Answer No 1

(i) A rational number $\frac{p}{q}$ is an equivalence class of pairs (p, s) with $q \neq 0$, where

$$\frac{p}{q} = \frac{p'}{q'} \Leftrightarrow pq' = p'q$$

Addition: $\frac{p}{q} + \frac{r}{s} = \frac{ps+rq}{qs}$

Multiplication: $\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}$

(ii) For $\frac{p}{q}, \frac{r}{s} \in \mathbb{Q}$:

$$\frac{p}{q} + \frac{r}{s} = \frac{ps+rq}{qs} \in \mathbb{Q}, \quad \frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs} \in \mathbb{Q}$$

Integers are closed under addition and multiplication, and $qs \neq 0$.

(iii) Associativity:

Addition: $\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u} = \frac{(ps+rq)u+tqs}{qsu}$

$$= \frac{psu+rqu+tqs}{qsu}$$

$$= \frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right)$$

Multiplication: $\left(\frac{p}{q}\cdot\frac{r}{s}\right)\cdot\frac{t}{s} = \frac{pr}{qs}\cdot\frac{t}{u} = \frac{prt}{qsu}$

$$= \frac{p}{q}\cdot\left(\frac{r}{s}\cdot\frac{t}{u}\right)$$

(iv) commutativity:

Addition: $\frac{p}{q}+\frac{r}{s} = \frac{ps+rq}{qs} = \frac{rq+ps}{sq}$

$$= \frac{r}{s}+\frac{p}{q}$$

Multiplication: $\frac{p}{q}\cdot\frac{r}{s} = \frac{pr}{qs} = \frac{rp}{sq} = \frac{r}{s}\cdot\frac{p}{q}$

(v) **Identities:**

Additive identity: $0 = \frac{0}{1}$

$$\frac{p}{q} + 0 = \frac{p}{q} + \frac{0}{1} = \frac{p.1 + 0.q}{q.1} = \frac{p}{q}$$

Multiplicative identity: $1 = \frac{1}{1}$

$$\frac{p}{q}.1 = \frac{p}{q}.\frac{1}{1} = \frac{p}{q}$$

(vi) **Inverse:**

Additive inverse: For $\frac{p}{q} \in \mathbb{Q}$,

$$-\frac{p}{q} = \frac{-p}{q}, \quad \frac{p}{q} + \frac{-p}{q} = 0$$

Multiplicative Inverse: For $\frac{p}{q} \notin \mathbb{Q}$, $p \neq 0$

$$\left(\frac{p}{q}\right)^{-1} = \frac{p}{q}, \quad \frac{p}{q}.\frac{q}{p} = 1$$