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**Course: Computer Vision (EE 417) Lab** 

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# INTRODUCTION

A camera, in the most basic terms, is a sensor that takes images from a 3D scene. There are myriads of camera types available today; however, they all rely on the basic principle. Mathematically, a camera could be modeled as a projective camera to get an idea of the processes involved in image formation. In modern cameras, contrary to the pinhole applied in projective cameras, lenses are used to focus images on the image plane. This brings us to the notion of perspective cameras.

Today's lab focuses on the fundamental approach to calibrating a camera. Camera calibration entails estimating the intrinsic and extrinsic parameters of a camera. Intrinsic or internal parameters include the effective focal lengths,  $\alpha$  and  $\beta$ , which are scaled versions of the camera's focal length, the coordinates of the principal point  $u_0$  and  $v_0$ , the skew parameter, s, and the radial distortion  $\kappa_1$ .

# EXPLANATION OF METHODS

### **Line Detection Using Hough Transform**

Firstly, a calibration pattern or rig is created. This could be made from a checkerboard image. In the most ideal case, 3 patterns aligned with arbitrary x-, y-, and z- axes are used. However, 2 planes are enough to form the calibration rig. A photo of the above pattern is taken and used as the input photo for the calibration process.

Whilst data generation is the focal point of this lab, a key task is to compare two methods of corner extraction – Harris corners and subpixel accuracy using Hough Transform for lines. Edge images are used in both cases.

For the line detection using Hough Transform, the underlying is as follows:

$$\rho = x \cos\theta + y \sin\theta$$
 ----(1)

A point in the image space translates to a sinusoid in the parameter/accumulator space.

P is the length of the normal from a line to the origin.

 $\theta$  is the orientation of the line.

The Hough Transform for line detection assumes a parameter space where the rho and theta are fixed and a point in the image space represents a sinusoid in the parameter space.

The function looks at collinear points in the image space and finds the number of times their corresponding sinusoids in the parameter space intersect. With the help of a threshold, any cumulative intersection that exceeds the threshold qualifies to be a line.

Once the lines are detected, with the aid of the naked eyes, intersecting lines are identified. These lines are highlighted alongside

their points of intersection (by solving the two line-equations resulting from equation (1) using matrices and matrix inverse).

For the sake of comparison, corners are also extracted using Harris detector. The aim of this is to confirm that the sub-pixel accuracy offered by Hough Transform performs significantly better at identifying the exact location of corners.

In MATLAB, a script was written to perform the above tasks. This can be found in the appendix.

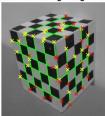
## **IN-LAB RESULTS**

### **Line Detection**

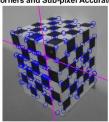
Original Image



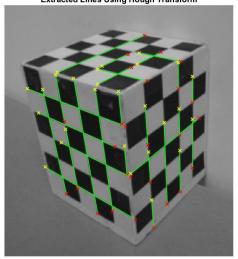
**Extracted Lines Using Hough Transform** 



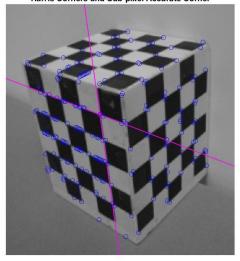
Harris Corners and Sub-pixel Accurate Corner



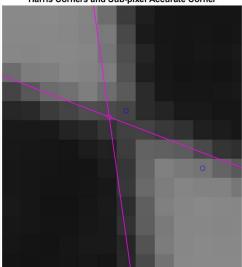
**Extracted Lines Using Hough Transform** 



Harris Corners and Sub-pixel Accurate Corner



Harris Corners and Sub-pixel Accurate Corner



The coordinates of the lines are:

Line 1: (x,y) = (36,129) and (187,187)Line 2: (x,y) = (140,130) and (167,317)

Harris corner

Chosen point: (146,170)

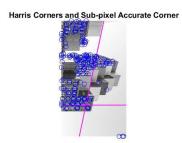
Real corner (by observation): (145.5, 170.5).

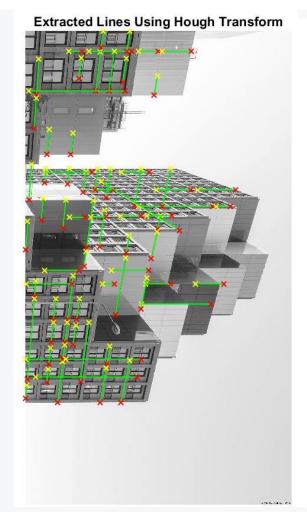
The distance between the Hough corner and the real corner is 0.4254. The distance between the Harris corner and the real corner is 0.7071.

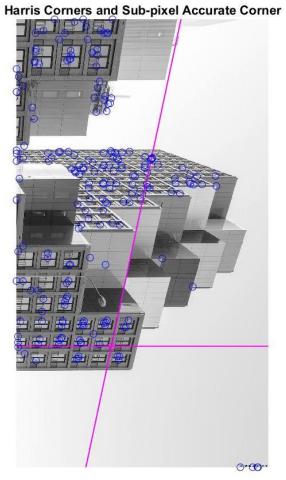
## **MULTIPLE RESULTS**

Using Hough Transform for line detection to obtain sub-pixel accurate corners in a random image;







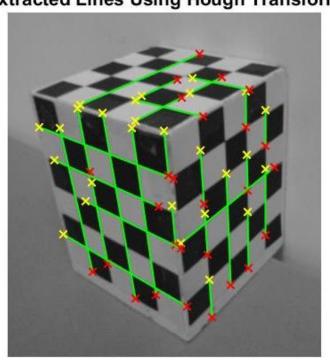


The coordinates of the lines are:

Line 1: (x,y) = (353,1113) and (323,1255)Line 2: (x,y) = (35,1169) and (378,1169)

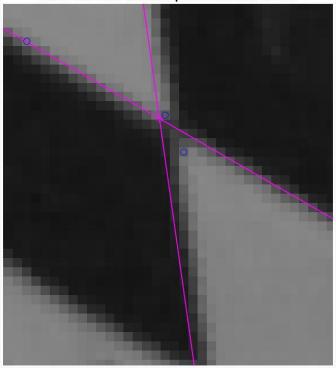
Going back to our original calibration image and changing the edge detector to 'zerocross', the following new corner was extracted:

**Extracted Lines Using Hough Transform** 



Harris Corners and Sub-pixel Accurate Corner

Harris Corners and Sub-pixel Accurate Corner



The coordinates of the lines are:

Line 1: (x,y) = (140,130) and (167,317)Line 2: (x,y) = (63,249) and (202,330)

Harris corner

Chosen point: (165,307)

Real corner (by observation): (165.5, 309.5).

The distance between the Hough corner and the real corner is 2.4096. The distance between the Harris corner and the real corner is 2.5495.

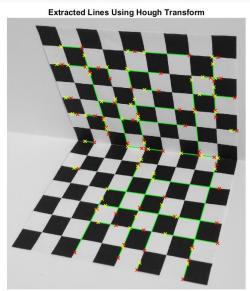
# POST-LAB QUESTIONS

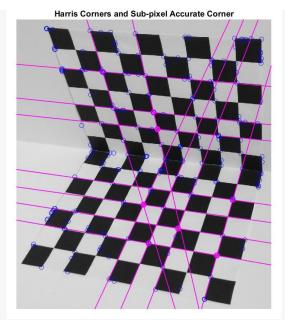
For the post-lab, the following calibration rig was created:



To extract the corners, the Laplacian of gaussian edge detector was applied first:







Eight corners were extracted using the Hough Transform. The equations of the 16 lines are as follows:

```
x_1 \cdot \cos(-1.4137) + y_1 \cdot \sin(-1.4137) = -1486
x_2 \cdot \cos(0.3665) + y_2 \cdot \sin(0.3665) = 1853
x_3 \cdot \cos(-1.43117) + y_3 \cdot \sin(-1.43117) = -1360
 x_4 \cdot \cos(0.4363) + y_4 \cdot \sin(0.4363) = 1.588e + 03
   x_5.\cos(-1.396) + y_5.\sin(-1.396) = -1621
   x_6.\cos(0.27925) + y_6.\sin(0.27925) = 2115
  x_7.\cos(-1.4661) + y_7.\sin(-1.4661) = -641
    x_8 \cdot \cos(-0.2269) + y_8 \cdot \sin(-0.2269) = 926
 x_9.\cos(-1.48353) + y_9.\sin(-1.48353) = -366
  x_{10}. \cos(-0.2967) + y_{10}. \sin(-0.2967) = 552
 x_{11} \cdot \cos(-1.4312) + y_{11} \cdot \sin(-1.4312) = -1219
     x_{12} \cdot \cos(0.279) + y_{12} \cdot \sin(0.279) = 2115
  x_{13} \cdot \cos(-1.396) + y_{13} \cdot \sin(-1.396) = -1621
x_{14} \cdot \cos(0.4014) + y_{14} \cdot \sin(0.4014) = 1.717e + 03
 x_{15}. \cos(-1.4661) + y_{15}. \sin(-1.4661) = -774
  x_{16} \cdot \cos(-0.2269) + y_{16} \cdot \sin(-0.2269) = 926
```

The coordinates of the end points of the above lines are:

```
Line 1: (x,y) = (863,1642), (1472,1739)

Line 2: (x,y) = (1357,1638), (1204,2037)

Line 3: (x,y) = (1386,1891), (1845,1972)

Line 4: (x,y) = (1822,1325), (1626,2008)

Line 5: (x,y) = (880,1498), (1380,1568)

Line 6: (x,y) = (1147,1300), (954,1714)

Line 7: (x,y) = (1086,760), (1513,804)

Line 8: (x,y) = (1116,712), (1146,846)

Line 9: (x,y) = (700,430), (803,439)

Line 10: (x,y) = (679,329), (723,476)

Line 11: (x,y) = (1279,1412), (1861,1493)

Line 12: (x,y) = (1822,1325), (1626,2008)
```

```
Line 13: (x,y) = (1080,1837), (1200,1858)
Line 14: (x,y) = (1129,1739), (1025,1984)
Line 15: (x,y) = (1153,900), (1563,943)
Line 16: (x,y) = (1150,863), (1199,1072)
```

• The intersection of lines 13 and 14 was compared with the closest Harris corner in terms of their distance to the real corner:

Harris corner

Chosen point: (1086,1838)

Real corner (by observation): (1085.5, 1837.5).

The distance between the Hough corner and the real corner is 0.1836. The distance between the Harris corner and the real corner is 0.7071.

#### Discussion

The discussion can be found in the following section.

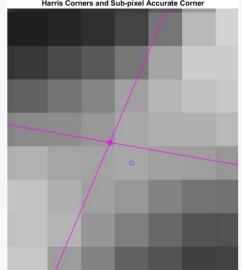
## **DISCUSSION**

### **General Comments on Hough Transform**

- The main idea is that we map a difficult pattern problem into a simple peak detection problem.
- Hough Transform is a really robust algorithm as it could be used to detect any curve that can be expressed in parametric form.

$$Y = f(x, a1,a2,...ap)$$
  
a1, a2, ... ap are the parameters

- I noticed that the number of lines extracted is really dependent on the edge detection algorithm used and also on the orientation of the image.
- In some cases, editing the photo and changing its orientation changed my results significantly.
- Corner extraction using Hough lines is significantly more accurate than using Harris corners. This can be credited to the fact that while Harris corners is restricted to integer corners, intersection of Hough lines allows for sub-pixel accuracy.



 As seen in the above picture, the real corner falls on the border (to the right) of the within which the Hough intersection corner lies, while the Harris corner is at half a pixel diagonal distance away. • Hough transform is robust to undesirable features like noise, occlusion, etc.

# Which Corner Extraction Method Would I Prefer for Camera Calibration?

• I would opt for the corners extracted using the intersection of Hough lines due to the sub-pixel accuracy it affords us.

#### **Better Methods**

- As found in the literature, we can apply certain methods to already existing corner detectors (like Harris, Kanade-Tomasi, etc.) to achieve sub-pixel accuracy.
- These methods can be classified into interpolation method, classification method, and fitting method.
- The fitting method is widely used in the field of digital image processing. There are various kinds of surface fitting, such as Gaussian surface fitting, polynomial fitting method, ellipse fitting method. Gaussian surface fitting algorithm is particularly common in achieving positioning precision at level of the subpixel.

## **REFERENCES**

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https://www.designtrends.com/graphic-web/patterns/circle-pattern.html

https://www.dreamstime.com/seamless-abstract-pattern-black-overlapping-circles-dots-bubbles-bulbs-background-vector-illustration-calm-classic-design-image185529825

https://download.atlantis-press.com/article/25866.pdf

## **APPENDIX**

#### **CAMERA CALIBRATION**

A single m-file was created for this lab:

#### In-Lab Code

```
clear all; close all; clc;
% Read the image to be preprocessed
im = imread('calibrationObject.png');
tic
% The row, column, and channels of the image are obtained
along with the cardinality of the image.
[r, c, ch] = size(im);
Card = r*c;
img = im;
% This is added in case the image introduced is an RGB
% It functions to convert it to a gray-scale image.
if ch == 3
    img = rgb2gray(img);
end
% Create a Black-white Edge Image
I = edge(imq, 'log');
% Find the space matrix, theta, and rho parameters.
[H,T,R] = hough(I);
% Threshold
t = 0.5*max(H(:));
% Find the peaks
P = houghpeaks(H, 20, 'threshold', t);
% Extract the line segments based on Hough Transform
lines = houghlines(I,T,R,P,'FillGap',10,'MinLength',40);
figure %1
```

```
subplot(2,2,1)
imshow(im)
title('Original Image')
subplot(2,2,2)
imshow(img), hold on
title ('Extracted Lines Using Hough Transform')
for k = 1:length(lines)
    % Obtain the endpoints of each line found earlier.
   xy = [lines(k).point1; lines(k).point2];
   plot(xy(:,1),xy(:,2),'LineWidth',1,'Color','green');
   % Plot beginnings and ends of lines
plot(xy(1,1),xy(1,2),'x','LineWidth',1,'Color','yellow');
   plot(xy(2,1),xy(2,2),'x','LineWidth',1,'Color','red');
   % r = lines(k).theta
   %a = xy(1), b = xy(2), c = xy(3), d = xy(4)
   if xy(1) == 140 \&\& xy(3) == 130 \&\& xy(2) == 167 \&\& xy(4)
== 317
       k1 = k;
       rho1 = lines(k).rho;
       theta1 = lines(k).theta*pi/180;
   end
   if xy(1) == 36 \&\& xy(3) == 129 \&\& xy(2) == 187 \&\& xy(4)
== 187
       k2 = k;
       rho2 = lines(k).rho;
       theta2 = lines(k).theta*pi/180;
   end
end
x1 = 0:1:386;
y1 = (rho1 - x1*cos(theta1))/sin(theta1);
x2 = 0:1:386;
y2 = (rho2 - x2*cos(theta2))/sin(theta2);
subplot(2,2, [3,4])
```

```
imshow(img), hold on
title ('Harris Corners and Sub-pixel Accurate Corner')
plot(x1, y1, 'LineWidth', 1, 'Color', 'magenta');
plot(x2, y2,'LineWidth',1,'Color','magenta');
% Find the intersection point of the two lines
intersection = [cos(theta1) sin(theta1); cos(theta2)
sin(theta2) \ \ \ [rho1; rho2];
% Plot the intersection point
plot(intersection(1), intersection(2), 'mo')
% Find the Harris corners
C H = corner(I, 'Harris');
plot(C_H(:,1),C H(:,2),'bo');
% Find the distance between the two points
len = norm(intersection - [145.5; 170.5]);
len2 = norm([146; 170] - [145.5; 170.5]);
fprintf('The distance between the Hough corner and the real
corner is %.4f.\n', len)
fprintf('The distance between the Harris corner and the
real corner is %.4f.\n', len2)
toc
```

#### **Post-Lab Code**

```
clear all; close all; clc;
% Read the image to be preprocessed
im = imread('3D_Calibration_Object.jpg');
tic
% The row, column, and channels of the image are obtained
along with the cardinality of the image.
[r, c, ch] = size(im);
Card = r*c;
img = im;
```

```
% This is added in case the image introduced is an RGB
image.
% It functions to convert it to a gray-scale image.
if ch == 3
    img = rgb2gray(img);
end
% Create a Black-white Edge Image
I = edge(img, 'log');
% Find the space matrix, theta, and rho parameters.
[H,T,R] = hough(I);
% Threshold
t = 0.5*max(H(:));
% Find the peaks
P = houghpeaks(H, 20, 'threshold', t);
% Extract the line segments based on Hough Transform
lines = houghlines(I,T,R,P,'FillGap',10,'MinLength',40);
figure %1
subplot(1,2,1)
imshow(im)
title('Original Image')
subplot(1,2,2)
imshow(imq), hold on
title ('Extracted Lines Using Hough Transform')
for k = 1:length(lines)
    % Obtain the endpoints of each line found earlier.
   xy = [lines(k).point1; lines(k).point2];
   plot(xy(:,1),xy(:,2),'LineWidth',1,'Color','green');
   % Plot beginnings and ends of lines
plot(xy(1,1),xy(1,2),'x','LineWidth',1,'Color','yellow');
   plot(xy(2,1),xy(2,2),'x','LineWidth',1,'Color','red');
   % Obtain rho and theta for the two intersecting lines
   % First intersection
```

```
if xy(1) == 863 \&\& xy(3) == 1642 \&\& xy(2) == 1472 \&\&
xy(4) == 1739
       k1 = k;
   elseif xy(1) == 1357 \&\& xy(3) == 1638 \&\& xy(2) == 1204
&& xy(4) == 2037
      k2 = k;
   end
   if xy(1) == 880 \&\& xy(3) == 1498 \&\& xy(2) == 1380 \&\&
xy(4) == 1568
       k3 = k;
   elseif xy(1) == 1147 \&\& xy(3) == 1300 \&\& xy(2) == 954 \&\&
xy(4) == 1714
      k4 = k;
   end
   if xy(1) == 1386 \&\& xy(3) == 1891 \&\& xy(2) == 1845 \&\&
xy(4) == 1972
      k5 = k;
   elseif xy(1) == 1822 \&\& xy(3) == 1325 \&\& xy(2) == 1626
&& xy(4) == 2008
   k6 = k;
   end
   if xy(1) == 1086 \&\& xy(3) == 760 \&\& xy(2) == 1513 \&\&
xy(4) == 804
      k7 = k;
   elseif xy(1) == 1116 \&\& xy(3) == 712 \&\& xy(2) == 1146 \&\&
xy(4) == 846
      k8 = k;
   if xy(1) == 700 \&\& xy(3) == 430 \&\& xy(2) == 803 \&\& xy(4)
== 439
       k9 = k;
   elseif xy(1) == 679 \&\& xy(3) == 329 \&\& xy(2) == 723 \&\&
xy(4) == 476
     k10 = k;
   end
   if xy(1) == 1279 \&\& xy(3) == 1412 \&\& xy(2) == 1861 \&\&
xy(4) == 1493
       k11 = k;
   elseif xy(1) == 1822 \&\& xy(3) == 1325 \&\& xy(2) == 1626
&& xy(4) == 2008
       k12 = k;
```

```
end
   if xy(1) == 1080 \&\& xy(3) == 1837 \&\& xy(2) == 1200 \&\&
xy(4) == 1858
      k13 = k;
   elseif xy(1) == 1129 \&\& xy(3) == 1739 \&\& xy(2) == 1025
&& xy(4) == 1984
       k14 = k;
   end
   if xy(1) == 1153 \&\& xy(3) == 900 \&\& xy(2) == 1563 \&\&
xy(4) == 943
       k15 = k;
   elseif xy(1) == 1150 \&\& xy(3) == 863 \&\& xy(2) == 1199 \&\&
xy(4) == 1072
      k16 = k;
   end
end
% Parameters for the 16 Lines
rho1 = lines(k1).rho; theta1 = lines(k1).theta*pi/180; %
Line 1
rho2 = lines(k2).rho; theta2 = lines(k2).theta*pi/180; %
Line 2
rho3 = lines(k3).rho; theta3 = lines(k3).theta*pi/180; %
Line 3
rho4 = lines(k4).rho; theta4 = lines(k4).theta*pi/180; %
Line 4
rho5 = lines(k5).rho; theta5 = lines(k5).theta*pi/180; %
Line 5
rho6 = lines(k6).rho; theta6 = lines(k6).theta*pi/180; %
Line 6
rho7 = lines(k7).rho; theta7 = lines(k7).theta*pi/180; %
Line 7
rho8 = lines(k8).rho; theta8 = lines(k8).theta*pi/180; %
Line 8
rho9 = lines(k9).rho; theta9 = lines(k9).theta*pi/180; %
Line 9
rho10 = lines(k10).rho; theta10 = lines(k10).theta*pi/180;
% Line 10
rho11 = lines(k11).rho; theta11 = lines(k11).theta*pi/180;
% Line 11
```

```
rho12 = lines(k12).rho; theta12 = lines(k12).theta*pi/180;
% Line 12
rho13 = lines(k13).rho; theta13 = lines(k13).theta*pi/180;
% Line 13
rho14 = lines(k14).rho; theta14 = lines(k14).theta*pi/180;
% Line 14
rho15 = lines(k15).rho; theta15 = lines(k15).theta*pi/180;
% Line 15
rho16 = lines(k16).rho; theta16 = lines(k16).theta*pi/180;
% Line 16
% Define the equations of each pair of lines
x1 = 0:1:2505;
y1 = (rho1 - x1*cos(theta1))/sin(theta1); y2 = (rho2 -
x1*cos(theta2))/sin(theta2);
y3 = (rho3 - x1*cos(theta3))/sin(theta3); y4 = (rho4 -
x1*cos(theta4))/sin(theta4);
y5 = (rho5 - x1*cos(theta5))/sin(theta5); y6 = (rho6 -
x1*cos(theta6))/sin(theta6);
y7 = (rho7 - x1*cos(theta7))/sin(theta7); y8 = (rho8 -
x1*cos(theta8))/sin(theta8);
y9 = (rho9 - x1*cos(theta9))/sin(theta9); y10 = (rho10 - x1*cos(theta9))/sin(theta9) + (rho10 - x1*cos
x1*cos(theta10))/sin(theta10);
y11 = (rho11 - x1*cos(theta11))/sin(theta11); y12 = (rho12)
- x1*cos(theta12))/sin(theta12);
y13 = (rho13 - x1*cos(theta13))/sin(theta13); y14 = (rho14)
- x1*cos(theta14))/sin(theta14);
y15 = (rho15 - x1*cos(theta15))/sin(theta15); y16 = (rho16)
- x1*cos(theta16))/sin(theta16);
figure %2
imshow(img), hold on
title ('Harris Corners and Sub-pixel Accurate Corner')
plot(x1, y1, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y2, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y3,'LineWidth',1,'Color','magenta');
plot(x1, y4, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y5, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y6, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y7, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y8,'LineWidth',1,'Color','magenta');
```

```
plot(x1, y9,'LineWidth',1,'Color','magenta');
plot(x1, y10, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y11, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y12, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y13,'LineWidth',1,'Color','magenta');
plot(x1, y14, 'LineWidth', 1, 'Color', 'magenta');
plot(x1, y15,'LineWidth',1,'Color','magenta');
plot(x1, y16,'LineWidth',1,'Color','magenta');
% Find the intersection point of the two lines
intersection1 = [cos(theta1) sin(theta1); cos(theta2)
sin(theta2)] \ [rho1; rho2];
intersection2 = [cos(theta3) sin(theta3); cos(theta4)
sin(theta4) \ [rho3; rho4];
intersection3 = [cos(theta5) sin(theta5); cos(theta6)
sin(theta6)] \setminus [rho5; rho6];
intersection4 = [cos(theta7) sin(theta7); cos(theta8)
sin(theta8)] \setminus [rho7; rho8];
intersection5 = [cos(theta9) sin(theta9); cos(theta10)
sin(theta10)] \ [rho9; rho10];
intersection6 = [cos(theta11) sin(theta11); cos(theta12)
sin(theta12)] \setminus [rho11; rho12];
intersection7 = [cos(theta13) sin(theta13); cos(theta14)
sin(theta14) | \ | rho13; rho14 |;
intersection8 = [cos(theta15) sin(theta15); cos(theta16)
sin(theta16) | \ [rho15; rho16];
% Plot the intersection point
plot(intersection1(1), intersection1(2), 'mo',
'LineWidth',2)
plot(intersection2(1), intersection2(2), 'mo',
'LineWidth',2)
plot(intersection3(1), intersection3(2), 'mo',
'LineWidth',2)
plot(intersection4(1), intersection4(2), 'mo',
'LineWidth',2)
plot(intersection5(1), intersection5(2), 'mo',
'LineWidth',2)
plot(intersection6(1), intersection6(2), 'mo',
'LineWidth',2)
plot(intersection7(1), intersection7(2), 'mo',
'LineWidth',2)
```

```
plot(intersection8(1), intersection8(2), 'mo',
'LineWidth',2)
% H = detectHarrisFeatures(I);
% plot(H.selectStrongest(100));
% Find the Harris corners
C H = corner(I, 'Harris');
plot(C H(:,1),C H(:,2),'bo');
% Find the distance between the two points
% The real corner from inspection lies around (1085.5,
1837.5), while the
% Harris corner lies on (1086, 1838)
len = norm(intersection7 - [1085.5; 1837.5]);
len2 = norm([1086; 1838] - [1085.5; 1837.5]);
fprintf('The distance between the Hough corner and the real
corner is %.4f.\n', len)
fprintf('The distance between the Harris corner and the
real corner is %.4f.\n', len2)
toc
```