Mathematical calculations made for the Circuit Diagrams proposed

This document presents non-formal mathematical calculations for the proposed Circuit Diagrams with, a most likely, unusual method for most (or maybe even all the readers of this document) with which the author (i.e., Cesar Miranda Meza) uses as his main methodology for electronic circuit designs.

The formal and professional way with which the author practices this methodology might be detailly and formally introduced to the public on another independent project in the future, since this is probably a non-used and is also considered as a non-practical methodology by most electronic engineers, but where the author has found a way, or at least is convinced, that this can actually be made in a very practical way. However, just to give a slight idea on how the author's methodology works is that the author already has formal detailed characterizations for each electronic component used in such a way that the author has also developed precise mathematical models for each characterizations made with 95%, 99% and 99.9% confidence intervals that basically guarantee the regions at which all the components will behave. Then, the author basically uses those mathematical models as the core means to represent the behavior of each particular component together with the well-known mathematical laws within the electronics field to basically obtain incredibly reliable results with respect to the actual physical circuits (i.e., The author can truly predict the behavior of his electronic circuit designs from the very start, whenever he calculates the mathematical part of his circuit designs).

Having cleared the overall concept of the methodology used, know that for now, both the mathematical models and characterizations made will not be shown in this project or document because the author wishes to formally introduce this on a future project independent to this one. Therefore, what will be made is the electronic component parameter values that were taken from these mathematical models and characterizations will be simply proposed in this document, but it will be indicated that those values came from there. However, know that although the results were precise, they can be way more accurate since the author did not want to spent a lot of time into the calculations to be shown in this document due to that he considered this project as a simple prototype, whose main purpose is to show a proof-of-concept hands-on of applying his recently introduced ETX OTA Protocol for managing and handling Firmware Updates in STMicroelectronics devices.

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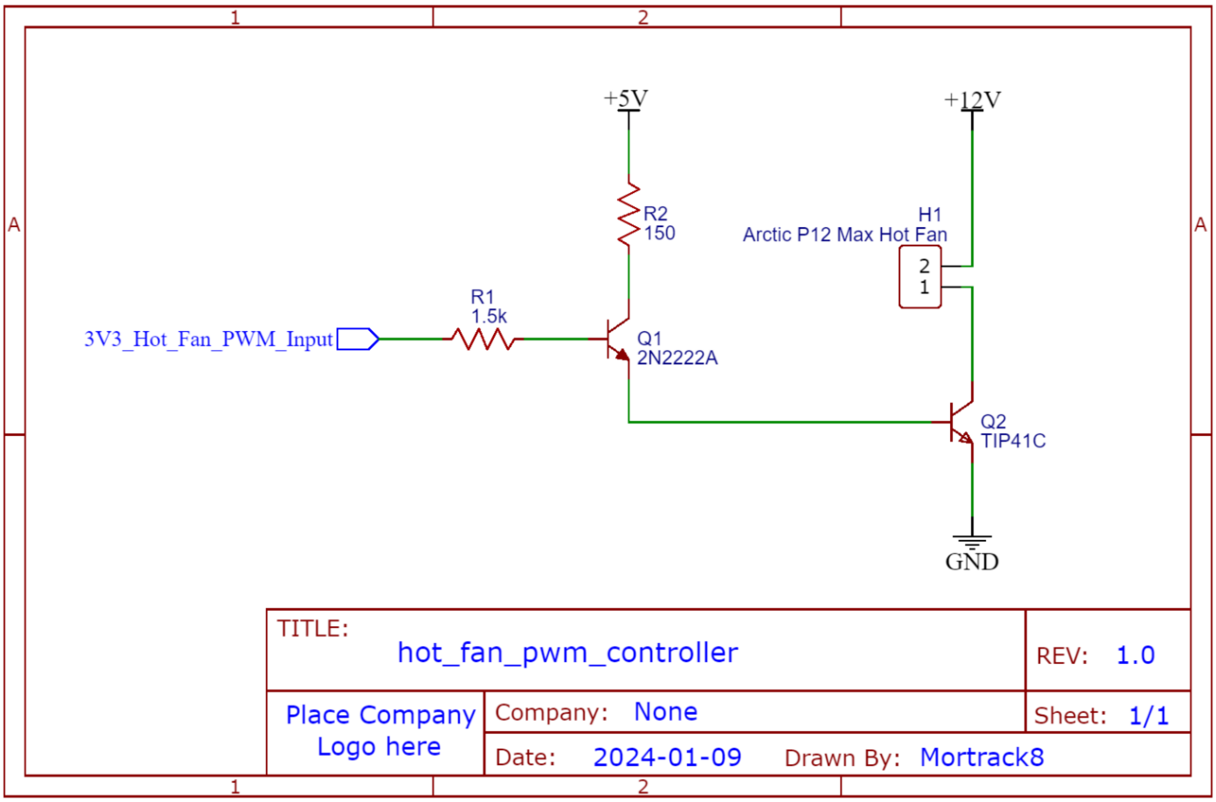
1. Considerations, limitations and/or nominal values to have in mind for the mathematical calculations

For all the circuit designs made, know that the following was considered:

- STM32F108C8T6 Microcontroller (MCU) was proposed.
- Chosen MCU requires either a 5Vdc or 3.3Vdc Power Supply, where a 5Vdc Power Supply was proposed.
- From what the author was able to quickly grasp from the corresponding STMicroelectronics datasheet for the chosen MCU, this device can:
    - Hold up to 150mA from its GPIO pins together.
    - Hold up to 25mA from each of its GPIO pins.
    - Manage 0 up to 3.3 Volts from the GPIO pins that were chosen for the circuit designs made.
- The circuit designs were calculated so that the heat that they could generate was negligible and they were auto-sufficient in such a way that they would not require a cooling system.
- The characterizations and mathematical models previously made by the author independently to this project will be labeled as Mortrack Characterizations from now on.
- All the parameter values taken from the Mortrack Characterizations will correspond to the expected average values.

2. Mathematical calculations for both the Hot and Cold Fan Circuits

Since for the circuit for both the Hot and Cold Fans are the same, the circuit diagram for the Hot Fan will be used as a reference for the mathematical calculation made for both of them:



From the simple characterizations made in this project for the Artic P12 Max Fans, the maximum average current of the Fans $I_{c_{(Q_2)}}$ is the following:

$$I_{c_{(Q_2)}} = 0.89A$$

From the Mortrack Characterizations, it is known that the TIP41C Transistor will be well saturated whenever used in conmutation mode with a Beta $\beta_{(Q_2)}$ as follows:

$$\beta_{(Q_2)} \approx 34$$

Where that same transistor is expected to have a base-emitter voltage $V_{be_{(Q_2)}}$ of:

$$V_{be_{(Q_2)}} \approx 0.75V$$

Under these circumstances, the Q_2 transistor is expected to work with a specific base current $I_{b_{(Q_2)}}$ of:

$$I_{b_{(Q_2)}} \to 26.176mA$$

However, a similar and also proposed current that was decided to drive the base of the Q_2 transistor was of $26.5mA$ because, from the Mortrack Characterizations, there was already some characterized data for the Q_1 transistor under that Colector current whenever using that transistor in conmutation mode and while also having that transistor well saturated, and where also, the base current for that transistor was also within the working specifications for the MCU proposed for the system:

$$I_{c_{(Q1)}} = I_{b_{(Q_2)}} = 26.5mA$$

Where from the data of the Mortrack Characterizations, it was known the following additional information from the Q_1 transistor:

$$\beta_{(Q_1)} \approx 27$$

$$V_{be_{(Q_1)}} \approx 0.72V$$

$$I_{b_{(Q_1)}} = 0.9815mA \approx 1mA$$

$$V_{CE_{(Q_1)}} \approx 0.0418v$$

Now, with all these electronic parameters at hand, the following mathematical procedure was followed in order to identify the values that R_1 and R_2 must have in order to make the proposed circuit design to work as expected.

First, we will get the value for R_2:

$$5V = I_{C_{(Q_1)}} R_2 + V_{CE_{(Q_2)}} + V_{be_{(Q_2)}}$$

$$R_2 = \frac{5 - V_{CE_{(Q_2)}} - V_{be_{(Q_2)}}}{I_{C_{(Q_1)}}}$$

$$R_2 = \frac{5 - (0.0418) - (0.75)}{(26.5x10^{-3})}$$

$$R_2 = \frac{4.2082}{26.5x10^{-3}}$$

$$\boxed{R_2 = 158.8\Omega \approx 150\Omega}$$

Note that although the ideal value for R_2 is 158.8Ω, 150Ω was proposed instead because that was the nearest Resistor value the author had at his disposal.

Moving on, let's now get the value for R_1, where "3V3_Hot_Fan_PWM_Input" will be substituted by 3.3v since that is what the Input PWM will give in its On-State:

$$3V3\_Hot\_fan\_PWM\_Input \ = \ I_{b_{(Q_1)}} R_1 + V_{be_{(Q_1)}} + V_{be_{(Q_2)}}$$

$$I_{b_{(Q_1)}} R_1 = 3V3\_Hot\_fan\_PWM\_Input \ - V_{be_{(Q_1)}} - V_{be_{(Q_2)}}$$

$$R_1 = \frac{3V3\_Hot\_fan\_PWM\_Input \ - V_{be_{(Q_1)}} - V_{be_{(Q_2)}}}{I_{b_{(Q_1)}}}$$

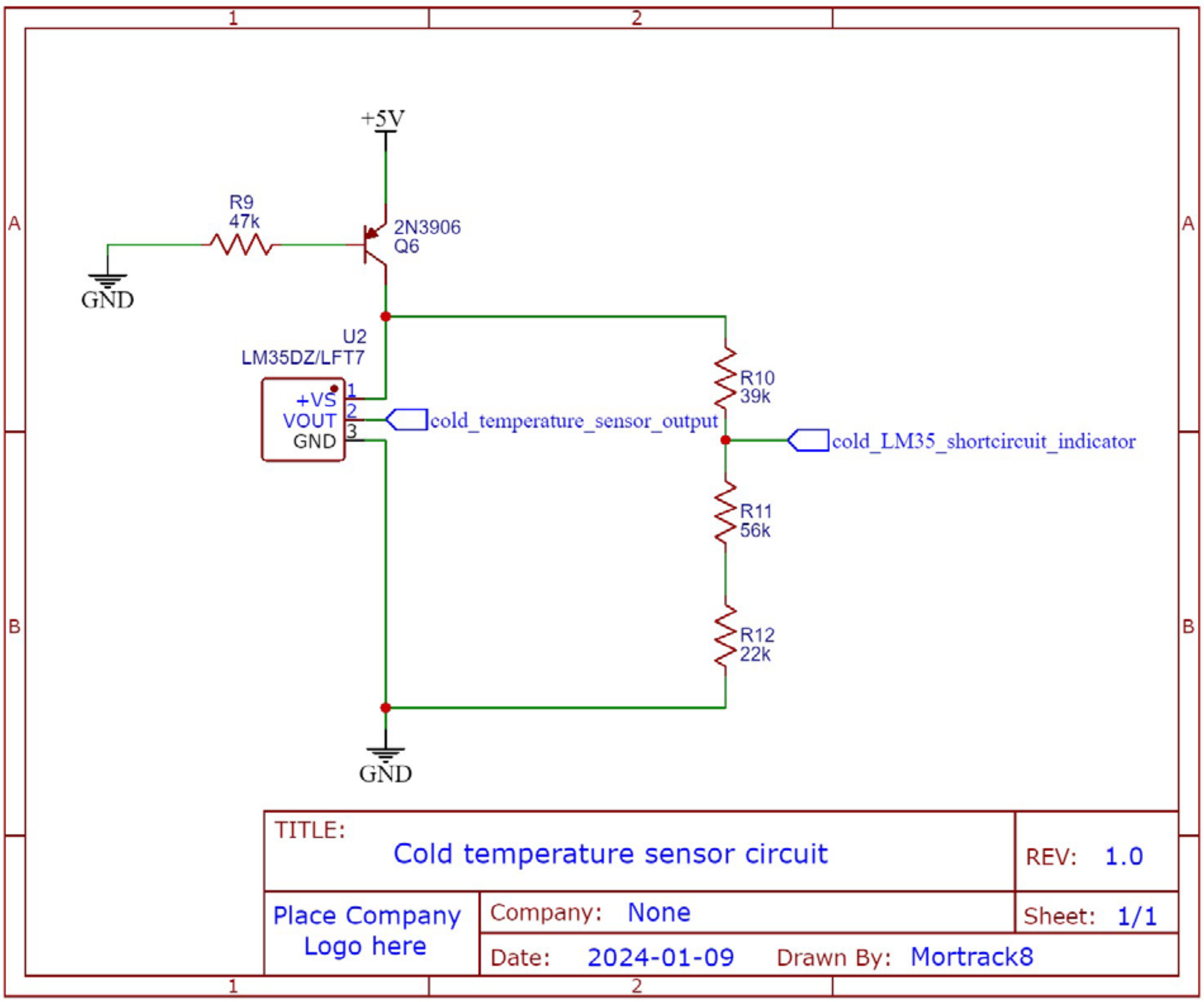
$$R_1 = \frac{(3.3)\ - (0.72) - (0.75)}{(1x10^{-3})}$$

$$\boxed{R_1 = 1830\Omega \approx 1500\Omega}$$

Note that although the ideal value for R_1 is 1830Ω, 1500Ω was proposed instead because that was the nearest Resistor value the author had at his disposal.

3. Mathematical calculations for both the Hold and Cold Temperature Sensor Circuits

Since for the circuit for both the Hold and Cold Temperature Sensors are the same, the circuit diagram for the Cold Temperature Sensor will be used as a reference for the mathematical calculation made for both of them:



Now, before heading straight to the mathematical calculations made for this circuit, it is important to discuss some aspects that will allow the reader to understand the purpose of this circuit diagram and the reason on why this circuit strategy was chosen.

First, there are two main goals for this circuit: 1) To supply the power source required by the LM35 Temperature Sensor to work, and 2) To provide a short-circuit protection in consideration that these temperature sensors (both for the cold and hot water deposits of the system) are meant to be submerged into water and that both sensors have been surrounded by some thermofit material manually by the author. This means that these water isolation attempts are not professionally made and that although they were tested beforehand, there is a risk of having some water getting into the connectors of the actual temperature sensors.

Regarding the first main goal described, in order to guarantee that there is not added voltage value into the output of the LM35 sensor with respect to ground, it was determined that the best type of BJT transistor to propose for this circuit diagram was of a PNP type.

As for the second goal that was described, the idea to provide this short-circuit protection is based on setting a working point on the proposed PNP transistor so that it works in conmutation mode and is indefinitely sending the 5V power supply to the LM35 Sensor V_{cc} terminal so that it is electrically energized, but where at the same time, this transistor will limit the current up to a bit higher of what is expected that the LM35 Sensor works with, according to the datasheet, and where that current limit point is a also a safe working point for the PNP transistor (to prevent it from stressing or getting damaged due to being exposed at that current limited working point for long periods of time).

At the same time, a divisor resistor is used to provide a means for the MCU to know whether this circuit is working normally or if it is under a short-circuit situation.

Now, with these explanations in mind, the concept of the proposed circuit should now make sense and with that, let us now start with the mathematical process.

First, from the LM35 datasheet, it is known that this sensor is supposed to work with 10mA at the most. Therefore, a quickly proposed value by the author (without investing a lot of time into identifying the actually best or most efficient value) to make the Q_6 transistor to limit the current of the circuit so that any possible LM35 sensor is able to work as expected, but to also quickly limit the current in case of a short-circuit, is the following:

$$I_{c_{(Q_6)max}} \approx 23.8mA$$

Since this proposed value was chosen from the Mortrack Characterizations, the following additional parameter values are also known from there:

$$\beta_{(Q_6)} \approx 235$$

$$V_{be_{(Q_6)}} \approx 0.66V$$

$$I_{b_{(Q_6)}} = \frac{I_{c_{(Q_6)max}}}{\beta_{(Q_6)}}$$

$$I_{b_{(Q_6)}} = \frac{23.8x10^{-3}}{235}$$

$$I_{b_{(Q_6)}} = 101.276\mu A \approx 100\mu A$$

With these parameter values in mind, it is now possible to determine the required value for the R_9 Resistor:

$$5v = V_{be_{(Q_6)}} + I_{b_{(Q_6)}} R_9$$

$$I_{b_{(Q_1)}} R_9 = 5V - V_{be_{(Q_6)}}$$

$$R_9 = \frac{5V - V_{be_{(Q_6)}}}{I_{b_{(6)}}}$$

$$R_9 = \frac{5 - (0.66)}{(101.276x10^{-6})}$$

$$\boxed{R_9 = 42'853.193\Omega \approx 47k\Omega}$$

Note that although the ideal value for R_9 is $42'853.193\Omega$, $47k\Omega$ was proposed instead because that was the nearest Resistor value the author had at his disposal.

As a result of placing the calculated R_9 Resistor value, this circuit should now be able to give almost the entirety of the $5V$ power supply to the LM35 sensor due to that from the Mortrack Characterization's data, it can be concluded that the Q_6 Transistor should be well saturated from up to around $18mA$ in its Collector current. However, since each LM35 is expected to drain a different amount of current from the 5V power supply, it is expected for each different LM35 sensor (although being from the same model) to be delivered a slightly different amount of Power Supply Voltage, which will depend on the current drained from one LM35 sensor to another one. Nonetheless, the difference in that voltage should be negligible due to that, according to the Mortrack Characterizations, the Q_6 Transistor is expected to retain around $120mV$ at the most from the $5V$ Power Supply whenever the LM35 is under a functional normal operation.

Moving on, regarding the voltage divider generated at the "$cold\_LM35\_shortcircuit\_indicator$" label from this circuit's R_{10}, R_{11} and R_{12} Resistors, the values proposed there were a result of several empirical mathematically calculated attempts to: 1) Drain a negligible amount of current with respect to what an LM35 sensor is expected to consume from the $5V$ Power Supply in order to not alter the previously calculations made for this circuit so far, and 2) To basically provide a Voltage Divider that delivers around $3.3V$ at the most at the "$cold\_LM35\_shortcircuit\_indicator$" label in order to provide a safe means for the MCU to read that voltage and to allow it to learn whether this circuit is currently under the case where the LM35 sensor is under a short-circuit or not.

To prove that those resistor values should allow the circuit to work as expected, let us first find out the expected amount of current that they should drain at the most whenever we are at the case where the Q_6 Transistor completely delivers the $5V$ to the load of the circuit:

$$5V = I_{(R_{10}+R_{11}+R_{12})} R_{10} + I_{(R_{10}+R_{11}+R_{12})} R_{11} + I_{(R_{10}+R_{11}+R_{12})} R_{12}$$

$$5V = I_{(R_{10}+R_{11}+R_{12})} (R_{10} + R_{11} + R_{12})$$

$$I_{(R_{10}+R_{11}+R_{12})} (R_{10} + R_{11} + R_{12}) = 5V$$

$$I_{(R_{10}+R_{11}+R_{12})} = \frac{5V}{R_{10} + R_{11} + R_{12}}$$

$$I_{(R_{10}+R_{11}+R_{12})} = \frac{5V}{(39'000) + (56'000) + (22'000)}$$

$$I_{(R_{10}+R_{11}+R_{12})} = \frac{5V}{117'000}$$

$$\boxed{I_{(R_{10}+R_{11}+R_{12})} = 42.735\ \mu A}$$

With this $I_{(R_{10}+R_{11}+R_{12})}$ value, let us now know the proportional relationship it has with respect to the maximum expected current that the LM35 sensor is to drain:

$$\frac{I_{(R_{10}+R_{11}+R_{12})}}{I_{(LM35_{max})}} = \frac{42.735\ x10^{-6}}{10x10^{-3}}$$

$$\boxed{\frac{I_{(R_{10}+R_{11}+R_{12})}}{I_{(LM35_{max})}} = 4.2735x10^{-3} \rightarrow 0.427\%}$$

From where it is possible to see that $I_{(R_{10}+R_{11}+R_{12})}$ represents a 0.427% of $I_{(LM35_{max})}$. With this in mind, the author assumed that the values proposed for the R_{10}, R_{11} and R_{12} Resistors should have negligible effects on the circuit's previous calculations. However, it is important to know that it is unknown for the author the range at which an LM35 could drain current from a $5V$ Power Supply since the author has not characterized or modeled that sensor. However, for simplicity and due to the very large proportional relationship obtained with the proposed Resistor values, it was considered that these Resistor values should work well, which were later proved to work as expected in a prototype circuit before getting to this conclusion.

One last thing to determine is the voltage range at which the "$cold\_LM35\_shortcircuit\_indicator$" label is expected to work at, for which it is necessary to first get to the mathematical equation that will provide that information:

$$cold\_LM35\_shortcircuit\_indicator\ = V_C \frac{R_{11} + R_{12}}{R_{10} + R_{11} + R_{12}}$$

$$cold\_LM35\_shortcircuit\_indicator\ = V_C \frac{(56'000) + (22'000)}{(39'000) + (56'000) + (22'000)}$$

$$cold\_LM35\_shortcircuit\_indicator\ = V_C \left(\frac{2}{3}\right)$$

Where, if we evaluate the case for whenever the LM35 is expected to work correctly and if we consider the best possible case where the Q_6 Transistor delivers the full $5V$ of the Power Supply, then we would have the following expected voltage at the "$cold\_LM35\_shortcircuit\_indicator$" label:

$$cold\_LM35\_shortcircuit\_indicator_{max}\ = (5)\left(\frac{2}{3}\right)$$

$$\boxed{cold\_LM35\_shortcircuit\_indicator_{max}\ = 3.333}$$

And for the case where the LM35 is expected to be at short-circuit:

$$cold\_LM35\_shortcircuit\_indicator_{min}\ = (0)\left(\frac{2}{3}\right)$$

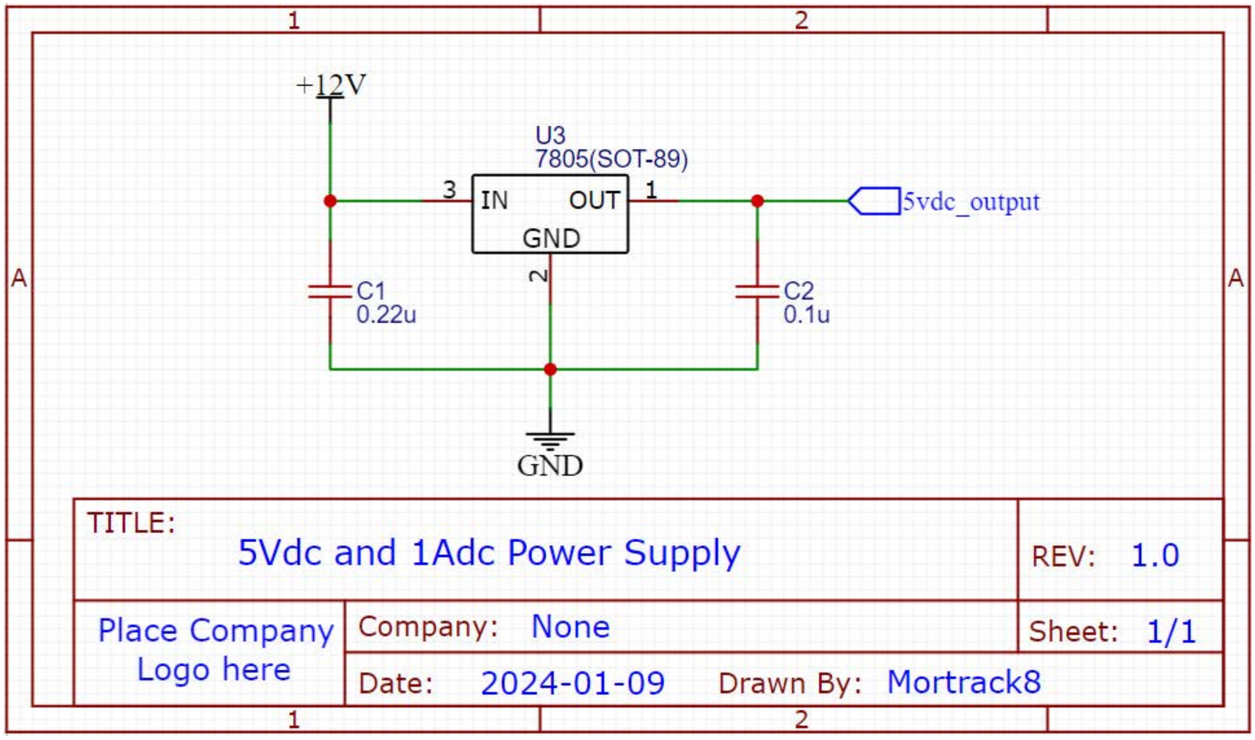
$$\boxed{cold\_LM35\_shortcircuit\_indicator_{min}\ = 0V}$$

Therefore, the expected voltage range at the "$cold\_LM35\_shortcircuit\_indicator$" label is from $0V$ up to $3.333V$, which should definitively be a safe voltage range from which the MCU can read a voltage from. Also, because of all the information discussed for this Voltage Divider, it can be inferred that it should be possible to read this label from the MCU as an Input GPIO Pin such that:

$$\boxed{\begin{aligned} &Input\ GPIO\ Pin\ of\ "cold\_LM35\_shortcircuit\_indicator" \\ &= \begin{cases} High\ State, & if\ LM35\ is\ operational. \\ Low\ State, & if\ LM35\ is\ expected\ to\ be\ under\ a\ shortcircuit. \end{cases} \end{aligned}}$$

4. Mathematical calculations for the 5V and 1A Power Supply Circuit

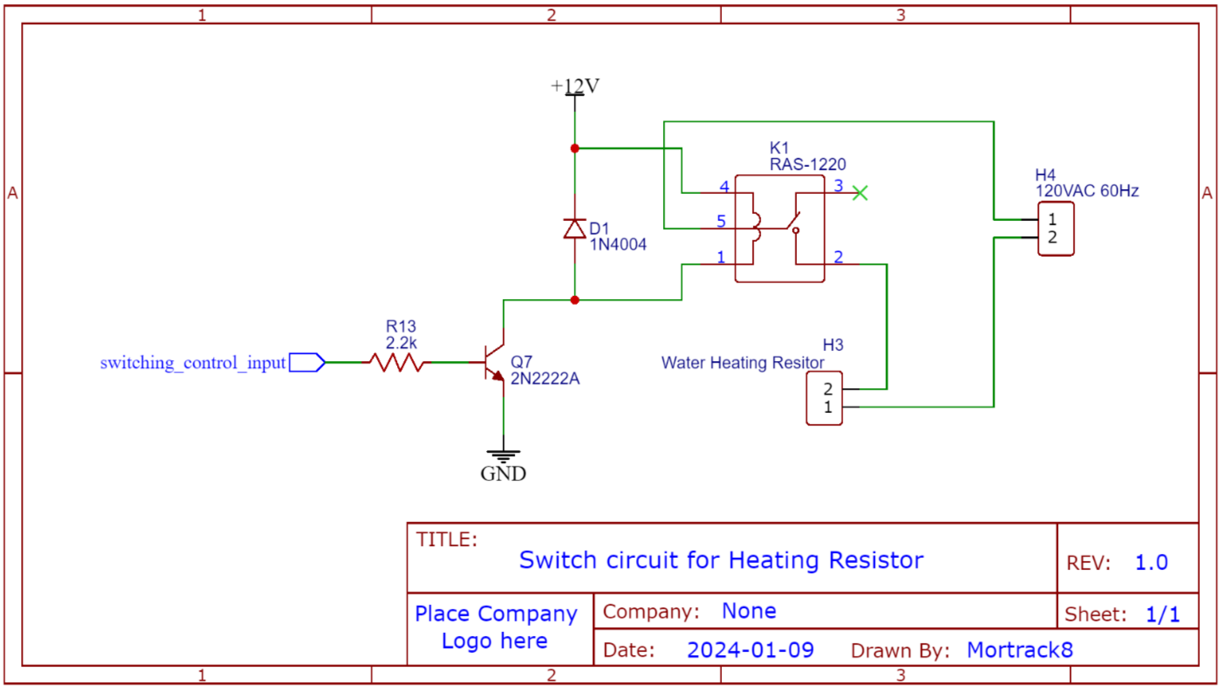
The following image shows the circuit diagram for the 5v and 1A Power Supply Circuit:



Where it is important to know that no mathematical calculations were required for this circuit design and where the proposed Capacitors C_1 and C_2were chosen because that the datasheet of the LM7805 voltage regulator suggests them to be used whenever it is desired that this Integrated Circuit behaves as foretold by that datasheet.

5. Mathematical calculations for the Switch Circuit for the Water Heating Resistor

The following image shows the circuit diagram for the Switch Circuit for the Water Heating Resistor:



Where the purpose of that circuit is to basically provide a means to the MCU to be able to turn On and Off the Water Heating Resistor of the System whenever it is desired.

To start with the Mathematical Calculations made, lets us first state that the RAS-1220 Relay was chosen since, according to its datasheet, it is supposedly able to handle a current of $15A$, i.e., the current that our 900W Water Heating Resistor is supposed to consume with respect to the Current, which should be:

$$I_{waterHeatingResistor} = \frac{P_{waterHeatingResistor}}{V_{AC}}$$

$$I_{waterHeatingResistor} = \frac{(900)}{(120)}$$

$$\boxed{I_{waterHeatingResistor} = 7.5A}$$

And where the datasheet of that Relay states that the current that it should drain under a $12V$ Power Supply is that of:

$$I_{(RAS-1220\ coil)} = I_{C_{(Q_7)}} = 30mA$$

Although the author empirically proved that current to be accurate with only one sample, it is hard to tell if this would be true for most or all possible samples for that Relay Model since the author does not have characterized that Relay. Nonetheless, it will be considered in this project that this current consumption is true and representative for an RAS-1220 Relay.

Now that this have been cleared up, from the Mortrack Characterizations for the Q_7 Transistor are the following:

$$\beta_{(Q_7)} \approx 30$$

$$V_{be_{(Q_7)}} \approx 0.73V$$

$$I_{b_{(Q_7)}} = \frac{I_{C_{(Q_7)}}}{\beta_{(Q_7)}}$$

$$I_{b_{(Q_7)}} = \frac{30x10^{-3}}{30}$$

$$\boxed{I_{b_{(Q_7)}} = 1mA}$$

Where it is important to know that the author tried several combinations before choosing this $I_{b_{(Q_7)}}$ as the final result. This is because it was desired to achieve a functional circuit able to properly commutate the Relay of this circuit but to also consume a small and negligible $I_{b_{(Q_7)}}$ with respect to the maximum working current values of the MCU.

Now, with the parameter values that we know so far, we can now identify the right value for the R_{13} Resistor:

$$switching\_control\_input_{At\_On\_State} = I_{b_{(Q_7)}} R_{13} + V_{be_{(Q_7)}}$$

$$I_{b_{(Q_7)}} R_{13} = switching\_control\_input_{At\_On\_State} - V_{be_{(Q_7)}}$$

$$R_{13} = \frac{switching\_control\_input_{At\_On\_State} - V_{be_{(Q_7)}}}{I_{b_{(Q_7)}}}$$

$$R_{13} = \frac{(3.3) - (0.73)}{(1x10^{-3})}$$

$$\boxed{R_{13} = 2'570\Omega \approx 2.2\text{k}\Omega}$$

Note that although the ideal value for R_{13} is $2'570\Omega$, $2.2k\Omega$ was proposed instead because that was the nearest Resistor value the author had at his disposal.

And basically, that's it for this circuit since regarding the D_1 Diode, it was proposed simply because it was the Diode that the author had at his disposal and that had the least amount of nominal Current but were at the same time it was enough to properly work in this circuit.

However, as an additional note for those who may not know, the D_1 Diode was placed there to work as what is well known as a Flyback Diode.