

### Answer No 1

A set  $G$  with a binary operation  $*$  is an abelian group if all these hold:

1. closure: For all  $a, b \in G$ ,  $a * b \in G$
2. Associativity: For all  $a, b, c \in G$ ,  $(a * b) * c = a * (b * c)$
3. Identity Element: There exist  $e \in G$  such that  $a * e = e * a$  for all  $a \in G$
4. Inverse Element: For all  $a \in G$ , there exist  $b \in G$  such that  $a * b = b * a = e$ .
5. Commutativity: For all  $a, b \in G$ ,  $a * b = b * a$

Define the set and operation

$$\text{Set } O = \{\dots, -5, -3, -1, 1, 3, \dots\}$$

operation = + (usual addition)

① closure under +:

sum of two odd numbers:

$$(2m+1) + (2n+1) = 2(m+n+1)$$

this is even, closure fails

② Associativity:

Addition of integers is associative:

$$(a+b)+c = a+(b+c)$$

③ Identity element:

The identity element for addition is 0

$$a+0=0$$

0 is even, not odd

No identity element in the set of odd numbers

④ Inverse Element:

Inverse of a under addition is  $-a$

$-a$  is odd if  $a$  is odd

## 5. Commutativity:

Addition is commutative :  $a+b = b+a$

So, (Odd numbers, +) is NOT  
an abelian group.

○ Identity element:

The identity element for addition is zero.

Inverse element

Every odd number has an inverse element.

The identity element in the set of odd

numbers is zero.

Zero is the identity element.