

Answer No 1

A set G with a binary operation $*$ is an abelian group if all these hold:

1. closure: For all $a, b \in G$, $a * b \in G$
2. Associativity: For all $a, b, c \in G$, $(a * b) * c = a * (b * c)$
3. Identity Element: There exist $e \in G$ such that $a * e = e * a$ for all $a \in G$
4. Inverse Element: For all $a \in G$, there exist $b \in G$ such that $a * b = b * a = e$.
5. Commutativity: For all $a, b \in G$, $a * b = b * a$

Define the set and operation

$$\text{Set } G = \{\dots, -5, -3, -1, 1, 3, \dots\}$$

$$\text{operation} = + \text{ (usual addition)}$$

① closure under +:

Sum of two odd numbers:

$$(2m+1) + (2n+1) = 2(m+n+1)$$

this is even, closure fails

② Associativity:

Addition of integers is associative:

$$(a+b) + c = a + (b+c)$$

③ Identity element:

The identity element for addition is 0

$$a + 0 = a$$

0 is even, not odd

No identity element in the set of odd numbers

④ Inverse Element:

Inverse of a under addition is $-a$

$-a$ is odd if a is odd

5. Commutativity:

Addition is commutative: $a + b = b + a$

So, (Odd numbers, +) is NOT
an abelian group.

⑥ Identity element:

The identity element for addition is 0.

Inverse

are even not odd

No identity element in the set of odd numbers.

Identity element