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Final Year Project

Earnings Events and Market Microstructure Noise: Fitting Parametric Models to Statistical Bases

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Declaration of Authorship

The research reported here is an original work that has not been published for any other purposes.

Abstract

We study the characteristics of market Microstructure Noise (MSN), the discrepancy between observed and efficient prices because of trading mechanisms, specifically around earnings dates. We compare the performance of established parametric and statistical volatility estimates using across various periods using real-world market data. Our findings indicate that certain parametric microstructure models can reliably capture substantial portions of statistically labelled noise, suggesting their potential for MSN corrections under changing market conditions. We also uncover a dynamic aspect to MSN corrections that is linked to abnormal price action, underscore the practicality of parametric models, and expand on existing work by testing assumptions about MSN at different time periods. This adds to previous work explaining the link between economic theory and statistical noise corrections, and points to areas of future research in creating a comprehensive theory of market microstructure.

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ABBREVIATIONS / NOMENCLATURES

MSN	Microstructure Noise
RV	Realized Variance
IV	Integrated Variance
Y	Observed log price
X	Underlying efficient log price
ε	Noise
σ^2	Variance
γ_i	i-th order lag autocorrelation

Chapter 1 Introduction

1.1. Project Background and Motivation

Noise is a prevalent feature of stock markets, representing the deviation between efficient price and observed price. On a small timescale, price action appears random with unpredictable fluctuations – over a longer time period, these movements form a price discovery mechanism. If we take noise as the random fluctuations in price that are not related to an asset's fundamental value, Microstructure Noise (MSN) is a subset of this process, a specific type of noise that caused by trade execution mechanisms – specifically, who is executing trades and so how they can execute trades, and what liquidity conditions these can be executed under.

Distinguishing between signal and noise is particularly valuable in financial markets, and the study of market MSN is one way to create a more comprehensive picture of the market by examining the sub-second activity of market participants.

MSN is small and transient. Couleau et al. (2018) suggest that the effects of MSN fade after around 10 minutes, and price movement is largely contained within narrow bands. In recent years, the increasing availability of high frequency data has allowed more in-depth analysis of the underlying microstructure effects. This comes from many sources, capturing market properties such as the bid-ask bounce and discreteness of transaction prices, along with the actions of market participants (Clinet & Potiron, 2019; Hansen & Lunde, 2006).

Sampling at the highest possible frequency theoretically captures the maximum amount of information, but the effects of MSN mean that naively calculated volatility estimates are unreliable and significantly overestimate variance. At the same time, naively eliminating noise by increasing sampling periods or smoothing reduces the amount of potentially useful information that can be learnt.

A large amount of recent work focuses on robust statistical volatility estimators to solve this problem even when sampling at high frequencies (Da & Xiu, 2021; Jacod, et al., 2019;

Li & Linton, 2022; Varneskov, 2016). However, purely statistical methods cannot capture the underlying mechanisms of MSN as they are numerical rather than econometric methods. If MSN results from the activity of market participants, it should be important to understand the conditions under which they act. Several other researchers have therefore tried to explain MSN through various indicators (such as liquidity or volume) or by fitting estimated noise to various hypothetical parametric frameworks (Clinet & Potiron, 2019; Diebold & Strasser, 2013; Virgilio, 2022; Doman, 2010; Chaker, 2017).

Although intuitively the latter approach potentially contains more information, they ultimately create observable proxies for an unobservable noise process. Models are easily mis-specified and estimated with a significant probability of error – for example, trade direction used in bid-ask models must be inferred due to the lack of data. In comparison, statistical MSN models are significantly more complex and robust as long as certain assumptions about market structure are satisfied.

To bridge this gap, Diebold and Strasser (2013) pioneered an approach to combine economic insights with statistical models: by treating MSN as a result of market agent decisions, they pointed out the advantages of being able to explain and predict patterns. However, there has not been significant research since then exploring the implications of their findings.

1.2 Objectives

The main contributions of this paper are: First, a practical application of microstructure theory to examine how specific news events affect market activity. Second, a comparison between basic parametric models and more recent statistical models to examine whether we can attribute MSN to a certain pattern of underlying market participant behaviour. This will allow us to identify which base models are a better fit to the real-world conditions, which opens up the path for more complicated parametric estimators that can explain market participant behaviour.

To my best knowledge, there is a gap in the recent literature in trying to address this problem: Recent statistical estimators do not account for the real-time price discovery process on the release of new information as many models make the assumption of perfect information. Meanwhile, Couleau et al. (2018) are the only example that investigate the effects of market participants incorporating real-time information release into the variance of agricultural futures markets.

Through comparing microstructure characteristics over earnings release dates for liquid stocks, this paper aims to contribute to uniting mathematical models with practical real-time application.

Chapter 2 Literature Review

Section 2 begins by defining MSN in the relevant literature, followed by a discussion on explanations for the noise in empirical data. Section 2.3 then covers a more in-depth review of existing parametric and statistical estimators of volatility which are used later in the study.

2.1 Definitions and Sources of Microstructure Noise

Market Microstructure Noise is defined slightly differently in many papers, but the common definition relates to the difference between latent return and observed market return (Diebold & Strasser, 2013). Couleau et al. (2018) frame it as the difference between the permanent and transitory component of price variance: the former reflects the efficient price variance because of information flows, while the latter captures the effect of MSN. We narrow the definition slightly in this study, specifically targeting transient noise on the sub-second level and focusing on noise spanning individual trades.

When examining the sources of MSN, Diebold and Strasser (2013) provided a framework to model it as a result of financial economic decisions from traders and market makers and derive a series of cross-correlation patterns between latent return and MSN depending on the active participants. This includes conditions such as the presence of informed traders or the risk tolerance of market makers. These observations were followed up in Clinet and Potiron (2019), where it was suggested that MSN can almost entirely be explained by a linear model. When trade direction was multiplied by the dynamic bid-ask spread, they found that it explained around 99% of observed variance.

Meanwhile, Virgilio (2022) attempted to examine driving factors behind sub-second price changes using Granger-causality tests. Their results support a "common sense" theory: Previous volatility generates fear, which can exacerbate volatility; low liquidity results in larger price variations; and stop-loss orders can trigger cascading effects. Although the literature has identified some characteristics of MSN, it is impossible to account for every factor due to imperfect information and randomness – the patterns mentioned above rely on assumptions about price movement and statistical conditions.

2.2 Noise in empirical data

Previous research has identified that noise bias from arrival of information dissipates quickly in agricultural futures markets, having a limited duration of 10 minutes or less (Couleau, et al., 2018). Increased variance is observed following information being released, and attempts to mitigate this noise in the agricultural futures markets have not been particularly fruitful. Doman (2010) attempts to explain variations in noise because of liquidity factors by using the Noise-to-Signal ratio. They found that liquidity measures such as volume and number of transactions poorly explained MSN, but that days with high Noise-to-Signal ratio showed a very regular pattern of returns – this was attributed to market maker activity.

Virgilio (2022) studied the Flash Crash of 2010 using Granger causality tests, demonstrating causality between previous volatility and lack of liquidity with excess volatility. They also found no evidence of a relationship between trading volume and volatility, consistent with the findings of Doman (2010).

Other parametric models try to model behaviour of market participants instead, as shown in Diebold and Strasser (2013). These used a similar evaluation metric: proportion of noise explained by a model, where the magnitude of noise is measured using a statistical estimator. Their results represented the first step in showing that minimal microstructure models could have explanatory power for various scenarios.

Hansen and Lunde (2006) highlighted that the key characteristic of microstructure noise in empirical data was autocovariance – contrary to previous models that assumed identical independently distributed variations, they demonstrated the dependence of noise on previous lags.

More recently, the ReMeDI estimator by Li and Linton (2022) allows for estimation of arbitrary moments of microstructure noise. Building on the same framework as the local averaging methodology (Li et al., 2017), they use a differencing method to convert the time series to a stationary one. The results here also show a serial short-term dependence in microstructure noise, which is in line with previous microstructure models.

There have also been efforts to identify the timescale over which we can neglect microstructure noise in volatility calculations (Aït-Sahalia & Xiu, 2019). They concluded that for most stocks, sampling above the 5-minute level allows one to assume noise-free datapoints. This was paired with a new testing framework to verify the absence of significant noise.

Despite the large amount of investigation into this area, there is still a lack of literature seeking to reconcile empirical econometric analysis with statistical MSN properties. Since Clinet and Potiron's paper, there have been improved statistical MSN estimators, liquidity, market participants, and explanatory theories. All of these would benefit more research into the field, and extending their models to include news events would expand our knowledge of the interaction between real world events and market microstructure.

2.3 Volatility in Microstructure Noise

Volatility estimation in the presence of MSN is by far the most popular application in the market microstructure literature as it has direct practical implications. In this paper, we deal with the integrated variance (IV), the integral of σ^2 over a certain period. Unless otherwise mentioned, any reference to volatility estimates will be IV.

Observed time series prices are often modelled in the literature as the sum of an underlying latent process (fully efficient) and deviations from this series (Varneskov, 2016; Diebold & Strasser, 2013; Li & Linton, 2022; Jacod, et al., 2017; Da & Xiu, 2021).

$$Y = X + \varepsilon \tag{2.1}$$

Y is the observed price, X is the underlying, and ε covers the noise.

Under perfect conditions, IV can be estimated by Realized Variance (RV), defined here to be the sum of squared intraday returns as in Hansen & Lunde (2006). This gives a perfect estimate of volatility when prices are continuously observed without measurement error or noise.

$$RV = Var(\Delta Y_t) = \sum_{t=1}^{T} \Delta {Y_t}^2 \tag{2.2}$$

When there is noise, the RV becomes a biased estimator of σ^2, as shown in Diebold & Strasser (2013).

$$\begin{aligned} Var(\Delta Y_t) &= Var(X + \varepsilon_c + \varepsilon_u) \\ &= \sigma^2 + Var(\varepsilon_c) + Var(\varepsilon_u) + 2Cov(X, \varepsilon_c) \end{aligned} \tag{2.3}$$

We divide the noise component into ε_c, which is correlated with the underlying "true" price, and ε_u, which is uncorrelated. Each component can be associated with different types of noise, such as ε_c reflecting the presence of participants acting with asymmetric information, or ε_u capturing the bid-ask bounce during uninformed trading.

Under an assumption of dependence between noise and underlying price as shown in Hansen & Lunde (2006), corrections under the independent noise assumption remove only the second and third term, leaving us with an often negative fourth term and a reduced volatility estimate. Irregularly spaced data also causes biases in the estimates (Jacod, et al., 2017).

Autocorrelation of noise and cross correlation of underlying price and noise (past price affecting future noise components) mean that more robust estimators under these conditions have been the subject of investigation for the last two decades.

2.4 Statistical and Parametric Estimators of Volatility

To estimate IV, there are two common approaches in the literature. Here we define them in broadly two categories. Statistical estimators rely on purely statistical analysis, while Parametric estimators assume a certain model of market participant behaviour exists, and by specifying these parameters we can estimate volatility.

Using a specific parametric model of noise gives useful insight into its source by fitting to an econometric model but runs the risk of model misspecification. Statistical estimators aim to eliminate this by acting as a generally robust system that still allows information about noise characteristics to be calculated. In general, statistical estimators have been far more complex and robust. We will be comparing these parametric estimates with robust statistical estimators: the similarity between estimators can tell us which specific parametric model performs the best in different situations, and we can also try to predict whether statistical estimation is eliminating potentially useful noise that can inform us about the market microstructure – this would happen if there is a significant and consistent difference between the statistical and parametric estimates.

2.4.1 Parametric Models

Although we model the latent price as a stochastic process, we can view transaction prices as the end result of market participants optimizing for their own gain. Diebold and Strasser

(2013) divide these participants into informed traders, uninformed traders, and market makers. In this paper, we use several basic volatility estimators derived from this framework to model microstructure noise created in the following situations.

The following section will lay out the theoretical model framework, followed by brief summaries of the basic parametric models examined in this paper.

For simplicity, prices are modelled as standard Brownian motion with no jumps, time-varying volatility, or time-varying sampling intervals.

$$X_t = \begin{cases} X_{t-1} + \sigma i_t & \forall t = kT \\ X_{t-1} & otherwise \end{cases} \tag{2.4}$$

$$i_t \sim N(0,1)$$

X_t is the efficient price at timestep t, which can change at times kT with size equal to standard deviation σ multiplied by a random value i_t, which is independent and identically distributed.

Trading follows a simple model with three parties:

1. Uninformed traders, who always trade with a probability β in every time period with equal probability in either direction.
2. Informed traders, who always know the efficient price, and act with a probability α when there is a discrepancy between the market and efficient price.
3. Market makers, who are the counterparty to all trades: they earn money from facilitating trades but are guaranteed to lose when trading against Informed traders.

The market maker's goal is to minimize loss from being the counterparty to informed traders. They can do this by using transaction data and external information to adjust their perception of the efficient price.

In this case, the latent price can be modelled as one of two options: the full-information price (strong efficient), and the price at which a market maker is willing to keep asset on accounts (semi-strong-form efficient price). There are different cross-correlation patterns depending on which one we choose as the underlying price, but for simplicity we will only consider strong form efficient price as the true underlying price here.

Fully parametric model makes sampling as often as possible optimal. Overall, the estimators are built on the equation

$$\sigma^2 = RV + 2\sum_{i=1}^{k} \gamma_i - 2E[\varepsilon_t \Delta\varepsilon_{t-k}] - 2E[\Delta X_t \varepsilon_{t-k}] \tag{2.5}$$

The main assumptions here are that the latent price changes every time period and remains unobserved for one or more periods.

Bid-Ask Bounce Estimator

The Bid-Ask Bounce Estimator considers a situation in which MSN is explained by the spread of quoted prices, regardless of what type of market participant there is.

Assuming all information is revealed after one period, constant spread, and symmetric quotes around the underlying price, the observed transaction price is

$$Y = X + s_t q_t \tag{2.6}$$

Where s_t is one-half of the bid-ask spread and q_t is the inferred trade direction. This means that the noise ε_t has the form

$$\Delta\varepsilon_t = \sigma(i_t - i_{t-1}) + s(q_t - q_{t-1}) \tag{2.7}$$

Using this, we get an unbiased estimator for IV as

$$\widehat{IV} = E[(\Delta Y_t - \Delta\varepsilon_t)^2] = RV + 2\gamma_1 \tag{2.8}$$

Where γ_1 is the first-order autocorrelation of market returns.

Restricted Learning

The Restricted Learning Estimator applies when market makers observe a noisy signal – they do not know whether the trade originated from an informed trader, or if there are any active.

In the absence of exogenous noise, we impose an exponential model on market maker learning – in other words, the unbiased estimator here is under very restrictive conditions.

This leads us to a simple estimator equal to scaled RV.

$$\widehat{IV} = \frac{e^{\hat{r}} + 1}{e^{\hat{r}} - 1} = \frac{RV + \gamma_1}{RV - \gamma_1} RV \tag{2.9}$$

Non-strategic incompletely informed traders (Noisy)

The Restricted Learning estimator is a special case in which $\beta = -\sigma, \nu_t = 0$ (no white noise). Diebold and Strasser (2013) generalize this to noise of the form:

$$\varepsilon_t = \alpha\varepsilon_{t-1} + \beta(i_t + v_t) \tag{2.10}$$

i_t and v_t here are both independent and identically distributed white noise variables to capture noise trading, while the α term captures the role of informed traders in the noise. Using this form, the estimator under the general case becomes

$$\widehat{IV} = \gamma_0 + 2\frac{{\gamma_1}^2}{\gamma_1 - \gamma_2} \tag{2.11}$$

Non-strategic informed traders

A similar case arises when the MSN is purely due to the activity of informed traders. Both this and the case above try to model a geometric decay of MSN as the market maker learns the efficient price from non-strategic trades.

$$\widehat{IV} = \gamma_0 + 2\gamma_1 + 2\frac{{\gamma_2}^2}{\gamma_2 - \gamma_3} \tag{2.12}$$

Strategic Informed Traders

Strategic informed traders are an extension of the previous cases. Informed traders can seek to maximize profits by disguising their trades from market makers, with the original sequential auction model pioneered in Kyle (1985).

We refer to this situation as the presence of strategic informed traders, which results in an approximately linear learning rate of maker makers as opposed the exponential function seen above. Order flow is a mixture of insider and noise traders, and market makers do not have extra information about the sources of the trades or inside information. When informed traders act strategically, they minimize the learning rate of market makers from the order flow.

Based off current and previous prices, an insider decides the quantity to trade based on expected market depth at current and future time periods. If market depth in the future is greater than current, the insider is incentivized to "save" private information for future rounds and thus trade in smaller quantities in the present. This is directly related to the amount of noise trading done on a certain day.

Interestingly, a large part of volatility is determined here by noise traders due to the relative size of their volume in this situation. However, because the insider is the only one with trades that are positively correlated across periods, they determine the final established price.

To calculate the volatility estimator here, we first estimate private information period as

$$\hat{S} = \sqrt{\left(\frac{3\gamma_1 - \gamma_2}{2(\gamma_2 - \gamma_1)}\right)^2 + \frac{2}{\gamma_2 - \gamma_1}\sum_{i=1}^{\infty}\gamma_i} - \frac{3\gamma_1 - \gamma_2}{2(\gamma_2 - \gamma_1)} \tag{2.13}$$

This allows us to calculate the following, as derived in Diebold and Strasser (2013).

$$\widehat{IV} = \gamma_0 + S(3 - S)\gamma_1 + S(S - 1)\gamma_2 \tag{2.14}$$

2.4.2 Statistical Models

Statistical estimators tend to account for microstructure noise through various smoothing algorithms. This can take the form of realized kernel estimators, maximum likelihood estimators, and pre-averaging methods, which were later improved upon to include features such as autocorrelation of noise (Hansen & Lunde, 2006; Jacod, et al., 2019; Da & Xiu, 2021; Li & Linton, 2022). Here we consider three statistical estimators to act as benchmark values.

Standard Statistical

Here we set a very basic baseline statistical estimator as the "Standard". In this case, it is the flat-top kernel estimator of Hansen and Lunde (2006). This reduces the observed prices to a martingale process – in theory, we expect this to eliminate more noise than desired, as it gets rid of both the correlated and uncorrelated aspects of noise referred to above.

This is a simple variation of the flat-top estimator covered later in this section.

$$\widehat{IV} = \gamma_0 + 2\sum_{i=1}^{30} \gamma_i + 2\sum_{i=1}^{30} \frac{30-i}{30}\gamma_{30+i} \tag{2.15}$$

Pre-averaging

The improved pre-averaging volatility estimator developed by Jacod et al. (2019) has the advantages of allowing for irregular observation times, dependent noise, and latent price jumps. Compared to the traditional pre-averaging estimator, this approach was shown to give a nearly unbiased estimator in the presence of coloured noise.

Pre-averaging is based around the idea that averaging K observations of the latent price X (as in Equation 1.1) reduces variance by a factor of 1/K. The resulting volatility estimate from these averaged samples will then be closer to the underlying semi-martingale.

The implementation of the improved estimator involves changing the underlying model to allow dependent noise, averaging over different windows to eliminate bias, then truncating pre-averaged values to allow the underlying to jump. The authors note that one possible weakness in this approach is that the noise and the underlying process are dependent but still uncorrelated. Full details on derivation can be found in Jacod et al. (2019), and code implementation is in the supplementary materials of this paper.

It is important to note that local averaging as seen in Jacod et al. (2019) relies heavily on the noise-to-signal ratio and sample size, making it challenging to implement in practice.

Flat-Top Realized Kernel

The Flat-Top Realized Kernel estimator allows for weaker assumptions about the MSN. To counteract the explosion in variances as sampling frequency increases, realized kernel estimators smooth the observations by weighting autocovariances differently.

The estimator presented in Varneskov (2016) is a multivariate extension of their previous paper, with the key improvement being the use of a shrinking flat-top support near the origin: the width of the flat-top region decreases, which helps obtain rate-optimal estimators.

Although these estimators are robust to a differing extent over a range of tuning parameters, there is still the need for adjustment to the market data, which could either give more information about the market structure, or simply leave the estimators more prone to overfitting.

Chapter 3 Methodology

The main features examined here will be a comparison between different volatility estimators, the fit of various parametric models to their statistical counterparts, and the autocovariance profile for both microstructure-adjusted and raw calculations.

To do so, we take statistical models as a baseline: the volatility estimates from both the *Pre-averaging* and *Flat Top Realized Kernel* models are treated as accurate estimates of IV, corresponding to the movement of the underlying efficient price.

The third statistical model is used in a similar way, but we also follow the method of Diebold & Strasser (2013) to claim that the *Standard Statistical* estimate removes all deviations of the transaction price from a martingale sequence. This might be different from the true latent price process, which only accounts for MSN. In other words, this is a simplified version of a statistical MSN correction that should eliminate *too much* noise.

The volatility estimates from parametric models can then be compared against these three benchmarks: how much of the eliminated noise can we explain with each MSN model? The initial hypothesis is that under different circumstances, there will be different parametric models being the most suitable to fit with the results of the statistical models. This would inform us that a particular market microstructure is present at a certain time and encourage further development along those lines to create a better parametric estimator. For example, if the *Strategic Informed* estimate captures 99% of the difference between *Standard RV* and the *Pre-averaging estimator*, we can conclude that the *Strategic Informed* model of market participants is a good fit for a certain time period, suggesting that adding more complex models of strategic informed trader activity would be useful.

To see if there is any effect of external market conditions on their interactions, we will look at earnings weeks, which are a consistent source of new information associated with abnormal volume and price changes (Berkman & Truong, 2009).

Unless otherwise mentioned, all references to prices are the logarithmic prices. Three different earnings dates in 2021 across the three largest composite stocks of the S&P 500 were selected – this allows us to be relatively sure of good liquidity and thus price discovery around new events.

The volatility estimators are as follows, divided into three categories:

Table 1: Summary of volatility estimators

Category	**Name**	**Description**
Baseline	Standard Realized Variance	Sum of squared intraday returns
Parametric	Bid-Ask Estimator	Volatility under the assumption of a constant spread of discrete prices

Parametric	Restricted Learning	Constrained case of nonstrategic noisy trader case, with $\beta = -\sigma, \nu_t = 0$
Parametric	Nonstrategic noisy	Nonstrategic, incompletely informed traders.
Parametric	Nonstrategic Informed	Informed traders and autoregressive noise.
Parametric	Strategic Informed	Informed traders strategically trying to maximize profit.
Statistical	Pre-averaging	Local averaging statistical estimator of Jacod et al (2019).
Statistical	Flat Top Realized Kernel	Flat Top Realized Kernel of Varneskov (2016).
Statistical	Standard Statistical (Rectangular Triangular Realized Kernel)	Rectangular Triangular Realized Kernel of Hansen and Lunde (2006).

We also study the ReMeDI estimators of autocovariance across different days and tick sizes and compare them to autocovariance calculated without a microstructure adjustment. This allows us to see how the characteristics of noise dependence changes – whether it matches any of the theoretical profiles of parametric estimators, and whether changing the sampling frequency will significantly affect our results.

We anticipate increased volatility around these events. However, our study also aims to explore the nature of this volatility. We seek to answer questions such as: When does volatility increase? What type of microstructure can we attribute to it? How does this change compare to random fluctuations in normal market conditions?

The following calculations were carried out and visualized in python, for which the source code can be found in the appendix.

Chapter 4 Case Study

The case study focuses on the top 3 liquid stocks in the SPY index – in this case, AAPL, MSFT, and AMZN. Specifically, the samples analysed are from the three largest stocks in the S&P 500 on the three days prior to and after earnings are released after hours in April, July, and October 2021. Prices are sampled from 09:30:00 until 16:00:00, as extended trading is assumed to have lower liquidity and different market conditions, which could be an avenue for further research. Unless otherwise mentioned, earnings dates here refer to the day prior to the actual release – characteristic of potentially market moving information, earnings data is normally published after market close.

A preliminary examination of the data shows that the number of intraday trades has been on an uptrend, likely due to the increasing number of market participants and availability of technology. This may also affect the characteristics of MSN in comparison to prior years in which there would have been far less activity.

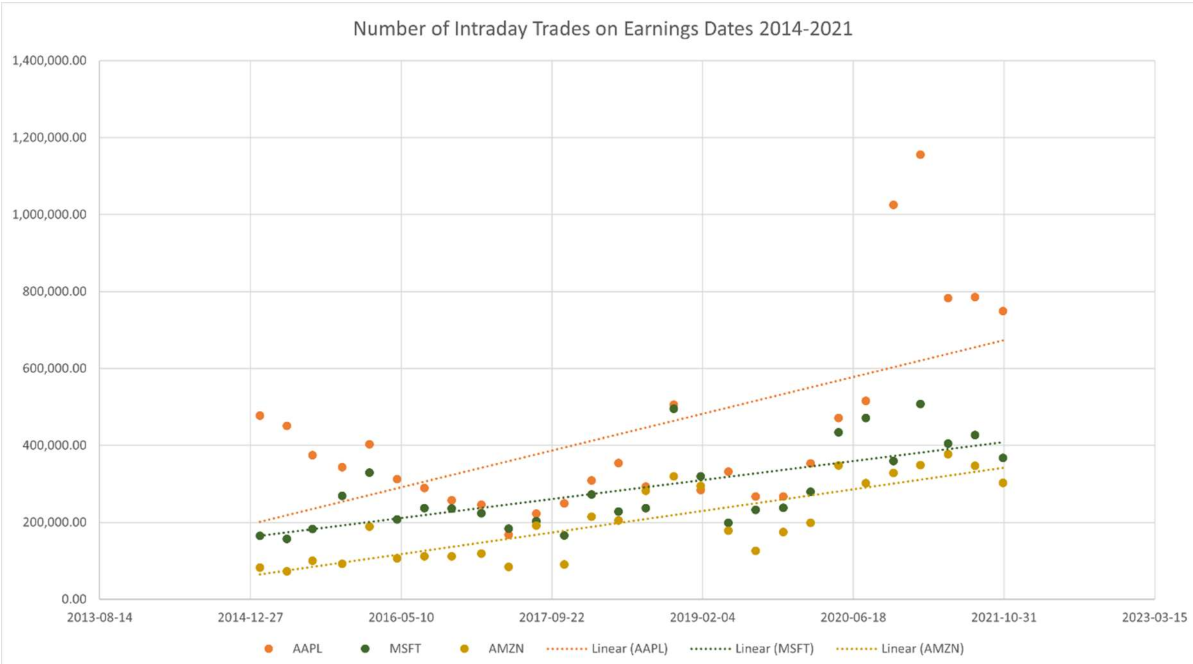

Figure 1: Number of intraday trades before earnings announcements

Chapter 5 Results & Discussion

The results are broken down into several sections: 5.1 explores the effect of sampling at different frequencies; 5.2 compares the IV estimates across different time periods; 5.3 breaks down the explanatory power of each parametric model; 5.4 briefly discusses changes when sampling every second, while 5.5 discusses the autocovariance profiles across different days for both a traditional calculation and the ReMeDI-adjusted estimates.

5.1 Volatility signature plots

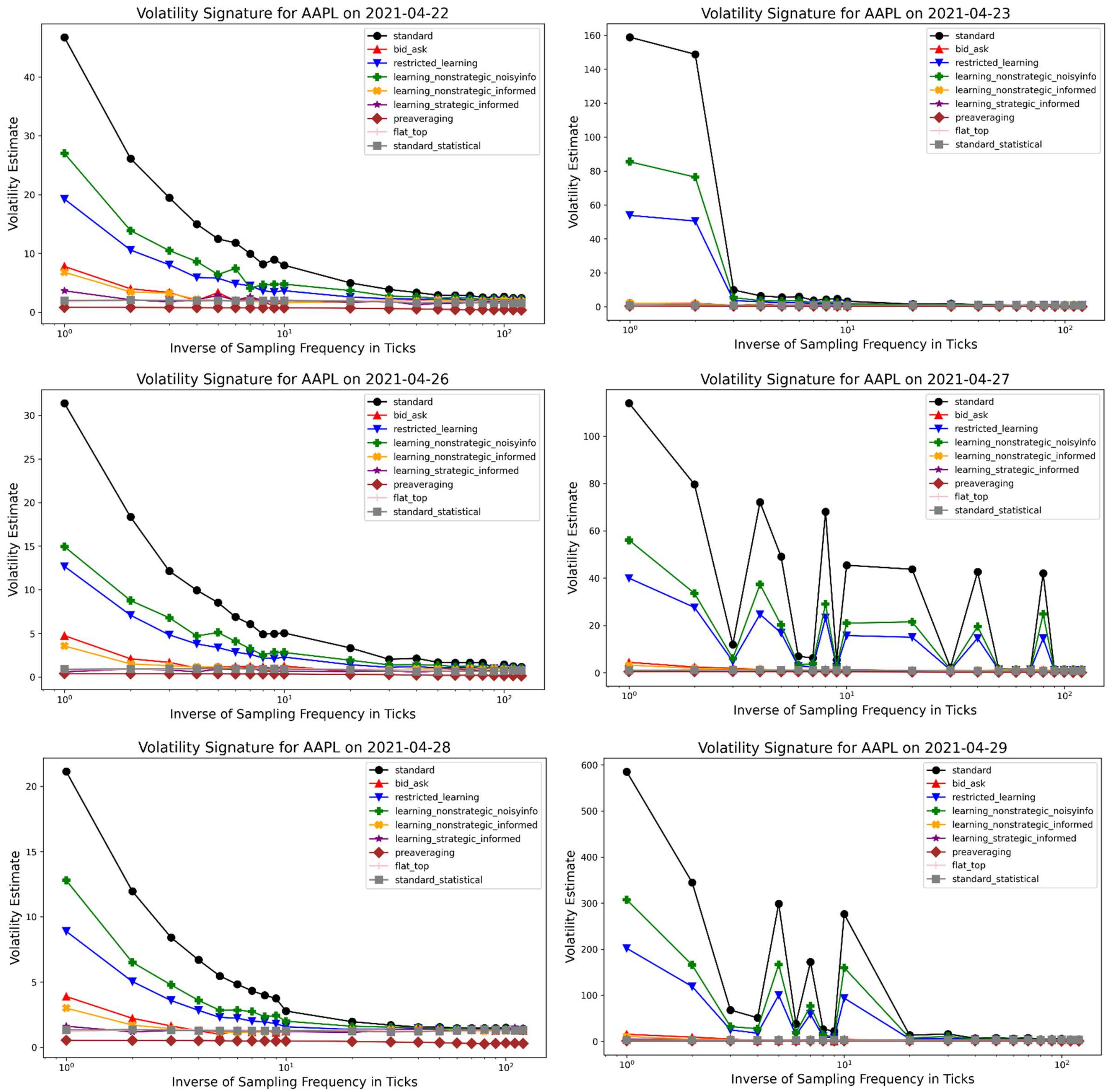

Figure 2: Volatility signature plots for the period 22/04-29/04/2021

Consistent with previous literature, volatility signature plots illustrate the effect of higher sampling frequency on the Realized Volatility, with an upwards explosion in magnitude. As expected, several parametric estimators show similar patterns but on a damped level: as they adjust for a certain level of MSN, estimates should generally be lower. The statistical estimates also show relative stability alongside the *Strategic Informed* parametric model. Due to their ability to adjust through smoothing methods, this pattern is also expected. Volatility estimates tend to stabilize around the 1000-tick mark. In the sample shown above, there is an average of around 180 ticks per second.

On days with higher volatility, volatility signature plots seem to also contain more irregularities. Figure 2 presents a side-by-side comparison of the comparatively low and high volatility days and their corresponding volatility signatures. On days with relatively lower absolute volatility estimates on the left-hand side, the signature follows a consistent slope along the log scale. On higher volatility days opposite, there are irregular spikes in volatility estimates even as sampling intervals increase – more chaotic conditions may mean that microstructural noise effects are more prominent on these days. This suggests that there may be a dynamic component to the sampling frequency at which we can claim to have eliminated the effects of noise: for example, on higher volatility days we cannot make the independent noise assumption at even the scale of ~100 ticks on the 27th of April. However, this could also be linked to the amount of trades happening within a set time period, and further research could look into how the number of individual trades could affect noise.

5.2 Integrated Volatility Estimates Across Days

We expected abnormal trading volume and prices in the day after earnings are released. However, although there are clear peaks 2 to 3 times that of the second largest volatility spike throughout the sampled period, there is no specific rule for when this day appears. For all samples, there was an uptick in volatility on earnings day.

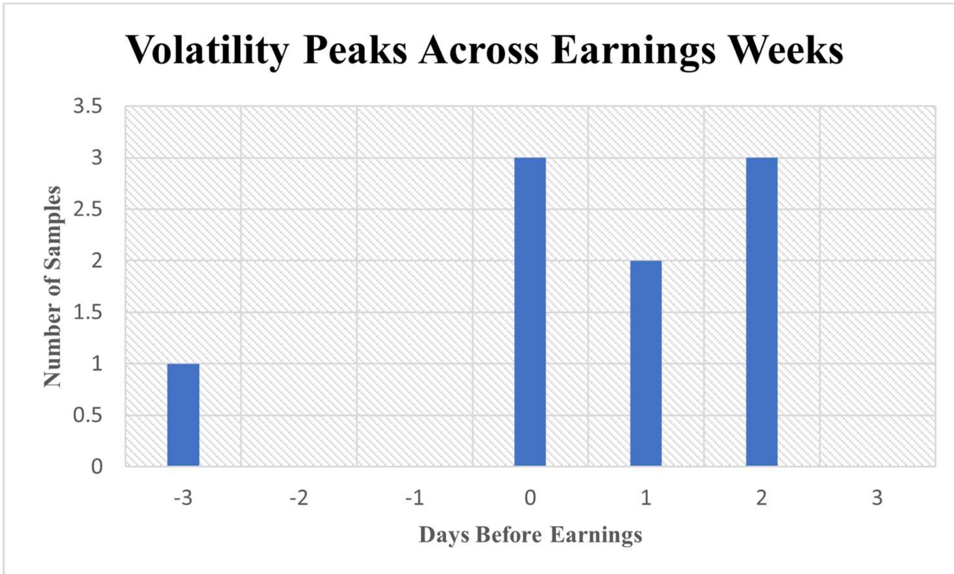

Figure 3: Days before/after earnings on which IV estimates peak

The maximum peak characteristic was not unique to earnings weeks. When comparing with the estimated IV profile from two weeks before the earnings date, similar spikes are found but with a more random distribution. The following figure shows one such sample for AAPL in April 2021, with earnings released after market close on the 27th. The next figure shows the same calculation run two weeks before – the absolute value of peaks is smaller in the non-earnings week.

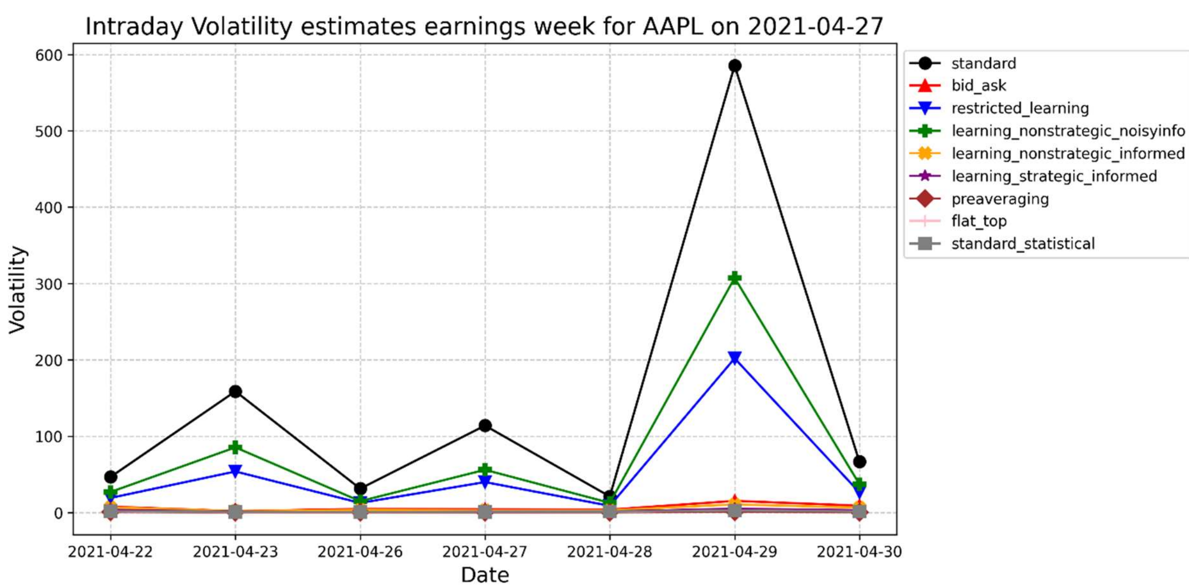

Figure 4: Intraday IV Estimates 27/04/2021

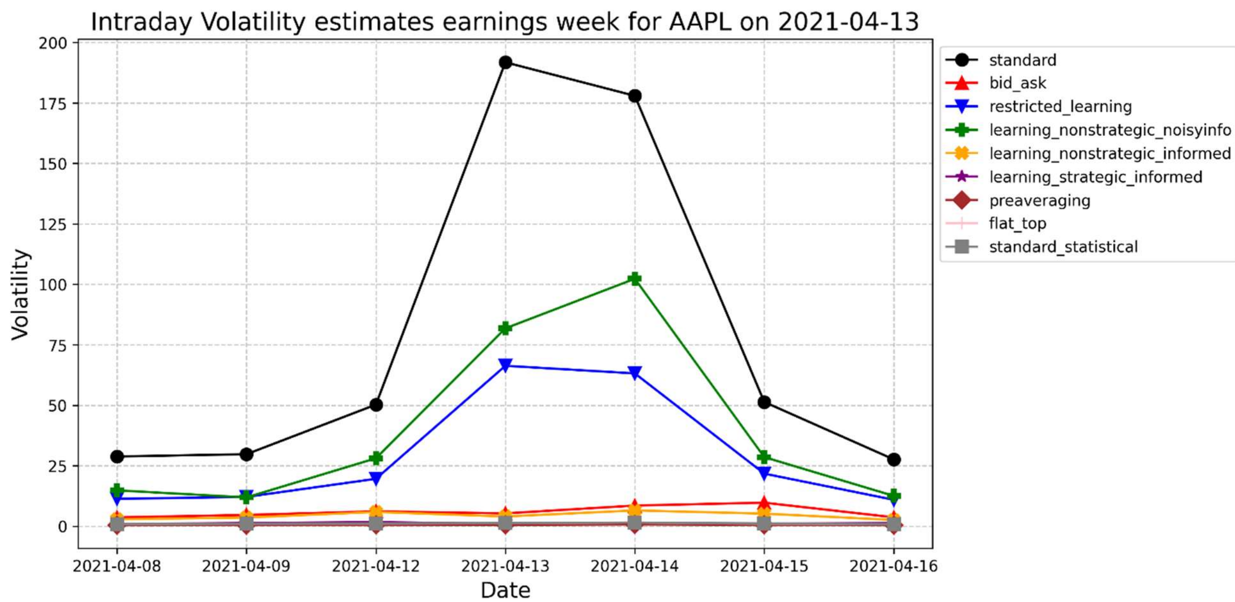

Figure 5: Intraday IV Estimates 13/04/2021

We can also see that relatively much simpler estimates approximately generate the same results as more advanced smoothing methods – both the *Flat Top Realized Kernel* and *Standard Statistical* estimators deliver similar results (full figures can be found in the Appendix). Although they both use kernel functions, the former is much more robust to different noise settings, and this may suggest that under certain conditions a simpler estimate can be substituted for practicality.

5.3 Comparison of market microstructure

In order to understand the characteristics of microstructure noise, we extend Diebold and Strasser's (2013) method to compare the parametric and statistical estimates. *Standard Statistical*, *Pre-averaging*, and *Flat Top Realized Kernel* estimates are used as baseline estimates – the difference between the Standard Realized Variance and each respective statistical estimator is the total noise.

We are trying to find whether parametric models explain a large proportion of this total noise, which can justify an MSN correction as we can fit an explanatory agent-based model to the noise calculated from statistical methods.

In general, the percentage of noise captured by each parametric estimator is relatively consistent across comparisons to the *Pre-averaging, Flat Top Realized Kernel*, and *Standard Statistical* volatility estimators, and given the smoothing nature of each of these functions it is relatively safe to assume they deliver similar results – at least in the region of comparing with parametric MSN models.

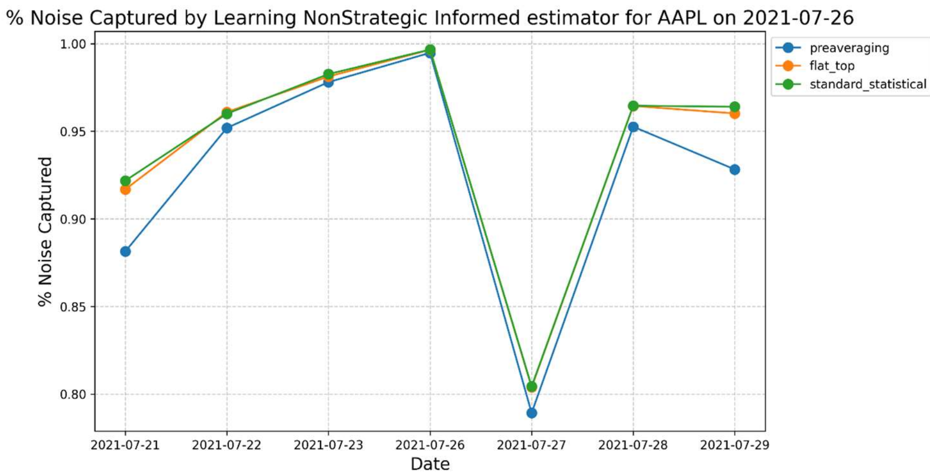

Figure 6: Comparison of noise captured by Nonstrategic Informed estimator across three statistical baselines

In the samples analysed, the *Bid-Ask*, *Nonstrategic Informed*, and *Strategic Informed* estimators captured an average of 83%, 84.5%, and 89% respectively, with the result for all three statistical estimators being within a 1% variation either way. As such, these parametric models justify the majority of noise removed, with the most likely explanation being the market activity of strategic informed traders. However, this may also be in part due to the circularity – if more traders demand a certain price, it then becomes the fair price.

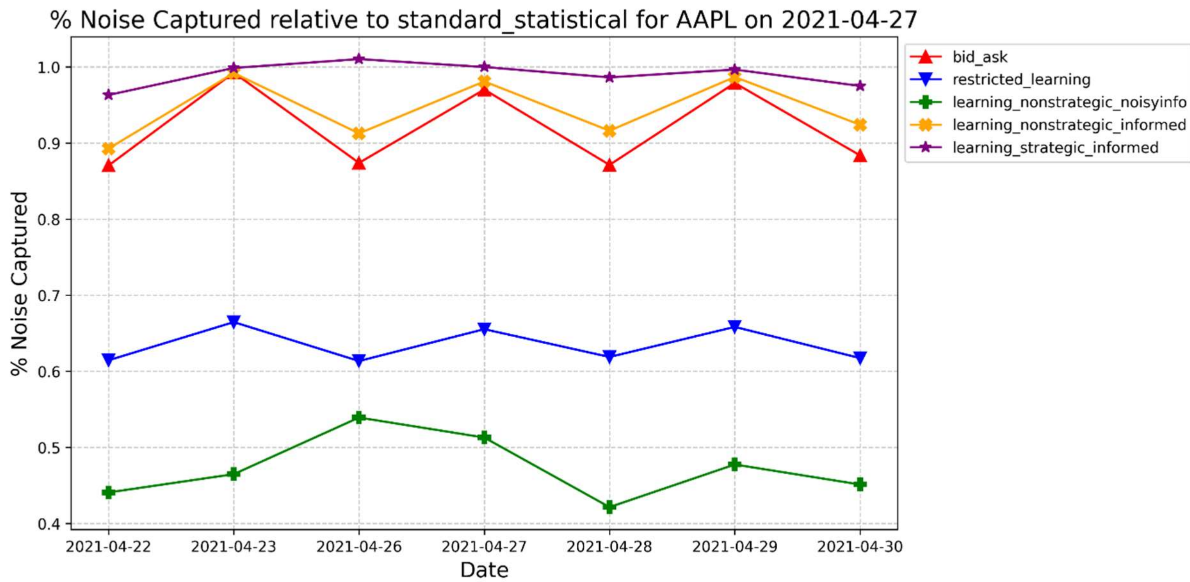

Figure 7: Percentage of total noise captured by each parametric estimator on 27/04/2021

Throughout each sample, *Strategic Informed*, *Nonstrategic Informed*, and *Bid-Ask* estimators consistently capture a significant portion of the noise, suggesting a good fit of informed traders driving towards a certain value. However, the Bid-Ask model also captures a large portion of the noise, in agreement with the findings of Client and Potiron (2019).

The noise captured by the *Bid-Ask*, *Restricted Learning*, and *Nonstrategic Informed* estimators rise and fall inversely to the estimated volatility. This is expected as the *Standard RV* estimator is likely to rise more proportionally when compared to statistical estimators that reduce MSN due to increased noise in volatile conditions. Interestingly, the *Nonstrategic Noisy* and *Strategic Informed* estimators do not show the exact same pattern but have more variation across each day.

It is also worth exploring the changes in MSN on earnings day. Table 2 shows the difference between the average baseline percentage of noise captured (measured over 7 trading days) and the percentage of noise captured on the single earnings day. Across all samples there was a small positive skew. Notably, the *Bid-Ask* and *Nonstrategic Informed* models improve almost twice as much as the other estimators on earnings day, suggesting the increased effect of price discreteness and noise trading.

Table 2: Difference between percent noise captured on earnings day versus the entire week

	% Difference in Noise Captured from Average		
	Standard	**Pre-averaging**	**Flat Top**
Bid-Ask	2.25	2.74	2.18
Learning	0.89	1.23	0.84
Nonstrategic Noisy	0.83	1.11	0.79
Nonstrategic Informed	2.13	2.64	2.06
Strategic Informed	0.19	0.74	0.10

5.4 Second Sampling

There are several observed differences when sampling data every second as opposed to in tick-time. Although MSN effects are still present, we expect them to be significantly diminished alongside volatility as we skip over many samples.

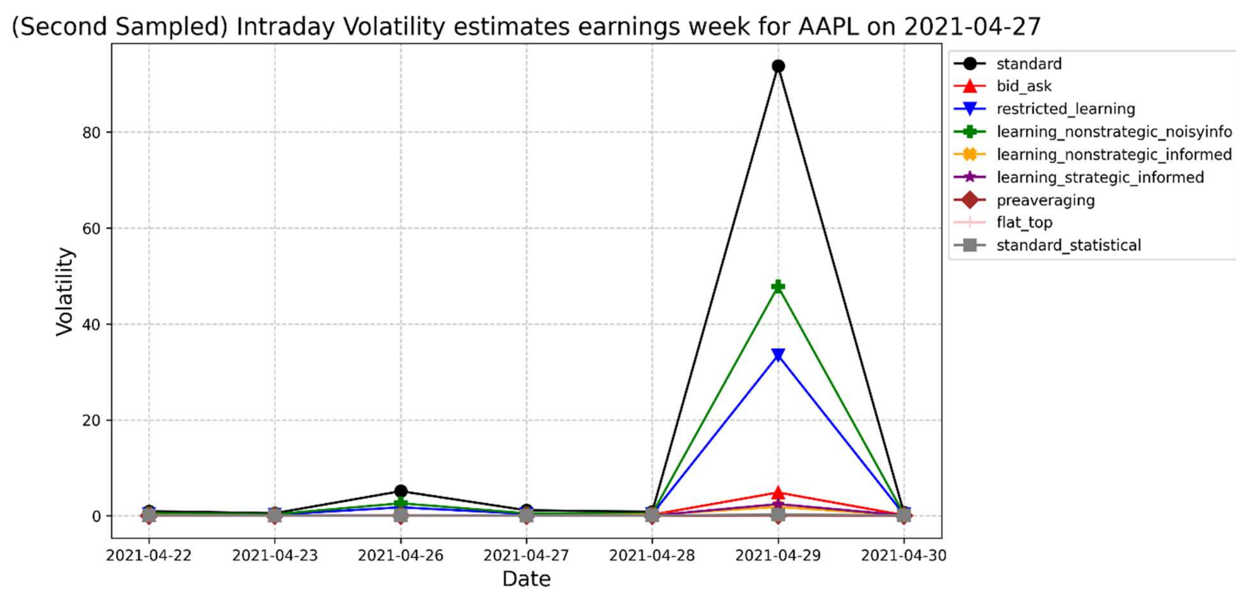

Figure 8:Second sampled intraday IV Estimates 27/04/2021

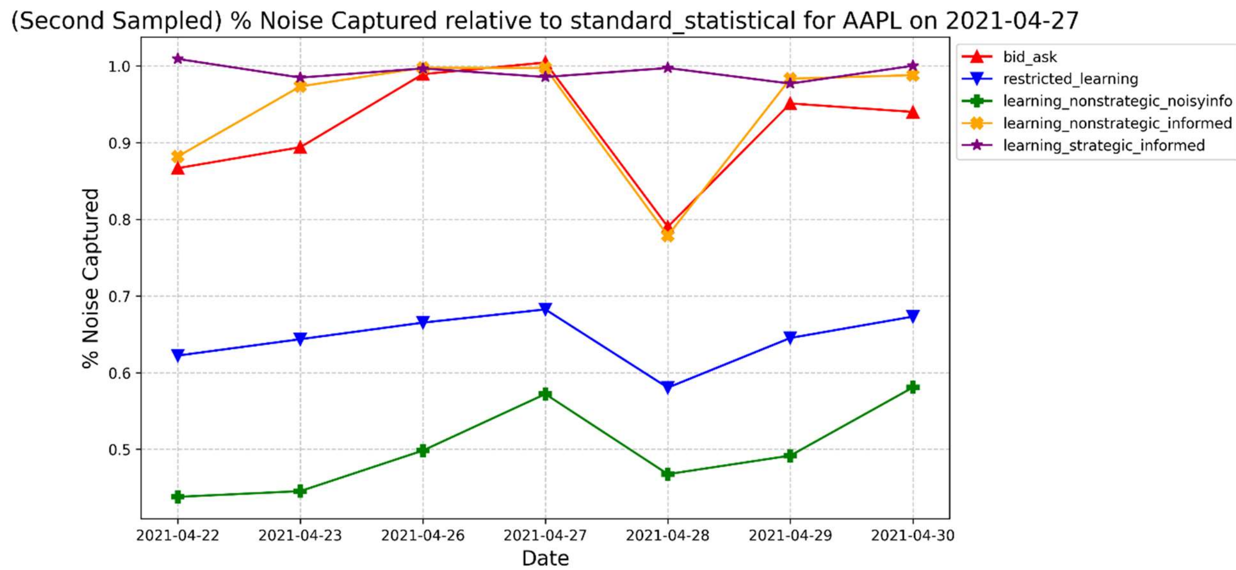

Figure 9: Percentage of total noise captured by each parametric estimator when sampled every second on 27/04/2021

The results support this idea, with significantly lower peak sizes. However, this also resulted in a larger difference between high and low volatility days. One possible explanation again refers to the idea that high volatility results in stickier MSN, even at higher timeframes.

The percentage of noise captured by each estimator shows a similar ranking to the tick-sampled profile, but the trends are largely altered – apart from the main dip corresponding to the spike of *Standard RV* on the 28th, there is no clear pattern. As such, further research investigating the causes and timeframes on which these changes occur would be useful.

5.5 Autocovariance Variation

Examining the estimated autocovariance of prices across these days also gives us more clues about the MSN.

Standard autocovariance calculations are consistent across different days – a negative first lag autocovariance dominates the terms, with a clear relationship between sampling frequency and magnitude of autocovariances, supporting MSN dependence as the autocovariance decays rapidly with a larger period between samples.

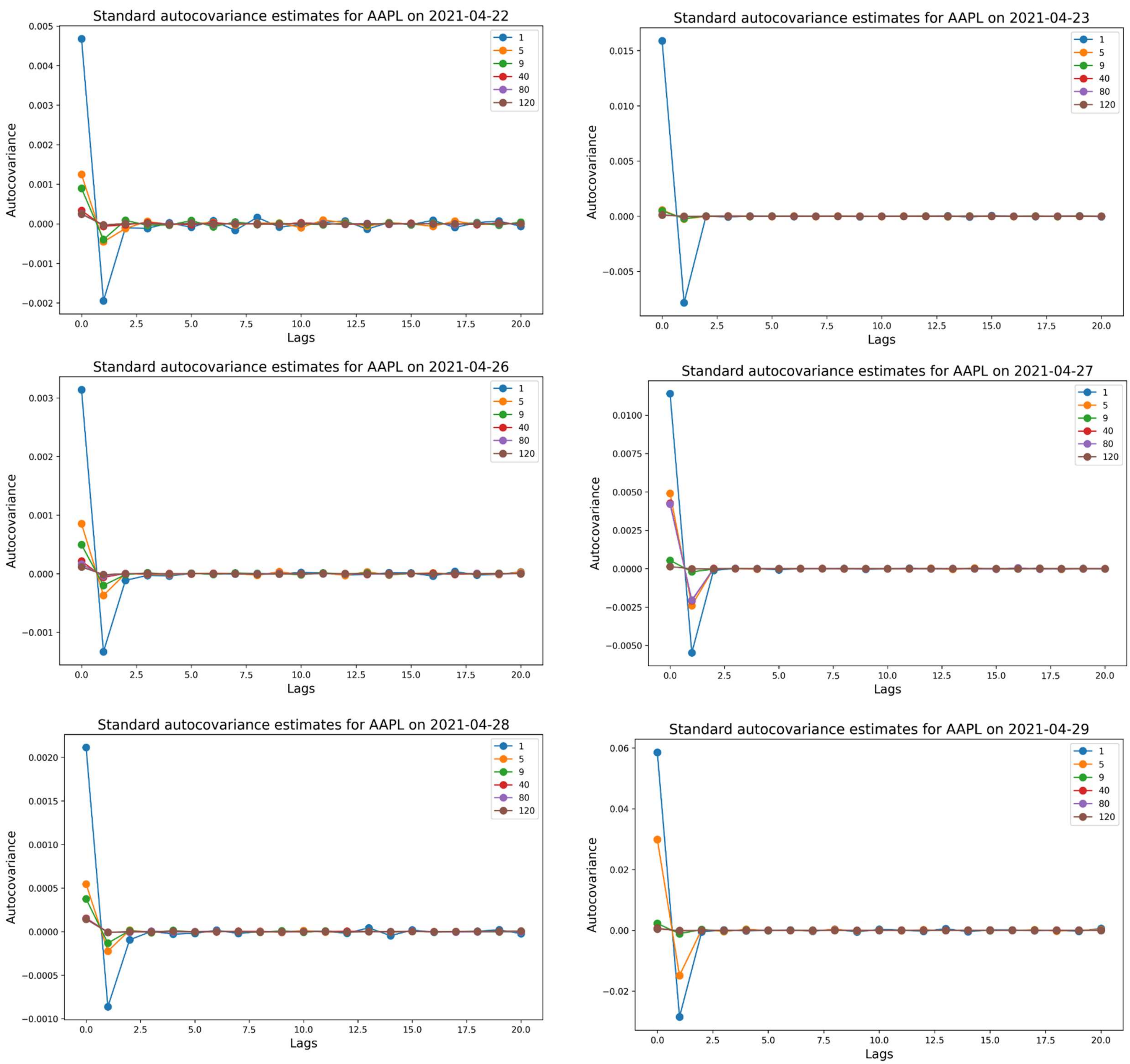

Figure 10: Standard autocovariance estimates of prices for the period 22/04-29/04/2021

In contrast, there does not seem to be a clear pattern with the ReMeDI estimators. Given that unpredictable behaviour mostly arises from calculations using a slower sampling period, it is likely that the statistical adjustment for MSN is no longer valid at these

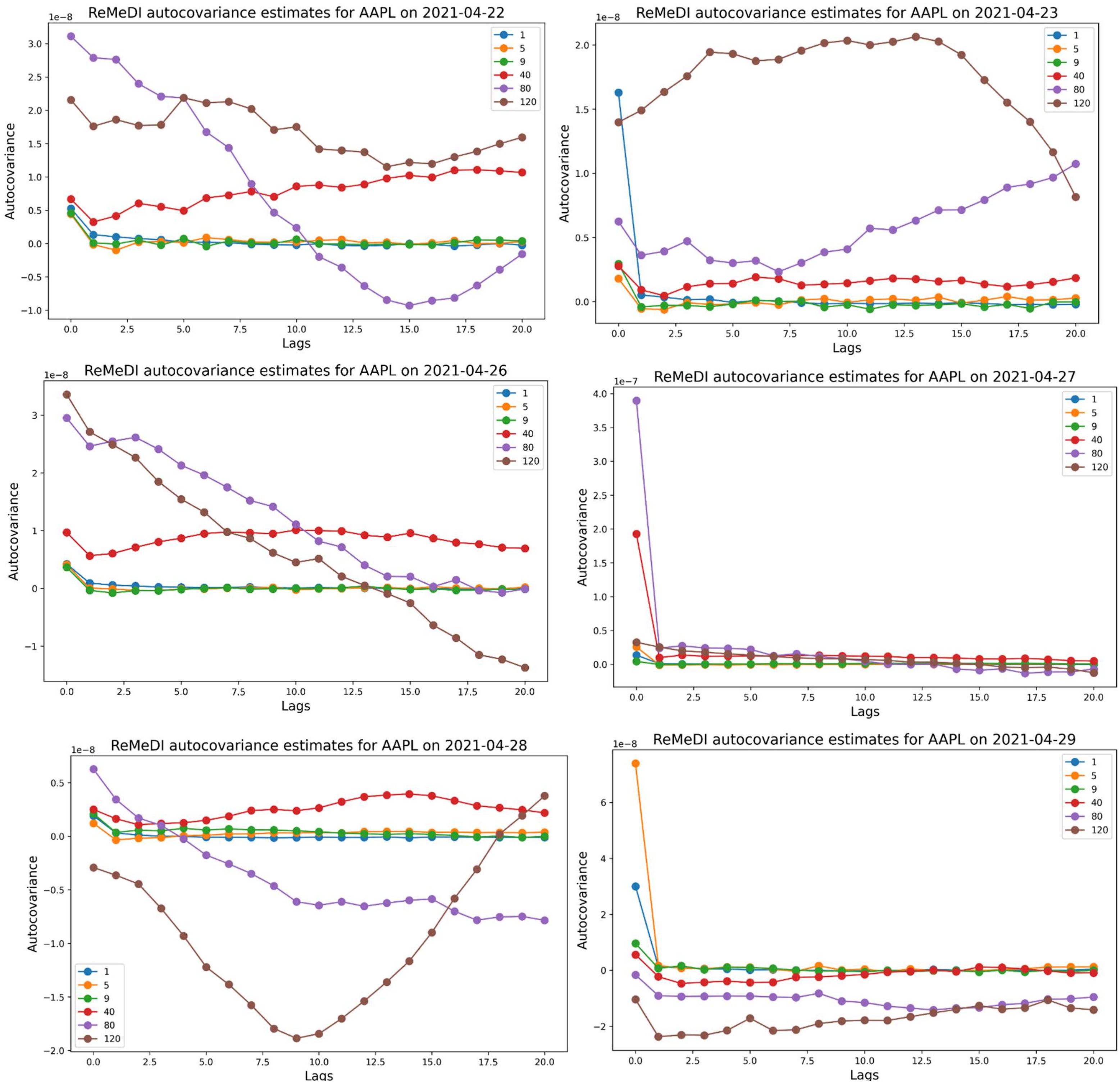

Figure 11: ReMeDI autocovariance estimates of prices for the period 22/04-29/04/2021

timeframes. The differences in how much these estimates deviate between days also suggests that MSN is present to different degrees depending on the day: for example, on April 27 it may be that even sampling at a much higher tick time (longer intervals between each sample), there is still enough of a significant MSN effect to justify the use of a statistical correction. However, the autocovariance profile of April 26 just one day before suggests a sampling interval of 40 ticks or above may not be suitable.

Chapter 6 Conclusion

6.1 Conclusions

The main findings of this study are as follows: first, there is a dynamic aspect to the level at which MSN is present which correlates with volatility. Second, treating MSN as a result of market participant decisions is still applicable in recent years with the increase in high frequency data. Third, abnormal price movement in days surrounding new events may influence the validity of MSN assumptions.

Other conclusions are largely verification of previous work. The *Strategic Informed* model seems to fit best with tick-time data across multiple time periods, in accordance with the findings of Diebold & Strasser (2013) between 1987 and 2010 for oil futures. However, there are significant differences in the present findings for stock price behaviour. The original authors found parametric estimates to be lower than both statistical and standard RV estimates when sampling every tick. Our samples show that parametric estimates are virtually always higher than their statistical counterparts. This effect is found when sampled per second in the original literature, which may suggest that the increase in market activity has changed MSN characteristics over time such that the MSN models examined here are a better fit even at lower timeframes.

6.2 Limitations & Further Research

There are several important limitations to this study.

The first is that the parametric models examined here are very basic – given the increasingly sophisticated level of market actors, it is likely that models combining more specific agents in a more realistic setting will be necessary. The stochastic nature of financial markets may also mean that it is fundamentally impossible to completely capture every small variation. However, more complex MSN models are certainly possible and should be investigated. This, however, comes with the trade-off that more specific models would only be applicable in even more narrow market conditions: we have shown a preliminary example of what analysis could be undertaken to find suitable models for different economic events.

Second, we cannot claim any causality about the findings here. Both a more rigorous statistical analysis on a larger dataset and more in-depth economic theory is required to make the first steps towards a comprehensive picture of MSN. This study acts as a preliminary step to suggest future avenues of research, and there is a significant amount of extension and verification that can be done: the most obvious of which would be a large-scale analysis of the trends mentioned here to identify whether they are consistent across liquid stocks.

Third, the characteristics of the stocks examined here are limited, and any results cannot necessarily be generalized to the entire market. For example, stocks with lower liquidity or market activity may exhibit significantly different characteristics: the most obvious contrast being the number of datapoints which restrict the possible sampling frequencies and corrections.

Finally, we do not consider several robust statistical estimators for IV that have been recently introduced. This is largely because on the scale studied here, the estimators provide very similar results when compared to parametric estimates. A complete review could verify these conclusions against all available estimators in the literature, although it may be more worth trying to expand the parametric side of research.

Further research aimed at solving these problems will help create a more complete image of market activity at a sub-second level. Supporting statistical estimates with theory maximises the amount of practical information we can gain in a semi-random market, and we hope that this can be a contribution towards uniting both sides of the literature.

Acknowledgements

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Appendix

Code

All the code for the project, including plots and calculations, can be found on Github. https://github.com/MoseslGit/MSN_Calc

Supplementary Results

A full set of supplementary results can be found in the Github Repository attached with this project. The following presents selected plots of findings from analysis of AAPL, MSFT, and AMZN from three earnings days across 2021.

1. Complete Tables for % Noise Captured

Standard Statistical			
	Average	Earnings	Difference
Bid Ask	83.63	85.88	2.25
Learning	57.45	58.33	0.89
Nonstrategic Noisy	46.49	47.32	0.83
Nonstrategic Informed	84.99	87.12	2.13
Strategic Informed	89.76	89.95	0.19

Pre-Averaging			
	Average	Earnings	Difference
Bid Ask	82.56	85.29	2.74
Learning	56.70	57.93	1.23
Nonstrategic Noisy	45.89	47.00	1.11
Nonstrategic Informed	83.89	86.52	2.64
Strategic Informed	88.58	89.32	0.74

Flat Top Realized Kernel			
	Average	Earnings	Difference
Bid Ask	83.72	85.90	2.18

Learning	57.51	58.35	0.84
Nonstrategic Noisy	46.55	47.33	0.79
Nonstrategic Informed	85.08	87.14	2.06
Strategic Informed	89.87	89.97	0.10

2. IV estimates for AAPL between 04/2021 and 10/2021

Date	Standard	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed	Pre-averaging	Flat-Top	Standard Statistical
4/8/2021	28.8028607	3.61849832	11.2794654	14.7972309	2.85274139	0.81428346	0.37688942	0.82496606	0.9092448
4/9/2021	29.7415002	4.62679771	12.0826536	11.8731472	3.5286464	1.3143618	0.44739401	0.98667234	1.04696966
4/12/2021	50.2471754	6.10766073	19.5782041	28.0642454	5.86893251	1.65943082	0.49665283	1.06089875	1.14152247
4/13/2021	191.826925	5.27028179	66.3063053	81.7948044	3.98585152	1.04236663	0.50856915	1.21294012	1.28243667
4/14/2021	177.970484	8.5101768	63.1670593	102.296395	6.49758085	1.04792702	0.67163778	1.49329837	1.47333999
4/15/2021	51.3416171	9.69623817	21.7128269	28.5458147	5.16449179	1.01356683	0.49523785	1.01561127	1.14131481
4/16/2021	27.6019462	3.63976003	10.8926943	12.5147357	2.55089311	1.27407839	0.36551289	0.73437601	0.82057821
4/22/2021	46.7628339	7.79818161	19.257465	27.0333534	6.80555336	3.64947249	0.82766691	1.80462183	1.98871563
4/23/2021	158.85116	2.10617446	53.8906196	85.4501085	2.20083286	1.13977041	0.3744418	0.77868964	0.89086699
4/26/2021	31.3974725	4.74395812	12.6860718	14.951154	3.56102615	0.58257019	0.37813403	0.82583867	0.88920032
4/27/2021	113.981619	4.42527119	39.986447	56.0682276	3.22312615	1.0812519	0.46461798	0.97788146	1.04402829
4/28/2021	21.1417062	3.892544	8.89037248	12.7973913	3.00298162	1.61533496	0.5392665	1.2064643	1.33844941
4/29/2021	585.403801	15.3564749	202.019906	307.345353	10.7680269	5.05158842	1.09376922	2.86725464	2.91038935
4/30/2021	66.6472968	9.11384385	26.4598172	37.2746669	6.4938342	3.17721469	0.62286931	1.73736725	1.53529139
7/7/2021	94.865326	4.66902659	33.7315102	49.2220496	3.55730766	1.76285474	0.5527932	1.23693171	1.34493311
7/8/2021	127.218333	9.78334851	46.868659	83.0149237	10.915792	2.96708183	0.82479669	1.93066033	1.96943266
7/9/2021	19.6251412	3.10285466	7.99748219	10.2572555	2.83562354	0.81684982	0.42254827	0.88435296	0.94478045
7/12/2021	53.6972646	3.27233501	19.3836152	24.143612	2.99486279	1.05475506	0.4167957	0.8906258	0.94922348
7/13/2021	358.189466	5.76217377	121.971261	106.843539	4.42713316	1.5585339	0.65155642	1.24183642	1.41952063
7/14/2021	58.1706167	3.65020678	21.047178	25.1768754	2.54599957	1.52529253	0.64189661	1.28479058	1.41625195
7/15/2021	40.2444181	5.0834019	15.7734031	20.608964	3.48023963	1.6741256	0.70088812	1.40655091	1.57584543
7/21/2021	13.5353948	2.10206056	5.49705098	7.33521156	1.97666011	1.0079041	0.42059787	0.92765469	0.99540151
7/22/2021	66.5139087	3.94798507	23.9613801	29.9955074	3.55133304	0.79285901	0.3682102	0.99142322	0.92882609
7/23/2021	95.9970976	2.63523775	33.1810654	40.1162236	2.40937648	1.16214613	0.30796324	0.6188631	0.75343409
7/26/2021	273.134166	2.04276078	91.9548848	135.875176	1.79279324	0.98466638	0.36070708	0.84176094	0.87192727
7/27/2021	44.7390403	10.4869762	19.9689331	27.9197389	9.94532996	1.41630913	0.6524029	1.45988408	1.48009884
7/28/2021	142.70121	13.0942895	53.5703758	68.9294132	7.94146665	1.74087368	1.23472743	2.9736863	2.98873907

7/29/2021	14.0624497	1.8053181	5.52571726	8.03271647	1.3612531	0.92092145	0.37874798	0.83419474	0.88669712
10/11/2021	23.5310357	1.59946889	8.57103373	11.9180558	1.64834616	0.89118273	0.4097466	0.99299736	0.99779615
10/12/2021	46.5277557	6.47720941	18.5280979	22.3549246	4.37248014	1.63245795	0.56375596	1.70353287	1.40667288
10/13/2021	191.521456	10.2024861	68.4568967	96.9621677	11.4571198	-1.1869603	0.55066211	2.55652358	1.53646053
10/14/2021	55.0799244	4.52197477	20.4262884	20.6412034	3.24662048	0.78215134	0.27209085	0.86923647	0.71466233
10/15/2021	41.3144267	2.96013808	15.1192822	21.0190497	2.30471872	0.91733917	0.27221996	0.80703534	0.66935052
10/18/2021	102.898158	9.61370524	38.7094882	50.170357	9.20840634	0.61802815	0.3818968	1.10165041	1.06823265
10/19/2021	56.1566558	3.70309572	20.401695	31.1617312	3.54442097	0.8467141	0.33017797	0.83245748	0.8346182
10/25/2021	76.2976347	7.15063674	28.7130902	37.5845954	7.21727615	0.00917782	0.32946445	0.99599098	1.1017175
10/26/2021	66.3502188	8.3624559	25.9963768	32.0775999	6.33415314	-0.136572	0.42223346	1.38418773	0.88119278
10/27/2021	24.059531	4.15092064	9.97726678	12.2201534	3.5304732	0.9169258	0.35358377	1.07283157	0.87949431
10/28/2021	155.403281	8.64338154	55.7151622	78.9660303	8.44318866	2.65782512	0.46439286	1.7476145	1.28898281
10/29/2021	1138.158	61.8967721	407.403572	585.051369	51.2548527	12.6866321	1.5331389	11.9158569	4.18524458
11/1/2021	267.543063	36.8948893	106.368831	141.136734	32.8625613	4.14727165	0.61891045	3.1002096	1.54500326
11/2/2021	114.701626	8.12045333	41.9301943	68.4600302	7.81102857	-0.9429794	0.51359602	1.82110726	1.11377514

3. IV estimates for MSFT between 04/2021 and 10/2021

Date	Standard	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed	Pre-averaging	Flat-Top	Standard Statistical
4/22/2021	21.8644026	2.56325042	8.47368558	9.9829506	2.05775862	1.5961849	0.65031203	1.42965959	1.51985607
4/23/2021	201.75005	1.36069714	67.8561336	87.5288184	1.19913801	0.57187532	0.28976427	0.68266895	0.70977334
4/26/2021	214.777667	43.7370094	92.4468025	104.396755	32.5780554	7.54032825	0.41808829	2.70232828	1.14431172
4/27/2021	695.899596	3.31490601	233.442167	262.892291	2.26924355	1.13799973	0.32449302	0.78219212	0.7559095
4/28/2021	116.860518	7.63056088	42.4203233	38.4323521	5.80128693	1.5692139	0.58916764	1.56004897	1.50873492
4/29/2021	18.9381196	4.3905488	8.42748959	10.2804169	3.2613697	0.97967322	0.44971476	1.30019419	1.11714224
4/30/2021	30.1841471	4.64831077	12.2390859	13.8067822	4.47933349	0.97895739	0.41930946	1.2600352	1.11693427
7/21/2021	147.874442	1.69865643	50.0493409	79.1166406	1.60108661	0.48584932	0.24178327	0.58091782	0.59498043
7/22/2021	41.7720554	1.17944457	14.4531965	18.0155222	0.94303351	0.51739031	0.25107104	0.60920473	0.60799661
7/23/2021	17.4261222	1.42101423	6.45791585	9.40873715	2.13589479	1.19497535	0.20680346	0.60286777	0.52828907
7/26/2021	26.2756557	3.38359937	10.3298195	13.3349896	2.78404985	0.83236136	0.29286572	0.66682771	0.66738205
7/27/2021	35.188324	3.02735814	13.1146587	17.1032845	2.5305766	1.04629674	0.46725267	1.12936318	0.99804749
7/28/2021	23.878264	6.54610927	11.161406	13.1796846	4.99080803	2.03844614	0.80341358	1.92139689	1.91720666
7/29/2021	9.63888454	1.24062709	3.78906836	5.35857163	1.29334512	0.6500717	0.30380397	0.81418614	0.69340079
10/21/2021	63.7433116	2.01664057	22.1536079	27.0939417	1.55114395	0.48114637	0.21724934	0.50645957	0.51640038
10/22/2021	17.39466	2.26138361	6.84880636	7.85162407	1.97829147	1.00587685	0.29776309	0.84436042	0.71656168
10/25/2021	35.5847007	2.99406607	13.230661	18.9858477	2.59340418	0.46646416	0.23959051	0.70787988	0.61607759
10/26/2021	47.9951773	5.37092145	18.477961	23.5992448	3.4625215	0.58068232	0.33448174	0.84731929	0.8586689
10/27/2021	202.744689	17.5089274	75.5939573	97.4348404	19.974772	6.23696303	0.80890491	1.98726707	1.68066479
10/28/2021	390.007412	8.25117779	133.695706	218.499447	6.00369003	1.96644355	0.4350482	1.14327259	1.23126619
10/29/2021	129.635368	35.1947389	60.4103053	65.9091755	28.0862612	2.54171929	0.78705974	4.02415564	1.95002251

4. IV estimates for AMZN between 04/2021 and 10/2021

Date	Standard	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed	Pre-averaging	Flat-Top	Standard Statistical
4/26/2021	55.902697	6.50566194	21.6423264	27.4855918	4.87350804	1.50442683	0.68439252	1.54474097	1.4779864
4/27/2021	82.3803216	5.48284148	29.9522132	37.9749276	5.13220246	0.97095552	0.58610806	1.15351183	1.26878373
4/28/2021	63.8850926	7.83633882	24.9263232	35.5885985	5.72156748	1.99060958	0.74613041	1.6948558	1.68099205
4/29/2021	269.107797	5.7580643	92.2801222	131.295365	4.71084937	2.0646682	0.71328301	1.61773832	1.70397698
4/30/2021	263.096576	7.7518749	91.1783093	152.269214	7.70251141	2.28406616	1.08517794	2.13132323	2.33274698
5/3/2021	216.318383	5.44773391	74.5478401	91.6917656	3.38871493	2.91697714	0.88367535	1.57589126	1.94368107
5/4/2021	77.4950795	7.41995315	29.2381706	35.7152201	6.00220395	2.33129188	1.03217268	2.1983859	2.34958109
7/26/2021	27.6327458	2.85133947	10.5233182	11.7178944	2.24154539	1.36935332	0.46995806	0.93377649	1.11751347
7/27/2021	52.3474331	5.62646275	20.0427162	29.3489761	4.43997807	2.07508438	0.83215015	1.78530018	1.95179152
7/28/2021	50.4737901	5.24983908	19.241659	23.6417516	3.92513823	1.58717742	0.70286405	1.61904202	1.71781244
7/29/2021	36.2659101	4.19493321	14.0278209	18.3445472	2.96987555	0.72326725	0.37156963	0.91501449	0.89462749
7/30/2021	308.252455	9.65894446	107.088994	104.979781	18.7062768	4.03041239	1.23067385	4.20062948	2.48555868
8/2/2021	575.750048	6.82434908	194.961758	219.831616	4.93051633	-0.76472371	0.56377182	0.1216132	1.38137602
8/3/2021	185.028673	4.68315607	63.7753368	79.1781225	3.70275757	1.66404777	0.6035013	1.3275601	1.45652976
10/25/2021	80.5171467	5.28552177	29.240722	31.8133771	4.69274617	0.93628095	0.38562656	1.17277586	0.95435284
10/26/2021	86.3344019	5.00229258	31.0451596	41.1512788	3.89811196	-0.92005346	0.55407973	1.245459	1.29309574
10/27/2021	71.0068978	3.87456559	25.4228968	37.2567718	3.58680585	0.46274209	0.5327473	1.28225064	1.2841933
10/28/2021	132.681818	3.58364177	45.834472	60.3206385	2.61721384	1.21755153	0.60894118	1.47054927	1.4444676
10/29/2021	527.804801	35.9373501	192.278013	290.722909	30.7548937	14.1276445	1.4616129	4.904545	4.10634634
11/1/2021	123.41713	5.63617556	43.6827317	61.0420185	4.64445059	2.42876177	0.50250084	1.32821004	1.27617614
11/2/2021	67.3218425	3.07264368	23.8273307	38.1363289	2.83869666	1.95413173	0.41046933	1.00937119	0.90422734

5. % Noise Captured relative to Flat-Top Realized Kernel Estimator between 04/2021 and 10/2021 for AAPL

			% Noise Captured For Flat-Top Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			AAPL			
4/8/2021	28.8028607	0.90015216	0.62633002	0.50059627	0.92752223	1.00038182
4/9/2021	29.7415002	0.8734082	0.61411763	0.62140358	0.91159836	0.98860402
4/12/2021	50.2471754	0.89739492	0.623527	0.45099836	0.90224847	0.98783132
4/13/2021	191.826925	0.97871435	0.65850688	0.57725104	0.98545274	1.00089486
4/14/2021	177.970484	0.96023918	0.65052842	0.42880381	0.97164346	1.00252368
4/15/2021	51.3416171	0.8275121	0.58873717	0.45296268	0.91755991	1.00004062
4/16/2021	27.6019462	0.89186279	0.62191154	0.56153982	0.93238997	0.9799125
4/22/2021	46.7628339	0.86668598	0.61179855	0.43884042	0.88876489	0.95896521

4/23/2021	158.85116	0.99160205	0.66400266	0.46435063	0.99100322	0.99771573
4/26/2021	31.3974725	0.87183807	0.61205106	0.53796008	0.91053185	1.00795733
4/27/2021	113.981619	0.96949314	0.65480287	0.51249094	0.98013124	0.99908525
4/28/2021	21.1417062	0.86525974	0.61455656	0.41857104	0.90988234	0.97949006
4/29/2021	585.403801	0.97856062	0.65812849	0.47732361	0.98643729	0.99625031
4/30/2021	66.6472968	0.88635827	0.61912684	0.45251366	0.92672204	0.97781776
7/7/2021	94.865326	0.96334344	0.65294098	0.48749396	0.97521718	0.99438287
7/8/2021	127.218333	0.93732274	0.64132147	0.35281531	0.92828399	0.99172767
7/9/2021	19.6251412	0.88162175	0.62044663	0.49986615	0.89588108	1.00360194
7/12/2021	53.6972646	0.95489754	0.64979802	0.5596579	0.96015204	0.99689188
7/13/2021	358.189466	0.98733613	0.66177272	0.70415351	0.99107629	0.99911276
7/14/2021	58.1706167	0.95841818	0.65259558	0.57999933	0.97782912	0.9957722
7/15/2021	40.2444181	0.90532819	0.63008133	0.50557498	0.94660652	0.99311047
7/21/2021	13.5353948	0.90685041	0.63757214	0.49177594	0.91679671	0.99363491
7/22/2021	66.5139087	0.95487714	0.64943398	0.55734151	0.96093082	1.00303047
7/23/2021	95.9970976	0.97885917	0.65859923	0.58588707	0.98122723	0.99430391
7/26/2021	273.134166	0.9955893	0.665385	0.50408674	0.99650731	0.99947518
7/27/2021	44.7390403	0.79142172	0.57233341	0.3886236	0.80393689	1.00100683
7/28/2021	142.70121	0.92756901	0.63789032	0.52796897	0.96444666	1.00882298
7/29/2021	14.0624497	0.92658719	0.64534078	0.45582227	0.96015662	0.99344383
10/11/2021	23.5310357	0.9730912	0.66376682	0.51526134	0.97092254	1.00451746
10/12/2021	46.5277557	0.8935023	0.62465462	0.53928054	0.94045748	1.00158564
10/13/2021	191.521456	0.95953766	0.65125607	0.50040654	0.95289816	1.01981047
10/14/2021	55.0799244	0.93261959	0.63923992	0.63527548	0.95614547	1.00160642
10/15/2021	41.3144267	0.94684667	0.64667567	0.50102898	0.96302691	0.99727695
10/18/2021	102.898158	0.91638166	0.63055866	0.51797259	0.92036312	1.00475087
10/19/2021	56.1566558	0.94811243	0.64628069	0.45179009	0.95098052	0.99974231
10/11/2021	23.5310357	0.9730912	0.66376682	0.51526134	0.97092254	1.00451746
10/12/2021	46.5277557	0.8935023	0.62465462	0.53928054	0.94045748	1.00158564
10/13/2021	191.521456	0.95953766	0.65125607	0.50040654	0.95289816	1.01981047
10/14/2021	55.0799244	0.93261959	0.63923992	0.63527548	0.95614547	1.00160642
10/15/2021	41.3144267	0.94684667	0.64667567	0.50102898	0.96302691	0.99727695
10/18/2021	102.898158	0.91638166	0.63055866	0.51797259	0.92036312	1.00475087
10/19/2021	56.1566558	0.94811243	0.64628069	0.45179009	0.95098052	0.99974231

6. % Noise Captured relative to Flat-Top Realized Kernel Estimator between 04/2021 and 10/2021 for MSFT

			% Noise Captured For Flat-Top Estimator			
Date	**Standard RV**	**Bid-Ask**	**Restricted Learning**	**Nonstrategic Noisy**	**Nonstrategic Informed**	**Strategic Informed**
			MSFT			
4/22/2021	21.8644026	0.9445263	0.655291678	0.58143388	0.96926318	0.99185087
4/23/2021	201.75005	0.99662786	0.665915653	0.5680744	0.99743136	1.00055103

4/26/2021	214.777667	0.80650894	0.576827391	0.52047972	0.85912682	0.97718735
4/27/2021	695.899596	0.99635642	0.665293987	0.62292687	0.99786072	0.99948813
4/28/2021	116.860518	0.9473505	0.645619185	0.68020683	0.96321578	0.99992051
4/29/2021	18.9381196	0.82478922	0.595910788	0.4908572	0.88880917	1.01817226
4/30/2021	30.1841471	0.88285637	0.620418745	0.56621842	0.88869846	1.00971777
7/21/2021	147.874442	0.99241149	0.664150726	0.46680804	0.99307391	1.00064544
7/22/2021	41.7720554	0.98614674	0.663677525	0.57713528	0.99189005	1.00223052
7/23/2021	17.4261222	0.95136812	0.651966977	0.47656564	0.90887453	0.96480422
7/26/2021	26.2756557	0.89391269	0.622669501	0.50532051	0.91732452	0.99353607
7/27/2021	35.188324	0.94427326	0.648101551	0.53099211	0.95885918	1.0024389
7/28/2021	23.878264	0.78937285	0.579174519	0.48725437	0.86020724	0.99466913
7/29/2021	9.63888454	0.95167643	0.662891344	0.48503787	0.94570251	1.01859717
10/21/2021	63.7433116	0.97611866	0.657681437	0.57955715	0.98347982	1.00040029
10/22/2021	17.39466	0.91438081	0.637200166	0.57660805	0.93148577	0.99024088
10/25/2021	35.5847007	0.9344497	0.640942583	0.47592793	0.94593761	1.00692195
10/26/2021	47.9951773	0.90405498	0.626056358	0.51743459	0.9445319	1.00565534
10/27/2021	202.744689	0.9226845	0.633355074	0.52456267	0.91040179	0.97883169
10/28/2021	390.007412	0.98172137	0.659129192	0.44104855	0.98750099	0.99788314
10/29/2021	129.635368	0.75184872	0.55110576	0.50732885	0.80843983	1.01180178

7. % Noise Captured relative to Flat-Top Realized Kernel Estimator between 04/2021 and 10/2021 for AMZN

			% Noise Captured For Flat-Top Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			MSFT			
4/26/2021	55.902697	0.90873607	0.63027334	0.5227773	0.9387621	1.00074164
4/27/2021	82.3803216	0.94670073	0.64545325	0.54668396	0.95101752	1.00224749
4/28/2021	63.8850926	0.90124683	0.6264451	0.45499898	0.93525171	0.99524437
4/29/2021	269.107797	0.98452157	0.6610626	0.51520581	0.98843654	0.99832917
4/30/2021	263.096576	0.97846245	0.65877838	0.42468245	0.97865161	0.9994147
5/3/2021	216.318383	0.98196983	0.66018859	0.58035378	0.99155815	0.99375491
5/4/2021	77.4950795	0.93065343	0.64089015	0.55486977	0.94948227	0.9982349
7/26/2021	27.6327458	0.92817839	0.64082727	0.59608486	0.951018	0.98368563
7/27/2021	52.3474331	0.92403084	0.63891128	0.45485536	0.94749672	0.99426875
7/28/2021	50.4737901	0.9256818	0.63928548	0.54922069	0.95279689	1.00065223
7/29/2021	36.2659101	0.90721823	0.62906721	0.5069564	0.94187245	1.00542411
7/30/2021	308.252455	0.98204808	0.66160912	0.66854614	0.95229219	1.00055983
8/2/2021	575.750048	0.98835579	0.66151751	0.6183128	0.99164582	1.00153977
8/3/2021	185.028673	0.98173339	0.66005771	0.57621072	0.98707032	0.99816829
10/25/2021	80.5171467	0.94816588	0.64625158	0.61382766	0.9556368	1.00298061
10/26/2021	86.3344019	0.95584816	0.64978175	0.53101051	0.96882494	1.02544999

10/27/2021	71.0068978	0.96282068	0.6537717	0.48404872	0.96694777	1.0117535
10/28/2021	132.681818	0.9838955	0.66188938	0.55148601	0.99126093	1.00192817
10/29/2021	527.804801	0.94065253	0.64166499	0.45339793	0.95056352	0.98236165
11/1/2021	123.41713	0.96471453	0.65308464	0.51089904	0.9728375	0.99098565
11/2/2021	67.3218425	0.9688856	0.65590244	0.44012104	0.97241355	0.9857529

8. % Noise Captured relative to Pre-Averaging Estimator between 04/2021 and 10/2021 for AAPL

			% Noise Captured For Pre-Averaging Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			AAPL			
4/8/2021	28.8028607	0.88596313	0.61645722	0.49270541	0.91290176	0.98461287
4/9/2021	29.7415002	0.85732954	0.60281227	0.6099641	0.89481664	0.9704047
4/12/2021	50.2471754	0.88721711	0.61645526	0.44588336	0.89201561	0.97662782
4/13/2021	191.826925	0.97511105	0.65608247	0.57512579	0.98182463	0.9972099
4/14/2021	177.970484	0.95578911	0.64751366	0.42681659	0.96714055	0.99787766
4/15/2021	51.3416171	0.81904316	0.5827119	0.44832695	0.90816939	0.98980598
4/16/2021	27.6019462	0.87978429	0.61348899	0.55393488	0.91976261	0.96664154
4/22/2021	46.7628339	0.8482532	0.59878674	0.4295071	0.86986253	0.93856982
4/23/2021	158.85116	0.98907264	0.6623089	0.46316615	0.98847534	0.99517072
4/26/2021	31.3974725	0.85925476	0.60321727	0.53019566	0.89739007	0.9934094
4/27/2021	113.981619	0.9651096	0.6518422	0.51017373	0.9756996	0.99456792
4/28/2021	21.1417062	0.83723881	0.59465451	0.40501586	0.88041634	0.94776985
4/29/2021	585.403801	0.97559052	0.65613095	0.47587485	0.98344328	0.99322651
4/30/2021	66.6472968	0.87139647	0.60867593	0.44487519	0.9110789	0.96131212
7/7/2021	94.865326	0.96334344	0.65294098	0.48749396	0.97521718	0.99438287
7/8/2021	127.218333	0.93732274	0.64132147	0.35281531	0.92828399	0.99172767
7/9/2021	19.6251412	0.88162175	0.62044663	0.49986615	0.89588108	1.00360194
7/12/2021	53.6972646	0.95489754	0.64979802	0.5596579	0.96015204	0.99689188
7/13/2021	358.189466	0.98733613	0.66177272	0.70415351	0.99107629	0.99911276
7/14/2021	58.1706167	0.95841818	0.65259558	0.57999933	0.97782912	0.9957722
7/15/2021	40.2444181	0.90532819	0.63008133	0.50557498	0.94660652	0.99311047
7/21/2021	13.5353948	0.90685041	0.63757214	0.49177594	0.91679671	0.99363491
7/22/2021	66.5139087	0.95487714	0.64943398	0.55734151	0.96093082	1.00303047
7/23/2021	95.9970976	0.97885917	0.65859923	0.58588707	0.98122723	0.99430391
7/26/2021	273.134166	0.9955893	0.665385	0.50408674	0.99650731	0.99947518
7/27/2021	44.7390403	0.79142172	0.57233341	0.3886236	0.80393689	1.00100683
7/28/2021	142.70121	0.92756901	0.63789032	0.52796897	0.96444666	1.00882298
7/29/2021	14.0624497	0.92658719	0.64534078	0.45582227	0.96015662	0.99344383
10/11/2021	23.5310357	0.94854429	0.64702283	0.50226352	0.94643034	0.9791778
10/12/2021	46.5277557	0.87134598	0.60916495	0.52590791	0.9171368	0.97674915

10/13/2021	191.521456	0.94945916	0.6444156	0.49515052	0.94288939	1.00909889
10/14/2021	55.0799244	0.92245846	0.63227524	0.62835399	0.94572802	0.99069366
10/15/2021	41.3144267	0.93450844	0.63824893	0.49450014	0.95047784	0.98428157
10/18/2021	102.898158	0.90994786	0.62613159	0.51433597	0.91390137	0.99769664
10/19/2021	56.1566558	0.93958211	0.640466	0.44772527	0.9424244	0.99074747
10/25/2021	76.2976347	0.91826678	0.63191907	0.51410617	0.91738181	1.0131048
10/26/2021	66.3502188	0.89258589	0.62115295	0.52754676	0.92380687	1.02340854
10/27/2021	24.059531	0.8660926	0.61262663	0.5150534	0.89308419	1.00678243
10/28/2021	155.403281	0.95512195	0.64877607	0.49745807	0.95642481	0.9940763
10/29/2021	1138.158	0.95562152	0.64884309	0.49110809	0.96507057	0.99931562
11/1/2021	267.543063	0.87220422	0.60948606	0.47801	0.88745261	0.9960405
11/2/2021	114.701626	0.94419457	0.64467662	0.40965081	0.94693574	1.02448684

9. % Noise Captured relative to Pre-Averaging Estimator between 04/2021 and 10/2021 for MSFT

			% Noise Captured For Pre-Averaging Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			MSFT			
4/22/2021	21.8644026	0.90982699	0.631218057	0.5600736	0.93365511	0.95541299
4/23/2021	201.75005	0.99468415	0.664616929	0.56696649	0.99548609	0.99859967
4/26/2021	214.777667	0.79791469	0.570680653	0.51493343	0.84997187	0.96677433
4/27/2021	695.899596	0.99570081	0.664856214	0.62251697	0.99720411	0.99883045
4/28/2021	116.860518	0.93944	0.640228178	0.67452701	0.9551728	0.99157104
4/29/2021	18.9381196	0.78684835	0.568498479	0.46827743	0.84792334	0.97133563
4/30/2021	30.1841471	0.85791956	0.602894644	0.55022524	0.86359663	0.98119768
7/21/2021	147.874442	0.99013177	0.662625071	0.46573571	0.99079266	0.9983468
7/22/2021	41.7720554	0.97764086	0.657953063	0.57215727	0.98333463	0.99358591
7/23/2021	17.4261222	0.92948555	0.63697098	0.46560408	0.88796936	0.9426126
7/26/2021	26.2756557	0.88104689	0.613707618	0.4980476	0.90412176	0.97923642
7/27/2021	35.188324	0.92626652	0.635742634	0.5208664	0.9405743	0.98332298
7/28/2021	23.878264	0.7511275	0.551113343	0.46364675	0.81852994	0.94647712
7/29/2021	9.63888454	0.89964488	0.626648708	0.45851912	0.89399758	0.96290683
10/21/2021	63.7433116	0.97611866	0.657681437	0.57955715	0.98347982	1.00040029
10/22/2021	17.39466	0.91438081	0.637200166	0.57660805	0.93148577	0.99024088
10/25/2021	35.5847007	0.9344497	0.640942583	0.47592793	0.94593761	1.00692195
10/26/2021	47.9951773	0.90405498	0.626056358	0.51743459	0.9445319	1.00565534
10/27/2021	202.744689	0.9226845	0.633355074	0.52456267	0.91040179	0.97883169
10/28/2021	390.007412	0.98172137	0.659129192	0.44104855	0.98750099	0.99788314
10/29/2021	129.635368	0.75184872	0.55110576	0.50732885	0.80843983	1.01180178

10.% Noise Captured relative to Pre-Averaging Estimator between 04/2021 and 10/2021 for AMZN

			% Noise Captured For Pre-Averaging Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			AMZN			
4/26/2021	55.902697	0.89457718	0.62045314	0.51463198	0.92413538	0.98514923
4/27/2021	82.3803216	0.9401335	0.64097576	0.54289163	0.94442034	0.99529493
4/28/2021	63.8850926	0.8877047	0.61703215	0.44816217	0.92119862	0.98028984
4/29/2021	269.107797	0.98120386	0.65883491	0.51346963	0.98510563	0.99496493
4/30/2021	263.096576	0.9745557	0.65614805	0.4229868	0.9747441	0.99542429
5/3/2021	216.318383	0.97881466	0.65806733	0.57848904	0.98837216	0.99056187
5/4/2021	77.4950795	0.91645909	0.63111528	0.54640689	0.93500075	0.98300981
7/26/2021	27.6327458	0.91232927	0.62988482	0.58590641	0.93477888	0.9668887
7/27/2021	52.3474331	0.90693417	0.62708996	0.4464395	0.92996587	0.97587251
7/28/2021	50.4737901	0.90864194	0.62751758	0.53911069	0.9352579	0.98223233
7/29/2021	36.2659101	0.89348283	0.61954305	0.49928102	0.92761238	0.99020186
7/30/2021	308.252455	0.9725483	0.65520909	0.662079	0.94308025	0.99088098
8/2/2021	575.750048	0.98911557	0.66202604	0.61878812	0.99240812	1.00230968
8/3/2021	185.028673	0.97787908	0.6574663	0.5739485	0.98319505	0.99424945
10/25/2021	80.5171467	0.94816588	0.64625158	0.61382766	0.9556368	1.00298061
10/26/2021	86.3344019	0.95584816	0.64978175	0.53101051	0.96882494	1.02544999
10/27/2021	71.0068978	0.96282068	0.6537717	0.48404872	0.96694777	1.0117535
10/28/2021	132.681818	0.9838955	0.66188938	0.55148601	0.99126093	1.00192817
10/29/2021	527.804801	0.94065253	0.64166499	0.45339793	0.95056352	0.98236165
11/1/2021	123.41713	0.96471453	0.65308464	0.51089904	0.9728375	0.99098565
11/2/2021	67.3218425	0.9688856	0.65590244	0.44012104	0.97241355	0.9857529

11.% Noise Captured relative to Standard Statistical Estimator between 04/2021 and 10/2021 for AAPL

			% Noise Captured For Standard Statistical Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			AAPL			
4/8/2021	28.8028607	0.90287191	0.62822243	0.50210879	0.93032468	1.00340441
4/9/2021	29.7415002	0.87524354	0.6154081	0.62270937	0.91351395	0.99068142
4/12/2021	50.2471754	0.8988683	0.62455073	0.45173883	0.90372982	0.98945318
4/13/2021	191.826925	0.97907132	0.65874705	0.57746158	0.98581216	1.00125992
4/14/2021	177.970484	0.96013059	0.65045486	0.42875532	0.97153359	1.00241031
4/15/2021	51.3416171	0.82958423	0.59021139	0.45409691	0.91985751	1.00254477
4/16/2021	27.6019462	0.89473346	0.62391331	0.56334727	0.93539109	0.98306658

4/22/2021	46.7628339	0.87024946	0.61431403	0.44064476	0.89241915	0.96290811
4/23/2021	158.85116	0.99230625	0.66447421	0.4646804	0.99170699	0.99842427
4/26/2021	31.3974725	0.87364877	0.6133222	0.53907735	0.91242291	1.01005072
4/27/2021	113.981619	0.97006096	0.65518639	0.5127911	0.98070529	0.99967041
4/28/2021	21.1417062	0.87102654	0.61865247	0.42136074	0.91594654	0.98601818
4/29/2021	585.403801	0.97863309	0.65817722	0.47735896	0.98651034	0.99632408
4/30/2021	66.6472968	0.88360745	0.61720537	0.45110928	0.92384595	0.97478309
7/7/2021	94.865326	0.96445595	0.65369503	0.48805694	0.9763434	0.99553122
7/8/2021	127.218333	0.9376129	0.64152	0.35292453	0.92857135	0.99203467
7/9/2021	19.6251412	0.88447363	0.62245366	0.50148312	0.89877909	1.0068484
7/12/2021	53.6972646	0.95595834	0.65051988	0.56027962	0.96121867	0.99799933
7/13/2021	358.189466	0.98782786	0.66210231	0.70450421	0.99156988	0.99961036
7/14/2021	58.1706167	0.96063818	0.6541072	0.5813428	0.98009408	0.99807873
7/15/2021	40.2444181	0.9092918	0.63283988	0.50778844	0.95075085	0.9974584
7/21/2021	13.5353948	0.91174963	0.6410166	0.49443274	0.92174967	0.99900298
7/22/2021	66.5139087	0.95396577	0.64881413	0.55680956	0.96001367	1.00207314
7/23/2021	95.9970976	0.98024222	0.65952978	0.58671487	0.98261362	0.99570877
7/26/2021	273.134166	0.99569961	0.66545872	0.50414259	0.99661772	0.99958592
7/27/2021	44.7390403	0.79179154	0.57260086	0.3888052	0.80431257	1.0014746
7/28/2021	142.70121	0.92766894	0.63795904	0.52802586	0.96455057	1.00893167
7/29/2021	14.0624497	0.93027943	0.64791232	0.45763862	0.96398263	0.99740248
10/11/2021	23.5310357	0.97329844	0.66390818	0.51537108	0.97112932	1.00473138
10/12/2021	46.5277557	0.88762378	0.6205449	0.53573251	0.93427003	0.99499602
10/13/2021	191.521456	0.95438574	0.64775936	0.49771977	0.94778188	1.01433493
10/14/2021	55.0799244	0.92996792	0.6374224	0.63346923	0.95342691	0.9987586
10/15/2021	41.3144267	0.94363924	0.64448506	0.49933175	0.95976467	0.99389868
10/18/2021	102.898158	0.91608093	0.63035173	0.51780261	0.92006109	1.00442114
10/19/2021	56.1566558	0.94814946	0.64630593	0.45180774	0.95101766	0.99978135
10/25/2021	76.2976347	0.91955788	0.63280755	0.51482901	0.91867167	1.01452924
10/26/2021	66.3502188	0.8857282	0.61638067	0.52349364	0.91670931	1.01554574
10/27/2021	24.059531	0.8588688	0.60751691	0.5107575	0.88563526	0.99838518
10/28/2021	155.403281	0.95227958	0.64684536	0.49597767	0.95357857	0.99111801
10/29/2021	1138.158	0.94910678	0.64441974	0.48776007	0.95849141	0.99250301
11/1/2021	267.543063	0.86710472	0.60592259	0.47521523	0.88226396	0.99021696
11/2/2021	114.701626	0.9383149	0.64066211	0.40709984	0.941039	1.01810717

12. % Noise Captured relative to Standard Statistical Estimator between 04/2021 and 10/2021 for MSFT

			% Noise Captured For Standard Statistical Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			MSFT			

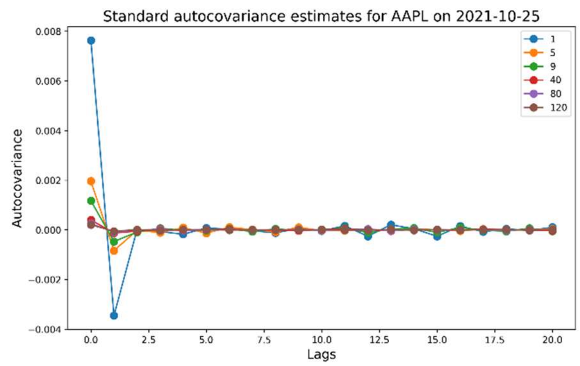
4/22/2021	21.8644026	0.94871381	0.658196879	0.58401164	0.97356036	0.99624819
4/23/2021	201.75005	0.99676222	0.666005432	0.56815099	0.99756584	1.00068592
4/26/2021	214.777667	0.80062712	0.57262062	0.51668389	0.85286126	0.97006078
4/27/2021	695.899596	0.99631875	0.665268833	0.62290331	0.99782299	0.99945034
4/28/2021	116.860518	0.94692907	0.645331982	0.67990424	0.96278729	0.9994757
4/29/2021	18.9381196	0.81631723	0.589789764	0.48581526	0.87967958	1.00771389
4/30/2021	30.1841471	0.87850997	0.617364359	0.56343087	0.8843233	1.00474682
7/21/2021	147.874442	0.99250625	0.664214141	0.46685261	0.99316873	1.00074098
7/22/2021	41.7720554	0.98611779	0.663658047	0.57711834	0.99186094	1.0022011
7/23/2021	17.4261222	0.94716925	0.649089517	0.47446232	0.9048632	0.96054605
7/26/2021	26.2756557	0.89393204	0.62268298	0.50533145	0.91734438	0.99355758
7/27/2021	35.188324	0.94064656	0.645612367	0.52895271	0.95517646	0.9985888
7/28/2021	23.878264	0.78922223	0.579064011	0.4871614	0.86004311	0.99447934
7/29/2021	9.63888454	0.93882653	0.65394073	0.4784887	0.93293327	1.00484368
10/21/2021	63.7433116	0.97627213	0.657784841	0.57964827	0.98363444	1.00055758
10/22/2021	17.39466	0.90737422	0.632317512	0.57218969	0.9243481	0.98265299
10/25/2021	35.5847007	0.93199651	0.639259932	0.47467848	0.94345426	1.00427851
10/26/2021	47.9951773	0.90427266	0.626207101	0.51755918	0.94475932	1.00589748
10/27/2021	202.744689	0.9212775	0.632389271	0.52376276	0.90901352	0.97733907
10/28/2021	390.007412	0.98194356	0.659278376	0.44114838	0.98772449	0.998109
10/29/2021	129.635368	0.73963562	0.542153545	0.49908776	0.79530745	0.99536598

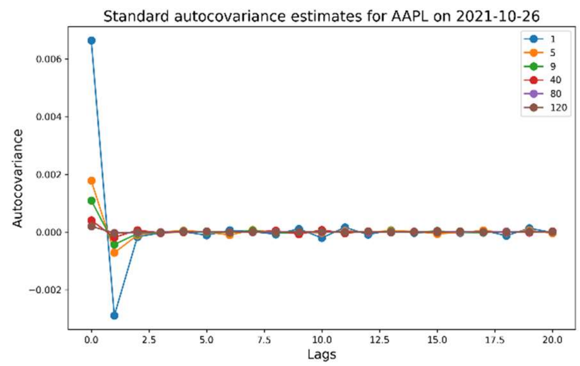
13.% Noise Captured relative to Standard Statistical Estimator between 04/2021 and 10/2021 for AMZN

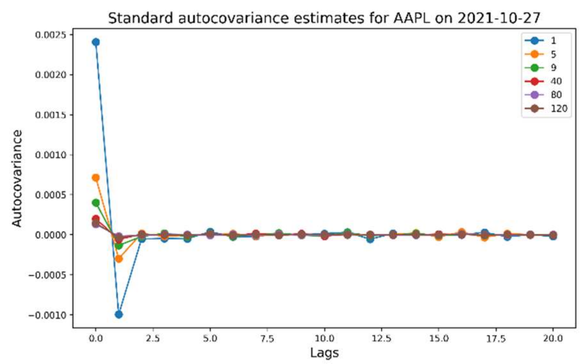
			% Noise Captured For Standard Statistical Estimator			
Date	Standard RV	Bid-Ask	Restricted Learning	Nonstrategic Noisy	Nonstrategic Informed	Strategic Informed
			AMZN			
4/26/2021	55.902697	0.90762146	0.62950028	0.52213608	0.93761066	0.99951418
4/27/2021	82.3803216	0.94804614	0.64637054	0.54746088	0.95236906	1.00367184
4/28/2021	63.8850926	0.90104596	0.62630549	0.45489757	0.93504326	0.99502255
4/29/2021	269.107797	0.98483908	0.6612758	0.51537196	0.98875531	0.99865114
4/30/2021	263.096576	0.97921825	0.65928725	0.42501049	0.97940756	1.00018669
5/3/2021	216.318383	0.98365454	0.66132124	0.58134946	0.99325931	0.99545984
5/4/2021	77.4950795	0.93252594	0.64217964	0.55598619	0.95139266	1.00024338
7/26/2021	27.6327458	0.93461019	0.64526787	0.60021542	0.95760807	0.99050207
7/27/2021	52.3474331	0.92708355	0.64102204	0.45635806	0.95062695	0.9975535
7/28/2021	50.4737901	0.92755705	0.64058055	0.55033331	0.95472707	1.00267936
7/29/2021	36.2659101	0.90669533	0.62870463	0.50666421	0.94132958	1.00484461
7/30/2021	308.252455	0.97653969	0.6578981	0.66479621	0.94695071	0.99494761
8/2/2021	575.750048	0.99052356	0.66296842	0.61966895	0.9938208	1.00373645
8/3/2021	185.028673	0.98242312	0.66052144	0.57661554	0.98776379	0.99886956

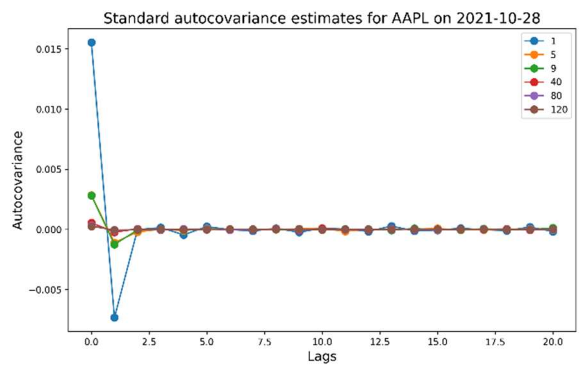
10/25/2021	80.5171467	0.94556289	0.64447743	0.61214253	0.9530133	1.00022714
10/26/2021	86.3344019	0.95638359	0.65014573	0.53130796	0.96936764	1.0260244
10/27/2021	71.0068978	0.96284751	0.65378991	0.4840622	0.96697471	1.01178169
10/28/2021	132.681818	0.98369996	0.66175784	0.55137641	0.99106393	1.00172905
10/29/2021	527.804801	0.93921883	0.64068699	0.45270688	0.94911471	0.98086437
11/1/2021	123.41713	0.96430354	0.65280642	0.51068139	0.97242305	0.99056348
11/2/2021	67.3218425	0.96735179	0.6548641	0.43942429	0.97087415	0.98419238

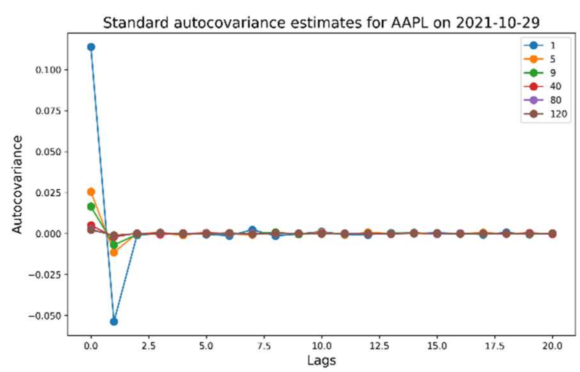
14.Sample autocovariance profiles for AAPL across 10/2021

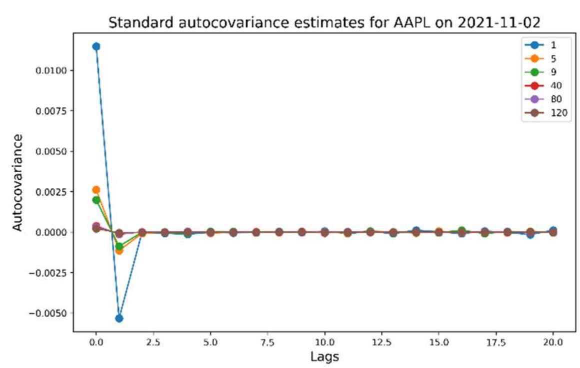


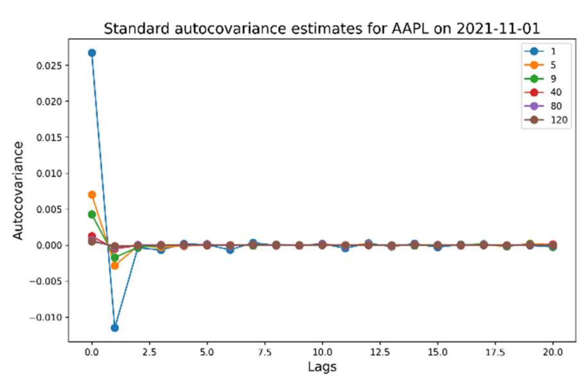


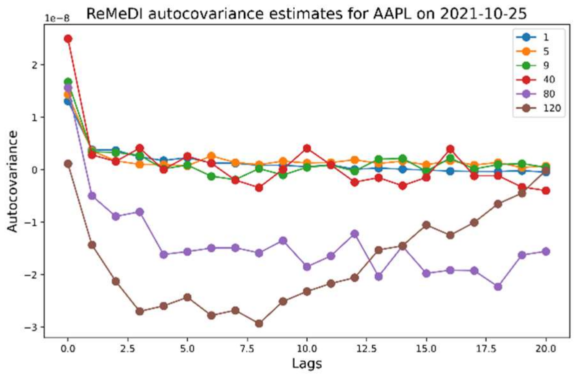


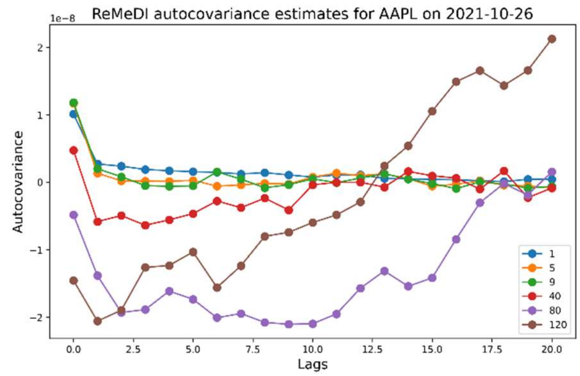


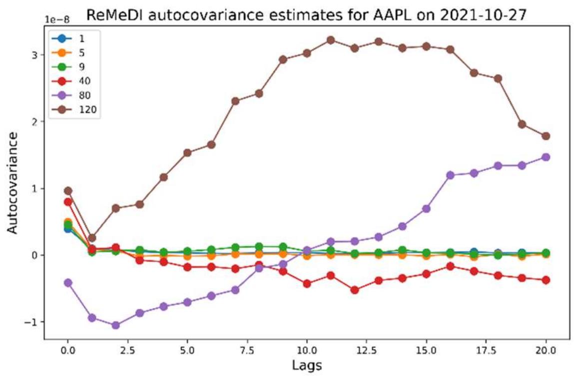


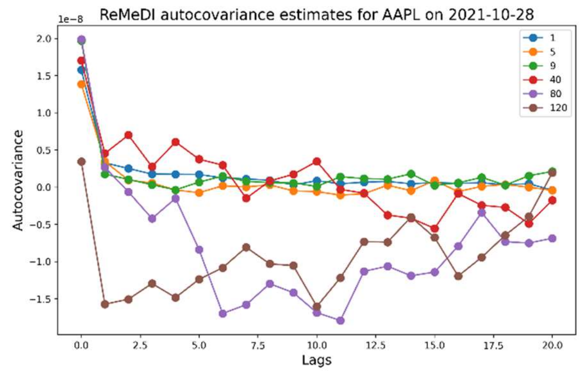


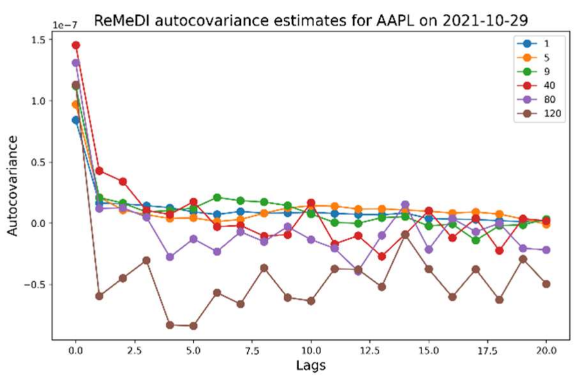



ReMeDI autocovariance estimates for AAPL on 2021-10-25
1e−8
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AAPL on 2021-10-26
1e−8
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AAPL on 2021-10-27
1e−8
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AAPL on 2021-10-28
1e−8
Autocovariance
Lags
1
5
9
40
80
120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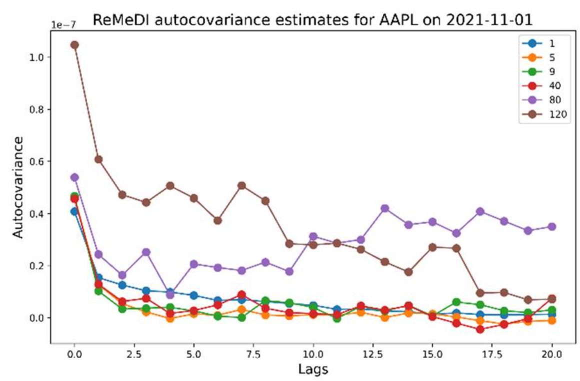
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AAPL on 2021-11-01
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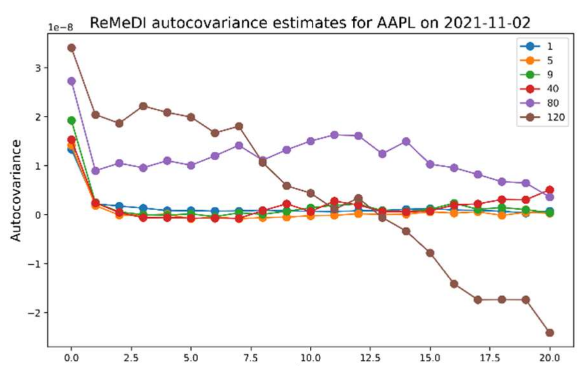
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AAPL on 2021-11-02
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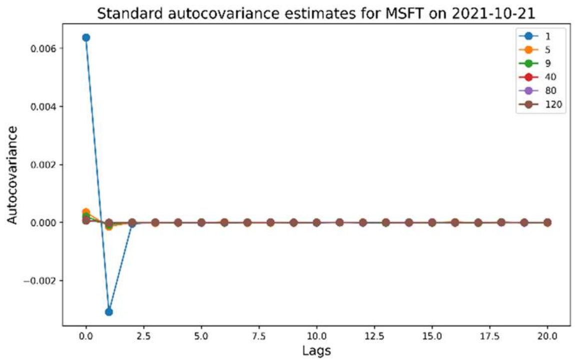
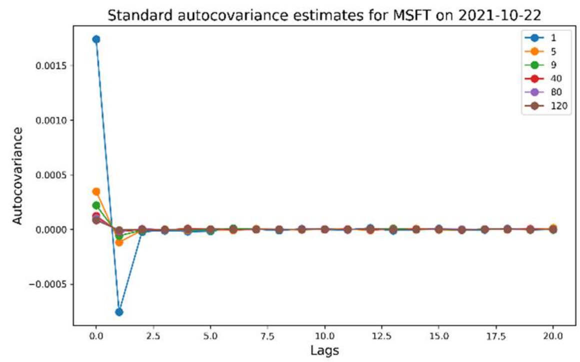
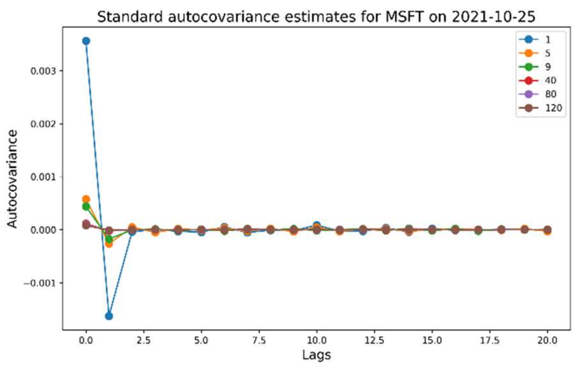
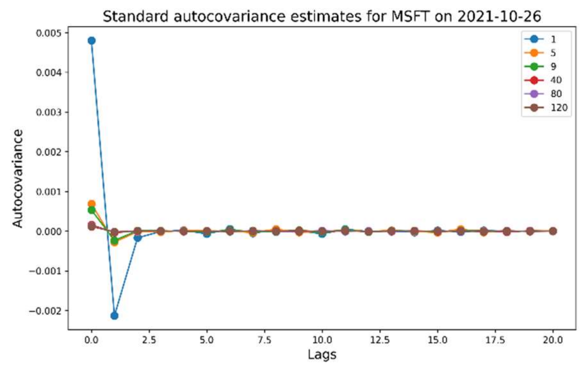
Autocovariance
Lags
1
5
9
40
80
120

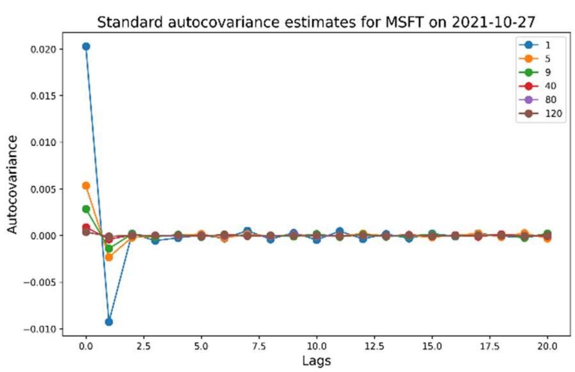
15. Sample autocovariance profiles for MSFT across 10/2021

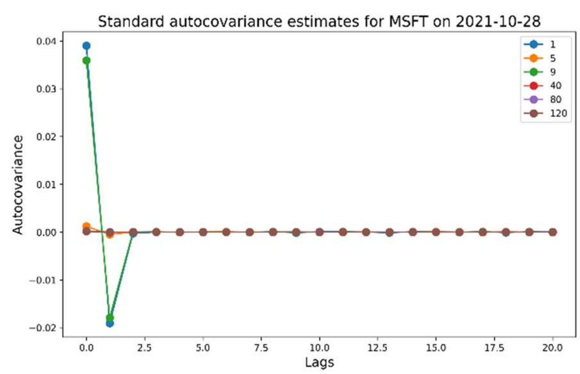


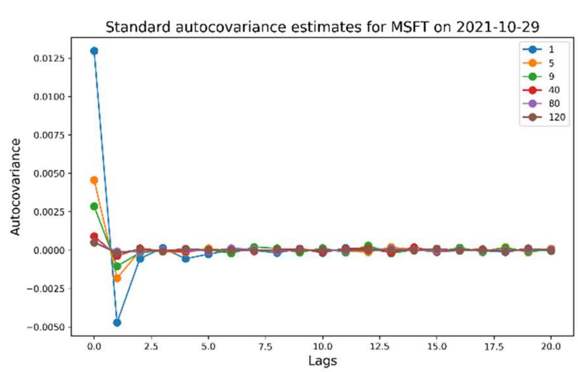


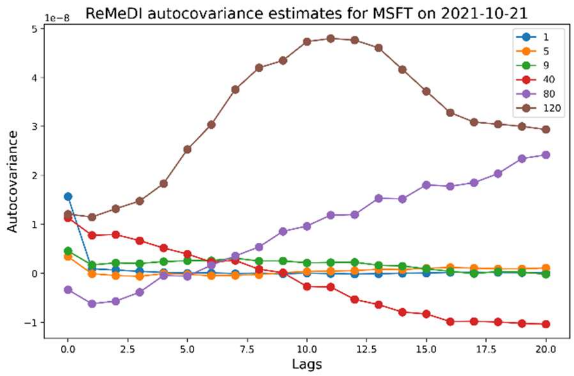


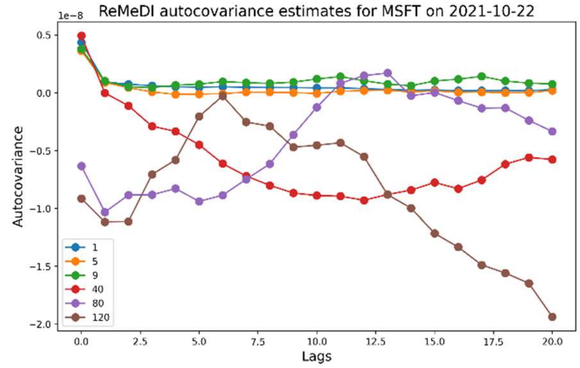


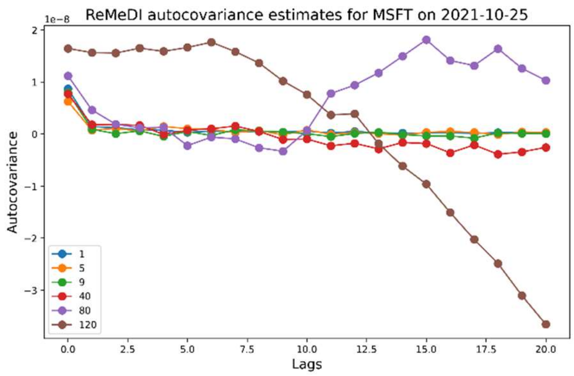


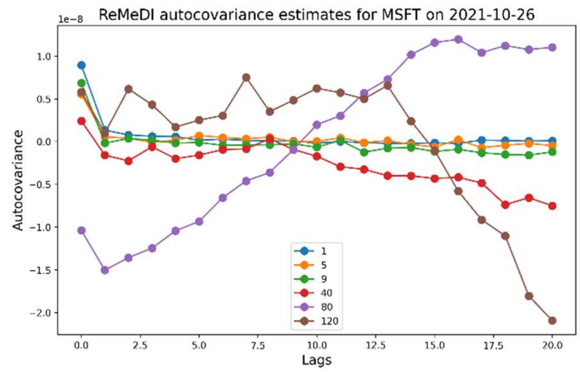


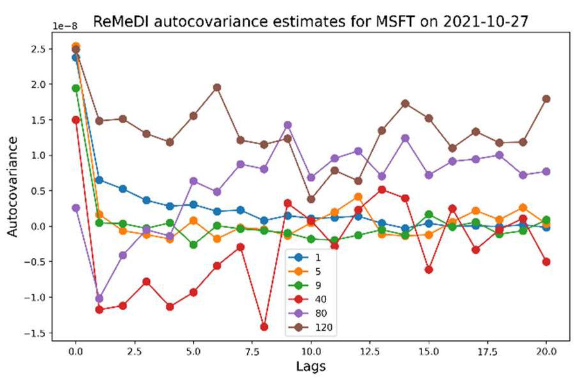


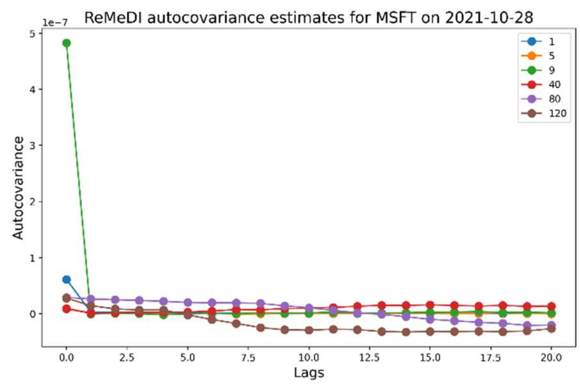

ReMeDI autocovariance estimates for MSFT on 2021-10-21
1e−8
Autocovariance
Lags
1
5
9
40
80
120

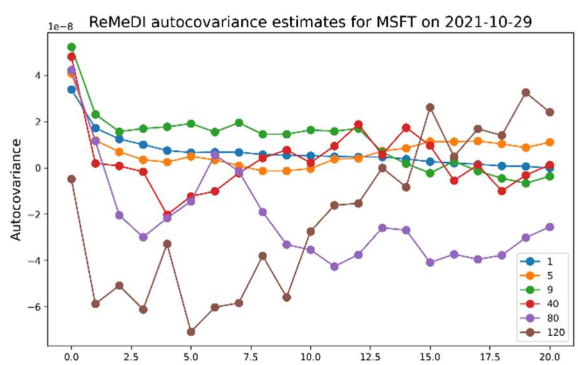

ReMeDI autocovariance estimates for MSFT on 2021-10-22
1e−8
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for MSFT on 2021-10-25
1e−8
Autocovariance
Lags
1
5
9
40
80
120

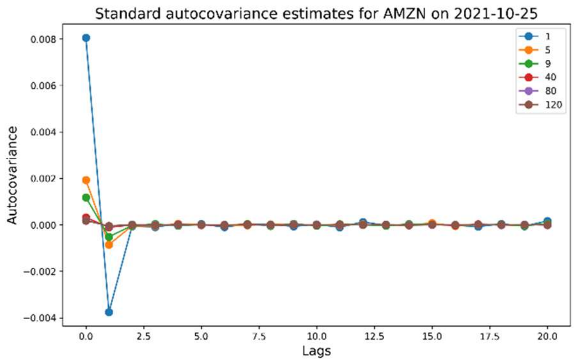

ReMeDI autocovariance estimates for MSFT on 2021-10-26
1e−8
Autocovariance
Lags
1
5
9
40
80
120

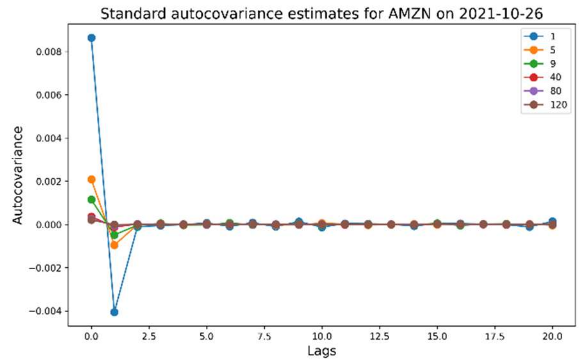

ReMeDI autocovariance estimates for MSFT on 2021-10-27
1e−8
Autocovariance
Lags
1
5
9
40
80
120

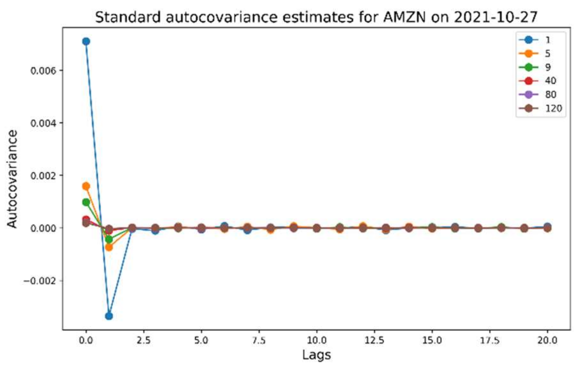

ReMeDI autocovariance estimates for MSFT on 2021-10-28
1e−7
Autocovariance
Lags
1
5
9
40
80
120

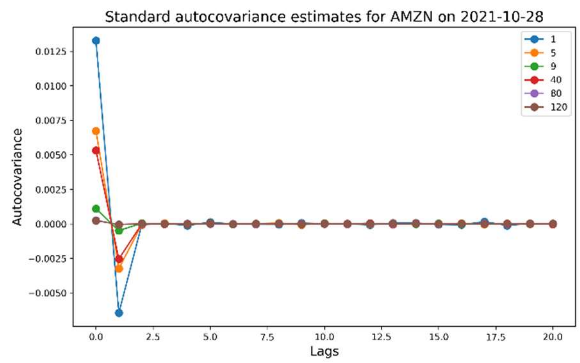

ReMeDI autocovariance estimates for MSFT on 2021-10-29
1e−8
Autocovariance
Lags
1
5
9
40
80
120

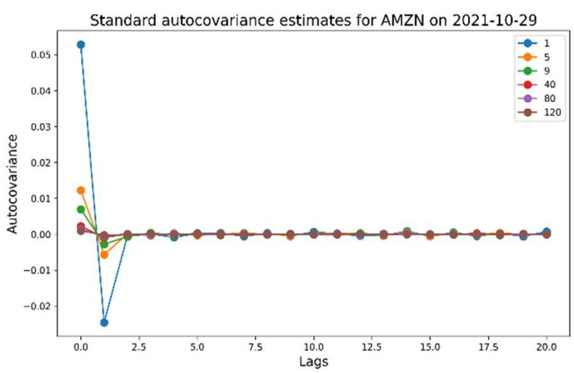
16.Sample autocovariance profiles for AMZN across 10/2021

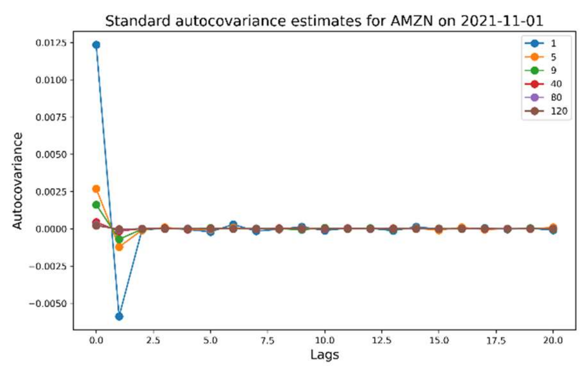


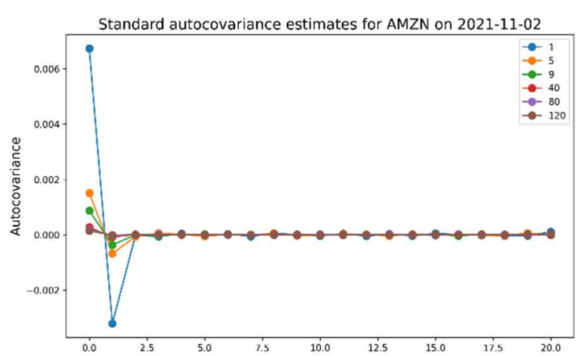


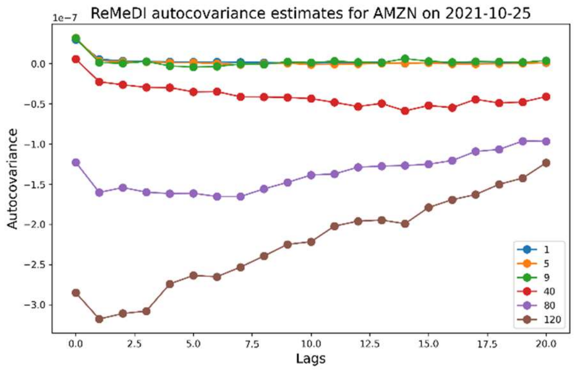


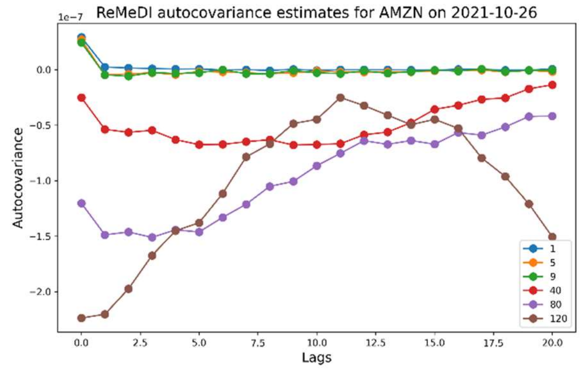


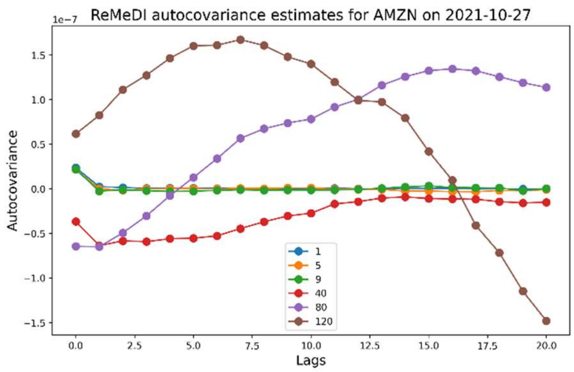


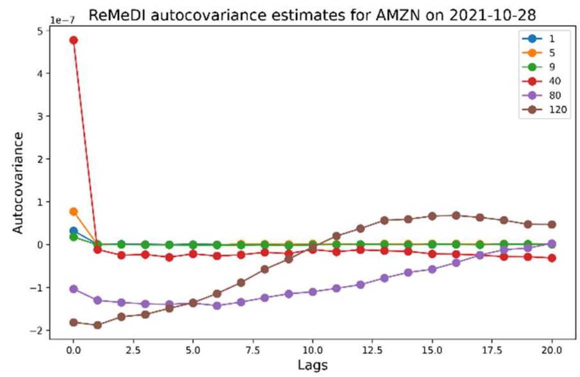


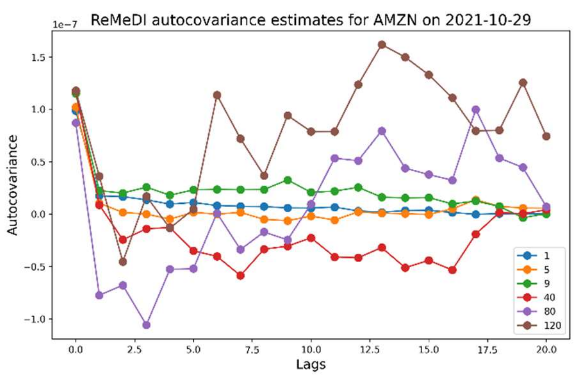


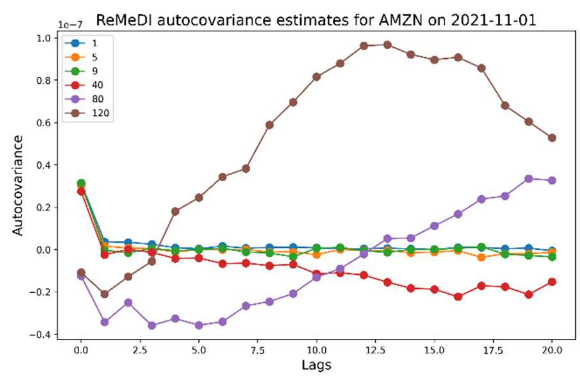

ReMeDI autocovariance estimates for AMZN on 2021-10-25
1e−7
Autocovariance
Lags
1
5
9
40
80
120

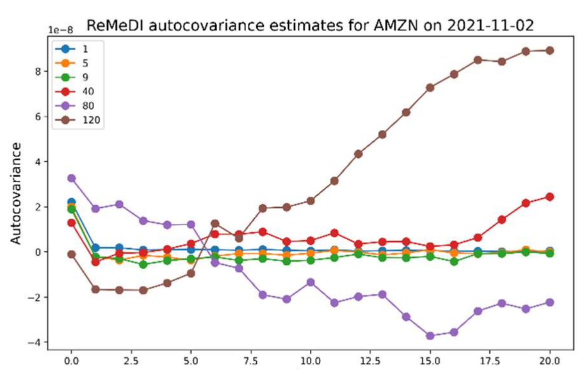

ReMeDI autocovariance estimates for AMZN on 2021-10-26
1e−7
Autocovariance
Lags
1
5
9
40
80
120


ReMeDI autocovariance estimates for AMZN on 2021-10-27
1e−7
Autocovariance
Lags
1
5
9
40
80
120

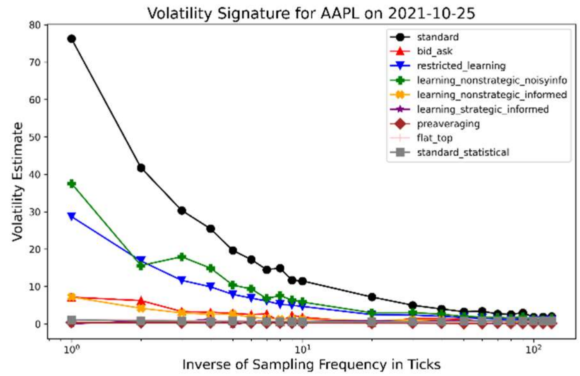

ReMeDI autocovariance estimates for AMZN on 2021-10-28
1e−7
Autocovariance
Lags
1
5
9
40
80
120

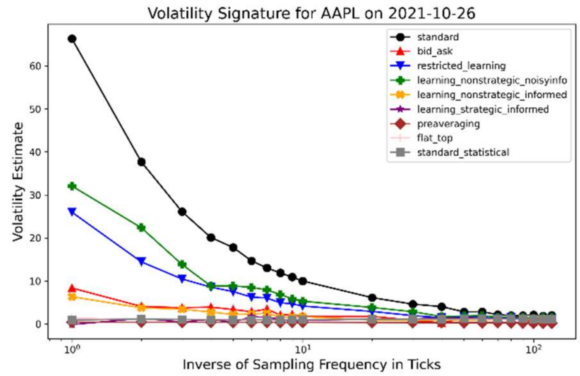

ReMeDI autocovariance estimates for AMZN on 2021-10-29
1e−7
Autocovariance
Lags
1
5
9
40
80
120

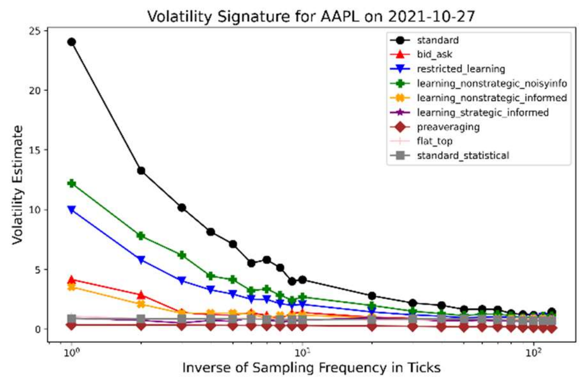

ReMeDI autocovariance estimates for AMZN on 2021-11-01
1e−7
Autocovariance
Lags
1
5
9
40
80
120

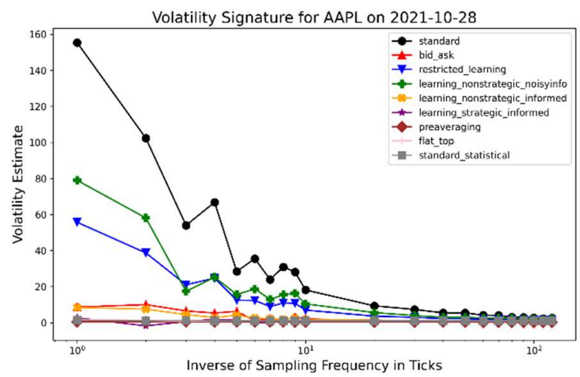

ReMeDI autocovariance estimates for AMZN on 2021-11-02
1e−8
Autocovariance
Lags
1
5
9
40
80
120

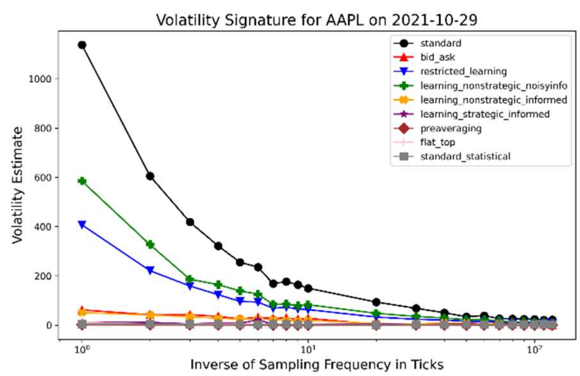
17. Sample Volatility Signatures for AAPL across 10/2021

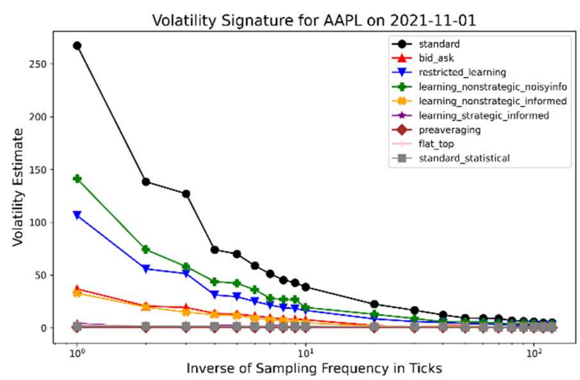


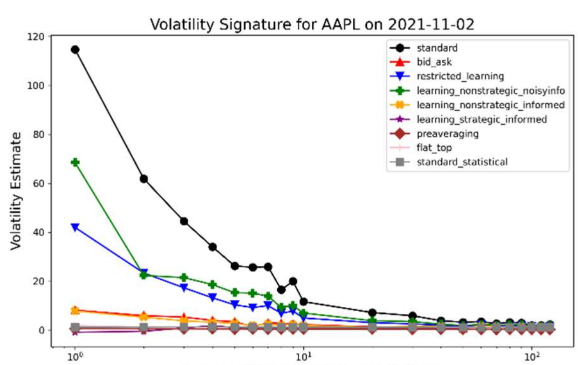




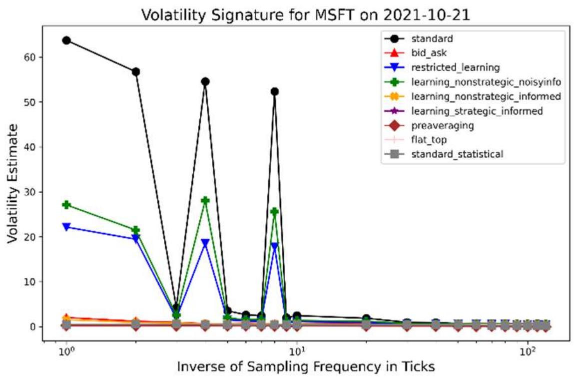


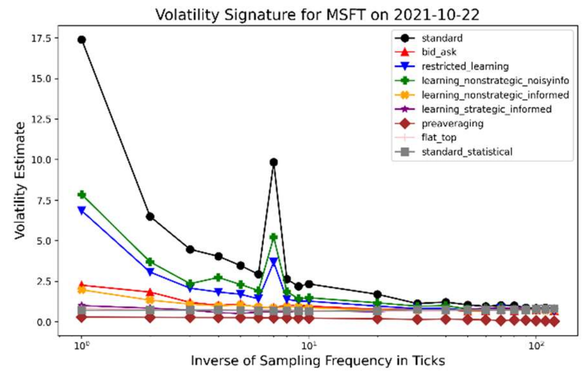


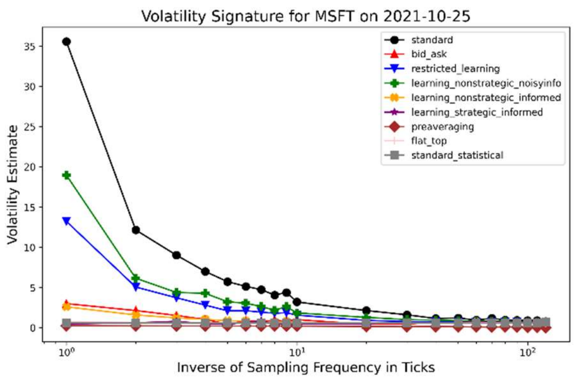


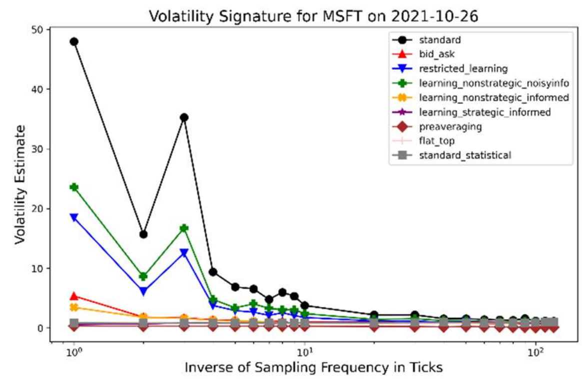


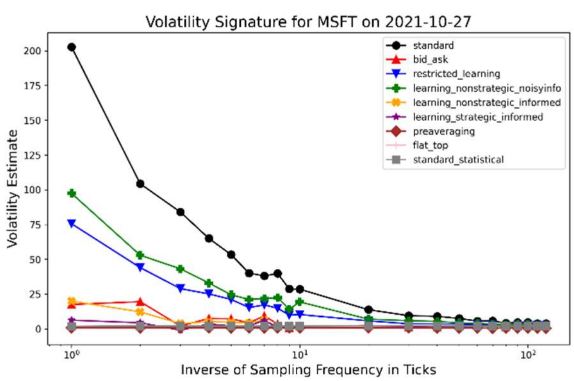
18.Sample Volatility Signatures for MSFT across 10/2021

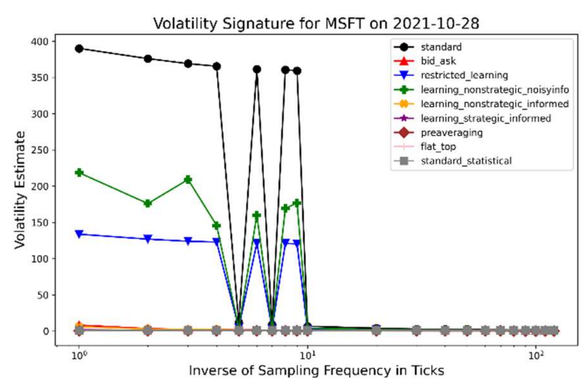


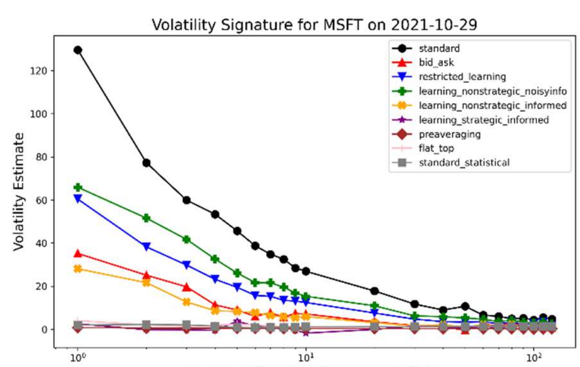












19. Sample Volatility Signatures for AMZN across 10/2021