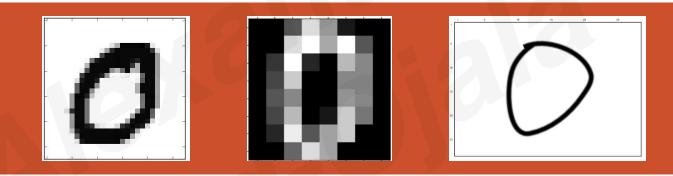
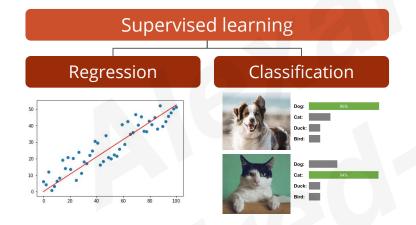
Introduction to Deep Neural Networks





Supervised ML Background

Start with a set of "observations"

 \mathcal{X}

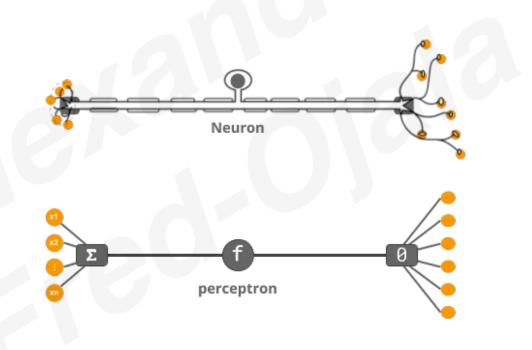
and a space of "targets" (or "labels")

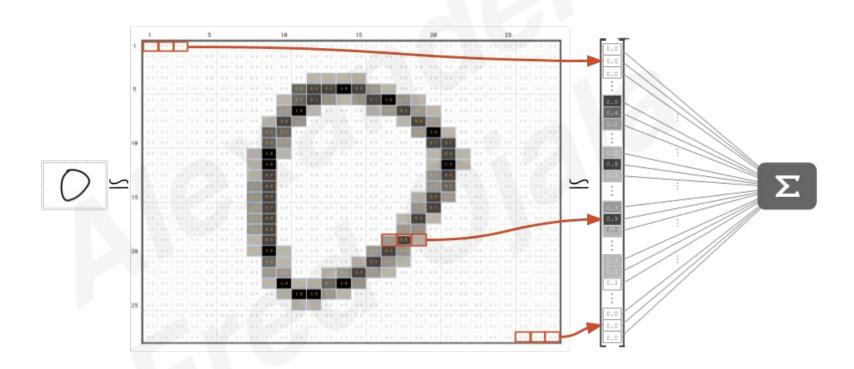
y

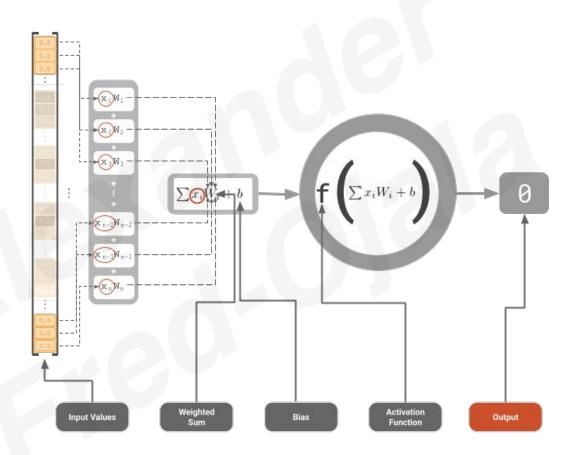
We are interested in finding a model that can map observations to determine its associated target (e.g. prediction, classification etc).

Neural:

Inspired by the way biological neurons work in the human brain





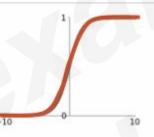


Activation functions

Sigmoid
$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

tanh

tanh(x)



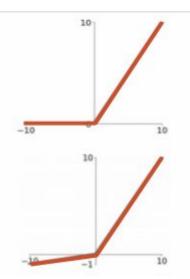


ReLU

 $\max(0, x)$

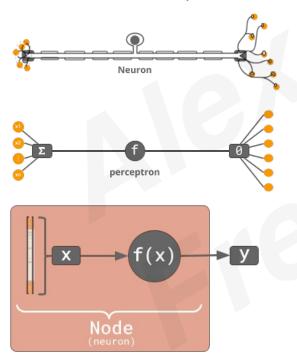


 $\max\left(\alpha x, x\right)$



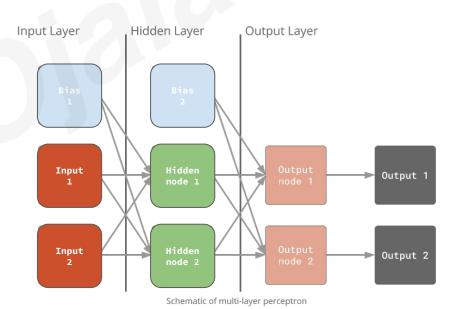
Neural:

Inspired by the way biological neurons work in the human brain process



Network:

A network of neurons/nodes connected by a set of weights



Neural Networks

How does the network know what good weight values are? — → it learns them.

Steps:

- Initialize the network with random weights
- Input a set of features
- Calculate the output of the network by feeding the example through all layers (forward pass)
- Calculate the value of the loss function
- Backpropagate the error across every layer, calculate the loss gradient and update the weights
- Repeat from 2 until desired (or acceptable) performance is achieved

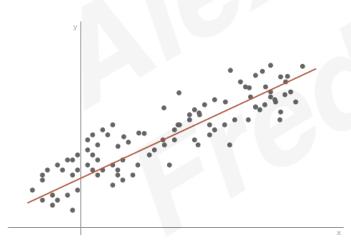
Multi-layer **Perceptron** (covered)

Neural Network (Deep if hidden layers > 2)

(covered next)

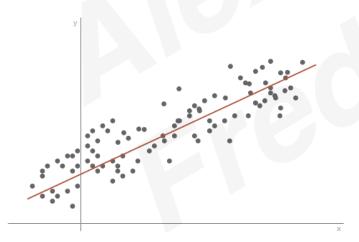
Prediction Functions

$$\hat{y} = f(x) = ax + b$$



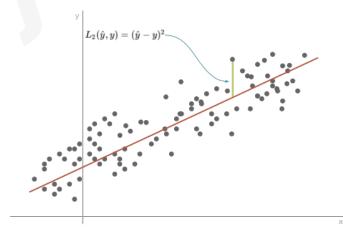
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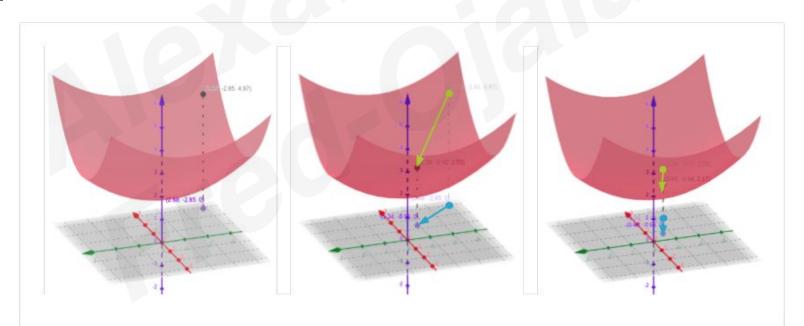


Loss Functions

$$L_2(\hat{y},y) = (\hat{y}-y)^2$$



Optimization: Gradient Descent



Backpropagation

Chain Rule

For input x, target y, and parameters W, our loss is of the form

$$L\left(f\left(x,W\right) ,y\right)$$

To compute the gradient of *L* with respect to *W* we need the chain rule

$$\frac{dL}{dW} = \frac{dL}{df} \frac{df}{dW}$$

given *f* is single-valued, *W* a single parameter.

if W is a vector of parameters, then

$$abla_W L = rac{dL}{df}
abla_W f$$

if *f* is vector-valued with *k* values and *W* is a vector of *m* parameters

$$\frac{\partial L}{\partial W_j} = \sum_{i=1}^k \frac{\partial L}{\partial f_i} \frac{\partial f_i}{\partial W_j}$$

Backpropagation

Jacobians

$$\frac{\partial L}{\partial W_j} = \sum_{i=1}^k \frac{\partial L}{\partial f_i} \frac{\partial f_i}{\partial W_j}$$

For j = 1, ..., m the above can be written as

$$J_L(W) = J_L(f)J_f(W)$$

where the Jacobian matrix is given by

$$J_f(W)_{ij} = rac{\partial f_i}{\partial \overline{W}_j}$$

Thus the Jacobian generalizes the gradient of a scalar-valued function f to a k-valued function -- which is used to represent a neural layer with m inputs and k outputs with.

$$J_f(W) = egin{bmatrix} rac{\partial f_1}{\partial W_1} & rac{\partial f_1}{\partial W_2} & \cdots & rac{\partial f_1}{\partial W_m} \ rac{\partial f_2}{\partial W_1} & rac{\partial f_2}{\partial W_2} & \cdots & rac{\partial f_2}{\partial W_m} \ dots & dots & \ddots & dots \ rac{\partial f_k}{\partial W_1} & rac{\partial f_k}{\partial W_2} & \cdots & rac{\partial f_k}{\partial W_m} \end{bmatrix}$$

Backpropagation

N-Step Chain Rule

Now suppose we have a deep network composed of several vector-valued functions

$$A, B, C, \dots$$

composed in a chain (Input 1, Hidden 1, etc.)

$$W o A o B o C o \cdots o L$$

Which is represented algebraically as

$$L(W) = L\left(\cdots C\left(B\left(A\left(W\right)\right)\right)\cdots\right)$$

Then we can get the gradient by simple matrix multiplication of the Jacobians

$$J_L(W) = J_L(K) * \cdots * J_C(B) * J_B(A) * J_A(W)$$

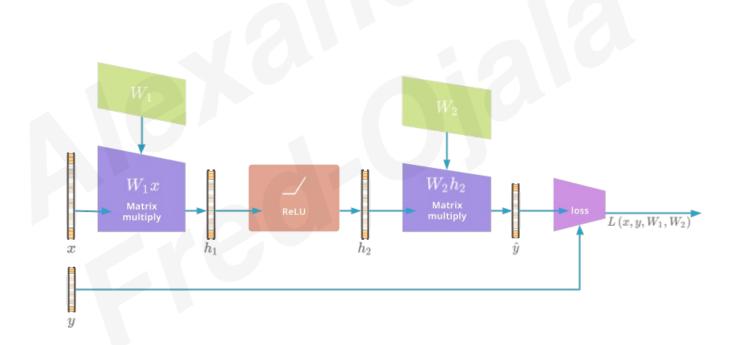
where

$$J_L(W) = \left(
abla_W L
ight)^T$$

is the gradient we need to minimize the loss over *W*.

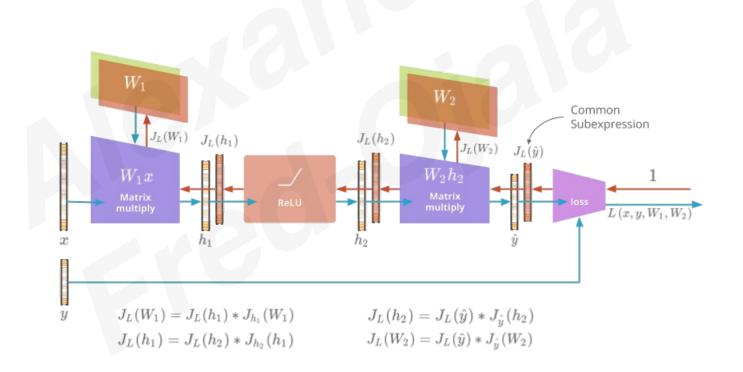
1-Layer Neural Network - forward pass

Zooming out: building a neural network (Tensorflow style)



1-Layer Neural Network - backpropagation

Zooming out: building a neural network (Tensorflow style)



Thank You!