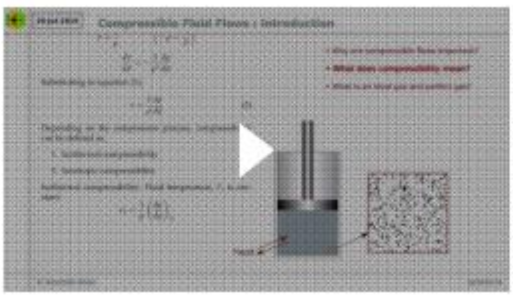
Chapter 9

Lecture 19 – Goals for today's class

1. Review of the thermodynamic properties for compressible gasses (must watch the video posted below)
2. Application of Compressible flow
3. Introduction to compressible duct flow with friction
4. Derivation of the relationships for compressible flow stated in 3
5. Solving a problem

Introductory Video to watch
01 Compressible Fluid Flows - Introduction (Part 1)



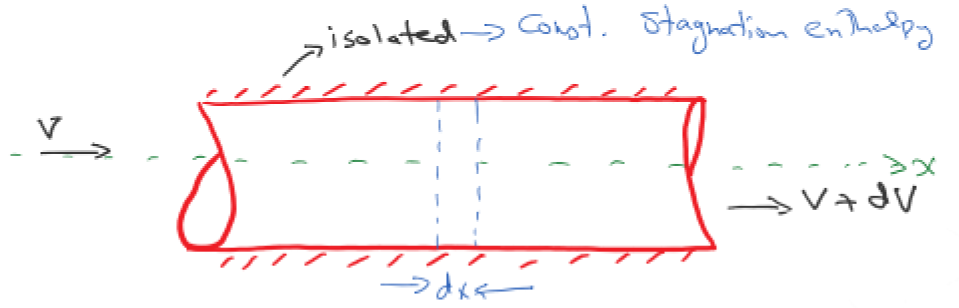
Watch this video from Min 1:00 to 51:00 to remind yourself of the preliminaries from Thermodynamics you need for Compressible flow:

Fluid Mechanics: Introduction to Compressible Flow (26 of 34)

The notes below do not include the material related to the review that was provided through YouTube videos above. Make sure that for that purpose you read sections 9.1 to 9.3 of the Whites textbook.

Section 9.7 – Compressible Duct Flow with Friction

Note that for compressible fluids, pressure is not simply the mechanical normal stress, but a well-defined thermodynamic variable. Therefore, the drop in pressure is caused not only by the friction of the fluid, but also by the change in the thermal state.



v + dv → no change in cross section, no change in mass flow → so other properties must change (e.g., ρ, T, P) to have v form inlet to change to v + dv in outlet.

Let's make these assumptions to find analytical solutions:

1. Steady 1D flow
2. No shaft work & potential changes
3. Wall shear stress is correlated by Dancy friction factor (f), i.e.,

$$\tau_w = \frac{1}{8} f \rho v^2$$

4. Adiabatic Flow

Considering the conservation laws: $\rho v = constant$

$$\frac{d\rho}{\rho} + \frac{dv}{v} = 0 \quad mass$$

$\frac{d\rho}{\rho}$ → appearing become of compressibility.

$$dP + \frac{4\tau_w dx}{D} + \rho V dV = 0$$

dx → element length (see Figure above)

x-momentum energy $(h + \frac{1}{2}v^2 = h_o = c_p T_o)$, where $h = c_p T_o$

despite being adiabatic, temperature can change → $c_p T + v dv = 0$

In above 3 equations we have P, ρ, T, v, τ_w; Assumption 3 above given a 4th equation AND if we restrict the analysis to gases, then ideal gas law PV=nRT can be read.

Integrating/solving above 5 equations will give:

Equations 9.64 a-e

$$\frac{dP}{P} = -kM_a^2 \frac{1 + (K-1)M_a^2}{2(1 - M_a^2)} f \frac{dx}{D}$$

$$\frac{d\rho}{\rho} = -\frac{kM_a^2}{2(1 - M_a^2)} f \frac{dx}{D} = -\frac{dV}{V}, where\ V = volume$$

$$\frac{dP_\circ}{P_\circ} = \frac{d\rho_\circ}{\rho_\circ} = -\frac{1}{2} k M_a^2 f \frac{dx}{D}, where\ P_\circ = Stag. Pressure$$

$$\frac{dT}{T} = -\frac{K(K-1)M_a^4}{2(1-M_a^2)} f \frac{dx}{D}$$

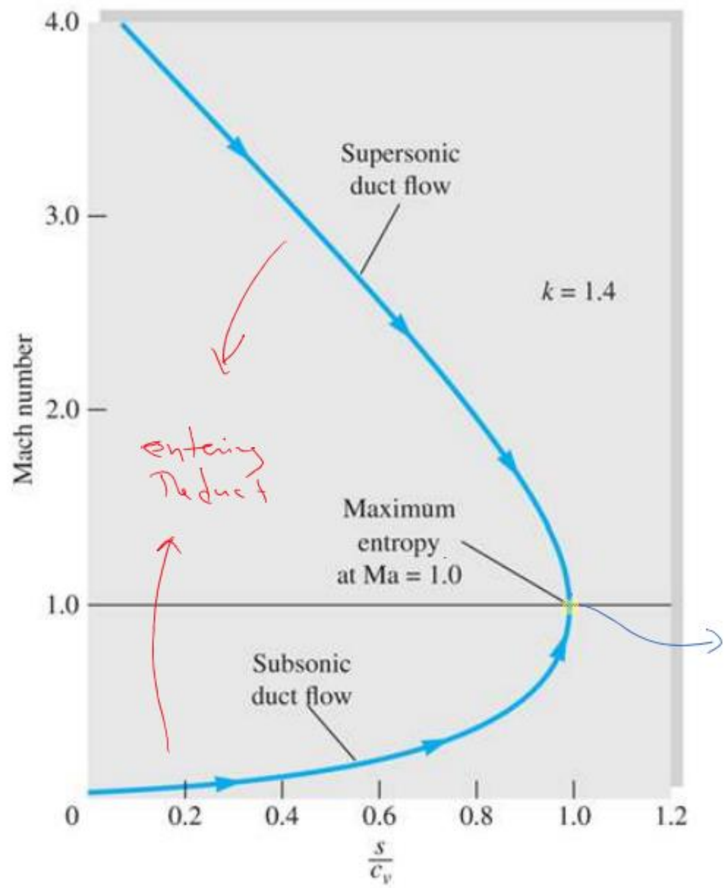
$$\frac{dM_a^2}{M_a^2} = kM_a^2 \frac{1+\frac{1}{2}(k-1)M_a^2}{1-M_a^2} f \frac{dx}{D}$$

This type of flow is called Fanno flow!
The term (1-M_a^2) appears in above equations, so depending on whether $M_a > 1$ (supersonic), $M_a < 1$ (sub-sonic), the flow properties vary diff., see table:

Property	Subsonic	Supersonic
p	Decreases	Increases
ρ	Decreases	Increases
V	Increases	Decreases
p_0, ρ_0	Decreases	Decreases
T	Decreases	Increases
Ma	Increases	Decreases
Entropy	Increases	Increases

$P_\circ$, $\rho_\circ$ → always decrease as there is losses (τ_w) at the wall
Entropy → S always increases otherwise violation of 2nd law of thermodynamics

Where do you see Fanno flow?
Emptying of pressured container through a relatively short tube, exhaust system of an internal combustion engine, compressed air systems, historically it raised from the need to explain the steam flow in turbines.



Sonic Properties

From 9.14 one will see that

1. S became S_{max} at $M_a = 1$
2. Since flow is non-isentropic (frictionless losses); $P_\circ$ & $\rho_\circ$ decreases (see table above), so ref. properties should be sonic properties
   i.e., $\rho^*, P^*, T^*, P_\circ^*, and\ \rho_\circ^*$

So, integrating between o and L^* (length of tube when $M_a = 1$) of say Eq. 9.64e

$$\frac{\bar{f}L^*}{D} = \frac{1-M_a^2}{kM_a^2} + \frac{k+1}{2k} ln \frac{(k+1)M_a^2}{2+(k-1)M_a^2} \quad (1)$$

Where, $\bar{f}$ = Average f between o & L^*
D → can use D_n
K → gas constant (air h = 1.4)

Table B.3 has numeric values of L^* for various M_a.

If M_a does not reach 1, i.e., the dual is short. Then eq. (2) can be used:

$$\bar{f} = \frac{\Delta L}{\Delta} = (\frac{\bar{f}L^*}{D})_1 - \left(\frac{\bar{f}L^*}{D}\right)_2 \quad (2)$$

Find ‘f’ from Moody diagram for Average R_e & $\frac{\varepsilon}{D}$ values.

Air flows from a reservoir and enters a uniform pipe with a diameter of 0.05 [m] and length of 10 [m]. The air exits to the atmosphere. The following conditions prevail at the exit: $P_2 = 1[bar]$ temperature $T_2 = 27°C$ $M_2 = 0.9^4$. Assume that the average friction factor to be $f = 0.004$ and that the flow from the reservoir up to the pipe inlet is essentially isentropic. Estimate the total temperature and total pressure in the reservoir under the Fanno flow model.

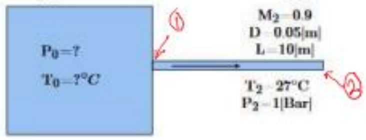


Fig. -10.3. Schematic of Example 10.1.

Solution

For isentropic, the flow to the pipe inlet, the temperature and the total pressure at the pipe inlet are the same as those in the reservoir. Thus, finding the star pressure and temperature at the pipe inlet is the solution. With the Mach number and temperature known at the exit, the total temperature at the entrance can be obtained by knowing the $\frac{4fL}{D}$. For given Mach number ($M = 0.9$) the following is obtained.

M	$\frac{4fL}{D}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0{}^*}$	$\frac{\rho}{\rho^*}$	$\frac{U}{U^*}$	$\frac{T}{T^*}$
0.90000	0.01451	1.1291	1.0089	1.0934	0.9146	1.0327

So, the total temperature at the exit is

$$T^*|_2 = \left.\frac{T^*}{T}\right|_2 T_2 = \frac{300}{1.0327} = 290.5[K]$$

In comp. flow Always use abso. Temp.

That is 1

To “move” to the other side of the tube the $\frac{4fL}{D}$ is added as

$$\left.\frac{4fL}{D}\right|_1 = \frac{4fL}{D} + \left.\frac{4fL}{D}\right|_2 = \frac{4 \times 0.004 \times 10}{0.05} + 0.01451 \simeq 3.21$$

The rest of the parameters can be obtained with the new $\frac{4fL}{D}$ either from Table (B3) Use interpolation if needed

OR you can use Eqs. 9.68 [Tables often easier]

Table B3

M	$\frac{4fL}{D}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0{}^*}$	$\frac{\rho}{\rho^*}$	$\frac{U}{U^*}$	$\frac{T}{T^*}$
0.35886	3.2100	3.0140	1.7405	2.5764	0.38814	1.1699

from Table B1

Note that the subsonic branch is chosen. The stagnation ratios has to be added for $M = 0.35886$

Table B1

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
0.35886	0.97489	0.93840	1.7405	0.91484	1.5922	0.78305

The total pressure P_{01} can be found from the combination of the ratios as follows:

$$P_{01} = \overbrace{\underbrace{P_2 \left.\frac{P^*}{P}\right|_2}_{P^*} \left.\frac{P}{P^*}\right|_1}^{P_1} \left.\frac{P_0}{P}\right|_1$$

$$= 1 \times \frac{1}{1.12913} \times 3.014 \times \frac{1}{0.915} = 2.91[Bar]$$

Table B1

$$T_{01} = \overbrace{\underbrace{T_2 \left.\frac{T^*}{T}\right|_2}_{T^*} \left.\frac{T}{T^*}\right|_1}^{T_1} \left.\frac{T_0}{T}\right|_1$$

$$= 300 \times \frac{1}{1.0327} \times 1.17 \times \frac{1}{0.975} \simeq 348K = 75°C$$

Table B1

Note 1: see ex. 9.10

Note 2: Equations 9.68a – 9.68d in the textbook give relationships for $\frac{P}{P^*}, \frac{\rho}{\rho^*}$, etc. on a function of M_a & K, similar to Equation (1) above.

The values, e.g., for Pressure, between two points, can be found using:

$$\frac{P_2}{P_1} = \frac{P_2}{P^*}\frac{P^*}{P_1}$$

Where $\frac{P}{P^*}, \frac{T}{T^*}$, etc. can be found from equations 9.68 or table B3 in appendix of textbook.

Lecture 20 – Goals for today's class

1. **Adiabatic** flow cont'd: Chocking of compressible flow in a duct due to friction
2. **Isothermal** Compressible flow with friction in a duct
3. Solving a problem

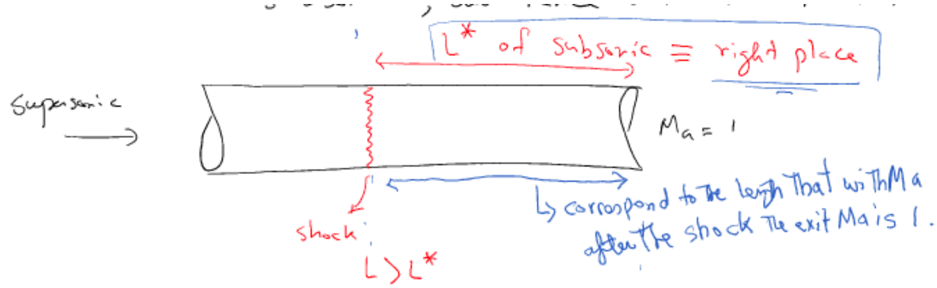
Chocking Due to Friction

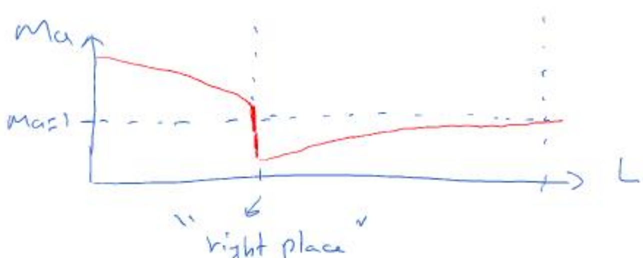
What happens if the length of the duct is longer than L^* for a given Ma at the inlet?

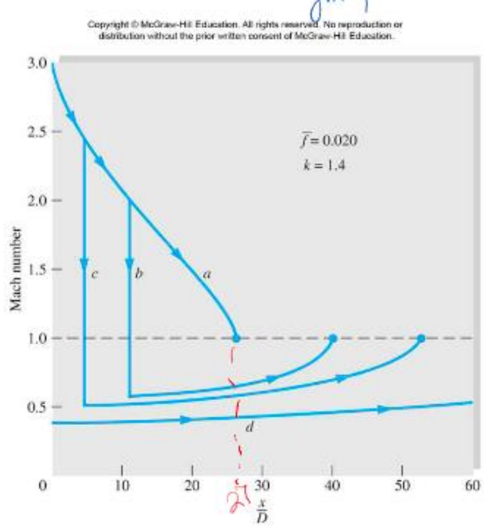
Subsonic: the <u>inlet</u> vel adjusts down so at exit (L^*), $Ma = 1$

- This means the length can determine the main flow rate

Supersonic: A shock will appear at the "right place", to turn flow subsonic, and hence at exit $Ma = 1$







Note: Increasing the Ma at inlet, beings the shock closer to the inlet; think of why?

Hint: Length of duct to have $Ma = 1$ at exit for a "subsonic" flow

Note: Minor losses for compressible flow are calculated as:

$$P_1 - P_2 = \frac{1}{2} k \delta_2 v_2^2$$

- P_2 **:** pressure after the fitting
- k **:** minor loss coefficient for comp flow (an approx. might use the k values from sec 6.9. of test) (3)
- V : vel after the fitting

Isothermal flow with Friction:

- For very long ducts, e.g natural gas lines, the flow takes the state of isothermal rather than adiabatic. In this case, the conservation of energy e.q is replaced by T = const or dT = 0 in the set of eqs given for adiabatic flow

It can be shown that

$\frac{\bar{f} Lmax}{D} = 1 - \frac{kMa^2}{kMa} + \ln(kMa^2)$ (4)

Two differences between the isothermal and adiabatic frictional flow:

1. In isothermal, the flow behavior does not depend on inlet Ma being sub- or super-sonic (I.e no more $(1 - Ma^2)$term)
2. The Lmax is not zero when Ma=0, but according to the above equation when $Ma = \frac{1}{\sqrt{k}}$. So the limiting Ma at the exit will be $\frac{1}{\sqrt{k}}$ (0.845 for air) for isothermal flow

What if $L > L\,max$?

- Similar to adiabatic flow, if inlet is subsonic, the Ma at inlet will decrease, so exit Ma is $\frac{1}{\sqrt{k}}$; and if inlet is supersonic, then a shock at the "right place: will appear, so the exit Ma is still $\frac{1}{\sqrt{k}}$

Note: The ref properties will be values of $P,\ T,\ \delta,$ etc at $Ma = \frac{1}{\sqrt{k}}$ (not $Ma = 1$), I.e. $\delta^1,\ P^1,\ T^1$

Some useful formulae for <u>isothermal flow</u>:

$\frac{P}{P^1} = \frac{1}{Ma\sqrt{k}}$; $\frac{V}{V^1} = \frac{\delta^1}{\delta} = Ma\sqrt{k}$

$$G^2 = \left(\frac{m}{A}\right)^2 = \frac{P_1^2 - P_2^2}{RT\left[\bar{f}\ \frac{L}{D} + 2\ln \quad \frac{P_1}{P_2}\right]}$$

- G^2 : mass flux at inlet P1 or ext P2 (static pressure)

Example:

Find the mass flow rate for $f = 0.05$, $L = 4[m]$, $D = 0.02[m]$ and pressure ratio P2/P1 = 0.1. The conditions at the entrance are 27 C and 3[bar] air. The pipeline can be considered a very long one delivering compressed air in a petro-chemical facility.

Flow should be treated as "isothermal" due to very long pipe!

$$\frac{P_2}{P_1} = 0.1 \rightarrow \frac{P_1}{P_{12}} = 10 \rightarrow 3 \times 10325pa \rightarrow P_2 = \frac{P_1}{10}$$

$$T = 27 + 273 = 300k$$

R for air = 287J/kg.k

$$\frac{f}{d}L = \frac{0.05 \times 4}{0.02} = 10$$

Now all value to use in Eq 5 above is known to find G

EXAMPLE 9.13

Air enters a pipe of 1-cm diameter and 1.2-m length at $p_1 = 220$ kPa and $T_1 = 300$ K. If $\bar{f} = 0.025$ and the exit pressure is $p_2 = 140$ kPa, estimate the mass flow for (*a*) isothermal flow and (*b*) adiabatic flow.

Solution

For isothermal flow Eq. (9.73) applies without iteration:

$$\frac{\bar{f}L}{D} + 2\ln\frac{p_1}{p_2} = \frac{(0.025)(1.2\text{ m})}{0.01\text{ m}} + 2\ln\frac{220}{140} = 3.904$$

$$G^2 = \frac{(220{,}000\text{ Pa})^2 - (140{,}000\text{ Pa})^2}{[287\text{ m}^2/(\text{s}^2\cdot\text{K})](300\text{ K})(3.904)} = 85{,}700 \qquad \text{or} \qquad G = 293\text{ kg}/(\text{s}\cdot\text{m}^2)$$

Since $A = (\pi/4)(0.01\text{ m})^2 = 7.85\text{ E-5 m}^2$, the isothermal mass flow estimate is

$$\dot{m} = GA = (293)(7.85\text{ E-5}) \approx 0.0230\text{ kg/s} \qquad \textit{Ans. (a)}$$

Check that the exit Mach number is not choked:

$$\rho_2 = \frac{p_2}{RT} = \frac{140{,}000}{(287)(300)} = 1.626\text{ kg/m}^3 \qquad V_2 = \frac{G}{\rho_2} = \frac{293}{1.626} = 180\text{ m/s}$$

or

$$\text{Ma}_2 = \frac{V_2}{\sqrt{kRT}} = \frac{180}{[1.4(287)(300)]^{1/2}} = \frac{180}{347} \approx 0.52$$

always must make this check.

note "chocking" for isothermal flow happens when $\text{Ma} = \frac{1}{\sqrt{k}}$ NOT $\text{Ma} = 1$

This is well below choking, and the isothermal solution is accurate.

For adiabatic flow, we can iterate by hand, in the time-honored fashion, using Eqs. (9.74) and (9.75) plus the definition of stagnation speed of sound. A few years ago the author

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when 1... NOT Ma=1

would have done just that, laboriously. However, EES makes handwork and manipulation of equations unnecessary, although careful programming and good guesses are required. If we ignore superfluous output such as T_2 and V_2, 13 statements are appropriate. First, spell out the given physical properties (in SI units):

```
k = 1.4
P1 = 220000
P2 = 140000
T1 = 300
```

Next, apply the adiabatic friction relations, Eqs. (9.66) and (9.67), to both points 1 and 2:

```
fLD1 = (1 - Ma1^2)/k/Ma1^2 + (k + 1)/2/k*LN((k + 1)*Ma1^2/(2 +
(k - 1)*Ma1^2))
fLD2 = (1 - Ma2^2)/k/Ma2^2 + (k + 1)/2/k*LN((k + 1)*Ma2^2/(2 +
(k - 1)*Ma2^2))
DeltafLD = 0.025*1.2/0.01
fLD1 = fLD2 + DeltafLD
```

Then apply the pressure ratio formula (9.68*a*) to both points 1 and 2:

```
P1/Pstar = ((k + 1)/(2 + (k - 1)*Ma1^2))^0.5/Ma1
P2/Pstar = ((k + 1)/(2 + (k - 1)*Ma2^2))^0.5/Ma2
```

These are *adiabatic* relations, so we need not further spell out quantities such as T_0 or a_0 unless we want them as additional output.

The above 10 statements are a closed algebraic system, and EES will solve them for Ma_1 and Ma_2. However, the problem asks for mass flow, so we complete the system:

```
V1 = Ma1*sqrt(1.4*287*T1)
Rho1 = P1/287/T1
Mdot = Rho1*(pi/4*0.01^2)*V1
```

If we apply no constraints, EES reports "cannot solve" because its default allows all variables to lie between $-\infty$ and $+\infty$. So we enter Variable Information and constrain Ma_1 and Ma_2 to lie between 0 and 1 (subsonic flow). EES still complains that it "cannot solve" but hints that "better guesses are needed." Indeed, the default guesses for EES variables are normally 1.0, too large for the Mach numbers. Guess the Mach numbers equal to 0.8 or even 0.5, and EES still complains, for a subtle reason: Since $f\Delta L/D = 0.025(1.2/0.01) = 3.0$, Ma_1 can be no larger than 0.36 (see Table B.3). Finally, then, we guess Ma_1 and $Ma_2 = 0.3$ or 0.4, and EES reports the solution:

$$\mathrm{Ma}_1 = 0.3343 \quad \mathrm{Ma}_2 = 0.5175 \quad \frac{fL}{D_1} = 3.935 \quad \frac{fL}{D_2} = 0.9348$$

$$p^* = 67{,}892 \text{ Pa} \quad \dot{m} = 0.0233 \text{ kg/s} \qquad \textit{Ans. (b)}$$

Though the programming is complicated, the EES approach is superior to hand iteration; and, of course, we can save this program for use again with new data.