# VIABLE MINIMAL-KERNEL CANDIDATES (all assume β / ann / id / void rules already fixed)

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1. "O-4" (4 OPERATORS) Core: void, delta, integrate, merge Extra rewrites: none How to get arithmetic: Church-style δ-chains, iteration via self-applied integrate/merge gadgets (no dedicated recursor). Equality: encode by exhaustive δ-unrolling inside a merge loop. Risks: • Strong-normalisation doubtful once self-application is allowed. • No clean internal equality ⇒ diagonal construction becomes meta only. Status: adequate for Robinson-Q-level arithmetic; Gödel I achievable meta-theoretically, Gödel II unlikely without extra operator.
2. "O-5A" (5 OPERATORS: recursion) Core: void, delta, integrate, merge, recΔ New rules: $Rrec_0$ recΔ b s void → b $Rrec_1$ recΔ b s (delta n) → s (recΔ b s n) Arithmetic & bounded search become primitive-recursive inside the system. Equality: still needs meta traversal → diagonal inside still blocked. SN proof: by δ-height measure. Confluence: trivial new peaks (recΔ vs ann). Suitability: Gödel I can be completed once an equality macro is derived (heavy). Gödel II still needs derivability conditions.
3. "O-5B" (5 OPERATORS: equality) Core: void, delta, integrate, merge, eqW New rules: $Req_1$ eqW a b → void if nf a = nf b $Req_2$ eqW a b → integrate(merge nfa nfb) otherwise (canonical order) Equality internal, diagonal easy; arithmetic forced to use very inefficient search (no recursor). SN proof: by multiset(nf-size, eqW flag). Confluence: join peaks eqW/ann, eqW/β. Suitability: Gödel I done, Gödel II needs primitive recursion for provability search → missing.
4. "O-6" (6 OPERATORS) [recommended] Core: void, delta, integrate, merge, recΔ, eqW Rules: $Rrec_0$, $Rrec_1$, $Req_1$, $Req_2$ (+ existing). Properties: • Arithmetic, bounded search, primitive recursion → recΔ • Internal decidable equality → eqW • Diagonal & $\Sigma_1$-provability internal, proofs remain SN/confluent (measures: δ-height + nf-size). Everything required for both incompleteness theorems can be carried inside the reduction system.
5. "O-6*" (keep 6, add discretionary lam/app privately) Extended lam/app only for metaprogramming; erased before extraction. Kernel unchanged; keeps axiom-free property.

– All options preserve cancellation negation provided complement-uniqueness lemma is proved.

IMMEDIATE HIGH-RISK PROOFS (uncertain even in O-6) P1 Complement-Uniqueness: ∀x y z, merge x z →* void ∧ merge y z →* void ⇒ x = y. Needed for sound negation, Boolean laws.

P2 Global Confluence with eqW + recΔ: Must enumerate and join every peak (β, ann, $rec_0$, $rec_1$, eq rules). Failure breaks Truth predicate (nf unique).

P3 Internal Derivability Conditions D1–D3: Formalise substitution inside Trace, show Prov encodes modus-ponens and $\Sigma_1$-reflection. Mandatory for Gödel II. No prior literature in a terminating TRS; feasibility open.

(if any of these three collapse, Gödel II collapses with them.)

TIMELINE ESTIMATE Optimistic 2–3 weeks is plausible only if: • you automate critical-pair generation, • re-use existing Lean termination packages, • and keep proof objects highly mechanical.

Otherwise typical completion times remain months.

NEXT ACTION Pick O-6, focus sequentially:

1. prove P2 (confluence) → ensures framework stability;
2. prove P1 (negation) → validates logic layer;
3. attack P3 (derivability) → unlocks second incompleteness.