Subtraction of Whole Numbers

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to subtract whole numbers. By the end of this module, students should be able to understand the subtraction process, subtract whole numbers, and use a calculator to subtract one whole number from another whole number.

Section Overview

- Subtraction
- Subtraction as the Opposite of Addition
- The Subtraction Process
- Subtraction Involving Borrowing

Borrowing From Zero

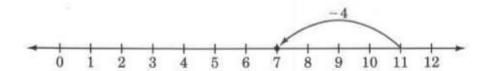
Calculators

Subtraction

Subtraction

Subtraction is the process of determining the remainder when part of the total is removed.

Suppose the sum of two whole numbers is 11, and from 11 we remove 4. Using the number line to help our visualization, we see that if we are located at 11 and move 4 units to the left, and thus remove 4 units, we will be located at 7. Thus, 7 units remain when we remove 4 units from 11 units.



The Minus Symbol

The **minus symbol** (-) is used to indicate subtraction. For example, 11 – 4 indicates that 4 is to be subtracted from 11.

Minuend

The number immediately in front of or the minus symbol is called the **minuend**, and it represents the *original* number of units.

Subtrahend

The number immediately following or below the minus symbol is called the **subtrahend**, and it represents the number of units *to be removed*.

Difference

The *result* of the subtraction is called the **difference** of the two numbers. For example, in 11 - 4 = 7, 11 is the minuend, 4 is the subtrahend, and 7 is the difference.

Subtraction as the Opposite of Addition

Subtraction can be thought of as the opposite of addition. We show this in the problems in Sample Set A.

Sample Set A

Example:

$$8 - 5 = 3$$
 since $3 + 5 = 8$.

Example:

$$9 - 3 = 6$$
 since $6 + 3 = 9$.

Practice Set A

Complete the following statements.

Exercise:

Problem:7 - 5 = since + 5 = 7.

Solution:

$$7 - 5 = 2$$
 since $2 + 5 = 7$

Exercise:

Problem:
$$9 - 1 = since + 1 = 9$$
.

Solution:

$$9 - 1 = 8$$
 since $8 + 1 = 9$

Exercise:

Problem:
$$17 - 8 = \text{since} + 8 = 17.$$

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17 - 8 = 9 since 9 + 8 = 17
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The Subtraction Process

We'll study the process of the subtraction of two whole numbers by considering the difference between 48 and 35.

$$\begin{array}{ccc}
48 & \text{means} & 4 \text{ tens} + 8 \text{ ones} \\
-35 & -3 \text{ tens} - 5 \text{ ones} \\
\hline
1 \text{ ten} & + 3 \text{ ones}
\end{array}$$

which we write as 13.

Example:

The Process of Subtracting Whole Numbers

To subtract two whole numbers,

The process

1. Write the numbers vertically, placing corresponding positions in the same column.

48

-35

2. Subtract the digits in each column. Start at the right, in the ones position, and move to the left, placing the difference at the bottom.

48

-35 13

Sample Set B

Perform the following subtractions.

Example:

275

-142

133

5 - 2 = 3.

7 - 4 = 3.

2 - 1 = 1.

Example:

46,042

- 1,031

45,011

2 - 1 = 1.

4 - 3 = 1.

0 - 0 = 0.

6 - 1 = 5.

4 - 0 = 4.

Example:

Find the difference between 977 and 235.

Write the numbers vertically, placing the larger number on top. Line up the columns properly. 977

-235

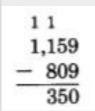
742

The difference between 977 and 235 is 742.

Example:

In Keys County in 1987, there were 809 cable television installations. In Flags County in 1987, there were 1,159 cable television installations. How many more cable television installations were there in Flags County than in Keys County in 1987?

We need to determine the difference between 1,159 and 809.



There were 350 more cable television installations in Flags County than in Keys County in 1987.

Practice Set B		
Perform the following subtractions. Exercise: 534 Problem:		
Solution:		
331		
Exercise:		
857 Problem:		
- 43		
Solution:		
814		
Exercise:		
95,628 Problem:		
-34,510		
Solution:		
61,118		
Exercise:		
11,005 Problem:		
- 1,005		
Solution:		
10,000		
Exercise:		
Problem: Find the difference between 88,526 and 26,412.		

Solution:

62,114

In each of these problems, each bottom digit is less than the corresponding top digit. This may not always be the case. We will examine the case where the bottom digit is greater than the corresponding top digit in the next section.

Subtraction Involving Borrowing

Minuend and Subtrahend

It often happens in the subtraction of two whole numbers that a digit in the **minuend** (top number) will be less than the digit in the same position in the **subtrahend** (bottom number). This happens when we subtract 27 from 84.

84 - 27

We do not have a name for 4 - 7. We need to rename 84 in order to continue. We'll do so as follows:

$$84 = 8 \text{ tens} + 4 \text{ ones}$$

 $-27 = 2 \text{ tens} + 7 \text{ ones}$

$$7 ext{ tens} + 1 ext{ ten} + 4 ext{ ones}$$
 $2 ext{ tens} + 7 ext{ ones}$

$$7 ext{ tens} + 10 ext{ ones} + 4 ext{ ones}$$
 $2 ext{ tens} + 7 ext{ ones}$

Our new name for 84 is 7 tens + 14 ones.

$$7 ext{ tens} + 14 ext{ ones}$$
 $2 ext{ tens} + 7 ext{ ones}$
 $5 ext{ tens} + 7 ext{ ones}$

= 57

Notice that we converted 8 tens to 7 tens + 1 ten, and then we converted the 1 ten to 10 ones. We then had 14 ones and were able to perform the subtraction.

Borrowing

The process of **borrowing** (converting) is illustrated in the problems of Sample Set C.

Sample Set C

Example:

 $\begin{array}{r}
 714 \\
 \hline
 84 \\
 \hline
 -27 \\
 \hline
 57
 \end{array}$

- 1. Borrow 1 ten from the 8 tens. This leaves 7 tens.
- 2. Convert the 1 ten to 10 ones.
- 3. Add 10 ones to 4 ones to get 14 ones.

Example:

 $\begin{array}{r}
 517 \\
 \cancel{6}\cancel{7}2 \\
 -91 \\
 \hline
 581
 \end{array}$

- 1. Borrow 1 hundred from the 6 hundreds. This leaves 5 hundreds.
- 2. Convert the 1 hundred to 10 tens.
- 3. Add 10 tens to 7 tens to get 17 tens.

Practice Set C

Perform the following subtractions. Show the expanded form for the first three problems. **Exercise:**

53 Problem:

-35

Exercise:

76 **Problem:** -28

Solution:

Exercise:

872 Problem:

-565

Exercise:
441 Problem:
-356
Solution:
85
Exercise:
775 Problem:
- 66
Solution:
709
Exercise:
5,663 Problem:
-2,559
Solution:
3,104
Borrowing More Than Once
Sometimes it is necessary to horrow more than once. This is shown in the problems in

Sometimes it is necessary to **borrow more than once**. This is shown in the problems in [link].

Sample Set D

Perform the Subtractions. Borrowing more than once if necessary

513 \$11 **64**1 - 358 - 283

- 1. Borrow 1 ten from the 4 tens. This leaves 3 tens.
- 2. Convert the 1 ten to 10 ones.
- 3. Add 10 ones to 1 one to get 11 ones. We can now perform 11 8.
- 4. Borrow 1 hundred from the 6 hundreds. This leaves 5 hundreds.
- 5. Convert the 1 hundred to 10 tens.
- 6. Add 10 tens to 3 tens to get 13 tens.
- 7. Now 13 5 = 8.
- 8. 5-3=2.

Example:

- 1. Borrow 1 ten from the 3 tens. This leaves 2 tens.
- 2. Convert the 1 ten to 10 ones.
- 3. Add 10 ones to 4 ones to get 14 ones. We can now perform 14 5.
- 4. Borrow 1 hundred from the 5 hundreds. This leaves 4 hundreds.
- 5. Convert the 1 hundred to 10 tens.

- 6. Add 10 tens to 2 tens to get 12 tens. We can now perform 12 8 = 4.
- 7. Finally, 4 0 = 4.

71529

- 6952

After borrowing, we have

Practice Set D

Perform the following subtractions.

Exercise:

526 Problem:

-358

Exercise:
63,419 Problem:
- 7,779
Solution:
55,640
Exercise:
4,312 Problem:
-3,123
Solution:
1,189
Borrowing from Zero
It often happens in a subtraction problem that we have to borrow from one or more zeros. This occurs in problems such as
503 - 37
1.
and
5000 - 37
2.
We'll examine each case.

Borrowing from a single zero.

Consider the problem 37

Since we do not have a name fo8-7, we must borrow from 0.

$$503 = 5 \text{ hundreds} + 0 \text{ tens} + 3 \text{ ones}$$

$$-37 \qquad 3 \text{ tens} + 7 \text{ ones}$$

Since there are no tens to borrow, we must borrow 1 hundred. One hundred = 10 tens.

$$4 \text{ hundreds} + 10 \text{ tens} + 3 \text{ ones}$$

 $3 \text{ tens} + 7 \text{ ones}$

We can now borrow 1 ten from 10 tens (leaving 9 tens). One ten = 10 ones and 10 ones + 3 ones = 13 ones.

$$\frac{4 \text{ hundreds} + 9 \text{ tens} + 13 \text{ ones}}{3 \text{ tens} + 7 \text{ ones}}$$

$$\frac{3 \text{ tens} + 7 \text{ ones}}{4 \text{ hundreds} + 6 \text{ tens} + 6 \text{ ones}} = 466$$

Now we can suggest the following method for borrowing from a single zero.

Borrowing from a Single Zero To borrow from a single zero,

- 1. Decrease the digit to the immediate left of zero by one.
- 2. Draw a line through the zero and make it a 10.
- 3. Proceed to subtract as usual.

Sample Set E

Perform this subtraction.

503

- 37

The number 503 contains a single zero

1. The number to the immediate left of 0 is 5. Decrease 5 by 1.



- 2. Draw a line through the zero and make it a 10.
- 3. Borrow from the 10 and proceed.

1 ten + 10 ones

10 ones + 3 ones = 13 ones

Practice Set E

Perform each subtraction. Exercise:

906 Problem:

- 18

Exercise:
5102 Problem:
- 559
Solution:
4,543
Exercise:
9055 Problem:
- 386
Solution:
8,669
Example:

Borrowing from a group of zeros

5000

Consider the problem

- 37

In this case, we have a group of zeros.

$$5000 = 5$$
 thousands $+ 0$ hundred $+ 0$ tens $+ 0$ ones $-37 = 3$ tens $+ 7$ ones

Since we cannot borrow any tens or hundreds, we must borrow 1 thousand. One thousand = 10 hundreds.

4 thousands
$$+$$
 10 hundreds $+$ 0 tens $+$ 0 ones 3 tens $+$ 7 ones

We can now borrow 1 hundred from 10 hundreds. One hundred = 10 tens.

4 thousands
$$+$$
 9 hundreds $+$ 10 tens $+$ 0 ones 3 tens $+$ 7 ones

We can now borrow 1 ten from 10 tens. One ten = 10 ones.

$$\frac{4 \text{ thousands} + 9 \text{ hundreds} + 9 \text{ tens} + 10 \text{ ones}}{3 \text{ tens} + 7 \text{ ones}}$$

$$\frac{3 \text{ tens} + 7 \text{ ones}}{4 \text{ thousands} + 9 \text{ hundreds} + 6 \text{ tens} + 3 \text{ ones}} = 4,963$$

From observations made in this procedure we can suggest the following method for borrowing from a group of zeros.

Borrowing from a Group of zeros To borrow from a group of zeros,

- 1. Decrease the digit to the immediate left of the group of zeros by one.
- 2. Draw a line through each zero in the group and make it a 9, except the rightmost zero, make it 10.
- 3. Proceed to subtract as usual.

Sample Set F

Perform each subtraction.

40,000

- 125

The number 40,000 contains a group of zeros.

1. The number to the immediate left of the group is 4. Decrease 4 by 1.

2. Make each 0, except the rightmost one, 9. Make the rightmost 0 a 10.

$$\begin{array}{r}
 39910 \\
 40,000 \\
 - 125
 \end{array}$$

3. Subtract as usual.

$$\begin{array}{r}
39 9910 \\
40,000 \\
- 125 \\
\hline
39,875
\end{array}$$

Example:

8,000,006 - 41,107

The number 8,000,006 contains a group of zeros.

1. The number to the immediate left of the group is 8. Decrease 8 by 1.

$$8 - 1 = 7$$

2. Make each zero, except the rightmost one, 9. Make the rightmost 0 a 10.

3. To perform the subtraction, we'll need to borrow from the ten.

$$\begin{array}{r} 9\\7\ 999\ 91016\\\$, \emptyset \emptyset \emptyset, \emptyset \emptyset 6\\- 41,107\\\hline 7,958,899\end{array}$$

$$1 \text{ ten} = 10 \text{ ones}$$

 $10 \text{ ones} + 6 \text{ ones} = 16 \text{ ones}$

Practice Set F

Perform each subtraction. Exercise:

21,007 **Problem:**

- 4,873

Solution:

16,134

Exercise:

10,004 **Problem:**

- 5,165

4,839	
Exercise:	
16,000,000 Problem:	
- 201,060	
Solution:	
15,789,940	
Calculators	

In practice, calculators are used to find the difference between two whole numbers.

Sample Set G

Find the difference between 1006 and 284.

Display Reads		
Туре	1006	1006
Press	-	1006
Туре	284	284
Press	=	722

The difference between 1006 and 284 is 722.

(What happens if you type 284 first and then 1006? We'll study such numbers in [link]Chapter 10.)

Practice Set G

Exercise:

Problem:

Use a calculator to find the difference between 7338 and 2809.

4,529
Exercise:
Problem:
Use a calculator to find the difference between 31,060,001 and 8,591,774.
Solution:
22,468,227
Exercises
For the following problems, perform the subtractions. You may check each difference with a calculator. Exercise:
15 Problem:
- 8
Solution:
7
Exercise:
19 Problem:
- 8
Exercise:
11 Problem:
- 5
Solution:
6
Exercise:
14 Problem:
- 6
Exercise:

12 Problem:	
- 9	
Solution:	
3	
Exercise:	
56 Problem:	
-12	
Exercise:	
74 Problem:	
-33	
Solution:	
41	
Exercise:	
80 Problem:	
-61	
Exercise:	
350 Problem:	
-141	
Solution:	
209	
Exercise:	
800 Problem:	
-650	
Exercise:	
35,002 Problem:	

Solution:	
21,001	
Exercise:	
5,000,566 Problem:	
-2,441,326	
Exercise:	
400,605 Problem:	
-121,352	
Solution:	
279,253 Exercise:	
46,400 Problem:	
- 2,012	
Exercise:	
77,893 Problem:	
- 421	
Solution:	
77,472	
Exercise:	
42 Problem:	
-18	
Exercise:	
51 Problem:	
-27	

Solution:		
24		
Exercise:		
622 Problem:		
- 88 Exercise:		
261 Problem:		
- 73		
Solution:		
188		
Exercise:		
242 Problem:		
-158		
Exercise:		
3,422 Problem:		
-1,045		
Solution:		
2,377		
Exercise:		
5,565 Problem:		
-3,985		
Exercise:		
42,041 Problem:		
-15,355		
Solution:		

26,686

Exercise:
304,056 Problem:
- 20,008
Exercise:
64,000,002 Problem:
- 856,743
Solution:
63,143,259 Exercise :
4,109 Problem:
- 856
Exercise:
10,113 Problem:
- 2,079
Solution:
8,034
Exercise:
605 Problem:
- 77
Exercise:
59 Problem:
-26
Solution:
33
Exercise:
36,107 Problem:

- 8,314
Exercise:
92,526,441,820 Problem:
-59,914,805,253
Solution:
32,611,636,567
Exercise:
1,605 Problem:
- 881
Exercise:
30,000 Problem:
-26,062
Solution:
3,938
Exercise:
600 Problem:
-216
Exercise:
9,000,003 Problem:
- 726,048
Solution:
8,273,955
For the following problems, perform each subtraction.
Exercise:

Problem: Subtract 63 from 92.

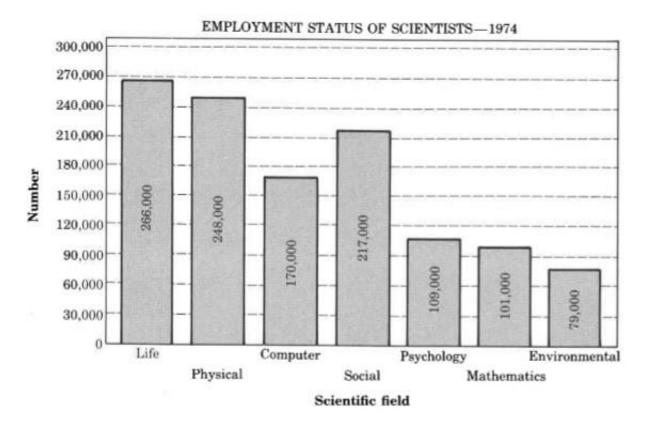
Note: The word "from" means "beginning at." Thus, 63 from 92 means beginning at 92 or 92 – 63.	,
Exercise:	
Problem: Subtract 35 from 86.	
Solution:	
51	
Exercise:	
Problem: Subtract 382 from 541.	
Exercise:	
Problem: Subtract 1,841 from 5,246.	
Solution:	
3,405	
Exercise:	
Problem: Subtract 26,082 from 35,040.	
Exercise:	
Problem: Find the difference between 47 and 21.	
Solution:	
26	
Exercise:	
Problem: Find the difference between 1,005 and 314.	
Exercise:	
Problem: Find the difference between 72,085 and 16.	
Solution:	_
72,069	

Exercise:
Problem: Find the difference between 7,214 and 2,049.
Exercise:
Problem: Find the difference between 56,108 and 52,911.
Solution:
3,197
Exercise:
Problem: How much bigger is 92 than 47?
Exercise:
Problem: How much bigger is 114 than 85?
Solution:
29
Exercise:
Problem: How much bigger is 3,006 than 1,918?
Exercise:
Problem: How much bigger is 11,201 than 816?
Solution:
10,385
Exercise:
Problem: How much bigger is 3,080,020 than 1,814,161?
Exercise:
Problem:
In Wichita, Kansas, the sun shines about 74% of the time in July and about 59% of the time in November. How much more of the time (in per cent) does the sun shine in July than in No vember?

Solution:
15%
Exercise:
Problem:
The lowest temperature on record in Concord, New Hampshire in May is 21°F, and in July it is 35°F. What is the difference in these lowest tem peratures?
Exercise:
Problem:
In 1980, there were 83,000 people arrested for prostitution and commercialized vice and 11,330,000 people arrested for driving while in toxicated. How many more people were arrested for drunk driving than for prostitution?
Solution:
11,247,000 Exercise:
Problem:
In 1980, a person with a bachelor's degree in ac counting received a monthly salary offer of \$1,293, and a person with a marketing degree a monthly salary offer of \$1,145. How much more was offered to the person with an accounting degree than the person with a marketing degree?
Exercise:
Problem:
In 1970, there were about 793 people per square mile living in Puerto Rico, and 357 people per square mile living in Guam. How many more people per square mile were there in Puerto Rico than Guam?
Solution:
436
Exercise:

The 1980 population of Singapore was 2,414,000 and the 1980 population of Sri Lanka was 14,850,000. How many more people lived in Sri Lanka than in Singapore in 1980?

Exercise:
Problem:
In 1977, there were 7,234,000 hospitals in the United States and 64,421,000 in Mainland China. How many more hospitals were there in Mainland China than in the United States in 1977?
Solution:
57,187,000 Exercise :
Problem:
In 1978, there were 3,095,000 telephones in use in Poland and 4,292,000 in Switzerland. How many more telephones were in use in Switzerland than in Poland in 1978?
For the following problems, use the corresponding graphs to solve the problems. Exercise:
Problem:
How many more life scientists were there in 1974 than mathematicians? ([link])
Solution:
165,000 Exercise :
Problem:
How many more social, psychological, mathe matical, and environmental scientists were there than life, physical, and computer scientists? ([link])



Exercise:

Problem:

How many more prosecutions were there in 1978 than in 1974? ([link])

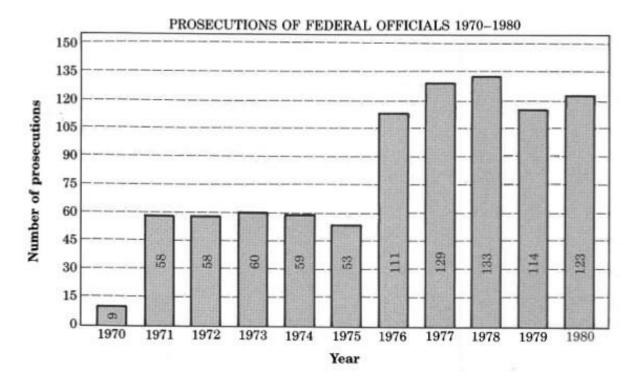
Solution:

74

Exercise:

Problem:

How many more prosecutions were there in 1976-1980 than in 19701975? ([link])



Exercise:

Problem:

How many more dry holes were drilled in 1960 than in 1975? ([link])

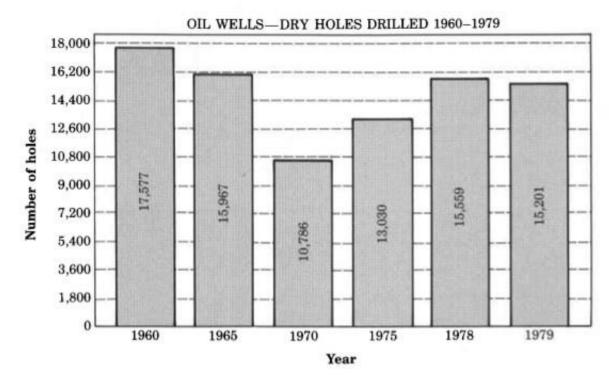
Solution:

4,547

Exercise:

Problem:

How many more dry holes were drilled in 1960, 1965, and 1970 than in 1975, 1978 and 1979? ([link])



For the following problems, replace the \square with the whole number that will make the subtraction true. **Exercise:**

14

Problem: -□

3

Solution:

11

Exercise:

21

Problem: -□

14

Exercise:

35

Problem: -□

25

Exercise:
51
Solution:
Problem: Subtract 37 from the difference between 263 and 175.
Exercise:
Problem: Subtract 1,127 from the sum of 2,161 and 387.
Exercise:
76
Solution:
Problem: Subtract 105 from the sum of 92 and 89.
Exercise:
Problem: Subtract 42 from the sum of 16 and 56.
For the following problems, find the solutions. Exercise:
12
Solution:
16
Problem: -□
28
Exercise:
9
Problem: -□
16
Exercise:
10
Solution:

Problem: Subtract 1,109 from the difference between 3,046 and 920.
Exercise:
Problem:
Add the difference between 63 and 47 to the difference between 55 and 11.
Solution:
60
Exercise:
Problem:
Add the difference between 815 and 298 to the difference between 2,204 and 1,016.
Exercise:
Problem:
Subtract the difference between 78 and 43 from the sum of 111 and 89.
Solution:
165
Exercise:
Problem:
Subtract the difference between 18 and 7 from the sum of the differences between 42 and 13, and 81 and 16.
Exercise:
Problem:
Find the difference between the differences of 343 and 96, and 521 and 488.
Solution:
214
Exercises for Review
Exercise:

([link]) In the number 21,206, how many hundreds are there?

Exercise:

Problem:

([link]) Write a three-digit number that has a zero in the ones position.

Solution:

330 (answers may vary)

Exercise:

Problem: ([link]) How many three-digit whole numbers are there?

Exercise:

Problem: ([link]) Round 26,524,016 to the nearest million.

Solution:

27,000,000 **Exercise:**

Problem: ([<u>link</u>]) Find the sum of 846 + 221 + 116.