Q1_A Selection Combining

$\bar{\gamma} = 12\,dB = 15.8$, $\gamma_0 = 10\,dB = 10$, $M = 1, 2, 4$

$$P_{out} = Pr\{\gamma_\Sigma < \gamma_0\} = \int_0^{\gamma_0} f_{\gamma_\Sigma}(\gamma)\,d\gamma$$

$$\xrightarrow[\text{SC Diversity}]{f..} f_{\gamma_\Sigma}(\gamma) = \frac{M}{\bar{\gamma}}\left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^{M-1} \cdot e^{-\frac{\gamma}{\bar{\gamma}}} \quad \xrightarrow{U = 1 - e^{-\frac{\gamma}{\bar{\gamma}}}}$$

$$\longrightarrow P_{out} = M \int_0^{1 - e^{\gamma_0/\bar{\gamma}}} U^{M-1}\,dU = \frac{M}{M} \cdot U^M \Big|_0^{1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}}$$

$$= \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^M = (0.469)^M$$

$$P_{out} \xrightarrow{M=?} \begin{cases} 0.4690 & ; M = 1 \\ 0.2199 & ; M = 2 \\ 0.0483 & ; M = 4 \end{cases}$$

B_ Maximum Ratio Combining

$$P_{out} = Pr\{\gamma_\Sigma < \gamma_0\} = \int_0^{\gamma_0} f_{\gamma_\Sigma}(\gamma)\,d\gamma \quad ; f_{\gamma_\Sigma}(\gamma) \sim \text{chi}(\text{sum of } 2M \text{ gaussian RVs})$$

$$= 1 - e^{-\gamma_0/\bar{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_0/\bar{\gamma})^{k-1}}{(k-1)!}$$

$$P_{out} \xrightarrow{M=?} \begin{cases} 0.469 & ; M = 1 \\ 0.1329 & ; M = 2 \\ 0.0041 & ; M = 4 \end{cases}$$

Q2 _ a

$\overline{P_e} = E\{Q(\sqrt{2\gamma})\}$: $\gamma = |h|^2 \rho$

$f_{|h|}(x) = e^{-x} u(x)$

$\longrightarrow \overline{P_e} = \int_0^\infty Q(\sqrt{2x\rho})\, e^{-x} dx$

SNR ↑

↳ $\rho \to \infty \Rightarrow x \to 0 \Rightarrow e^{-x} \to 1$

$\overline{P_e} \simeq \int_0^\infty Q(\sqrt{2x\rho})\, dx \overset{y = \sqrt{x\rho}}{=\!=} \frac{2}{\rho} \int_0^\infty y\, Q(\sqrt{2}y)\, dy$

$dy = \sqrt{\rho}\, \frac{dx}{2\sqrt{x}}$

$dx = dy \times \frac{1}{\sqrt{\rho}} \times 2\sqrt{x}$

$= \frac{2y\, dy}{\rho}$

$\longrightarrow \overline{P_e} = \frac{2}{\rho} \times \frac{1}{2 \times (\sqrt{2})^2} = \frac{1}{2\rho} \longrightarrow \lim_{\rho \to \infty} P_e \rho = \frac{1}{2}$

b _ $\overline{P_e} \simeq E\{Q(\sqrt{2|h|^2\rho})\} = \int_0^\infty Q(\sqrt{2x\rho})\, f(x) dx$!!

$\overset{\rho \to \infty}{\simeq} f(0) \int_0^\infty Q(\sqrt{2x\rho})\, dx \underset{\text{as in } A}{=\!=} \frac{f(0)}{2\rho}$

$\longrightarrow \lim_{\rho \to \infty} \overline{P_e}\, \rho = \frac{f(0)}{2\rho}$

Q3: $P_s = 10^{-3} \xrightarrow{QPSK} P_s = 2Q(\sqrt{\gamma_s}) = 10^{-3}$

$\longrightarrow \gamma_0 = \gamma_s = \left[Q^{-1}\left(\frac{10^{-3}}{2}\right) \right]^2 = 10.82$

Selection $\longrightarrow$ Combining $P_{out} = \prod_{i=1}^{M} \left(1 - e^{-\frac{\gamma_0}{\gamma_i}}\right)$

$P_{out} \rightarrow$ 0.6613 ; $M=1$

$\rightarrow$ 0.1917 ; $M=2$

$\rightarrow$ 0.0197 ; $M=3$

Q4 - A -

$P_b = 0.2 e^{-1.5 \frac{\gamma}{M-1}} = 0.2 e^{-\frac{15}{3}} = 0.0013$

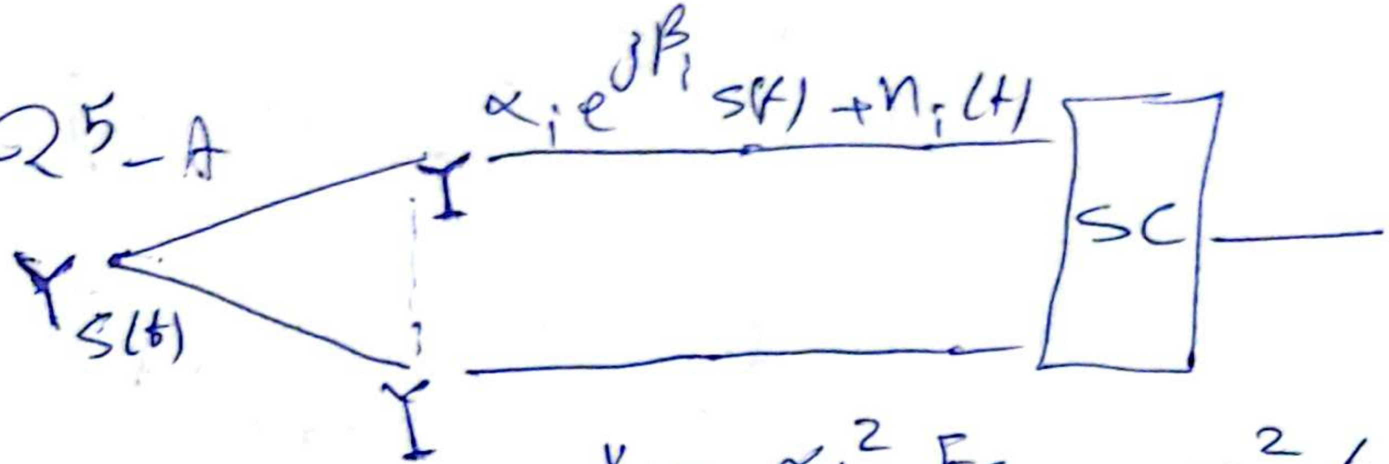
B -

$\gamma_\Sigma = \sum_{i=1}^{N} \gamma_i = N\gamma_i = 10N$

$\longrightarrow P_b = 0.2 e^{-1.5 \frac{\gamma_\Sigma}{M-1}} = 0.2 e^{-5N} \leq 10^{-6}$

$\longrightarrow N=3 \longrightarrow P_b = 6.12 \times 10^{-8} \leq 10^{-6}$

Q5_A



$$\gamma_i = \alpha_i^2 \frac{E_s}{N_0} = \alpha_i^2 (\gamma_c) \quad ; \quad \gamma_c = \frac{E_s}{N_0}$$

$$E(|h|^2) = 1 \rightarrow E\{h h^*\} = E\{\alpha^2\} = 1 \quad ; \quad \alpha \sim \text{uniform}(0, A)$$

$$\rightarrow f_\alpha(x) = \frac{1}{A} \quad ; x \in (0, A)$$

$$\rightarrow E\{\alpha^2\} = \frac{1}{A} \int_0^A x^2 \, dx = \frac{1}{A} \times \frac{A^3}{3} = \frac{A^2}{3} = 1 \rightarrow A = \sqrt{3}$$

$$\rightarrow \alpha \sim \text{uniform}(0, \sqrt{3}) \qquad \underset{\text{equiprob}}{\beta} \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi\right\}$$

$$P(\alpha < x) = \int_0^x \frac{1}{\sqrt{3}} \, dx = \frac{x}{\sqrt{3}} = P_\alpha(x)$$

Q5 - Cont'd

$$p_{\alpha^2}(x) = \frac{d}{dx}\left(P_{\alpha^2}(x)\right) = \left(\frac{\sqrt{3}}{3}\sqrt{x}\right)' = \frac{\sqrt{3}}{3} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{3x}} \; ; \quad x \in [0,3]$$

note that in SC $\longrightarrow$ $\gamma_\Sigma = \mathrm{Max}(\alpha_i^2)\, \gamma_c$

$\overset{\text{assuming}}{\underset{\text{equal energy on each branch}}{=}} \alpha^2 \gamma_c$; $\gamma_c = SNR$ of AWGN channel)

$$\longrightarrow E(\gamma_\Sigma) = \gamma_c \, E\{\alpha^2\}$$

$$P_{\gamma_\Sigma}(\gamma) = Pr\{\gamma_\Sigma < \gamma\} = Pr\{\max(\gamma_1, --, \gamma_L) < \gamma\}$$

$$\underset{\text{indep}}{\overset{\gamma_i}{=}} \prod_{i=1}^{L} Pr\{\gamma_i < \gamma\} = \prod_{i=1}^{L} P_{\gamma_i}(\gamma) \underset{\gamma_i\text{'s}}{\overset{\text{equal}}{=}} \left(P_{\gamma_i}(\gamma)\right)^L$$

$$\gamma_i = \alpha_i^2 \gamma_c \longrightarrow \bar{\gamma}_i = E\{\gamma_i\} = \gamma_c \underbrace{E(\alpha_i^2)}_{1} = \gamma_c$$

$$\longrightarrow P_{\gamma_\Sigma}(\gamma) = \left(P_{\gamma_i}(\gamma)\right)^L = \left(\frac{\sqrt{3\gamma}}{3}\right)^L \gamma_c$$

$$\longrightarrow p_{\gamma_\Sigma}(\gamma) = \frac{d}{d\gamma} P_{\gamma_\Sigma}(\gamma) = \frac{\sqrt{3}L}{3}\left(\frac{\sqrt{3\gamma}}{3}\right)^{L-1} \gamma_c$$

$$\longrightarrow \bar{\gamma}_\Sigma = \frac{\sqrt{3}}{3}\gamma_c L \int_0^3 \gamma \left(\frac{\sqrt{3\gamma}}{3}\right)^{L-1} d\gamma = \frac{\sqrt{3}}{3}\gamma_c L \int_0^3 \gamma \times \left(\frac{1}{\sqrt{3}}\right)^{L-1} \times \gamma^{\frac{L-1}{2}} d\gamma$$

$$= \left(\frac{1}{\sqrt{3}}\right)^L L \gamma_c \int_0^3 \gamma^{\frac{L+1}{2}} d\gamma = \frac{\left(\frac{1}{\sqrt{3}}\right)^L L \gamma_c}{\left(\frac{L+3}{2}\right)} \times \left(3^{\frac{L+3}{2}}\right)$$

$$= 3^{\frac{L}{2}+1} \times \frac{2L\gamma_c}{L+3}$$