Whole Numbers

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses many of aspects of whole numbers, including the Hindu-Arabic numeration system, the base ten positional number system, and the graphing of whole numbers. By the end of this module students should be able to: know the difference between numbers and numerals, know why our number system is called the HinduArabic numeration system, understand the base ten positional number system, and identify and graph whole numbers.

Section Overview

- Numbers and Numerals
- The Hindu-Arabic Numeration System
- The Base Ten Positional Number System
- Whole Numbers

Graphing Whole Numbers

Numbers and Numerals

We begin our study of introductory mathematics by examining its most basic building block, the **number**.

Number

A **number** is a concept. It exists only in the mind.

The earliest concept of a number was a thought that allowed people to mentally picture the size of some collection of objects. To write down the number being conceptualized, a **numeral** is used.

Numeral

A **numeral** is a symbol that represents a number.

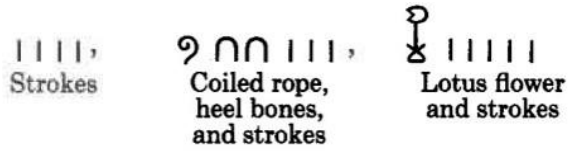
In common usage today we do not distinguish between a number and a numeral. In our study of introductory mathematics, we will follow this common usage.

Sample Set A

The following are numerals. In each case, the first represents the number four, the second repre sents the number one hundred twenty-three, and the third, the number one thousand five. These numbers are represented in different ways.

- Hindu-Arabic numerals 4, 123, 1005

- Roman numerals
  IV, CXXIII, MV
- Egyptian numerals



Practice Set A

Exercise:

Problem:

Do the phrases "four," "one hundred twenty-three," and "one thousand five" qualify as numerals? Yes or no?

Solution:

Yes. Letters are symbols. Taken as a collection (a written word), they represent a number.

The Hindu-Arabic Numeration System

Hindu-Arabic Numeration System

Our society uses the **Hindu-Arabic numeration system**. This system of numer ation began shortly before the third century when the Hindus invented the nu merals 0 1 2 3 4 5 6 7 8 9

Leonardo Fibonacci

About a thousand years later, in the thirteenth century, a mathematician named Leonardo Fibonacci of Pisa introduced the system into Europe. It was then popu larized by the Arabs. Thus, the name, Hindu-Arabic numeration system.

The Base Ten Positional Number System

Digits

The Hindu-Arabic numerals 0 1 2 3 4 5 6 7 8 9 are called **digits**. We can form any number in the number system by selecting one or more digits and placing them in certain positions. Each position has a particular value. The

Hindu mathematician who devised the system about A.D. 500 stated that "from place to place each is ten times the preceding."

Base Ten Positional Systems

It is for this reason that our number system is called a **positional** number system with **base ten**.

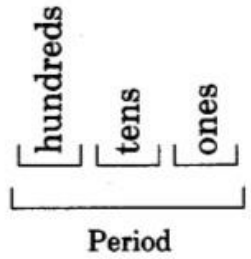
Commas

When numbers are composed of more than three digits, **commas** are sometimes used to separate the digits into groups of three.

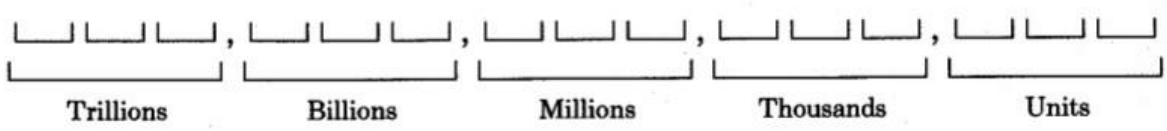
Periods

These groups of three are called **periods** and they greatly simplify reading numbers.

In the Hindu-Arabic numeration system, a period has a value assigned to each or its three positions, and the values are the same for each period. The position values are



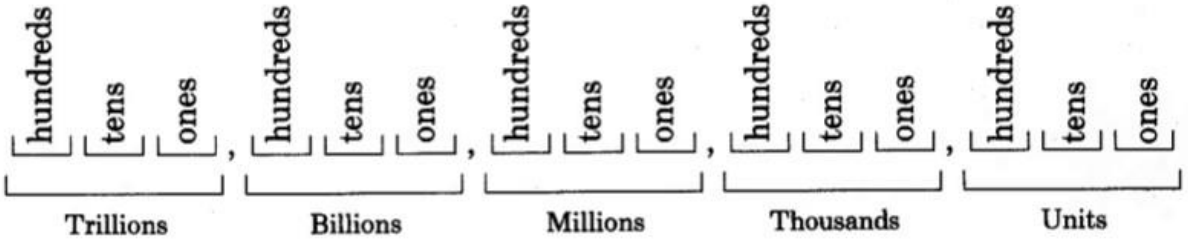
Thus, each period contains a position for the values of one, ten, and hundred. Notice that, in looking from right to left, the value of each position is ten times the preceding. Each period has a particular name.



As we continue from right to left, there are more periods. The five periods listed above are the most common, and in our study of introductory mathematics, they are sufficient.

The following diagram illustrates our positional number system to trillions.

(There are, to be sure, other periods.)



In our positional number system, the **value of a digit** is determined by its *position* in the number.

Sample Set B

Example:

Find the value of 6 in the number 7,261.

Since 6 is in the tens position of the units period, its value is 6 tens. 6 tens = 60

Example:

Find the value of 9 in the number 86,932,106,005.

Since 9 is in the hundreds position of the millions period, its value is 9 hundred millions.

9 hundred millions = 9 hundred million

Example:

Find the value of 2 in the number 102,001.

Since 2 is in the ones position of the thousands period, its value is 2 one thousands.

2 one thousands = 2 thousand

Practice Set B

Exercise:

Problem: Find the value of 5 in the number 65,000.

Solution:

five thousand

Exercise:

Problem: Find the value of 4 in the number 439,997,007,010.

Solution:

four hundred billion **Exercise:**

Problem: Find the value of 0 in the number 108.

Solution:

zero tens, or zero

Whole Numbers

Whole Numbers

Numbers that are formed using only the digits

0 1 2 3 4 5 6 7 8 9

are called **whole numbers**. They are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ...

The three dots at the end mean "and so on in this same pattern."

Graphing Whole Numbers

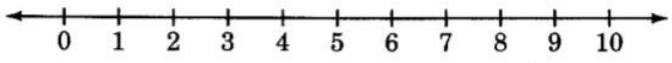
Number Line

Whole numbers may be visualized by constructing a **number line**. To construct a number line, we simply draw a straight line and choose any point on the line and label it 0.

Origin

This point is called the **origin**. We then choose some convenient length, and moving to the right, mark off consecutive intervals (parts) along the line starting at 0. We label each new interval endpoint with the next whole

number.

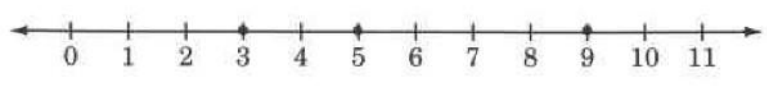


Graphing

We can visually display a whole number by drawing a closed circle at the point labeled with that whole number. Another phrase for visually displaying a whole number is graphing the whole number. The word graph means to "visually display."

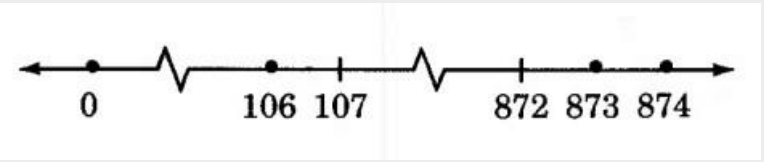
Sample Set C

Example:
Graph the following whole numbers: 3, 5, 9.



Example:

Specify the whole numbers that are graphed on the following number line. The break in the number line indicates that we are aware of the whole numbers between 0 and 106, and 107 and 872, but we are not listing them due to space limitations.



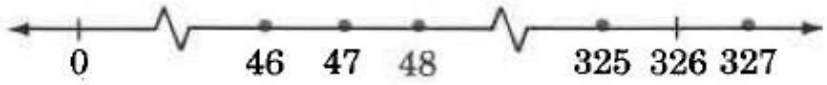
The numbers that have been graphed are
0, 106, 873,874

Practice Set C

Exercise:

Problem: Graph the following whole numbers: 46, 47, 48, 325, 327.

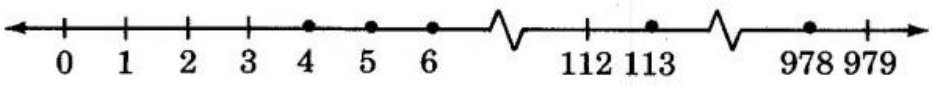
Solution:



Exercise:

Problem:

Specify the whole numbers that are graphed on the following number line.



Solution:

4, 5, 6, 113, 978

A **line** is composed of an endless number of points. Notice that we have labeled only some of them. As we proceed, we will discover new types of numbers and determine their location on the number line.

Exercises Exercise:

Problem: What is a number?

Solution:

concept **Exercise:**

Problem: What is a numeral?

Exercise:

Problem: Does the word "eleven" qualify as a numeral?

Solution:

Yes, since it is a symbol that represents a number.

Exercise:

Problem: How many different digits are there?

Exercise:

Problem:

Our number system, the Hindu-Arabic number system, is a number system with base .

Solution:

positional; 10 **Exercise:**

Problem:

Numbers composed of more than three digits are sometimes separated into groups of three by commas. These groups of three are called .

Exercise:

Problem:

In our number system, each period has three values assigned to it. These values are the same for each period. From right to left, what are they?

Solution:

ones, tens, hundreds **Exercise:**

Problem:

Each period has its own particular name. From right to left, what are the names of the first four?

Exercise:

Problem: In the number 841, how many tens are there?

Solution:

4

Exercise:

Problem: In the number 3,392, how many ones are there?

Exercise:

Problem: In the number 10,046, how many thousands are there?

Solution:

0

Exercise:

Problem:

In the number 779,844,205, how many ten mil lions are there?

Exercise:

Problem:

In the number 65,021, how many hundred thousands are there?

Solution:

0

For following problems, give the value of the indicated digit in the given number.
Exercise:

Problem: 5 in 599 **Exercise:**

Problem: 1 in 310,406

Solution:

ten thousand **Exercise:**

Problem: 9 in 29,827 **Exercise:**

Problem: 6 in 52,561,001,100

Solution:

6 ten millions = 60 million

Exercise:

Problem:

Write a two-digit number that has an eight in the tens position.

Exercise:

Problem:

Write a four-digit number that has a one in the thousands position and a zero in the ones position.

Solution:

1,340 (answers may vary) **Exercise:**

Problem: How many two-digit whole numbers are there?

Exercise:

Problem: How many three-digit whole numbers are there?

Solution:

900

Exercise:

Problem: How many four-digit whole numbers are there?

Exercise:

Problem: Is there a smallest whole number? If so, what is it?

Solution: yes; zero **Exercise:**

Problem: Is there a largest whole number? If so, what is it?

Exercise:

Problem: Another term for "visually displaying" is .

Solution:

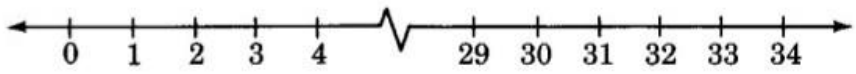
graphing **Exercise:**

Problem: The whole numbers can be visually displayed on a .

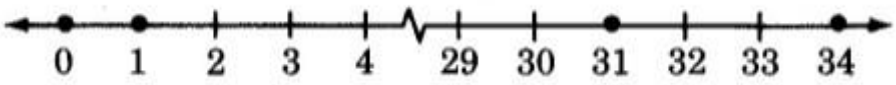
Exercise:

Problem:

Graph (visually display) the following whole numbers on the number line below: 0, 1, 31, 34.



Solution:



Exercise:

Problem:

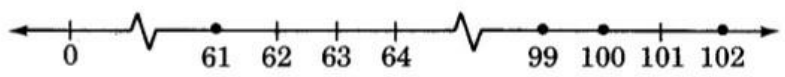
Construct a number line in the space provided below and graph (visually display) the following whole numbers: 84, 85, 901, 1006,

1007.

Exercise:

Problem:

Specify, if any, the whole numbers that are graphed on the following number line.

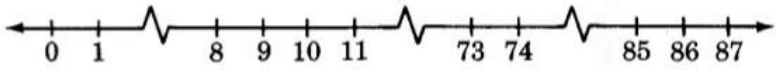


Solution:

61, 99, 100, 102 **Exercise:**

Problem:

Specify, if any, the whole numbers that are graphed on the following number line.



Reading and Writing Whole Numbers

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to read and write whole numbers. By the end of this module, students should be able to read and write whole numbers.

Section Overview

- Reading Whole Numbers
- Writing Whole Numbers

Because our number system is a positional number system, reading and writing whole numbers is quite simple.

Reading Whole Numbers

To convert a number that is formed by digits into a verbal phrase, use the following method:

1. Beginning at the right and working right to left, separate the number into distinct periods by inserting commas every three digits.

2. Beginning at the left, read each period individually, saying the period name.

Sample Set A

Write the following numbers as words.

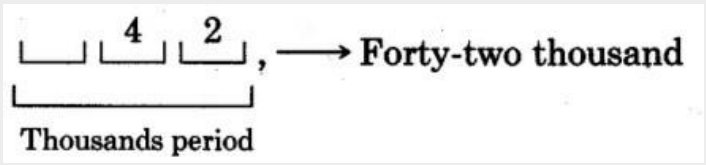
Example:

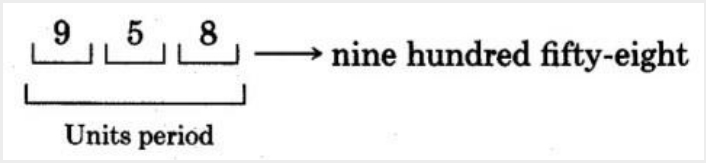
Read 42958.

1. Beginning at the right, we can separate this number into distinct periods by inserting a comma between the 2 and 9.

42,958

2. Beginning at the left, we read each period individually:

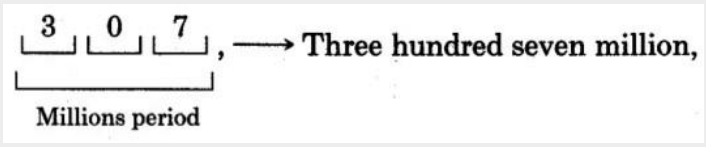


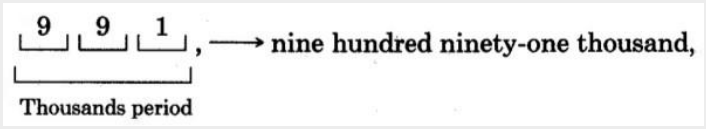


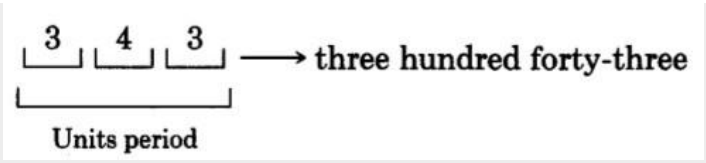
Forty-two thousand, nine hundred fifty-eight.

Example:
Read 307991343.

1. Beginning at the right, we can separate this number into distinct periods by placing commas between the 1 and 3 and the 7 and 9.
   307,991,343
2. Beginning at the left, we read each period individually.



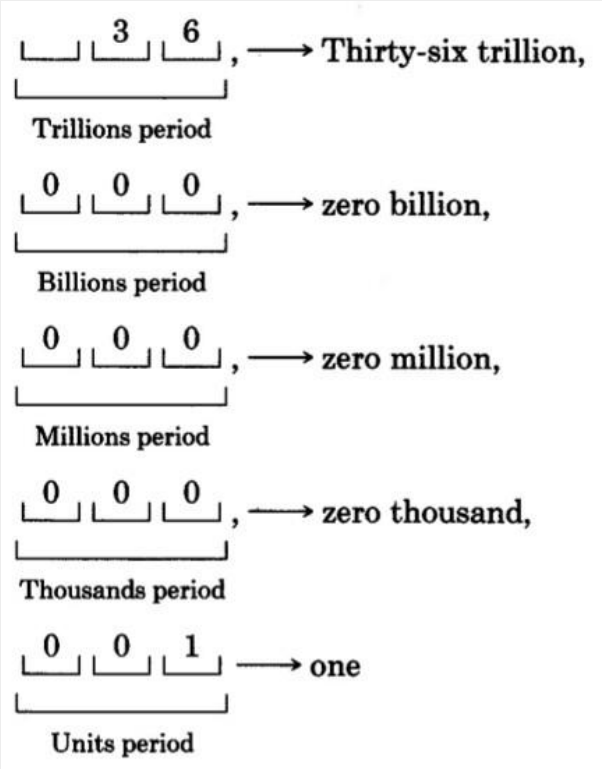




Three hundred seven million, nine hundred ninety-one thousand, three hundred forty-three.

Example:
Read 36000000000001.

1. Beginning at the right, we can separate this number into distinct periods by placing commas. 36,000,000,001
2. Beginning at the left, we read each period individually.



Thirty-six trillion, one.

Practice Set A

Write each number in words.

Exercise:

Problem: 12,542

Solution:

Twelve thousand, five hundred forty-two **Exercise:**

Problem: 101,074,003

Solution:

One hundred one million, seventy-four thousand, three **Exercise:**

Problem: 1,000,008

Solution:

One million, eight

Writing Whole Numbers

To express a number in digits that is expressed in words, use the following method:

1. Notice first that a number expressed as a verbal phrase will have its periods set off by commas.

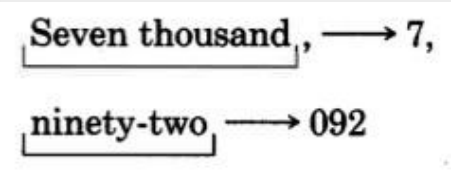
2. Starting at the beginning of the phrase, write each period of numbers individu ally.

3. Using commas to separate periods, combine the periods to form one number.

Sample Set B

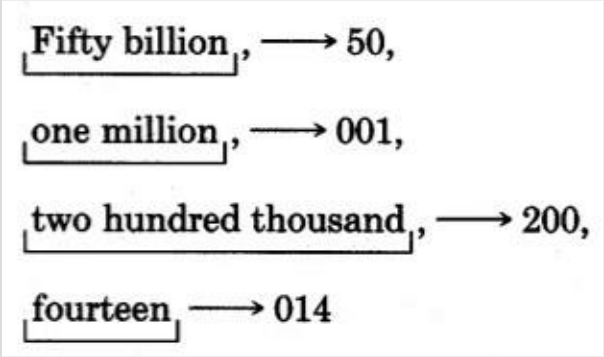
Write each number using digits.

Example:
Seven thousand, ninety-two.
Using the comma as a period separator, we have



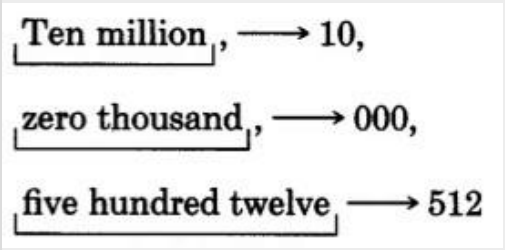
7,092

Example:
Fifty billion, one million, two hundred thousand, fourteen.
Using the commas as period separators, we have



50,001,200,014

Example:
Ten million, five hundred twelve.
The comma sets off the periods. We notice that there is no thousands period. We'll have to insert this ourselves.



10,000,512

Practice Set B

Express each number using digits.

Exercise:

Problem: One hundred three thousand, twenty-five.

Solution:

103,025 **Exercise:**

Problem: Six million, forty thousand, seven.

Solution:

6,040,007

Exercise:

Problem:

Twenty trillion, three billion, eighty million, one hundred nine thousand, four hundred two.

Solution:

20,003,080,109,402 **Exercise:**

Problem: Eighty billion, thirty-five.

Solution:

80,000,000,035

Exercises

For the following problems, write all numbers in words.

Exercise:

Problem: 912

Solution:

nine hundred twelve **Exercise:**

Problem: 84 **Exercise:**

Problem: 1491

Solution:

one thousand, four hundred ninety-one **Exercise:**

Problem: 8601 **Exercise:**

Problem: 35,223

Solution:

thirty-five thousand, two hundred twenty-three **Exercise:**

Problem: 71,006 **Exercise:**

Problem: 437,105

Solution:

four hundred thirty-seven thousand, one hundred five **Exercise:**

Problem: 201,040 **Exercise:**

Problem: 8,001,001

Solution:

eight million, one thousand, one

Exercise:

Problem: 16,000,053 **Exercise:**

Problem: 770,311,101

Solution:

seven hundred seventy million, three hundred eleven thousand, one hundred one **Exercise:**

Problem: 83,000,000,007 **Exercise:**

Problem: 106,100,001,010

Solution:

one hundred six billion, one hundred million, one thousand ten **Exercise:**

Problem: 3,333,444,777 **Exercise:**

Problem: 800,000,800,000

Solution:

eight hundred billion, eight hundred thousand **Exercise:**

Problem:

A particular community college has 12,471 students enrolled.

Exercise:

Problem:

A person who watches 4 hours of television a day spends 1460 hours a year watching T.V.

Solution:

four; one thousand, four hundred sixty **Exercise:**

Problem:

Astronomers believe that the age of the earth is about 4,500,000,000 years.

Exercise:

Problem:

Astronomers believe that the age of the universe is about 20,000,000,000 years.

Solution:

twenty billion **Exercise:**

Problem:

There are 9690 ways to choose four objects from a collection of 20.

Exercise:

Problem:

If a 412 page book has about 52 sentences per page, it will contain about 21,424 sentences.

Solution:

four hundred twelve; fifty-two; twenty-one thousand, four hundred twenty-four
Exercise:

Problem:

In 1980, in the United States, there was $1,761,000,000,000 invested in life insurance.

Exercise:

Problem:

In 1979, there were 85,000 telephones in Alaska and 2,905,000 telephones in Indiana.

Solution:

one thousand, nine hundred seventy-nine; eighty-five thousand; two million, nine hundred five thousand **Exercise:**

Problem:

In 1975, in the United States, it is estimated that 52,294,000 people drove to work alone.

Exercise:

Problem:

In 1980, there were 217 prisoners under death sentence that were divorced.

Solution:

one thousand, nine hundred eighty; two hundred seventeen **Exercise:**

Problem:

In 1979, the amount of money spent in the United States for regularsession college educa tion was $50,721,000,000,000.

Exercise:

Problem:

In 1981, there were 1,956,000 students majoring in business in U.S. colleges.

Solution:

one thousand, nine hundred eighty one; one million, nine hundred fifty-six thousand **Exercise:**

Problem:

In 1980, the average fee for initial and follow up visits to a medical doctors office was about $34.

Exercise:

Problem:

In 1980, there were approximately 13,100 smugglers of aliens apprehended by the Immigration border patrol.

Solution:

one thousand, nine hundred eighty; thirteen thousand, one hundred **Exercise:**

Problem:

In 1980, the state of West Virginia pumped 2,000,000 barrels of crude oil, whereas Texas pumped 975,000,000 barrels.

Exercise:

Problem: The 1981 population of Uganda was 12,630,000 people.

Solution:

twelve million, six hundred thirty thousand **Exercise:**

Problem:

In 1981, the average monthly salary offered to a person with a Master's degree in mathematics was $1,685.

For the following problems, write each number using digits.

Exercise:

Problem: Six hundred eighty-one

Solution:

681

Exercise:

Problem: Four hundred ninety **Exercise:**

Problem: Seven thousand, two hundred one

Solution:

7,201

Exercise:

Problem: Nineteen thousand, sixty-five **Exercise:**

Problem: Five hundred twelve thousand, three

Solution:

512,003 **Exercise:**

Problem:

Two million, one hundred thirty-three thousand, eight hundred fiftynine

Exercise:

Problem: Thirty-five million, seven thousand, one hundred one

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Solution:

35,007,101 **Exercise:**

Problem: One hundred million, one thousand **Exercise:**

Problem: Sixteen billion, fifty-nine thousand, four

Solution:

16,000,059,004

Exercise:

Problem:

Nine hundred twenty billion, four hundred sev enteen million, twentyone thousand
Exercise:

Problem: Twenty-three billion

Solution:

23,000,000,000 **Exercise:**

Problem:

Fifteen trillion, four billion, nineteen thousand, three hundred five **Exercise:**

Problem: One hundred trillion, one

Solution:

100,000,000,000,001

Exercises for Review Exercise:

Problem: ([link]) How many digits are there?

Exercise:

Problem: ([link]) In the number 6,641, how many tens are there?

Solution:

4

Exercise:

Problem: ([link]) What is the value of 7 in 44,763?

Exercise:

Problem: ([link]) Is there a smallest whole number? If so, what is it?

Solution:

yes, zero **Exercise:**

Problem:

([link]) Write a four-digit number with a 9 in the tens position.

Rounding Whole Numbers

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to round whole numbers. By the end of the module students should be able to understand that rounding is a method of approximation and round a whole number to a specified position.

Section Overview

- Rounding as an Approximation
-

The Method of Rounding Numbers

Rounding as an Approximation

A primary use of whole numbers is to keep count of how many objects there are in a collection. Sometimes we're only interested in the approximate number of objects in the collection rather than the precise number. For example, there are *approxi mately* 20 symbols in the collection below.

The *precise* number of symbols in the above collection is 18.

Rounding

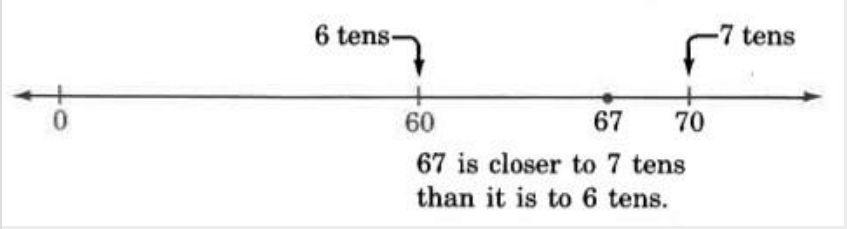
We often approximate the number of objects in a collection by mentally seeing the collection as occurring in groups of tens, hundreds, thousands, etc. This process of approximation is called **rounding**. Rounding is very useful in estimation. We will study estimation in Chapter 8.

When we think of a collection as occurring in groups of tens, we say we're *rounding to the nearest ten*. When we think of a collection as occurring in groups of hundreds, we say we're *rounding to the nearest hundred*. This idea of rounding continues through thousands, ten thousands, hundred thousands, millions, etc.

The process of rounding whole numbers is illustrated in the following examples.

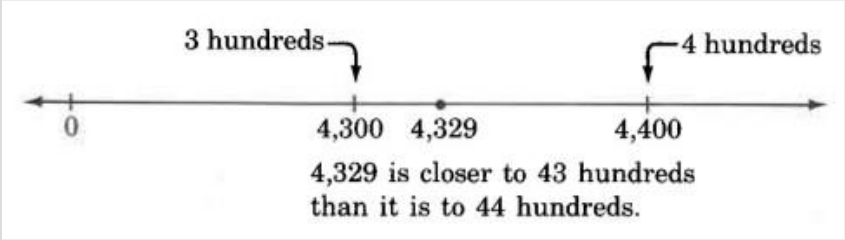
Example:

Round 67 to the nearest ten.
On the number line, 67 is more than halfway from 60 to 70. The digit immediately to the right of the tens digit, the round-off digit, is the indicator for this.



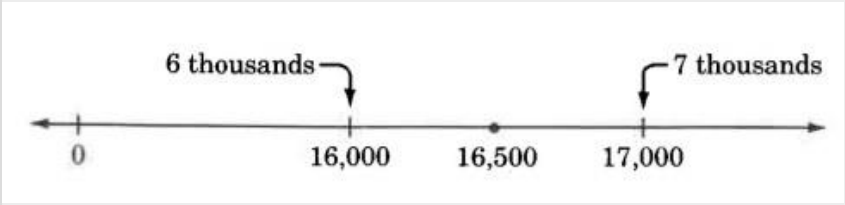
Thus, 67, rounded to the nearest ten, is 70.

Example:
Round 4,329 to the nearest hundred.
On the number line, 4,329 is less than halfway from 4,300 to 4,400. The digit to the immediate right of the hundreds digit, the round-off digit, is the indicator.



Thus, 4,329, rounded to the nearest hundred is 4,300.

Example:
Round 16,500 to the nearest thousand.
On the number line, 16,500 is exactly halfway from 16,000 to 17,000.



By convention, when the number to be rounded is *exactly halfway* between two numbers, it is rounded to the *higher* number.
Thus, 16,500, rounded to the nearest thousand, is 17,000.

Example:

A person whose salary is $41,450 per year might tell a friend that she makes $41,000 per year. She has rounded 41,450 to the nearest thousand. The number 41,450 is closer to 41,000 than it is to 42,000.

The Method of Rounding Whole Numbers

From the observations made in the preceding examples, we can use the following method to **round a whole number** to a particular position.

1. Mark the position of the round-off digit.
2. Note the digit to the immediate right of the round-off digit.
    a. If it is less than 5, replace it and all the digits to its right with zeros. Leave the round-off digit unchanged.
    b. If it is 5 or larger, replace it and all the digits to its right with zeros. Increase the round-off digit by 1.

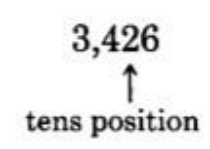
Sample Set A

Use the method of rounding whole numbers to solve the following problems.

Example:

Round 3,426 to the nearest ten.

1. We are rounding to the tens position. Mark the digit in the tens position



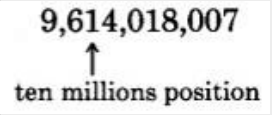
2. Observe the digit immediately to the right of the tens position. It is 6. Since 6 is greater than 5, we *round up* by replacing 6 with 0 and adding 1 to the digit in the tens position (the round-off position): 2+1=3 . 3,430

Thus, 3,426 rounded to the nearest ten is 3,430.

Example:

Round 9,614,018,007 to the nearest ten million.

1. We are rounding to the nearest ten million.



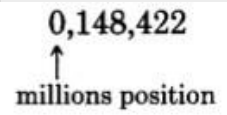
2. Observe the digit immediately to the right of the ten millions position. It is 4. Since 4 is less than 5, we *round down* by replacing 4 and all the digits to its right with zeros. 9,610,000,000

Thus, 9,614,018,007 rounded to the nearest ten million is 9,610,000,000.

Example:

Round 148,422 to the nearest million.

1. Since we are rounding to the nearest million, we'll have to *imagine* a digit in the millions position. We'll write 148,422 as 0,148,422.



2. The digit immediately to the right is 1. Since 1 is less than 5, we'll *round down* by replacing it and all the digits to its right with zeros.

0,000,000

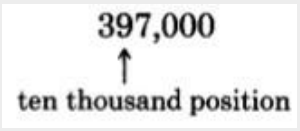
This number is 0.

Thus, 148,422 rounded to the nearest million is 0.

Example:

Round 397,000 to the nearest ten thousand.

1. We are rounding to the nearest ten thousand.



2. The digit immediately to the right of the ten thousand position is 7. Since 7 is greater than 5, we round up by replacing 7 and all the digits to its right with zeros and adding 1 to the digit in the ten thousands position. But 9+1=10 and we must carry the 1 to the next (the hundred thousands) position. 400,000

Thus, 397,000 rounded to the nearest ten thousand is 400,000.

Practice Set A

Use the method of rounding whole numbers to solve each problem.

Exercise:

Problem: Round 3387 to the nearest hundred.

Solution:

3400

Exercise:

Problem: Round 26,515 to the nearest thousand.

Solution:

27,000

Exercise:

Problem: Round 30,852,900 to the nearest million.

Solution:

31,000,000 **Exercise:**

Problem: Round 39 to the nearest hundred.

Solution:

0

Exercise:

Problem: Round 59,600 to the nearest thousand.

Solution:

60,000

Exercises

For the following problems, complete the table by rounding each number to the indicated positions.

Exercise: Problem: 1,642

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
1,600	2000	0	0

Exercise:

Problem: 5,221

hundred	thousand	ten thousand	million

Exercise: Problem: 91,803

Hundred	thousand	ten thousand	million

Solution:

Hundred	thousand	ten thousand	million
91,800	92,000	90,000	0

Exercise: Problem: 106,007

hundred	thousand	ten thousand	million

Exercise:

Problem: 208

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
200	0	0	0

Exercise: Problem: 199

hundred	thousand	ten thousand	million

Exercise:

Problem: 863

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise: Problem: 794

hundred	thousand	ten thousand	million

Exercise: Problem: 925

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise: Problem: 909

hundred	thousand	ten thousand	million

Exercise: Problem: 981

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
1,000	1,000	0	0

Exercise:

Problem: 965

hundred	thousand	ten thousand	million

Exercise: Problem: 551,061,285

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
551,061,300	551,061,000	551,060,000	551,000,000

Exercise:

Problem: 23,047,991,521

hundred	thousand	ten thousand	million

Exercise: Problem: 106,999,413,206

Hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
106,999,413,200	106,999,413,000	106,999,410,000	106,999,000,000

Exercise: Problem: 5,000,000

hundred	thousand	ten thousand	million

Exercise:

Problem: 8,006,001

hundred	thousand	ten thousand	million

Solution:

Hundred	Thousand	ten thousand	Million
8,006,000	8,006,000	8,010,000	8,000,000

Exercise: Problem: 94,312

hundred	thousand	ten thousand	million

Exercise:

Problem: 33,486

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million

33,500	33,000	30,000	0

Exercise: Problem: 560,669

hundred	thousand	ten thousand	million

Exercise: Problem: 388,551

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
388,600	389,000	390,000	0

Exercise: Problem: 4,752

hundred	thousand	ten thousand	million

Exercise: Problem: 8,209

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
8,200	8,000	10,000	0

Exercise:

Problem:

In 1950, there were 5,796 cases of diphtheria reported in the United States. Round to the nearest hundred.

Exercise:

Problem:

In 1979, 19,309,000 people in the United States received federal food stamps. Round to the near est ten thousand.

Solution:

19,310,000 **Exercise:**

Problem:

In 1980, there were 1,105,000 people between 30 and 34 years old enrolled in school. Round to the nearest million.

Exercise:

Problem:

In 1980, there were 29,100,000 reports of aggra vated assaults in the United States. Round to the nearest million.

Solution:

29,000,000

For the following problems, round the numbers to the posi tion you think is most reasonable for the situation. **Exercise:**

Problem:

In 1980, for a city of one million or more, the average annual salary of police and firefighters was $16,096.

Exercise:

Problem:

The average percentage of possible sunshine in San Francisco, California, in June is 73%.

Solution:

70% or 75% **Exercise:**

Problem:

In 1980, in the state of Connecticut, $3,777,000,000 in defense contract payroll was awarded.

Exercise:

Problem:

In 1980, the federal government paid $5,463,000,000 to Viet Nam veterans and depen dants.

Solution:

$5,500,000,000 **Exercise:**

Problem: In 1980, there were 3,377,000 salespeople em ployed in the United States.

Exercise:

Problem:

In 1948, in New Hampshire, 231,000 popular votes were cast for the president.

Solution:

230,000 **Exercise:**

Problem: In 1970, the world production of cigarettes was 2,688,000,000,000.

Exercise:

Problem:

In 1979, the total number of motor vehicle regis trations in Florida was 5,395,000.

Solution:

5,400,000

Exercise:

Problem: In 1980, there were 1,302,000 registered nurses the United States.

Exercises for Review

Exercise:

Problem:

([link]) There is a term that describes the visual displaying of a number. What is the term?

Solution:

graphing **Exercise:**

Problem: ([link]) What is the value of 5 in 26,518,206?

Exercise:

Problem: ([link]) Write 42,109 as you would read it.

Solution:

Forty-two thousand, one hundred nine **Exercise:**

Problem: ([link]) Write "six hundred twelve" using digits.

Exercise:

Problem: ([link]) Write "four billion eight" using digits.

Solution:

4,000,000,008

Addition of Whole Numbers

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add whole numbers. By the end of this module, students should be able to understand the addition process, add whole numbers, and use the calculator to add one whole number to another.

Section Overview

- Addition
- Addition Visualized on the Number Line
- The Addition Process
- Addition Involving Carrying

Calculators

Addition

Suppose we have two collections of objects that we combine together to form a third collection. For example,

We are combining a collection of four objects with a collection of three objects to obtain a collection of seven objects.

Addition

The process of combining two or more objects (real or intuitive) to form a third, the total, is called **addition**.

In addition, the numbers being added are called **addends** or **terms**, and the total is called the **sum**. The **plus symbol** (+) is used to indicate addition, and the **equal symbol** (=) is used to represent the word "equal." For example, 4 + 3 = 7 means "four added to three equals seven."

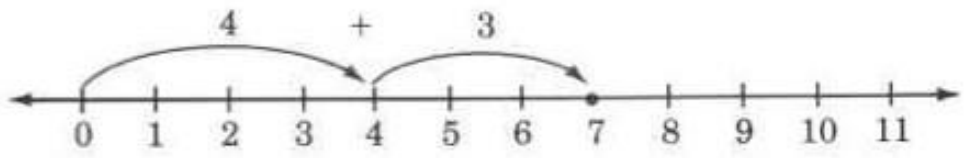
Addition Visualized on the Number Line

Addition is easily visualized on the number line. Let's visualize the addition of 4 and 3 using the number line.

To find 4 + 3,

1. Start at 0.
2. Move to the right 4 units. We are now located at 4.
3. From 4, move to the right 3 units. We are now located at 7.

Thus, 4 + 3 = 7.



The Addition Process

We'll study the process of addition by considering the sum of 25 and 43.

25 means +43

$$\begin{array}{r} 2 \text{ tens} + 5 \text{ ones} \\ +\,4 \text{ tens} + 3 \text{ ones} \\ \hline 6 \text{ tens} + 8 \text{ ones} \end{array}$$

We write this as 68.

We can suggest the following procedure for adding whole numbers using this example.

Example:

The Process of Adding Whole Numbers To **add whole numbers**, The process:

1. Write the numbers vertically, placing corresponding positions in the same column.

25 +43

2. Add the digits in each column. Start at the right (in the ones position) and move to the left, placing the sum at the bottom.

25

+43 68

Note: Confusion and incorrect sums can occur when the numbers are *not aligned* in columns properly. Avoid writing such additions as

25

+43

25

+43

Sample Set A

Example: Add 276 and 103.

276 6 + 3 = 9.

+103 7 + 0 = 7. 379 2 + 1 = 3.

Example:

Add 1459 and 130

9 + 0 = 9.

1459

5 + 3 = 8.

+130

4 + 1 = 5.

1589

1 + 0 = 1.

In each of these examples, each individual sum does not exceed 9. We will examine individual sums that exceed 9 in the next section.

Practice Set A

Perform each addition. Show the expanded form in problems 1 and 2.

Exercise:

Problem: Add 63 and 25.

Solution:

88

$$\begin{array}{r} 6 \text{ tens} + 3 \text{ ones} \\ +2 \text{ tens} + 5 \text{ ones} \\ \hline 8 \text{ tens} + 8 \text{ ones} \end{array}$$

Exercise:

Problem: Add 4,026 and 1,501.

Solution:

5,527

$$\begin{array}{r} 4 \text{ thousands} + 0 \text{ hundreds} + 2 \text{ tens} + 6 \text{ ones} \\ +1 \text{ thousand} \ + 5 \text{ hundreds} + 0 \text{ tens} + 1 \text{ one} \\ \hline 5 \text{ thousands} + 5 \text{ hundreds} + 2 \text{ tens} + 7 \text{ ones} \end{array}$$

Exercise:

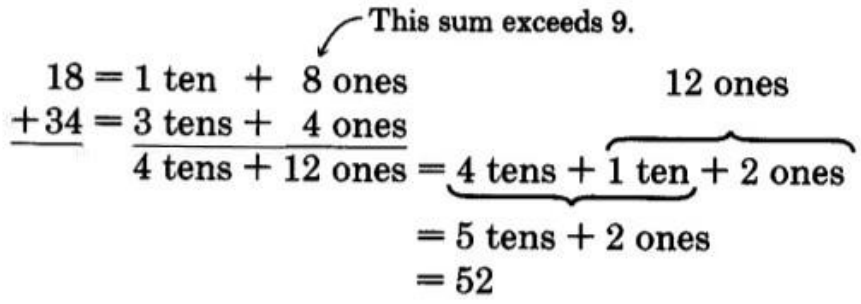
Problem: Add 231,045 and 36,121.

Solution:

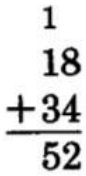
267,166

Addition Involving Carrying

It often happens in addition that the sum of the digits in a column will exceed 9. This happens when we add 18 and 34. We show this in expanded form as follows.



Notice that when we add the 8 ones to the 4 ones we get 12 ones. We then convert the 12 ones to 1 ten and 2 ones. In vertical addition, we show this conversion by **carrying** the ten to the tens column. We write a 1 at the top of the tens column to indicate the carry. This same example is shown in a shorter form as follows:



8 + 4 = 12 Write 2, carry 1 ten to the top of the next column to the left.

Sample Set B

Perform the following additions. Use the process of carrying when needed.

Example:

Add 1875 and 358.

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 1875 \\ +\ 358 \\ \hline 2233 \end{array}$$

5 + 8 = 13 Write 3, carry 1 ten.

1 + 7 + 5 = 13 Write 3, carry 1 hundred.

1 + 8 + 3 = 12 Write 2, carry 1 thousand.

1 + 1 = 2

The sum is 2233.

Example:

Add 89,208 and 4,946.

$$\begin{array}{r} {\scriptstyle 1\,1\ \ 1} \\ 89{,}208 \\ +\ 4{,}946 \\ \hline 94{,}154 \end{array}$$

8 + 6 = 14 Write 4, carry 1 ten.

1 + 0 + 4 = 5 Write the 5 (nothing to carry).

2 + 9 = 11 Write 1, carry one thousand.

1 + 9 + 4 = 14 Write 4, carry one ten thousand.

1 + 8 = 9

The sum is 94,154.

Example:
Add 38 and 95.

$$\begin{array}{r} {\scriptstyle 11} \\ 38 \\ +\ 95 \\ \hline 133 \end{array}$$

$8 + 5 = 13$ Write 3, carry 1 ten.

$1 + 3 + 9 = 13$ Write 3, carry 1 hundred.

$1 + 0 = 1$

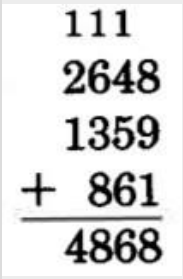
As you proceed with the addition, it is a good idea to keep in mind what is actually happening.

$$\begin{array}{rcrcl} 38 & \text{means} & 3 \text{ tens} & + & 8 \text{ ones} \\ +95 & & +\ 9 \text{ tens} & + & 5 \text{ ones} \\ \hline & & 12 \text{ tens} & + & 13 \text{ ones} \\ & = & 12 \text{ tens} + 1 \text{ ten} & + & 3 \text{ ones} \\ & = & 13 \text{ tens} & + & 3 \text{ ones} \\ & = 1 \text{ hundred} + & 3 \text{ tens} & + & 3 \text{ ones} \\ & = 133 & & & \end{array}$$

The sum is 133.

Example:

Find the sum 2648, 1359, and 861.



$$\begin{array}{r} {\scriptstyle 111} \\ 2648 \\ 1359 \\ +\ 861 \\ \hline 4868 \end{array}$$

8 + 9 + 1 = 18 Write 8, carry 1 ten.

1 + 4 + 5 + 6 = 16 Write 6, carry 1 hundred.

1 + 6 + 3 + 8 = 18 Write 8, carry 1 thousand.

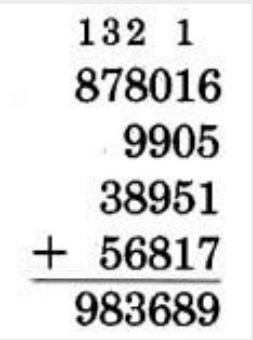
1 + 2 + 1 = 4

The sum is 4,868.

Numbers other than 1 can be carried as illustrated in [link].

Example:

Find the sum of the following numbers.



$$\begin{array}{r} {\scriptstyle 132\ 1} \\ 878016 \\ 9905 \\ 38951 \\ +\ 56817 \\ \hline 983689 \end{array}$$

6 + 5 + 1 + 7 = 19	Write 9, carry the 1. Write 8.
1 + 1 + 0 + 5 + 1 = 8	Write 6, carry the 2.
0 + 9 + 9 + 8 = 26	Write 3, carry the 3.
2 + 8 + 9 + 8 + 6 = 33	Write 8, carry the 1.
3 + 7 + 3 + 5 = 18	Write 9.
1 + 8 = 9	
The sum is 983,689.	

Example:

The number of students enrolled at Riemann College in the years 1984, 1985, 1986, and 1987 was 10,406, 9,289, 10,108, and 11,412, respectively. What was the total number of students en rolled at Riemann College in the years 1985, 1986, and 1987?

We can determine the total number of students enrolled by adding 9,289, 10,108, and 11,412, the number of students enrolled in the years 1985, 1986, and 1987.

$$\begin{array}{r} {\scriptstyle 1\quad 11} \\ 9{,}289 \\ 10{,}108 \\ +11{,}412 \\ \hline 30{,}809 \end{array}$$

The total number of students enrolled at Riemann College in the years 1985, 1986, and 1987 was 30,809.

Practice Set B

Perform each addition. For the next three problems, show the expanded form.

Exercise:

Problem: Add 58 and 29.

Solution:

87

$$\begin{array}{r} 5 \text{ tens} + 8 \text{ ones} \\ +2 \text{ tens} + 9 \text{ ones} \\ \hline 7 \text{ tens} + 17 \text{ ones} \end{array}$$

= 7tens + 1ten + 7ones = 8tens + 7ones = 87

Exercise:

Problem: Add 476 and 85.

Solution:

561

$$\begin{array}{r} 4 \text{ hundreds} + 7 \text{ tens} + 6 \text{ ones} \\ + \qquad\qquad 8 \text{ tens} + 5 \text{ ones} \\ \hline 4 \text{ hundreds} + 15 \text{ tens} + 11 \text{ ones} \end{array}$$

= 4 hundreds + 15 tens + 1 ten + 1 one

= 4 hundreds + 16 tens + 1 one

= 4 hundreds + 1 hundred + 6 tens + 1 one = 5 hundreds + 6 tens + 1 one = 561

Exercise:

Problem: Add 27 and 88.

Solution:

115

$$\begin{array}{r} 2 \text{ tens} + 7 \text{ ones} \\ + 8 \text{ tens} + 8 \text{ ones} \\ \hline 10 \text{ tens} + 15 \text{ ones} \end{array}$$

= 10 tens + 1 ten + 5 ones = 11 tens + 5 ones = 1 hundred + 1 ten + 5 ones = 115

Exercise:

Problem: Add 67,898 and 85,627.

Solution:

153,525

For the next three problems, find the sums.

Exercise:

57

Problem: 26

84

Solution:

167

Exercise:

847

Problem: 825

796

Solution:

2,468

Exercise:

16,945 8,472

Problem: 387,721

21,059 629

Solution:

434,826

Calculators

Calculators provide a very simple and quick way to find sums of whole numbers. For the two problems in Sample Set C, assume the use of a calculator that does not require the use of an ENTER key (such as many Hewlett-Packard calculators).

Sample Set C

Use a calculator to find each sum.

Example:

34+21		Display Reads
Type	34	34
Press	+	34
Type	21	21
Press	=	55

The sum is 55.

Example:

106+85+322+406		Display Reads	
Type	106	106	The calculator keeps a running subtotal
Press	+	106	
Type	85	85	
Press	=	191	← 106 + 85

Type	322	322	
Press	+	513	← 191 + 322
Type	406	406	
Press	=	919	← 513 + 406
The sum is 919.			

Practice Set C

Use a calculator to find the following sums. **Exercise:**

Problem: 62 + 81 + 12

Solution:

155

Exercise:

Problem: 9,261 + 8,543 + 884 + 1,062

Solution:

19,750

Exercise:

Problem: 10,221 + 9,016 + 11,445

Solution:

30,682

Exercises

For the following problems, perform the additions. If you can, check each sum with a calculator. **Exercise:**

Problem:14 + 5

Solution:

19

Exercise:

Problem: 12 + 7

Exercise:

Problem: 46 + 2

Solution:

48

Exercise:

Problem: 83 + 16 **Exercise:**

Problem: 77 + 21

Solution:

98

Exercise:

321 **Problem:**

+ 42

Exercise:

916 **Problem:**

+ 62

Solution:

978

Exercise:

104 **Problem:**

+561

Exercise:

265 **Problem:**

+103

Solution:

368

Exercise:

Problem: 552 + 237 **Exercise:**

Problem: 8,521 + 4,256

Solution:

12,777

Exercise:

16,408 **Problem:**

+ 3,101

Exercise:

16,515 **Problem:**

+42,223

Solution:

58,738

Exercise:

Problem: 616,702 + 101,161 **Exercise:**

Problem: 43,156,219 + 2,013,520

Solution:

45,169,739 **Exercise:**

Problem: 17 + 6

Exercise:

Problem: 25 + 8

Solution:

33

Exercise:

84 **Problem:**

+ 7 **Exercise:**

75 **Problem:**

+ 6

Solution:

81

Exercise:

Problem: 36 + 48 **Exercise:**

Problem: 74 + 17

Solution:

91

Exercise:

Problem: 486 + 58 **Exercise:**

Problem: 743 + 66

Solution:

809

Exercise:

Problem: 381 + 88

Exercise:

687 **Problem:**

+175

Solution:

862

Exercise:

931 **Problem:**

+853

Exercise:

Problem: 1,428 + 893

Solution:

2,321

Exercise:

Problem: 12,898 + 11,925

Exercise:

631,464 **Problem:**

+509,740

Solution:

1,141,204 **Exercise:**

805,996 **Problem:**

+ 98,516

Exercise:

38,428,106 **Problem:**

+522,936,005

Solution:

561,364,111 **Exercise:**

Problem: 5,288,423,100 + 16,934,785,995 **Exercise:**

Problem: 98,876,678,521,402 + 843,425,685,685,658

Solution:

942,302,364,207,060 **Exercise:**

Problem: 41 + 61 + 85 + 62

Exercise:

Problem: 21 + 85 + 104 + 9 + 15

Solution:

234

Exercise:

116 27

Problem: 110

110 + 8

Exercise:

75,206

Problem: 4,152

+16,007

Solution:

95,365

Exercise:

8,226 143 92,015

Problem:

8

487,553 5,218

Exercise:

50,006 1,005 100,300 20,008 1,000,009 800,800

Problem:

Solution:

1,972,128 **Exercise:**

616 42,018 1,687 225 **Problem:** 8,623,418 12,506,508 19 2,121 195,643

For the following problems, perform the additions and round to the nearest hundred.

Exercise:

1,468 **Problem:**

2,183

Solution:

3,700

Exercise:

928,725 **Problem:**

15,685

Exercise:

82,006 **Problem:**

3,019,528

Solution:

3,101,500 **Exercise:**

18,621 **Problem:**

5,059

Exercise:

92 **Problem:**

48

Solution:

100

Exercise:

16 **Problem:**

37

Exercise:

21 **Problem:**

16

Solution:

0

Exercise:

11,172

Problem: 22,749

12,248

Exercise:

240

280 **Problem:**

210 310

Solution:

1,000

Exercise:

9,573

Problem: 101,279

122,581

For the next five problems, replace the letter m with the whole number that will make the addition true.

Exercise:

62

Problem: + m

67

Solution:

5

Exercise:

106

Problem: + m

113

Exercise:

432

Problem: + m

451

Solution:

19

Exercise:

803

Problem: + m

830

Exercise:

1,893

Problem: + m

1,981

Solution:

Exercise:

Problem:

The number of nursing and related care facilities in the United States in 1971 was 22,004. In 1978, the number was 18,722. What was the total num ber of facilities for both 1971 and 1978?

Exercise:

Problem:

The number of persons on food stamps in 1975, 1979, and 1980 was 19,179,000, 19,309,000, and 22,023,000, respectively. What was the total number of people on food stamps for the years 1975, 1979, and 1980?

Solution:

60,511,000 **Exercise:**

Problem:

The enrollment in public and nonpublic schools in the years 1965,

1970, 1975, and 1984 was 54,394,000, 59,899,000, 61,063,000, and 55,122,000, respectively. What was the total en rollment for those years?

Exercise:

Problem:

The area of New England is 3,618,770 square miles. The area of the

Mountain states is 863,563 square miles. The area of the South

Atlantic is 278,926 square miles. The area of the Pacific states is 921,392 square miles. What is the total area of these regions?

Solution:

5,682,651 square miles **Exercise:**

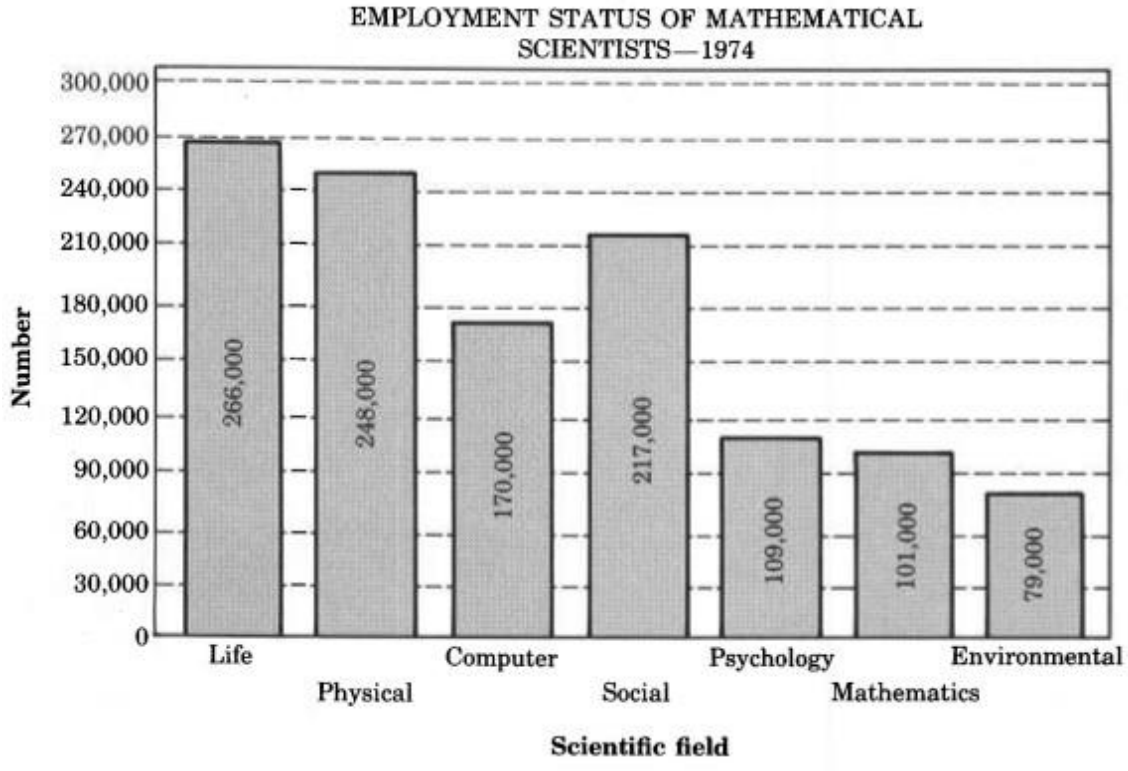
Problem:

In 1960, the IRS received 1,188,000 corporate income tax returns. In 1965, 1,490,000 returns were received. In 1970, 1,747,000 returns were received. In 1972 —1977,

1,890,000; 1,981,000; 2,043,000; 2,100,000; 2,159,000; and 2,329,000 re turns were received, respectively. What was the total number of corporate tax returns received by the IRS during the years 1960, 1965, 1970, 1972 —1977?

Exercise:

Problem: Find the total number of scientists employed in 1974.

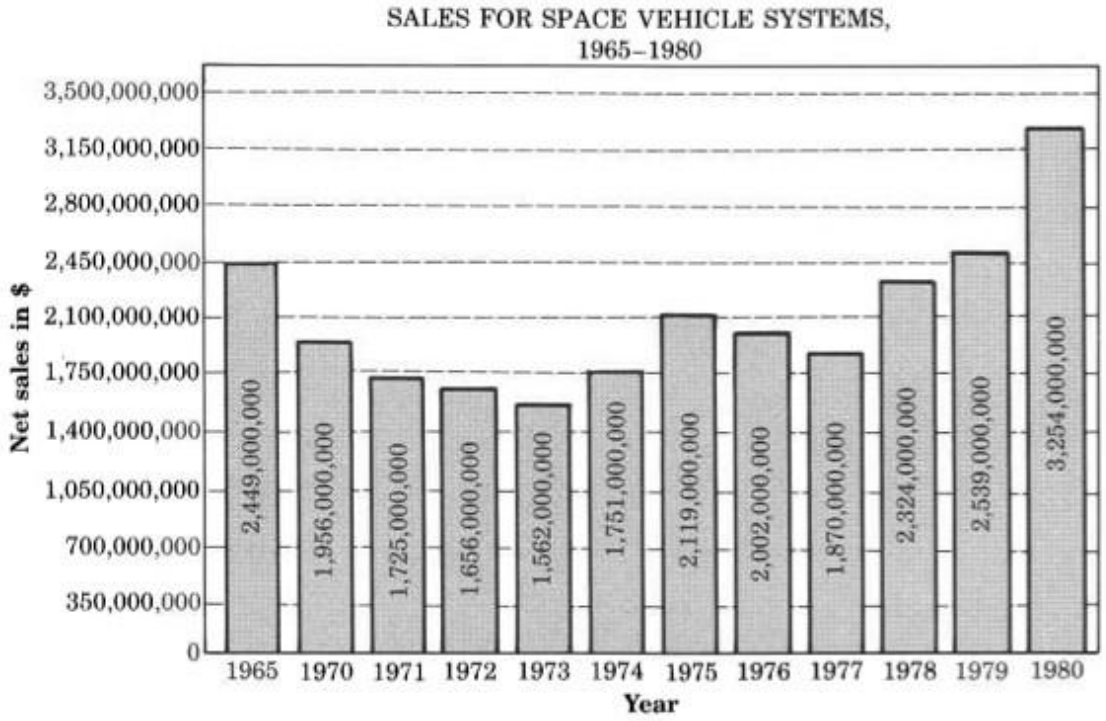


Solution:

1,190,000 **Exercise:**

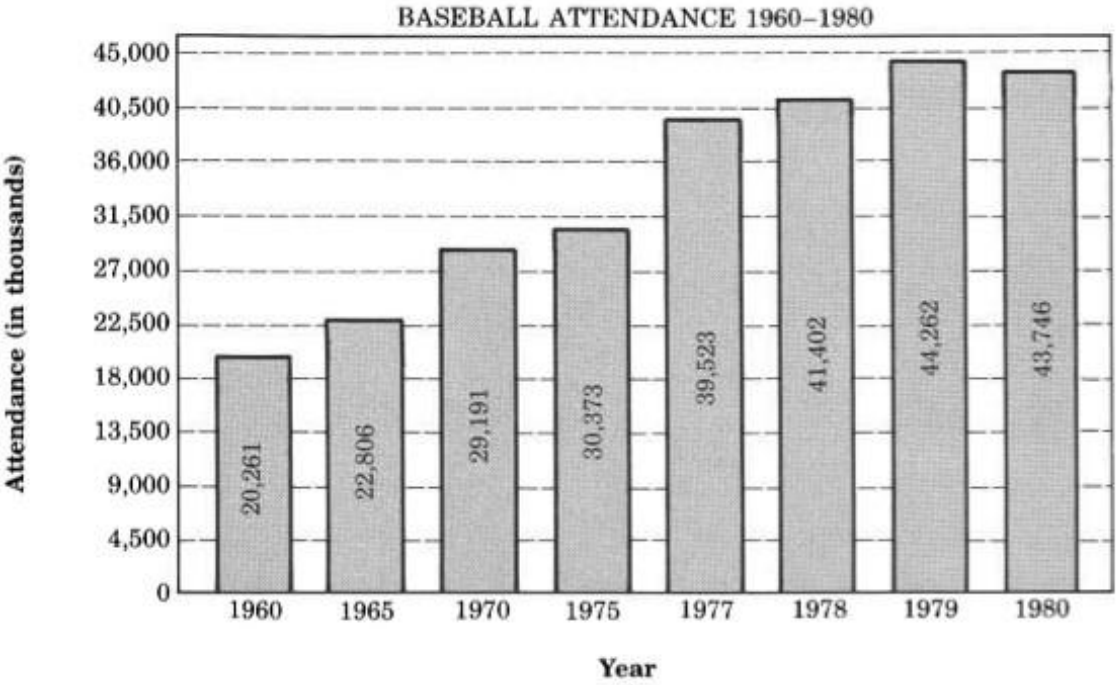
Problem:

Find the total number of sales for space vehicle systems for the years 1965-1980.



Exercise:

Problem: Find the total baseball attendance for the years 1960-1980.

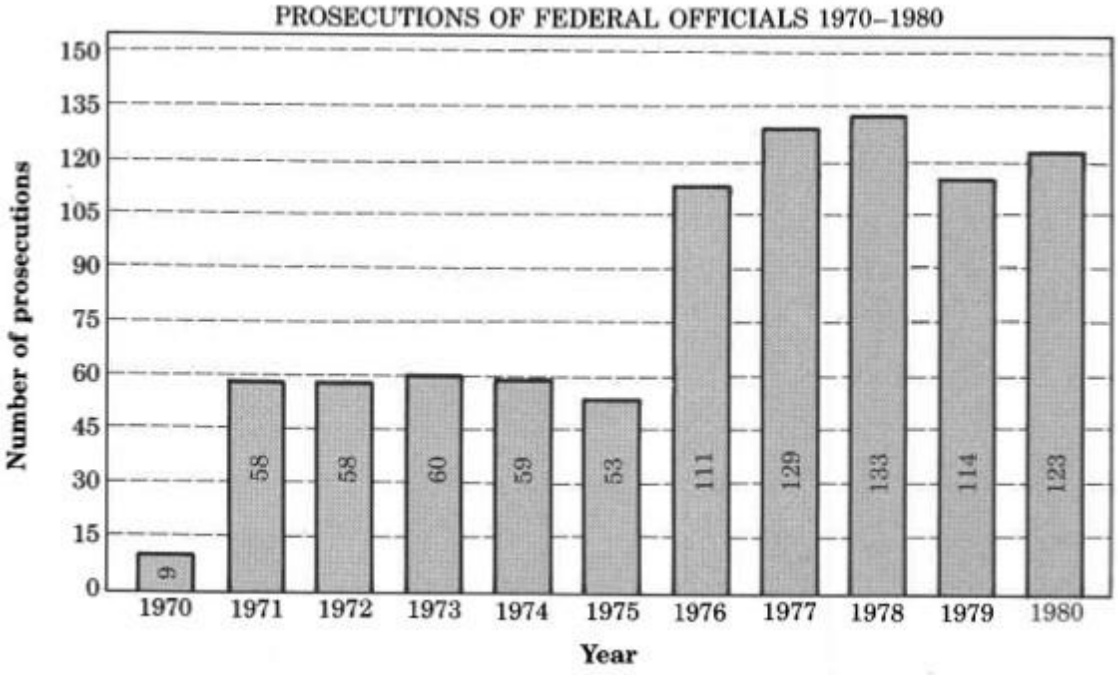


Solution:

271,564,000 **Exercise:**

Problem:

Find the number of prosecutions of federal officials for 1970-1980.



For the following problems, try to add the numbers mentally.

Exercise:

5

5 **Problem:**

3

7

Solution:

20

Exercise:

8

2 **Problem:**

6

4

Exercise:

9

1

Problem: 8

5

2

Solution:

25

Exercise:

5

2

5 **Problem:**

8

3

7 **Exercise:**

6

4

3

1 **Problem:**

6

7

9

4

Solution:

40

Exercise:

20 **Problem:**

30 **Exercise:**

15 **Problem:**

35

Solution:

50

Exercise:

16 **Problem:**

14 **Exercise:**

23 **Problem:**

27

Solution:

50

Exercise:

82 **Problem:**

18

Exercise:

36 **Problem:**

14

Solution:

50

Exercises for Review

Exercise:

Problem:

([link]) Each period of numbers has its own name. From right to left, what is the name of the fourth period?

Exercise:

Problem:

([link]) In the number 610,467, how many thousands are there?

Solution:

0

Exercise:

Problem: ([link]) Write 8,840 as you would read it.

Exercise:

Problem: ([link]) Round 6,842 to the nearest hundred.

Solution:

6,800

Exercise:

Problem: ([link]) Round 431,046 to the nearest million.