Hw 1.

bayes theorem

$$P(w_j|x) = \frac{P(x|w_j)P(w_j)}{P(x)}$$

2.3) consider minimax critereon for the zero-one loss function ($\lambda_{11} = \lambda_{22} = 0$ and $\lambda_{12} = \lambda_{21} = 1$)

a) $$\int_{R2} p(\vec{x}|w_1)\,d\vec{x} = \int_{R1} P(\vec{x}|w_2)\,d\vec{x}$$

$$\frac{P(\vec{x}|w_1)}{P(\vec{x}|w_2)} \underset{\alpha_1}{\overset{\alpha_2}{\lessgtr}} \frac{(\overset{1}{\lambda_{12}} - \overset{0}{\lambda_{22}})\,P(w_2)}{(\underset{1}{\lambda_{21}} - \underset{0}{\lambda_{11}})\,P(w_1)}$$

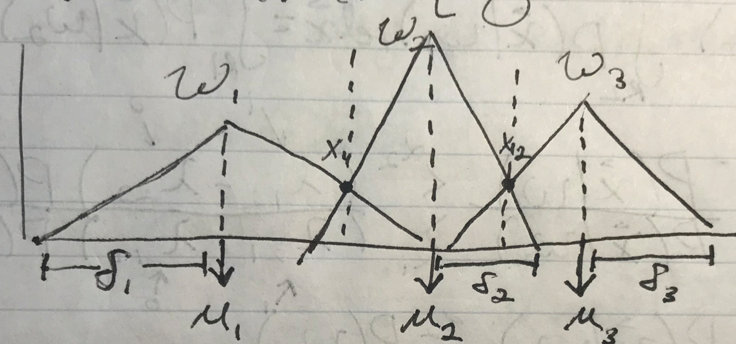
$$\frac{P(\vec{x}|w_1)}{P(\vec{x}|w_2)} \underset{\alpha_1}{\overset{\alpha_2}{\lessgtr}} \frac{P(w_2)}{P(w_1)}$$

- given the loss is 1 for either wrong choice and ϕ for either corect choice, I believe the area of the decision regions for both classes will be equally weighted by loss.
- That being said, whether they will be equal is dependant on likelihood and prior probabilities of w_1 & w_2

b) The uniqueness is dependant on the likelihood of w_1 & w_2 and the prior probabilities.

2.5) Generalize the minimax decision rule in order to classify

$$P(x|\omega_i) = T(\mu_i, \delta_i) = \begin{cases} (\delta_i - |x-\mu_i|)/\delta_i^2 & \text{for } |x-\mu_i| < \delta_i \\ 0 & \text{otherwise} \end{cases}$$



a) in terms of $P(\omega_i)$, μ_i, δ_i, find optimal decision points x_1 & x_2 under a 0-1 loss
- assuming $P(\omega_1) = P(\omega_2) = P(\omega_3)$

<u>for $x > \mu_2$</u> ... (x_2)

$$\frac{P(x|\omega_2)}{P(x|\omega_3)} \underset{\omega_2}{\overset{\omega_3}{\lessgtr}} \frac{(\cancel{\lambda_{23}} - \cancel{\lambda_{33}})\cancel{P(\omega_3)}}{(\cancel{\lambda_{32}} - \cancel{\lambda_{22}})\cancel{P(\omega_2)}}$$

$\lambda_{23} = 1$, $\lambda_{33} = 0$, $\lambda_{32} = 1$, $\lambda_{22} = 0$

under a 0-1 loss
- assuming $P(\omega_1) = P(\omega_2) = P(\omega_3)$

for $x > \mu_2$ ... (x_2)

$$\frac{P(x|\omega_2)}{P(x|\omega_3)} \underset{>_{\alpha_2}}{\overset{<^{\alpha_3}}{}} \frac{(\overset{1}{\cancel{\lambda_{23}}} - \overset{0}{\cancel{\lambda_{33}}})\,P(\cancel{\omega_3})}{(\underset{1}{\cancel{\lambda_{32}}} - \underset{0}{\cancel{\lambda_{22}}})\,P(\cancel{\omega_2})}$$

$$\frac{(\delta_2 - |x-\mu_2|)/\delta_2^2}{(\delta_3 - |x-\mu_3|)/\delta_3^2} \underset{>_{\alpha_2}}{\overset{<^{\alpha_3}}{}} \frac{1}{1}$$

for $x < \mu_2$ ... (x_1)

$$\frac{P(x|\omega_1)}{P(x|\omega_2)} \underset{>_{\alpha_1}}{\overset{<^{\alpha_2}}{}} \frac{(\cancel{\lambda_{12}} - \cancel{\lambda_{22}})\,P(\cancel{\omega_2})}{(\cancel{\lambda_{21}} - \cancel{\lambda_{11}})\,P(\cancel{\omega_1})}$$

$$\frac{(\delta_1 - |x-\mu_1|)/\delta_1^2}{(\delta_2 - |x-\mu_2|)/\delta_2} \underset{>_{\alpha_1}}{\overset{<^{\alpha_2}}{}} 1$$

8) Generally, the decision rule wants you to minimize the loss/risk and maximize the probability. With two points, you want to be as far away as possible from x_1 & x_2, but close to any of the μ_i expected vals

μ, δ

c) $\{0, 1\}$ 1 $\quad (\delta_i - |x - \mu_i|)/\delta_i^2 \Rightarrow (1-|x|)/1$

$\{.5, .5\}$ 2 $\quad \Rightarrow (.5-|x-.5|)/.25$

$\{1, 1\}$ 3 $\quad \Rightarrow (1-|x-1|)$

for x_1: $\dfrac{(1-|x|)4}{(.5-|x-.5|)} \mathop{\gtrless}^{\alpha_2}_{\alpha_1} 1 \Rightarrow 1-|x| \mathop{\gtrless}^{\alpha_2}_{\alpha_1} 4(.5-|x-.5|)$

$= 1-|x| \mathop{\gtrless}^{\alpha_2}_{\alpha_1} 2-|4x-2|$

for x_2: $\dfrac{(.5-|x-.5|)/.25}{(1-|x-1|)} \mathop{\gtrless}^{\alpha_3}_{\alpha_2} 1 \Rightarrow (.5-|x-.5|) \mathop{\gtrless}^{\alpha_3}_{\alpha_2} \dfrac{(1-|x-1|)}{4}$

? I'm so lost...

d) I have no idea what a minimax risk is. Assuming its the $R(\alpha_i|x)$ formula we talked about in class, it would be 3 sums, each having the likelihood $\lambda_{ij} P(x|w_j)$ for the two incorrect actions, for each action.

d) I have no idea what a minimax risk is. Assuming its the $R(\alpha_i|x)$ formula we talked about in class, it would be 3 sums, each having the likelihood $\lambda_{ij} P(x|w_j)$ for the two incorrect actions, for each action.

2.7) consider two Cauchy distributions

$$P(x/w_i) = \left(\frac{1}{\pi b}\right) \frac{1}{1+\left(\frac{x-a_i}{b}\right)^2} \qquad i = 1, 2$$

$a_1 < a_2$, $P(w_1) = P(w_2)$, $b_1 = b_2$

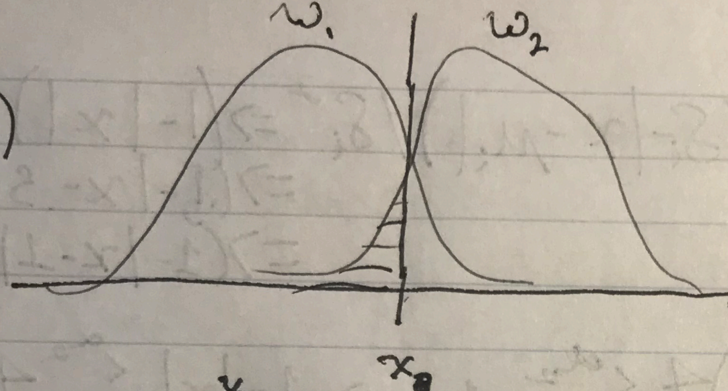
a) $$\cancel{\left(\frac{1}{\pi b}\right)} \frac{1}{1+\left(\frac{x-a_1}{b}\right)^2} = \cancel{\frac{1}{(\pi b)}} \cdot \frac{1}{1+\left(\frac{x-a_2}{b}\right)^2}$$

$$\cancel{1} + \frac{(x-a_2)^2}{b^2} = \cancel{1} + \frac{(x-a_1)^2}{b^2}$$
$$-1 \qquad\qquad -1$$

$$\cancel{x^2} - 2xa_2 + a_2^2 = \cancel{x^2} - 2xa_1 + a_1^2$$
$$-a_2^2 \qquad\qquad +2xa_1$$

$$2xa_1 - 2xa_2 = a_1^2 - a_2^2$$
$$2x(a_1 - a_2) = a_1^2 - a_2^2 \Rightarrow \boxed{x = \left(\frac{1}{2}\right)\frac{a_1^2 - a_2^2}{(a_1 - a_2)}}$$

b)



find error rate for choosing ω_2 when its ω_1.

$$\text{error} = \int_{-\infty}^{x_0} P(x|\omega_2)\,dx \qquad \text{cdf} = \frac{1}{\pi}\arctan\left(\frac{x-a_i}{b}\right)+\frac{1}{2}$$

inverse this

$$E_1 = \frac{1}{\pi}\arctan\left[\frac{x-a_i}{b}\right]+\frac{1}{2}$$
$$-\frac{1}{2}$$

$$\pi\left(E_1-\frac{1}{2}\right) = \arctan\left[\frac{x-a_i}{b}\right]$$

$$\tan\left[\pi\left(E_1-\frac{1}{2}\right)\right] = \frac{1}{b}(x-a_i)$$

$$b\tan\left[\pi\left(E_1-\frac{1}{2}\right)\right] = x-a_i$$

$$\boxed{a_1+(b)\tan\left[\pi\left(E_1-\frac{1}{2}\right)\right] = x}$$

$\tan\left[\pi\left(E_1 - \frac{1}{2}\right)\right] = \frac{1}{b}(x - a_i)$

$b \tan\left[\pi\left(E_1 - \frac{1}{2}\right)\right] = x - a_i$

$$\boxed{a_i + (b)\tan\left[\pi\left(E_1 - \frac{1}{2}\right)\right] = x}$$

c) overall? $\dfrac{P(x|\omega_1)}{P(x|\omega_2)} \underset{\alpha_3}{\overset{\alpha_1}{\lessgtr}} \dfrac{\cancel{(\lambda_{12} - \lambda_{11})}^{1}}{(\lambda_{21} - \lambda_{22})} \dfrac{\cancel{P(\omega_1)}^{1}}{\cancel{P(\omega_2)}}$

d) $\dfrac{\left(\frac{1}{\pi b}\right)\frac{1}{1+\left(\frac{x-a_1}{b}\right)^2}}{\left(\frac{1}{\pi b}\right)\frac{1}{1+\left(\frac{x-a_2}{b}\right)^2}} \underset{\alpha_1}{\overset{\alpha_2}{\lessgtr}} 1 \Rightarrow \dfrac{1+(x+1)^2}{1+(x-1)^2} \overset{\alpha_2}{\underset{\alpha_1}{\lessgtr}}$

e) it seems the error rates are similar, the only differences are what the $P(x|\omega_j)$ equals, due to the distributions