ProofSystem

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Overview

Proof system definitions and rules

Source Code

```
import OperatorKernelO6.Kernel
import OperatorKernelO6.Meta.Arithmetic
import OperatorKernelO6.Meta.Equality

open OperatorKernelO6 Trace

namespace OperatorKernelO6.Meta

-- Encode proofs as traces
inductive ProofTerm : Type
| axiom : Trace → ProofTerm
| mp : ProofTerm → ProofTerm → ProofTerm  -- Modus ponens
| gen : (Trace → ProofTerm) → ProofTerm   -- Generalization

-- Convert proof terms to traces
def proof_to_trace : ProofTerm → Trace
| ProofTerm.axiom t => t
| ProofTerm.mp p q => merge (proof_to_trace p) (proof_to_trace q)
| ProofTerm.gen f => integrate (proof_to_trace (f void))  -- Rough encoding

-- Provability predicate using bounded search via recΔ
def provable (formula : Trace) (bound : Trace) : Trace :=
  recΔ void (search_step formula) bound
where
  search_step (f : Trace) : Trace :=
    eqW f void  -- Check if formula equals void (proven)

-- Σ₁ characterization of provability
theorem provable_sigma1 (formula : Trace) :
  (∃ proof : Trace, ∃ bound : Trace,
    StepStar (provable formula bound) void) ↔
  (∃ n : Nat, ∃ proof_term : ProofTerm,
    StepStar (proof_to_trace proof_term) formula) := by
  sorry -- Complex encoding/decoding argument

-- Soundness: if provable then true (in some model)
theorem soundness (formula : Trace) :
  (∃ bound, StepStar (provable formula bound) void) →
  formula = void := by  -- void represents "true"
  sorry -- Requires model-theoretic argument

-- Consistency: not both A and ¬A are provable
theorem consistency (formula : Trace) :
  ¬(∃ b₁ b₂, StepStar (provable formula b₁) void ∧
              StepStar (provable (complement formula) b₂) void) := by
  sorry -- Follows from soundness and complement properties

-- Reflection principle encoding
def reflection (formula : Trace) : Trace :=
  eqW (provable formula (numeral 100)) formula

end OperatorKernelO6.Meta
```