Modular Interpretive Decompilation of Low-Level Code by Partial Evaluation

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joint work with Miguel Gómez-Zamalloa¹ and Germán Puebla²

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Motivation

$Low-level\ code\ \Rightarrow\ Intermediate\ representations$

- Mobile environments: only low-level code available.
- Analysis tools unavoidably more complicated.
 - unstructured control flow,
 - use of operand stack,
 - use of heap, etc.

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Low-level code ⇒ Intermediate representations

- Mobile environments: only low-level code available.
- Analysis tools unavoidably more complicated.
 - unstructured control flow,
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- Decompiling to intermediate representations:
 - abstracts away particular language features.
 - simplifies development of analyzers, model checkers, etc.
 - variants: clause-based, BoogiePL, Soot, etc.

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 - variants: clause-based, BoogiePL, Soot, etc.

High-level (declarative) languages

- Convenient intermediate representation:
 - ▶ iterative constructs (loops) ⇒ recursion.
 - ▶ all variables in local scope of methods represented uniformly.
- Advanced tools (for declarative) languages re-used.

Interpretive Decompilation

- Most of the approaches develop hand-written decompilers.
- Appealing alternative: interpretive decompilation
- PE allows specializing a program w.r.t. some part of its input.

Interpretive Decompilation

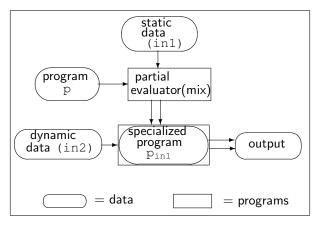
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Definition (1st Futamura Projection)

A program P written in L_S can be compiled into another language L_O by specializing an interpreter for L_S written in L_O w.r.t. P.

Partial Evaluation and the Interpretive Approach

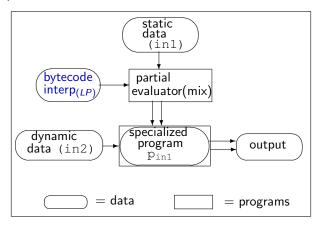
$$p(in1, in2) = output$$



$$[[p]][in1,in2] = [[[mix]][p,in1]][in2]$$

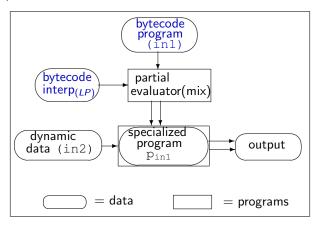
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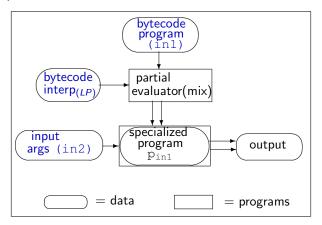
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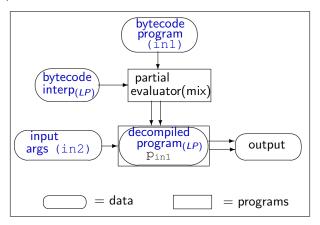
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int gcd(int x,int y){

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int res;
while (y != 0) {
  res = x mod y;
  x = y;
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bytecode

```
0:load(1)

1:if0eq(11)

2:load(0)

3:load(1)

4:rem

5:store(2)

6:load(1)

7:store(0)

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bytecode interpreter

```
main(Method, InArgs, Top) :-
   build_s0(InArgs,S0),
                          step(push(X), S1, S2) :-
   execute(S0.Sf).
                             S1 = st(PC, S, L)),
   Sf = st(_{-}, [Top|_{-}],_{-})).
                             next (PC, PC2).
                             S2 = st(PC2, [X|S], L))
execute(S1,Sf) :-
   S1 = st(PC, ...))
                          step(store(X),S1,S2) :-
   bytecode (PC, Inst, _),
                             S1 = st(PC,[I|S],LV))
    step(Inst, S1, S2),
                             next (PC, PC2),
   execute (S2, Sf).
                             localVar_update(LV,X,I,
                             S2 = st(PC2, S, LV2)).
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    step(Inst, S1, S2),
                             next (PC, PC2),
   execute (S2, Sf).
                             localVar_update(LV,X,I,
                             S2 = st(PC2, S, LV2)).
```

Contributions in Interpretive Decompilation

Advantages w.r.t. dedicated (de-)compilers:

- flexibility: interpreter easier to modify;
- more reliable: easier to trust that the semantics preserved;
- easier to maintain: new changes easily reflected in interpreter;
- easier to implement: provided a partial evaluator is available.

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- flexibility: interpreter easier to modify;
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Only proofs-of-concept in interpretive decompilation:

- e.g. in [PADL'07] we decompile a subset of Java Bytecode to Prolog.
- Open issues we have answered in this work:
 - Scalability: first modular decompilation scheme by PE
 - ▶ Structure preservation: of the original program
 - Quality: equivalent to hand-written decompilers

- We have provided mechanisms to positively answer these issues:
 - ▶ Method optimality: Code for each method is decompiled only once ⇒ Big-step interpreter and PE annotations.
 - ▶ Block optimality: Code for each instruction is emitted and evaluated at most once ⇒ PE annotations and pre-analysis.

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- Implemented an interpretive decompiler of Java Bytecode to Prolog.
- Future work: Special handling for the heap, exploit instrumented decompilation, improve efficiency, applications, etc.

Question to SCAM Audience

 Are we happy with hand-written decompilers or we would like more flexible approaches?

Contributions

Contribution 1

- Modular decompilation: decompile a method at a time
- First modular decompilation scheme by PE:
 - ▶ compositional treatment to method invocation ⇒ consider a big-step interpreter;
 - "residualize" calls in decompiled program, we automatically generate program annotations for this purpose;

Contributions

Contribution 1

- Modular decompilation: decompile a method at a time
- First *modular* decompilation scheme by PE:
 - ▶ compositional treatment to method invocation ⇒ consider a big-step interpreter;
 - "residualize" calls in decompiled program, we automatically generate program annotations for this purpose;

Proposition (modular optimality)

We decompile the code corresponding to each method in P_{bc} exactly once.

Decompilation of Low-level Code

Contribution 2

quality is equivalent to dedicated decompilers?

Is possible to obtain by interpretive decompilation programs whose

- Idea: since decompilers first build a CFG for the method, study how a similar notion can be used for controlling PE of the interpreter
- Block-level decompilation produce a rule for each block in the CFG.

Decompilation of Low-level Code

Contribution 2

- Is possible to obtain by interpretive decompilation programs whose quality is equivalent to dedicated decompilers?
- Idea: since decompilers first build a CFG for the method, study how a similar notion can be used for controlling PE of the interpreter
- Block-level decompilation produce a rule for each block in the CFG.

Proposition (block optimality)

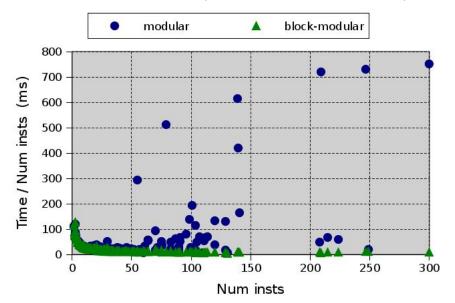
- residual code for each bytecode instruction emitted once;
- each bytecode instruction evaluated at most once;

- Open issues: scalability, structure preservation, quality ...
- We have provided mechanisms to positively answer these issues:
 - ▶ Method optimality: Code for each method is decompiled only once ⇒ Big-step semantics and PE annotations.
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- Average improvements: 10 times faster decompilations and 5 times smaller decompiled program sizes (even we get ∞ gains).

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- Future work: Special handling for the heap, exploit instrumented decompilation, improve efficiency, applications, etc.

Experimental Evaluation (JOIden benchmarks suite)



Intraprocedural Decompilation

• We consider the \mathcal{L}_{bc} -bytecode language ($\mathcal{L}_{bc} \subset \mathsf{Java}$ bytecode).

```
\begin{array}{ll} \textit{Inst} ::= & \mathsf{push}(\mathsf{x}) \mid \mathsf{load}(\mathsf{v}) \mid \mathsf{store}(\mathsf{v}) \mid \mathsf{add} \mid \mathsf{sub} \mid \mathsf{mul} \mid \mathsf{div} \mid \mathsf{rem} \mid \mathsf{neg} \mid \\ & \mathsf{if} \diamond (\mathsf{pc}) \mid \mathsf{if0} \diamond (\mathsf{pc}) \mid \mathsf{goto}(\mathsf{pc}) \mid \mathsf{return} \end{array}
```

• State $\equiv \langle PC, OpStack, LocalVars \rangle$

The \mathcal{L}_{bc} -bytecode interpreter

```
step(push(X),S1,S2) :-
main (Method, InArgs, Top) :-
                                       S1 = st(PC, S, L)),
   build_s0(InArgs,S0),
                                       next (PC, PC2),
   execute(S0,Sf),
                                       S2 = st(PC2, [X|S], L)).
   Sf = st(_,[Topl_],_)).
                                   step(store(X),S1,S2) :-
execute(S,S) :-
                                       S1 = st(PC, [I|S], LV)),
   S = st(PC, [Top]_], )),
                                       next (PC, PC2),
   bytecode (PC, return, _) .
                                       localVar_update(LV, X, I, LV2),
execute(S1,Sf) :-
                                       S2 = st(PC2, S, LV2).
   S1 = st(PC, ..., ...))
   bytecode (PC, Inst,_),
                                   step (goto (PC), S1, S2) :-
   step(Inst, S1, S2),
                                       S1 = st(\_, S, LV)),
   execute (S2, Sf).
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```
int gcd(int x,int y) {
  int res;
  while (y != 0) {
    res = x mod y;
    x = y;
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\mathcal{L}_{bc}-bytecode 0:load(1) 7:
```

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Unfolding trees

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main(gcd, [X, Y], Z)
```

\mathcal{L}_{bc} -bytecode 0:load(1)

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\label{eq:main(gcd,[X,Y],Z)} \begin{split} & \psi \\ & \text{exec(st(0,[],[X,Y,0]),S}_f) \end{split}
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$$\label{eq:main(gcd, [X, Y], Z)} \begin{split} & \underset{\qquad \qquad \qquad }{\text{wain(gcd, [X, Y], Z)}} \\ & \underset{\qquad \qquad \qquad }{\text{exec(st(0, [], [X, Y, 0]), S_f)}} \\ & \underset{\qquad \qquad \qquad \qquad }{\text{exec(st(1, [Y], [X, Y, 0]), S_f)}} \\ & \underset{\qquad \qquad \qquad \qquad \qquad }{\text{exec(st(11, [], [X, 0, 0]), S_f)}} \\ & \underset{\qquad \qquad \qquad \qquad \qquad }{\text{true}} \end{split}$$

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Unfolding trees

$$\begin{array}{c} \text{main}(\text{gcd}, [\textbf{X}, \textbf{Y}], \textbf{Z}) \\ & \psi \\ & \text{exec}(\text{st}(\textbf{0}, [], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_f) \\ & \psi \\ & \text{exec}(\text{st}(\textbf{1}, [Y], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_f) \\ & \{Y = 0\} \\ & \text{exec}(\text{st}(\textbf{11}, [], [\textbf{X}, \textbf{0}, \textbf{0}]), \textbf{S}_f) \\ & \psi \\ & \text{true} \end{array}$$

Decompiled code

main(gcd,[X,0],X).

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Unfolding trees

$$\label{eq:main(gcd, [x, y], z)} \begin{split} & \text{main(gcd, [x, y], z)} \\ & & \text{ψ} \\ & \text{exec(st(0, [], [x, y, 0]), $S_f)$} \\ & & \text{ψ} \\ & \text{exec(st(1, [y], [x, y, 0]), $S_f)$} \\ & \text{exec(st(11, [], [x, 0, 0]), $S_f)$} \\ & \text{exec(st(2, [], [x, y, 0]), $S_f)$} \\ & \text{ψ} \\ & \text{true} \end{split}$$

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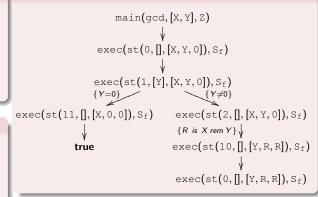
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```

Unfolding trees



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$$\begin{array}{c} \text{main}(\text{gcd}, [\textbf{X}, \textbf{Y}], \textbf{Z}) \\ & \psi \\ & \text{exec}(\text{st}(\textbf{0}, [], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{\text{f}}) \\ & \psi \\ & \text{exec}(\text{st}(\textbf{1}, [\textbf{Y}], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{\text{f}}) \\ & \{Y \neq 0\} \\ & \text{exec}(\text{st}(\textbf{11}, [], [\textbf{X}, \textbf{0}, \textbf{0}]), \textbf{S}_{\text{f}}) \\ & \psi \\ & \text{true} \\ & \text{exec}(\text{st}(\textbf{10}, [], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{\text{f}}) \\ & \psi \\ & \text{true} \\ & \text{exec}(\text{st}(\textbf{10}, [], [\textbf{Y}, \textbf{R}, \textbf{R}]), \textbf{S}_{\text{f}}) \\ & \psi \\ & \text{exec}(\text{st}(\textbf{0}, [], [\textbf{Y}, \textbf{R}, \textbf{R}]), \textbf{S}_{\text{f}}) \end{array}$$

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main(gcd,[X,0],X).
main(gcd,[X,Y],Z) :- Y = 0,
   R is X rem Y. exec 1 (Y.R.Z)
```

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Unfolding trees

```
exec(st(0, [], [Y, R, R]), S_f)
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\mathcal{L}_{bc} -bytecode

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Unfolding trees

```
exec(st(0, [], [Y, R, R]), S_f)
                  exec(st(1,[R],[Y,R,R]),S_f)
                  \{R=0\}
exec(st(12,[],[Y,0,0]),S_f) exec(st(2,[],[Y,R,R]),S_f)
                                    \{R' \text{ is } Y \text{ rem } R\}_{V}
                                    exec(st(10, [], [R, R', Z]), S_f)
             true
                                    exec(st(0,[],[R,R',Z]),S_f)
```

Decompiled code

6:load(1)

```
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```

7:store(0)

```
int gcd(int x, int y) {
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    x = v;
    y = res;
  return x:}
```

\mathcal{L}_{bc} -bytecode

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1:if0eq(11)
             8:load(2)
2:load(0)
             9:store(1)
3:load(1)
             10:goto(0)
4:rem
             11:load(0)
5:store(2)
             12:return
6:load(1)
```

Unfolding trees

```
exec(st(0,[],[Y,R,R]),S_f)
                 exec(st(1,[R],[Y,R,R]),S_f)
                 {R=0}
exec(st(12,[],[Y,0,0]),S_f) exec(st(2,[],[Y,R,R]),S_f)
                                   \{R' \text{ is } Y \text{ rem } R\}_{V}
                                  exec(st(10, [], [R, R', Z]), S_f)
             true
                                   exec(st(0,[],[R,R',Z]),S_f)
```

Decompiled code

```
main(gcd,[X,0],X).
                                exec_1(Y, 0, Y).
main(gcd, [X,Y], Z) :- Y \setminus= 0, exec_1(Y,R,Z) :- R \setminus= 0,
   R is X rem Y. exec 1(Y,R,Z). R' is Y rem R. exec 1(R,R',Z)
```

Elvira Albert (UCM)

```
int gcd(int x,int y) {
  int res;
  while (y != 0) {
    res = x mod y;
    x = y;
    y = res; }
  return x; }
```

```
\mathcal{L}_{\mathit{bc}}	ext{-bytecode}
```

```
0:load(1)

1:if0eq(11)

2:load(0)

3:load(1)

4:rem

5:store(2)

6:load(1)

7:store(0)

8:load(2)

9:store(1)

10:goto(0)

11:load(0)

12:return
```

```
int gcd(int x, int y) {
  int res;
  while (y != 0){
    res = x mod y;
    x = y;
    y = res;
  return x;}
```

Unfolding trees

```
main(gcd, [X, Y], Z)
```

\mathcal{L}_{bc} -bytecode

```
0:load(1)
             7:store(0)
1:if0eq(11)
             8:load(2)
2:load(0)
             9:store(1)
3:load(1)
             10:goto(0)
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             11:load(0)
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```

```
int gcd(int x,int y) {
  int res;
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    y = res; }
  return x; }
```

Unfolding trees

```
\label{eq:main(gcd,[X,Y],Z)} \begin{split} & & \psi \\ & \text{exec(st(0,[],[X,Y,0]),S}_f) \end{split}
```

\mathcal{L}_{bc} -bytecode 0:load(1)

```
7:store(0)

1:if0eq(11)

2:load(0)

3:load(1)

4:rem

5:store(2)

6:load(1)

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8:load(2)

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11:load(0)

12:return
```

int gcd(int x, int y) {

```
int res;
while (y != 0){
  res = x mod y;
  x = y;
  y = res;}
return x;}
```

Unfolding trees

\mathcal{L}_{bc} -bytecode

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$\mathcal{L}_{\mathit{bc}}$ -bytecode

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Unfolding trees

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\mathcal{L}_{bc} -bytecode

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Unfolding trees

Decompiled code

int gcd(int x, int y) {

```
int res;
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\mathcal{L}_{bc} -bytecode

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8:load(2)

9:store(1)

10:goto(0)

11:load(0)

12:return
```

Unfolding trees

$$\begin{array}{c} \text{main(gcd, [X, Y], Z)} \\ & \psi \\ & \text{exec(st(0, [], [X, Y, 0]), S}_f) \\ & \psi \\ & \text{exec(st(1, [Y], [X, Y, 0]), S}_f) \\ & \{Y = 0\} \\ & \text{exec(st(11, [], [X, 0, 0]), S}_f) \\ & \text{exec(st(2, [], [X, Y, 0]), S}_f) \\ & \psi \\ & \text{true} \end{array}$$

Decompiled code

int gcd(int x, int y) {

```
int res;
while (y != 0){
  res = x mod y;
  x = y;
  y = res;}
return x;}
```

\mathcal{L}_{bc} -bytecode

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0:load(1)

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```

Unfolding trees

Decompiled code

int gcd(int x, int y) {

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int res;
while (y != 0){
  res = x mod y;
  x = y;
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```

Unfolding trees

Decompiled code

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int gcd(int x, int y) {
  int res:
  while (y != 0){
    res = x mod y;
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```

\mathcal{L}_{bc} -bytecode

```
0:load(1)
             7:store(0)
1:if0eq(11)
             8:load(2)
2:load(0)
             9:store(1)
3:load(1)
             10:goto(0)
4:rem
             11:load(0)
5:store(2)
             12:return
6:load(1)
```

Unfolding trees

$$\begin{array}{c} \text{main}(\text{gcd}, [\textbf{X}, \textbf{Y}], \textbf{Z}) \\ \downarrow \\ \text{exec}(\text{st}(\textbf{0}, [], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{f}) \\ \downarrow \\ \text{exec}(\text{st}(\textbf{1}, [\textbf{Y}], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{f}) \\ \{Y \neq \textbf{0}\} \\ \text{exec}(\text{st}(\textbf{11}, [], [\textbf{X}, \textbf{0}, \textbf{0}]), \textbf{S}_{f}) \\ \downarrow \\ \text{true} \\ \text{exec}(\text{st}(\textbf{10}, [], [\textbf{X}, \textbf{Y}, \textbf{0}]), \textbf{S}_{f}) \\ \downarrow \\ \text{true} \\ \text{exec}(\text{st}(\textbf{10}, [], [\textbf{Y}, \textbf{R}, \textbf{R}]), \textbf{S}_{f}) \\ \downarrow \\ \text{exec}(\text{st}(\textbf{0}, [], [\textbf{Y}, \textbf{R}, \textbf{R}]), \textbf{S}_{f}) \\ \end{pmatrix}$$

```
main(gcd,[X,0],X).
main(gcd,[X,Y],Z) :- Y = 0,
   R is X rem Y. exec 1 (Y.R.Z)
```

```
int gcd(int x, int y) {
  int res;
  while (y != 0){
    res = x mod y;
    x = y;
    y = res;
  return x;}
```

Unfolding trees

```
exec(st(0, [], [Y, R, R]), S_f)
```

\mathcal{L}_{bc} -bytecode

```
0:load(1)
             7:store(0)
1:if0eq(11)
             8:load(2)
2:load(0)
             9:store(1)
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```
main(gcd,[X,0],X).
main(gcd,[X,Y],Z) :- Y \setminus= 0,
```

```
int gcd(int x, int y) {
  int res:
  while (y != 0){
    res = x mod y;
    x = y;
    y = res;
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```

\mathcal{L}_{bc} -bytecode 0:load(1)

1:if0eq(11)

```
8:load(2)
2:load(0)
             9:store(1)
3:load(1)
             10:goto(0)
4:rem
             11:load(0)
5:store(2)
             12:return
```

Unfolding trees

Decompiled code

6:load(1)

```
main(gcd,[X,0],X).
main(gcd,[X,Y],Z) :- Y = 0,
   R is X rem Y. exec 1 (Y.R.Z)
```

7:store(0)

```
int gcd(int x, int y) {
  int res;
  while (y != 0) {
    res = x mod y;
    x = y;
    y = res; }
  return x; }
```

$\mathcal{L}_{\mathit{bc}}$ -bytecode

```
0:load(1)

1:if0eq(11)

2:load(0)

3:load(1)

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5:store(2)

6:load(1)

7:store(0)

8:load(2)

9:store(1)

10:goto(0)

11:load(0)

12:return
```

Unfolding trees

```
main(gcd,[X,0],X). exec_1(Y,0,Y).

main(gcd,[X,Y],Z):-Y \= 0, exec_1(Y,R,Z):-R \= 0,

R is X rem Y, exec_1(Y,R,Z). R' is Y rem R, exec_1(R,R',Z)
```