# پسگرد

# backtracking

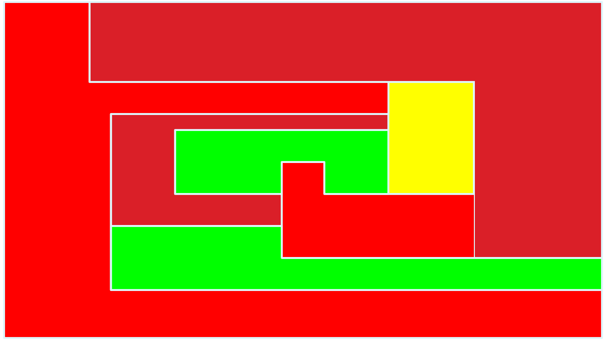
# Backtracking

- Suppose you have to make a series of *decisions,* among various *choices,* where
  - ✓ You don't have enough information to know what to choose
  - ✓ Each decision leads to a new set of choices
  - ✓ Some sequence of choices (possibly more than one) may be a solution to your problem
- Backtracking is a methodical way of trying out various sequences of decisions, until you find one that "works"

# Solving a maze

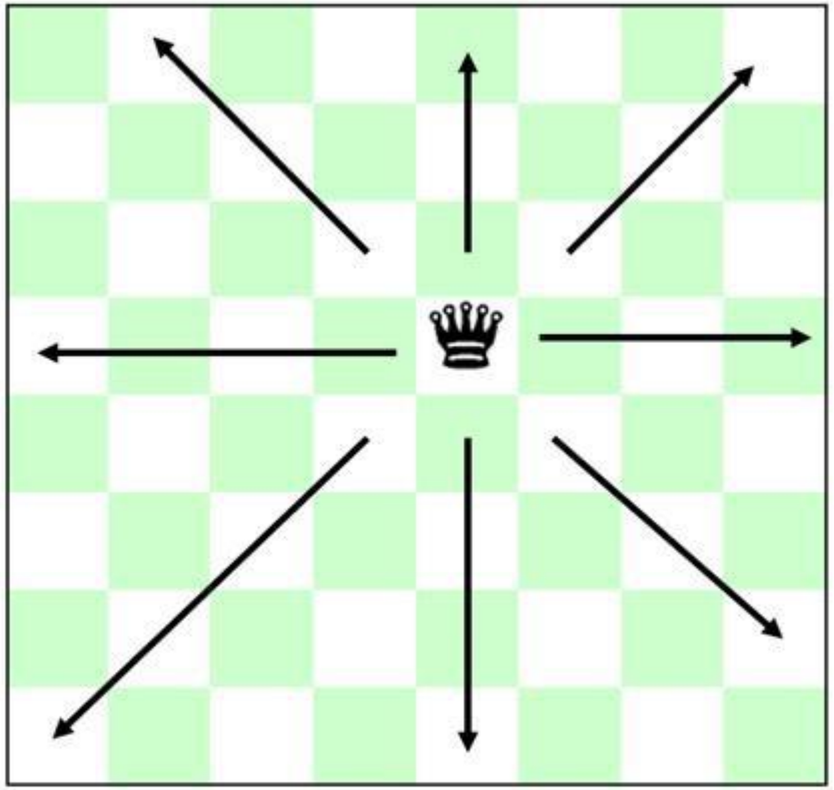
- Given a maze, find a path from start to finish
- At each intersection, you have to decide between three or fewer choices:
  - ✓ Go straight
  - ✓ Go up
  - ✓ Go down
- You don’t have enough information to choose correctly
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many types of maze problem can be solved with backtracking

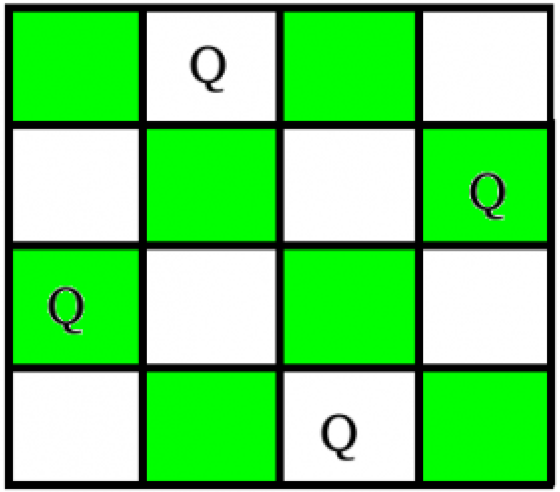
# Coloring a map



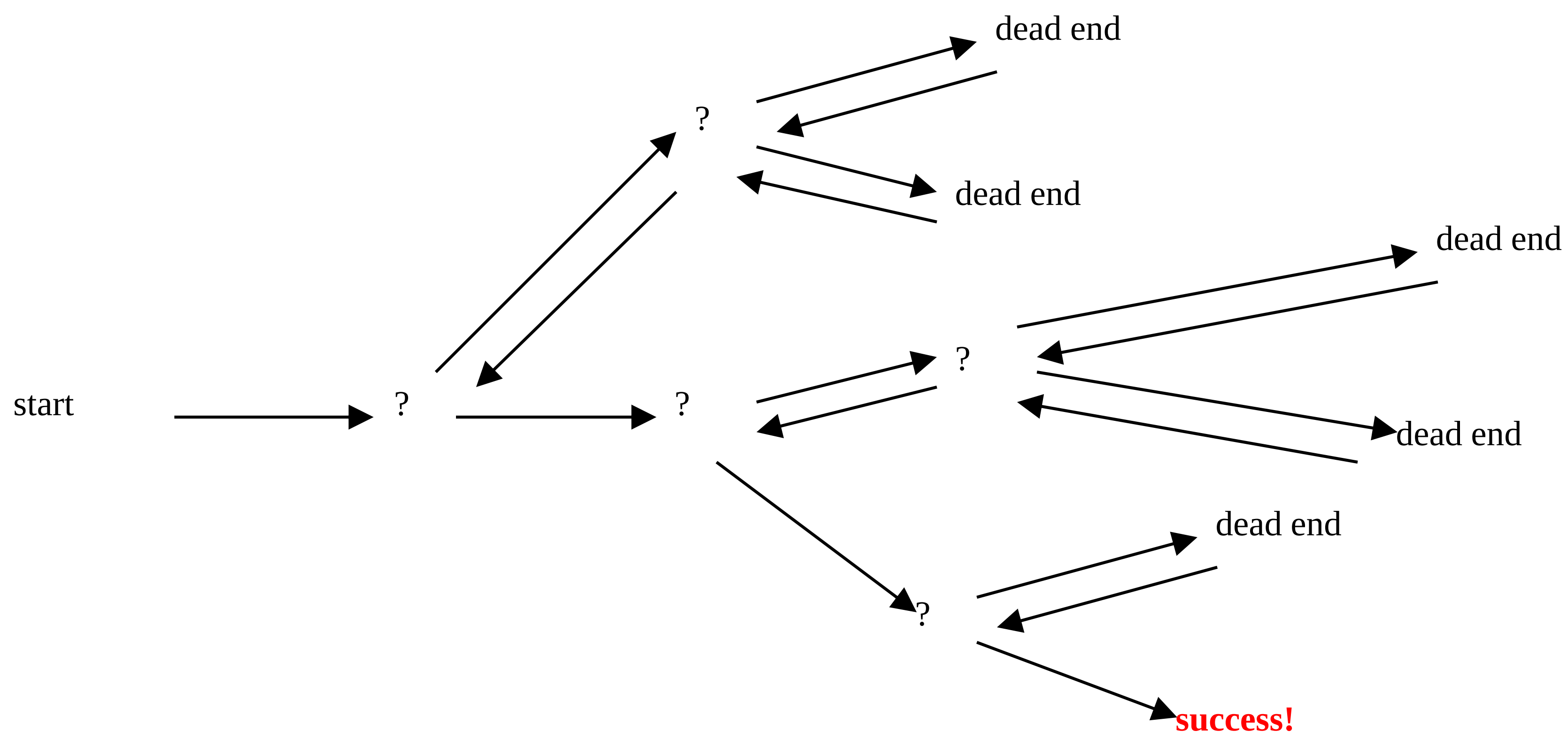
- You wish to color a map with not more than four colors
  - ✓ red, yellow, green, blue
- Adjacent countries must be in different colors
- You don't have enough information to choose colors
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many coloring problems can be solved with backtracking

# N-Queens



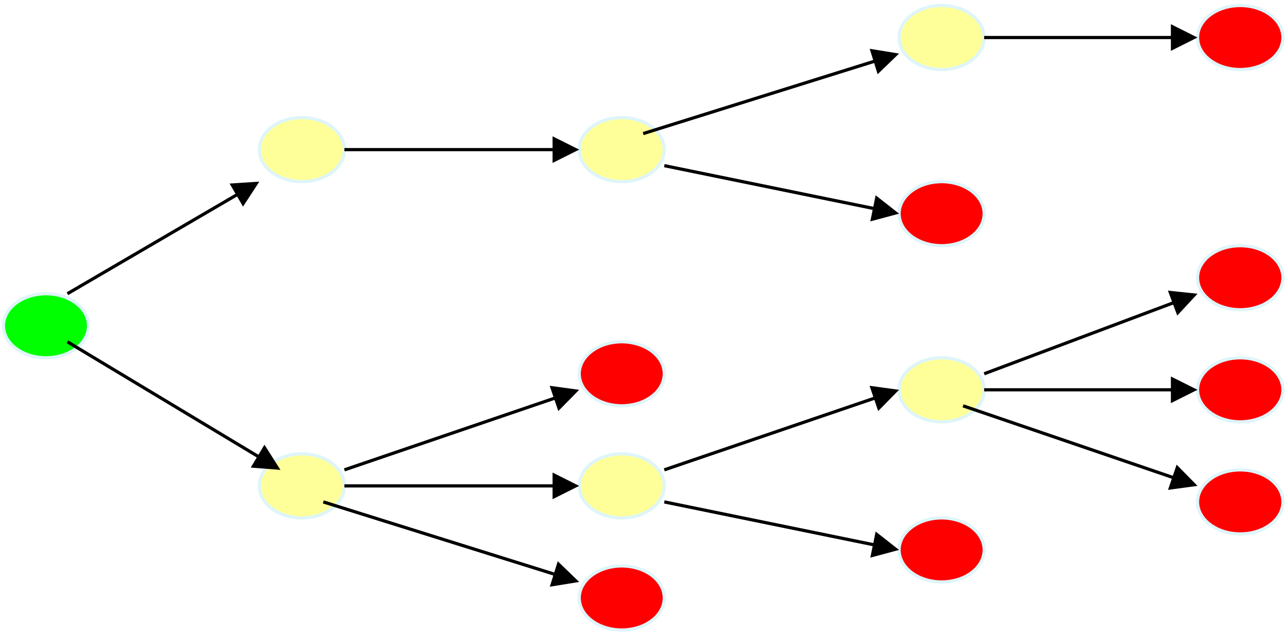


# Backtracking (animation)

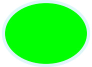


# Terminology I

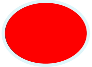
A tree is composed of nodes



There are three kinds of nodes:

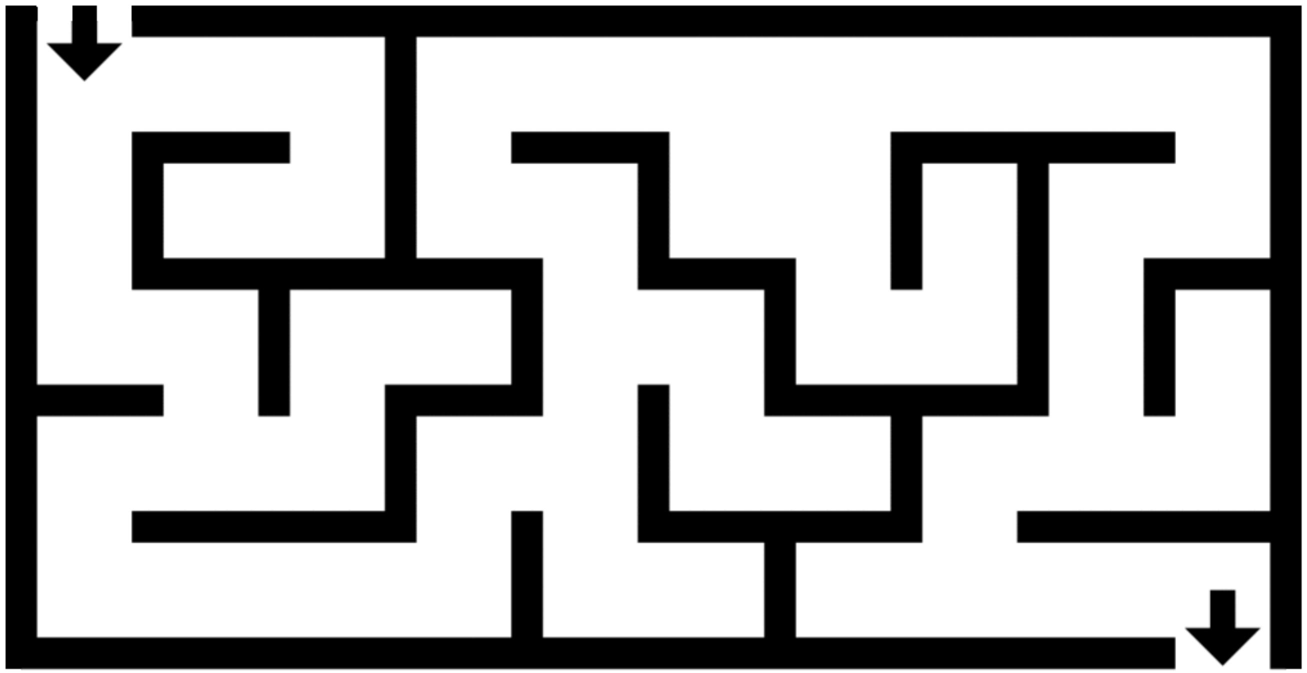
 The (one) root node

 Internal nodes

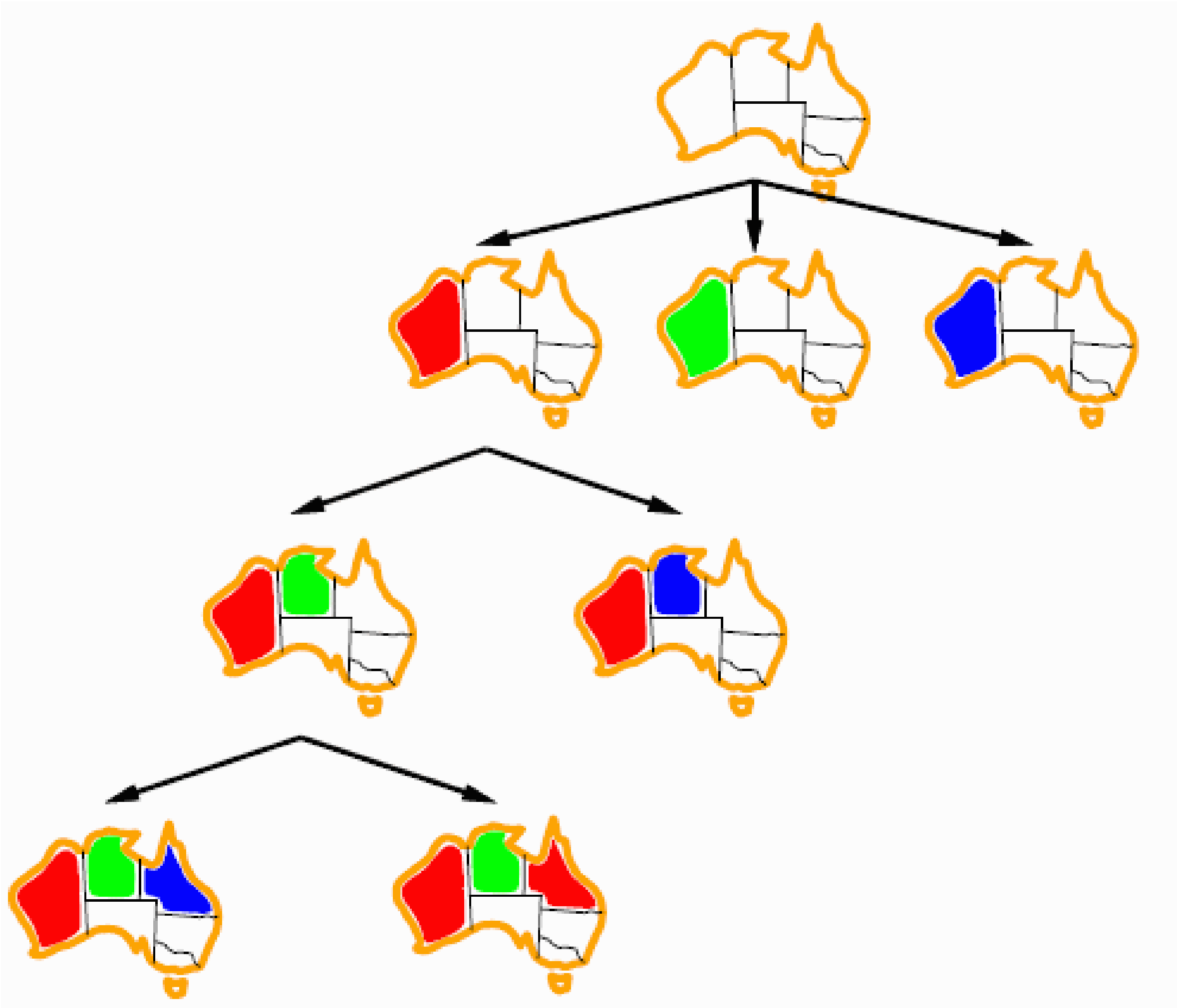
 Leaf nodes

*Backtracking* can be thought of as searching a tree for a particular “goal” leaf node

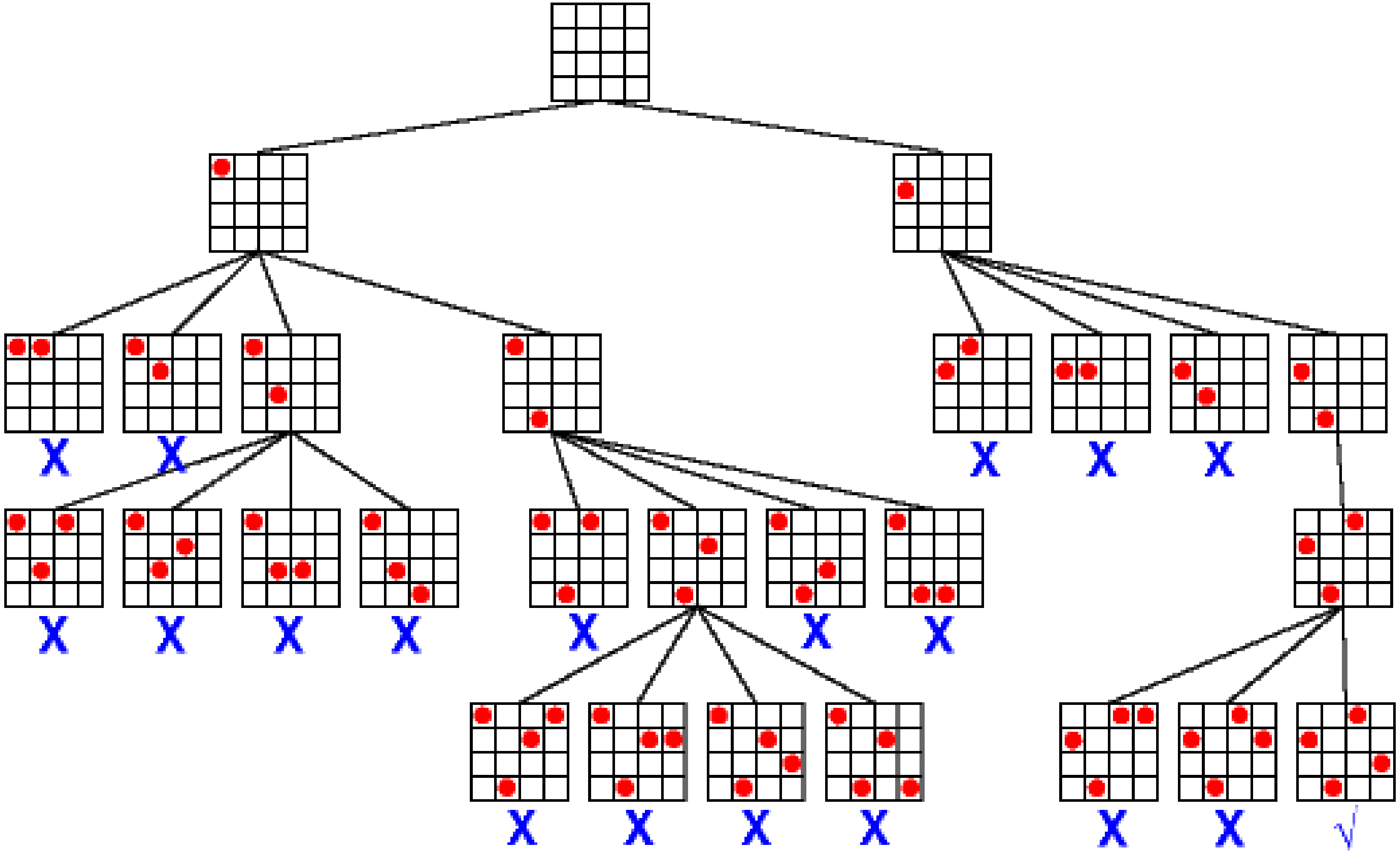
# Maze



# Graph coloring

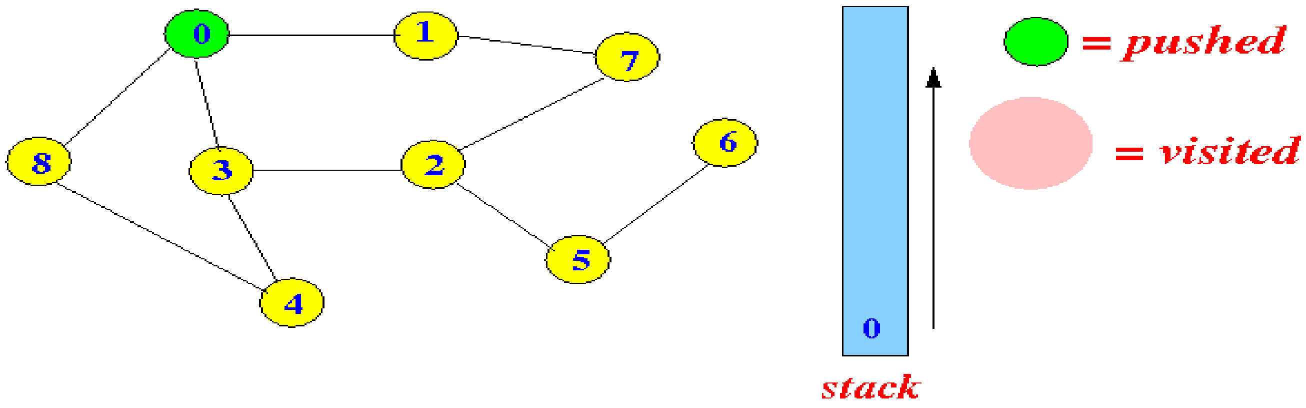


# N-Queens

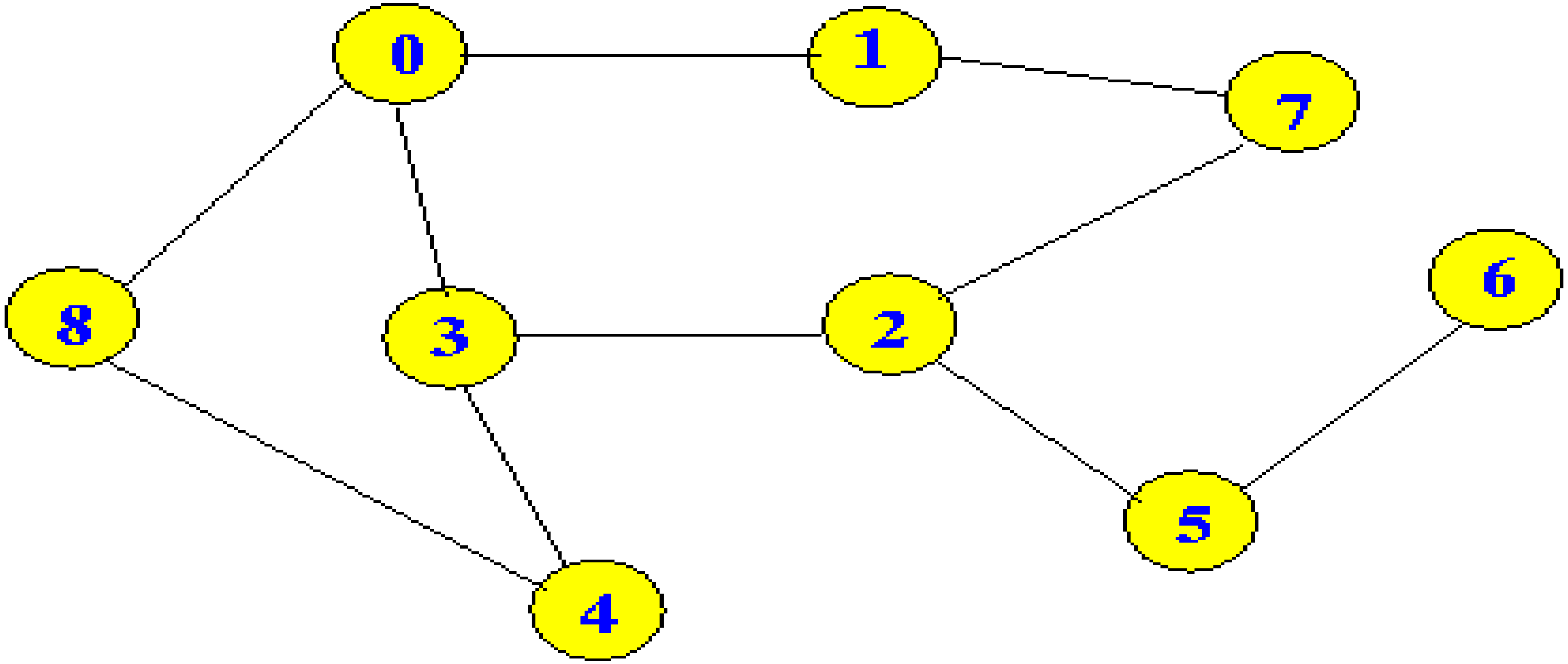


# جستجوی اول‌عمق

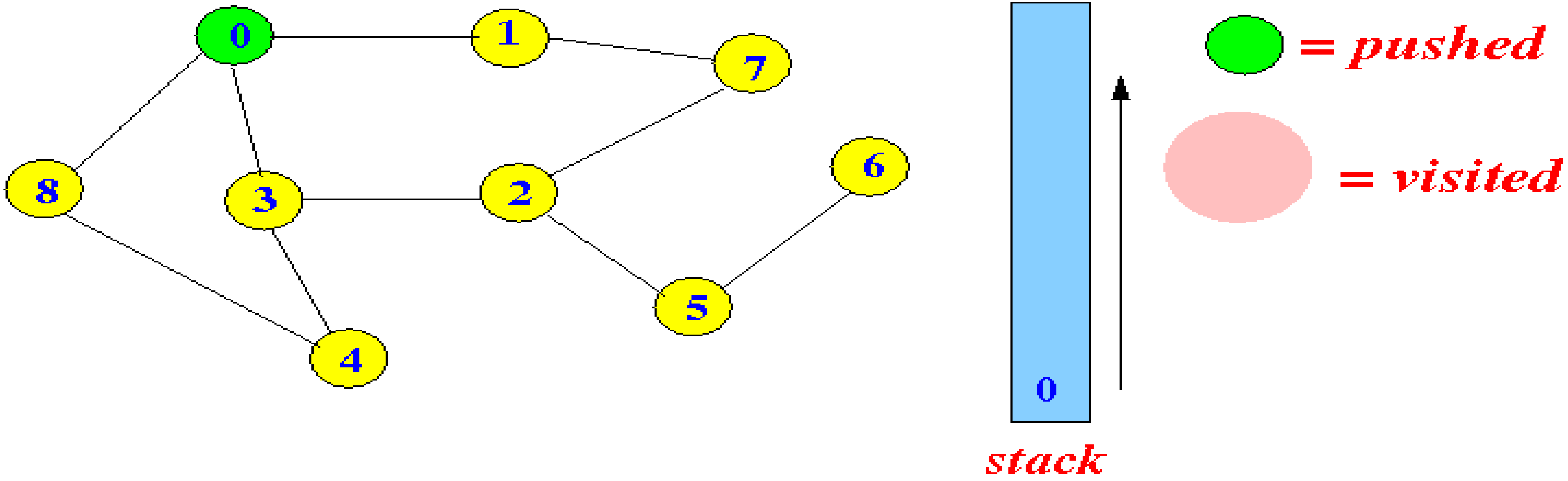
- بسط عمیق‌ترین گره در frontier فعلی درخت جستجو،  تاجایی که گره‌ها فاقد جانشین باشند.
- حذف این گره‌ها از frontier درخت.
  - ✓ استفاده از صف LIFO (پشته)



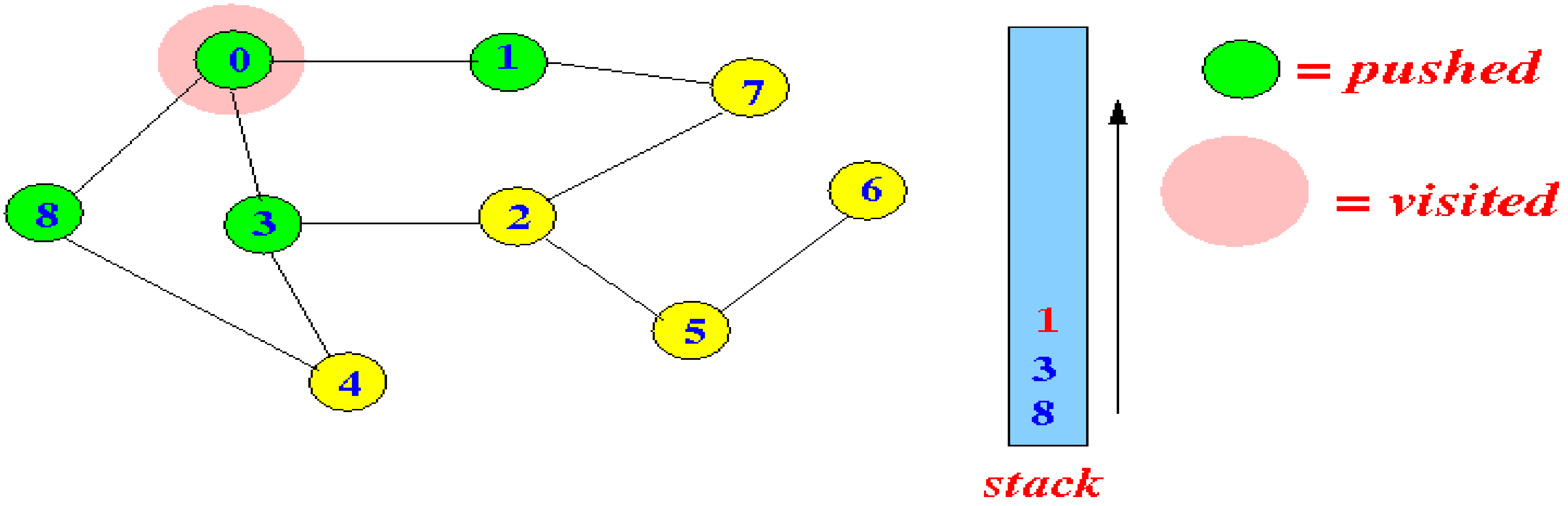
# DFS



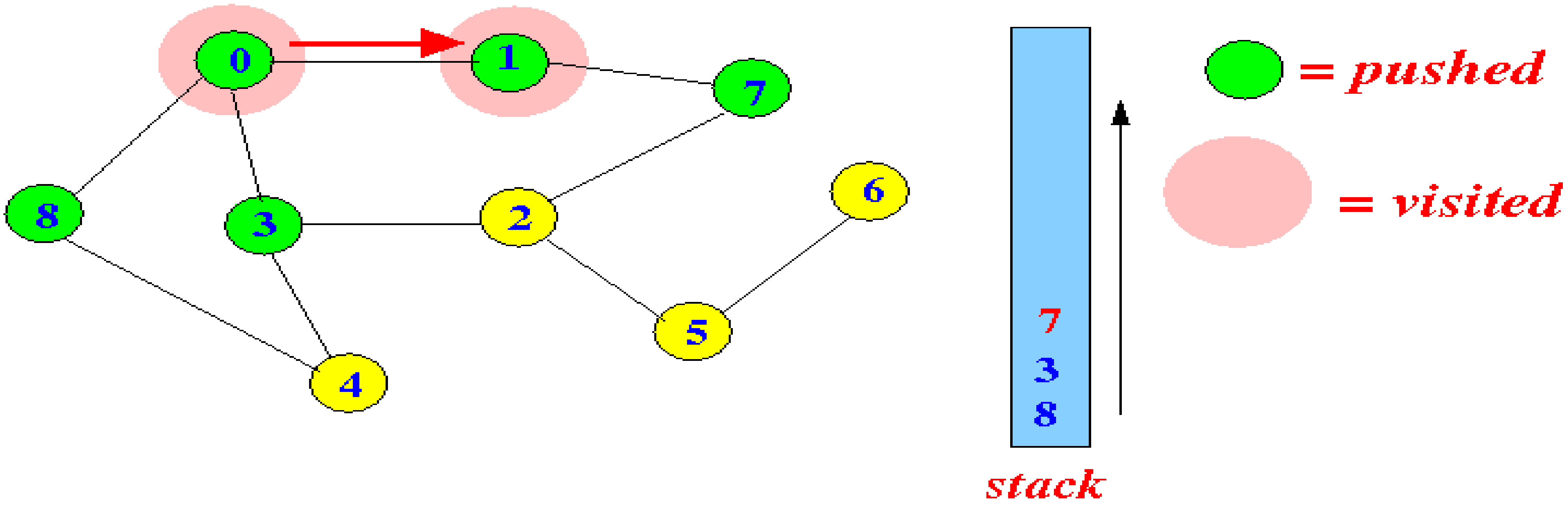
# DFS



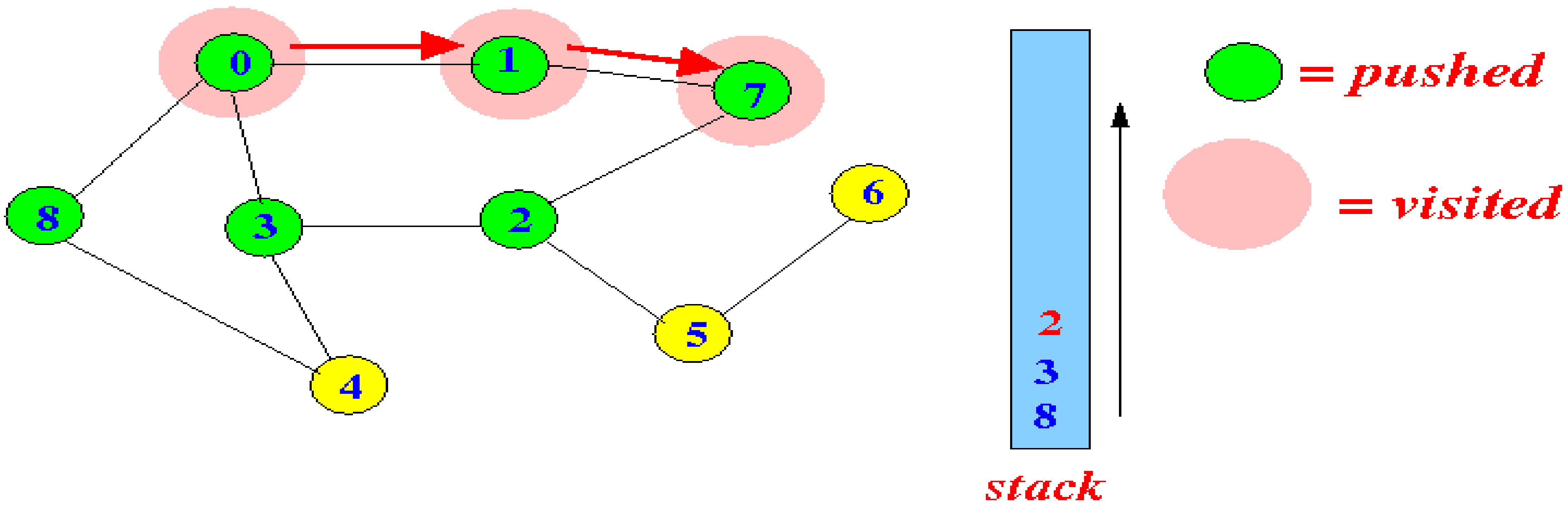
# DFS



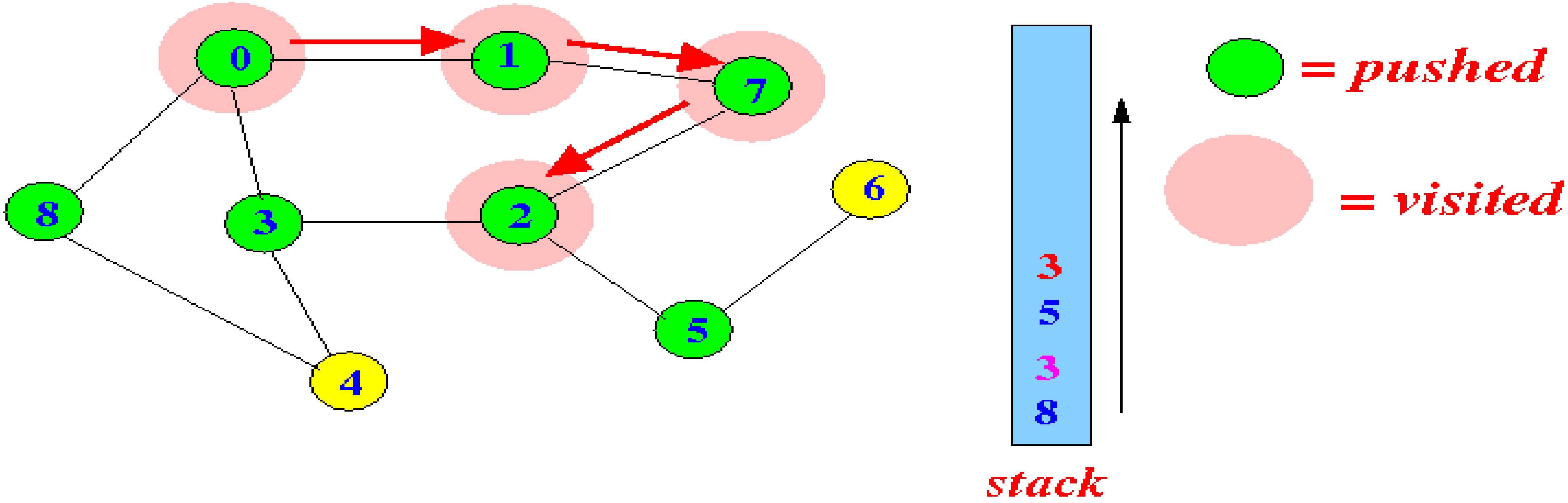
# DFS



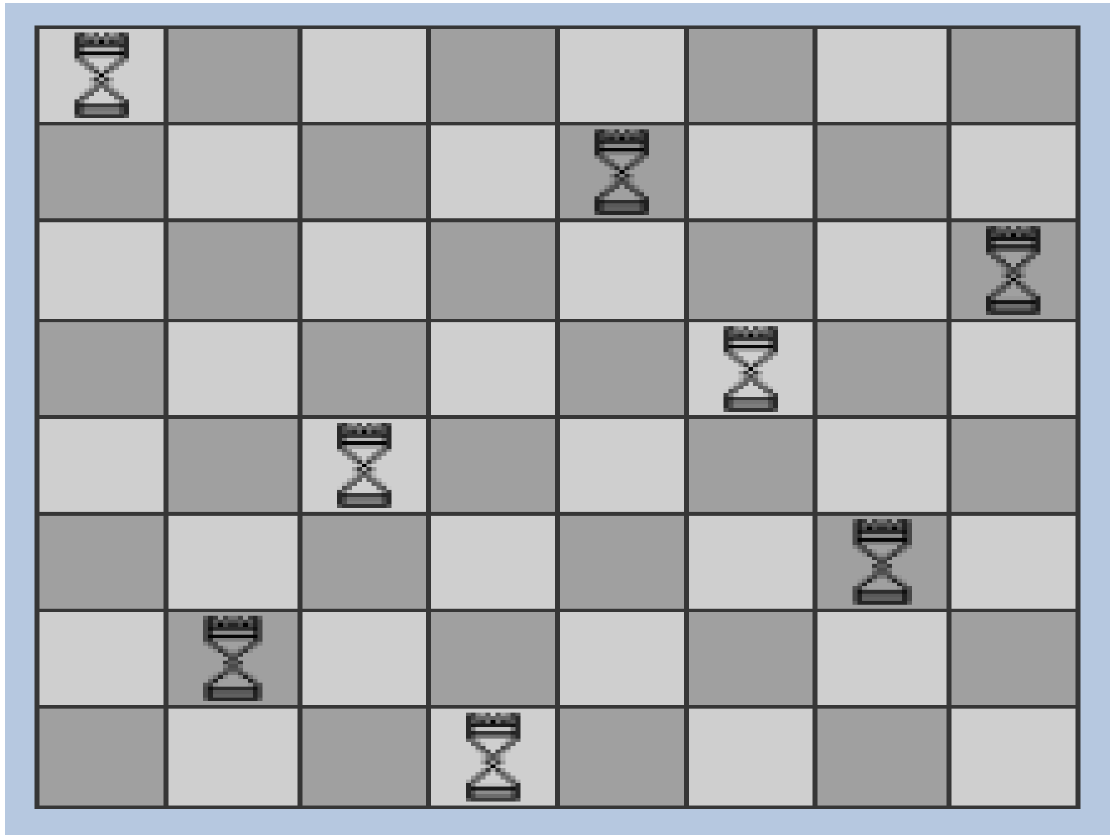
# DFS

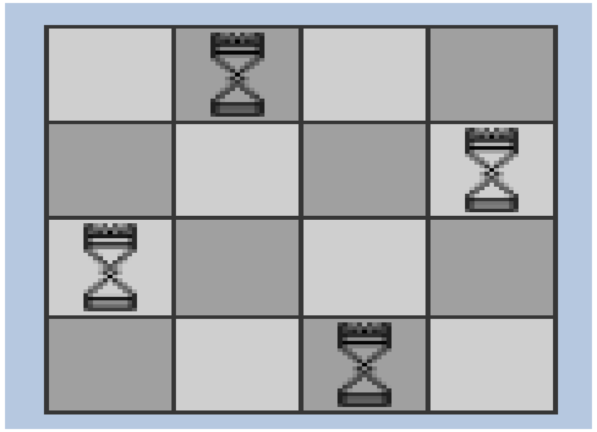


# DFS



# N-Qeens





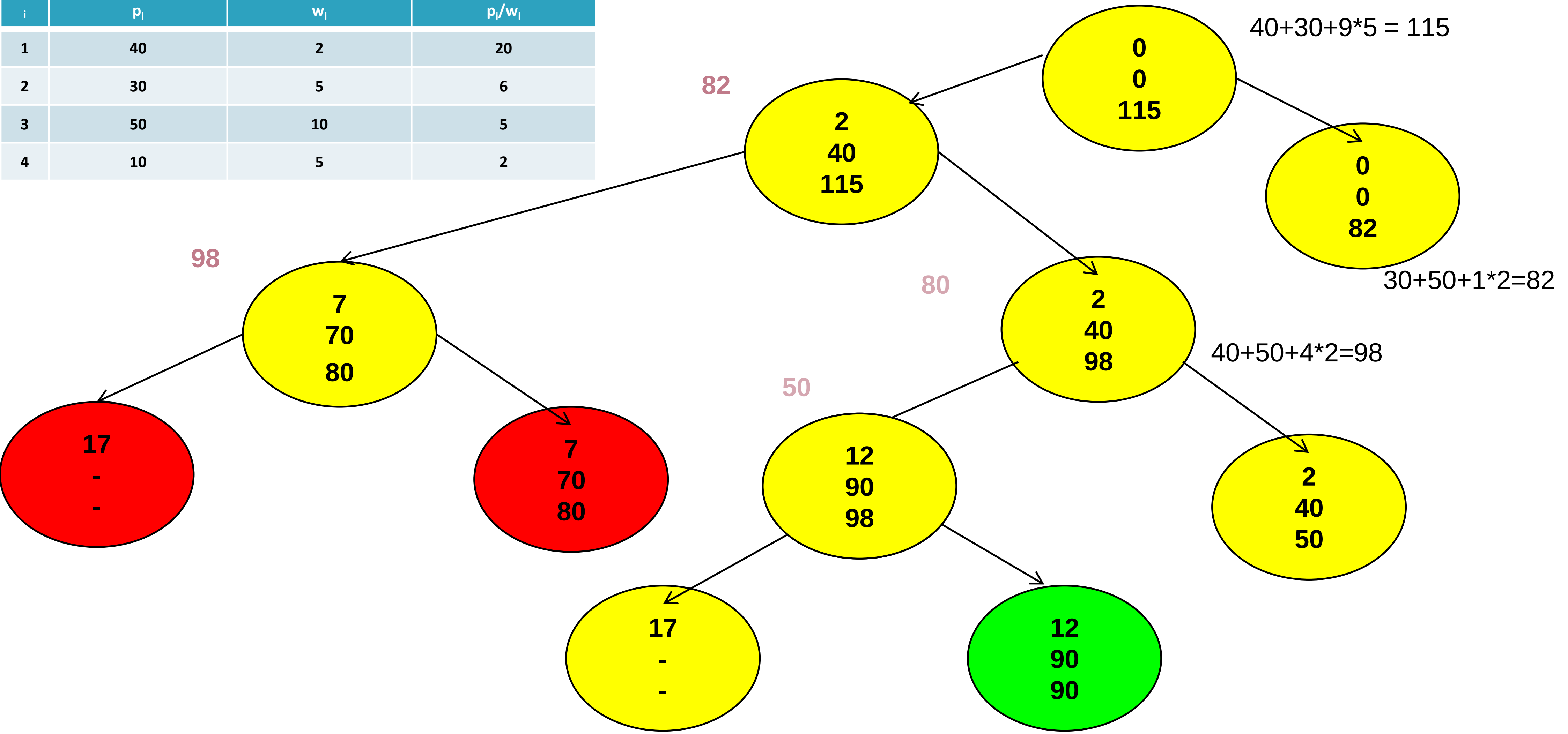
- Given some items, pack the knapsack to get the maximum total value.
- Each item $i$ has some weight denoted by $w_i$ and some value denoted by $p_i$.
- Total weight that we can carry is no more than some fixed number $W$.
- So we must consider weights of items as well as their values.

| $i$ | $p_i$ | $w_i$ | $p_i/w_i$ |
|---|---|---|---|
| 1 | 40 | 2 | 20 |
| 2 | 30 | 5 | 6 |
| 3 | 50 | 10 | 5 |
| 4 | 10 | 5 | 2 |

W = 16

# کوله‌پشتی ۰-۱

| i | $p_i$ | $w_i$ | $p_i/w_i$ |
|---|---|---|---|
| 1 | 40 | 2 | 20 |
| 2 | 30 | 5 | 6 |
| 3 | 50 | 10 | 5 |
| 4 | 10 | 5 | 2 |



- رابطه بازگشتی:

$$V[k,w] = \begin{cases} V[k-1,w] & \text{if } w_k > w \\ \max\{V[k-1,w], V[k-1,w-w_k]+b_k\} & \text{else} \end{cases}$$

- It means, that the best subset of $S_k$ that has total weight $w$ is:

✓ 1) the best subset of $S_{k-1}$ that has total weight $\leq w$, **or**

✓ 2) the best subset of $S_{k-1}$ that has total weight $\leq w-w_k$ plus the item $k$

# Example (1)

- می‌خواهیم مسئله زیر را به روش برنامه‌نویسی پویا حل کنیم:

| Item # | Weight | Value |
|---|---|---|
| 1 | 2 | 3 |
| 2 | 3 | 4 |
| 3 | 4 | 5 |
| 4 | 5 | 6 |

$$W = 5$$

# Example (2)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |

for w = 0 to W

$\quad V[0,w] = 0$

# Example (3)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | | | | | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

for i = 1 to n

  V[i,0] = 0

# Example (4)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | | | | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=1$

$b_i=3$

$w_i=2$

$w=1$

$w-w_i=-1$

Items:

1: (2,3)

2: (3,4)

3: (4,5)

4: (5,6)

if $w_i <= w$ // item i can be part of the solution

    if $b_i + V[i-1,w-w_i] > V[i-1,w]$

        $V[i,w] = b_i + V[i-1,w- w_i]$

    else

        $V[i,w] = V[i-1,w]$

else **V[i,w] = V[i-1,w]** // $w_i > w$

# Example (5)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | **3** | | | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

i=1

$b_i$=3

$w_i$=2

w=2

$w-w_i$ =0

Items:

1: (2,3)

2: (3,4)

3: (4,5)

4: (5,6)

if $w_i$ <= w // item i can be part of the solution

    if $b_i$ + V[i-1,w-$w_i$] > V[i-1,w]

        **V[i,w] = $b_i$ + V[i-1,w- $w_i$]**

    else

        V[i,w] = V[i-1,w]

else V[i,w] = V[i-1,w]  // $w_i$ > w

# Example (6)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | **3** | | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

i=1

$b_i$=3

$w_i$=2

w=3

w-$w_i$ =1

Items:

1: (2,3)

2: (3,4)

3: (4,5)

4: (5,6)

if $w_i$ <= w // item i can be part of the solution

if $b_i$ + V[i-1,w-$w_i$] > V[i-1,w]

**V[i,w] = $b_i$ + V[i-1,w- $w_i$]**

else

V[i,w] = V[i-1,w]

else V[i,w] = V[i-1,w] // $w_i$ > w

# Example (7)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | **3** | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

i=1

$b_i=3$

$w_i=2$

w=4

$w-w_i=2$

Items:

1: (2,3)

2: (3,4)

3: (4,5)

4: (5,6)

if $w_i$ <= w // item i can be part of the solution

    if $b_i$ + V[i-1,w-$w_i$] > V[i-1,w]

        **V[i,w] = $b_i$ + V[i-1,w- $w_i$]**

    else

        V[i,w] = V[i-1,w]

else V[i,w] = V[i-1,w]  // $w_i$ > w

# Example (8)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | **3** |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=1$

$b_i=3$

$w_i=2$

$w=5$

$w-w_i=3$

Items:

1: (2,3)

2: (3,4)

3: (4,5)

4: (5,6)

if $w_i <= w$ // item i can be part of the solution

    if $b_i + V[i-1,w-w_i] > V[i-1,w]$

        **$V[i,w] = b_i + V[i-1,w- w_i]$**

    else

        $V[i,w] = V[i-1,w]$

else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (9)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | **0** | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

i=2

$b_i=4$

$w_i=3$

w=1

$w-w_i = -2$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i$ <= w // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        $V[i,w] = b_i + V[i-1,w- w_i]$
    else
        $V[i,w] = V[i-1,w]$
else **V[i,w] = V[i-1,w]** // $w_i > w$

# Example (10)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | **3** | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=2$

$b_i=4$

$w_i=3$

$w=2$

$w-w_i=-1$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i <= w$ // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        $V[i,w] = b_i + V[i-1,w- w_i]$
    else
        $V[i,w] = V[i-1,w]$
else **V[i,w] = V[i-1,w]** // $w_i > w$

# Example (11)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | **4** | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=2$

$b_i=4$

$w_i=3$

$w=3$

$w-w_i=0$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i <= w$ // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        **$V[i,w] = b_i + V[i-1,w- w_i]$**
    else
        $V[i,w] = V[i-1,w]$
else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (12)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | **4** | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=2$

$b_i=4$

$w_i=3$

$w=4$

$w-w_i=1$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i <= w$ // item i can be part of the solution
  if $b_i + V[i-1,w-w_i] > V[i-1,w]$
    **$V[i,w] = b_i + V[i-1,w- w_i]$**
  else
    $V[i,w] = V[i-1,w]$
else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (13)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |

$i=2$

$b_i=4$

$w_i=3$

$w=5$

$w-w_i=2$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i \le w$ // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        **$V[i,w] = b_i + V[i-1,w-w_i]$**
    else
        $V[i,w] = V[i-1,w]$
else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (14)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | **0** | **3** | **4** | | |
| 4 | 0 | | | | | |

$i=3$

$b_i=5$

$w_i=4$

$w= 1..3$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i$ <= w // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        $V[i,w] = b_i + V[i-1,w- w_i]$
    else
        $V[i,w] = V[i-1,w]$
else **V[i,w] = V[i-1,w]** // $w_i > w$

# Example (15)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | 0 | 3 | 4 | **5** | |
| 4 | 0 | | | | | |

$i=3$

$b_i=5$

$w_i=4$

$w= 4$

$w- w_i=0$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i <= w$ // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        **$V[i,w] = b_i + V[i-1,w- w_i]$**
    else
        $V[i,w] = V[i-1,w]$
else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (16)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | 0 | 3 | 4 | 5 | **7** |
| 4 | 0 | | | | | |

$i=3$

$b_i=5$

$w_i=4$

$w= 5$

$w- w_i=1$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i <= w$ // item i can be part of the solution
    if $b_i + V[i-1,w-w_i] > V[i-1,w]$
        $V[i,w] = b_i + V[i-1,w- w_i]$
    else
        **V[i,w] = V[i-1,w]**
else $V[i,w] = V[i-1,w]$ // $w_i > w$

# Example (17)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | 0 | 3 | 4 | 5 | 7 |
| 4 | 0 | **0** | **3** | **4** | **5** | |

$i=4$

$b_i=6$

$w_i=5$

$w= 1..4$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

if $w_i$ <= w // item i can be part of the solution
  if $b_i + V[i-1,w-w_i] > V[i-1,w]$
    $V[i,w] = b_i + V[i-1,w- w_i]$
  else
    $V[i,w] = V[i-1,w]$
else **V[i,w] = V[i-1,w]** // $w_i > w$

# Example (18)

| i\W | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 2 | 0 | 0 | 3 | 4 | 4 | 7 |
| 3 | 0 | 0 | 3 | 4 | 5 | 7 |
| 4 | 0 | 0 | 3 | 4 | 5 | **7** |

$i=4$

$b_i=6$

$w_i=5$

$w= 5$

$w- w_i=0$

Items:

1: (2,3)
2: (3,4)
3: (4,5)
4: (5,6)

```
if wi <= w // item i can be part of the solution
    if bi + V[i-1,w-wi] > V[i-1,w]
        V[i,w] = bi + V[i-1,w- wi]
    else
        V[i,w] = V[i-1,w]
else V[i,w] = V[i-1,w]  // wi > w
```