ON THE ASTIGMATISM

OF

ROWLAND'S CONCAVE GRATINGS.

BY DR. J. L. SIRKS, *Groningen.*

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By Dr. **J. L. SIRKS**, *Groningen*.

In a well-known paper of Mr. J. S. Ames, On Concave Gratings for Optical Purposes [1]), the following passage occurs. „Owing to „the astigmatism of the grating, it is not possible to adopt the usual „method of illuminating part of the slit with the solar image and „part with the spark or arc; and so a different and far better plan „is adopted. A compound photograph of the two spectra is taken „in the following manner."

Yet this new plan, devised and executed by Professor Rowland with his wonted success, is only applicable by means of photography, as the photographs of the different spectra must be taken one after another; and if the precited statement, — which, so far as I see, neither Mr. Ames nor Prof. Rowland, at whose request he wrote, has recalled or modified — were to be accepted in its apparent purport, the beautiful instrument with which Mr. Rowland has endowed the spectroscopist would be unfit for the direct comparison of spectra from different sources by ocular observation, that was always regarded as a precious function of the dioptric spectroscope.

Fortunately however, though in the literal acceptation of the words it is useless to illuminate *part* of the slit with one source of light and *part* with another, it is certainly possible to institute the intended comparison, at least with the first and second spectra, by a slight modification of the common method: the prisms or other equivalent contrivances that are generally used to introduce lateral beams of light, need only be placed *not against the slit*, in A (Fig. 1), *but at a distance* $\varrho \sec \nu - \varrho \cos \nu$ *from the slit*, $\varrho \sec \nu$ from the grating, viz. at a point Q, being the intersection of B A and the tangent in the focus C.

[1]) Phil. Mag. XXVII, p. 381, 1889; cf. Astr. and Astro-Ph. 1892, p. 39.

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In order to demonstrate the truth of this assertion let us consider the pencil of monochromatic rays that will, after the reflection at the grating, concur in the focus at C. It may be divided into what I may be allowed to call vertical „fans'' of rays, each of them being limited by two vertical planes passing through the slit and including an infinitesimally narrow strip of the grating. Now all the rays contained in such a „fan", in order to concur at C without any difference of path, must issue from an apex situated in the line CQ, being the axis of the spherical surface, part of whose equatorial region is occupied by the grating.

On the other hand the horizontal fans of rays into which the pencil may be divided, by theory of diffraction have their apices in the slit. So *all* the rays that concur at C must have passed successively through two caustics: the one realised by the slit, the other only virtual, lying along the line GH, where it may be realised by another slit, if the source of light be placed at a sufficient distance. It will be easily seen that the length of the first caustic, the available part of the slit, is $b \times QA / QB = b \sin^2 v$, that of the second $GH = a \times QA / BA = a \operatorname{tg}^2 v$, a and b being the horizontal and vertical dimensions of the grating.

The existence of the second caustic, that is of great importance for the complete theory of the instrument, may be very simply demonstrated ad oculos by stretching a thin wire in Q along GH across an incident beam of sunlight: the result is a *perfectly defined* narrow black band passing horizontally across the field of the eyepiece. Any other horizontal strip of the field has its own conjugate horizontal strip, of a somewhat greater width, in the proportion $\varrho \sec v / \varrho$, a little above or below Q in a vertical plane passing through GH. Yet every single *point* in the strip of the field, belonging to one single λ, derives from the conjugate horizontal caustic // GH in its full *length;* conversely every *point* of a horizontal slit above or below GH has its horizontal *linear* image in the field depicted by rays of different λ's.

If the horizontal band, seen in the field, is required to have a width h, the horizontal slit in GH must be replaced by a rectangular diaphragm height $h \times QB / BC = h \sec v$, length as before $GH = a \operatorname{tg}^2 v$. At the same time the vertical slit ought to be lengthened by the quantity $h \cos v$, until it gives passage to all the rays issuing from the diaphragm that can reach the grating; so the full length becomes $h \cos v + b \sin^2 v$. All the rays that are obstructed by the diaphragm, if admitted would only tend to increase the disadvantageous illumination of the field by scattered light.

Any incident ray passing through the diaphragm *over* (under) the line CQ and through the slit will come at a focus in the *lower* (upper) half of the field. A short but rather broad prism, 2 or 3 mm. in height, placed at Q and reflecting lateral solar light, will give a narrow solar spectrum with perfectly defined edges, passing through the centre of the field; at the same time it will obstruct *none* of such rays, emanating from a sodium-flame or arc-light placed somewhere about T, as may concur in forming a sodium- or metal-spectrum in the remaining part of the field. Of course if we wish to get the metal-spectrum as bright as possible, the cone of light furnished by the condensing lens S must be wide enough to fill up the wedge formed by the rectangular diaphragm and the slit.

With the third and ulterior spectra and with a very large grating the condensing lens should be of rather great dimensions, so I think the method will only be quite applicable with the first and second spectra. I may add that probably the very best plan would be to have a bicylindrical lens, or two cylindrical lenses put crosswise, of such a curvature that both its orthogonal caustics might coincide with the above named caustics of the grating; but every different angle v or at least every successive spectrum would require its especial lens.

Through the kind permission and efficacious assistance of Professor HAGA I have been able to control the above by a provisional experiment. A narrow central band of the field on a black ground showed the first sodium-spectrum originating from a strip of mirror-glass, height 2,5 mm., placed along the caustic at Q, at 171 mm. from the slit, and upon which the light of a lateral Bunsen-flame was concentrated through a lens, $f = 150$ mm. The strip of glass just arrested the superfluous central part of a direct beam of sunlight that filled out the upper and lower parts of the field with its spectrum. The sunlight had to be passed through several layers of wire-gauze in order to bring down its intensity to that of the reflected sodium-light. Now in the compound spectrum the two positive sodium-lines ended abruptly where the negative sodium-lines began; yet two very narrow sharp black lines, about 0,1 mm. wide, separated the three contiguous spectral bands: this was occasioned by the strip of glass having been simply cut with a diamond without any ulterior grinding or polishing; so the somewhat rugged edges, while they were unable to take part in the reflection of the sodium-flame only acted as a barrier against the sunlight grazing them.

In order to try to what limit, if need be, the method can be applied, we turned the moveable girder of the spectroscope on to the last or fourth spectrum with $v = 68°$, $\sin v = 0.928$. A knitting-needle held in the horizontal caustic, that now lay at 714 cm. from the slit, was accurately represented by a narrow black line across the solar spectrum. This proves that the definition in the images of horizontal lines, produced by the vertical fans holds good even at this great angle of incidence.

I still may remark that the whole action of the hollow grating with a radius ϱ, may *for these fans* be regarded as the result of three successive operations: one being that of a first concave mirror, with a radius 2ϱ, but reduced by astigmatism to a radius $2\varrho \sec v$, that brings the incident rays to parallelism; the second that of a plane grating, which occasions the diffraction at an angle v; the third that of an other concave mirror 2ϱ, which makes the diffracted parallel rays converge into a focus. The distances and dimensions of two conjugate images may be simply calculated by the formulae for one mirror with $f = \varrho/(1 + \cos v)$, as may be proved in the following manner.

Let B K (Fig. 2) be part of a very narrow vertical strip, and B the centre of the mirror, C the centre of curvature; D and E two conjugate foci determined by their height $z_1 = \mathrm{DM}$, $z_2 = \mathrm{EL}$ over the horizontal plane LBM, by $\mathrm{BM} = \mathrm{R}$, $\mathrm{BL} = r$ and $\angle \mathrm{MBL} = v$; ϱ being the radius BC of the sphere, $\mathrm{KI} = l$.

Now with a sufficient degree of approximation we successively find

$$\mathrm{BI} = \frac{l^2}{2\varrho}, \quad \mathrm{IM} = \mathrm{R} - \frac{l^2 \cos v}{2\varrho},$$

$$\mathrm{KD}^2 = \mathrm{IM}^2 + (l - z_1)^2 = \mathrm{R}^2 - \frac{\mathrm{R}\, l^2 \cos v}{\varrho} + (l - z_1)^2$$

$$\mathrm{KD} = \mathrm{R} - \frac{l^2 \cos v}{2\varrho} + \frac{l^2 - 2\, l\, z_1 + z_1^2}{2\,\mathrm{R}};$$

consequently

$$\mathrm{KE} = r - \frac{l^2}{2\varrho} + \frac{l^2 - 2\, l\, z_2 + z_2^2}{2\, r}.$$

For the point B, $l = 0$, we have

$$-\,\mathrm{BD} = -\,\mathrm{R} \qquad\qquad - \frac{z_1^2}{2\,\mathrm{R}},$$

$$-\,\mathrm{BE} = -\,r \qquad\qquad - \frac{z_2^2}{2\, r}.$$

Hence by addition we find for the difference Δ of the two paths $DKE - DBE$

$$\Delta = l^2 \left(- \frac{1 + \cos v}{2 \varrho} + \frac{1}{2 R} + \frac{1}{2 r} \right) - l \left(\frac{z_1}{R} + \frac{z_2}{r} \right).$$

Now if indeed D and E be conjugate foci, Δ must vanish for every value of l, and both the factors included in brackets must be $= 0$. So the first factor gives for the relation of the distances

$$\frac{1}{R} + \frac{1}{r} = \frac{1 + \cos v}{\varrho}$$

as with a mirror of $\varrho/(1 + \cos v)$ focus; the second makes

$$\frac{z_1}{z_2} = - \frac{R}{r}$$

so that the heights of the images are proportional to the distances as in common optics.

I think I have shown that the astigmatism of the grating, while securing to the instrument some precious qualities, is no impediment against a method of observation that seems to be reputed incompatible with astigmatism. On the other hand the valued quality of the concave grating, that it shows no dust-lines, and that the image of a star or a spark on the slit is broadened out into a band, may be imparted to a dioptric spectroscope by giving a slight convex spherical curvature to one side of one of the prisms, so that the instrument becomes slightly astigmatic.

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Fig. 1.

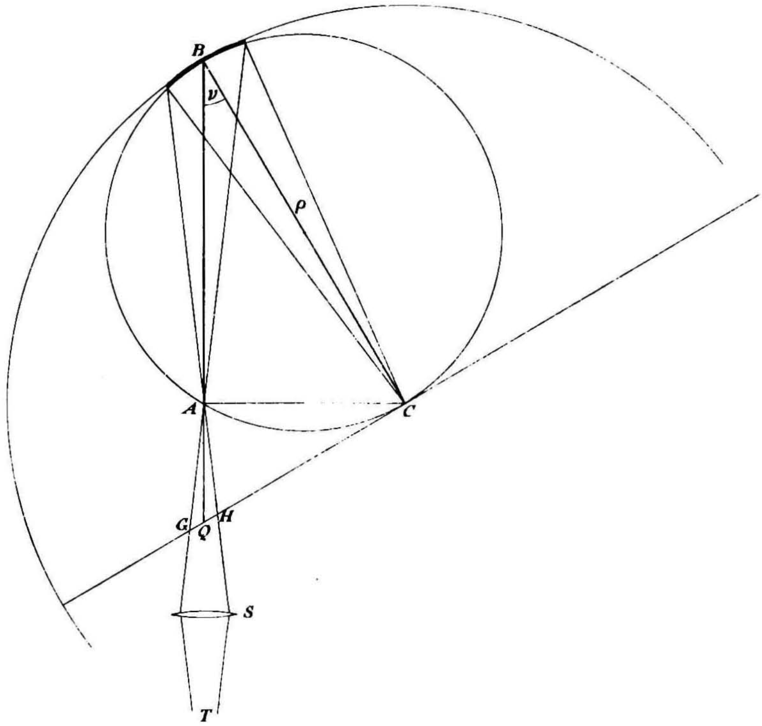
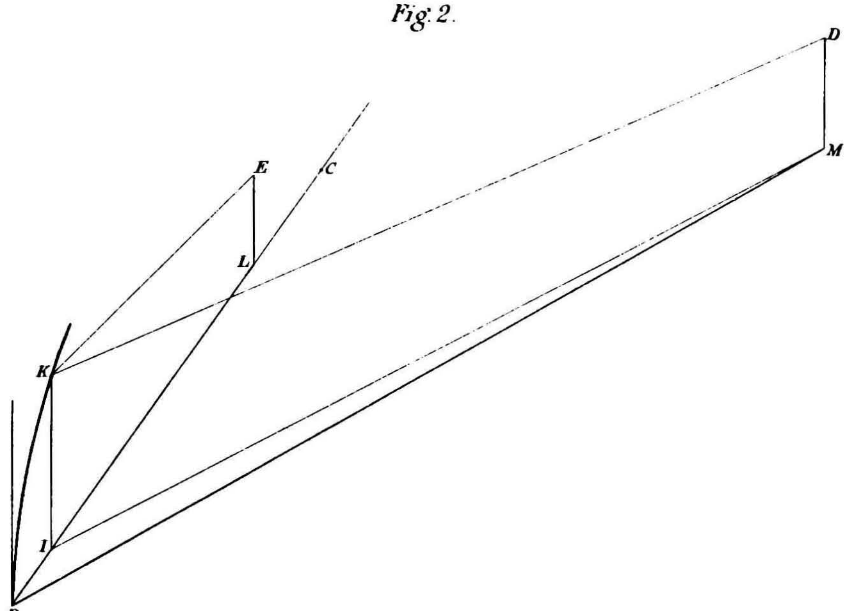


Fig. 2.



J. Bijtel, lith. Leiden.