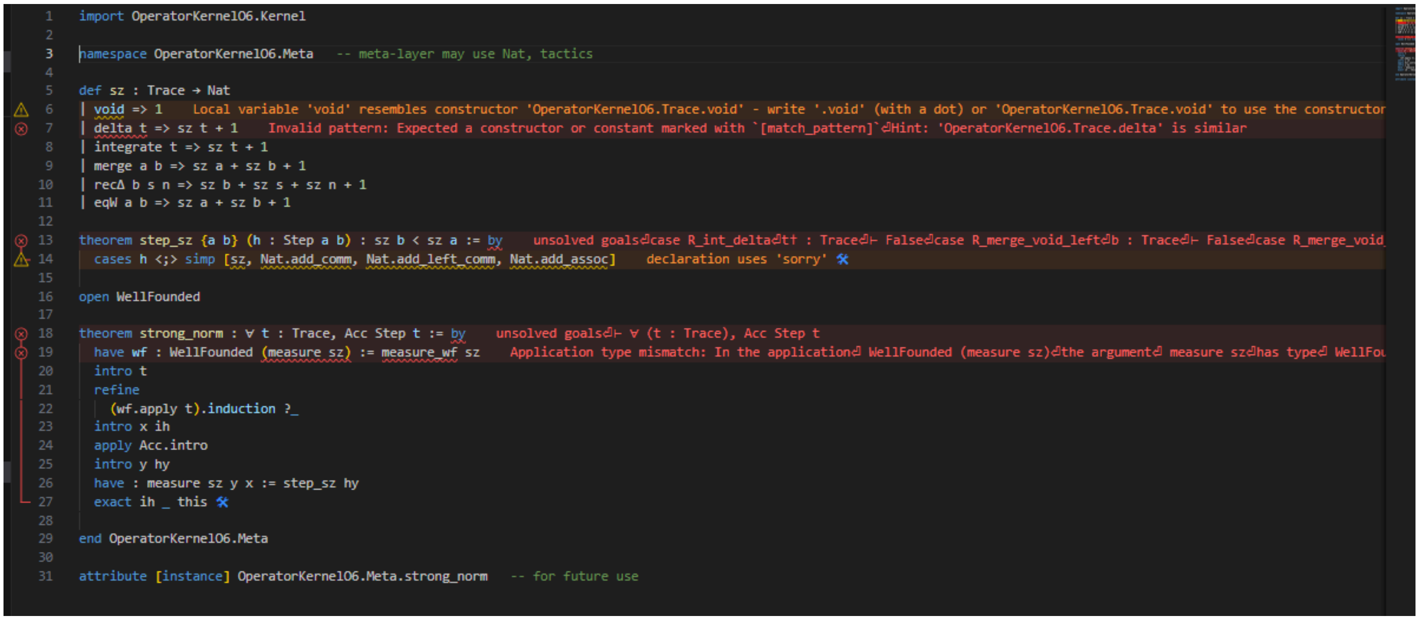
Untitled

```
import OperatorKernelO6.Kernel open OperatorKernelO6 Trace Step
namespace OperatorKernelO6.Meta  -- meta-layer may use Nat, tactics
def sz : Trace → Nat | void => 1 | delta t => sz t + 1 | integrate t => sz t + 1 | merge a b => sz a + sz b + 1 |
recΔ b s n => sz b + sz s + sz n + 1 | eqW a b => sz a + sz b + 1
theorem step_sz {a b} (h : Step a b) : sz b < sz a := by cases h <;> simp [sz, Nat.add_comm,
Nat.add_left_comm, Nat.add_assoc]
open WellFounded
theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure sz) := measure_wf sz
intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure sz y x := step_sz hy
exact ih _ this
end OperatorKernelO6.Meta
attribute [instance] OperatorKernelO6.Meta.strong_norm  -- for future use
```

```
Termination.lean:20:24
Application type mismatch: In the application
  WellFounded (measure sz)
the argument
  measure sz
has type
  WellFoundedRelation Trace : Type
but is expected to have type
  ?m.793 → ?m.793 → Prop : Sort (max 1 ?u.792)
Termination.lean:20:40
unknown identifier 'measure_wf'
Termination.lean:19:49
unsolved goals
⊢ ∀ (t : Trace), Acc Step t
```



```
import OperatorKernel06.Kernel

namespace OperatorKernel06.Meta   -- meta-layer may use Nat, tactics

def sz : Trace → Nat
| void => 1    Local variable 'void' resembles constructor 'OperatorKernel06.Trace.void' - write '.void' (with a dot) or 'OperatorKernel06.Trace.void' to use the constructor
| delta t => sz t + 1    Invalid pattern: Expected a constructor or constant marked with `[match_pattern]`⏎Hint: 'OperatorKernel06.Trace.delta' is similar
| integrate t => sz t + 1
| merge a b => sz a + sz b + 1
| recΔ b s n => sz b + sz s + sz n + 1
| eqW a b => sz a + sz b + 1

theorem step_sz {a b} (h : Step a b) : sz b < sz a := by    unsolved goals⏎case R_int_delta⏎t† : Trace⏎⊢ False⏎case R_merge_void_left⏎b : Trace⏎⊢ False⏎case R_merge_void_
  cases h <;> simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]    declaration uses 'sorry' 🛠

open WellFounded

theorem strong_norm : ∀ t : Trace, Acc Step t := by    unsolved goals⏎⊢ ∀ (t : Trace), Acc Step t
  have wf : WellFounded (measure sz) := measure_wf sz    Application type mismatch: In the application⏎ WellFounded (measure sz)⏎the argument⏎ measure sz⏎has type⏎ WellFou
  intro t
  refine
    (wf.apply t).induction ?_
  intro x ih
  apply Acc.intro
  intro y hy
  have : measure sz y x := step_sz hy
  exact ih _ this 🛠

end OperatorKernel06.Meta

attribute [instance] OperatorKernel06.Meta.strong_norm   -- for future use
```

CONTAINS SORRY AND ADMIT

import OperatorKernelO6.Kernel import Mathlib.Tactic.Linarith open OperatorKernelO6 Trace Step

namespace OperatorKernelO6.Meta

def size : Trace → Nat | void => 1 | delta t => size t + 1 | integrate t => size t + 1 | merge a b => size a + size b + 1 | recΔ b s n => size b + size s + size n + 1 | eqW a b => size a + size b + 1

theorem step_size {a b : Trace} (h : Step a b) : size b < size a := by cases h <;> simp [size] <;> linarith

open WellFounded

theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure size) := measure_wf size intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure size y x := step_size hy exact ih _ this

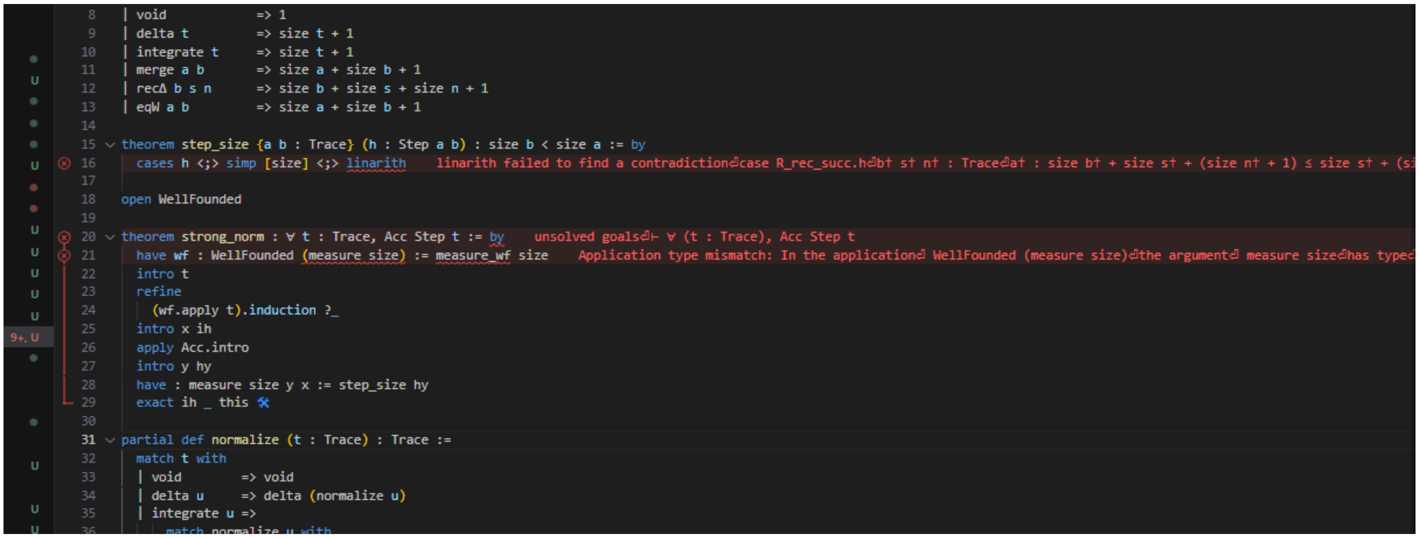
partial def normalize (t : Trace) : Trace := match t with | void => void | delta u => delta (normalize u) | integrate u => match normalize u with | delta v => void | w => integrate w | merge a b => let na := normalize a let nb := normalize b if na = void then nb else if nb = void then na else if na = nb then na else merge na nb | recΔ b s n => match normalize n with | void => normalize b | delta n' => let nb := normalize b let ns := normalize s merge ns (normalize (recΔ nb ns n')) | n' => recΔ (normalize b) (normalize s) n' | eqW a b => let na := normalize a let nb := normalize b if na = nb then void else integrate (merge na nb) termination_by size t decreasing_by simp_wf_tac first | apply step_size | simp [size] ; linarith

theorem normalize_idempotent : ∀ t : Trace, normalize (normalize t) = normalize t := by intro t have : size (normalize t) ≤ size t := by have h := strong_norm t cases h with | intro _ go => have rec : ∀ x, Acc Step x → size (normalize x) ≤ size x := by intro x acc cases acc with | intro _ k => induction x with | void => simp [normalize, size] | delta x ih => have := rec x (k _ (Step.int_delta x))? sorry admit

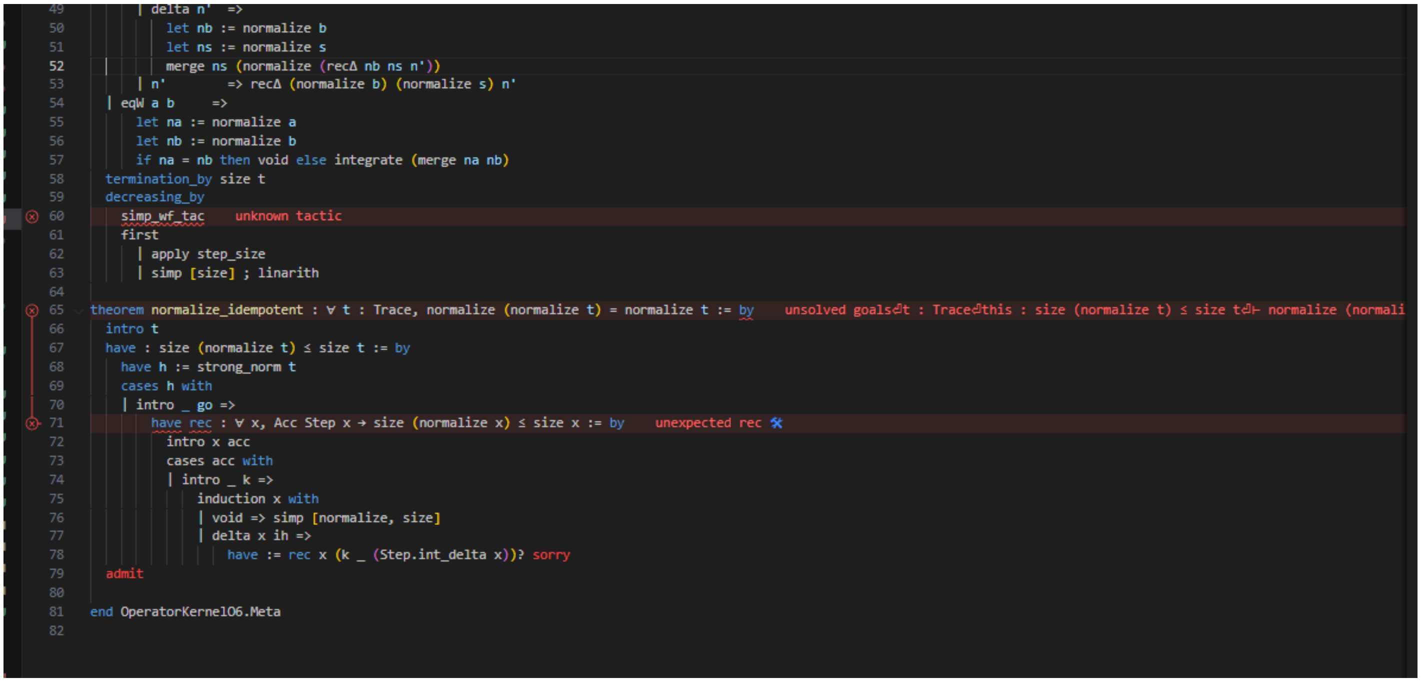
end OperatorKernelO6.Meta

Termination.lean:16:30 linarith failed to find a contradiction case R_rec_succ.h b† s† n† : Trace a† : size b† + size s† + (size n† + 1) ≤ size s† + (size b† + size s† + size n† + 1) ⊢ False

failed Termination.lean:16:30 linarith failed to find a contradiction case R_eq_refl.h a†^{1} : Trace a† : size a†^{1} ≤ 0 ⊢ False failed Termination.lean:16:30 linarith failed to find a contradiction case R_eq_diff a† b† : Trace ⊢ False failed Termination.lean:21:24 Application type mismatch: In the application WellFounded (measure size) the argument measure size has type WellFoundedRelation Trace : Type but is expected to have type ?m.889 → ?m.889 → Prop : Sort (max 1 ?u.888) Termination.lean:21:42 unknown identifier 'measure_wf' Termination.lean:20:49 unsolved goals ⊢ ∀ (t : Trace), Acc Step t Termination.lean:60:5 unknown tactic Termination.lean:71:8 unexpected syntax failed to pretty print term (use 'set_option pp.rawOnError true' for raw representation) Termination.lean:65:85 unsolved goals t : Trace this : size (normalize t) ≤ size t ⊢ normalize (normalize t) = normalize t Termination.lean:71:13 unexpected rec



```
 8    | void            => 1
 9    | delta t         => size t + 1
10    | integrate t     => size t + 1
11    | merge a b       => size a + size b + 1
12    | recΔ b s n      => size b + size s + size n + 1
13    | eqW a b         => size a + size b + 1
14
15  theorem step_size {a b : Trace} (h : Step a b) : size b < size a := by
16    cases h <;> simp [size] <;> linarith    linarith failed to find a contradiction⏎case R_rec_succ.h⏎b† s† n† : Trace⏎a† : size b† + size s† + (size n† + 1) ≤ size s† + (si
17
18  open WellFounded
19
20  theorem strong_norm : ∀ t : Trace, Acc Step t := by    unsolved goals⏎⊢ ∀ (t : Trace), Acc Step t
21    have wf : WellFounded (measure size) := measure_wf size    Application type mismatch: In the application⏎ WellFounded (measure size)⏎the argument⏎ measure size⏎has type⏎
22    intro t
23    refine
24      (wf.apply t).induction ?_
25    intro x ih
26    apply Acc.intro
27    intro y hy
28    have : measure size y x := step_size hy
29    exact ih _ this
30
31  partial def normalize (t : Trace) : Trace :=
32    match t with
33    | void        => void
34    | delta u     => delta (normalize u)
35    | integrate u =>
36        match normalize u with
```

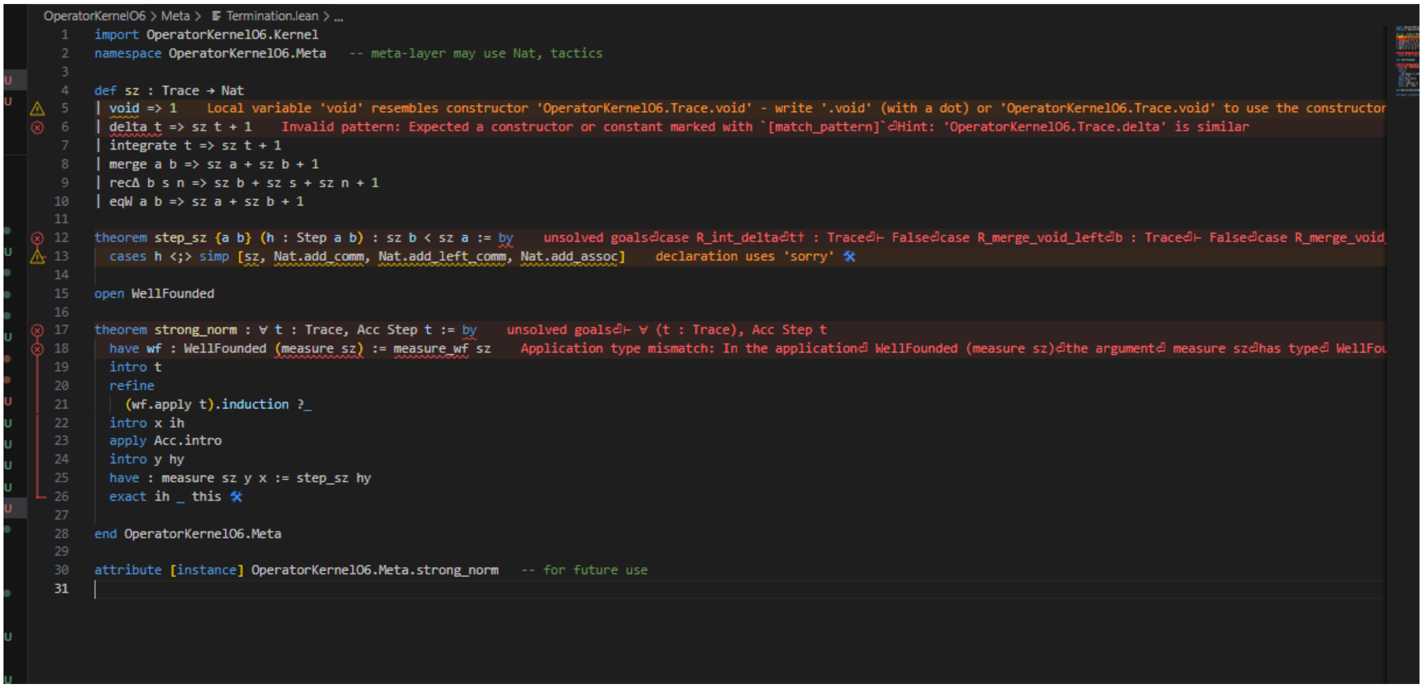


```
49        | delta n'  =>
50            let nb := normalize b
51            let ns := normalize s
52            merge ns (normalize (recΔ nb ns n'))
53        | n'        => recΔ (normalize b) (normalize s) n'
54    | eqW a b     =>
55        let na := normalize a
56        let nb := normalize b
57        if na = nb then void else integrate (merge na nb)
58    termination_by size t
59    decreasing_by
60      simp_wf_tac    unknown tactic
61      first
62        | apply step_size
63        | simp [size] ; linarith
64
65  theorem normalize_idempotent : ∀ t : Trace, normalize (normalize t) = normalize t := by    unsolved goals⏎t : Trace⏎this : size (normalize t) ≤ size t⏎⊢ normalize (normali
66    intro t
67    have : size (normalize t) ≤ size t := by
68      have h := strong_norm t
69      cases h with
70      | intro _ go =>
71          have rec : ∀ x, Acc Step x → size (normalize x) ≤ size x := by    unexpected rec
72            intro x acc
73            cases acc with
74            | intro _ k =>
75                induction x with
76                | void => simp [normalize, size]
77                | delta x ih =>
78                    have := rec x (k _ (Step.int_delta x))? sorry
79    admit
80
81  end OperatorKernel06.Meta
82
```

import OperatorKernelO6.Kernel namespace OperatorKernelO6.Meta -- meta-layer may use Nat, tactics def sz : Trace → Nat | void => 1 | delta t => sz t + 1 | integrate t => sz t + 1 | merge a b => sz a + sz b + 1 | recΔ b s n => sz b + sz s + sz n + 1 | eqW a b => sz a + sz b + 1

```
theorem step_sz {a b} (h : Step a b) : sz b < sz a := by cases h <;> simp [sz, Nat.add_comm,
Nat.add_left_comm, Nat.add_assoc]
open WellFounded
theorem strong_norm : ∀ t : Trace, Acc Step t := by have wf : WellFounded (measure sz) := measure_wf sz
intro t refine (wf.apply t).induction ?_ intro x ih apply Acc.intro intro y hy have : measure sz y x := step_sz hy
exact ih _ this
end OperatorKernelO6.Meta
attribute [instance] OperatorKernelO6.Meta.strong_norm  -- for future use
```

```
All Messages (10) Termination.lean:5:2 Local variable 'void' resembles constructor
'OperatorKernelO6.Trace.void' - write '.void' (with a dot) or 'OperatorKernelO6.Trace.void' to use the
constructor.
Note: This linter can be disabled with set_option linter.constructorNameAsVariable false
Termination.lean:6:2 Invalid pattern: Expected a constructor or constant marked with [match_pattern]
Hint: 'OperatorKernelO6.Trace.delta' is similar Termination.lean:13:20 declaration uses 'sorry'
Termination.lean:13:24 This simp argument is unused: Nat.add_comm
Hint: Omit it from the simp argument list. simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]
Note: This linter can be disabled with set_option linter.unusedSimpArgs false Termination.lean:13:38
This simp argument is unused: Nat.add_left_comm
Hint: Omit it from the simp argument list. simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]
Note: This linter can be disabled with set_option linter.unusedSimpArgs false Termination.lean:13:57
This simp argument is unused: Nat.add_assoc
Hint: Omit it from the simp argument list. simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]
Note: This linter can be disabled with set_option linter.unusedSimpArgs false Termination.lean:12:54
unsolved goals case R_int_delta t† : Trace ⊢ False
case R_merge_void_left b : Trace ⊢ False
case R_merge_void_right b : Trace ⊢ False
case R_merge_cancel b : Trace ⊢ False
case R_rec_zero b s† : Trace ⊢ False
case R_rec_succ b† s† n† : Trace ⊢ False
case R_eq_refl a† : Trace ⊢ False
case R_eq_diff a† b† : Trace ⊢ False Termination.lean:18:24 Application type mismatch: In the application
WellFounded (measure sz) the argument measure sz has type WellFoundedRelation Trace : Type but is
expected to have type ?m.67 → ?m.67 → Prop : Sort (max 1 ?u.66) Termination.lean:18:40 unknown identifier
'measure_wf' Termination.lean:17:49 unsolved goals ⊢ ∀ (t : Trace), Acc Step t
```



OperatorKernelO6 > Meta > Termination.lean > ...

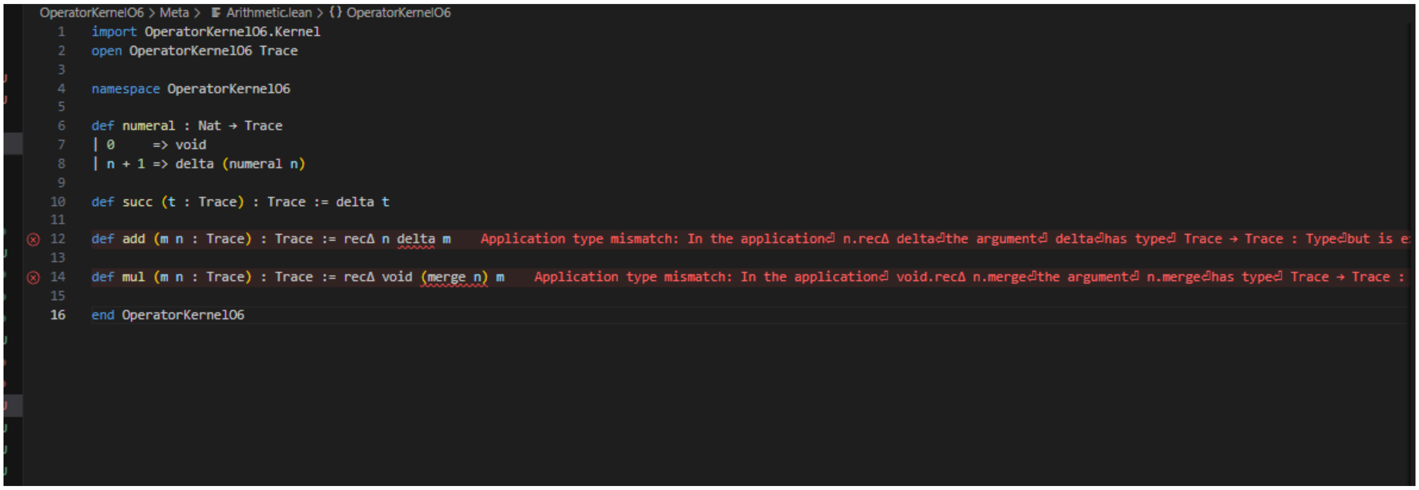
```
import OperatorKernelO6.Kernel
namespace OperatorKernelO6.Meta   -- meta-layer may use Nat, tactics

def sz : Trace → Nat
| void => 1    Local variable 'void' resembles constructor 'OperatorKernelO6.Trace.void' - write '.void' (with a dot) or 'OperatorKernelO6.Trace.void' to use the constructor
| delta t => sz t + 1    Invalid pattern: Expected a constructor or constant marked with `[match_pattern]`⏎Hint: 'OperatorKernelO6.Trace.delta' is similar
| integrate t => sz t + 1
| merge a b => sz a + sz b + 1
| recΔ b s n => sz b + sz s + sz n + 1
| eqW a b => sz a + sz b + 1

theorem step_sz {a b} (h : Step a b) : sz b < sz a := by    unsolved goals⏎case R_int_delta⏎t† : Trace⏎⊢ False⏎case R_merge_void_left⏎b : Trace⏎⊢ False⏎case R_merge_void
  cases h <;> simp [sz, Nat.add_comm, Nat.add_left_comm, Nat.add_assoc]    declaration uses 'sorry' 🛠

open WellFounded

theorem strong_norm : ∀ t : Trace, Acc Step t := by    unsolved goals⏎⊢ ∀ (t : Trace), Acc Step t
  have wf : WellFounded (measure sz) := measure_wf sz    Application type mismatch: In the application⏎ WellFounded (measure sz)⏎the argument⏎ measure sz⏎has type⏎ WellFou
  intro t
  refine
    (wf.apply t).induction ?_
  intro x ih
  apply Acc.intro
  intro y hy
  have : measure sz y x := step_sz hy
  exact ih _ this 🛠

end OperatorKernelO6.Meta

attribute [instance] OperatorKernelO6.Meta.strong_norm   -- for future use
```

Arithmetic.lean

```
import OperatorKernelO6.Kernel
open OperatorKernelO6 Trace
namespace OperatorKernelO6
def numeral : Nat → Trace
| 0     => void
| n + 1 => delta (numeral n)
def succ (t : Trace) : Trace := delta t
def add (m n : Trace) : Trace := recΔ n delta m
def mul (m n : Trace) : Trace := recΔ void (merge n) m
end OperatorKernelO6
```

```
All Messages (2)
Arithmetic.lean:12:40
Application type mismatch: In the application
  n.recΔ delta
the argument
  delta
has type
  Trace → Trace : Type
but is expected to have type
  Trace : Type
Arithmetic.lean:14:43
Application type mismatch: In the application
  void.recΔ n.merge
the argument
  n.merge
```

has type
  Trace → Trace : Type
but is expected to have type
  Trace : Type



OperatorKernelO6 > Meta > Arithmetic.lean > {} OperatorKernelO6

```
import OperatorKernelO6.Kernel
open OperatorKernelO6 Trace

namespace OperatorKernelO6

def numeral : Nat → Trace
| 0     => void
| n + 1 => delta (numeral n)

def succ (t : Trace) : Trace := delta t

def add (m n : Trace) : Trace := recΔ n delta m    Application type mismatch: In the application⏎ n.recΔ delta⏎the argument⏎ delta⏎has type⏎ Trace → Trace : Type⏎but is e

def mul (m n : Trace) : Trace := recΔ void (merge n) m    Application type mismatch: In the application⏎ void.recΔ n.merge⏎the argument⏎ n.merge⏎has type⏎ Trace → Trace :

end OperatorKernelO6
```