

# Untitled

```
import OperatorKernelO6.Kernel open OperatorKernelO6 Trace Step
namespace OperatorKernelO6.Meta -- meta-layer may use Nat, tactics
def sz : Trace → Nat | void => 1 | delta t => sz t + 1 | integrate t => sz t + 1 | merge a b => sz a + sz b + 1 |
recΔ b s n => sz b + sz s + sz n + 1 | eqW a b => sz a + sz b + 1
theorem step_sz {a b} (h : Step a b) : sz b < sz a := by cases h <|> simp [sz, Nat.add_comm,
Nat.add_left_comm, Nat.add_assoc]
open WellFounded
theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure sz) := measure_wf sz
intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure sz y x := step_sz hy
exact ih _ this
end OperatorKernelO6.Meta
attribute [instance] OperatorKernelO6.Meta.strong_norm -- for future use
```

Termination.lean:20:24

Application type mismatch: In the application

WellFounded (measure sz)

the argument

measure sz

has type

WellFoundedRelation Trace : Type

but is expected to have type

?m.793 → ?m.793 → Prop : Sort (max 1 ?u.792)

Termination.lean:20:40

unknown identifier 'measure\_wf'

Termination.lean:19:49

unsolved goals

⊢ ∀ (t : Trace), Acc Step t

CONTAINS SORRY AND ADMIT

```
import OperatorKernelO6.Kernel import Mathlib.Tactic.Linarith open OperatorKernelO6 Trace Step
namespace OperatorKernelO6.Meta
def size : Trace → Nat | void      => 1 | delta t      => size t + 1 | integrate t      => size t + 1 | merge a b
=> size a + size b + 1 | recΔ b s n  => size b + size s + size n + 1 | eqW a b      => size a + size b + 1
theorem step_size {a b : Trace} (h : Step a b) : size b < size a := by cases h <|> simp [size] <|> linarith
open WellFounded
theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure size) := measure_wf size
intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure size y x := step_size
```

```

hy exact ih _ this
partial def normalize (t : Trace) : Trace := match t with | void      => void | delta u  => delta (normalize u) |
integrate u => match normalize u with | delta v => void | w      => integrate w | merge a b => let na :=
normalize a let nb := normalize b if na = void then nb else if nb = void then na else if na = nb then na else
merge na nb | recΔ b s n => match normalize n with | void      => normalize b | delta n'  => let nb :=
normalize b let ns := normalize s merge ns (normalize (recΔ nb ns n')) | n'      => recΔ (normalize b)
(normalize s) n' | eqW a b  => let na := normalize a let nb := normalize b if na = nb then void else integrate
(merge na nb) termination_by size t decreasing_by simp_wf_tac first | apply step_size | simp [size] ; linarith
theorem normalize_idempotent : ∀ t : Trace, normalize (normalize t) = normalize t := by intro t have : size
(normalize t) ≤ size t := by have h := strong_norm t cases h with | intro _ go => have rec : ∀ x, Acc Step x →
size (normalize x) ≤ size x := by intro x acc cases acc with | intro _ k => induction x with | void => simp
[normalize, size] | delta x ih => have := rec x (k _ (Step.int_delta x))? sorry admit
end OperatorKernelO6.Meta

```

```

Termination.lean:16:30 linarith failed to find a contradiction case R_rec_succ.h b† st† nt† : Trace at† : size b† +
size st† + (size nt† + 1) ≤ size st† + (size b† + size st† + size nt† + 1) ⊢ False
failed Termination.lean:16:30 linarith failed to find a contradiction case R_eq_refl.h a†¹ : Trace at† : size a†¹ ≤ 0
⊢ False
failed Termination.lean:16:30 linarith failed to find a contradiction case R_eq_diff a† b† : Trace ⊢ False
failed Termination.lean:21:24 Application type mismatch: In the application WellFounded (measure size) the
argument measure size has type WellFoundedRelation Trace : Type but is expected to have type ?m.889 → ?
m.889 → Prop : Sort (max 1 ?u.888) Termination.lean:21:42 unknown identifier 'measure_wf'
Termination.lean:20:49 unsolved goals ⊢ ∀ (t : Trace), Acc Step t Termination.lean:60:5 unknown tactic
Termination.lean:71:8 unexpected syntax failed to pretty print term (use 'set_option pp.rawOnError true' for
raw representation) Termination.lean:65:85 unsolved goals t : Trace this : size (normalize t) ≤ size t ⊢
normalize (normalize t) = normalize t Termination.lean:71:13 unexpected rec

```

```

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def sz : Trace → Nat | void => 1 | delta t => sz t + 1 | integrate t => sz t + 1 | merge a b => sz a + sz b + 1 |
recΔ b s n => sz b + sz s + sz n + 1 | eqW a b => sz a + sz b + 1
theorem step_sz {a b} (h : Step a b) : sz b < sz a := by cases h <|> simp [sz, Nat.add_comm,
Nat.add_left_comm, Nat.add_assoc]
open WellFounded
theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure sz) := measure_wf sz
intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure sz y x := step_sz hy
exact ih _ this
end OperatorKernelO6.Meta
attribute [instance] OperatorKernelO6.Meta.strong_norm -- for future use

```

All Messages (10) Termination.lean:5:2 Local variable 'void' resembles constructor  
 'OperatorKernelO6.Trace.void' - write '.void' (with a dot) or 'OperatorKernelO6.Trace.void' to use the constructor.  
 Note: This linter can be disabled with `set_option linter.constructorNameAsVariable false`  
 Termination.lean:6:2 Invalid pattern: Expected a constructor or constant marked with `[match_pattern]`  
 Hint: 'OperatorKernelO6.Trace.delta' is similar Termination.lean:13:20 declaration uses 'sorry'  
 Termination.lean:13:24 This simp argument is unused: `Nat.add_comm`  
 Hint: Omit it from the simp argument list. `simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]`  
 Note: This linter can be disabled with `set_option linter.unusedSimpArgs false` Termination.lean:13:38  
 This simp argument is unused: `Nat.add_left_comm`  
 Hint: Omit it from the simp argument list. `simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]`  
 Note: This linter can be disabled with `set_option linter.unusedSimpArgs false` Termination.lean:13:57  
 This simp argument is unused: `Nat.add_assoc`  
 Hint: Omit it from the simp argument list. `simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]`  
 Note: This linter can be disabled with `set_option linter.unusedSimpArgs false` Termination.lean:12:54  
 unsolved goals case `R_int_delta t† : Trace`  $\vdash$  False  
 case `R_merge_void_left b : Trace`  $\vdash$  False  
 case `R_merge_void_right b : Trace`  $\vdash$  False  
 case `R_merge_cancel b : Trace`  $\vdash$  False  
 case `R_rec_zero b s† : Trace`  $\vdash$  False  
 case `R_rec_succ b† s† n† : Trace`  $\vdash$  False  
 case `R_eq_refl a† : Trace`  $\vdash$  False  
 case `R_eq_diff a† b† : Trace`  $\vdash$  False Termination.lean:18:24 Application type mismatch: In the application `WellFounded (measure sz)` the argument `measure sz` has type `WellFoundedRelation Trace : Type` but is expected to have type `?m.67  $\rightarrow$  ?m.67  $\rightarrow$  Prop : Sort (max 1 ?u.66)` Termination.lean:18:40 unknown identifier 'measure\_wf' Termination.lean:17:49 unsolved goals  $\vdash \forall (t : \text{Trace}), \text{Acc Step } t$

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### Arithmetic.lean

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```
import OperatorKernelO6.Kernel
open OperatorKernelO6 Trace
namespace OperatorKernelO6
def numeral : Nat  $\rightarrow$  Trace
| 0 => void
| n + 1 => delta (numeral n)
def succ (t : Trace) : Trace := delta t
def add (m n : Trace) : Trace := rec $\Delta$  n delta m
def mul (m n : Trace) : Trace := rec $\Delta$  void (merge n) m
end OperatorKernelO6
```

All Messages (2)

Arithmetic.lean:12:40

Application type mismatch: In the application

n.recΔ delta  
the argument  
delta  
has type  
Trace → Trace : Type  
but is expected to have type  
Trace : Type  
Arithmetic.lean:14:43  
Application type mismatch: In the application  
void.recΔ n.merge  
the argument  
n.merge  
has type  
Trace → Trace : Type  
but is expected to have type  
Trace : Type