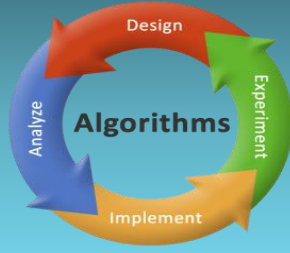


برنامه‌نویسی پویا

بزرگترین زیردنباله مشترک (Longest Common Subsequence)

بلند ترین زیر دنباله مشترک

Longest common subsequence (LCS)



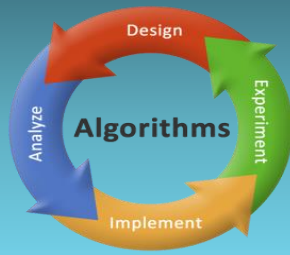
□ یک رشته DNA شامل یک دنباله از مولکول‌هایی است که به آنها Base می‌گویند.
برخی از این Base ها عبارتند از

Guanine, Cytosine, Thymine, Adenine

□ هر کدام از این Base ها با حرف اول خود نشان داده می‌شوند.
✓ به عنوان مثال $\{A, G, C, T\}$ می‌تواند یک رشته DNA باشد.

بلند ترین زیر دنباله مشترک

Longest common subsequence (LCS)



□ شباهت دو رشته DNA را می توان به یکی از سه روش زیر پیدا کرد:

✓ حداقل تعداد تغییراتی روی Base ها که یک رشته را به رشته دیگر تبدیل کند.

✓ طول زیررشته مشترک دو رشته

✓ طول زیردنباله مشترک دو رشته

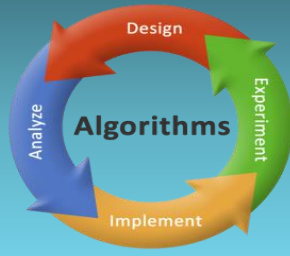
▪ برای دو رشته S_1 و S_2 ، رشته جدیدی مانند S_3 پیدا کنید طوری که، Base ها ظاهر شوند ، اما ترتیب حضور این base ها در S_1 و S_2 باید همان ترتیب حضور در S_3 باشد.

$S_1 = \text{ACCGGTCGAGTGC GCGGAAGCCGGCCGAA}$

$S_2 = \text{GTCGTTCGGAATG CCGTTGCTCTGTAAA}$

$S_3 = \text{GTCGTCGGAAGCCGGCCGAA}$

زیردنباله مشترک



□ یک زیردنباله حاوی حروف مشترک دو دنباله است که متوالی نیستند ولی ترتیب حضور در دنباله‌های اولیه را حفظ کرده‌اند.

$X: ABCBDAB$

$Y: BDCABA$

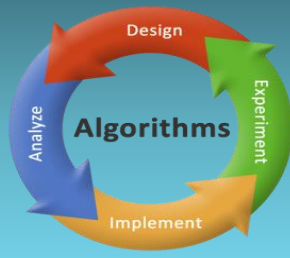
CS1: **BCBA**

CS2: **BCA**

CS3: **CA**

CS1: **BDAB**

چگونه مسئلہ را حل کنیم

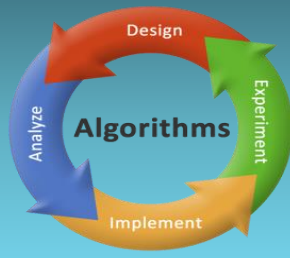


$X: ABCBDAB$

$Y: BDCABA$

$X: ABCBDABC$

$Y: BDCABAC$



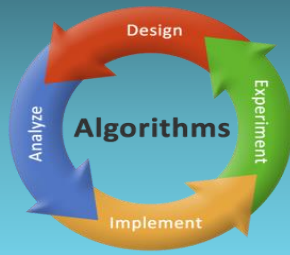
Theorem 15.1 (Optimal substructure of an LCS)

Let $X = \langle x_1, x_2, \dots, x_m \rangle$ and $Y = \langle y_1, y_2, \dots, y_n \rangle$ be sequences, and let $Z = \langle z_1, z_2, \dots, z_k \rangle$ be any LCS of X and Y .

1. If $x_m = y_n$, then $z_k = x_m = y_n$ and Z_{k-1} is an LCS of X_{m-1} and Y_{n-1} .
2. If $x_m \neq y_n$, then $z_k \neq x_m$ implies that Z is an LCS of X_{m-1} and Y .
3. If $x_m \neq y_n$, then $z_k \neq y_n$ implies that Z is an LCS of X and Y_{n-1} .

$$c[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0, \\ c[i - 1, j - 1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j, \\ \max(c[i, j - 1], c[i - 1, j]) & \text{if } i, j > 0 \text{ and } x_i \neq y_j. \end{cases}$$

بزرگترین زیر دنباله مشترک (LCS)



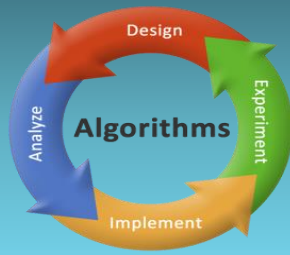
LCS-LENGTH(X, Y)

```

1   $m = X.length$ 
2   $n = Y.length$ 
3  let  $b[1..m, 1..n]$  and  $c[0..m, 0..n]$  be new tables
4  for  $i = 1$  to  $m$ 
5       $c[i, 0] = 0$ 
6  for  $j = 0$  to  $n$ 
7       $c[0, j] = 0$ 
8  for  $i = 1$  to  $m$ 
9      for  $j = 1$  to  $n$ 
10         if  $x_i == y_j$ 
11              $c[i, j] = c[i - 1, j - 1] + 1$ 
12              $b[i, j] = "\nwarrow"$ 
13         elseif  $c[i - 1, j] \geq c[i, j - 1]$ 
14              $c[i, j] = c[i - 1, j]$ 
15              $b[i, j] = "\uparrow"$ 
16         else  $c[i, j] = c[i, j - 1]$ 
17              $b[i, j] = "\leftarrow"$ 
18  return  $c$  and  $b$ 
    
```

		j	0	1	2	3	4	5	6
		y_j		B	D	C	A	B	A
0	x_i		0	0	0	0	0	0	0
1	A		0	\uparrow	\uparrow	\uparrow	\nwarrow 1	\leftarrow 1	\nwarrow 1
2	B		0	\nwarrow 1	\nwarrow 1	\nwarrow 1	\uparrow 1	\nwarrow 2	\leftarrow 2
3	C		0	\uparrow 1	\uparrow 1	\nwarrow 2	\nwarrow 2	\uparrow 2	\uparrow 2
4	B		0	\nwarrow 1	\uparrow 1	\uparrow 2	\uparrow 2	\nwarrow 3	\leftarrow 3
5	D		0	\uparrow 1	\nwarrow 2	\uparrow 2	\uparrow 2	\nwarrow 3	\uparrow 3
6	A		0	\uparrow 1	\uparrow 2	\uparrow 2	\nwarrow 3	\uparrow 3	\nwarrow 4
7	B		0	\nwarrow 1	\uparrow 2	\uparrow 2	\uparrow 3	\nwarrow 4	\uparrow 4

بزرگترین زیر دنباله مشترک (LCS)



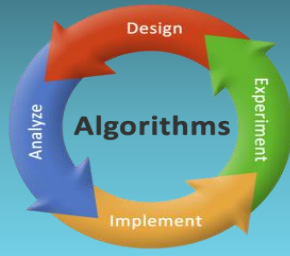
PRINT-LCS(b, X, i, j)

```

1  if  $i == 0$  or  $j == 0$ 
2      return
3  if  $b[i, j] == \nwarrow$ 
4      PRINT-LCS( $b, X, i - 1, j - 1$ )
5      print  $x_i$ 
6  elseif  $b[i, j] == \uparrow$ 
7      PRINT-LCS( $b, X, i - 1, j$ )
8  else PRINT-LCS( $b, X, i, j - 1$ )
    
```

		j	0	1	2	3	4	5	6
i	x_i	y_j		B	D	C	A	B	A
0		x_i	0	0	0	0	0	0	0
1	A		0	0	0	0	1	←1	1
2	B		0	1	←1	←1	1	2	←2
3	C		0	1	1	2	←2	2	2
4	B		0	1	1	2	2	3	←3
5	D		0	1	2	2	2	3	3
6	A		0	1	2	2	3	3	4
7	B		0	1	2	2	3	4	4

مثال



□ حال راه حل مذکور را روی یک مثال بینیم:

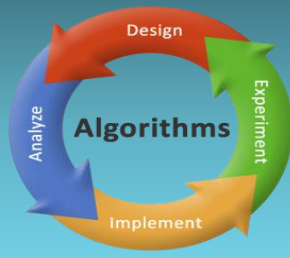
✓ $X = ABCB$

✓ $Y = BDCAB$

□ به راحتی می توان دریافت که:

$$LCS(X, Y) = BCB$$

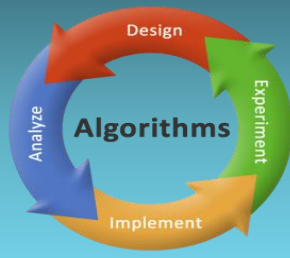
LCS Example (0)



$X = \text{ABCB}; m = |X| = 4$
 $Y = \text{BDCAB}; n = |Y| = 5$
Allocate array $c[6,5]$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i	0						
	A	1						
	B	2						
	C	3						
	B	4						

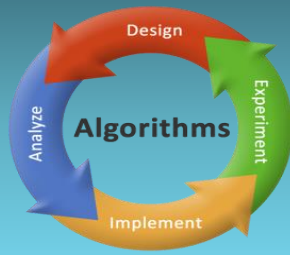
LCS Example (1)



for $i = 1$ to m $c[i,0] = 0$

		j	0	1	2	3	4	5
		Y_j		B	D	C	A	B
i	X_i	0						
1	A	0						
2	B	0						
3	C	0						
4	B	0						

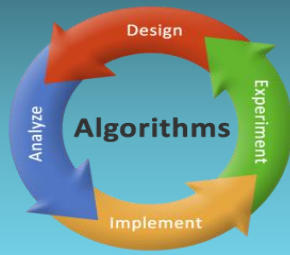
LCS Example (2)



for $j = 0$ to n $c[0,j] = 0$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0					
2	B		0					
3	C		0					
4	B		0					

LCS Example (3)



case $i=1$ and $j=1$

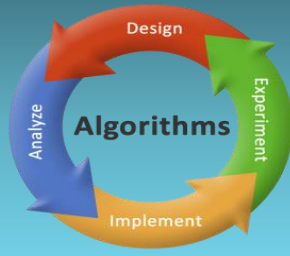
$A \neq B$

but, $c[0,1] \geq c[1,0]$

so $c[1,1] = c[0,1]$, and $b[1,1] = \uparrow$

		j	0	1	2	3	4	5
i		Y _j		B	D	C	A	B
	X _i							
0			0	0	0	0	0	0
1	A	0	0	0				
2	B	0						
3	C	0						
4	B	0						

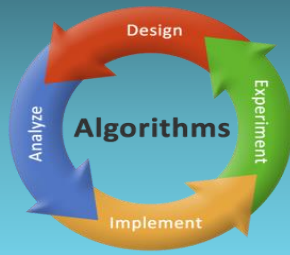
LCS Example (4)



case $i=1$ and $j=2$
 $A \neq D$
 but, $c[0,2] \geq c[1,1]$
 so $c[1,2] = c[0,2]$, and $b[1,2] = \uparrow$

		j	0	1	2	3	4	5
i	Xi	Yj		B	D	C	A	B
0			0	0	0	0	0	0
1	A		0	0	0			
2	B							
3	C		0					
4	B		0					

LCS Example (5)



case $i=1$ and $j=3$

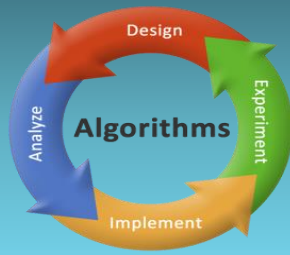
$A \neq C$

but, $c[0,3] \geq c[1,2]$

so $c[1,3] = c[0,3]$, and $b[1,3] = \uparrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	0	0	0	0	0	0	0	0
	0	1	0	0	0	0		

LCS Example (6)



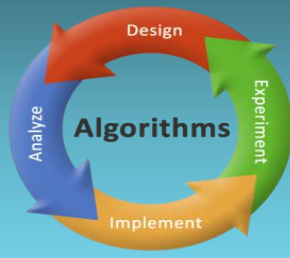
case $i=1$ and $j=4$

$A = A$

so $c[1,4] = c[0,2] + 1$, and $b[1,4] = \nwarrow$

		j	0	1	2	3	4	5
		Yj		B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0 \uparrow	0 \uparrow	0 \uparrow	1 \nwarrow	
2	B		0					
3	C		0					
4	B		0					

LCS Example (7)



case $i=1$ and $j=5$

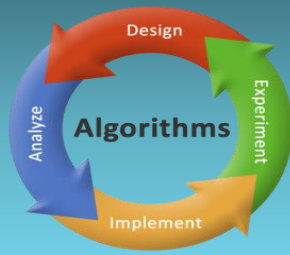
$A \neq B$

this time $c[0,5] < c[1,4]$

so $c[1,5] = c[1,4]$, and $b[1,5] = \leftarrow$

		j	0	1	2	3	4	5
i		Y _j		B	D	C	A	B
	X _i							
0			0	0	0	0	0	0
1	A		0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←
2	B		0					
3	C		0					
4	B		0					

LCS Example (8)



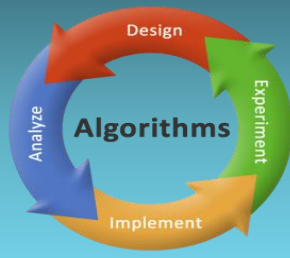
case $i=2$ and $j=1$

$B = B$

so $c[2, 1] = c[1, 0] + 1$, and $b[2, 1] = \nwarrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1				
3	C		0					
4	B		0					

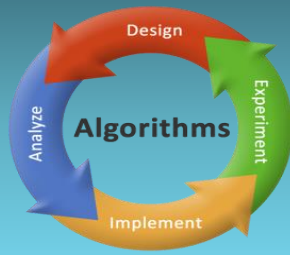
LCS Example (9)



case $i=2$ and $j=2$
 $B \neq D$
 and $c[1, 2] < c[2, 1]$
 so $c[2, 2] = c[2, 1]$ and $b[2, 2] = \leftarrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1			
3	C		0					
4	B		0					

LCS Example (10)



case $i=2$ and $j=3$

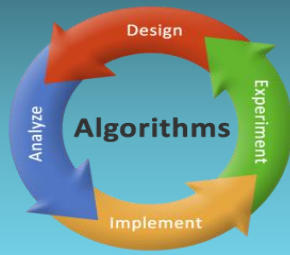
$B \neq D$

and $c[1, 3] < c[2, 2]$

so $c[2, 3] = c[2, 2]$ and $b[2, 3] = \leftarrow$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i	0						
0		0	0	0	0	0	0	0
1	A	0	0	0	0	0	1	1
2	B	0	1	1	1			
3	C	0						
4	B	0						

LCS Example (11)



case $i=2$ and $j=4$

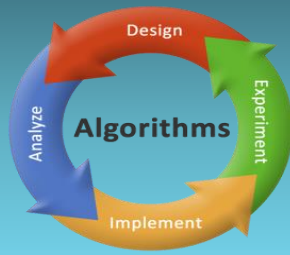
$B \neq A$

and $c[1, 4] = c[2, 3]$

so $c[2, 4] = c[1, 4]$ and $b[2, 2] = \uparrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	
3	C		0					
4	B		0					

LCS Example (12)



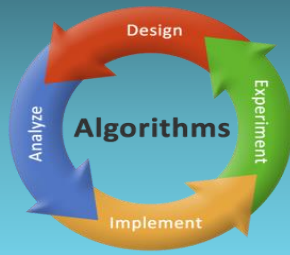
case $i=2$ and $j=5$

$B = B$

so $c[2, 5] = c[1, 4] + 1$ and $b[2, 5] = \swarrow$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i							
0			0	0	0	0	0	0
1	A		0	0 \uparrow	0 \uparrow	0 \uparrow	1 \swarrow	1 \leftarrow
2	B		0	1 \swarrow	1 \leftarrow	1 \leftarrow	1 \uparrow	2 \nearrow
3	C		0					
4	B		0					

LCS Example (13)



case $i=3$ and $j=1$

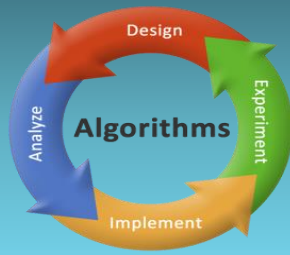
$C \neq B$

and $c[2, 1] > c[3, 0]$

so $c[3, 1] = c[2, 1]$ and $b[3, 1] = \uparrow$

		j	0	1	2	3	4	5
		Yj		B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1				
4	B		0					

LCS Example (14)



case $i=3$ and $j=2$

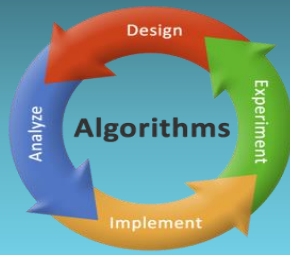
$C \neq D$

and $c[2, 2] = c[3, 1]$

so $c[3, 2] = c[2, 2]$ and $b[3, 2] = \uparrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i	0						
0	Xi	0	0	0	0	0	0	0
1	A	0	0	0	0	0	1	1
2	B	0	1	1	1	1	1	2
3	C	0	1	1	1			
4	B	0						

LCS Example (15)



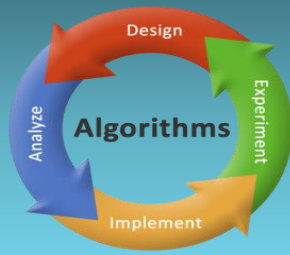
case $i=3$ and $j=3$

$C = C$

so $c[3, 3] = c[2, 2] + 1$ and $b[3, 3] = \nwarrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i	0	0	0	0	0	0	0
1	A	0	0	↑	0	↑	1	↖
2	B	0	1	↖	1	←	1	↑
3	C	0	1	↑	1	↑	2	↖
4	B	0						

LCS Example (16)



case $i=3$ and $j=4$

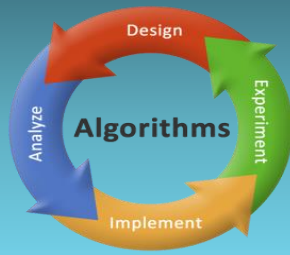
$C \neq A$

$c[2, 4] < c[3, 3]$

so $c[3, 4] = c[3, 3]$ and $b[3, 3] = \leftarrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0 \uparrow	0 \uparrow	0 \uparrow	1 \swarrow	1 \leftarrow
2	B		0	1 \swarrow	1 \leftarrow	1 \leftarrow	1 \uparrow	2 \swarrow
3	C		0	1 \uparrow	1 \uparrow	2 \swarrow	2 \leftarrow	
4	B		0					

LCS Example (17)



case $i=3$ and $j=5$

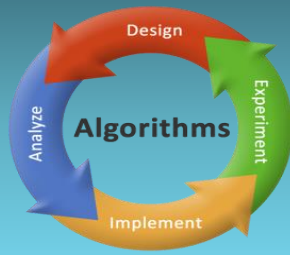
$C \neq B$

$c[2, 5] = c[3, 4]$

so $c[3, 5] = c[2, 5]$ and $b[3, 5] = \uparrow$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i							
0			0	0	0	0	0	0
1	A		0	0 \uparrow	0 \uparrow	0 \uparrow	1 \swarrow	1 \leftarrow
2	B		0	1 \swarrow	1 \leftarrow	1 \leftarrow	1 \uparrow	2 \swarrow
3	C		0	1 \uparrow	1 \uparrow	2 \swarrow	2 \leftarrow	2 \uparrow
4	B		0					

LCS Example (18)



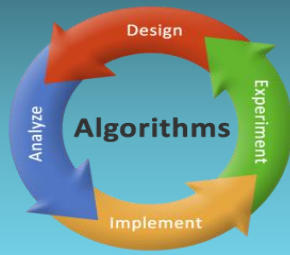
case $i=4$ and $j=1$

$B = B$

so $c[4, 1] = c[3, 0] + 1$ and $b[4, 1] = \swarrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1				

LCS Example (19)



case $i=4$ and $j=2$

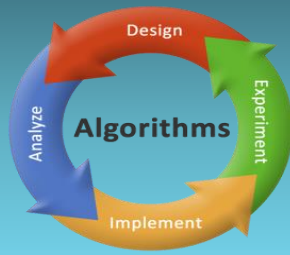
$B \neq D$

$c[3, 2] = c[4, 1]$

so $c[4, 2] = c[3, 2]$ and $b[4, 2] = \uparrow$

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i	0						
0	Xi	0	0	0	0	0	0	0
1	A	0	0	0	0	0	1	1
2	B	0	1	1	1	1	1	2
3	C	0	1	1	2	2	2	2
4	B	0	1	1				

LCS Example (20)



case $i=4$ and $j=3$

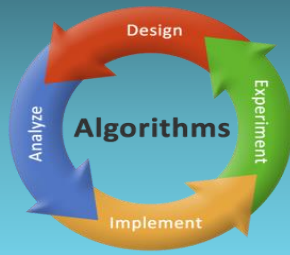
$B \neq C$

$c[3, 3] > c[4, 2]$

so $c[4, 3] = c[3, 3]$ and $b[4, 3] = \uparrow$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i	0						
0		0	0	0	0	0	0	0
1	A	0	0	\uparrow	0	\uparrow	1	\swarrow
2	B	0	1	\swarrow	1	\leftarrow	1	\uparrow
3	C	0	1	\uparrow	1	\uparrow	2	\swarrow
4	B	0	1	\swarrow	1	\uparrow	2	\leftarrow

LCS Example (21)



case $i=4$ and $j=4$

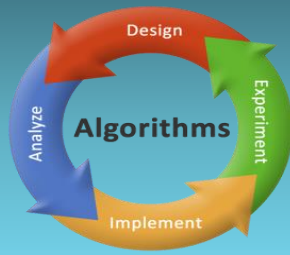
$B \neq A$

$c[3, 4] = c[4, 3]$

so $c[4, 4] = c[3, 4]$ and $b[3, 5] = \uparrow$

		j	0	1	2	3	4	5
			Y_j	B	D	C	A	B
i	X_i	0						
0		0	0	0	0	0	0	0
1	A	0	0	\uparrow	0	\uparrow	0	\uparrow
2	B	0	1	\swarrow	1	\swarrow	1	\swarrow
3	C	0	1	\uparrow	1	\uparrow	2	\swarrow
4	B	0	1	\swarrow	1	\uparrow	2	\uparrow

LCS Example (22)



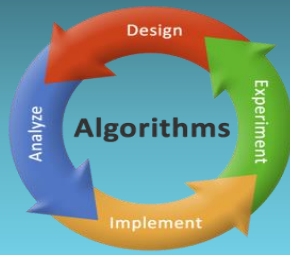
case $i=4$ and $j=5$

$B = B$

so $c[4, 5] = c[3, 4] + 1$ and $b[4, 5] = \nearrow$

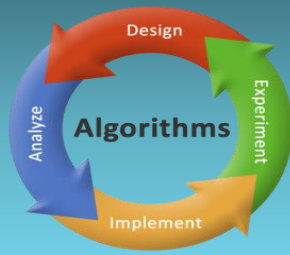
		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0 \uparrow	0 \uparrow	0 \uparrow	1 \nwarrow	1 \leftarrow
2	B		0	1 \nwarrow	1 \leftarrow	1 \leftarrow	1 \uparrow	2 \nwarrow
3	C		0	1 \uparrow	1 \uparrow	2 \nwarrow	2 \leftarrow	2 \uparrow
4	B		0	1 \nwarrow	1 \uparrow	2 \uparrow	2 \uparrow	3 \nwarrow

Finding LCS



		j	0	1	2	3	4	5
			Yj	B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Finding LCS (2)



LCS (reversed order): **B C B**

LCS (straight order): **B C B**

(this string turned out to be a palindrome)

		j	0	1	2	3	4	5
i	Xi	Yj		B	D	C	A	B
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3