F_{k_1}} . \mathrm{F_{k_2}} \times \prod_{\mathrm{k} \neq \mathrm{k_1,k_2}} \left(1 - \mathrm{U_{t,k}} \times \mathrm{F_k}\right) \right) \times \left(\left(\mathrm{L_t^p} + \mathrm{C_{k_1}} + \mathrm{C_{k_2}} - \mathrm{IC_t}\right) . \mathrm{IP_t} \right) \right] \tag{16}$$

According to (14), a product of two or more binary variables U and D is a nonlinear expression which must be linearized. This linearization results in a final binary variable called LB. The product of the binary variable LB and the continuous variable IC is then linearized using a new continuous variable LC [23]. After linearization, the resultant formulation would look like (17).

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$$LOLP_t = A'' + B'' + \cdots \; ; \forall t \in \Omega_t \tag{17}$$

where:

$$A'' = \sum_{k_1} \left[\left(LB_{k_1}^t . \left(L_t^p + C_{k_1} \right) - LC_{k_1}^t \right) \cdot IP_t . F_{k_1} . NL_{t,1}^{apx} \right] \tag{18}$$

$$B'' = \sum_{k_1 \in \Omega_k} \sum_{k_2 \in \Omega_k} \left[\left(LB_{k_1,k_2}^t \cdot \left(L_t^p + C_{k_1} + C_{k_2} \right) - LC_{k_1,k_2}^t \right) \times IP_t . F_{k_1} . F_{k_2} . NL_{t,2} \right] \tag{19}$$

$$NL_{t,1} = \prod_{k \neq k_1} \left(1 - U_{t,k} \times F_k \right) \; ; \; \forall t \in \Omega_t \tag{20}$$

$$NL_{t,2} = \prod_{k \neq k_1, k_2} \left(1 - U_{t,k} \times F_k \right) \; ; \; \forall t \in \Omega_t \tag{21}$$

where, $NL_{t,1}$ and $NL_{t,2}$ are the only terms remained non-linear in the LOLP formulation, which correspond to single and double outages, respectively. This is where the accurate approximations are needed to eliminate the non-linear terms. For instance, $NL_{t,1}$ is equivalent to:

$$NL_{t,1} = \frac{\prod_k \left(1 - U_{t,k} \times F_k \right)}{1 - F_{k_1}} ; \; \forall t \in \Omega_t \tag{22}$$

As can be seen, in (22) the denominator is not $\left(1 - U_{t,k_1} \times F_{k_1}\right)$, unlike what may be expected and may rise the question that (22) is not completely equivalent to (20). In this regard, it should be noted that binaries $U_{t,k}$ are included to indicate if unit k is available at stage t (either exists at stage t or constructed up to stage t) or not. Considering that equation (22) is a part of equation (19), it is clear that $U_{t,k}$ is included in (19) and linearized as LB binary variable, therefore where $U_{t,k_1} = 0$, so $LB_{k_1}^t = 0$ and thus the whole equation for outage of unit k_1 will be forced to zero. This results in equation (22) to have no effect in case $U_{t,k} = 0$, and therefore (22) is accurately equivalent to (20).

It can be seen that the denominator of (22) is linear, but the numerator still remains nonlinear. This nonlinearity is studied and resolved in next section.

B. *Approximations applied to LOLP formulation: First type approximations*

Up to this point, most of the LOLP formulation is linearized; The goal is now to find a mathematical alternative for the only non-linear part of (22). To do so, it's necessary to get a good realization of the numerator of (22). It's clear that this term depends on the product of the Forced Outage Rates (FOR) of the units available at stage t. This concept is redefined using several approximations. Let NU_t be the number of units with average capacity AC_t needed to supply peak load plus the reserve at stage t. The nonlinear part of (22) can be approximately expressed as 1 minus the average value of FORs of the units which can be potentially available at stage t (i.e. (1- AF_t)), to the power of NU_t. Therefore, the numerator of (22) can be replaced with (23) as follows:

$$j_t = (1 - AF_t)^{NU_t} ; \; \forall t \in \Omega_t \tag{23}$$

where:

$$AF_t = \frac{\sum_k F_k}{k} \;\; \forall \, k \in S_{t,k} \, , t \in \Omega_t \tag{24}$$

$$NU_t = \frac{(1 + ER^{low}) \times L_t^p}{AC_t} \;\; ; \; \forall t \in \Omega_t \tag{25}$$

$$AC_t = \frac{\sum_k C_k}{k} \;\; ; \; \forall \, k \in S_{t,k} \, , t \in \Omega_t \tag{26}$$

$S_{t,k}$ is the set of all existing plus potential candidate units k at stage t. Potential candidate units are those that have the possibility to be installed at stage t depending on the tunnel limit. Parameter ER^{low} is equal to R^{low} if $R^{low} \neq 0$, i.e. a minimum reserve is defined for the problem, otherwise, it will be given a predefined value which will be discussed throughout the simulated case study in detail.

Finally, the approximation of (22) is written as follows:

$$NL_{t,1}^{apx} = \frac{j_t}{1 - F_{k_1}} \;\; ; \; \forall t \in \Omega_t \tag{27}$$

and for double outages:

$$NL_{t,2}^{apx} = \frac{j_t}{\left(1 - F_{k_1}\right) \times \left(1 - F_{k_2}\right)} \;\; ; \; \forall t \in \Omega_t \tag{28}$$

This procedure can be applied to the higher order outages too. By replacing the non-linear parts of (17) with the corresponding approximations proposed in (27) and (28), the final linearized LOLP formulation will be as given in (29):

$$LOLP_t = A''' + B''' + \cdots \;\; ; \; \forall t \in \Omega_t \tag{29}$$

where:

$$A''' = \sum_{k_1} \left[\left(LB_{k_1}^t . \left(L_t^p + C_{k_1} \right) - LC_{k_1}^t \right) \cdot IP_t . F_{k_1} . NL_{t,1}^{apx} \right] \tag{30}$$

$$B''' = \sum_{k_1 \in \Omega_k} \sum_{k_2 \in \Omega_k} \left[\left(LB_{k_1,k_2}^t \cdot \left(L_t^p + C_{k_1} + C_{k_2} \right) - LC_{k_1,k_2}^t \right) \times IP_t . F_{k_1} . F_{k_2} . NL_{t,2}^{apx} \right] \tag{31}$$

C. *Second type approximation for large-scale systems*

Up to this point, a fairly accurate formulation for LOLP was presented. This method works fine with small-scale generation systems with low order outages; however, for large-scale power systems considering higher order outages, this approximation fails to give accurate results with a reasonable computational burden since trying to obtain the linearized form of higher order outages makes the problem size larger. To create a further approximation of LOLP under higher order outages (e.g. four or more simultaneous outages) another procedure is developed.

An averaging method similar to the one utilized in part B of this section is used to obtain an approximation of the different terms in (9) associated with the higher order outages (e.g. four or more simultaneous outages). In order to simplify the equations, a parameter is needed to approximate the amount of capacity outages in MW from order d at stage t, therefore the capacity outages will be independent from index k, the advantage of which will be discussed in next equations. Therefore, the amount of capacity outage caused by d

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simultaneous outages at each stage t is denoted by $C_{t,d}$ which replaces parameter C_k in the coming equations. Parameter C_{td}, in an accurate expression, is the average of capacity outages from order d at stage t in MW, that are possible to occur (i.e. only considering all existing plus potential candidate units at stage t) and would cause loss of load. That is because those outages should always be considered that cause loss of load, and the outages that don't cause loss of load should not affect the approximated calculations. Also, the approximated production of FORs of unavailable units (i.e. unavailable due to forced outage) at stage t, $(AF_t)^d$, will replace parameter F_k in (9). Parameter $C_{t,d}$ can be obtained for single and double outages by (32) and (34), respectively.

$$C_{t,d} = \frac{\sum_{i\in\Omega_g}^{N_g} \binom{N_i^e + t.N_i^{new}}{d} \times (d.C_i)}{\sum_{i\in\Omega_g}^{N_g} \binom{N_i^e + t.N_i^{new}}{d}} \;;\; \forall\, t \in \Omega_t \tag{32}$$

subject to the set of constraints (33):

$$\begin{gathered} d = 1 \\ ER^{low}.L_t^p < d.C_i < (1 + ER^{low} - L^{min}).L_t^p \\ N_i^e + t.N_i^{new} \geq d \end{gathered} \tag{33}$$

$$C_{t,d} = \sum_{r=1}^{d-1} \sum_{i=1}^{N_g} \sum_{j=i+1}^{N_g} \frac{M.\left(r.C_i + (d-r).C_j\right)}{M} \;;\forall\, t \in \Omega_t \tag{34}$$

subject to the set of constraints (35):

$$\begin{gathered} d = 2 \\ ER^{low}.L_t^p < r.C_i + (d-r).C_j < (1 + ER^{low} - L^{min}).L_t^p \\ N_i^e + t.N_i^{new} \geq r \\ N_j^e + t.N_j^{new} \geq d - r \\ N_i^e + t.N_i^{new} + N_j^e + t.N_j^{new} \geq d \\ M = \binom{N_i^e + t.N_i^{new}}{r}.\binom{N_j^e + t.N_j^{new}}{d-r} \end{gathered} \tag{35}$$

When $C_{t,d} = 0$, it means that d number of simultaneous outages does not result in any loss of load at stage t (equivalent to $D_{t,k_1,k_2,\dots k_d} = 0$). Again, parameter $C_{t,d}$ corresponding to higher order outages can be obtained using a similar procedure given in (32) and (34). d is left as a parameter in (32) and (34) only to make the process of obtaining the formulations corresponding to higher order outages easier for the reader; in other words, $d = 1$ number of simultaneous outages means single outages and $d = 2$ number of simultaneous outages means double outages.

Considering the above descriptions, the formulation corresponding to the term dedicated to the d^{th} order outage of (9) at stage t is as follows:

$$\begin{aligned} OUT_{t,d} = \sum_{k_1} \dots \sum_{k_d} \Big[& U_{t,k_1} \times U_{t,k_2} \times \dots \times U_{t,k_d}.D_{t,k_1,k_2,\dots k_d} \\ & \times \left(L_t^p + C_{t,d} - IC_t\right) \\ & \times IP_t.(AF_t)^d.\frac{j_t}{(1-AF_t)^d} \Big] \end{aligned} \tag{36}$$

In (36), the following term is independent of index k:

$$X_{t,d} = \left(L_t^p + C_{t,d} - IC_t\right) \times IP_t.(AF_t)^d.\frac{j_t}{(1-AF_t)^d} \tag{37}$$

So, (36) can be re-written as below:

$$\begin{aligned} OUT_{t,d} = X_{t,d} \times \sum_{k_1} \dots \sum_{k_d} \big[& U_{t,k_1} \times U_{t,k_2} \times \dots U_{t,k_d} \\ & \times D_{t,k_1,k_2,\dots k_d} \big] \end{aligned} \tag{38}$$

In (38), the non-linear terms under summation can be approximated as follows:

$$\begin{aligned} & \sum_{k_1} \dots \sum_{k_d} \left[U_{tk_1} \times U_{t,k_2} \times \dots U_{t,k_d} \times D_{t,k_1,k_2,\dots k_d} \right] \\ & = \begin{cases} \binom{NU_t}{d};\; C_{t,d} \neq 0 \\ 0;\; C_{t,d} = 0 \end{cases} \end{aligned} \tag{39}$$

where, $\binom{NU_t}{d}$, i.e. d possible outage combinations of NU_t approximated available units, replaces the summation on the product of U binaries and the condition $C_{t,d} \neq 0$ ensures that (39) is only calculated when $D_{t,k_1,k_2,\dots k_d} = 1$.

Using(37), (38) and (39), the final formulation for the second type of approximations is presented in (40):

$$OUT_{t,d} = \binom{NU_t}{d}.X_{t,d};\; \forall\, t \in \Omega_t \tag{40}$$

Now the only remained variable in (40) is IC_t and the values of all other parameters are deterministically calculated, therefore, the size of the optimization problem is reduced significantly. This results in the computation burden to become less which is useful if higher order outages are to be considered. It's clear that d is the desired outage order to be considered by the second type approximations and should be chosen higher than the number of outages calculated by the first-type approximations and lower than the desired degree of outages. For example, if three outages are to be considered for LOLP calculation by the first-type approximations presented in (29), d should be chosen higher than 3 and lower than a desired value depending on the desired accuracy. Using this new approximation, the number of outages for LOLP calculations is not as limited as before and a good number of outages, for instance up to 15, can be calculated.

With the newly obtained approximations for higher order outages, the final LOLP formulation for large-scale problems will be extracted as given in (41).

$$LOLP_t = \overbrace{A'' + B'' + \cdots}^{1^{st}\text{type approx.}} + \overbrace{\sum_d OUT_{t,d}}^{2^{nd}\text{type approx.}} \;;\; \forall\, t \in \Omega_t \tag{41}$$

IV. Case Study

A. Test System

The proposed LOLP constrained GEP model is applied to the test system given in [13] with some modifications. The test system includes 12 types of installed power plants and 5 types of candidate plants. The planning horizon is 14 years divided into 7 planning stages, i.e. each stage is 2 years. The forecasted

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peak load of each stage is presented in Table I. The data for the existing and candidate power plants are given in Table II and Table III, respectively.

Other criteria and system-related values are assumed as follows. In order to signify the accuracy of LOLP calculations, the lower bound of reserve margin is assumed to be zero and no capacity mix constraints are considered; however, an upper reserve margin equal to 60% of peak load is considered for faster solution convergence. $ER^{low} = 0.15$ is chosen as a typical value for practical power systems. ER^{low} (abbreviated of Estimated Reserve) in (25) is in fact an estimate by the planner of how much reserve will be available at each stage. Considering the proposed formulation, the lower this parameter is chosen, the higher the reliability will be, resulting in a more conservative plan and vice versa. This provides a desired flexibility for the planner. The discount rate is 8.5% and the upper bound for LOLP value is 0.01 [13].

TABLE I
THE AMOUNT OF FORECASTED PEAK LOAD FOR THE PLANNING HORIZON

Stage	1 2018	2 2020	3 2022	4 2024	5 2026	6 2028	7 2030
Peak Load (MW)	8000	10000	11500	13000	14500	15500	17000

TABLE II
DATA OF THE EXISTING PLANTS

Name	No. of Units	Unit Capacity (MW)	FOR (%)	Operating Cost ($/kWh)	Maintenance Cost ($/kW-month)
Oil#1	1	200	7.0	0.024	2.25
Oil#2	1	200	6.8	0.027	2.25
Oil#3	1	150	6.0	0.030	2.13
LNG#1	3	50	3.0	0.043	4.52
LNG#2	1	400	10.0	0.038	1.63
LNG#3	1	400	10.0	0.040	1.63
LNG#4	1	450	11.0	0.035	2.00
Coal#1	2	250	15.0	0.023	6.65
Coal#2	1	500	9.0	0.019	2.81
Coal#3	1	500	8.5	0.015	2.81
Nuclear#1	1	1000	9.0	0.005	4.94
Nuclear#2	1	1000	8.8	0.005	4.63

TABLE III
DATA OF THE CANDIDATE PLANTS

Name	Tunnel limit	Unit Capacity (MW)	FOR (%)	Operating Cost ($/kWh)	Maintenance Cost ($/kW-month)	Capital Cost ($/kW)
Oil	5	200	7.0	0.021	2.20	812.5
LNG	4	450	10.0	0.035	0.90	500.0
Coal	3	500	9.5	0.014	2.75	1062.5
Nuc. (PWR)	3	1000	9.0	0.004	4.60	1625.0
Nuc. (PHWR)	3	700	7.0	0.003	5.50	1750.0

Minimum load and average load at each stage are assumed to be 30% and 70% of the peak load, respectively. The developed MIP formulation for dynamic LOLP-constrained GEP model is solved using CPLEX solver in GAMS software [24]. The simulation was carried out using Intel CPU Core i7-3.60 GHz with 12 GB of memory and simulation time was 1 hour and 56 minutes for the main and most accurate case of the proposed method which is Case 5. This is a reasonable CPU time considering a dynamic 7-stage GEP with a total number of possible expansion units equal to 125.

B. *Results and Discussion*

The proposed MIP formulation was applied to the described test system and the results are compared to the conventional LOLP calculation method utilized in [19-21]. The conventional method uses a simple approximation by calculating LOLP as the sum of the probability of outages considering up to two simultaneous outages; however, it suggests that higher order outages can be simulated too.

In this study, outages up to the 10th level were considered where up to 3rd level of outages were obtained by the first type of approximations presented in (29) and the rest were calculated by (40). The number of outages is flexible and the 10 outages used in this paper can be increased or decreased to make a trade-off between reliability and economic benefits/computational burden. There is also a flexibility in choosing the number of outages to be calculated by each type of approximations of the proposed method. It should be noted that assigning a high level of outages to the first type approximations increases the accuracy slightly but the computational burden significantly. Therefore, the first type of approximation was utilized to approximate the LOLP for up to 3rd order outages.

In this study, the results of five different cases for the proposed method PM (i.e. Cases 1-5) and two cases for the conventional method CM (i.e. Cases 6-7) are presented. The cases 1-5 reported for the proposed method are carried out considering different outage orders; i.e. cases 1-5 correspond to outage orders considered equal to 2, 3, 5, 7 and 10, respectively. The conventional method includes cases 6 and 7 which correspond to outage orders considered equal to 2 and 3, respectively. The reason behind considering only two cases for the conventional method is that, as will be discussed in the following results, case 6 results in a very unreliable plan, and case 7 results in a very conservative plan, therefore, considering higher outages will have no positive effect but only would lead to more conservative plans. In all cases, the main GEP problem was formulated similarly and only the LOLP formulation differs between the proposed method and the conventional method.

The dynamic generation expansion plan using Case-5 (i.e. the case in which LOLP is determined for outages up to 10th order) of the proposed method is presented in Table IV. This is the main and most accurate case of the proposed method and it can be seen that expensive Nuclear plants were rarely chosen for construction while other less-expensive units are constructed frequently.

Cumulative number of installed units of each type for different cases is presented in Table V. The difference between the obtained configurations of expansion plans of simulated cases proves that the accuracy of the utilized LOLP formulation has a vital role in dynamic GEP problem. Also for a better comparison the installed capacities obtained by the proposed

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method (i.e. case-5) and the conventional method (i.e. cases 6 and 7) are visually illustrated in Fig. 3. Comparing the results reported in Table V and Fig. 3, it can be seen that while case 5 and case 7 result in a close cumulative installed capacity, they have different GEP configurations at some stages which clearly demonstrates the impact of the proposed method on decision-making.

TABLE IV
THE EXPANSION PLAN OBTAINED BY THE PROPOSED METHOD

Stage	Plant Type				
	Oil	LNG C/C	Coal	Nuc. (PWR)	Nuc. (PHWR)
1	0	4	1	2	0
2	0	7	3	2	0
3	0	7	6	2	0
4	2	9	7	2	0
5	3	11	8	2	0
6	4	13	8	2	0
7	8	15	8	2	0

Table VI presents the reliability oriented results obtained for different cases. As mentioned above, Cases 1-5 present the results of the proposed method for different outage orders, amongst which case 5 is the main and best case of the proposed method and the rest are presented only to show the improvement of results with increasing the outage orders. Case 6 and case 7 report the results of the conventional method for double and triple outages, respectively. To have a fair and clear comparison, the LOLP corresponding to the configuration of each case was calculated by the exact COPT-based formulation [22]. The difference between the exact LOLP and the approximated LOLP is interpreted as the violation (i.e. a measure of accuracy).

Analysis and comparison of the amount of LOLP violations and the total cost of each method gives a good insight of efficiency of the methods. According to Case 6 in Table VI, which considers up to 2nd order outages and presents the results of the conventional method, it can be seen that the inaccurate calculations resulted in an unreliable condition for the power system. In this case, the LOLP violations are about 2 or 3 times the permissible bound at some stages of the planning horizon. Case 7 presents the conventional method considering up to third order outages. The reverse situation holds true for Case 7. In this case, the inaccurate LOLP calculations led to a nearly conservative schedule which resulted in an expensive plan.

These results verify that considering higher order outages using the conventional method is ineffective since by including higher order outages the obtained plan becomes too conservative leading to exorbitant costs and the computational burden/time would grow exponentially which makes the method useless for large-scale problems. On the other hand, according to Table VI, the results of the proposed method which are carried out under different outage orders in 5 different cases are more satisfying.

The importance of considering higher-order outages in a long-term planning study can also be seen from Table V and Table VI where, for instance, comparing cases 2 and 5 proves a significant difference in the obtained expansion plans and reliability violations. From Table V, it can be seen that considering up to 10th order outages has resulted in more capacity to be installed to improve the reserves margin. Reliability-wise, by looking at the results corresponding to stage 7, it’s also clear in Table VI that including higher order outages has significantly improved the reliability form case 1 to case 5. This considerable decrement in reliability violations can also visually be seen in Fig. 4.

Fig. 4 shows the sum of LOLP violations in percent for different cases. It’s clear that case 5 of the proposed method has led to the second-least LOLP violations with a value very close to the conservative and non-economical Case 7. In Case 5, the violation of LOLP has only occurred in two stages with an acceptable error. The values for other stages were all under the predetermined threshold and close to the bound, which presents a good accuracy that gives the most economical results.

LOLP is graphically presented in Fig. 5 for Cases 5, 6 and 7. It is evident that, as expected, Case 6 resulted in unacceptable probability of load loss whereas Case 7 gave a highly conservative plan in some stages and a violating result at other stages; questioning the efficiency of the conventional method. In this regard, the LOLP obtained by Case 5 is reasonable and shows acceptable results in most stages.

TABLE V
CUMULATIVE NUMBER OF INSTALLED UNITS FOR DIFFERENT CASES

Plant Type	Case no.	Planning Stages						
		1	2	3	4	5	6	7
OIL	1	0	0	0	0	1	1	1
	2	0	0	0	0	1	1	1
	3	0	0	0	1	1	1	2
	4	0	0	0	2	3	4	8
	5	0	0	0	2	3	4	8
	6	4	4	4	4	5	6	6
	7	1	2	2	2	2	3	4
LNG	1	2	5	6	8	10	12	16
	2	3	5	6	8	10	12	16
	3	3	7	7	8	11	13	16
	4	3	7	7	9	11	13	15
	5	4	7	7	9	11	13	15
	6	4	5	5	5	8	10	13
	7	3	7	7	7	10	12	15
Coal	1	0	2	4	6	7	7	7
	2	0	2	4	6	7	7	7
	3	0	1	4	6	7	7	7
	4	0	1	4	5	6	6	6
	5	1	3	6	7	8	8	8
	6	3	6	9	12	12	12	12
	7	1	1	4	7	7	7	7
Nuc. (PWR)	1	3	3	3	3	3	3	3
	2	3	3	3	3	3	3	3
	3	3	3	3	3	3	3	3
	4	3	3	3	3	3	3	3
	5	2	2	2	2	2	2	2
	6	0	0	0	0	0	0	0
	7	3	3	3	3	3	3	3
Nuc. (PHWR)	1	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0

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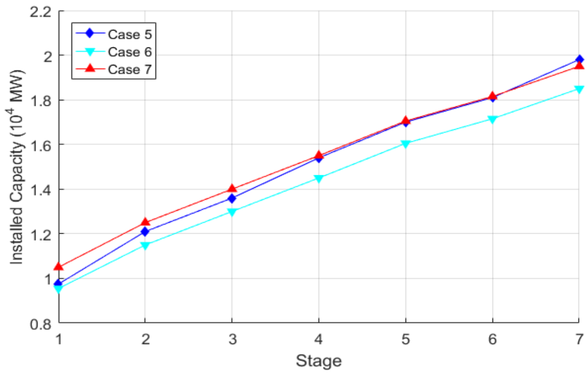

Fig. 3. Installed capacity obtained by Cases 5-7 at different stages

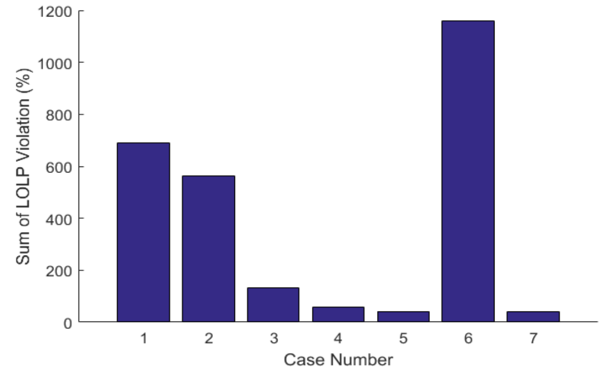

Fig. 4. Sum of LOLP violations (%) of different cases

TABLE VI
RELIABILITY ORIENTED RESULTS OF DIFFERENT CASES

Method	**Case 1- PM - 2 outages**		**Case 2- PM - 3 outages**		**Case 3- PM - 5 outages**		**Case 4- PM - 7 outages**		**Case 5- PM - 10 outages**		**Case 6- CM- 2 Outages**		**Case 7- CM- 3 Outages**	
LOLP	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)
Stage 1	0.025	150%	0.0126	26%	0.0126	26%	0.0126	26%	0.0124	24%	0.0129	29%	0.0035	0
Stage 2	0.0187	87%	0.0187	87%	0.0103	3%	0.0103	3%	0.0094	0	0.0194	94%	0.0051	0
Stage 3	0.0236	136%	0.0236	136%	0.0126	26%	0.0126	26%	0.0118	18%	0.0238	138%	0.0068	0
Stage 4	0.0163	63%	0.0163	63%	0.0122	22%	0.0096	0	0.0090	0	0.0283	183%	0.0093	0
Stage 5	0.0168	68%	0.0168	68%	0.0092	0	0.0102	2%	0.0096	0	0.0309	209%	0.0095	0
Stage 6	0.0212	112%	0.0212	112%	0.0120	20%	0.0101	1%	0.0095	0	0.0299	199%	0.0093	0
Stage 7	0.0173	73%	0.0173	73%	0.0134	34%	0.0089	0	0.0084	0	0.0406	306%	0.0139	39%
Cost × 10^{10}	1.7298249 $		1.7335279 $		1.7491921 $		1.7571704 $		1.7580609 $		1.7326114 $		1.7720148 $	

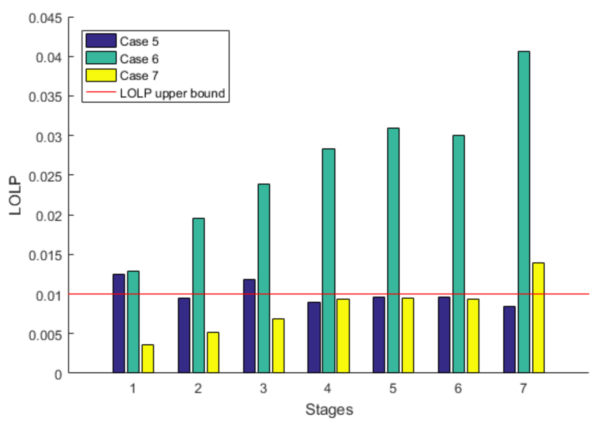

Fig. 5. LOLP obtained by Cases 5-7

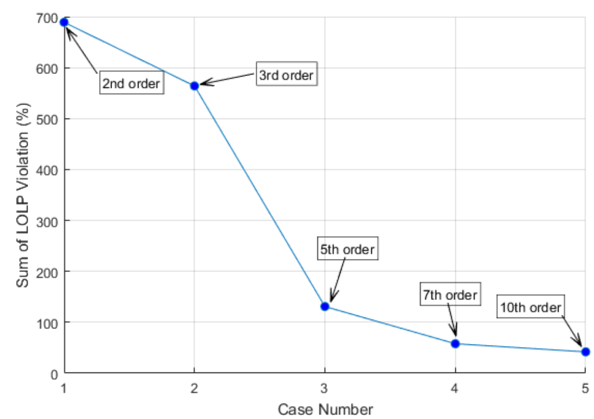

Fig. 6. Sum of LOLP violations (%) for different cases of the proposed method

TABLE VII
RESULTS OF CASE 5 (i.e. PROPOSED METHOD CONSIDERING UP TO 10TH DEGREE OUTAGES) FOR DIFFERENT LOLP THRESHOLDS

LOLP Threshold	**0.0095**		**0.01**		**0.012**		**0.015**		**0.018**		**0.02**	
LOLP	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)	value	Vio. (%)
Stage 1	0.0126	32%	0.0124	24%	0.0126	6%	0.0126	0	0.0126	0	0.0126	0
Stage 2	0.0103	8%	0.0094	0	0.0103	0	0.0103	0	0.0187	7%	0.0187	0
Stage 3	0.01	5%	0.0118	18%	0.0126	6%	0.0126	0	0.0126	0	0.0126	0
Stage 4	0.0104	9%	0.0090	0	0.0122	2%	0.0122	0	0.0122	0	0.0163	0
Stage 5	0.0091	0	0.0096	0	0.0102	0	0.0102	0	0.0122	0	0.0122	0
Stage 6	0.0090	0	0.0095	0	0.0104	0	0.0101	0	0.0120	0	0.0124	0
Stage 7	0.0089	0	0.0084	0	0.0089	0	0.0114	0	0.0134	0	0.0139	0
Cost × 10^{10}	1.7691648 $		1.7580609 $		1.7546185 $		1.7510202 $		1.7426003 $		1.7416157 $	

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Comparing case 2 from the proposed method and case 7 from the conventional method since 2nd order outages are considered in both, this question may arise that case 2 has led to violations too. In order to further clear this issue, it should be noted that the authors never propose case 2 as an appropriate or final case of the proposed method. Cases 1-3 are mostly included to show the reasonable and natural improvement of the proposed method with increasing the outage orders. That is, case 2 results in considerable violation because it only considers up to 2nd order outages and if this does not occur, it means the method is not accurate. In other words, it is clear that as we increase the considered outage orders, we should obtain a better optimized expansion plan (i.e. a better plan is the one with least costs and least violations). Considering the conventional method; increasing the outage order to 10 would result in a heavily conservative method which would increase costs significantly, that is while the computation time for case 7 , which considers only up to 3rd order outages, was about 8 hours on the same hardware and calculation of 10th order outages would be probably impossible to be done with the conventional method. Whereas, in the proposed method, the violations and costs improve with increasing outage orders and a good plan, i.e. case5, is obtained with reasonable computation time.

Fig. 6 shows sum of LOLP violations in percentage for different cases of the proposed method. Cases 1 to 5 are implemented with up to 2nd, 3rd, 5th, 7th, and 10th order outages, respectively. As can be seen, the violations have decreased considerably when considering more than 7 simultaneous outages (case 4). It shows that the accuracy of the proposed method significantly increases as the order of simultaneous outages is increased.

Different LOLP thresholds were applied to the proposed method and the results obtained considering up to 10th order outages are presented in Table VII. The LOLP violation is acceptable at all stages of different threshold cases. It should be noted that the problem becomes infeasible with an LOLP threshold lower than 0.0093 considering the upper reserve margin equal to 60% of peak load. It is therefore evident that accuracy of the proposed method is acceptable even at a strict LOLP limit of 0.0095. The results obtained by the conventional method considering up to two outages for all LOLP thresholds are equal to the results presented in Case 6 of Table VI, which again indicates the inaccuracy of the conventional method.

Our main purpose in this article was to show the impact of LOLP criteria on GEP problem. In order to fulfill this goal, we had to increase the yearly peak loads and also considered an upper reserve limit of 60% for faster convergence of the problem. In order to verify the capability of the proposed method for problems with stricter LOLP thresholds, another case was run with some modified data in which the FOR of units are divided by 2 and the upper limit of installed capacity is chosen 5 times the peak load. In this case, the LOLP threshold was chosen equal to 0.0003 which is a real world standard and up to 10th order outages were calculated for obtaining the LOLP value. The results of this case can be seen in Table VIII and it's clear that the violations are again acceptable.

TABLE VIII
THE RESULTS OBTAINED BY THE PROPOSED METHOD FOR LOLP CRITERIA OF 0.0003

Stage	Plant Type					LOLP	
	Oil	LNG C/C	Coal	Nuc. (PWR)	Nuc. (PHWR)	value	Vio. (%)
1	5	4	2	1	0	0.000308	3%
2	7	8	2	1	0	0.000234	0
3	8	11	2	1	0	0.000273	0
4	9	12	4	1	0	0.000242	0
5	10	15	4	1	0	0.000280	0
6	10	17	4	1	0	0.000434	44%
7	12	18	4	2	0	0.000273	0

V. CONCLUSIONS

A dynamic GEP formulation considering an accurate LOLP calculation was presented in this paper. In GEP problem, the priority is given to the globality of the expansion solutions, so the traditional MINLP formulation of GEP problem was converted to a MIP one with reasonable CPU time of calculations. The developed linearized LOLP-constrained GEP model will guarantee the global solution for expansion plan. It was shown that ignoring the higher order outages in LOLP calculation inside the optimization model causes significant errors in final costs and expansion plans. Therefore, two types of approximations were proposed to obtain an optimal result in an acceptable computation time.

By comparing to other conventional methods, the obtained results verified the accuracy of the proposed method in calculating LOLP reliability index. The utilized linear approximation of LOLP index has no limitation in considering simultaneous higher order outages.

Considering the flexibility of approximations, the proposed method has also the capability to be applied to real large-scale generation expansion planning problems with a compromise between LOLP value accuracy and computation burden/time.

The uncertainty of the input parameters such as yearly peak loads was not considered in this study. Such uncertainties create a new dimension to the optimization model of dynamic GEP problem which needs powerful and efficient computational tools. Also the impact of nonlinear LDC on LOLP calculation is still an open question for future works. Another challenge of GEP problem and especially the reliability-constrained GEP is the consideration of intermittent renewable power generation technologies. Their consideration is twofold. First, an accurate reliability model must be developed and second, the LOLP should be reformulated to consider the model. In [25-27] , some reliability models of renewable resources have been proposed. Another important aspect is their uncertainties which need to be investigated. Consideration of intermittent power generation technologies may be investigated in future research works.

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