BDCDB

DBDCD

2/3

13/8

0

e

8

16. (1). $P(AB) = P(A) + P(B) - P(A+B)$.

$0.7 \le P(A+B) \le 1$. 当 $A \subset B$ 时. $P(A+B) = 0.7$.

当 $A+B = \Omega$ 时. $P(A+B) = 1$.

$P(AB)_{max} = 0.6$ $P(AB)_{min} = 0.3$.

17. $X \sim U[-\pi,\pi]$.

$$f(x)=\begin{cases}\frac{1}{2\pi}. & -\pi\le x\le\pi.\\ 0, & \text{其他}.\end{cases}$$

$$E[\min(|X|,1)]=\int_{-\infty}^{+\infty}\frac{\min(|X|,1)}{2\pi}\,dx$$

$$=2\int_0^1\frac{x}{2\pi}+\int_{-\pi}^{-1}\frac{1}{2\pi}dx+\int_1^{\pi}\frac{1}{2\pi}dx$$

$$=\frac{2\pi-1}{2\pi}.$$

18. (1). $f_X(x)=\int_{-x}^{x}f(x,y)\,dy$

$$=\frac{3}{2}xy\Big|_{-x}^{x}=3x^2\cdot(0<x<1).$$

$$f_X(x)=\begin{cases}3x^2, & 0<x<1\\ 0, & \text{其他}.\end{cases}$$

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}\frac{1}{2x}, & 0<x<1,\ |y|<x\\ 0 & \text{其他}.\end{cases}$$

(2). $$f_{Y|X=\frac{1}{2}}(y|x)=\begin{cases}1 & |y|<\frac{1}{2}.\\ 0 & \text{其他}\end{cases}$$

$$P\{Y\le\tfrac{1}{4}\,|\,X=\tfrac{1}{2}\}=\int_{-\frac{1}{2}}^{\frac{1}{4}}1\,dy=\frac{3}{4}.$$

(3). $$P\{X+Y>1\}=\int_{\frac{1}{2}}^{1}dx\int_{1-x}^{x}\frac{3}{2}x\,dy=\frac{5}{16}.$$

19. (1). $EZ = EX - EY = 0$.

$DZ = DX + DY - 2Cov(X,Y) = 3\sigma^2$.

$f_Z(z) = \frac{1}{\sqrt{6\pi\sigma^2}} e^{-\frac{z^2}{6\sigma^2}}$.

(2). $L(\sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{6\pi\sigma^2}} e^{-\frac{z_i^2}{6\sigma^2}}$.

$= (6\pi\sigma^2)^{-\frac{n}{2}} e^{\frac{-\sum_{i=1}^{n} z_i^2}{6\sigma^2}}$

$\ln L(\sigma^2) = -\frac{n}{2}(\ln 6\pi + \ln\sigma^2) - \frac{\sum_{i=1}^{n} z_i^2}{6\sigma^2}$.

$\frac{d\ln L(\sigma^2)}{d\sigma^2} = \frac{-\frac{n}{2}}{\sigma^2} + \frac{\sum_{i=1}^{n} z_i^2}{6\sigma^4} = 0$.

$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} z_i^2}{3n}$.

(3). $E\hat{\sigma}^2 = E\frac{\sum_{i=1}^{n} z_i^2}{3n}$.

$= \frac{1}{3n} \cdot E\sum_{i=1}^{n} z_i^2$

$= \frac{1}{3} EZ^2$

$= \frac{1}{3}[DZ + (EZ)^2]$

$= \frac{1}{3}\{3\sigma^2 + 0]$

$= \sigma^2$. 是无偏估计量.

20 (1). $f_X(x)=\int_0^{+\infty}[(1+\rho y)(1+\rho x)-\rho]e^{-x-y-\rho xy}\,dy$

$=e^{-x}[\int_0^{+\infty}e^{-y(1+\rho x)}(1+\rho y)(1+\rho x)\,dy-\int_0^{+\infty}\rho e^{-y(1+\rho x)}dy]$

$=e^{-x}\{\int_0^{+\infty}-(1+\rho y)\,d e^{-y(1+\rho x)}dy-\int_0^{+\infty}\rho e^{-y(1+\rho x)}dy]$

$=e^{-x}[-(1+\rho y)e^{-y(1+\rho x)}\Big|_0^{+\infty}+\int_0^{+\infty}\rho e^{-y(1+\rho x)}dy-\int_0^{+\infty}\rho e^{-y(1+\rho x)}dy]$

$=e^{-x}.\quad(x>0)\qquad f_X(x)=\begin{cases}e^{-x} & x>0\\ 0 & x\le 0.\end{cases}$

同理. $f_Y(y)=\begin{cases}e^{-y}, & y>0\\ 0, & y\le 0\end{cases}$

$F(x)=\int e^{-x}dx=-e^{-x}+C\quad(x>0)\quad \lim_{x\to\infty}F(x)=0\Rightarrow C=1.$

$F(x)=\begin{cases}0 & ;\ x<0\\ 1-e^{-x} & ;\ x\ge 0.\end{cases}\quad F(y)=\begin{cases}0 & ;\ y<0\\ 1-e^{-y} & ;\ y\ge 0\end{cases}$

(2) $\rho=0.\ f(x,y)=e^{-x-y}.\ (x>0,y>0).$

$EY=\int_0^{+\infty}ye^{-y}dy=1$

$EX=\int_0^{+\infty}xe^{-x}dx=1$

$EXY=\int_0^{+\infty}\int_0^{+\infty}xy\,e^{-x-y}dx\,dy=1.$

$Cov(X,Y)=EXY-EX\cdot EY=0.$

$\rho_{XY}=\dfrac{Cov(X,Y)}{\sqrt{DX}\cdot\sqrt{DY}}=0.$

(3) $Z=2(X+Y).$

$f_Z(z)=\int_0^z f_X(2x)f_Y(z-2x)\,dx.$

$\cancel{=\int_0^z e^{-2x}\cdot e^{-(z-2x)}dx}=\int_0^z e^{-2x}e^{2x-z}dx$

$=xe^{-z}\Big|_0^z$

$=ze^{-z}.\quad(z>0).$

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