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Math for Machine Learning

Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix

W2 Lesson 1

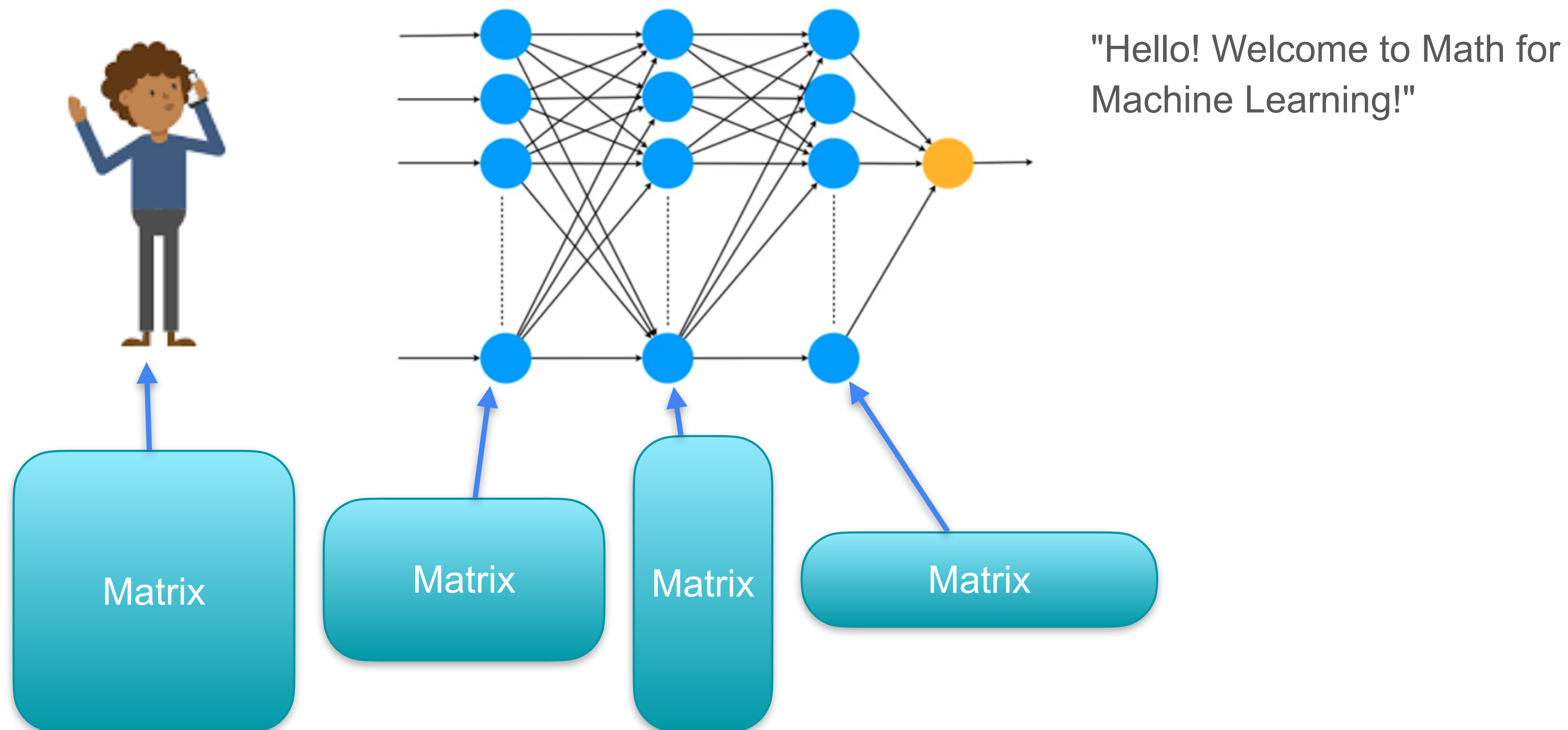


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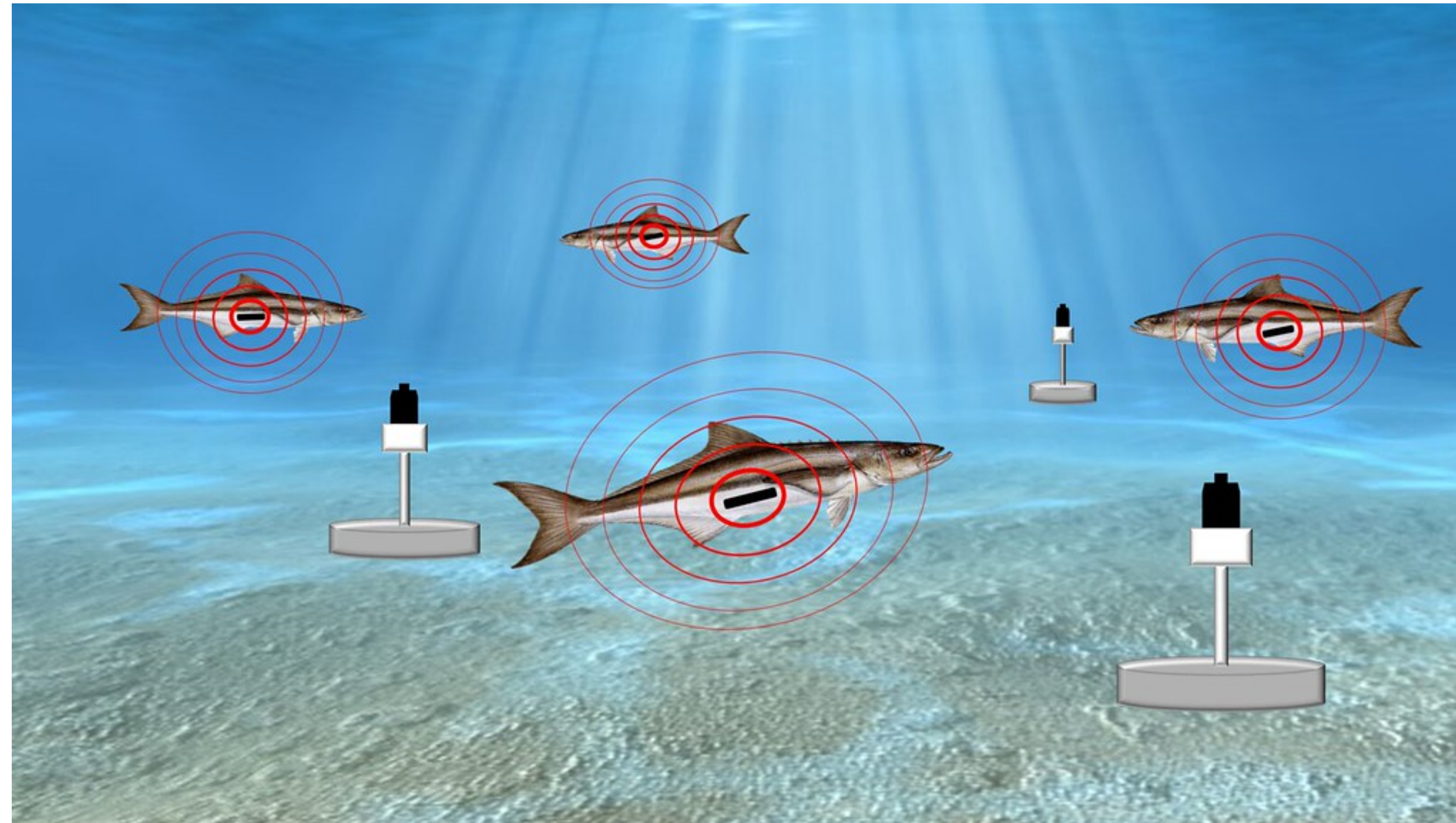
Solving System of Linear Equations

Machine learning motivation

Neural networks - Matrix operations



Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.





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Solving System of Linear Equations



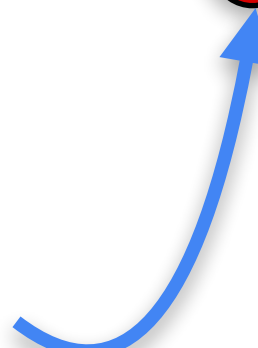

**Solving non-singular system
of linear equations**




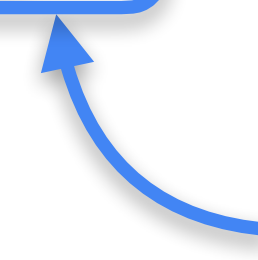
Solving systems of equations

System

- $a + b = 10$
 

- $a + 2b = 12$
 

 +  = \$10
\$8  \$2 

 +  +  = \$12
 \$2

Solving systems of equations

System

- $a + b = 10$



- $a + 2b = 12$



Some process

Manipulating equations

Swapping equations

Adding equations

Multiplying equations by a constant

Solved system

- $a = 8$



- $b = 2$



Solving systems of equations

System

- $a + b = 10$



- $a + 2b = 12$



Eliminate 'a' from this equation

Solved system

- $a = 8$



- $b = 2$



Manipulating equations

Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 22 \\ \hline 3a + 4b = 32 \end{array}$$

Systems of equations

System

- $5a + b = 17$
- $4a - 3b = 6$

Eliminate 'a'
from this equation

Divide by coefficient of a

- $a + 0.2b = 3.4$
- $a - 0.75b = 1.5$

Subtract equation 1 from equation 2

$$a - 0.75b = 1.5$$

$$\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline \end{array}$$

$$0a - 0.95b = -1.9$$

$$-0.95b = -1.9$$

$$b = 2$$

Solved system

- $a = ?$ 3
- $b = ?$ 2

$$a + 0.2(2) = 3.4$$

$$a + 0.4 = 3.4$$

$$a = 3$$

What if one of the coefficients of a is zero?



Quiz

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- Solve the following system of equations

System

- $2a + 5b = 46$
- $8a + b = 32$

Solution

- $a = 3$
- $b = 8$

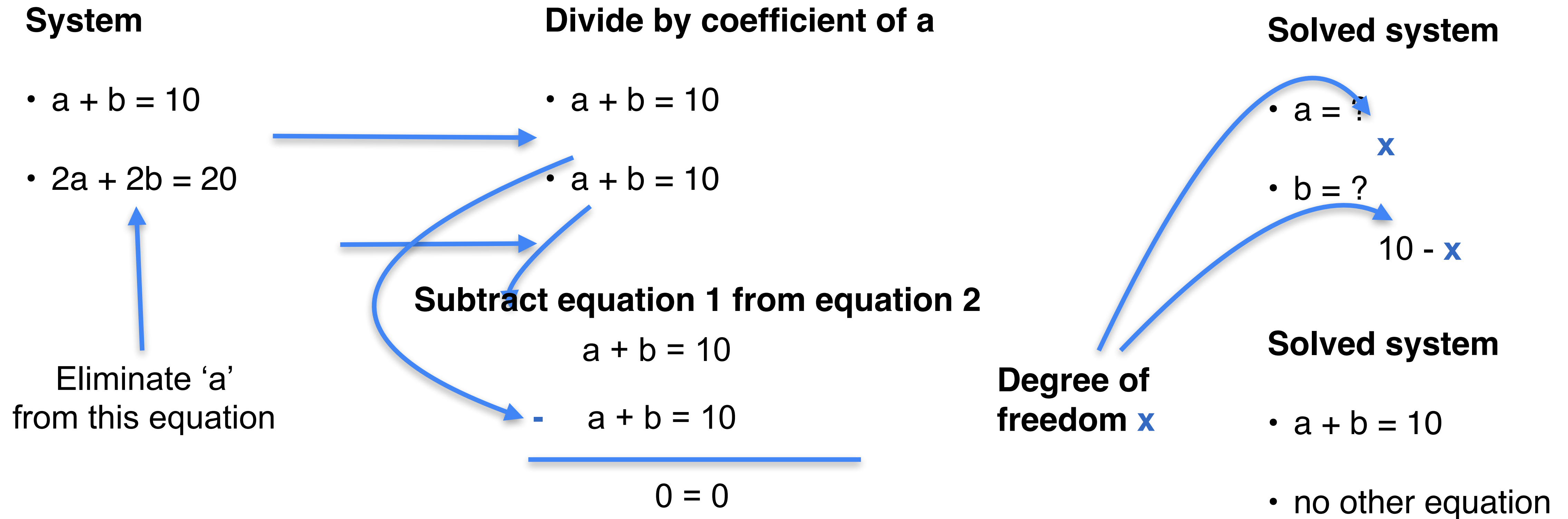


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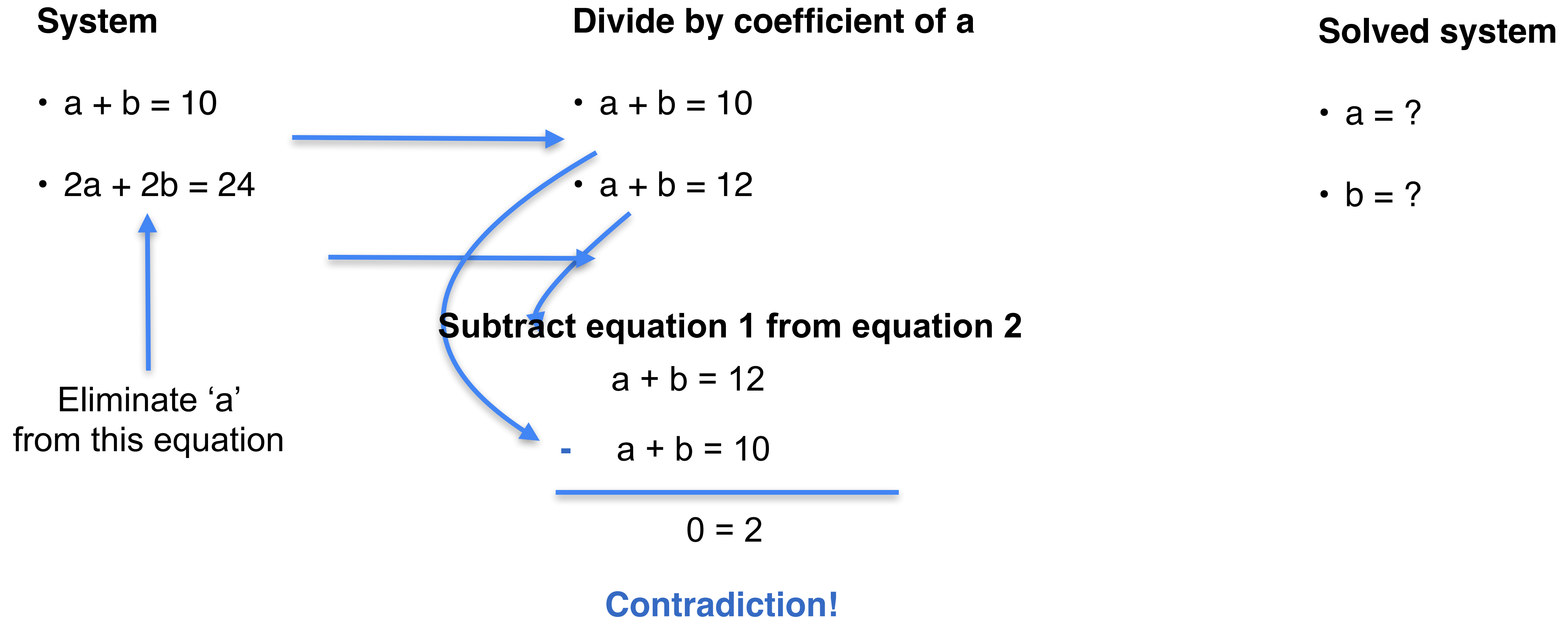
Solving System of Linear Equations

**Solving singular system of
linear equations**

What if the system is singular (redundant)?



What if the system is singular (contradictory)?



Quiz

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution

- Solve the following system of equations

System

- $5a + b = 11$
- $10a + 2b = 22$

Solution: If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



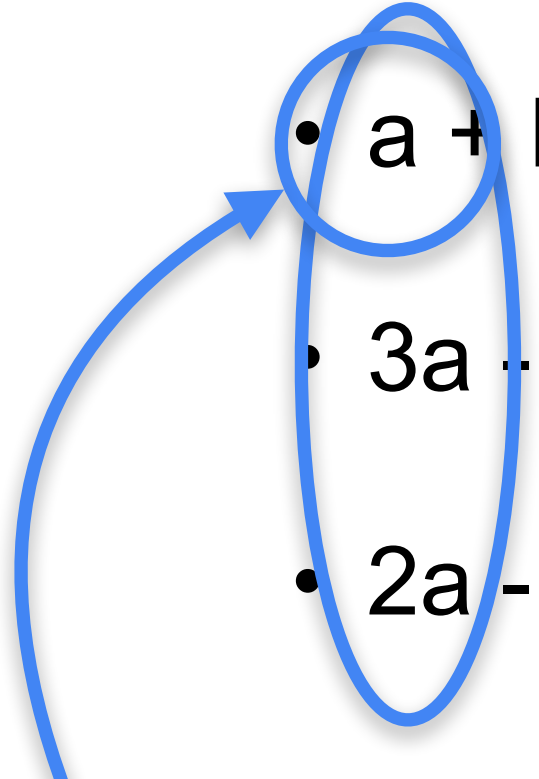
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Solving System of Linear Equations

**Solving system of equations
with more variables**

Elimination method

System



- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by
itself

Divide each row
by the
coefficient of 'a'

- $a + b + 2c = 12$

- $a - b - \frac{1}{3}c = 1$

- $a - \frac{b}{2} + 3c = 12$

Use the first
equation to
remove 'a' from
the others



- $a + b + 2c = 12$

- $-2b - \frac{7}{3}c = -11$

- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new
system of 2
equations

Elimination method

System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$

Elimination method

System

- $a + b + 2c = 12$ 
- $b + \frac{7}{6}c = \frac{11}{2}$ 
- $c = 3$

$a + 2 + 6 = 12$
 $a = 4$

$b + \frac{7}{2} = \frac{11}{2}$
 $b = 2$

Replace $c = 3$
in the second
equation, get
 $b = 2$

Replace $c = 3$
and $b = 2$ in the
first equation,
get $a = 4$

The solution is
 $a = 4$
 $b = 2$
 $c = 3$



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Solving System of Linear Equations

Matrix row reduction

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Systems of equations to matrices

Original system

- $a + b = 10$
- $2a + 2b = 20$

Intermediate System

- $a + b = 10$
- $0a + 0b = 0$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

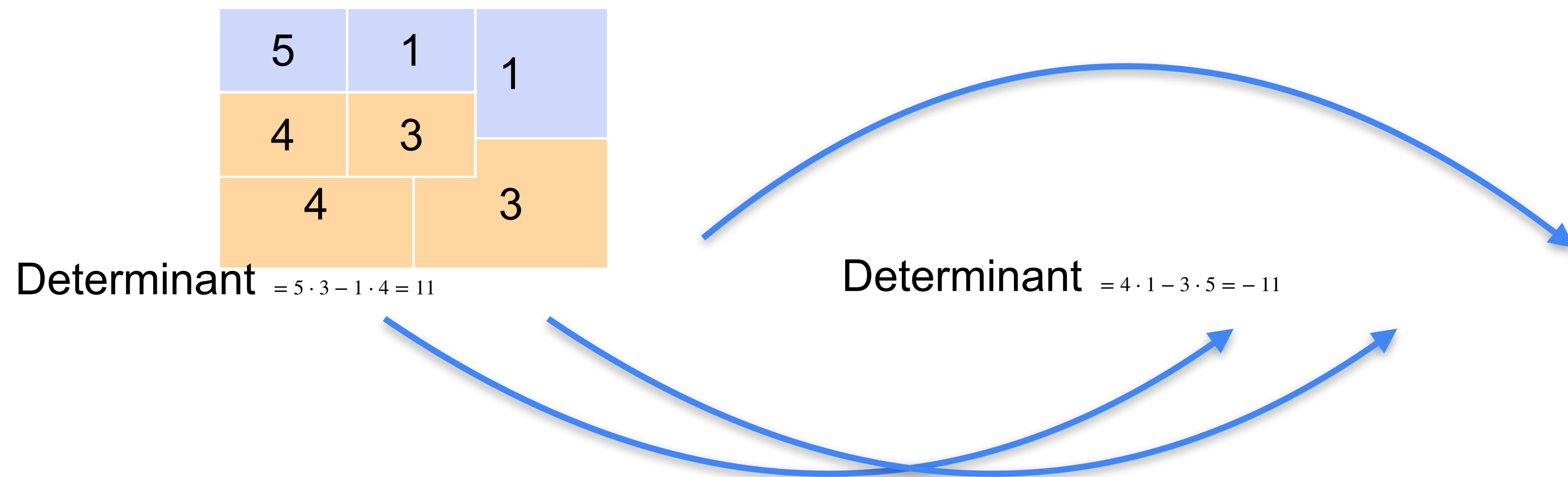


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Solving System of Linear Equations

**Row operations that preserve
singularity**

Switching rows



Multiplying a row by a (non-zero) scalar

5	1	1
4	3	
4		-3

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

$\times 10 =$

50	10
----	----

Determinant $= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$

$= 10 \cdot 11$

Adding a row to another row



The diagram shows a 3x3 matrix. The top row consists of three blue blocks with values 5, 1, and 1. The middle row consists of two orange blocks with values 4 and 3, followed by a blue block with value 1. The bottom row consists of two orange blocks with values 4 and 3.

Determinant $= 5 \cdot 3 - 1 \cdot 4$

$= 11$

+



The diagram shows a 2x2 matrix with two teal blocks. The top row has blocks with values 9 and 4. The bottom row has blocks with values 4 and 3.

Determinant $= 9 \cdot 3 - 4 \cdot 4$

$= 11$

W2 Lesson 2



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Solving System of Linear Equations

Rank of a matrix

Compressing Images - Reducing rank

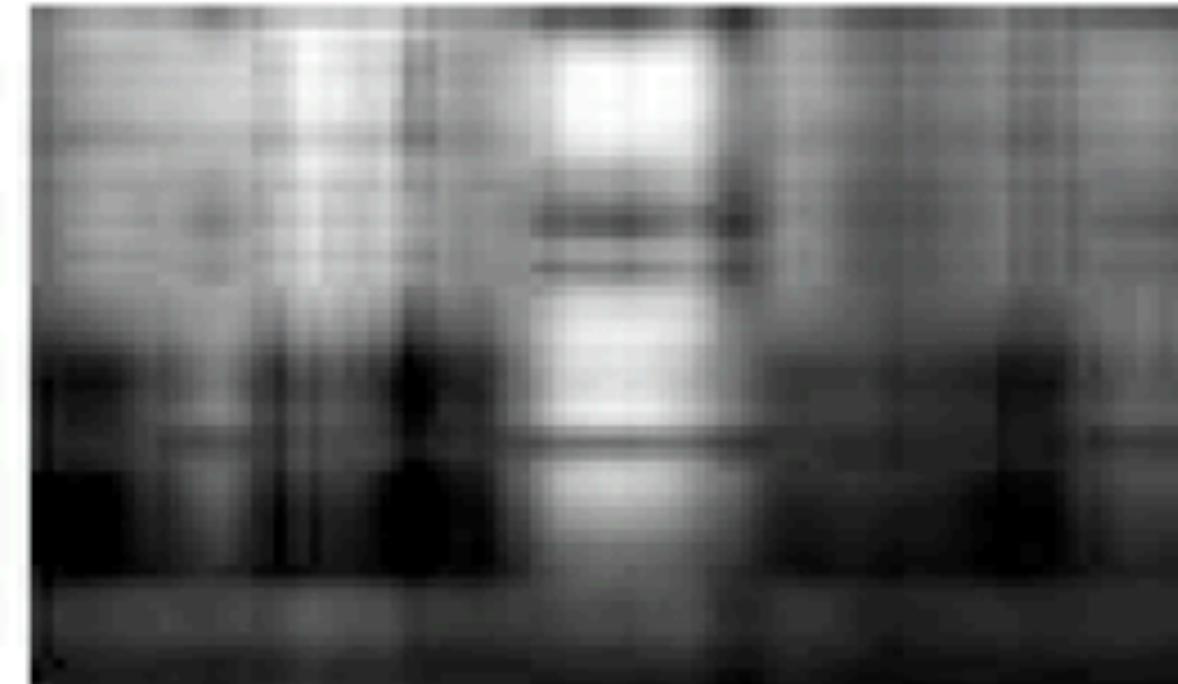
Original (Rank 200)



Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2


 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3

 The dog
 The dog

Two sentences

Zero pieces of information

Rank = 0

Systems of equations

System 1

$a + b = 0$
 
 $a + 2b = 0$
 

	
1	1
1	2





Rank = 2

Two equations

Two pieces of information

Rank = 2

System 2

$a + b = 0$
 
 $2a + 2b = 0$
 

	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

System 3

$0a + 0b = 0$
 $0a + 0b = 0$

	
0	0
0	0



Rank = 0

Two equations

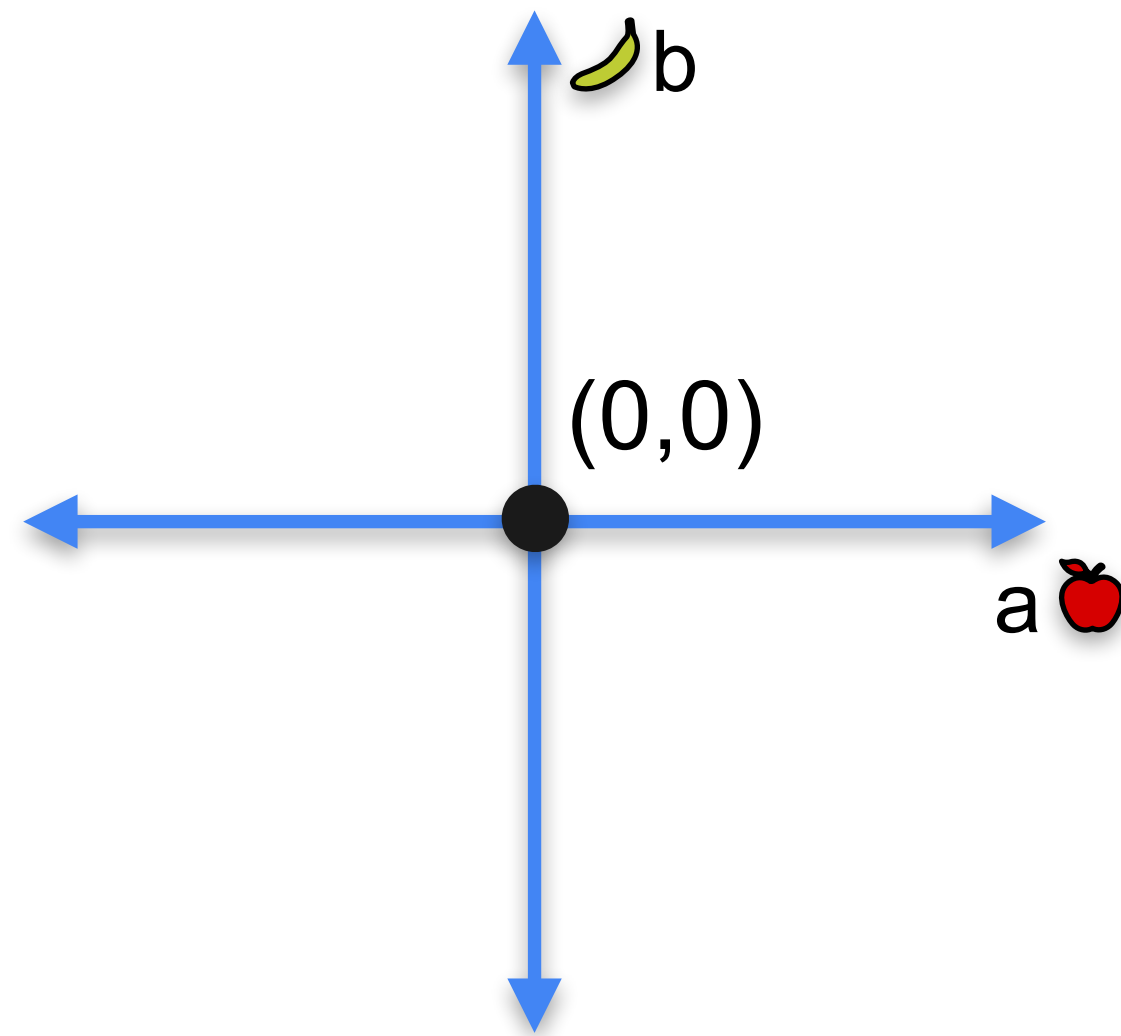
Zero pieces of information



Rank = 0

Rank and solutions to the system

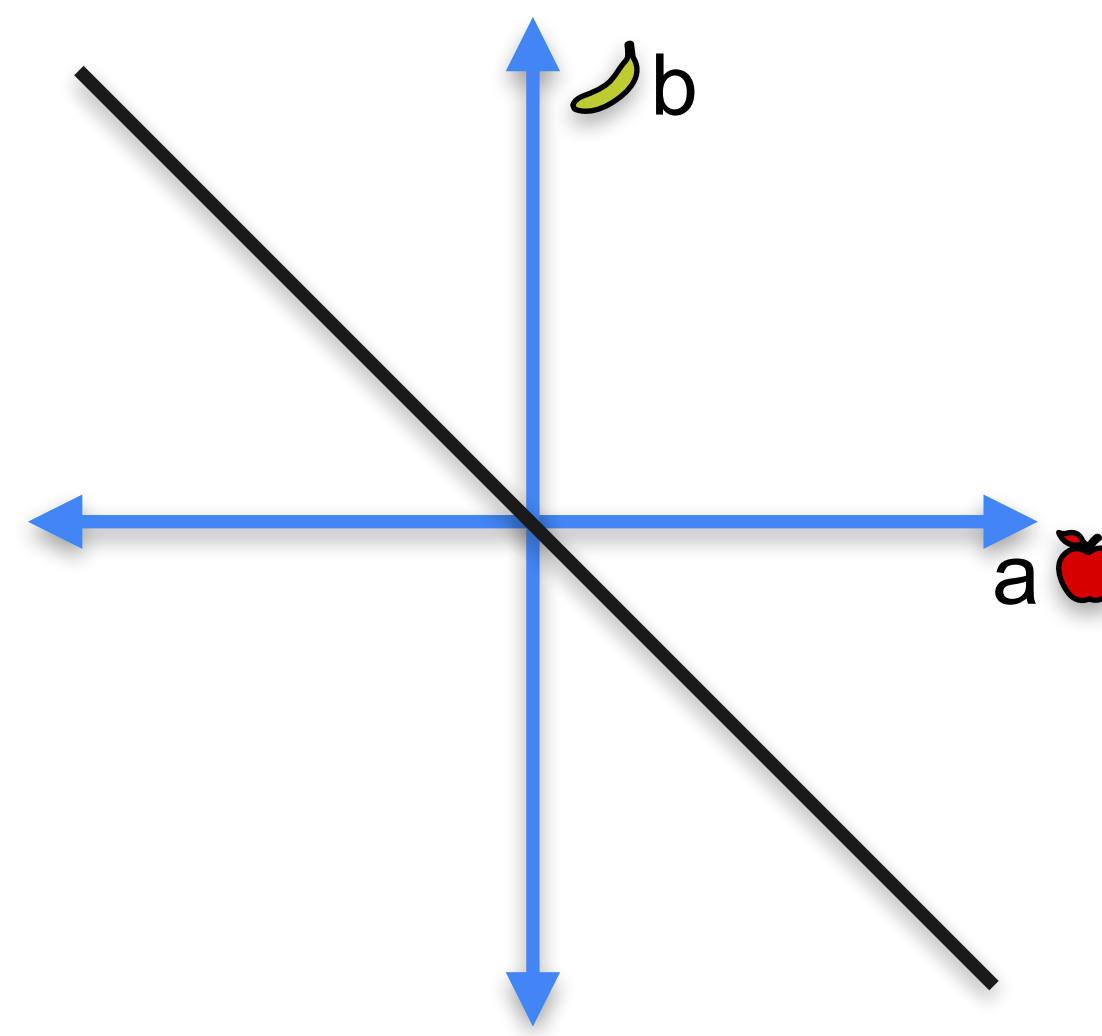
	
1	1
¹ Rank = 2	2



Dimension of solution space = 0



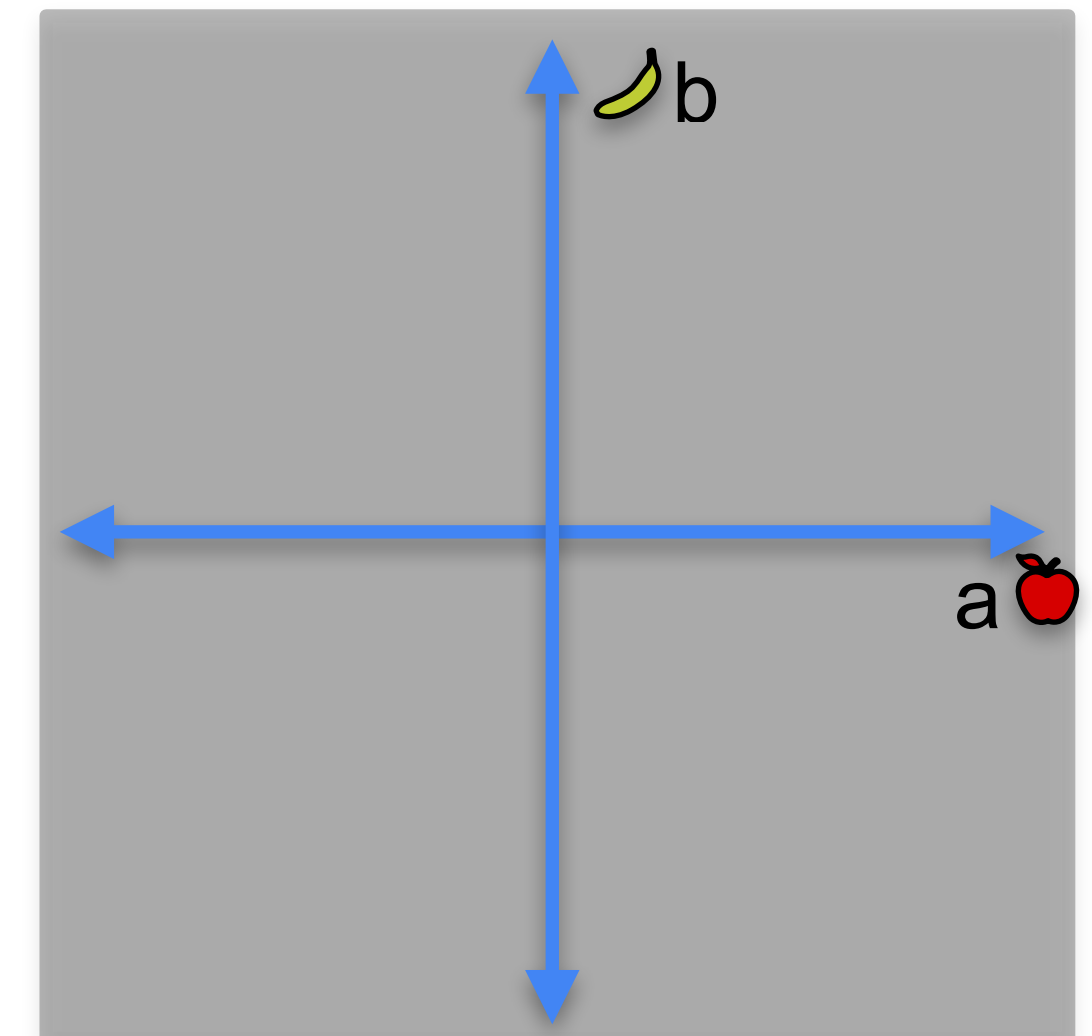
	
1	1
² Rank = 1	2

Dimension of solution space = 1



	
0	0
⁰ Rank = 0	0

Dimension of solution space = 2



Rank of a matrix

	
1	1
¹ Rank = 2	2

Dimension of solution space = 0

	
1	1
² Rank = 1	2

Dimension of solution space = 1

	
0	0
⁰ Rank = 0	0



Dimension of solution space = 2

$$\text{Rank} = 2 - (\text{Dimension of solution space})$$



Rank and singularity

	
1	1
¹ Rank = 2	2

Non-singular

	
1	1
² Rank = 1	2

Singular

	
0	0
⁰ Rank = 0	0

Singular

Quiz: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Rank of a matrix

Determine the rank of the following two matrices

Matrix 1: Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

Matrix 2: Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



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Solving System of Linear Equations

**Rank of a matrix:
General case**

Rank for matrices

System 1

$a + b + c = 0$ ✓
 $a + 2b + c = 0$ ✓
 $a + b + 2c = 0$ ✓

3 Equations
3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$ ✓
 $a + b + 2c = 0$ ✗
 $a + b + 3c = 0$ ✓

3 Equations
2 Pieces of information

Rank 2

1	1	1
1	1	2
1	1	3

System 3

$a + b + c = 0$ ✓
 $2a + 2b + 2c = 0$ ✗
 $3a + 3b + 3c = 0$ ✗

3 Equations
1 Piece of information

Rank 1

1	1	1
2	2	2
3	3	3

System 4

$0a + 0b + 0c = 0$ ✗
 $0a + 0b + 0c = 0$ ✗
 $0a + 0b + 0c = 0$ ✗

3 Equations
0 Pieces of information

Rank 0

0	0	0
0	0	0
0	0	0

Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



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Solving System of Linear Equations

Row echelon form

Row echelon form of a matrix

Original matrix

5	1
4	-3
5	1
10	2
0	0
0	0

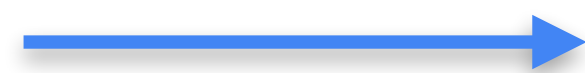
Row echelon form

1	0.2
0	1
1	1
0	0
0	0
0	0

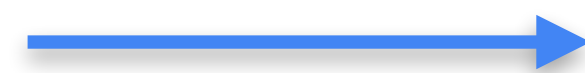
Row echelon form

Original matrix

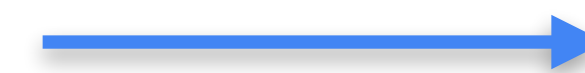
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Row echelon form

1	0.2
0	1

Divide each row by
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by
the leftmost non-zero coefficient

Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by
the leftmost coefficient

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

Divide the second row by
the leftmost non-zero coefficient

1	0.2
?	?

-	1	0.2
	1	0.2
<hr/>		
	0	0

Row echelon form for singular matrices

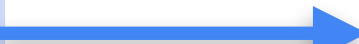
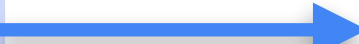
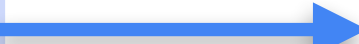
Row echelon form

Original matrix



Divide each row by
the leftmost coefficient

Row echelon form, singularity, and rank

Non-singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>-3</td></tr></table> 	5	1	4	-3	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>1</td></tr></table>	1	0.2	0	1	Rank 2 2 ones in the diagonal
5	1										
4	-3										
1	0.2										
0	1										
Singular matrix	<table><tr><td>5</td><td>1</td></tr><tr><td>10</td><td>2</td></tr></table> 	5	1	10	2	<table><tr><td>1</td><td>0.2</td></tr><tr><td>0</td><td>0</td></tr></table>	1	0.2	0	0	Rank 1 1 one in the diagonal
5	1										
10	2										
1	0.2										
0	0										
Singular matrix	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table> 	0	0	0	0	<table><tr><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td></tr></table>	0	0	0	0	Rank 0 0 ones in the diagonal
0	0										
0	0										
0	0										
0	0										



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Solving System of Linear Equations

**Row echelon form:
General case**

Row echelon form

System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Matrix

1	1	2
3	-3	-1
2	-1	6



System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

Row echelon form matrix

1	1	2
0	-6	7
0	0	6

Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

Rank 5

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

Row echelon form

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

$\div 3$

$\div 1$

$\div (-4)$

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Note:

- In general, pivots different than 1 are allowed
- For this class, pivots are 1. This makes no mathematical difference.

Another example

Matrix

1	1	1
1	2	1
1	1	2

Row echelon form

1	1	1
0	1	0
0	0	1

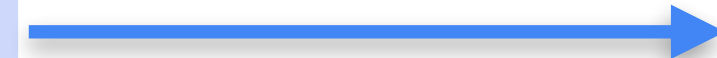
Subtract the first row
from the second and
the third ones

What if the matrix is singular?

Matrix

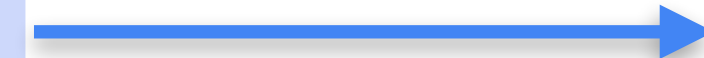
1	1	1
1	1	2
1	1	3

Subtract the first row
from the second and
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the
second row from the
third one



Row echelon form

1	1	1
0	0	1
0	0	0

What if the matrix is singular?

Matrix

1	1	1
2	2	2
3	3	3

Subtract twice the first
row from the second
row



1	1	1
0	0	0
3	3	3

Subtract three times
the first row from the
third row



Row echelon form

1	1	1
0	0	0
0	0	0

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0



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Solving System of Linear Equations

Reduced row echelon form

Systems of equations to matrices

Original system

- $5a + b = 17$
- $4a - 3b = 6$

Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

Original matrix

5	1
4	-3

Upper diagonal matrix

1	0.2
0	1

Row echelon form

Diagonal matrix

1	0
0	1

Reduced row echelon form

Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

\times **0.2**

0	0.2
---	-----

Reduced row echelon form

1	0.2
0	1

1	0.2
0	0.2

$-$

1	0
---	---

Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by the value of the pivot

Reduced row echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a pivot to 0

Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one

1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one

1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one

Reduced row echelon form

1	0	0
0	1	0
0	0	1



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Solving System of Linear Equations

The Gaussian Elimination Algorithm

Augmented matrix

$$\begin{array}{rcl} 2a - b + c & = & 0 \\ 2a + 2b + 4c & = & 0 \\ 4a + b & = & 0 \end{array}$$



Augmented matrix

2	-1	1
2	2	4
4	1	0

Proceed with the elimination method

Augmented matrix

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

 R_1

2

-1

1

1

 R_2

2

2

4

-2

 R_3

4

1

0

-1



Pivoting

$$2a - b + c = 1$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$



R_1	2	-1	1		1
R_2	2	2	4		-2
R_3	4	1	0		-1

$$R_1 \leftarrow \frac{1}{2}R_1$$

$$R_1 \leftarrow \frac{1}{2}$$

=	1	-1/2	1/2		1/2
---	---	------	-----	--	-----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$2a + 2b + 4c = -2$$

$$4a + b = -1$$

R_1	1	-1/2	1/2	1/2
R_2	2	2	4	-2
R_3	4	1	0	-1

$$R_2 \leftarrow R_2 - 2R_1$$

	2	2	4	-2
-2	1	-1/2	1/2	1/2
<hr/>				
$R_2 \leftarrow$	0	3	3	-3

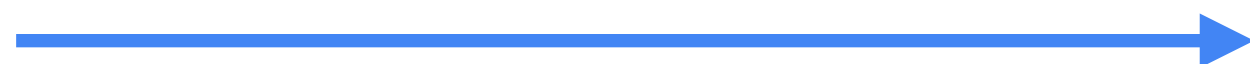
$$R_3 \leftarrow R_3 - 4R_1$$

	4	1	0	-1
-4	1	-1/2	1/2	1/2
<hr/>				
$R_3 \leftarrow$	0	3	-2	2

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$
$$3b + 3c = -3$$

$$3b - 2c = 2$$



R_1	1	-1/2	1/2	1/2
R_2	0	3	3	-3
R_3	0	3	-2	2

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$3b + 3c = -3$$

$$3b - 2c = 2$$



R_1	1	-1/2	1/2		1/2
R_2	0	3	3		-3
R_3	0	3	-2		2

$$R_2 \leftarrow \frac{1}{3}R_2$$

$$R_2 \leftarrow \frac{1}{3}$$

0	3	3		-3
---	---	---	--	----

=

0	1	1		-1
---	---	---	--	----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$

 R_1

1	-1/2	1/2	1/2
---	------	-----	-----

 R_2

0	1	1	-1
---	---	---	----

 R_3

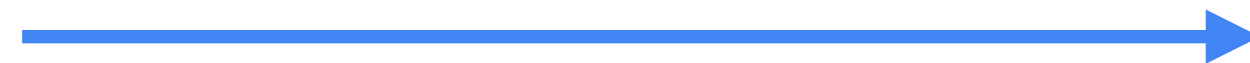
0	3	-2	2
---	---	----	---

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$3b - 2c = 2$$



R_1	1	-1/2	1/2		1/2
R_2	0	1	1		-1
R_3	0	3	-2		2

$$R_3 \leftarrow R_3 - 3R_2$$

	0	3	-2		2
-3	0	1	1		-1
<hr/>					
$R_3 \leftarrow$	0	0	-5		5

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$-5c = 5$$

 R_1

1	-1/2	1/2		1/2
---	------	-----	--	-----

 R_2

0	1	1		-1
---	---	---	--	----

 R_3

0	0	-5		5
---	---	----	--	---

$$R_3 \leftarrow -\frac{1}{5}R_3$$

$$R_3 \leftarrow -\frac{1}{5} \begin{array}{|ccc|c} 0 & 0 & -5 & 5 \end{array} = \begin{array}{|ccc|c} 0 & 0 & 1 & -1 \end{array}$$

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 R_1

1	-1/2	1/2	1/2
---	------	-----	-----

 R_2

0	1	1	-1
---	---	---	----

 R_3

0	0	1	-1
---	---	---	----

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 R_1 R_2 R_3

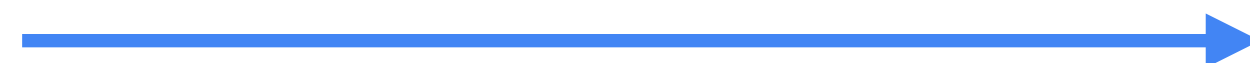
1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

Pivoting

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

 R_1

1	-1/2	1/2	1/2
0	1	1	-1
0	0	1	-1

 R_2 R_3

Back substitution

$$a - \frac{1}{2}b + \frac{1}{2}c = \frac{1}{2}$$

$$b + c = -1$$

$$c = -1$$

$$\begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{|ccc|c} \hline 1 & -1/2 & 1/2 & 1/2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ \hline \end{array}$$

$$R_2 \leftarrow R_2 - R_3$$

$$\begin{array}{r} \begin{array}{|ccc|c} \hline 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ \hline \end{array} \\ - \\ \begin{array}{|ccc|c} \hline 0 & 0 & 1 & -1 \\ \hline \end{array} \end{array}$$

$$R_2 \leftarrow$$

$$\begin{array}{|ccc|c} \hline 0 & 1 & 0 & 0 \\ \hline \end{array}$$

$$R_1 \leftarrow R_1 - \frac{1}{2}R_3$$

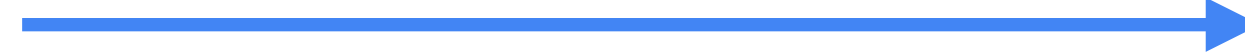
$$\begin{array}{r} \begin{array}{|ccc|c} \hline 0 & 0 & 1 & -1 \\ 1 & -1/2 & 1/2 & 1/2 \\ \hline \end{array} \\ -\frac{1}{2} \\ \begin{array}{|ccc|c} \hline 1 & -1/2 & 0 & 1 \\ \hline \end{array} \end{array}$$

Back substitution

$$a - \frac{1}{2}b = 1$$

$$b = 0 \quad -1$$

$$c = -1$$



R_1	1	-1/2	0		1
R_2	0	1	0		0
R_3	0	0	1		-1

$$R_1 \leftarrow R_1 + \frac{1}{2}R_2$$

$$+\frac{1}{2}$$

$R_1 \leftarrow$	1	0	0		1
------------------	---	---	---	--	---

The result

$a = 1$
 $b = 0$
 $c = -1$



R_1

1	0	0	1
---	---	---	---

R_2

0	1	0	0
---	---	---	---

R_3

0	0	1	-1
---	---	---	----

Solution to the system

Identity matrix

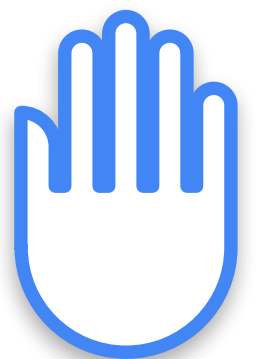
What if the system is singular?

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



1	2	-1	5
0	0	-7	9
0	0	0	0



There is no need to worry!

Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	6

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0



$$0a + 0b + 0c = 0$$

Infinitely many
solutions

Checking if it has infinitely many or no solutions

1	2	-1	5
2	4	5	1
3	6	4	10

After row reduction...



Look at the column of constants

1	2	-1	5
0	0	-7	9
0	0	0	0

4

$$0a + 0b + 0c = 4$$

The system has
no solutions

Checking if it has infinitely many or no solutions

- Row full of zeroes in row echelon form
- Constant in that row is zero
- **Infinitely many solutions**

1	2	-1	5
0	0	-7	9
0	0	0	0

- Row full of zeroes in row echelon form
- Constant in that row is not zero
- **No solutions**

1	2	-1	5
0	0	-7	9
0	0	0	4

Gaussian Elimination - Summary

1. Create the augmented matrix
2. Get the matrix into reduced row echelon form
3. Complete back substitution
4. Stop if you encounter a row of 0s



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Solving System of Linear Equations

Conclusion