Conformal Modeling of Space-Charge-Limited Emission from Curved Boundaries in Particle Simulations

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Outline

Motivation

- electromagnetic Particle-In-Cell simulations are used to investigate particle physics in accelerators (e.g. FD, FDTD, FIT on orthogonal grids)
- modeling curved material boundaries on such grids presents difficulties
- low accuracy of boundary fields at curved emission surfaces greatly affects overall performance
- boundary-conformal approach retains advantages and provides accurate solutions for curved emission boundaries

Mathematical Model

- the model equations read

$$\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = \mathbf{v}_i$$
$$\frac{\mathrm{d}m_e\mathbf{v}_i}{\mathrm{d}t} = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \quad i = 1 \ldots N$$

  where e is the electron charge, m_e is the relativistic electron mass; $\mathbf{r}_i$ and $\mathbf{v}_i$ denote particle positions and velocities respectively

Mathematical Model

- only taking into account electrostatic particle-particle interactions the space-charge field **E** is determined by

$$\nabla(\varepsilon \mathbf{E}) = \frac{1}{\varepsilon_0} \sum_i^N q_i \delta(\mathbf{r} - \mathbf{r}_i)$$
$$\mathbf{E} = -\nabla \varphi,$$

where ε is the permittivity and q_i the charge of the i-th particle

Space Charge Limited Emission

- the Child-Langmuir diode equation reads

$$J_{\mathrm{CL}} = \left(\frac{4\varepsilon_0}{9}\right)\sqrt{\frac{2e}{m_e}}\frac{\delta\varphi_b^{3/2}}{\delta d^2},$$

where J_{CL} is the current at the emission surface and $\delta\varphi_b$ the local potential difference at a distance δd from the surface

Discrete Field Equations

- using the Finite Integration Technique the discrete formulation of the equations reads

$$\mathbf{S}\widehat{\widehat{\mathbf{d}}} = \mathbf{q}$$

$$\widehat{\widehat{\mathbf{d}}}_i = \iint_{\Delta A_i} \varepsilon(\nabla\varphi)\mathrm{d}A$$

- $\widehat{\widehat{\mathbf{d}}}$ denotes the vector of electrostatic fluxes, $\mathbf{S}$ the discrete div-operator, $\mathbf{q}$ the total charge contained in each dual cell and ΔA_i the area element of the i-th dual cell face
- discretization error stems only from approximating the flux integrals $\widehat{\widehat{\mathbf{d}}}_i$ using potential grid values ϕ_i
- conformal approach takes into account the geometry of the boundary and minimizes error

Particle Injection

- emitted particles are assigned to predetermined positions
- local currents associated to the individual particles are calculated using Child's law
- consistency between the currents and the field solution is enforced through iteration until constant emission current is established

Results

- steady state emission current is observed on the source surface for different mesh resolutions
- using a staircase approximation 1.5 mio. mesh nodes are needed for convergence
- staircase model also introduces numerical oscillations caused by low order field approximation at the emission surface
- the relative error for fine to moderate mesh sizes varies between 10-25%
- using the conformal method only 130k nodes are needed for convergence
- computational effort can be reduced since grid-field-equations contain fewer nodes

Conclusion

- boundary conformal discretization of electrostatic field equations and boundary conformal particle injection technique
- method enforces field-space-charge consistency from Child's law
- solution of a Pierce gun model shows advantages over staircase approximation

List of References

[1] E. Gjonaj, T. Lau and T. Weiland. Conformal Modeling of Space-Charge-Limited Emission from Curved Boundaries in Particle Simulations. In Proceedings of the 2003 Particle Accelerator Conference May 12-16, 2003 in Portland, OR, USA.

[2] B. Krietenstein, R. Schuhmann, P. Thoma and T. Weiland. The Perfect Boundary Approximation Technique Facing the Big Challenge of High Precision Field Computation. In Proceedings of the XIX International Linear Accelerator Conference, 1998.