Mathematical Formulae

Root Type Determinant Formulae
$(b^2 - 4ac)$

Quadratic Equation Formulae
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Cubic Equation Formulae

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^2}} + \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^2}} - \frac{b}{3a}$$

Binomial Theorem
$(a + b)^n = C_{(n,0)} a^n b^0 + C_{(n,1)} a^{n-1} b^1 + C_{(n,2)} a^{n-2} b^2 + \cdots + C_{(n,n)} a^{n-n} b^n$

Polar Form Conversion	
$x + iy = r\cos\theta + ir\sin\vartheta$	$x + iy = r(\cos\theta + i\sin\vartheta)$

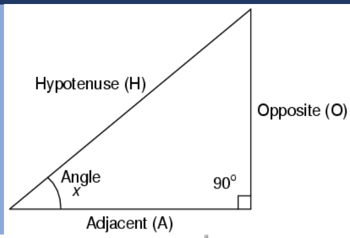
Sequence and Series	
Arithmetic Progression	**Geometric Progression**
$\{x_n\} = x_1 + (n-1)d$	$\{x_n\} = x_1 r^{n-1}$
$\sum_k^n or \{S_n\} = \frac{n}{2}(2x_1 + (n-1)d)$	$\sum_k^n or \{S_n\} = \frac{x_1(1 - r^n)}{1 - r}$

Combinatorials	
Permutations:	$P_{(n,r)} = \frac{n!}{(n-r)!}$
Combinations:	$C_{(n,r)} = \frac{n!}{(n-r)!\,r!}$

Vectors	
Scalar Product:	**Vector Product:**
$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert\boldsymbol{a}\rvert \cdot \lvert\boldsymbol{b}\rvert \cos\theta$	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \sin\theta\, \hat{\boldsymbol{n}}$
$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$	$\boldsymbol{a} \cdot \boldsymbol{b} = (a_2 b_3 - a_3 b_2)i - (a_3 b_1 - a_1 b_3)j + (a_1 b_2 - a_2 b_1)k$

Mathematical Formulae

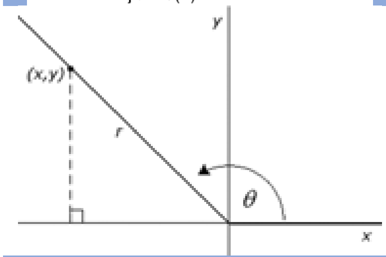
Trigonometric Functions and Identities



Right triangle definitions, where $0 < \theta < \frac{\pi}{2}$.

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

$$\csc\theta = \frac{hypotenuse}{opposite} \qquad \sec\theta = \frac{hypotenuse}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$



Circular function definitions, where θ is any angle.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Reduction Formulae:

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Sum or Difference of two angles:

$$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$$
$$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$$
$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta\tan\varphi}$$

Half-Angle Formulae:

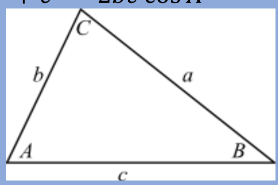
$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

Double-Angle Formulae:

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$= \cos^2\theta - \sin^2\theta$$

Laws of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$



Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reduction Formulae:

Function:	Quadrant II	Quadrant III	Quadrant IV
$\sin\theta$	$\sin(180° - \theta)$	$-\sin(\theta - 180°)$	$-\sin(360° - \theta)$
$\cos\theta$	$-\cos(180° - \theta)$	$-\cos(\theta - 180°)$	$\cos(360° - \theta)$
$\tan\theta$	$-\tan(180° - \theta)$	$\tan(\theta - 180°)$	$-\tan(360° - \theta)$
$\cot\theta$	$-\cot(180° - \theta)$	$\cot(\theta - 180°)$	$-\cot(360° - \theta)$
$\sec\theta$	$-\sec(180° - \theta)$	$\sec(\theta - 180°)$	$-\sec(360° - \theta)$
$\csc\theta$	$-\csc(180° - \theta)$	$\csc(\theta - 180°)$	$-\csc(360° - \theta)$

Hyperbolic Functions and Identities

Hyperbolic Function:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$

Hyperbolic Identities:

$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\sinh(x + y) = \sinh x \,.\, \cosh y + \sinh y \,.\, \cosh x$$
$$\cosh(x + y) = \sinh x \,.\, \sinh y + \cosh x \,.\, \cosh y$$
$$\cosh^2\frac{x}{2} = \frac{1 + \cosh x}{2}$$
$$\sinh^2\frac{x}{2} = \frac{\cosh x - 1}{2}$$

Mathematical Formulae

Interest & Depreciation	
Simple Interest	$F_v = P_v(1+rt)$
Compound Interest	$F_v = P_v(1+r)^t$
Compound Interest Rate	$r = (1+\frac{i}{m})^m - 1$
Straight Line Depreciation	$F_v = P_v(1-rt)$
Reducing Balance Depreciation	$F_v = P_v(1-r)^t$

Annuities	
Ordinary Certain	**Overdue**
$F_v = R(\frac{(1+i)^n - 1}{i})$	$F_v = R(\frac{((1+i)^n - 1)(1+i)}{i})$
$P_v = R(\frac{1-(1+i)^{-n}}{i})$	$P_v = R(\frac{(1-(1+i)^{-n})(1+i)}{i})$

Standard Derivatives		
$\frac{d}{dx}cu = c\frac{du}{dx}$	$\frac{d}{dx}u \pm v = \frac{du}{dx} \pm \frac{dv}{dx}$	$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}c = 0$	$\frac{d}{dx}x = 1$
$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$	$(If\ u \neq 0)\frac{d}{dx}\lvert u\rvert = \frac{u}{\lvert u\rvert}\frac{du}{dx}$	$\frac{d}{dx}a^x = (ln\, a)\, a^x$
$\frac{d}{dx}a^u = (ln\, a)\, a^u\frac{du}{dx}$	$(If\ a = e)\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}e^u = e^u\frac{du}{dx}$
$\frac{d}{dx}log_a x = \frac{1}{(ln\, a)x}$	$\frac{d}{dx}log_a u = \frac{1}{(ln\, a)u}\frac{du}{dx}$	$(If\ a = e)\frac{d}{dx}ln\, x = \frac{1}{x}$
$\frac{d}{dx}ln\, u = \frac{1}{u}\frac{du}{dx}$		

$\frac{d}{dx}csc\, u = -csc\, u\, cot\, u\frac{du}{dx}$	$\frac{d}{dx}sin^{-1}u = \frac{d}{dx}arcsin\, u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
$\frac{d}{dx}cos^{-1}u = \frac{d}{dx}arccos\, u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$	$\frac{d}{dx}tan^{-1}u = \frac{d}{dx}arctan\, u = \frac{1}{1+u^2}\frac{du}{dx}$
$\frac{d}{dx}cot^{-1}u = \frac{d}{dx}arccot\, u = \frac{-1}{1+u^2}\frac{du}{dx}$	$\frac{d}{dx}sec^{-1}u = \frac{d}{dx}arcsec\, u = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$
$\frac{d}{dx}csc^{-1}u = \frac{d}{dx}arccsc\, u = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$	

Standard Trigonometry Derivatives		
$\frac{d}{dx}sin\, u = cos\, u\frac{du}{dx}$	$\frac{d}{dx}cos\, u = -sin\, u\frac{du}{dx}$	$\frac{d}{dx}tan\, u = sec^2\, u\frac{du}{dx}$
$\frac{d}{dx}cot\, u = -csc^2\, u\frac{du}{dx}$	$\frac{d}{dx}sec\, u = sec\, u\, tan\, u\frac{du}{dx}$	$\frac{d}{dx}csc\, u = -csc\, u\, cot\, u\frac{du}{dx}$

Standard Inverse Trigonometry Derivatives	
$\frac{d}{dx}sin^{-1}u = \frac{d}{dx}arcsin\, u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$	$\frac{d}{dx}cos^{-1}u = \frac{d}{dx}arccos\, u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$
$\frac{d}{dx}tan^{-1}u = \frac{d}{dx}arctan\, u = \frac{1}{1+u^2}\frac{du}{dx}$	$\frac{d}{dx}cot^{-1}u = \frac{d}{dx}arccot\, u = \frac{-1}{1+u^2}\frac{du}{dx}$
$\frac{d}{dx}sec^{-1}u = \frac{d}{dx}arcsec\, u = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$	$\frac{d}{dx}csc^{-1}u = \frac{d}{dx}arccsc\, u = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}$

Mathematical Formulae

Standard Hyperbolic Derivatives		
$\frac{d}{dx}\sinh u = \cosh u\,\frac{du}{dx}$	$\frac{d}{dx}\cosh u = -\sinh u\,\frac{du}{dx}$	$\frac{d}{dx}\tanh u = sech^2\, u\,\frac{du}{dx}$
$\frac{d}{dx}\coth u = -csch^2\, u\,\frac{du}{dx}$	$\frac{d}{dx}sech\, u = sech\, u\tanh u\,\frac{du}{dx}$	$\frac{d}{dx}csch\, u = -csch\, u\coth u\,\frac{du}{dx}$

Standard Integrals	
$\int kf(u)\,du = k\int f(u)\,du$	$\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du$
$\int du = u + c$	$(If\ n \neq -1)\int u^n\,dx = \frac{u^{n+1}}{n+1} + c$
$\int \ln u\,du = u\ln(u) - u + c$	$\int e^{kx}\,dx = \frac{e^{kx}}{k} + c$
$\int e^u\,du = e^u + c$	$\int x^{-1}\,dx = \int \frac{1}{x}dx = \ln\lvert x\rvert + c$

Standard Trigonometry Integrals	
$\int \sin u\,du = -\cos u + c$	$\int \cos u\,du = \sin u + c$
$\int \tan u\,du = -\ln\lvert\cos u\rvert + c$	$\int \cot u\,du = \ln\lvert\sin u\rvert + c$
$\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + c$	$\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + c$
$\int \sec^2 u\,du = \tan u + c$	$\int \csc^2 u\,du = -\cot u + c$
$\int \sec u\tan u\,du = \sec u + c$	$\int \csc u\cot u\,du = -\csc u + c$
$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ac+b\rvert + c$	$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}(\frac{u}{a}) + c$
$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + c$	$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + c$

Standard Hyperbolic Integrals	
$\int \sinh u\,du = \cosh u + c$	$\int \cosh u\,du = \sinh u + c$
$\int \tanh u\,du = \ln\lvert\cosh u\rvert + c$	$\int \coth u\,du = \ln\lvert\sinh u\rvert + c$
$\int sech\, u\,du = \tan^{-1}\lvert\sinh u\rvert + c$	$\int csch\, u\,du = \ln\left\lvert\tanh\frac{u}{2}\right\rvert + c$

Mathematical Formulae

Differentiation and Integration Application

$$Rate\ of\ change = \frac{Change\ in\ Position}{Change\ in\ Time}$$

$$Velocity\ Function = \frac{d}{dx}(Position\ Function)$$

$$Acceleration\ Function = \frac{d^2}{dx^2}(Position\ Function)$$

$$Mariginal\ revenue = \frac{d}{dx}(Revenue\ Function)$$

$$Mariginal\ cost = \frac{d}{dx}(Cost\ Function)$$

$$Mariginal\ profit = \frac{d}{dx}(Profit\ Function)$$

$$Volume\ of\ Solids\ of\ Revolution = \int_a^b \pi y^2\,dx$$

$$Arc\ Lenght\ of\ Surface\ of\ Revolution = \int_a^b \sqrt{1+\left(\frac{d}{dx}\right)^2}\,dx$$

$$Area\ of\ Surface\ of\ Revolution = 2\pi\int_a^b (Radius\ Function)\sqrt{1+\left(\frac{d}{dx}\right)^2}\,dx$$

$$Work\ done\ by\ constant\ Force = (Force)(Distance)$$

$$Work\ Done\ by\ Variable\ Force = \int_a^b F(x)\,dx$$

$$Work\ done\ by\ Expanding\ Gas = \int_{V_0}^{V_1} \frac{k}{V}\,dV$$

$$Law\ of\ Universal\ Gravitation: F = k\frac{m_1 m_2}{d^2}$$

$$Hooke's\ Law: F = kd$$

$$Coulomb's\ Law: F = k\frac{q_1 q_2}{d^2}$$

$$Moment\ about\ the\ origin\ (One\ Dimension): M_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n$$

$$Centre\ of\ mass\ (One\ Dimension): \bar{x} = \frac{M_0}{m}$$

$$Moment\ about\ the\ y-axis\ and\ \ x-axis\ (Two\ Dimension):$$

$$M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$$

$$M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$$

$$Centre\ of\ mass\ (Two\ Dimension): \bar{x} = \frac{M_y}{m} \text{ and } \bar{y} = \frac{M_x}{m}$$

$$Moment\ about\ the\ y-axis\ and\ \ x-axis\ (Planar\ Lamina):$$

$$M_x = p\int_a^b \left(\frac{f(x)+g(x)}{2}\right)\big(f(x)-g(x)\big)\,dx$$

$$M_y = p\int_a^b x\big(f(x)-g(x)\big)\,dx$$

$$Fluid\ Pressure = wh$$

$$Fluid\ Force = PA$$

$$Force\ exerted\ by\ fluid = w\int_c^d h(y)L(y)\,dy$$

Mathematical Formulae

Geometry Formulae

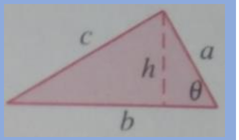
Triangle

$h = a \sin\theta$

$Area = \frac{1}{2}bh$

(Law of Cosines)

$c^2 = a^2 + b^2 - 2ab\cos\theta$



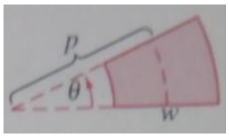
Sector of Circular Ring

$(p = average\ radius$

$w = width\ of\ ring,$

$\theta\ in\ radians)$

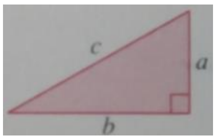
$Area = \theta pw$



Right Triangle

(Pythagorean Theorem)

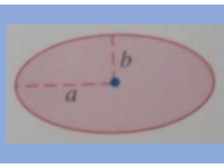
$c^2 = a^2 + b^2$



Ellipse

$Area = \pi ab$

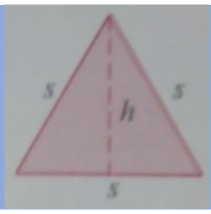
$Circumference \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$



Equilateral Triangle

$h = \frac{\sqrt{3}x}{2}$

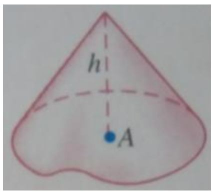
$Area = \frac{\sqrt{3}x^2}{4}$



Cone

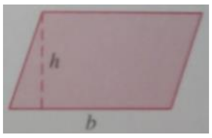
$(A = area\ of\ base$

$Volume = \frac{Ah}{3}$



Parallelogram

$Area = bh$

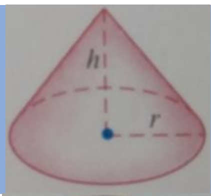


Right Circular Cone

$Volume = \frac{\pi r^2 h}{3}$

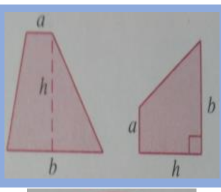
$Lateral\ Survace\ Area$

$= \pi r\sqrt{r^2 + h^2}$



Trapezoid

$Area = \frac{h}{2}(a + b)$

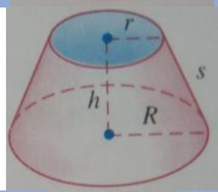


Frustum of Right Circular Cone

$Volume = \frac{\pi(r^2 + rR + R^2)h}{3}$

$Lateral\ Survace\ Area$

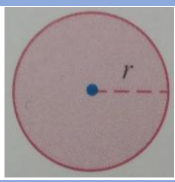
$= \pi s(R + r)$



Circle

$Area = \pi r^2$

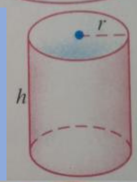
$Circumference = 2\pi r$



Right Circular Cylinder

$Volume = \pi r^2 h$

$Lateral\ Surface\ Area = 2\pi rh$

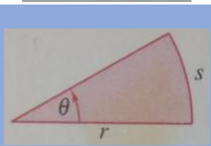


Sector of Circle

$(\theta\ in\ radians)$

$Area = \frac{\theta r^2}{2}$

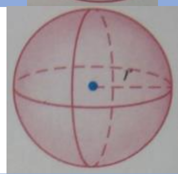
$s = r\theta$



Sphere

$Volume = \frac{4}{3}\pi r^3$

$Surface\ Area = 4\pi r^2$



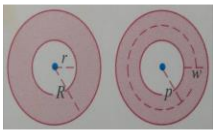
Circular Ring

$(p = average\ radius$

$w = width\ of\ ring)$

$Area = \pi(R^2 - r^2)$

$= 2\pi pw$



Wedge

$(A = area\ of\ upper\ face,$

$B = area\ of\ base)$

$A = B\sec\theta$